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D.T.: N° 110

Marzo, 2012

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Optimal nondiscriminatory auctions with favoritism¹

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March 2012

Abstract

In many auction settings, there is favoritism: the seller's welfare depends positively on the utility of a subset of potential bidders. However, laws or regulations may not allow the seller to discriminate among bidders. We find the optimal nondiscriminatory auction in a private value, single-unit model under favoritism. At the optimal auction there is a reserve price, or an entry fee, which is decreasing in the proportion of preferred bidders and in the intensity of the preference. Otherwise, the highest-valuation bidder wins. We show that, at least under some conditions, imposing a no-discrimination constraint raises expected seller revenue.

Keywords: auctions, favoritism, nondiscriminatory mechanisms.

JEL classification: C72, D44.

¹We are grateful to Daniel Aromí, Juan Dubra, Peter Klibanoff, and to participants at several seminars, for their comments on an earlier version of this paper. Arozamena and Weinschelbaum acknowledge financial support from FONCyT, PICT 2255.

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1 Introduction

It is frequently the case when auctions are used that the seller is not indifferent as to which of the bidders will be the winner. A seller runs an auction to enhance competition among bidders but, for a given selling price, she would prefer some of the bidders to win rather than others. This may occur when some of the bidders' welfare positively influences the seller's welfare. For example, in a government-run auction, domestic firms may generate more tax revenue than their foreign rivals. Alternatively, the seller and some of the bidders may be firms in the same conglomerate. We say that there is favoritism when the seller has such a preference for some bidders over others.

Favoritism usually motivates the design of discriminatory auctions.⁵ Since the bidders' identities are relevant to the seller, auction rules are specified in such a way that not only bids matter, but also who makes them. For example, price preferences are frequently introduced: to win, a non-preferred bidder may have to beat the highest bid made by a preferred bidder by at least some previously specified margin. Another usual way to discriminate, known as *right of first refusal*, awards one of the preferred bidders the right to match the highest bid that any of her rivals may submit.⁶

However, in many situations discrimination is not possible. This happens quite often in public procurement, where laws and regulations sometimes forbid favoring some bidders over others to level the field and thus to foster competition. There may be higher-level regulations that explicitly prevent local authorities from favoring local firms.⁷ In general, this constraint may be interpreted as one imposed by a principal on an agent who is in charge of the auction.⁸

⁵The justification for discrimination that we examine, which derives from the fact that the seller values some of the bidders' utilities, is not the only possible one. With a fixed number of bidders, biasing the auction against strong bidders raises revenue, as shown by optimal auction theory. With endogenous entry, discriminating against strong bidders may also be optimal for the seller, since it could encourage the entry of weak bidders. Those arguments require asymmetry among bidders, while our model is (but for the possibly unequal consideration of bidders' utilities by the seller) symmetric.

⁶This right has been studied in Walker (1999), Bikhchandani et al. (2005), Arozamena and Weinschelbaum (2006), Lee (2008), Burguet and Perry (2009), and Choi (2009).

⁷For example, see Maasland et al. (2004) for a discussion on whether discriminating among bidders may be viewed as state aid -and thus be prohibited- under European Union rules or not.

⁸Violating this constraint can result in unwanted scrutiny and other costs for auction designers. See, for example, Hu (2010) for criticism of the Texas Lottery Commission for perceived conflict of interest in its organizing the procurement auction for Texas' state lottery vendors. A more complete analysis would embed the auction design problem within a principal-agent model, but here we are simply concerned with the effects

Our aim here is to examine the auction design problem faced by a seller who places positive weight on some of the bidders' welfare,⁹ but faces a no-discrimination constraint. We conclude that the seller will choose an auction where the highest-valuation bidder wins unless her valuation is too low. Thus, there will be a reserve price or an entry fee adequately chosen to exclude bidders with lower valuations, just as in the standard, revenue-maximizing auction. However, we find that the set of valuations excluded is smaller when there is favoritism: there will be a lower reserve price or entry fee. Furthermore, the set of excluded valuations becomes smaller when the weight attached to the utility of any favored bidder grows. Hence, favoritism raises the probability of selling the object.

One possible justification for ruling out discriminatory auctions may be that doing so raises expected revenue. Indeed, we find that, in some cases, the optimal mechanism that the seller would select under favoritism when she *cannot* discriminate among bidders generates more revenue than the mechanism she would use if discrimination were allowed.

Our work contributes to the literature on favoritism in auctions. Laffont and Tirole (1991) and Vagstad (1995) study the case of multidimensional auctions, where favoritism may appear when the auctioneer assesses product quality. McAfee and McMillan (1989), Branco (1994), and Naegelen and Mougeot (1998) examine single-dimensional auctions, where price-preferences may be used. The main result of this literature is that the optimal allocation rule follows from comparing the maximum valuation among preferred bidders with the maximum “virtual” valuation among non-preferred bidders. In all these papers, the seller is allowed to use discriminatory mechanisms. Arozamena and Weinschelbaum (2011) extend the analysis of the single-dimensional case to a situation where the number of bidders is endogenous, and conclude that the optimal auction in that setting is indeed nondiscriminatory. Thus, not being allowed to discriminate is irrelevant when entry is endogenous. Here, however, we examine the case where the number of bidders is fixed.

In the following section we present the model and characterize the optimal mechanism for a seller that may discriminate among bidders. We derive the optimal mechanism under a no-discrimination constraint in section 3. In section 4, we examine the effect of that constraint on expected revenue. We conclude in section 5.

of a no-discrimination constraint on auction design under seller favoritism among bidders.

⁹In Section 3, we also analyze the case where the seller's welfare rises by a fixed value if certain bidders win.

2 The model

The owner of a single, indivisible object is selling it through an auction.¹⁰ For simplicity, we assume the seller attaches no value to the object. There are $N \geq 2$ bidders whose valuations for the object are given by v_i , $i = 1, \dots, N$. Each v_i is bidder i 's private information. These valuations are distributed identically and independently according to the c.d.f. F with support on an interval that is normalized to $[\underline{v}, \bar{v}]$ and a density f that is positive and bounded on the whole support. The context is therefore one of symmetric independent private values. All parties to the auction are risk-neutral, and we assume that the virtual valuation of any bidder, $J(v) = v - \frac{1-F(v)}{f(v)}$, is increasing in her actual valuation.¹¹

Our aim is to characterize a selling mechanism that maximizes the utility of a seller who values positively the welfare of a subset of the set of bidders, in addition to her own expected revenue. Specifically, we assume that the seller's objective function follows from adding to the seller's revenue each bidder's welfare, where bidder i 's welfare is weighted according to an exogenous parameter α_i , $i = 1, \dots, N$. We assume as well that $\alpha_i \in [0, 1]$ for all i . That is, the seller attaches a weakly positive weight to each bidder's welfare, but cannot value the latter more than her own, "private" utility (i.e., her revenue). Note that if $\alpha_i = 0$ for all i , we have a standard, revenue-maximizing seller. However, if $\alpha_i = 1$ for all i , the seller will behave as she would when pursuing efficiency in the absence of favoritism.

In this specific context, as we mentioned above, we intend to characterize the optimal mechanism for a seller that cannot discriminate among bidders. However, the best choice for a seller who is allowed to treat different bidders in an asymmetric way will be useful to us as a reference point later on. Thus, we describe here the optimal mechanism when discrimination is indeed possible.

As described so far, our problem is a slight modification of the standard optimal auction problem with independent private values.¹² Let $H_i(v_1, \dots, v_N)$ ($P_i(v_1, \dots, v_N)$) be the probability that bidder i gets the object (respectively, the expected price bidder i has to pay to the seller) if bidder valuations are given by $(v_1, \dots, v_N)$. Then, the seller has to choose a mechanism $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ such that, for all $(v_1, \dots, v_N)$, $0 \leq H_i(v_1, \dots, v_N) \leq 1$ for all i and $\sum_{i=1}^N H_i(v_1, \dots, v_N) \leq 1$. In addition, let $h_i(v_i)$ ($p_i(v_i)$) be the expected probability that bidder i gets the object (respectively, the expected price she pays) when her valuation is v_i , and the

¹⁰All of our results, however, are applicable as well to the case of procurement auctions.

¹¹This is what the literature calls the "regular case" after Myerson (1981).

¹²See Myerson (1981) and Riley and Samuelson (1981).

valuations of all other bidders are unknown.

Bidder i 's expected utility when her valuation is v_i and she announces that it is v'_i is

$$\tilde{U}_i(v_i, v'_i) = h_i(v'_i)v_i - p_i(v'_i).$$

Additionally, let

$$U_i(v_i) = \tilde{U}_i(v_i, v_i) = h_i(v_i)v_i - p_i(v_i).$$

Then, the seller's problem is

$$\max_{\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N} \sum_{i=1}^N \left(\int_{\underline{v}}^{\bar{v}} p_i(v_i) f(v_i) dv_i + \alpha_i \int_{\underline{v}}^{\bar{v}} U_i(v_i) f(v_i) dv_i \right) \quad (1)$$

subject to the standard incentive compatibility and participation constraints

$$U_i(v_i) \geq \tilde{U}_i(v_i, v'_i) \quad \text{for all } i, \text{ for all } v_i, v'_i \quad (2)$$

$$U_i(v_i) \geq 0 \quad \text{for all } i, \text{ for all } v_i. \quad (3)$$

A slightly different version of this problem has been studied before, for instance, in Naegelen and Mougeot (1998).¹³ Since we will use it later on as a reference point, let us characterize the solution. We follow the usual steps in the literature.

Let $\tilde{v}_i(v_i)$ be the valuation that bidder i announces optimally when her true valuation is v_i . Clearly, by incentive compatibility, it has to be true that $\tilde{v}_i(v_i) = v_i$ and $U_i(v_i) = \tilde{U}_i(v_i, \tilde{v}_i(v_i))$. The envelope theorem then implies that

$$U'_i(v_i) = \frac{\partial}{\partial v_i} \tilde{U}_i(v_i, \tilde{v}_i(v_i)) = h_i(v_i).$$

Therefore, $U_i(v_i) = \int_{\underline{v}}^{v_i} h_i(s) ds + U_i(\underline{v})$. Stated in a way that is more convenient for us in what follows, and noting that, in the solution to our problem, $U_i(\underline{v}) = 0$ for all i ,¹⁴ we have

$$p_i(v_i) = h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s) ds$$

¹³This problem can be thought of as an extension to the N -bidder context of a particular case of the analysis in Naegelen and Mougeot (1998), when there is no consumer surplus and the shadow cost of public funds is zero.

¹⁴ $U_i(\underline{v})$ may be zero or positive for those bidders i with $\alpha_i = 1$. Given that we are adding the expected utilities of the seller and these bidders, how much they pay (as long as incentive compatibility holds) does not affect the seller's objective function. However, by no-discrimination, $U_i(v_i) = U(v_i)$ for all i , so $U(\underline{v}) > 0$ is only possible if $\alpha_i = 1$ for all i . But even in this case there exists a solution where $U_i(\underline{v}) = 0$ for all i .

for all i . Substituting for $p_i(v_i)$ and $U_i(v_i)$ in the seller's objective function yields

$$\sum_{i=1}^N \left[\int_{\underline{v}}^{\bar{v}} \left[h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s)ds \right] f(v_i)dv_i + \alpha_i \int_{\underline{v}}^{\bar{v}} \int_{\underline{v}}^{v_i} h_i(s)ds f(v_i)dv_i \right].$$

Integrating by parts, we have

$$\sum_{i=1}^N \int_{\underline{v}}^{\bar{v}} h(v_i) \left[v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] f(v_i)dv_i.$$

Then, the seller solves

$$\max_{\{H_i(\cdot)\}_{i=1}^N} E_{v_1, \dots, v_N} \left[\sum_{i=1}^N H_i(v_1, \dots, v_N) \left[v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \right] \right]. \quad (4)$$

In the absence of favoritism, the seller would maximize the expected value of the winner's virtual valuation. Here, virtual valuations are adjusted to account for favoritism. Only a fraction $(1 - \alpha_i)$ of bidder i 's rent is subtracted from that bidder's actual valuation in the seller's objective function. In an extreme case, if $\alpha_i = 1$ —i.e., when bidder i 's utility is as important to the seller as her own revenue—it is bidder i 's actual valuation that enters into the seller's objective function.

The solution to this problem is straightforward, and is described in the following proposition.

Proposition 1 *If discrimination is allowed, the optimal allocation rule for the seller is¹⁵*

$$H_i^D(v_1, \dots, v_N) = \begin{cases} 1 & \text{if } i = \arg \max_j v_j - (1 - \alpha_j) \frac{1 - F(v_j)}{f(v_j)} \\ & \text{and } v_i - (1 - \alpha_i) \frac{1 - F(v_i)}{f(v_i)} \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

The seller thus defines individual minimum acceptable valuations r_i that solve

$$r_i - (1 - \alpha_i) \frac{1 - F(r_i)}{f(r_i)} = 0 \quad (5)$$

$i = 1, \dots, N$. In order to be awarded the object, bidder i 's valuation must be larger than r_i , and her adjusted virtual valuation must be higher than all other bidders'.

¹⁵If there is a tie, the object may be allocated randomly among the bidders who have the highest adjusted virtual valuation.

3 No discrimination

As mentioned above, we are interested in the case where the seller *cannot* discriminate among bidders. Hence, we add a new constraint on the set of mechanisms $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ that the seller can select.

No-discrimination constraint: *The seller has to choose a mechanism $\{H_i(\cdot), P_i(\cdot)\}_{i=1}^N$ that, for any permutation $\pi : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$, satisfies*

$$\begin{aligned} H_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= H_{\pi(i)}(v_1, \dots, v_N) \\ P_i(v_{\pi(1)}, \dots, v_{\pi(N)}) &= P_{\pi(i)}(v_1, \dots, v_N) \end{aligned}$$

for all i .

Remark 1 *The no-discrimination constraint may be viewed as one imposed on the specific indirect mechanism that the seller will use. Here, we present it as a constraint on the direct mechanisms that we are examining under the revelation principle. We may do that given that we are in a symmetric context, as described above: a nondiscriminatory auction with a symmetric equilibrium implements a nondiscriminatory direct revelation mechanism. If the distributions of valuations were not symmetric, though, clearly we would not necessarily be able to associate a nondiscriminatory auction with a symmetric direct mechanism.*

The seller's problem, then, is to maximize (1) subject to the incentive compatibility and participation constraints described in (2) and (3), and subject to the no-discrimination constraint. Following the steps described in the previous section, we can incorporate the incentive compatibility and participation constraints into the objective function and describe the seller's problem as that of maximizing (4) subject to the no-discrimination constraint.

The fact that the seller cannot discriminate among bidders, however, allows us to restate this problem in a more convenient way.

Remark 2 *Let $v_{(n)}$ be the n th order statistic associated with the vector of valuations $(v_1, \dots, v_N)$.¹⁶ For any vector $\mathbf{v} = (v_1, \dots, v_N)$, we may define a function $\gamma_{\mathbf{v}} : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$ that assigns to each position $1, \dots, N$ the identity of the bidder whose valuation ranks in that position.*

¹⁶We follow the convention by which $v_{(1)}$ is the highest value in the vector, $v_{(2)}$ the second-highest, and so on.

That is, $\gamma_{\mathbf{v}}(n) = i$ if $v_{(n)} = v_i$.¹⁷ The no-discrimination constraint implies that for any two vectors $\mathbf{v} = (v_1, \dots, v_N)$, $\mathbf{v}' = (v'_1, \dots, v'_N)$ such that $(v_{(1)}, \dots, v_{(N)}) = (v'_{(1)}, \dots, v'_{(N)})$ we must have

$$H_{\gamma_{\mathbf{v}}(n)}(v_1, \dots, v_N) = H_{\gamma_{\mathbf{v}'}(n)}(v'_1, \dots, v'_N), \quad n = 1, \dots, N.$$

To show this, assume $H_{\gamma_{\mathbf{v}}(n^*)}(v_1, \dots, v_N) \neq H_{\gamma_{\mathbf{v}'}(n^*)}(v'_1, \dots, v'_N)$ for some n^* . Let $\tilde{\pi} : \{1, \dots, N\} \longrightarrow \{1, \dots, N\}$ be such that $\tilde{\pi}(i) = \gamma_{\mathbf{v}}(\gamma_{\mathbf{v}'}^{-1}(i))$. The functions $\gamma_{\mathbf{v}}$ and $\gamma_{\mathbf{v}'}$ are permutations, so $\tilde{\pi}$ is a permutation as well. Note that $(v'_1, \dots, v'_N) = (v_{\tilde{\pi}(1)}, \dots, v_{\tilde{\pi}(N)})$. Then,

$$H_{\tilde{\pi}(i^*)}(v_1, \dots, v_N) \neq H_{i^*}(v_{\tilde{\pi}(1)}, \dots, v_{\tilde{\pi}(N)})$$

for $i^* = \gamma_{\mathbf{v}'}(n^*)$, which violates the no-discrimination constraint.

Thus, if for any two vectors of valuations the corresponding vectors of order statistics coincide, then the seller has to allocate the good with the same probability to those bidders that occupy each ordered position in the vectors of order statistics. In other words, the probability that any given bidder wins must depend only on the vector of order statistics and on her valuation's position in that vector. This, in turn, implies the following lemma.

Lemma 1 *The seller's problem can be expressed in terms of order statistics: she has to choose an allocation function*

$$\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N.$$

That is, all valuation vectors that generate the same vector of order statistics have to be treated equally. Then, we can focus only on which allocations the seller chooses when the vector of valuations is ordered. Allocations in all other cases follow from the no-discrimination constraint.

The seller, though, cares about the identities of the bidders. Given a vector of order statistics $(v_{(1)}, \dots, v_{(N)})$, since valuations are independently drawn from the same distribution, the probability that bidder i 's valuation ranks in position n is the same for all bidders. Then, for that vector of order statistics, the seller's objective function will take the following expected value

$$\sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[\sum_{i=1}^N \left[v_{(n)} - (1 - \alpha_i) \frac{1 - F(v_{(n)})}{f(v_{(n)})} \right] \right]$$

¹⁷Since we are using continuous distributions, ties will occur with probability zero. Still, in the event of a tie, positions have to be allocated with equal probabilities among those bidders whose valuations coincide so that the no-discrimination constraint is satisfied.

or,

$$\sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[Nv_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i) \right].$$

The seller's problem is then

$$\max_{\{H_n(v_{(1)}, \dots, v_{(N)})\}_{n=1}^N} E_{v_{(1)}, \dots, v_{(N)}} \left\{ \sum_{n=1}^N \frac{1}{N} H_n(v_{(1)}, \dots, v_{(N)}) \left[Nv_{(n)} - \frac{1 - F(v_{(n)})}{f(v_{(n)})} \sum_{i=1}^N (1 - \alpha_i) \right] \right\}.$$

The solution to this problem is simple. Since $J(v) = v - \frac{1-F(v)}{f(v)}$ is increasing in v , it is easy to show that $Nv - \frac{1-F(v)}{f(v)} \sum_{i=1}^N (1 - \alpha_i)$ is also increasing in v . Thus, the seller should allocate the object with probability 1 to the bidder with the highest valuation whenever $Nv_{(1)} - \frac{1-F(v_{(1)})}{f(v_{(1)})} \sum_{i=1}^N (1 - \alpha_i) > 0$. Otherwise, she should keep the object. We therefore have the following result.

Proposition 2 *The optimal allocation rule when discrimination is not allowed is¹⁸*

$$H_i^{ND}(v_1, \dots, v_N) = \begin{cases} 1 & \text{if } v_i > \max_{j \neq i} v_j \text{ and } Nv_i - \frac{1-F(v_i)}{f(v_i)} \sum_{j=1}^N (1 - \alpha_j) > 0 \\ 0 & \text{otherwise.} \end{cases}$$

This direct mechanism can be implemented by any auction where the highest-valuation bidder wins, with an adequately chosen reserve price or entry fee. For example, the seller may choose a first-price or a second-price auction with reserve price r such that

$$Nr - \frac{1 - F(r)}{f(r)} \sum_{i=1}^N (1 - \alpha_i) = 0.^{19} \quad (6)$$

Note that if $\alpha_i = 0$ for all i , the optimal mechanism for the seller described in Proposition 2 coincides with the standard, revenue-maximizing direct mechanism: the object is awarded to the highest-valuation bidder and all bidders with valuations below r such that $r - \frac{1-F(r)}{f(r)} = 0$ are excluded. At the same time, if $\alpha_i = 1$ for all i , then $r = 0$, no bidders are excluded and the seller chooses an auction where the highest-valuation bidder always wins.

Therefore, for any vector of weights $(\alpha_1, \dots, \alpha_N)$ that the seller attaches to the bidders' utilities, she chooses a mechanism that neither sells the good with probability one nor attains

¹⁸In order to satisfy the no-discrimination constraint, as we mentioned above, if there is a tie all bidders with the highest valuation win with the same probability.

¹⁹As the left-hand side of this equation is increasing in r , there is a unique solution.

revenue maximization. Furthermore, she selects a mechanism that falls in between these two extreme cases.²⁰

Example 1 *Consider the case of two bidders with valuations drawn from the uniform distribution on the unit interval with seller favoritism for bidder 1 given by $\alpha_1 = 1/2$ and no favoritism for bidder 2, so that $\alpha_2 = 0$. It is well-known that the standard, revenue-maximizing reserve price in this case is $r = 1/2$. Taking into consideration the seller's favoritism toward bidder 1 and her inability to discriminate between the bidders, the optimal mechanism's minimum acceptable valuation is $r = 3/7$. Finally, the reserve price that ensures that the good is sold with probability one is $r = 0$.*

Note as well that, as long as $\alpha_i > 0$ for some i (i.e., there is favoritism), the optimal mechanism's minimum acceptable valuation may increase when the number of bidders changes. That valuation depends on $(1/N) \sum_{i=1}^N \alpha_i$. Then, if a new bidder enters the auction with a weight in the seller's objective function that is higher (lower) than the average weight that the seller attaches to the existing bidders' utilities, the optimal mechanism's minimum acceptable valuation will fall (respectively, rise). This differs from the standard, revenue-maximizing reserve price, which is independent of N .

It is also interesting to examine, given N , the effect of a change in the vector of weights $(\alpha_1, \dots, \alpha_N)$ on the mechanism selected by the seller and on the welfare of each of the parties involved in the auction. First, notice that r , the minimum valuation that is not excluded from the mechanism, is decreasing in α_i for any i . If the seller places a larger weight on a given bidder's welfare, the only instrument she has to enhance that bidder's welfare is to reduce the reserve price or entry fee that she employs in any auction that implements the optimal mechanism. Doing so benefits not only the bidder whose corresponding weight has risen, but all other bidders as well. Therefore, all bidders' expected utilities are increasing in any α_i .

It is not the case, though, that the seller is "sharing" her gains from having a higher α_i . There are actually two effects. First, for any given value of r , the seller's utility straightforwardly grows with α_i . Second, the seller increases her utility by reducing r . This second effect raises the utilities of all bidders, too.

²⁰When the number of bidders is endogenous, as Arozamena and Weinschelbaum (2011) show, even with favoritism it is optimal for the seller to use a nondiscriminatory mechanism which maximizes revenue and at the same time sells the good with probability one.

Remark 3 *Our results also hold if we model favoritism in a different way. Assume, for example, that the seller attaches a fixed value w_i to bidder i winning the auction ($i = 1, \dots, N$). Then, the seller's objective function is given by*

$$\sum_{i=1}^N \left(\int_{\underline{v}}^{\bar{v}} p_i(v_i) f(v_i) dv_i + w_i \int_{\underline{v}}^{\bar{v}} h_i(v_i) f(v_i) dv_i \right).$$

In this case, it can be shown that, under a no-discrimination constraint, the bidder with the highest valuation wins and the optimal minimum acceptable valuation solves

$$N \left(r - \frac{1 - F(r)}{f(r)} \right) + \sum_{i=1}^N w_i = 0.$$

We could then find how that valuation changes with N and with each w_i . Our results would be analogous to the ones that follow when the seller cares about bidders' profits.

4 Revenue

In our setup, one justification for imposing the use of nondiscriminatory mechanisms may be that allowing preferential treatment to some bidders could reduce revenue. In the two previous sections, we have characterized the optimal mechanisms for the seller both with and without discrimination. We examine in what follows how those mechanisms compare in terms of expected revenue. As is well known, expected revenue is given by

$$E_{v_1, \dots, v_N} \left[\sum_{i=1}^N H_i(v_1, \dots, v_N) J(v_i) \right].$$

For any given weights $(\alpha_1, \dots, \alpha_N)$, let $R^D(\alpha_1, \dots, \alpha_N)$ ($R^{ND}(\alpha_1, \dots, \alpha_N)$) be the expected revenue generated by mechanism $(H_1^D(\cdot), \dots, H_N^D(\cdot))$ (respectively, $(H_1^{ND}(\cdot), \dots, H_N^{ND}(\cdot))$).

Figure 1 below helps visualize how discrimination influences revenue for the case where $N = 2$. Assume $\alpha_1 > \alpha_2$.

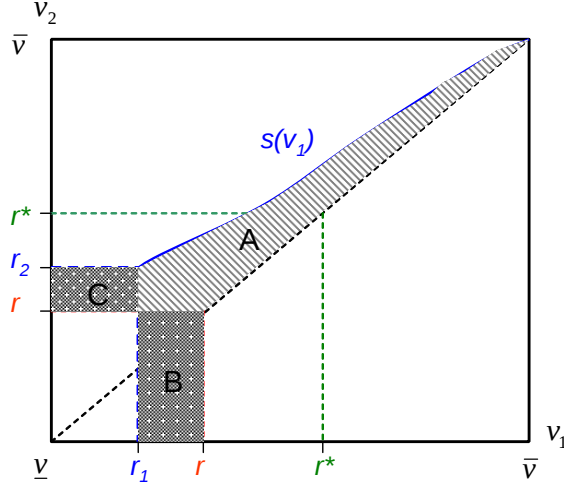


Figure 1

With discrimination, the seller sets individual minimum valuations r_1, r_2 as defined in (5). In the figure, $s(v_1)$ is the value of v_2 such that the adjusted virtual valuations of both bidders coincide, i.e.,

$$s(v_1) - (1 - \alpha_2) \frac{1 - F(s(v_1))}{f(s(v_1))} = v_1 - (1 - \alpha_1) \frac{1 - F(v_1)}{f(v_1)},$$

as long as $v_1 > r_1$. If $v_1 > r_1$ and $v_2 > r_2$, then she awards the object to bidder 1 if $v_2 < s(v_1)$, and to bidder 2 otherwise –clearly, if only one bidder’s valuation is above her own minimum acceptable level, that bidder is the winner. When discrimination is forbidden, the seller sets a uniform minimum valuation r defined in (6), and awards the object to the bidder with the highest valuation as long as it is larger than r . Finally, r^* is the minimum valuation that would be optimally selected in the absence of favoritism, defined in the usual way: $J(r^*) = 0$.

To examine the effect of the no-discrimination constraint on revenue, we focus on the pairs (v_1, v_2) where the allocation rule differs in the optimal discriminatory and nondiscriminatory mechanisms under favoritism. Those pairs are depicted in the figure in regions A, B and C. For (v_1, v_2) in region A, the object is awarded to a bidder under both mechanisms. With discrimination, bidder 1 wins, since her adjusted virtual valuation is higher. Without discrimination, bidder 2 wins, since her actual valuation is higher. Since $J(v)$ grows with v , in region A revenue is larger when discrimination is not allowed. In region B, the seller awards the object to bidder 1 when she can discriminate. Once she cannot discriminate the seller keeps the object, since $v_1, v_2 < r$. Given that $J(v_1) < 0$ in region B (as $v_1 < r^*$), forbidding discrimination raises revenue. Finally, in region C the object remains unsold with discrimination, but is awarded to

bidder 2 when the no-discrimination constraint applies. Since $J(v_2) < 0$ in region C, revenue is larger with discrimination.

These opposing effects apply as well when $N > 2$. When the object is sold in both mechanisms, under $(H_1^{ND}(\cdot), \dots, H_N^{ND}(\cdot))$ the object is allocated to the bidder with the highest actual and virtual valuation: revenue can only improve when discrimination is forbidden. Since under favoritism the object is awarded in cases where the winner has a negative virtual valuation, whenever the no-discrimination constraint causes the object to remain unsold it generates a positive effect on revenue. However, the new constraint also makes the seller allocate the object to a bidder in cases where she would not if she were allowed to treat bidders differently.

Can we ascertain whether there is a positive or negative net effect on revenue? In some cases we can.²¹ If $\alpha_i = 1$ for some i , then $r_i = \underline{v}$: the seller never keeps the object when discrimination is allowed. In Figure 1, this means that region C vanishes, and so do the cases where the no-discrimination constraint could reduce revenue. We then have the following proposition, that applies for any number of bidders and any distribution of valuations.

Proposition 3 *If $\alpha_i = 1$ for some i , then $R^{ND}(\alpha_1, \dots, \alpha_N) \geq R^D(\alpha_1, \dots, \alpha_N)$, and the inequality is strict unless $\alpha_1 = \alpha_2 = \dots = \alpha_N$.*

When $\alpha_i < 1$ for all i , we can provide a result along the same lines for the two-bidder case and under some conditions on the distribution of valuations. These conditions are satisfied, for example, by power-function distributions, $F(v) = v^k$, with $k \geq 1$.

Proposition 4 *Assume $N = 2$ and the density $f(\cdot)$ is differentiable. Then, if $F(\cdot)$ has a monotone increasing hazard rate and $J(\cdot)$ is convex, then $R^{ND}(\alpha_1, \alpha_2) \geq R^D(\alpha_1, \alpha_2)$, and the inequality is strict if $\alpha_1 \neq \alpha_2$.*

Proof. Let $\bar{\alpha} = \alpha_1 + \alpha_2$, and assume without loss of generality that $\alpha_1 \geq \alpha_2$ –if $\alpha_1 = \alpha_2$ we know that forbidding discrimination has no effect on the allocation rule. Recall that, while the optimal mechanism with discrimination depends on individual weights, the optimal mechanism without discrimination only depends on $\bar{\alpha}$, and both mechanisms coincide only when $\alpha_1 = \alpha_2$. We will then show that if we could choose α_1 and α_2 so as to maximize expected revenue keeping $\bar{\alpha}$ constant, we would select $\alpha_1 = \alpha_2 = \bar{\alpha}/2$. This means that expected revenue without discrimination is larger for any possible values of α_1, α_2 , and for any $\bar{\alpha}$.

²¹We have not been able to find an example where the no-discrimination constraint *reduces* expected revenue.

For a given pair (α_1, α_2) , we define an allocation rule that will be useful as a reference point.

Let

$$\hat{H}_1(v_1, v_2) = \begin{cases} 1 & \text{if } v_1 > v_2 \text{ and } v_1 \geq r_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{H}_2(v_1, v_2) = \begin{cases} 1 & \text{if } v_2 > v_1 \text{ and } v_2 \geq r_2 \\ & \text{or } r_2 > v_2 > v_1 \geq r_1 \\ 0 & \text{otherwise} \end{cases}$$

where r_i is defined as in (5), $i = 1, 2$. This allocation rule combines the optimal mechanisms with and without discrimination, as shown in Figure 2, where $\alpha_1 > \alpha_2$.

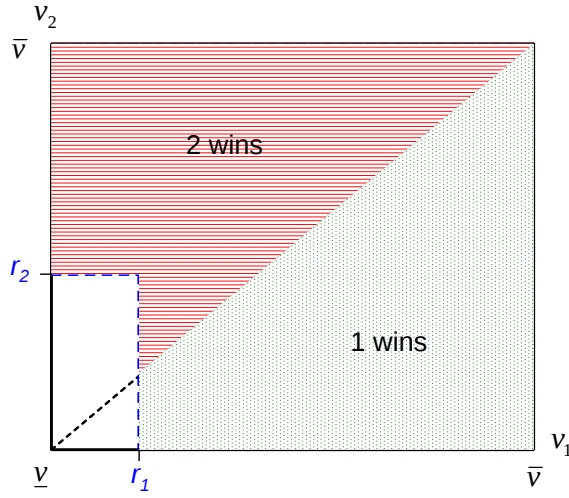


Figure 2

The seller keeps the object in the same cases as in the optimal discriminatory mechanism (namely, when $v_1 < r_1$ and $v_2 < r_2$). However, when the object is allocated to a bidder, the bidder with the highest valuation wins. Let

$$\hat{R}(\alpha_1, \alpha_2) = E_{v_1, v_2} \left[\sum_{i=1}^2 \hat{H}_i(v_1, v_2) J(v_i) \right].$$

Clearly, we have $R^D(\alpha_1, \alpha_2) \leq \hat{R}(\alpha_1, \alpha_2)$, and the inequality is strict if $\alpha_1 > \alpha_2$ —recall that $(\hat{H}_1(\cdot), \hat{H}_2(\cdot))$ allocates the object to a bidder in the exact same cases as $(H_1^D(\cdot), H_2^D(\cdot))$, but the former does so efficiently while the latter does not. Furthermore, $(H_1^D(\cdot), H_2^D(\cdot)) = (\hat{H}_1(\cdot), \hat{H}_2(\cdot)) = (H_1^{ND}(\cdot), H_2^{ND}(\cdot))$ if $\alpha_1 = \alpha_2 = \bar{\alpha}/2$, so $R^D(\bar{\alpha}/2, \bar{\alpha}/2) = \hat{R}(\bar{\alpha}/2, \bar{\alpha}/2) = R^{ND}(\bar{\alpha}/2, \bar{\alpha}/2)$. We will now prove that, if $\alpha_1 \geq \alpha_2$, $\hat{R}(\alpha_1, \alpha_2)$ falls weakly if α_1 grows

while keeping $\bar{\alpha}$ constant, and it does so strictly if $\alpha_1 > \alpha_2$. This means that $R^D(\alpha_1, \alpha_2) < \hat{R}(\alpha_1, \alpha_2) < \hat{R}(\bar{\alpha}/2, \bar{\alpha}/2) = R^{ND}(\bar{\alpha}/2, \bar{\alpha}/2) = R^{ND}(\alpha_1, \alpha_2)$ if $\alpha_1 > \alpha_2$.

We can write

$$\begin{aligned} \hat{R}(\alpha_1, \alpha_2) = & \int_{\underline{v}}^{r_1} \int_{r_2}^{\bar{v}} J(v_2) f(v_2) f(v_1) dv_2 dv_1 + \\ & \int_{r_1}^{\bar{v}} \left[\int_{\underline{v}}^{v_1} J(v_1) f(v_2) dv_2 + \int_{v_1}^{\bar{v}} J(v_2) f(v_2) dv_2 \right] f(v_1) dv_1 \end{aligned}$$

Then, we differentiate this expression with respect to α_1 keeping $\alpha_1 + \alpha_2$ constant, and after a few steps of algebra we get

$$\begin{aligned} \left. \frac{d\hat{R}(\alpha_1, \alpha_2)}{d\alpha_1} \right|_{\alpha_1 + \alpha_2 = \bar{\alpha}} = & -f(r_1) \frac{dr_1}{d\alpha_1} \int_{r_1}^{r_2} J(v_2) f(v_2) dv_2 \\ & - F(r_1) \left[J(r_2) f(r_2) \frac{dr_2}{d\alpha_2} \frac{d\alpha_2}{d\alpha_1} \right|_{\alpha_1 + \alpha_2 = \bar{\alpha}} + J(r_1) f(r_1) \frac{dr_1}{d\alpha_1} \right] \end{aligned} \quad (7)$$

From (5), we have

$$\frac{dr_i}{d\alpha_i} = - \frac{\frac{1-F(r_i)}{f(r_i)}}{1 - (1 - \alpha_i) \left(\frac{1-F(r_i)}{f(r_i)} \right)'} < 0$$

Since $J(v) < 0$ for $v \in (r_1, r_2)$, the first term on the right-hand side of (7) is strictly negative if $\alpha_1 > \alpha_2$, and zero if $\alpha_1 = \alpha_2$ (since $r_1 = r_2$ in that case). Then, a sufficient condition for the derivative to be nonpositive is

$$J(r_2) f(r_2) \frac{dr_2}{d\alpha_2} \frac{d\alpha_2}{d\alpha_1} \Big|_{\alpha_1 + \alpha_2 = \bar{\alpha}} + J(r_1) f(r_1) \frac{dr_1}{d\alpha_1} \geq 0 \quad (8)$$

The second term on the left-hand side of (8) is strictly positive. If $\alpha_1 > \alpha_2 = 0$, $J(r_2) = 0$, the first term vanishes and the sufficient condition is automatically satisfied. Note that

$$\frac{dr_2}{d\alpha_2} \frac{d\alpha_2}{d\alpha_1} \Big|_{\alpha_1 + \alpha_2 = \bar{\alpha}} = - \frac{dr_2}{d\alpha_2} > 0$$

This implies that, if $\alpha_2 > 0$, the first term on the left-hand side of (8) is negative. However, since $r^* > r_2 > (=) r_1$ and $J(\cdot)$ is increasing, $|J(r_1)| > (=) |J(r_2)|$ if $\alpha_1 > (=) \alpha_2$. Furthermore, the monotone-hazard-rate condition (by which $(\frac{1-F(r_i)}{f(r_i)})' < 0$, $i = 1, 2$) and the fact that $J(\cdot)$ is convex (in other words, that $(\frac{1-F(r_i)}{f(r_i)})'$ is decreasing), yield $0 > (\frac{1-F(r_1)}{f(r_1)})' > (=) (\frac{1-F(r_2)}{f(r_2)})'$ if $\alpha_1 > (=) \alpha_2$. Therefore,

$$\left| f(r_2) \frac{dr_2}{d\alpha_2} \right| = \left| \frac{1 - F(r_2)}{1 - (1 - \alpha_2) \left(\frac{1-F(r_2)}{f(r_2)} \right)'} \right| < (=) \left| \frac{1 - F(r_1)}{1 - (1 - \alpha_1) \left(\frac{1-F(r_1)}{f(r_1)} \right)'} \right| = \left| f(r_1) \frac{dr_1}{d\alpha_1} \right|$$

if $\alpha_1 > (=)\alpha_2$. We conclude then that (8) is satisfied, and that $\left. \frac{d\hat{R}(\alpha_1, \alpha_2)}{d\alpha_1} \right|_{\alpha_1 + \alpha_2 = \bar{\alpha}} < (=)0$ if $\alpha_1 > (=)\alpha_2$. Thus, $R^D(\alpha_1, \alpha_2) < R^{ND}(\alpha_1, \alpha_2)$ whenever $\alpha_1 > \alpha_2$. ■

In our setting, at least in the cases described in Propositions 2 and 3, there is a revenue-based justification for a principal to prevent an agent in charge of running an auction from discriminating among bidders when that agent's preferences exhibit favoritism.

5 Conclusion

We have examined the problem faced by a seller that cannot discriminate among bidders despite her preferences exhibiting favoritism among bidders. We have characterized the optimal mechanism, and concluded that, at least in some cases, it yields higher expected revenue than the optimal mechanism when discrimination is allowed.

We have carried out our examination of this issue in a symmetric context. It would be interesting to analyze the effect of a no-discrimination constraint with bidder asymmetries, both with and without favoritism. That problem, though, is technically harder and we leave it for future research.

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