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Performance in Economies with Cash Constraints.
Chang and Velasco Revisited”**

Enrique Kawamura

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Vito Dumas 284, (B1644BID) Victoria, Pcia. de Buenos Aires, Argentina
Tel: 4725-7053 Fax: 4725-7010
Email: economia@udesa.edu.ar

Exchange Rate Regimes, Monetary Policy and Banking Performance in Economies with Cash Constraints. Chang and Velasco Revisited. *

Enrique Kawamura
Universidad de San Andrés
Vito Dumas 284
Victoria, Buenos Aires (1644)
Argentina
kawa@udesa.edu.ar

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Abstract

This paper analyzes a two-good version of the Diamond and Dybvig model in a small open economy. This structure is used to analyze the interaction between banks as liquidity insurers, real exchange rates and monetary policies. It is shown that fixed exchange rates with a Central Bank providing liquidity in pesos implements the efficient allocation. The conditions for a run equilibrium in this system are stronger than in the literature. In a flexible exchange rate regime, multiple equilibria cannot be eliminated. In particular, there is an equilibrium where a fraction of patient consumers purchases dollars in the interim period. This can be interpreted as a *partial currency run* event. A dollarized banking system may also implement the efficient allocation under some conditions.

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1 Introduction

A vast recent theoretical as well as empirical literature has recently emphasized the role of the banking sector fragility to explain recent currency crisis observed in Asia and Russia. One well - known branch of this literature identifies *illiquidity* of banks as an important source of banking fragility. The main theoretical contributions in this venue are found in successive papers by Chang and Velasco (1998, 2000a, 2000b and 2001), which constitute small-open-economy extensions of the Diamond and Dybvig (1983) banking model. In particular, Chang and Velasco (2000b, denoted as CV hereon) emphasize the interaction of the exchange rate regime with the possibility of equilibrium banking or currency crisis due to a so-called *international illiquidity* condition (see Chang and Velasco (1998) for a definition). Perhaps the strongest result of the CV paper is that *flexible* exchange rates eliminate the possibility of currency or bank runs, provided that the Central Bank serves as a lender of last resort. This was considered a point to favor flexible exchange rate regimes against fixed rates. (See, for example, Chang and Velasco (2000c), section III-A).

However this model has been criticized in different dimensions. One of the main objections is the way fiat money (especially, local currency) is modelled. They use a simple assumption of money - in - the - utility function. Indeed, they assume that patient consumers carry over a certain amount of local currency (called *peso* in the model) between periods 1 and 2. They also assume that the stock of real balances in pesos (deflated by the nominal exchange rate) provides some utility to those consumers. However, it is well-known that this is the most primitive way of modelling money, and what the money-in-the-utility-function assumption really means is unclear. On the other hand, even though the authors emphasize that it is possible to introduce local currency holdings in the utility function of the impatient agents, this amount of pesos must also be carried over between periods 1 and 2. A question is then why an impatient person would be interested in doing this given that she cares only about consumption at date 1.

This criticism calls for a more serious modelling of fiat money in this framework. This has already been noticed by the authors in the conclusions of the article. There exists a series of papers with overlapping generation models where money has positive value embedded in a framework similar to the Diamond - Dybvig structure¹. This allows to take money seriously as a store of value within the mentioned scheme. However, to my knowledge the

¹See, for example, Betts and Smith (1997), Champ, Smith and Williamson (1996), Schreft and Smith (1997) and Williamson (1998).

only theoretical model within the Diamond - Dybvig tradition where the local currency is used as a mean of purchasing goods is given by Chang and Velasco (2000a, section 8). They assume that consumers withdraw pesos from banks to purchase the unique consumption good. The results in the CV paper also hold here. In this regard, the results in CV may not depend on the money - in - the - utility function condition. But then why do the authors assume this condition in first place? One way to interpret this assumption is to say that patient consumers need pesos to purchase some units of a good different from the tradeable commodity of the model, and that these transactions must take place at the end of period 1. However this second good is of course never seen in the CV model.

On the other hand, Chang and Velasco (2000a, section 6) also consider a version of the model in a world with a tradeable good and a non tradeable good. The objective here was to study the behavior of the real exchange rate in the no - run equilibrium as well as in the bank run equilibrium. However there is no reference to monetary issues when speaking about the equilibrium RER. Thus, it seems natural to combine the presence of a non - tradeable good with the necessity of using local currency to buy these goods in order to rationalize the money - in - the - utility function assumption.

In this paper I present a model that intends to fulfill the latter objective. The model studies the interaction between the different exchange rate and monetary arrangements with the properties of allocations implemented by banks in equilibrium. The main interest is to characterize those equilibria in terms of optimality and financial fragility as analyzed in CV. Two different crucial assumptions are introduced. The economy assumes a tradeable as well as a non - tradeable good. There are also two currencies, the *dollar* (foreign) and the *peso* (local). The second important element of the model is that some currency *must* be exchanged for these goods and viceversa. Therefore, a type of *cash-in-advance* constraint is imposed in the economy. Given that this is a three-dates economy, this type of constraint forces to introduce frictions in transactions technologies. The paper explores the consequences of different exchange arrangements, each of which must be consistent with the exchange rate and monetary policy addressed. In some cases the story resembles the traditional shopper - seller division of the household as in, for example, Lucas (1980, 1982). This is in fact used only under a currency board or dollarized regime. A Central Exchange institution centralizing non-tradeables exchange is introduced as in Magill and Quinzii (1992, 1996) when the Central Bank is allowed to work as a local lender for banks.

The introduction of these new assumptions yields quite different results than in the CV model, at least in certain cases. The first case considered is the fixed exchange rate regime, where a Central Bank may issue pesos not

backed by dollars. The financial and exchange system is arranged so that the exchange of non - tradeable goods occur after a first round of withdrawals and the purchase of dollars occur after a second round of withdrawals. In this model the main feature is the fact that the Central Bank only needs to give liquidity unbacked by dollars within a period, not between periods. The use of this *liquidity* in local currency is to allow the exchange of non - tradeables. There is no other role for the Central Bank in this case.

Although the optimal allocation can be implemented by a Diamond and Dybvig banking system in this case, it is also shown that the conditions for a run equilibrium to exist are stronger than in CV. In particular, international illiquidity may not be sufficient to generate a currency crisis due to self fulfilling expectations. In fact this illiquidity conditions depend now on features of the Central Banks policy. This shows how this modelling of money introduces an extra condition for a currency run equilibrium. However, with logarithmic preferences conditions for a run are the same as in CV.

One of the most important results is that, within the same banking system, an exchange rate policy contingent on the pesos sold in every period does not necessarily imply a unique equilibrium. This policy is what is called a *flexible* exchange rate regime in the CV model. If the monetary policy in period 1 is sufficiently loose, there is an equilibrium where it is indifferent for patient consumers to purchase dollars in either period, and so a fraction of those patient depositors decide to buy dollars in period 1 instead of period 2. This event may be called a *partial* currency run. This result contrasts sharply with Chang and Velasco (2000b), showing that the assumption of money in the utility function with only one commodity may have been too important to generate the unique equilibrium result with *flexible* exchange rates.

I also consider a dollarized banking system within this model, which may also be interpreted as a banking system within a currency board regime. First, implementation of the optimal allocation is possible under certain conditions. In the logarithmic preferences case, implementation could hold if the share of tradeables is strictly greater than one half. This system may also be subject to runs, if the international illiquidity in the CV model holds for the optimal allocation. This condition is in fact sufficient to have a bank run equilibrium, contrary to what happens in the case where the Central Bank provides liquidity in pesos. Hence, in a dollarized banking system the liquidity crisis works as in the currency board system in CV.

The rest of the paper is organized as follows. In section 2 I present the general environment to be used throughout the paper. Section 3 shows the characterization of the optimal allocation. Section 4 discusses the implementation through a banking system with peso denominated deposits within a

fixed exchange rate regime with a Central Bank acting as a lender of last resort, as well as the emergence of a run equilibrium. It also presents a contingent exchange rate policy and the impossibility of runs. Section 5 shows a dollarized banking system, discusses its optimality and also its fragility. Section 6 shows concluding remarks and extensions while section 7 contains all the proofs of lemmas and propositions.

2 The Environment

The economy lasts for three periods, $t = 0, 1, 2$. There are two consumption goods in periods 1 and 2. One good is called *tradable* and the other, *non-tradable*. In period 0 there are only tradeables. Non-tradeables are produced with a constant-returns-to-scale technology. For every period of maturity, the per-unit gross return is $A > 1$ non tradeables per unit of tradable invested at date 0. In this same period there is a continuum of ex-ante identical consumers, with names in the unit interval. As it is standard in this literature, at the beginning of date 1 there exists a preference shock that determines the ex-post type of each consumer. With probability π the consumer becomes *impatient* and with the remaining probability she becomes *patient*. An impatient agent has preferences represented by the utility function $u(c_{1T}) + v(c_{1N})$, where c_{1l} is the consumption by an impatient consumer (in period 1) of good $l = T, N$, where T stands for *tradables* and N for *non-tradables*. A patient consumer has utility function $u(c_{2T}) + v(c_{2N})$, where c_{2l} stands for the consumption at date 2 by the impatient agent of good l . Hence, the ex-ante utility function is

$$\pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2N})]$$

There exist two other investment technologies. There is a long term investment project that gives $R > 1$ units of tradable goods at date 2 per unit of the T good invested in period 0. As in the literature, assume that if liquidated at date 1 the gross return in terms of tradeables is $r < 1$. The other investment corresponds to the fact that the tradable good is assumed to be storable with net return equal to 0. There is no endowment of none of the goods in this economy.

3 A planner's problem with limited international credit.

This section characterizes the planner's problem assuming the existence of credit at date 0 that allows the planner to borrow tradable goods directly in the first period, at a net interest rate equal to 0. This planner allocates this amount in the different available technologies. Let d be the amount of tradeables borrowed by the planner at date 0, let x be the amount of tradeables invested in the long term project, y be the amount of tradeables stored between periods 0 and 1. Let z be the amount of tradeables invested in 0 to produce non-tradeables. The planner's problem can be written as follows.

$$\max \quad \pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2N})]$$

subject to

$$\begin{aligned} x + y + z &\leq d \\ \pi c_{1T} &\leq y \\ (1 - \pi) c_{2T} &\leq Rx - d \\ \pi c_{1N} &\leq \alpha Az \\ (1 - \pi) c_{2N} &\leq (1 - \alpha) A^2 z \end{aligned}$$

The problem can be written as

$$\begin{aligned} \max_{x, z_1, z_2} \quad & \pi \left\{ u \left(\frac{d - x - z}{\pi} \right) + v \left(\frac{\alpha Az}{\pi} \right) \right\} \\ & + (1 - \pi) \left\{ u \left(\frac{Rx - d}{1 - \pi} \right) + v \left(\frac{(1 - \alpha) A^2 z}{1 - \pi} \right) \right\} \end{aligned}$$

The next result characterizes the optimal α for a given z . (All the proofs are in the appendix).

Lemma 1 *The optimal amount of investment liquidated at date t in the non-tradeables technology is such that $v'(c_{1N}) = Av'(c_{2N})$. The optimal α is independent of z .*

The main reason for this last statement to hold is the fact that v is an homothetic function. The other first order conditions for an interior solution are the constraints and

$$\begin{aligned} u'(c_{1T}) &= Ru'(c_{2T}) \\ u'(c_{1T}) &= Av'(c_{1N}) \end{aligned}$$

Since $R > 1$ and $A > 1$, it is easy to show that $c_{1T} < c_{2T}$ and $c_{1N} < c_{2N}$. Therefore impatient consumers obtain always a strictly less ex - post utility than patient consumers.

This will be useful when discussing the conditions for runs in a banking system. For illustrative purposes consider the following example.

Example 2 Assume $u(c_{tT}) = \theta \ln c_{tT}$, $v(c_{tN}) = (1 - \theta) \ln c_{tN}$. The equation $v'(c_{1N}) = Av'(c_{2N})$ in this case implies that $\alpha^* = \pi$. It can be shown then that the optimal allocation satisfies the linear system

$$\begin{aligned} Rx + (1 - \pi) Rz &= d(R(1 - \pi) + \pi) \\ (1 - \theta)x + (1 - \theta + \theta\pi)z &= d(1 - \theta) \end{aligned}$$

whose solution is

$$\begin{aligned} x^* &= \frac{d(R\theta(1 - \pi) + (1 - \theta) + \theta\pi)}{R} \\ z^* &= \frac{d(R - 1)(1 - \theta)}{R} \end{aligned}$$

and so, consumption allocations are

$$\begin{aligned} c_{1T}^* &= \frac{d\theta(R - 1)}{R} \\ c_{2T}^* &= d\theta(R - 1) \\ c_{1N}^* &= \frac{(1 - \theta)Ad(R - 1)}{R} \\ c_{2N}^* &= \frac{(1 - \theta)A^2d(R - 1)}{R} \end{aligned}$$

This example will be kept for future reference.

4 Implementation through a banking system with peso - denominated deposits.

4.1 Preliminaries.

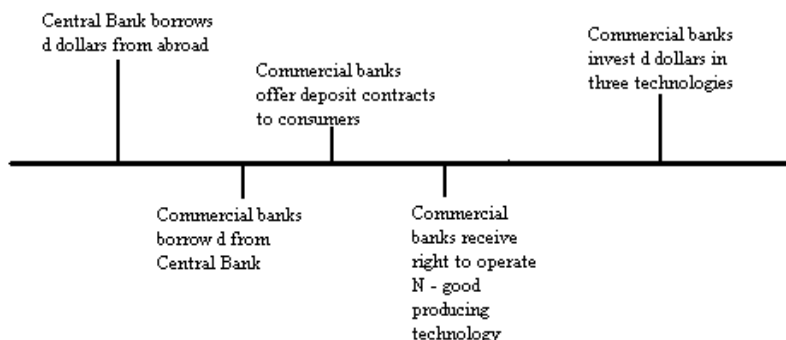
This section presents a banking system similar to that in Chang and Velasco (2000b) embedded in a monetary system with two currencies, the *dollar* and the *peso* (foreign and local currency respectively). Assume that the traded good is the numeraire in terms of dollars. This implies immediately that the

price of the T - good is equal to 1 dollar for every period. There exists a large number of commercial banks that compete to get customers (alternatively, we can assume that these banks act directly on behalf of consumers). There is also a Central Bank that borrows dollars from abroad and lends to commercial banks. Throughout the rest of the paper (for simplicity) I will assume that the Central Bank is the only creditor to commercial banks.

Following the cash-in-advance literature ² I assume that each consumer is in fact a household constituted by two parts, the shopper and the entrepreneur. How these two parts split and interact will be discussed below, and it will depend on how transactions take place in this economy. Also, to simplify the interpretation I assume that the two technologies producing tradeables (the liquid short run technology and the long term, illiquid project) produce directly dollars instead of physical tradable goods. In this regard, $R > 1$ is the dollar-gross return on the long term project if liquidated at date 2, while the dollar net return on the storage technologies is equal to zero.

The basic sequence of actions by the households and banks in the first model is as follows. The Central Bank borrows d dollars in the first period, through a two-period loan (at zero-net-interest-rate). The Central Bank lends this amount of dollars to commercial banks that invest in the technologies. At the same time commercial banks offer a deposit contract denominated in pesos. This contract specifies that the commercial bank gives the right to withdraw some amount of pesos to those showing up in period 1 and another amount of pesos in period 2. Those who withdraw in $t = 1$ cannot do it in period 2. At the same time, by this contract the consumers give to commercial banks the right to operate the non - tradeable production technology, where the banks commit to give to each household the rights over the amount of non - tradeables produced by each technology at the beginning of every period.

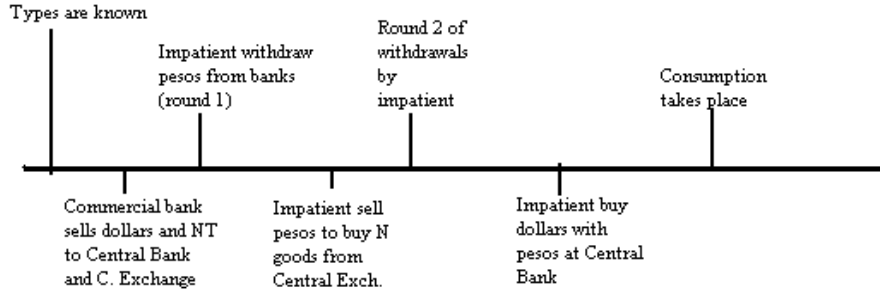
²See, for example, Lucas (1980, 1982) or Lucas and Stokey (1987).



Timing: date 0

In periods 1 and 2 the exchange process for both commodities is centralized at the Central Bank. This institution buys and sells dollars in exchange for pesos and also constitutes a Central Exchange (as in Magill and Quinzii, (1992,1996)) where all non - tradeables are exchanged. The idea is that all agents wanting to consume non - tradeable goods must purchase them from the Central Exchange with pesos. On the other hand, the supply of non - tradeables coming from the commercial banks 's investments are purchased by the Central Exchange with pesos issued by the Central Bank.

At the beginning of period 1, each household learns its type. Any impatient consumer goes to the bank to get the amount of pesos specified in the contract. For every consumer showing up in period 1, the bank sells the corresponding amount of liquid dollars to the Central Bank and also sells the amount of non - tradeables belonging to this consumer to the Central Exchange, both in exchange for pesos, which constitute what the impatient depositor withdraws. Then impatient agents sell these pesos in exchange for non-tradable goods and dollars (to buy tradeables) at the Central Exchange and the Central Bank respectively. The timing of withdrawals may change and more will be said below. At the end of period 1 shopper and entrepreneur get together again and consume what is left for them. Note that in period 1 all patient households do nothing (in the absence of runs).



Timing: date 1

In period 2 a similar sequence of actions is observed. All banks receive the amount of dollars from the long term investment. A portion of these dollars are returned to the Central Bank to repay the period 0 debt. (This institution then uses a portion of all dollars to repay the outstanding foreign debt.) The remaining dollars are purchased by the Central Bank. Commercial banks also sell the remaining amount of non - tradeables (owned by patient consumers) to the Central Exchange. Patient consumers withdraw the pesos from commercial banks, which are sold afterwards to buy non-tradeables and dollars (to buy then tradeables). At the end of this last period consumption takes place.

4.2 Commercial banks with peso - denominated deposits with fixed exchange rates and a Central Bank.

This section assumes that the Central Bank fixes the exchange rate to one for all periods, although it may issue more pesos than the amount of dollars in reserves (i.e., it is not a currency board regime). The price of non - tradeables in pesos is endogenous in every period. It is also assumed that the pattern of withdrawals and trading are divided in two subperiods. In subperiod 1 each agent withdraws a certain amount of pesos and immediately trade in non - tradeables only is open. In subperiod 2 each agent must go back to the commercial bank to withdraw another amount of pesos and immediately only dollars are sold by the Central Bank. Let p_t be the price of non - tradeables

in pesos in period t , w_t^1 the amount of pesos to be withdrawn by consumers in period t , subperiod 1. Then the problem of the commercial bank can be written as follows.

$$\max \quad \pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2T})]$$

subject to

$$x + y + z \leq d \tag{1}$$

$$\pi w_1^1 \leq p_1 \alpha A z \tag{2}$$

$$\pi w_1^2 \leq y \tag{3}$$

$$\pi c_{1T} \leq y \tag{4}$$

$$(1 - \pi) w_2^1 \leq p_2 (1 - \alpha) A^2 z \tag{5}$$

$$(1 - \pi) w_2^2 \leq R x - d \tag{6}$$

$$(1 - \pi) c_{2T} \leq R x - d \tag{7}$$

$$p_t c_{tN} \leq w_t^1 \tag{8}$$

$$c_{tT} \leq w_t^2 + (w_t^1 - p_t c_{tN}) \tag{9}$$

Equation (1) shows the constraint faced by the bank at date 0. Equation (2) is the constraint faced by the commercial bank in period 1, subperiod 1, whereas equation (3) is the corresponding constraint in period 1, subperiod 2. Inequality (4) states that the consumption of dollars by impatient depositors must be constrained to the amount of liquid dollars available in that period. Equations (5), (6) and (7) are the corresponding counterparts in period 2. Equation (8) shows that purchasing non-tradeables must be done using the pesos withdrawn from the commercial banks. Finally inequality (9) is the same constraint for the purchasing of dollars.

The last two expressions are the cash - in - advance constraints and constitute key elements in this analysis. The main difference with the standard

cash - in - advance models is the separation between the non - tradeable and the tradeable good shopping sessions. Although this is somewhat artificial, it captures the idea of separation between people purchasing non - tradeables and those buying tradeables. This separation resembles the same idea as in the liquidity models (e.g., Lucas (1990) and Fuerst (1992)) as well as in recent developments in the random matching literature (Lagos and Wright, 2002). This assumption also allows to implement the Pareto efficient allocation quite easily. Indeed, the next result shows this.

Proposition 3 *The solution to the banking problem satisfies the incentive - compatibility constraint*

$$u(c_{1T}) + v(c_{1N}) \leq u(c_{2T}) + v(c_{2T})$$

This banking system implements the optimal allocation as an equilibrium. The date 1 and 2 prices for non - tradeables are undetermined.

The following paragraphs present a sharper characterization for the logarithmic utility example.

Example 4 *Assume that preferences are of the log type as before. Recall that the efficient allocation is*

x^*	z^*	c_{1T}^*	c_{2T}^*	c_{1N}^*	c_{2N}^*
$\frac{d(R\theta(1-\pi)+(1-\theta)+\theta\pi)}{R}$	$\frac{d(R-1)(1-\theta)}{R}$	$\frac{d\theta(R-1)}{R}$	$d\theta(R-1)$	$\frac{(1-\theta)Ad(R-1)}{R}$	$\frac{(1-\theta)A^2d(R-1)}{R}$

and where $\alpha^* = \pi$. By the proof of the last proposition these quantities satisfy the first order conditions of the banking problem. The value of pesos withdrawn in each period can be computed using the date 1 constraints:

$$\pi w_1^{1*} = p_1 \frac{\pi(1-\theta)Ad(R-1)}{R}$$

Then the value of w_1^{1*} depends on the price p_1 , which can be written as

$$w_1^{1*}(p_1) = \frac{d(R-1)(\theta + p_1A(1-\theta))}{R}$$

also

$$w_1^{2*} = \frac{y^*}{\pi} = \frac{d\theta(R-1)}{R}$$

Similarly it can be shown that

$$w_2^{1*}(p_2) = \frac{d(R-1)(R\theta + p_2(1-\theta)A^2)}{R}; \quad w_2^{2*} = d\theta(R-1)$$

It is clear that the monetary policy of the Central Bank must imply that the corresponding per-capita amount of pesos not backed by dollars is just equal to $w_t^{1*}(p_t)$. What is the real exchange rate? In our case it is just $1/p_t$ (the price of tradeables in terms of non - tradeables). Hence in this equilibrium the real exchange rate is undetermined too. It is clear that the price of non - tradeables satisfies the simplest quantitative theory of the money demand. This is the direct consequence of assuming cash - in - advance constraints for all goods.

4.3 Financial and currency crisis under fixed exchange rates with a local lender of last resort.

This section explores when this banking system with the Central Bank that acts as a lender of last resort induces some type of run or crisis in the financial system. As Chang and Velasco (2000b) state, it is obvious that the only type of crisis that could arise in equilibrium is a *currency* crisis. Commercial banks can always get enough liquidity in pesos to satisfy any withdrawal pattern. The problem will be faced by the Central Bank, when trying to sell dollars in exchange for pesos. It arises when the long term, dollar-denominated asset is illiquid enough. Suppose that the Central Bank also has the authority to tax the commercial banks, being able to get all dollar investments at date 1 (including the illiquid long term projects). Assuming that the banking system and the Central Bank have perfect commitment to repay all foreign debt, the next proposition shows the conditions for a crisis to happen.

Proposition 5 *In a banking system with the exchange rate regime described in section 4 there is a currency crisis equilibrium if the following conditions hold*

$$w_1^{2*} > y^* + r \left(x^* - \frac{d}{R} \right) \quad \text{or} \quad (1 - \pi) w_1^{1*}(p_1) > r \left(x^* - \frac{d}{R} \right) \quad \text{or} \\ (1 - \pi) (w_1^{1*}(p_1) + w_1^{2*}) > r \left(x^* - \frac{d}{R} \right)$$

and

$$u(c_{1T}^*) + v(c_{1N}^*) > u(0) + v(c_{2N}^*) \quad (10)$$

If the last inequality fails to hold then there is no equilibrium with a currency crisis.

Note that in the CV model, only the first inequality was necessary and sufficient for a run equilibrium to exist. Why in this case the last inequality

is also needed? The intuition is very simple. Even though the Central Bank could run out of dollars if all patient consumers withdraw early, this does not prevent patient consumers to get c_{2N}^* units of N goods in period 2. Even though they would consume 0 of T goods, if the utility consuming c_{2N}^* units of N goods more than compensates the utility perceived if withdrawing early, then it is not individually rational to run. The second inequality basically implies that when all depositors withdraw $w_1^{1*}(p_1)$ in the first round then, even though commercial banks transfer all dollar investments to the Central Bank, reserves become too scarce compared to the amount of dollars available (at a fixed exchange rate of one peso, one dollar). This is a second illiquidity condition (alternative to the first one). The third inequality states that still patient agents are allowed to withdraw in *both* rounds of date 1. If the amount of pesos withdrawn are strictly greater than the amount of dollars left at the Central Bank, then this could also threat a currency crisis. Only if all the first three conditions fail to hold then there is no illiquidity problem.

In any case, this proposition shows that the assumptions of pesos used to purchase non tradeables in the original framework may introduce a change in the conditions for multiple equilibria. This is not trivial, since international illiquidity may not then be a sufficient condition to get a fragile banking system. It depends heavily on how monetary policy is conducted regarding financing the non - traded goods transactions (as well as how transactions are arranged). Indeed, both the presence of a second (non - tradeable) commodity as well as the fact that consumers must use pesos to purchase them are the key to understand this last proposition.

Note that the absence of international illiquidity may not be sufficient to eliminate the run equilibrium, being the incentive - compatibility constraint a key condition to guarantee a non - fragile banking system. The idea is that, even if $w_1^{1*}(p_1) + w_1^{2*}$ is less than or equal to $y^* + rx^*$, it could be the case that when a fraction $\hat{\pi} > \pi$ of depositors (which clearly includes a proportion of patient consumers) decides to withdraw in period 1 in both rounds, even though no bank fails (including the Central Bank) at date 1, the amount of dollars left at date 2 may be scarce enough to get a lower consumption of tradeables by patient agents, implying that these prefer to withdraw at date 1. Therefore the illiquidity conditions may not be necessary conditions to obtain a run equilibrium in this case.

In the logarithmic case, the condition $r < 1$ is sufficient to get the existence of a run equilibrium.

Example 6 *With logarithmic preferences, we have that the first inequality*

is equivalent to

$$\frac{d\theta (R-1)}{R} (1-\pi) > r \left(\frac{d\theta (1-\pi) (R-1)}{R} \right)$$

or

$$r < 1$$

which is always true. The second inequality is

$$(1-\pi) \frac{d(R-1)(\theta + p_1 A(1-\theta))}{R} > r \left(\frac{d\theta (1-\pi) (R-1)}{R} \right)$$

or

$$r < \frac{(\theta + p_1 A(1-\theta))}{\theta}$$

but $p_1 > 0$ so this is also true always. The second condition is trivially true given the fact that $\ln(0)$ is minus infinity.

Note that in this logarithmic example **any** monetary policy consistent with issuing some pesos unbacked by dollars implies the possibility of a run equilibrium.

The question about the behavior of the date-1 real exchange rate in period 1 has an obvious answer here. When the crisis occurs, that is, after the last consumer is able to sell pesos for dollars at the one - to - one rate the (implicit) nominal exchange rate *jumps to infinity*, which makes the real exchange rate also *jump to infinity*. Since this is not interesting, I delay a more complete discussion for the subsequent work in the following sections.

4.4 Devaluation contingent rules with peso - denominated deposits.

Chang and Velasco (2000b, section 6) demonstrated that flexible exchange rates eliminates equilibrium currency crisis when coupled with suitable monetary policies. We explore whether this result still holds in this banking system with peso-denominated deposits, as in the previous section.

Consider this alternative exchange rate policy. In period 0 the exchange rate is equal to one. At the beginning of date 1 the Central Bank buys all dollars sold by commercial banks also at an exchange rate equal to unity. When consumers go to the Central Bank to sell their pesos for dollars in this period, the exchange rate is set by the Central Bank in the following

way. The exchange rate is equal to the ratio of total offered pesos to the total amount of dollars in reserves. As before, the Central Bank lends to commercial banks any amount of pesos to satisfy any withdrawal pattern.

The exchange rate rule in period 2 is similar to that of period 1. The exchange rate rule is just set as the relative ratio of total pesos offered at the Central Bank at date 2 over the total amount of dollars in reserves in that same period. The question is whether this is enough to eliminate an equilibrium where at least a fraction of patient depositors withdraw early from commercial banks and sell pesos for dollars in period 1. This is not a standard run equilibrium in the sense that the Central Bank does not fail in any period, but what it could happen is that a non - trivial fraction of patient depositors decide to move one period forward the decision of buying dollars. The next proposition in fact shows that this equilibrium may exist under certain conditions.

Proposition 7 *Assume the exchange rate and monetary policy described above. Assume that p_1 is large enough. Then there is an equilibrium where all patient consumers withdraw pesos from banks in both rounds of period 1 and a fraction β of patient consumers sells pesos for dollars at date 1, while the remaining fraction $1 - \beta$ does so at date 2.*

More precisely, the expression for β satisfying the equality is

$$\beta = \frac{(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^*}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, \beta)) [\pi + \Delta^* (1 - \pi)]}$$

where $\Delta^* \equiv \frac{c_{2T}^*}{c_{1T}^*} > 1$ and $\tau(p_1, \beta)$ is the optimal fraction of pesos that each patient consumer reserves to purchase non tradeables in period 2. For this to be strictly positive it is needed that

$$(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] > \Delta^* y^*$$

which holds for a sufficiently high p_1 . Hence, this defines a function $\beta(p_1)$, so for every p_1 satisfying the last inequality it is always possible to find a value of $\beta(p_1)$ for which in equilibrium a fraction $\beta(p_1)$ anticipates the purchasing of dollars.

Example 8 *In the logarithmic case, the expression for τ^* satisfies $\frac{1-\theta}{\tau^*} = \frac{\theta}{1-\tau^*}$, which implies that*

$$\tau^* = 1 - \theta, \quad 1 - \tau^* = \theta$$

for every (p_1, β) . Therefore the value of β is

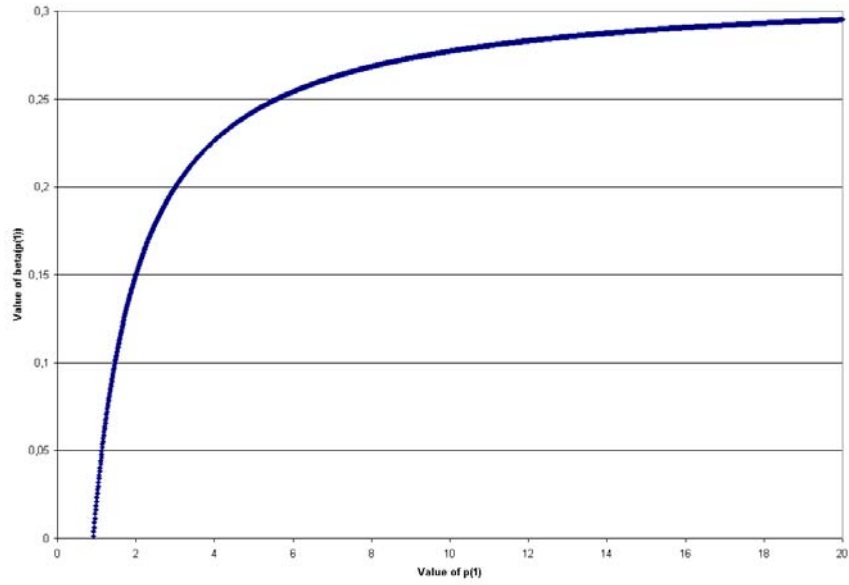
$$\beta(p_1) = \frac{\pi(\theta + p_1(1 - \theta)A) - R\pi}{(\theta + p_1(1 - \theta)A)[\pi + R(1 - \pi)]}$$

which is well defined for

$$p_1 > \frac{R - \theta}{A(1 - \theta)}$$

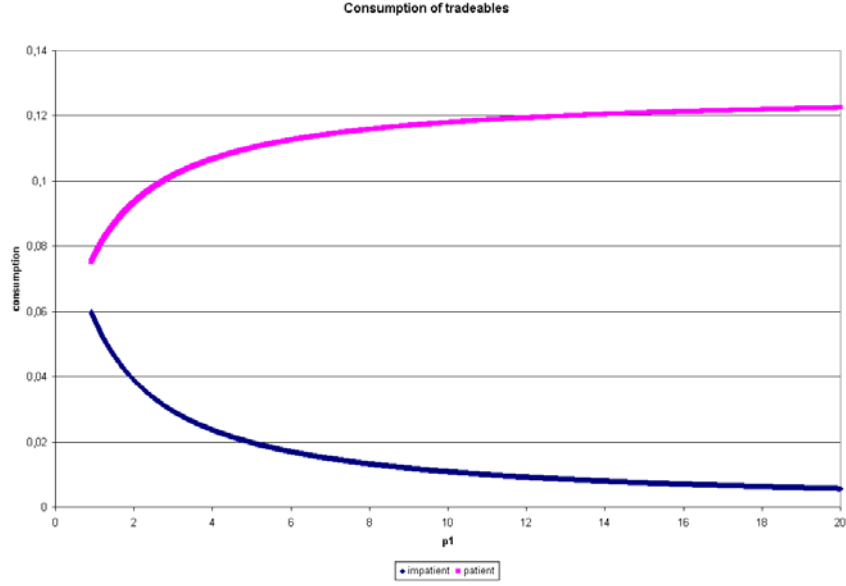
The picture below presents how β increases with p_1 for the following values of parameters (the lower bound for p_1 in this case is 0.90476)

π	θ	A	R
0.4	0.3	1.5	1.25



Graph of $\beta(p_1)$ for the log case

The two charts below show the consumption pattern for impatient and patient consumers respectively. Note that the former decreases while the second increases with the value of p_1 .



This result can be interpreted in the following way. Even though nominal exchange rates are flexible, so that Central Bank can never fail, it is possible to identify an equilibrium where commercial banks must obtain a credit in pesos in period 1. (This will be paid with the delivery of banks' period 2 investment revenues). This credit is used to pay to patient depositors who decide to withdraw in period 1 instead of period 2. In this equilibrium a fraction β of these patient depositors decide to purchase dollars at time 1, since it is indifferent for each of these patient consumers to randomize on the date of selling pesos. A key element is that this is an equilibrium at the expense of impatient consumers. Indeed, the larger p_1 , the larger β , but also smaller is the amount of tradeable goods (or dollars) consumed by impatient agents. In fact impatient consumers should get a very small fraction of all available dollars since the patient consumers who go early to the Central Bank must get an amount of dollars way above the average (per capita) reserves.

Hence, even though neither commercial banks nor the Central Bank can fail here (due to the way exchange rates and prices are modelled) it is true in this equilibrium that a fraction of patient depositors choose not to wait to purchase dollars until date 2, but they do so in period 1. This can be interpreted as a partial *run* since the efficient allocation implies no patient consumer doing anything in period 1. This type of *run* resembles that in Allen and Gale (1998), although in this case the reason for a run equilibrium is a solvency shock over the long - term investment, while here it is only one possible equilibrium.

This is an important difference with respect to the CV model. One of the main points of this paper was that flexible exchange rates ensured uniqueness of equilibrium (in which case the only possible equilibrium was that implementing the efficient allocation). The combination of two commodities with cash - in - advance constraints for each good changes that result dramatically. It is clear that there may be more than one equilibrium where at least part of the patient depositors anticipate their dollar purchases one period forward. The main driving force of this multiplicity of equilibria is the timing of purchasing first the non - tradeable good and only afterwards the tradeable good (dollars), which is clearly absent in the CV model since they ignore cash - in - advance constraints. It is apparent then that the original assumption used in CV where patient consumers derived utility from holding pesos (in a one - good world) may have been too important to generate the uniqueness of equilibrium result with flexible nominal exchange rates.

4.5 Exogenous monetary policy and partial currency runs.

Until now monetary policy was taken as endogenous. Indeed, the Central Bank was assumed to issue as much pesos as needed to finance purchases of non-tradeable goods and observe a given market clearing price p_1 at $t = 1$. This is one form of what was known in the seventies as *passive* money (see Olivera (1970)). However this assumption is clearly not very appealing. It is at least doubtful to believe in a Central Bank that accommodates the stock of money supply to fulfill any kind of inflation rate. Hence it is important to see what would happen with the equilibrium obtained in the last subsection when an exogenous monetary policy is assumed.

Suppose therefore that the Central Bank just issues a per capita amount of M_1 pesos to purchase the non - tradeable goods in period 1 sold by commercial banks. The monetary authority also is able to lend as much pesos as commercial banks need to fulfill all deposit withdraws in period 1, round 1. These pesos are given to the consumers withdrawing in this first round. Hence each consumer gets in this round M_1 / π pesos. (This is the amount of pesos that every impatient consumer gets in the equilibrium that implements the efficient allocation). The rest of the assumptions stated above holds here. Thus the following result can be shown.

Proposition 9 *Suppose that the efficient allocation satisfies*

$$\frac{R}{A}c_{1N}^* > c_{2N}^* + R[c_{2T}^* - c_{1T}^*]$$

Then for a sufficiently large M_1 there exists an equilibrium where all patient consumers withdraw pesos in period 1 in both rounds. A fraction $\hat{\beta} \in (0, 1)$ of these patient consumers purchases dollars in period 1 whereas the rest waits until date 2. All patient consumers purchase non tradeables in period 2.

This result implies that, although conditions for this partial run equilibrium to hold are stronger in the case of active monetary policy, there still exists a non - trivial set of economies where this equilibrium still exists under this Central Bank behavior.

Example 10 In the log case the expression for $\tau^{imp}(M_1, \beta)$ can be shown to come from the following impatient consumer's maximization:

$$\max_{\tau^{imp} \in [0, 1]} \ln \tau^{imp} + \ln [y^* + M_1 (1 - \tau^{imp})]$$

which gives the solution:

$$\tau^{imp} = \frac{y^* + M_1}{2M_1}$$

and so

$$(1 - \tau^{imp}) = \frac{M_1 - y^*}{2M_1}$$

so M_1 should be at least equal to y^* . From the proof of proposition 9 we obtain the equilibrium expression for β :

$$\beta = \frac{(M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) y^* - \pi \{y^* + M_1 [1 - \tau^{imp}[M_1, \beta]]\} c_{2T}^*}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) y^*}$$

Replacing by the efficient allocations this becomes

$$\beta = \frac{\pi \left[\frac{\theta}{R} - \frac{1}{2} \right]}{\theta \left[(1 - \pi) + \frac{\pi}{2} \right]}$$

which does not depend on M_1 , unlike the former case when p_1 was indeterminate. Note that $\left[\frac{\theta}{R} - \frac{1}{2} \right] > 0$ if and only if $R < 2\theta$. This imposes the necessary condition $\theta > 0.5$. Therefore, the values for θ and R in the case with p_1 indeterminate do not satisfy this restriction. Suppose that $\theta = 0.7$, $\pi = \frac{1}{4}$ and $R = 1.25$. Then it can be shown that for any value of $M_1 > y^*$ the value of β is equal to 0.0245. Clearly β increases as R decreases.

This result clarifies the nature of the *indeterminacy* of nominal variables. The way the bank contract is designed imposes constraints on banks such that, when offering this contract, either p_1 or M_1 is indeterminate. Although this not surprising at all, it is important to note that, at least for Cobb-Douglas logarithmic preferences, the outcomes differ substantially between one case and the other (reflected in the expression for β).

4.6 A note on the relationship with Chang and Velasco (2000b)

It seems apparent that these last two results contradict what Chang and Velasco expected in the referred paper (2000b). Quoting from the conclusions:

“We believe that, with the qualifications just mentioned, our other results are robust to changes in the way demand for *pesos* is introduced. This is because our analysis is largely based on the relationships between depositors, the commercial bank, and the Central Bank; the precise source of the demand for pesos does not make much of a difference”. (Page 31)

The last propositions suggest that how money is modelled may actually alter the relationship between depositors, private and the Central banks. Clearly, this model has shown that cash-in-advance constraints introduce such changes. This is one of the key ingredients that induce the difference in results. This is even more important in the flexible exchange rate regime, since the original paper showed the elimination of multiple equilibria with such a regime. The last two propositions show that this result is not robust to a change in the transactions role for local currency.

There is a point that merits some further research. Throughout this section, the flexible exchange rate regime has been chosen to be almost identical to that in the CV model. However this is clearly not the only flexible regime to be considered. The scope of this paper does not include to study other types of flexible regimes. However, we believe that this regime is easily generalizable. For example, we can think of a Central Bank offering a nominal exchange rate equal to the ratio between the pesos offered to the Central Bank and the dollar supply of the monetary authority, multiplied by an arbitrary constant, κ . From the proofs of both propositions 7 and 9 it is easy to see that this kind of generalization does not change the essence of those results. In such a case, the equilibrium value of β may depend on κ , and also the minimum value of p_1 or M_1 for a partial currency run to exist may also depend on κ , but it can be conjectured from that reasoning that the source

of the multiplicity of equilibria seems not to be the local currency depreciation mechanism. Rather, multiple equilibria does depend more crucially on the transactions technology, which is the same point stressed by Chang and Velasco as stated in the last quotation.

5 Dollarized (or currency board) banking systems.

When discussing a dollarized version of the former banking system, it is obvious that the only way to get exactly the same outcome as above is when the Central Bank as a local lender of last resort is replaced by an international lender of last resort. This institution is assumed to be willing to lend any dollar at the beginning of period 1 to be returned at the end of period 1 at zero net interest rate. Also, the institution centralizing the non - tradeable goods exchanges is replaced by a Central Exchange, as in Magill and Quinzii (1992, 1996). With these new assumptions, it is very easy to show that the same allocation is implementable when only dollars are traded. The next proposition shows this

Proposition 11 *When there exists an international lender of last resort, willing to lend dollars at zero net interest rate in intra - period loans, then there exists a version of the former banking system fully dollarized that implements the optimal allocation as an equilibrium.*

The problem with this banking system is that there may still be a run equilibrium if default is not allowed. For example, suppose that all patient consumers decide to withdraw in period 1 (in both rounds) and that the international lender provides the funds for this. It can be completely possible that the amount of dollars promised to be paid to patient consumers is strictly greater than the amount of dollars available at date 2 in the economy. This will call for a natural borrowing constraint, namely, that the total amount of dollars lent in period 1 to be repaid in period 2 is at most equal to the amount of dollars coming from the long term investment, net of the repayment of the date 0 loan. Depending upon the value of p_1 this may imply a bank run equilibrium or not. The following result characterizes this feature.

Proposition 12 *Suppose that the international lender follows a passive monetary policy relative to the date-1 nominal price of non-tradeables. Assume that:*

$$p_1 \leq \frac{c_{2T}^* - c_{1T}^*}{c_{2N}^*}$$

Then with an international lender of last resort as presented in the last proposition there is no run equilibrium. Otherwise, if the loan size is restricted by the amount of dollars available in the system in period 2 there is a run equilibrium.

Example 13 *In the logarithmic case, the condition above is reduced to*

$$p_1 \leq \frac{(R-1)}{A^2} \frac{\theta}{1-\theta}$$

Note that, as $R \downarrow 1$, the condition demands that to avoid a run equilibrium non tradeables must transact almost for free. This is in fact true also for the general case.

This states then that for a sufficiently low p_1 (maybe a deflationary state) the presence of the lender of last resort does prevent the emergence of a run equilibrium. If p_1 violates the former inequality, agents may think that the international lender of last resort is too generous in financing a high *inflation* event (a high value of p_1). This implies that some patient agents may anticipate the fact that the banking system does not have enough resources at time 2 to honor the foreign debt *and* all deposits, which implies that it may be individually rational (at least for some patient depositors) to withdraw and purchase dollars early.

This result seems surprising, since it is believed that, as long as the international lender only provides funds for liquidity reasons, then illiquidity crises should disappear. However this is not the case here (provided that default with the international lender is not possible) because the deposit contract and the transactions arrangements leave the value of p_1 undetermined. Hence if all domestic agents anticipate a high p_1 then it may be the case that patient depositors consider the amount of liquidity in dollars (provided by the international lender) too high. Hence, for these values of p_1 , these agents anticipate a future failure of the banking system, triggering a bank run equilibrium.

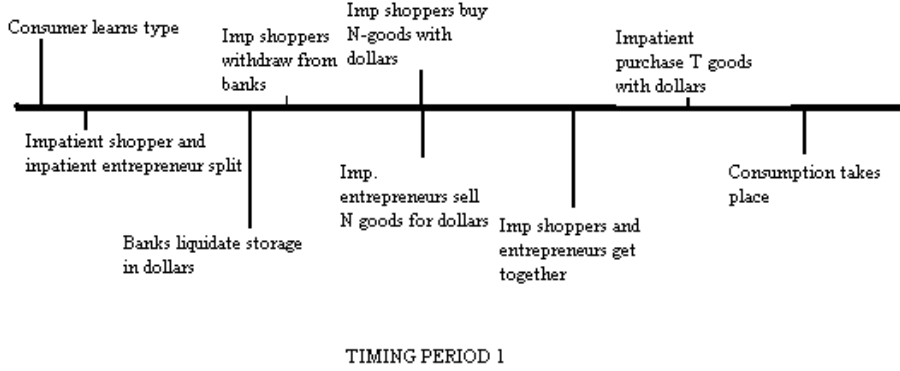
Therefore, either because of this possible bank failure, or because banks could declare default over the foreign loans, or due to non - modelled moral hazard problems, the US Federal Reserve is usually reluctant to act as a lender of last resort in foreign countries. The rest of this section assumes a different dollarized banking system without such a lender ³. The following analysis suggests that the discussion on the relationship between banking, exchange rates and optimality can be very sensitive to changes in the process of exchange of goods and assets.

³Alternatively, it can also be interpreted as a two - currency banking system where the Central Bank becomes a currency board at a one - to - one rate.

5.1 A dollarized banking system with no Central Exchange and no international lender of last resort.

Consider this alternative banking system. Suppose that in period 0 commercial banks borrow d dollars from abroad. These banks decide the amount to be invested in the liquid (short run), the illiquid (long run) dollar denominated investments and the amount of tradable inputs to be invested in the non - tradeables technology. At the beginning of date 1 impatient entrepreneurs and shoppers split. Commercial banks obtain y dollars by liquidating the storage technology. The dollars are paid to the impatient shoppers. These dollars should be enough to cover the final consumption of dollars (tradeable goods) by impatient households at the end of period 1. Commercial banks deliver $\alpha^e A z^e$ units of N goods to impatient entrepreneurs, who sell the non-tradable goods to shoppers at a nominal price of $\bar{p}_1$ dollars. After this first session shoppers and entrepreneurs get together. Shoppers bring non-tradeables to be consumed at the end of the period and entrepreneurs bring y dollars. Then consumers use these dollars to purchase tradable goods abroad. Then consumption takes place and period 1 ends.

In period 2 actions are similar. At the beginning of this period patient entrepreneurs and shoppers split. Then commercial banks obtain the results of the long term project (in dollars), and repay to the foreign creditors the outstanding debt. The remaining dollars are paid to patient shoppers who use them to purchase non - tradeables at a price equal to $\bar{p}_2$ from patient entrepreneurs. The same restrictions as for period 1 withdrawals apply. The N goods are provided by the commercial banks. After this, shoppers and entrepreneurs meet again. They use the remaining dollars to buy tradable goods. The period ends with the consumption of tradeables and non-tradeables.



Timing: Date 1

5.2 Efficiency of the dollarized banking system.

This subsection analyzes whether this banking system can decentralize the optimal allocation studied in section 3. The problem of the bank can be formalized as follows.

$$\max \quad \pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2N})] \quad (11)$$

subject to

$$x + y + z \leq d \quad (12)$$

$$\pi w_1 \leq y \quad (13)$$

$$p_1 c_{1N} \leq \lambda_1 w_1, \quad \lambda_1 \in [0, 1] \quad (14)$$

$$c_{1T} = \frac{\alpha p_1 A z}{\pi} + w_1 - p_1 c_{1N} \quad (15)$$

$$(1 - \pi) c_{1T} \leq y \quad (16)$$

$$(1 - \pi) w_2 \leq Rx - d \quad (17)$$

$$p_2 c_{2N} \leq \lambda_2 w_2, \quad \lambda_2 \in [0, 1] \quad (18)$$

$$c_{2T} = \frac{(1 - \alpha) p_2 A^2 z}{1 - \pi} + w_2 - p_2 c_{2N} \quad (19)$$

$$(1 - \pi) c_{2T} \leq Rx - d \quad (20)$$

The next proposition shows the characterization of the equilibrium allocation.

Proposition 14 *If the efficient allocation satisfies*

$$\frac{Ac_{1T}^*}{c_{1N}^*} > 1 \quad \text{and} \quad \frac{c_{2T}^*}{c_{2N}^*} > \frac{R}{A^2}$$

then the efficient allocation can be implemented as an equilibrium of this dollarized banking system.

This result shows formally that under certain conditions a dollarization of a banking system with this transactions mechanism has at least the efficient allocation as an equilibrium. However those conditions may not always hold.

Example 15 *In the logarithmic case the conditions can be written as:*

$$\frac{Ac_{1T}^*}{c_{1N}^*} = \frac{Ad\theta(R-1)}{(1-\theta)Ad(R-1)} = \frac{\theta}{1-\theta} > 1$$

$$\frac{c_{2T}^*}{c_{2N}^*} = \frac{d\theta R(R-1)}{(1-\theta)A^2d(R-1)} = \frac{\theta R}{(1-\theta)A^2} > \frac{R}{A^2}$$

which reduces to the inequality $\theta > \frac{1}{2}$. If we parametrize the economy through the value of θ then only for the economies with half Lebesgue measure such that $\theta > \frac{1}{2}$ the dollarized banking system can be implemented. In this case the price system (p_1, p_2) must satisfy:

$$\frac{1}{A} < p_1 < \frac{\theta}{A(1-\theta)}; \quad \frac{R}{A^2} < p_2 < \frac{\theta R}{(1-\theta)A^2}$$

Note that when the banking system is dollarized prices of non - tradeables depend entirely on real fundamentals. This is obvious since the relevant money supply is controlled by no local agent. In the logarithmic case, the inflation in terms of non - tradeables $\frac{p_2}{p_1}$ is bounded above by $\frac{\theta A}{(1-\theta)R}$ and below by $\frac{R(1-\theta)}{A^3\theta}$. Then the change in the real exchange rate between periods 1 and 2 may depend entirely on the marginal productivity of the non - tradeable production technology and the marginal productivity of the long - term tradeable - generating investment technology.

5.3 Banking system fragility

The second point in this section is whether this contract implies a form of bank runs. It is obvious to see that now we are referring to bank runs and not to currency crisis. The first obvious result is the following.

Proposition 16 *The necessary and sufficient condition for an equilibrium run to occur in a dollarized banking system is $c_{1T}^* > y^* + r \left(x^* - \frac{d}{R} \right)$.*

The proof of this is standard and left to the reader. It is clear that in the logarithmic example the condition $r < 1$ is sufficient to generate the inequality above. Hence, the dollarized banking system, although it may implement the efficient allocation, also has a run equilibrium under the same conditions as in the CV model. However, recall that the former system also had a second condition for a run to exist, while here that condition is not present. The reason is obvious. The timing in the former system allowed for any patient who waited until date 2 to withdraw some pesos to purchase non tradeables in $t = 2$. In the latter system this is not possible if all patient consumers withdraw early since there is nothing to withdraw in period 2 in this case. In this sense one can say that this dollarized banking system may be more fragile than a peso - denominated banking system.

Clearly, the transaction arrangements differ between the fixed exchange rate regime with a local lender with the dollarized (or currency board) regime (without an international lender of last resort). This is the reason of why the price level at time 1 may imply conditions for having a currency run, while in a dollarized banking system this is not true. This result suggests again that cash-in-advance constraints limit the validity of the results since they depend on the *timing* of transactions.

6 Concluding remarks and extensions

This paper has developed an extension of the Chang and Velasco (2000b) small-open-economy version of the Diamond and Dybvig (1983) model with cash-in-advance constraints, in order to give money a more serious role for consumers. In this way the results in the CV model concerning the interaction between banks, exchange rate regimes and monetary policy can be restated and analyzed with a more proper modelling of fiat currency. The main conclusion is that how non - tradeable goods exchanges are organized has a very strong influence on the properties of each regime and the banking system that can be designed.

First, the banking system with fixed exchange rates and a Central Bank providing credit in unbacked pesos does implement the optimal allocation. This system may be vulnerable to equilibrium currency runs, although the new assumptions imply more demanding conditions for a run to occur than in the original model. A flexible exchange rate regime, as developed in the CV model, may not constitute an effective threat against currency runs. Even

though this policy eliminates bank failures (by construction), it does not rule out the possibility of having some of the patient depositors purchasing dollars before the last period. This results in a cost to impatient consumers and in an inefficient allocation. The arising of this new equilibrium depends on the monetary policy conducted by the Central Bank in the interim period.

In a dollarized banking system the optimal allocation can be implemented under certain conditions, even though there may be no international lender of last resort. This result differs from the CV model. However, there may be a bank run equilibrium under similar conditions as in CV (the *international illiquidity condition*). Preferences over date-1 and date-2 consumption also matter.

Given that some of the discussion on exchange rate regimes refer to the CV paper as a fundamental reference of a case in favor of flexible regimes (and against fixed nominal exchange rates) the results in this paper rise some caveats about the virtues of flexible exchange rates over nominal pegs. In particular, there may be cases within the model where multiple equilibria arise with flexible exchange rates (with, for example, a loose monetary policy, i.e., very high M_1) but not with fixed exchange rates, being this true, for instance, when condition (10) in proposition 5 fails to hold.

The assumptions used throughout the paper may be subject to various criticisms. First, a reader familiar with the CV model may have the following point against the results described above. In CV, impatient consumers only needed pesos to purchase dollars (or tradeables) to be consumed in period 1, while patient people derived some utility from pesos holdings. In the model of this paper this is different. Impatient consumers have identical ex - post preferences than patient consumers. Therefore, comparing the results in this paper with those in CV may sound somehow unfair since preferences here do not seem to generate the same reduced-form type of model as in CV. In fact, timing between the two papers differ a lot.

As a consequence, an important next step should be to construct a version of this banking system with preferences that are closer to those in the CV model, although without the money-in-the-utility assumption. In particular, the original model implied an asymmetry between preferences of impatient consumers versus those of patient depositors. Hence, analyzing a version of this paper where impatient consumers only value tradeables in period 1 while patient depositors do so for consumption of both goods in period 2 may be a case worth to study, so that conclusions of this version may be more comparable to those in CV.

Perhaps one of the most fundamental limitations of this model is the assumption of ad - hoc cash - in - advance constraints. It is crucial to clarify that this paper does not want to lead with the question of why money is used

for transactions. The model takes this assumption as given (as in most of the cash - in - advance literature) to address other issues on the relationship between banks and monetary and exchange rate policies. Even though this is not completely satisfactory, the cash - in - advance approach seems to add at least a more explicit role for money, which was absent in the original framework.

Still, this framework can be extended to consider other questions that seem relevant. Specially important seems to consider solvency shocks to banks's investment and crisis due to fundamentals, as analyzed by Allen and Gale (2000b). In this model, the return on the long-run investment is random. Hence crisis are triggered by low realizations of this return. They show that flexible exchange rates may imply better risk-sharing than nominal pegs. However, there is no fiat money in the model. They only work with units of account. Thus, it seems relevant to adapt such a model to a framework considering fiat currency as a mean-of-exchange, like in this paper, to evaluate the robustness of those results to changes in the way money is introduced.

Another relevant question is what happens when there is some international aggregate (locally non - diversifiable) shock in the different arrangements. Given the riskiness in the international price of commodities, this may affect the optimal design of the banking contract as well as the properties of different monetary and exchange rate policies. This framework has also a natural two - country extension. This is interesting to study the interaction between international banks, monetary and exchange rate policies and the possibility of contagion, as studied in the real version of the Diamond and Dybvig framework in Allen and Gale (2000a).

A deeper question is the fact that money in this economy cannot be considered as essential, at least in the sense of Wallace (2000). The ad - hoc assumption may not be satisfactory if we want to study the interaction between monetary and exchange rate policies, banking stability and currency substitution, for example. This calls for endogeneizing the use of money for transactions, as studied by the already broad literature on search models starting from Kiyotaki and Wright (1989). However this implies a banking system with possibly infinitely lived households, issue that to my knowledge was not addressed yet. This last extension seems much more complicated to construct.

7 Appendix: Proofs

Proof. of Lemma 1. If z is fixed then the problem in terms of how much of z is liquidated early is the solution to

$$\max_{\alpha \in [0,1]} \pi v \left(\frac{\alpha A z}{\pi} \right) + (1 - \pi) v \left(\frac{(1 - \alpha) A^2 z}{1 - \pi} \right)$$

whose FOC is $v' \left(\frac{\alpha A z}{\pi} \right) = A v' \left(\frac{(1 - \alpha) A^2 z}{1 - \pi} \right)$. Strict concavity of v ensures that the solution to this problem is unique. Given that v is homothetic, the solution to this equation is independent of z . ■

Proof. of Proposition 3. Note that the problem is a standard concave programming. Hence we know that the first order conditions are necessary as well as sufficient to characterize the optimum. Let the Lagrangian be

$$\begin{aligned} \mathcal{L} = & \pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2N})] + \\ & + \phi_0 [d - x - y - z] + \phi_1^1 [\alpha p_1 A z - \pi w_1^1] + \phi_1^2 [y - \pi w_1^2] \\ & + \lambda_1 [y - \pi c_{1T}] + \psi_1^1 [w_1^1 - p_1 c_{1N}] + \psi_1^2 [w_1^2 + (w_1^1 - p_1 c_{1N}) - c_{1T}] \\ & + \phi_2^1 [(1 - \alpha) p_2 A^2 z - (1 - \pi) w_2^1] + \phi_2^2 [R x - d - (1 - \pi) w_2^2] \\ & + \lambda_2 [R x - d - (1 - \pi) c_{2T}] + \psi_2^1 [w_2^1 - p_2 c_{2N}] + \psi_2^2 [w_2^2 + (w_2^1 - p_2 c_{2N}) - c_{2T}] \end{aligned}$$

The decision variables are $\left(c_{1T}, c_{2T}, c_{1N}, c_{2N}, x, y, z, \alpha, (w_1^j, w_2^j)_{j=1}^2 \right)$. The FOC's for an interior solution are

$$\begin{aligned} \pi u'(c_{1T}) &= \pi \lambda_1 + \psi_1^2 \\ (1 - \pi) u'(c_{2T}) &= (1 - \pi) \lambda_2 + \psi_1^1 \\ \pi v'(c_{1N}) &= p_1 (\psi_1^1 + \psi_1^2) \\ (1 - \pi) v'(c_{2N}) &= p_2 (\psi_2^1 + \psi_2^2) \\ \phi_0 &= R (\phi_2^2 + \lambda_2) \\ \phi_0 &= \phi_1^2 + \lambda_1 \\ \phi_0 &= \phi_1^1 \alpha A p_1 + (1 - \alpha) \phi_2^1 A^2 p_2 \\ \phi_1^1 p_1 &= \phi_2^1 p_2 A \\ \pi \phi_1^1 &= \psi_1^1 + \psi_1^2 \\ \pi \phi_1^2 &= \psi_1^2 \\ (1 - \pi) \phi_2^1 &= \psi_2^1 + \psi_2^2 \\ (1 - \pi) \phi_2^2 &= \psi_2^2 \end{aligned}$$

Therefore we obtain $\phi_1^2 + \lambda_1 = R (\phi_2^2 + \lambda_2)$, but since also $u'(c_{tT}) = \lambda_t + \phi_t^2$

(because $\pi\phi_1^2 = \psi_1^2$ and $(1 - \pi)\phi_2^2 = \psi_2^2$) we then obtain

$$u'(c_{1T}) = Ru'(c_{2T})$$

Also we do have $v'(c_{tN}) = \phi_t^1 p_t$ for $t = 1, 2$. Given that $\phi_1^1 p_1 = \phi_2^1 p_2 A$ this of course implies that

$$v'(c_{1N}) = Av'(c_{2N})$$

The two equalities, together with $R > 1$ and $A > 1$, imply that the equilibrium allocation satisfies the incentive - compatibility constraint. Given that $\phi_0 = \phi_1^1 \alpha A p_1 + (1 - \alpha) \phi_2^1 A^2 p_2$ and that $\phi_1^1 p_1 = \phi_2^1 p_2 A$ then $\phi_0 = \phi_1^1 p_1 A (\alpha + 1 - \alpha) = A \phi_1^1 p_1$. But $\phi_0 = \phi_1^2 + \lambda_1 = u'(c_{1T})$. Therefore

$$u'(c_{1T}) = Av'(c_{1N})$$

Hence the three marginality conditions of the Pareto problem are satisfied. Next we need to show that the constraints shrink to those in the planner's problem. Period 0 constraint of the bank's problem is identical to that of the planner's problem. I will first show that banks's constraints in periods 1 and 2 hold with equality. To do this, note that from the FOC it is true that $v'(c_{tN}) = \phi_t^1 p_t$, and since p_t is assumed to be strictly positive then $\phi_t^1 > 0$. Given that $u'(c_{tT}) = \lambda_t + \phi_t^2$ then clearly $(\lambda_t + \phi_t^2) > 0$. Given that $\phi_t^1 > 0$ so is $\psi_t^1 + \psi_t^2$. To simplify I focus on $t = 1$. The arguments for $t = 2$ are identical. If $(\lambda_1 + \phi_1^2) > 0$ and $\psi_1^1 + \psi_1^2 > 0$ then the following must be the case

$$\left(\begin{array}{l} \text{either } \lambda_1 > 0 \\ \text{or } \phi_1^2 > 0 \end{array} \right) \quad \text{and} \quad \left(\begin{array}{l} \text{either } \psi_1^1 > 0 \\ \text{or } \psi_1^2 > 0 \end{array} \right)$$

The first pair of cases implies that either $\pi c_{1T} = y$ or $y = \pi w_1^2$ or both), while the second implies that either $w_1^1 = p_1 c_{1N}$ or $c_{1T} = w_1^2 + (w_1^1 - p_1 c_{1N})$ or both. Since $\phi_1^1 > 0$ then we will have always that $\pi w_1^1 = \alpha p_1 A z$.

Suppose then that $\lambda_1 > 0$ and so $\pi c_{1T} = y$. We still know that $\pi w_1^2 \leq y$ so $w_1^2 \leq c_{1T}$. In this case, if $\psi_1^1 > 0$ then $w_1^1 = p_1 c_{1N}$ and since it is always true that $c_{1T} \leq w_1^2 + (w_1^1 - p_1 c_{1N}) = w_1^2$ then $c_{1T} = w_1^2$ as well. If otherwise $\psi_1^2 > 0$ then $c_{1T} = w_1^2 + (w_1^1 - p_1 c_{1N})$ and so $w_1^2 \leq w_1^2 + (w_1^1 - p_1 c_{1N})$, or $p_1 c_{1N} \leq w_1^1$. But if $\psi_1^2 > 0$, given that $\pi \phi_1^2 = \psi_1^2$ then $\phi_1^2 > 0$ must also hold. Then $y = \pi w_1^2$ and in fact then $w_1^2 = c_{1T}$, which gives in this case that $p_1 c_{1N} = w_1^1$. Hence when $\lambda_1 > 0$ all constraints hold with equality.

Assume now that $\phi_1^2 > 0$. By the same argument as above we need $\psi_1^2 > 0$. Then we have that $y = \pi w_1^2$ and $c_{1T} = w_1^2 + (w_1^1 - p_1 c_{1N})$. Recall that it is always true that $\pi w_1^1 = \alpha p_1 A z$. It is always true that $\pi c_{1T} \leq y = \pi w_1^2$ so

$c_{1T} \leq w_1^2$ so $(w_1^1 - p_1 c_{1N}) \leq 0$ or $w_1^1 \leq p_1 c_{1N}$. But recall that we must always have that $p_1 c_{1N} \leq w_1^1$ so $p_1 c_{1N} = w_1^1$. If this is so then in fact $c_{1T} = w_1^2$ and therefore $\pi c_{1T} = \pi w_1^2 = y$. Hence when $\phi_1^2 > 0$ all constraints in period 1 also hold with equality. The arguments for restrictions in period 2 are identical.

Given this fact we just get that

$$\pi c_{1T} = y; \quad (1 - \pi) c_{2T} = Rx - d$$

and since $p_t c_{tN} = w_t^1$, $\pi w_1^1 = \alpha A p_1 z$ and $(1 - \pi) w_2^1 = (1 - \alpha) A^2 p_2 z$ then $p_1 \pi c_{1N} = \pi w_1^1 = \alpha A p_1 z$ and $p_1 (1 - \pi) c_{2N} = (1 - \pi) w_2^1 = (1 - \alpha) A^2 p_2 z$. Therefore

$$\pi c_{1N} = \alpha A z; \quad (1 - \pi) c_{2N} = (1 - \alpha) A^2 z$$

Therefore, all constraints corresponding to the planner's problem must also hold. Given the three marginality conditions we obtain that the efficient allocation constitute an equilibrium for this system.

Note that all the arguments above are independent of any particular value for p_1 and p_2 . They should just be strictly positive values. Therefore, the price of non - tradeables in every period can be any strictly positive real number.

■

Proof. of proposition 5. Focusing in symmetric run equilibria (all patient depositors make the same decisions) the proof is standard. Suppose that the condition

$$w_1^{2*} > y^* + r \left(x^* - \frac{d}{R} \right)$$

holds. Assume that a patient consumer is evaluating whether to withdraw (in the second round of) period 1, given that all other depositors decide to withdraw in this round. Note that the patient depositor can always purchase the amount c_{2N}^* with any fraction of w_1^{2*} , provided that p_2 is equal to

$$p_2(\tau) = \frac{(1 - \pi) \tau w_1^{2*}}{(1 - \alpha^*) A^2 z}$$

where τ is the fraction of the w_1^{2*} pesos withdrawn in period 1 by patient depositors. Hence τ can be made as small as desired (although never equal to 0). Clearly the inequality above implies that total demand for dollars (measured in pesos) is strictly greater than the supply of dollars at the end of period 1 (for a value of $1 - \tau$ sufficiently close to one). If the patient consumer stays home, she will get 0 dollars at date 2 with probability 1. If she otherwise withdraws w_1^{2*} pesos then there is a chance that she may get

c_{1T}^* dollars. If the third inequality is true then the patient depositor finds optimal to withdraw early.

If instead the second inequality holds, then there is also a currency run equilibrium if also the last inequality is true. The reason is simple. Suppose that all depositors withdraw in the first round of date 1. Assume moreover that all patient depositors but one use the fraction $1 - \tau$ of these pesos (with τ very small) to purchase dollars at the Central Bank⁴. If the patient depositor decides to stay at home, she will see how all reserves will be exhausted (the second inequality ensures this). If she also tries to sell pesos for dollars there is a chance that she may get c_{1T}^* dollars. If the last inequality holds then it is optimal for this patient depositor to follow the same strategy as the other consumers.

If the third inequality holds then a currency run is an equilibrium when the following occurs. Suppose that all depositors withdraw in both rounds. The total (per capita) amount of pesos withdrawn by patient agents is equal to $(1 - \pi) (w_1^{1*}(p_1) + w_1^{2*})$. The last inequality ensures that if all but one patient consumer sell these pesos for dollars at a unit exchange rate then the Central Bank runs out of dollars (provided that the fraction of pesos stored until date 2 is small enough)⁵. Given that the last inequality also holds then for that patient consumer is individually rational to do this instead of waiting until date 2.

Clearly, if the last inequality fails to hold, then it is not individually rational to run to the bank (for a patient depositor) in any of the two cases. The failure of the second condition implies that even when the Central Bank fails in period 1, any patient gets higher utility by waiting to withdraw until period 2 than by running early. ■

Proof. of proposition 7. Suppose that all patient depositors withdraw in period 1 (both rounds). Then they get a total of $w_1^{1*}(p_1) + w_1^{1*} = \frac{\alpha^* A z^* p_1 + y^*}{\pi}$ pesos. Note that this is possible because the Central Bank provides liquidity in pesos to the commercial banks in period 1 in both rounds of withdrawals. We will show that given the equality in the statement of this proposition all consumers are indifferent between selling pesos for dollars in either period, but they prefer strongly to purchase all non tradeable goods in period 2 instead of period 1. Suppose that each patient depositor reserves a fraction $\tau \in [0, 1]$ of all these pesos to purchase N goods while the rest is

⁴In this case the value of p_2 that allows every patient consumer to purchase $A^2 (1 - \alpha^*) z^*$ units of non-tradeables in $t = 2$ is equal to $p_2(\tau, p_1) = \frac{(1 - \pi) \tau w_1^{1*}(p_1)}{(1 - \alpha^*) A z^*}$.

⁵In this case the value of p_2 consistent with market clearing of non-tradeables is equal to $p_2(\tau^1, \tau^2, p_1) = \frac{(1 - \pi)(\tau^1 w_1^{1*}(p_1) + \tau^2 w_1^{2*})}{(1 - \alpha^*) A^2 z^*}$, where at least one τ^j is strictly positive (although small).

dedicated to purchase dollars.

If all patient depositors decide to destine $\tau [w_1^{1*}(p_1) + w_1^{1*}]$ to buy non - tradeables in period 2 then the market clearing condition implies

$$(1 - \pi) \tau [w_1^{1*}(p_1) + w_1^{1*}] = (1 - \alpha^*) A^2 z^* p_2$$

since in this equilibrium the supply of non - tradeables coincide with the efficient quantity. Therefore it must be the case that

$$p_2(p_1, \tau) = \frac{(1 - \pi) \tau [w_1^{1*}(p_1) + w_1^{1*}]}{(1 - \alpha^*) A^2 z^*} = \frac{\tau (1 - \pi) A p_1 z^* \alpha^* + y^*}{\pi A^2 z^* (1 - \alpha^*)}$$

Therefore each patient consumer obtains

$$c_{2N}^* = \frac{\tau [w_1^{1*}(p_1) + w_1^{1*}]}{p_2(p_1, \tau)}$$

It can be shown that if any individual patient consumer decides to purchase non - tradeables in period 1 she would obtain $c_{1N}^* < c_{2N}^*$, therefore there is no incentive to deviate from this decision in terms of the utilities of non - tradeables.

Let a fraction β of patient depositors sells the fraction $(1 - \tau)$ of pesos to the Central Bank in exchange for dollars in period 1. Recall that also all impatient depositors sell their pesos for dollars. Impatient depositors destine $\frac{y^*}{\pi}$ pesos to purchase dollars because in equilibrium all pesos withdrawn in round 1 by impatient consumers must completely be spent in purchasing non - tradeables. The reader can check that if this were not the case there is no market clearing in the non - tradeable market in period 1. Thus the exchange rate at date 1 is set as follows:

$$e_1^*(p_1, \tau) = \frac{y^* + (1 - \pi) \beta (1 - \tau) \left(\frac{[A p_1 z^* \alpha^* + y^*]}{\pi} \right)}{y^*}$$

Therefore, each impatient consumer gets

$$c_{1T}^{imp} = \frac{y^*}{\pi e_1^*(p_1, \tau)} = c_{1T}^* \left(\frac{y^*}{y^* + (1 - \pi) \beta (1 - \tau) \left(\frac{[A p_1 z^* \alpha^* + y^*]}{\pi} \right)} \right)$$

which is of course less than y^* . Each patient consumer who sold pesos to the Central Bank in period 1 gets an amount of dollars equal to:

$$c_{1T}^p = (1 - \tau) \frac{(A p_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi}$$

For any given τ it is easy to see that

$$\begin{aligned} c_{1T}^p &= \frac{y^* (1 - \tau)}{\pi} \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{y^* + (1 - \pi) \beta (1 - \tau) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \\ &= c_{1T}^* \left(\frac{(1 - \tau) [Ap_1 z^* \alpha^* + y^*]}{y^* + (1 - \pi) \beta (1 - \tau) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \end{aligned}$$

and so $c_{1T}^p > c_{1T}^{imp}$ since $Ap_1 z^* \alpha^* + y^* > y^*$, so for τ small enough $(1 - \tau) [Ap_1 z^* \alpha^* + y^*] > y^*$. Hence no patient depositor wants to replicate the behavior of impatient depositors. I will show that τ is *small enough* later on. On the other hand, the date 2 nominal exchange rate is

$$e_2^*(p_1, \tau) = \frac{(1 - \pi) (1 - \theta) (1 - \tau) (Ap_1 z^* \alpha^* + y^*)}{\pi (Rx^* - d)}$$

Therefore each of these patient consumers obtains the following amount of dollars in period 2:

$$c_{2T}^p = \frac{(1 - \tau) (Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau)} = \frac{(1 - \tau) (Ap_1 z^* \alpha^* + y^*)}{\pi} \frac{\pi (Rx^* - d)}{(1 - \pi) (1 - \theta) (1 - \tau) (Ap_1 z^* \alpha^* + y^*)}$$

In equilibrium

$$\begin{aligned} \frac{\tau [w_1^{1*}(p_1) + w_1^{1*}]}{p_2(p_1)} &= \frac{\bar{\tau} [Ap_1 z^* \alpha^* + y^*]}{\pi} \frac{\pi A^2 z^* (1 - \alpha^*)}{\bar{\tau} (1 - \pi) (Ap_1 z^* \alpha^* + y^*)} \\ &= \frac{A^2 z^* (1 - \alpha^*)}{1 - \pi} = c_{2N}^* \end{aligned}$$

$$c_{2T}^p = \frac{(Rx^* - d)}{(1 - \pi)} \frac{1}{(1 - \beta)} = \frac{c_{2T}^*}{1 - \beta}$$

So the last step is to show the behavior of the equilibrium τ . Suppose for a moment that each patient consumer randomizes between purchasing dollars in period 1 and doing this in period 2. The probability of purchasing dollars in $t = 1$ is equal to β . Hence the problem of choosing the optimal τ is

$$\begin{aligned} \max_{\tau \in [0,1]} \quad & v \left(\frac{\tau [w_1^{1*}(p_1) + w_1^{1*}]}{p_2(p_1, \tau)} \right) + \beta u \left((1 - \tau) \frac{(Ap_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi} \right) + \\ & (1 - \beta) u \left(\frac{(1 - \tau) (Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau)} \right) \end{aligned}$$

where the τ inside p_2 , e_1^* and e_2^* are taken as given. The FOC is:

$$\begin{aligned} & v' \left(\frac{\tau [w_1^{1*}(p_1) + w_1^{1*}]}{p_2(p_1, \tau)} \right) \left(\frac{w_1^{1*}(p_1) + w_1^{1*}}{p_2(p_1, \tau)} \right) \\ &= \beta u' \left((1 - \tau) \frac{(Ap_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi} \right) \frac{(Ap_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi} + \\ &+ (1 - \beta) u' \left(\frac{(1 - \tau) (Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau)} \right) \left(\frac{(Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau)} \right) \end{aligned}$$

Given that u and v are both strictly concave and C^2 , satisfying Inada conditions, it is clear that the optimal solution (this exists since the domain $[0, 1]$ is compact) is a C^1 function of p_1 , β and the value of the economy's τ . Let $\tau^*(p_1, \beta, \tau)$ be the decision rule. Hence in a symmetric equilibrium it must be the case that

$$\tau^*(p_1, \beta, \tau(p_1, \beta)) = \tau(p_1, \beta)$$

Is there a fixed point like this? For each (p_1, β) it is clear that $\tau^*(p_1, \beta, \cdot)$ is a continuous map from $[0, 1]$ into $[0, 1]$ (The reason is that τ^* is always strictly less than one and strictly positive, by the Inada conditions of the functions u and v). Hence by the Brower Fixed Point Theorem there is a τ that satisfies the equality above. This is true for each (p_1, β) so the fixed point is a function of (p_1, β) . Note that in equilibrium it must occur that

$$\begin{aligned} & v' \left(\frac{\tau^* [w_1^{1*}(p_1) + w_1^{1*}]}{p_2(p_1, \tau)} \right) \left(\frac{w_1^{1*}(p_1) + w_1^{1*}}{p_2(p_1, \tau)} \right) \\ &= v' \left(\tau(p_1, \beta) \frac{A^2 z^* (1 - \alpha^*)}{\tau(p_1, \beta) (1 - \pi)} \right) \left(\frac{A^2 z^* (1 - \alpha^*)}{\tau(p_1, \beta) (1 - \pi)} \right) = v'(c_N^*) \frac{c_N^*}{\tau(p_1, \beta)} \end{aligned}$$

and similarly:

$$\begin{aligned} & u' \left((1 - \tau(p_1, \beta)) \frac{(Ap_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi} \right) \frac{(Ap_1 z^* \alpha^* + y^*)}{e_1^*(p_1, \tau) \pi} \\ &= u' \left(\frac{(1 - \tau(p_1, \beta)) [Ap_1 z^* \alpha^* + y^*] c_{1T}^*}{y^* + (1 - \pi) \theta (1 - \tau(p_1, \beta)) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \\ & \quad \cdot \left(\frac{c_{1T}^* (Ap_1 z^* \alpha^* + y^*)}{y^* + (1 - \pi) \theta (1 - \tau(p_1, \beta)) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \\ & u' \left(\frac{(1 - \tau(p_1, \beta)) (Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau(p_1, \beta))} \right) \left(\frac{(Ap_1 z^* \alpha^* + y^*)}{\pi e_2^*(p_1, \tau(p_1, \beta))} \right) = u' \left(\frac{c_{2T}^*}{1 - \beta} \right) \frac{c_{2T}^*}{(1 - \tau(p_1, \beta)) (1 - \beta)} \end{aligned}$$

Therefore the FOC is equivalent in equilibrium to the expression

$$\begin{aligned}
& v' (c_{2N}^*) \frac{c_{2N}^*}{\tau (p_1, \beta)} \\
&= \beta u' \left(\frac{(1 - \tau (p_1, \beta)) [Ap_1 z^* \alpha^* + y^*] c_{1T}^*}{y^* + (1 - \pi) \theta (1 - \tau (p_1, \beta)) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \\
&\quad \cdot \left(\frac{c_{1T}^* (Ap_1 z^* \alpha^* + y^*)}{y^* + (1 - \pi) \theta (1 - \tau (p_1, \beta)) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) + \\
&\quad + (1 - \beta) u' \left(\frac{c_{2T}^*}{1 - \beta} \right) \frac{c_{2T}^*}{(1 - \tau (p_1, \beta)) (1 - \beta)}
\end{aligned}$$

First it is needed to know the behavior of $\tau (p_1, \beta)$ when $\beta \rightarrow 0$. Given that $\tau (p_1, \beta)$ is continuous in β (it is just a consequence of the application of the Implicit Function Theorem), then taking limits on both sides we have

$$\lim_{\beta \rightarrow 0} v' (c_{2N}^*) \frac{c_{2N}^*}{\tau (p_1, \beta)} = u' (c_{2T}^*) \lim_{\beta \rightarrow 0} \frac{c_{2T}^*}{(1 - \tau (p_1, \beta))}$$

Assuming that $\tau (p_1, 0)$ is well defined then

$$v' (c_N^*) \frac{c_N^*}{\tau (p_1, 0)} = u' \left(\frac{c_{2T}^*}{1 - \theta} \right) \frac{c_{2T}^*}{(1 - \tau (p_1, 0)) (1 - \theta)}$$

or

$$\begin{aligned}
\tau (p_1, 0) &= \frac{v' (c_{2N}^*) c_{2N}^*}{v' (c_{2N}^*) c_{2N}^* + u' (c_{2T}^*) c_{2T}^*} \\
1 - \tau (p_1, 0) &= \frac{u' (c_{2T}^*) c_{2T}^*}{v' (c_{2N}^*) c_{2N}^* + u' (c_{2T}^*) c_{2T}^*}
\end{aligned}$$

It will be shown that the only limit needed to be considered is that of β going to 0. Now, all this procedure assumed that each patient consumer randomizes between purchasing dollars in $t = 1$ or in $t = 2$.

On the other hand the equality in the statement is equivalent to:

$$c_{1T}^* \left(\frac{(1 - \tau (p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*]}{\pi y^* + (1 - \pi) \beta (1 - \tau (p_1, \beta)) (Ap_1 z^* \alpha^* + y^*)} \right) = \frac{c_{2T}^*}{(1 - \beta)}$$

or just simply $c_{1T}^p = c_{2T}^p$. Let $\Delta^* \equiv \frac{c_{2T}^*}{c_{1T}^*}$. This implies

$$(1 - \tau (p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] (1 - \beta) = \Delta^* [\pi y^* + (1 - \pi) \beta (1 - \tau (p_1, \beta)) (Ap_1 z^* \alpha^* + y^*)]$$

or

$$\begin{aligned} & (1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^* \\ &= \beta (Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, \beta)) [\pi + \Delta^* (1 - \pi)] \end{aligned}$$

So

$$\beta = \frac{(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^*}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, \beta)) [\pi + \Delta^* (1 - \pi)]}$$

We need to check if there is a β where this is true. First note that

$$(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] > \Delta^* \pi y^*$$

for a sufficiently high p_1 and any β . Note that when β goes to 0 the numerator converges to

$$\begin{aligned} & (1 - \tau(p_1, 0)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^* \\ &= \frac{u'(c_{2T}^*) c_{2T}^* \pi [Ap_1 z^* \alpha^* + y^*]}{v'(c_{2N}^*) c_{2N}^* + u'(c_{2T}^*) c_{2T}^*} - \Delta^* \pi y^* \\ &= \frac{u'(c_{2T}^*) c_{2T}^* \pi^2 [p_1^* c_{1N}^* + c_{1T}^*]}{v'(c_{2N}^*) c_{2N}^* + u'(c_{2T}^*) c_{2T}^*} - c_{2T}^* \pi^2 = c_{2T}^* \pi^2 \left(\frac{u'(c_{2T}^*) [p_1^* c_{1N}^* + c_{1T}^*]}{v'(c_{1N}^*) c_{1N}^* + u'(c_{2T}^*) c_{2T}^*} - 1 \right) \end{aligned}$$

Let p_1 be large enough so that:

$$p_1 > \frac{1}{c_{1N}^*} \left\{ \frac{v'(c_{2N}^*)}{u'(c_{2T}^*)} c_{2N}^* + c_{2T}^* - c_{1T}^* \right\}$$

Hence we have that

$$0 < \lim_{\beta \rightarrow 0} \left\{ \frac{(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^*}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, \beta)) [\pi + \Delta^* (1 - \pi)]} \right\}$$

Note that the limit of the left hand side as β goes to 1 is

$$\begin{aligned} & \frac{(1 - \tau(p_1, 1)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^*}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, 1)) [\pi + \Delta^* (1 - \pi)]} \\ & < \frac{(1 - \tau(p_1, 1)) \pi [Ap_1 z^* \alpha^* + y^*]}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, 1)) [\pi + \Delta^* (1 - \pi)]} = \frac{\pi}{\pi + \Delta^* (1 - \pi)} < 1 \end{aligned}$$

and so $1 > \lim_{\beta \rightarrow 1} \left\{ \frac{(1 - \tau(p_1, \beta)) \pi [Ap_1 z^* \alpha^* + y^*] - \Delta^* \pi y^*}{(Ap_1 z^* \alpha^* + y^*) (1 - \tau(p_1, \beta)) [\pi + \Delta^* (1 - \pi)]} \right\}$. Hence by the Intermediate Value Theorem there is at least a value of $\beta \in (0, 1)$ such that the equality is satisfied. Hence there exists a β where each patient consumer is indifferent between purchasing dollars in period 1 and doing this in period 2.

This implies that consumers randomize between these two options, where the probability of purchasing dollars in period 1 is β . Note that in equilibrium

$$\begin{aligned} c_{1T}^p &= c_{1T}^* \left(\frac{(1 - \tau(p_1, \beta)) [Ap_1 z^* \alpha^* + y^*]}{y^* + (1 - \pi) \beta (1 - \tau(p_1, \beta)) \left(\frac{[Ap_1 z^* \alpha^* + y^*]}{\pi} \right)} \right) \\ &= \frac{c_{2T}^*}{(1 - \beta)} > c_{2T}^* > c_{1T}^* > c_{1T}^{imp} \end{aligned}$$

This confirms that each patient person does not want to imitate the impatient consumer's behavior. This ends the proof. ■

Proof of Proposition 9. Suppose that all consumers withdraw $\frac{M_1}{\pi}$ pesos. Period 1 market clearing in non - tradeables is

$$\tau^{imp} M_1 = p_1 \alpha^* A z^*$$

so

$$p_1 (\tau^{imp} M_1) = \frac{\tau^{imp} M_1}{A \alpha^* z^*} = \frac{\tau^{imp} M_1}{\pi c_{1N}^*}$$

Let $\bar{\tau}^{imp}$ be the average value of τ^{imp} across all impatient consumers. Hence each impatient gets the following amount of non tradeables

$$c_{1N} [p_1] = \frac{\tau^{imp} M_1}{\pi p_1 (\bar{\tau}^{imp} M_1)}$$

In equilibrium of course this will be equal to c_{1N}^* .

In period 2 the market clearing condition is

$$\tau^{pat} (1 - \pi) \left(\frac{M_1 + y^*}{\pi} \right) = p_2 (1 - \alpha^*) A^2 z^*$$

therefore

$$p_2 (\tau^{pat}, M_1) = \frac{\tau^{pat} (1 - \pi) (M_1 + y^*)}{\pi (1 - \alpha^*) A^2 z^*} = \frac{\tau^{pat} (M_1 + y^*)}{\pi c_{2N}^*}$$

Define again $\bar{\tau}^{pat}$ the average across patient consumers of $\bar{\tau}^{pat}$. Each patient consumer gets

$$c_{2N} = \left(\frac{M_1 + y^*}{\pi p_2 (\bar{\tau}^{pat}, M_1)} \right) \tau^{pat}$$

which in equilibrium will be c_{2N}^* .

On the other hand, the date - 1 nominal exchange rate is equal to:

$$e_1(\bar{\tau}^{imp}, \bar{\tau}^{pat}, M_1, \beta) = \frac{y^* + M_1[1 - \bar{\tau}^{imp}] + \frac{(M_1 + y^*)(1 - \bar{\tau}^{pat})}{\pi}(1 - \pi)\beta}{y^*}$$

Each impatient consumer would get an amount of tradeables equal to:

$$\begin{aligned} c_{1T}^{imp} &= \frac{y^* + M_1[1 - \tau^{imp}]}{\pi e_1(\bar{\tau}^{imp}, \bar{\tau}^{pat}, M_1, \beta)} \\ &= \left\{ \frac{y^* + M_1[1 - \tau^{imp}]}{\pi[y^* + M_1[1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi)\beta} \right\} y^* \end{aligned}$$

and each patient consumer:

$$\begin{aligned} c_{1T}^p &= \frac{(M_1 + y^*)(1 - \tau^{pat})}{\pi e_1(\bar{\tau}^{imp}, \bar{\tau}^{pat}, M_1, \beta)} \\ &= \left[\frac{(M_1 + y^*)(1 - \tau^{pat})}{\pi[y^* + M_1[1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi)\beta} \right] y^* \end{aligned}$$

The date 2 nominal exchange rate is equal to:

$$e_2(M_1, \beta, \bar{\tau}^{pat}) = \frac{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})(M_1 + y^*)}{\pi(Rx^* - d)}$$

So each patient consumer who waits to buy dollars until date 2 gets an amount of tradeables equal to:

$$\begin{aligned} c_{2T}^p &= \frac{(1 - \tau^{pat})(M_1 + y^*)}{\pi e_2(M_1, \beta, \bar{\tau}^{pat})} \\ &= \frac{(Rx^* - d)(1 - \tau^{pat})(M_1 + y^*)}{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})(M_1 + y^*)} = \frac{(Rx^* - d)(1 - \tau^{pat})}{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})} \end{aligned}$$

Now we need to show the existence of optimal policies τ^{imp} and τ^{pat} that solves each consumer type's problem. We assume for now that the patient consumer is indifferent between purchasing dollars in period 1 and doing so in period 2. Therefore the patient consumer solves

$$\begin{aligned} &\max_{\tau^{pat} \in [0,1]} v \left[\left(\frac{M_1 + y^*}{\pi p_2(\bar{\tau}^{pat}, M_1)} \right) \tau^{pat} \right] + \\ &\beta u \left[\left[\frac{(M_1 + y^*)(1 - \tau^{pat})}{\pi[y^* + M_1[1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi)\beta} \right] y^* \right] \\ &+ (1 - \beta) u \left[\frac{(Rx^* - d)(1 - \tau^{pat})}{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})} \right] \end{aligned}$$

The solution clearly exists and by strict concavity and differentiability can be characterized from the equation

$$\begin{aligned}
& \left(\frac{M_1 + y^*}{\pi p_2(\bar{\tau}^{pat}, M_1)} \right) v' \left[\left(\frac{M_1 + y^*}{\pi p_2(\bar{\tau}^{pat}, M_1)} \right) \tau^{pat} \right] \\
= & \beta u' \left[\frac{(M_1 + y^*)(1 - \tau^{pat}) y^*}{\pi \{y^* + M_1 [1 - \bar{\tau}^{imp}]\} + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi) \beta} \right] \\
& \frac{(M_1 + y^*) y^*}{\pi \{y^* + M_1 [1 - \bar{\tau}^{imp}]\} + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi) \beta} \\
& + (1 - \beta) u' \left[\frac{(Rx^* - d)(1 - \tau^{pat})}{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})} \right] \left[\frac{(Rx^* - d)}{(1 - \pi)(1 - \beta)(1 - \bar{\tau}^{pat})} \right]
\end{aligned}$$

and from the Theorem of the Maximum the solution τ^{pat} is a continuous function of $[\bar{\tau}^{pat}, \bar{\tau}^{imp}, M_1, \beta]$.

Similarly, the problem of the impatient consumer is

$$\max_{\tau^{imp} \in [0,1]} v \left[\frac{\tau^{imp} M_1}{\pi p_1(\bar{\tau}^{imp}, M_1)} \right] + u \left[\left(\frac{y^* + M_1 [1 - \tau^{imp}]}{\pi [y^* + M_1 [1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi) \beta} \right) y^* \right]$$

whose FOC at an interior solution is

$$\begin{aligned}
& v' \left[\frac{\tau^{imp} M_1}{\pi p_1(\bar{\tau}^{imp}, M_1)} \right] \left[\frac{M_1}{\pi p_1(\bar{\tau}^{imp}, M_1)} \right] \\
= & u' \left[\left(\frac{y^* + M_1 [1 - \tau^{imp}]}{\pi [y^* + M_1 [1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi) \beta} \right) y^* \right] \\
& \left(\frac{y^* M_1}{\pi [y^* + M_1 [1 - \bar{\tau}^{imp}]] + (M_1 + y^*)(1 - \bar{\tau}^{pat})(1 - \pi) \beta} \right)
\end{aligned}$$

From here we obtain the decision rule $\tau^{imp} [\bar{\tau}^{pat}, \bar{\tau}^{imp}, M_1, \beta]$.

It is clear that the map

$$\tau [\bar{\tau}^{pat}, \bar{\tau}^{imp}, M_1, \beta] \equiv \begin{bmatrix} \tau^{pat} [\bar{\tau}^{pat}, \bar{\tau}^{imp}, M_1, \beta] \\ \tau^{imp} [\bar{\tau}^{pat}, \bar{\tau}^{imp}, M_1, \beta] \end{bmatrix}$$

for any fixed M_1 and β is a continuous map on the domain $[0, 1]^2$ into itself. Hence, applying again the Brouwer Fixed Point Theorem there exists a pair $[\tau^{pat} [M_1, \beta], \tau^{imp} [M_1, \beta]]$ such that

$$\tau [\tau^{pat} [M_1, \beta], \tau^{imp} [M_1, \beta], M_1, \beta] = \begin{bmatrix} \tau^{pat} [M_1, \beta] \\ \tau^{imp} [M_1, \beta] \end{bmatrix}$$

By the smoothness, strict concavity and Inada assumptions on both u and v then these maps are continuous on (M_1, β) .

In equilibrium then it happens from the FOC in the case of the patient consumer is:

$$\begin{aligned}
& \frac{c_{2N}^*}{\tau^{pat}[M_1, \beta]} v' [c_{2N}^*] \\
&= \beta u' \left[\frac{(M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) y^*}{\pi \{y^* + M_1 [1 - \tau^{imp}[M_1, \beta]]\} + (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) (1 - \pi) \beta} \right] \\
&+ (1 - \beta) u' \left[\frac{(Rx^* - d) (1 - \tau^{pat}[M_1, \beta])}{(1 - \pi) (1 - \beta) (1 - \tau^{pat}[M_1, \beta])} \right] \left[\frac{(Rx^* - d)}{(1 - \pi) (1 - \beta) (1 - \tau^{pat}[M_1, \beta])} \right] \\
&= \beta u' \left[\frac{(M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) y^*}{\pi \{y^* + M_1 [1 - \tau^{imp}[M_1, \beta]]\} + (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) (1 - \pi) \beta} \right] \\
&+ (1 - \beta) u' \left[\frac{c_{2T}^*}{1 - \beta} \right] \left(\frac{c_{2T}^*}{(1 - \beta) (1 - \tau^{pat}[M_1, \beta])} \right)
\end{aligned}$$

and from the impatient consumer's FOC:

$$\begin{aligned}
& v' \left[\frac{\tau^{imp}[M_1, \beta] M_1}{\pi p_1 (\tau^{imp}[M_1, \beta] M_1)} \right] \left[\frac{M_1}{\pi p_1 (\tau^{imp}[M_1, \beta] M_1)} \right] \\
&= v' (c_{1N}^*) \frac{c_{1N}^*}{\tau^{imp}[M_1, \beta]} \\
&= u' \left[\left(\frac{y^* + M_1 [1 - \tau^{imp}[M_1, \beta]]}{\pi [y^* + M_1 [1 - \tau^{imp}[M_1, \beta]] + (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) (1 - \pi) \beta} \right) y^* \right] \\
&\quad \left(\frac{y^* M_1}{\pi [y^* + M_1 [1 - \tau^{imp}[M_1, \beta]] + (M_1 + y^*) (1 - \tau^{pat}[M_1, \beta]) (1 - \pi) \beta} \right)
\end{aligned}$$

Applying continuity of $\tau^j[M_1, \beta]$, $j = imp$ or pat , then

$$\begin{aligned}
\lim_{\beta \rightarrow 0} \frac{c_{2N}^*}{\tau^{pat}[M_1, \beta]} v' [c_{2N}^*] &= \frac{c_{2N}^*}{\tau^{pat}[M_1, 0]} v' [c_{2N}^*] \\
&= u' [c_{2T}^*] \left(\frac{c_{2T}^*}{1 - \tau^{pat}[M_1, 0]} \right)
\end{aligned}$$

so

$$\tau^{pat}[M_1, 0] u' [c_{2T}^*] c_{2T}^* = c_{2N}^* v' [c_{2N}^*] [1 - \tau^{pat}[M_1, 0]]$$

or

$$\tau^{pat} [M_1, 0] = \frac{c_{2N}^* v' [c_{2N}^*]}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} > 0$$

and

$$1 - \tau^{pat} [M_1, 0] = \frac{u' [c_{2T}^*] c_{2T}^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} > 0$$

Similarly from the impatient consumer's FOC:

$$\lim_{\beta \rightarrow 0} v' (c_{1N}^*) \frac{c_{1N}^*}{\tau^{imp} [M_1, \beta]} = \frac{v' (c_{1N}^*) c_{1N}^*}{\tau^{imp} [M_1, 0]} = u' [c_{1T}^*] \left(\frac{c_{1T}^* M_1}{y^* + M_1 [1 - \tau^{imp} [M_1, 0]]} \right)$$

and so

$$v' (c_{1N}^*) c_{1N}^* [y^* + M_1 [1 - \tau^{imp} [M_1, 0]]] = u' (c_{1T}^*) c_{1T}^* M_1 \tau^{imp} [M_1, 0]$$

therefore

$$\tau^{imp} [M_1, 0] = \frac{v' (c_{1N}^*) c_{1N}^* [y^* + M_1]}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]}$$

Note that this expression may be strictly greater than one. This imposes a condition on M_1 :

$$(y^* + M_1) v' (c_{1N}^*) c_{1N}^* < M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]$$

or

$$M_1 > \frac{y^* v' (c_{1N}^*) c_{1N}^*}{u' (c_{1T}^*) c_{1T}^*}$$

which is the condition imposed in the statement of the proposition.

Therefore

$$1 - \tau^{imp} [M_1, 0] = \frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]}$$

Again recall that the patient consumer must be indifferent between purchasing dollars in period 1 or doing this in period 2, implying again that

$$\begin{aligned} & \frac{(M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*}{\pi \{y^* + M_1 [1 - \tau^{imp} [M_1, \beta]]\} + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) \beta} \\ & \qquad \qquad \qquad = \frac{c_{2T}^*}{1 - \beta} \end{aligned}$$

which is equivalent to:

$$(M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^* (1 - \beta) \\ = [\pi \{y^* + M_1 [1 - \tau^{imp} [M_1, \beta]]\} + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) \beta] c_{2T}^*$$

which is the same as

$$\beta = \frac{(M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^* - \pi \{y^* + M_1 [1 - \tau^{imp} [M_1, \beta]]\} c_{2T}^*}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*} \\ = \frac{(M_1 c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi [1 - \tau^{imp} (M_1, \beta)]) - (y^* c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi)}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*}$$

When taking limits for β going to 0, the right hand side converges to

$$\frac{(M_1) ((1 - \tau^{pat} [M_1, 0]) y^* - \pi c_{2T}^* [1 - \tau^{imp} (M_1, 0)]) - (y^*) ((1 - \tau^{pat} [M_1, 0]) y^* - \pi c_{2T}^*)}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, 0]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, 0]) y^*}$$

The numerator is equal to

$$(M_1) \left[\frac{u' [c_{2T}^*] c_{2T}^* y^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi c_{2T}^* \left(\frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right) \right] \\ - (y^*) \left[\frac{u' [c_{2T}^*] c_{2T}^* y^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi c_{2T}^* \right]$$

The reader notes that

$$\lim_{M_1 \rightarrow \infty} \left(\frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right) = \frac{[u' (c_{1T}^*) c_{1T}^*]}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]}$$

and recall that $u' (c_{2T}^*) = Ru' (c_{1T}^*)$, $v' (c_{2N}^*) = Av' (c_{1N}^*)$. Hence

$$\lim_{M_1 \rightarrow \infty} \left[\frac{u' [c_{2T}^*] c_{2T}^* y^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi c_{2T}^* \left(\frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right) \right] \\ = \frac{u' [c_{2T}^*] c_{2T}^* y^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi \frac{c_{2T}^* u' (c_{1T}^*) c_{1T}^*}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \\ = \left[\frac{Ru' (c_{1T}^*) y^*}{Ru' [c_{1T}^*] c_{2T}^* + c_{2N}^* Av' [c_{1N}^*]} - \pi \frac{u' (c_{1T}^*) c_{1T}^*}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right] c_{2T}^* \\ = \left[\frac{Ru' (c_{1T}^*) \pi c_{1T}^*}{Ru' [c_{1T}^*] c_{2T}^* + c_{2N}^* Av' [c_{1N}^*]} - \frac{u' (c_{1T}^*) \pi c_{1T}^*}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right] c_{2T}^* \\ = u' (c_{1T}^*) \pi c_{1T}^* c_{2T}^* \left[\frac{R}{Ru' [c_{1T}^*] c_{2T}^* + c_{2N}^* Av' [c_{1N}^*]} - \frac{1}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right]$$

We claim that if the efficient allocation satisfies the inequality in the statement this is strictly positive. Suppose that this is not the case. Then

$$\frac{R}{Ru' [c_{1T}^*] c_{2T}^* + c_{2N}^* Av' [c_{1N}^*]} - \frac{1}{[u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \leq 0$$

equivalent to

$$Ru' (c_{1T}^*) c_{1T}^* + Rv' (c_{1N}^*) c_{1N}^* \leq Ru' [c_{1T}^*] c_{2T}^* + c_{2N}^* Av' [c_{1N}^*]$$

or

$$Rc_{1N}^* v' (c_{1N}) \leq c_{2N}^* Av' (c_{1N}) + Ru' [c_{1T}^*] [c_{2T}^* - c_{1T}^*]$$

and since

$$u' (c_{1T}^*) = Av' (c_{1N}^*)$$

then the condition above implies

$$\frac{R}{A} c_{1N}^* \leq c_{2N}^* + R [c_{2T}^* - c_{1T}^*]$$

which contradicts the inequality in the statement. Hence

$$\lim_{M_1 \rightarrow \infty} \left[\frac{u' [c_{2T}^*] c_{2T}^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi \left(\frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right) \right] > 0$$

Therefore the limit of

$$(M_1 c_{2T}^*) \left[\frac{u' [c_{2T}^*] c_{2T}^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi \left(\frac{M_1 [u' (c_{1T}^*) c_{1T}^*] - y^* v' (c_{1N}^*) c_{1N}^*}{M_1 [u' (c_{1T}^*) c_{1T}^* + v' (c_{1N}^*) c_{1N}^*]} \right) \right] - (y^* c_{2T}^*) \left[\frac{u' [c_{2T}^*] c_{2T}^*}{u' [c_{2T}^*] c_{2T}^* + c_{2N}^* v' [c_{2N}^*]} - \pi \right]$$

is equal to plus infinity as $M_1 \rightarrow \infty$. Hence there exists a value $\bar{M}_1$ over which the numerator of the expression

$$\frac{(M_1 c_{2T}^*) (1 - \tau^{pat} [M_1, 0] - \pi [1 - \tau^{imp} (M_1, 0)]) - (y^* c_{2T}^*) (1 - \tau^{pat} [M_1, 0] - \pi)}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, 0]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, 0]) y^*}$$

is strictly positive. Hence for these values of M_1 then

$$\lim_{\beta \rightarrow 0} \frac{(M_1 c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi [1 - \tau^{imp} (M_1, \beta)]) - (y^* c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi)}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*} > \lim_{\beta \rightarrow 0} \beta = 0$$

and recalling that

$$\begin{aligned}
& \frac{(M_1 c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi [1 - \tau^{imp} (M_1, \beta)]) - (y^* c_{2T}^*) (1 - \tau^{pat} [M_1, \beta] - \pi)}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*} \\
&= \frac{(M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^* - \pi \{y^* + M_1 [1 - \tau^{imp} [M_1, \beta]]\} c_{2T}^*}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*} \\
&< \frac{(M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \beta]) y^*} < 1
\end{aligned}$$

for any β . Hence, there exists a $\bar{\beta}$ sufficiently close to 1 such that

$$\bar{\beta} > \frac{(M_1 + y^*) (1 - \tau^{pat} [M_1, \bar{\beta}]) y^* - \pi \{y^* + M_1 [1 - \tau^{imp} [M_1, \bar{\beta}]]\} c_{2T}^*}{c_{2T}^* (M_1 + y^*) (1 - \tau^{pat} [M_1, \bar{\beta}]) (1 - \pi) + (M_1 + y^*) (1 - \tau^{pat} [M_1, \bar{\beta}]) y^*}$$

Hence there is a value of $\hat{\beta}$ in $(0, 1)$ that satisfies the equality above, that is, such that a patient consumer is indifferent between purchasing dollars in period 1 or in period 2 given that a fraction $\hat{\beta}$ of patient consumers buys dollars in period 1.

Lastly, note that in this equilibrium an impatient consumer obtains a utility level strictly less than $v(c_{1N}^*) + u(c_{1T}^*)$ whereas the patient consumer gets a strictly higher utility of $v(c_{2N}^*) + u\left(\frac{c_{2T}^*}{1-\beta}\right)$. Hence patient agents do not have any incentive to behave as impatient agents. This shows the existence of a partial run equilibrium with active monetary policy. ■

Proof. of Proposition 11. In the dollarized banking system, all debt is now denominated in dollars. At the beginning of period 1, there is a Central Exchange that borrows from the international lender an amount M_1 dollars. Commercial banks sell the non - tradeable production to this Central Exchange at a price equal to p_1 (now this price is set in dollars per unit of good N). Hence banks have a per-capita amount of $M_1 = p_1^e A z^* \alpha^*$ dollars. These are paid to impatient shoppers, who spend them in buying non-tradeables. Non - tradeables are bought from the Central Exchange in exchange for M_1 dollars, who are used to repay the debt with the international lender of last resort. Then financial intermediaries also liquidate the storage technology. This is directly used to pay to impatient consumers who will purchase tradeable goods.

In period 2 the timing is also equivalent to that in the fixed exchange rate regime. At the beginning of period 2, the Central Exchange borrows M_2 dollars. These are used to purchase $(1 - \alpha^*) A^2 z^*$ units of good N at the

price p_2^e . The financial intermediaries receive these dollars against the output flow of non tradeables in period 2. Patient consumers withdraw M_2 dollars to purchase these non -tradeable goods. The dollars received by the Central Exchange are returned to the international lender. Then commercial banks liquidate the long - term investment technology. They set aside d dollars to be returned to the institutions who lent these dollars in period 0. The rest is paid to patient consumers, who spend this money in purchasing T - goods. The reader can easily verify that the equations involved in this system are exactly the same as in the banking system considered above. Hence the allocations must coincide. ■

Proof. of proposition 12. Suppose first that

$$p_1 \leq \frac{c_{2T}^* - c_{1T}^*}{c_{1N}^*}$$

Hence it is clear that

$$(1 - \pi) (p_1 c_{1N}^* + c_{1T}^*) \leq (1 - \pi) c_{2T}^*$$

and note that $p_1 c_{1N}^* = \frac{w_1^{1*}(p_1)}{\pi}$, $c_{1T}^* = \frac{y^*}{\pi}$ and $c_{2T}^* = \frac{Rx^* - d}{1 - \pi}$ therefore the last inequality is equivalent to

$$\frac{1 - \pi}{\pi} (w_1^{1*}(p_1) + w_1^{2*}) \leq Rx^* - d$$

Hence, suppose that a fraction β of patient consumers withdraw at date 1 and a fraction $1 - \beta$ withdraws in period 2. Assume without loss of generality that all patient consumers withdrawn in period 1 does that in both rounds, so that the total amount of dollars that is withdrawn in period 1 is $\frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} (1 - \pi) \beta$. Clearly, no patient person wants to spend any dollar in period 1 in any good since she may get higher consumption in period 2 as will be demonstrated. For, if spent $w_1^{1*}(p_1)$ in period 1 in purchasing non - tradeable goods then she will obtain c_{1N}^* units (it cannot be the case that less than $w_1^{1*}(p_1)$ is spent in purchasing N goods in period 1 in equilibrium, otherwise there is no market clearing) whereas if she spends a small fraction τ in period 2 in buying period 2 N - goods she will be able to get c_{2N}^* units. The same can be said in terms of T good consumption. Hence, let τ be the fraction of dollars spent in purchasing non - tradeable goods in period 2. Give that a fraction $1 - \beta$ withdraws $w_2^{1*}(p_2) = \frac{(1 - \alpha^*)A^2 z^* p_2}{1 - \pi}$ then market clearing implies

$$(1 - \pi) \beta \tau \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} + (1 - \beta) (1 - \alpha^*) A^2 z^* p_2 = (1 - \alpha^*) A^2 z^* p_2$$

or

$$(1 - \pi) \beta \tau \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} = \beta (1 - \alpha^*) A^2 z^* p_2$$

which implies

$$p_2^*(p_1) = \frac{(w_1^{1*}(p_1) + w_1^{2*})(1 - \pi) \tau}{(1 - \alpha^*) A^2 z^* \pi}$$

Therefore each patient consumer who withdraws in period 1 obtains

$$c_{2N}^{p1} = \tau \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi p_2(p_1)} = \frac{(1 - \alpha^*) A^2 z^*}{(1 - \pi)} = c_{2N}^*$$

while each patient consumer who withdraws late obtains

$$\frac{(1 - \alpha^*) A^2 z^* p_2(p_1)}{p_2(p_1)(1 - \pi)} = \frac{(1 - \alpha^*) A^2 z^*}{(1 - \pi)} = c_{2N}^*$$

and since $c_{2N}^* > c_{1N}^*$ then each patient consumer who withdraws early finds optimal to purchase N goods in period 2 instead of period 1. Note that the quantity $\tau \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi}$ of dollars is absorbed by the Central Exchange (when selling N goods in period 2) and returned to the international lender. On the other hand, the total available supply of dollars in per capita terms is $Rx^* - d - (1 - \tau)(1 - \pi) \beta \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi}$. This should be enough to cover the withdrawals of patient consumers who waited to period 2. Indeed, note that

$$\begin{aligned} Rx^* - d - (1 - \tau)(1 - \pi) \beta \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} \\ > Rx^* - d - (1 - \pi) \beta \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} \end{aligned}$$

and since $\frac{1 - \pi}{\pi} (w_1^{1*}(p_1) + w_1^{2*}) \leq Rx^* - d$ then

$$\begin{aligned} Rx^* - d - (1 - \pi) \beta \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} \\ \geq Rx^* - d - \beta (Rx^* - d) = (1 - \beta)(Rx^* - d) \end{aligned}$$

therefore each patient consumer who waited until date 2 gets an amount of

$$c_{2T}^{p2} > \frac{(Rx^* - d)(1 - \beta)}{(1 - \pi)(1 - \beta)} = c_{2T}^*$$

while all patient consumers who did not wait to withdraw obtain

$$\begin{aligned} c_{2T}^{1p} &= (1 - \tau) \frac{(w_1^{1*}(p_1) + w_1^{2*})}{\pi} \\ &< \frac{(Rx^* - d)}{(1 - \pi)} = c_{2T}^* \end{aligned}$$

It is clear then that each patient depositor prefers to wait until date 2 to withdraw. Since θ was arbitrary then this shows that under the inequality above there is no run equilibrium.

If the inequality is reversed:

$$p_1 > \frac{c_{2T}^* - c_{1T}^*}{c_{1N}^*}$$

which is equivalent to:

$$\frac{1 - \pi}{\pi} (w_1^{1*}(p_1) + w_1^{2*}) > Rx^* - d$$

then it is possible to find a run equilibrium. Suppose without loss of generality that

$$\frac{1 - \pi}{\pi} w_1^{1*}(p_1) \leq Rx^* - d$$

so that the run is threat only when patient depositors withdraw in both rounds in period 1. Assume therefore that all patient depositors try to withdraw in both rounds of period 1. Clearly in the first round commercial banks do not fail at all since the upper bound on the borrowing amount (equal to $Rx^* - d$) is not violated. However, if in the second round all patient consumers decide to withdraw as well, then the inequality in this paragraph implies that banks will fail to honor this deposit withdrawals since the upper bound of credit will be reached before all depositors obtain $\frac{y^*}{\pi}$ dollars. No patient consumer finds optimal to stay at home in this case since otherwise she gets 0 consumption of each good (banks failure implies the impossibility of withdrawing in date 2) so it is rational no run in this case. This ends the proof. ■

Proof. of proposition 14. The problem is

$$\begin{aligned} \max \quad & \pi \left\{ u \left(w_1 - p_1 c_{1N} + \frac{p_1 \alpha A z}{\pi} \right) + v(c_{1N}) \right\} \\ & + (1 - \pi) \left\{ u \left(w_2 - p_2 c_{2N} + \frac{p_2 (1 - \alpha) A^2 z}{1 - \pi} \right) + v(c_{2N}) \right\} \end{aligned}$$

subject to

$$x + y + z \leq d$$

$$\begin{aligned} \pi w_1 &\leq y \\ p_1 c_{1N} &\leq \lambda_1 w_1 \\ \pi \left(w_1 - p_1 c_{1N} + \frac{p_1 \alpha A z}{\pi} \right) &\leq y \end{aligned}$$

$$\begin{aligned} (1 - \pi) w_2 &\leq Rx - d \\ p_2 c_{2N} &\leq \lambda_2 w_2 \\ (1 - \pi) \left(w_2 - p_2 c_{2N} + \frac{p_2 (1 - \alpha) A^2 z}{1 - \pi} \right) &\leq Rx - d \end{aligned}$$

The Lagrangian then is the following expression,

$$\begin{aligned} L = & \pi \left\{ u \left(w_1 - p_1 c_{1N} + \frac{p_1 \alpha A z}{\pi} \right) + v(c_{1N}) \right\} \\ & + (1 - \pi) \left\{ u \left(w_2 - p_2 c_{2N} + \frac{p_2 (1 - \alpha) A^2 z}{1 - \pi} \right) + v(c_{2N}) \right\} \\ & + \phi_0 \{d - x - y - z\} + \phi_1 [y - \pi w_1] + \eta_1 [\lambda_1 w_1 - p_1 c_{1N}] \\ & + \psi_1 \left\{ y - \pi \left(w_1 - p_1 c_{1N} + \frac{p_1 \alpha A z}{\pi} \right) \right\} + \phi_2 [Rx - d - (1 - \pi) w_2] + \eta_2 [\lambda_2 w_2 - p_2 c_{2N}] \\ & + \psi_2 \left[Rx - d - (1 - \pi) \left(w_2 - p_2 c_{2N} + \frac{p_2 (1 - \alpha) A^2 z}{1 - \pi} \right) \right] \end{aligned}$$

and the first order conditions (necessary and sufficient conditions due to our assumptions) are:

$$\begin{aligned} L_{w_1} &= \pi u'(c_{1T}) - \pi \phi_1 + \eta_1 \lambda_1 + \pi \psi_1 p_1 = 0 \\ L_{c_{1N}} &= \pi v'(c_{1N}) - (\eta_1 + \pi u'(c_{1T})) p_1 + \psi_1 \pi p_1 = 0 \\ L_{w_2} &= (1 - \pi) u'(c_{2T}) - (1 - \pi) \phi_2 + \eta_2 \lambda_2 - \psi_2 (1 - \pi) = 0 \\ L_{c_{2N}} &= (1 - \pi) v'(c_{2N}) - (\eta_2 + (1 - \pi) u'(c_{2T})) p_2 + (1 - \pi) p_2 \psi_2 = 0 \\ L_x &= -\phi_0 + R(\phi_2 + \psi_2) = 0 \\ L_y &= -\phi_0 + \phi_1 + \psi_1 = 0 \\ L_z &= -\phi_0 + (u'(c_{1T}) - \psi_1) p_1 A \alpha + (u'(c_{2T}) - \psi_2) p_2 A^2 (1 - \alpha) = 0 \\ L_\alpha &= (u'(c_{1T}) - \psi_1) p_1 A z - (u'(c_{2T}) - \psi_2) A^2 p_2 z = 0 \\ L_{\lambda_1} &= \eta_1 \geq 0; \quad L_{\lambda_2} = \eta_2 \geq 0 \end{aligned}$$

where in the last two inequalities we have that $\lambda_t = 1$ whenever $\eta_t > 0$. Let us guess a solution to this problem where $\eta_t = 0$ for $t = 1, 2$. From these FOC we need to have

$$u'(c_{tT}) = \phi_t + \psi_t, \quad t = 1, 2$$

$$\phi_0 = \phi_1 + \psi_1 = R(\phi_2 + \psi_2)$$

so

$$u'(c_{1T}) = Ru'(c_{2T})$$

On the other hand

$$v'(c_{tN}) = (u'(c_{tT}) - \psi_t) p_t, \quad t = 1, 2$$

and since:

$$\begin{aligned} (u'(c_{1T}) - \psi_1) p_1 &= (u'(c_{2T}) - \psi_2) A p_2 \\ \phi_0 &= (u'(c_{1T}) - \psi_1) p_1 A \alpha + (u'(c_{2T}) - \psi_2) p_2 A^2 (1 - \alpha) \end{aligned}$$

therefore

$$\phi_0 = (u'(c_{1T}) - \psi_1) p_1 A = v'(c_{1N}) A$$

Hence

$$u'(c_{1T}) = \phi_1 + \psi_1 = \phi_0 = v'(c_{1N}) A$$

So it must also be true that

$$u'(c_{1T}) = Av'(c_{1N})$$

And note also that

$$v'(c_{2N}) = p_2 (u'(c_{2T}) - \psi_2) = \frac{(u'(c_{1T}) - \psi_1) p_1}{A} = \frac{v'(c_{1N})}{A}$$

so

$$v'(c_{1N}) = Av'(c_{2N}) \tag{21}$$

It has been shown then that the three marginality conditions that characterizes the efficient allocation hold here. It is needed then to show that the

feasibility conditions of the efficient allocations also hold here. In order to do that, note first that

$$v'(c_{tN}) = (u'(c_{tT}) - \psi_t) p_t = p_t \phi_t$$

Given that p_t is assumed to be strictly positive, then so is ϕ_t for $t = 1, 2$. Therefore $y = \pi w_1$. Recall also that

$$u'(c_{1T}) = \phi_1 + \psi_1 = \phi_0 = (u'(c_{1T}) - \psi_1) p_1 A$$

which implies that

$$A p_1 \psi_1 = (p_1 A - 1) u'(c_{1T})$$

Therefore, for any $p_1 > \frac{1}{A} > 0$ we have that $\psi_1 > 0$. Assume so (we will show later that under the inequalities in the statement of the proposition this is possible). Therefore

$$\pi c_{1T} = y$$

and so $w_1 = c_{1T}$, which implies

$$(1 - \pi) c_{1N} = \alpha A z$$

On the other hand we have that $R u'(c_{2T}) = R(\phi_2 + \psi_2) = \phi_0 = (u'(c_{1T}) - \psi_1) p_1 A = (u'(c_{2T}) - \psi_2) A^2 p_2$ so

$$A^2 p_2 \psi_2 = (A^2 p_2 - R) u'(c_{2T})$$

Hence for any $p_2 > \frac{R}{A^2}$ the multiplier ψ_2 is strictly positive. Recall also that

$$v'(c_{2N}) = p_2 (u'(c_{2T}) - \psi_2) = p_2 \phi_2$$

since $p_2 > \frac{R}{A^2} > 0$ then $\phi_2 > 0$. Therefore we both have that

$$R x - d = (1 - \pi) w_2 = (1 - \pi) c_{2T}$$

and therefore $c_{2T} = w_2$, which implies that

$$(1 - \pi) c_{2N} = (1 - \alpha) A^2 z$$

Therefore under the assumptions of $p_1 > \frac{1}{A}$, $p_2 > \frac{R}{A^2}$ and $\eta_t = 0$ all the feasibility conditions hold. Now we need to check under which conditions $\eta_t = 0$. This is the same as having

$$p_t \leq \frac{\lambda_t c_{tT}}{c_{tN}}$$

for $t = 1, 2$. If this price satisfies this then λ_t can be any number between 0 and 1. Without loss of generality assume that $\lambda_t = 1$. Therefore we must have that, to implement the efficient allocation as an equilibrium here, we need that date 1 and 2 prices satisfy

$$\frac{1}{A} < p_1 < \frac{c_{1T}^*}{c_{1N}^*}; \quad \frac{R}{A^2} < p_2 < \frac{c_{2T}^*}{c_{2N}^*}$$

Given that the inequalities in the statement of the proposition hold, then there exists a pair of prices p_1, p_2 satisfying these two inequalities (in fact the price system is not unique). This ends the proof. ■

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