Heterogeneous Ability, Career Choice and Firm Size*

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ABSTRACT. Entrepreneurial ability is the ability to innovate new products. Managerial ability is the ability to maintain the profitability of current operations. By assuming heterogeneous abilities of acquiring and maintaining an endogenous number of production processes, the model of this paper predicts a distribution of firm sizes, a diversity in the composition of R&D and chosen career. Firms conduct product improvement if and only if managerial ability is high relative to entrepreneurial ability. Individuals choose careers as either innovative entrepreneurs, managerial entrepreneurs or salaried employees depending on their abilities. An individual's entrepreneurial ability may not be high enough to choose a career as an innovative entrepreneur, but if managerial ability is sufficiently high, then a career as a managerial entrepreneur is optimal. Managerial ability has an effect on expected firm size if and only if the individual is a managerial entrepreneur.

1. Introduction

The entrepreneur has worn many hats in economic theory, such as innovator and source of creative destruction (Schumpeter, 1934), bearer of uncertainty (Knight, 1921), manager and coordinator of production (Clark, 1992) and arbitrageur (Walras, 1954; Kirzner, 1979).[1] However, the role of the entrepreneur has been ignored or missing entirely from economic theory for a number of years (Baumol, 1968; Barretto, 1989). The entrepreneur has recently re-emerged in economic theory as coordinator of production (Alchien and Demsetz, 1972; and Lucas, 1978), as risk bearer (Kihlstrom and Laffont, 1979), and as arbitrageur (Fisher, 1981). The entrepreneur has also been investigated in empirical studies (Evans and Leighton, 1989; Acs and Audretsch, 1990a and 1990b; Amel and Froeb, 1991).

The analysis presented below applies the results of Gifford and Wilson (1991) and Gifford (1992a) to the question of the optimal firm size and composition of innovative effort for entrepreneurs with limited and heterogeneous abilities in both acquiring projects and maintaining them after adoption. As in Sah and Stiglitz (1985), entrepreneurs are limited in their ability to make the right decisions. However, in the analysis below, entrepreneurs are also limited in another respect. As in Oi (1983) and Holmes and Schmitz (1990), the decisions which an entrepreneur makes, however imperfect, require the entrepreneur's attention for some period of time. This limits the number of decisions which can be made during any period of time. However, in this model attention must be allocated among an *endogenous* number of problems to be considered.[2] Thus, the model below is an effort to "encompass the determinants of the *allocation* of entrepreneurship among its competing uses" (Baumol, 1990, p. 918). The optimal allocation of entrepreneurial attention between the maintenance of current operations and the innovation of new projects determines the optimal size of the firm, its rate of growth and the composition of its innovative efforts.

The optimal allocation of attention will depend on the entrepreneurial and managerial abilities of the decision maker. Entrepreneurial ability is defined as the ability to recognize a new profit opportunity and to acquire resources, including financing, to begin the exploitation of the opportunity. Managerial ability is defined as the ability

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to maintain the profitability of current operations, that is, to manage projects after they have been adopted. Without any assumption of risk aversion or liquidity constraints, the endowments of entrepreneurial and managerial abilities determine the career choices of individuals between becoming an innovative entrepreneur who conducts only new product innovation, a managerial entrepreneur who conducts both new product innovation and product improvement, or a salaried employee.

The optimal allocation of attention of a self-employed entrepreneur takes two possible forms, an innovation rule and a managerial rule. The innovation rule prescribes that current operations are never evaluated and improved but are discontinued once profits fall to zero. The entrepreneur allocates all attention to innovating new products and conducts no product improvement. In this case, the rate of innovation depends only on the level of technological opportunity. The managerial rule prescribes that each current project is evaluated periodically for possible improvement and a new project is evaluated if no current project requires evaluation. In this case the rate of new product innovation decreases with firm size and increases in the bound on firm size, which is increasing in monopoly profits. In addition, the rate of product improvement increase with firm size. The managerial rule is optimal if the probability of successful product improvements is large relative to the probability of new product innovation.

The results for the choice of careers by individuals includes the rather mundane conclusion that those who have high entrepreneurial ability become innovative entrepreneurs rather than wage earners. However, there is a third alternative career. Those individuals whose entrepreneurial abilities are not high enough to justify a career as an innovative entrepreneur will choose to become managerial entrepreneurs if their managerial skills are high enough. A managerial entrepreneur allocates some effort to maintaining the profitability of current products through product improvement rather than always introducing new products. Thus a managerial entrepreneur may acquire only a few projects (perhaps only one) and the maintenance of these projects takes up most of the entrepreneur's time. The managerial entrepreneur will innovate a new product only if not evaluating and improving a current product line. In addition, the results imply that if entrepreneurial ability is sufficiently high, there is no incentive to increase managerial ability through, say, an MBA degree, unless managerial ability is increased sufficiently to make the entrepreneur become a managerial, rather than an innovative, entrepreneur. An individual with low entrepreneurial and managerial abilities becomes a salaried employee.

In addition to predicting career choices, the model also predicts a diversity of the composition of R&D effort. The composition of R&D comes from the fact that innovating new products carries an opportunity cost embodied in the neglect of current operations. Firms with high entrepreneurial ability conduct only new product innovation, independent of monopoly profits and firm size. Firms with high managerial ability, facing the same technological opportunity and entry of competition, conduct both new product innovation and process innovation (or product improvement). For these latter firms, the new product innovation will be increasing in monopoly profits and decreasing in firm size and process innovation will increase with firm size. In addition, for these firms new product innovation and process innovation are inversely related. The model also predicts that firm size is bounded, due to the limits of the fixed input of entrepreneurial attention. Thus the model formalizes the argument of Kaldor (1934) which attributes the limits of the growth of the firm to the fact that the entrepreneur is a fixed input in production.

Finally, this model incorporates a causal process of innovation in the reverse direction from the Schumpeterian hypothesis. As Dosi (1988) states: "Big firms happen to grow big because they innovated successfully in the past and continue to do so in the present without, however, finding a differential advantage in 'bigness' as such" (p. 1153). Dosi also argues that it is "different degrees of *tacitness* of the knowledge underlying innovative success" which helps explain why individuals with high entrepreneurial abilities will innovate successfully and acquire large firms which remain highly innovative. In other words, entrepreneurial ability is not something that can be sold or perhaps even taught.

Others have also analyzed the effects of limited attention and differing abilities. Inspired by

Kaldor, Oi (1983) investigates how differences in limited entrepreneurial and managerial abilities determine the supply of entrepreneurs. However, Oi ignores the question of optimal growth and does not allow the coordination activities which increase output to also increase the demands on the entrepreneur's attention. The coordinating function of the entrepreneur does not increase, or even affect, the number of activities which the entrepreneur has to supervise. In addition, the allocation of attention is exogenous, given the number of employees and hiring new employees requires no attention. Therefore, the allocation of attention has no affect on firm size or growth except to imply monitoring costs to hiring additional employees. The analysis below rectifies this by modeling the decision process of choosing projects to adopt and then maintaining them after their implementation. Attention is required not only to monitor current operations but also to adopt new projects. Since the allocation of attention is endogenous, the size of the firm, measured by the number of projects, is also endogenous and depends explicitly on the optimal allocation of attention between monitoring current projects and acquiring new ones.

Without specifying just what it is that managers do, Lucas (1978) analyzes the implications of heterogeneous and limited managerial talent for the distribution of firm sizes. Assuming diminishing returns to the span of control of management, Lucas derives the optimal firm size for any particular level of managerial ability. However, Lucas does not address the question of how a manager acquires the optimal size firm. Similarly, Calvo and Wellizc (1978) and Williamson (1967) derive the optimal firm size due to shirking on the part of supervised employees. In the analysis below, the acquisition of projects and firm growth are modeled explicitly. The assumption of limited attention implies a limited span of control rather than assuming it. The maintenance of current operations introduces a cost of growth embodied in the neglect of current operations while adopting new projects. This cost implies that, if managerial ability if sufficiently high, then firm size is bounded below the optimal firm size under the assumptions of these other models.

Another model which considers limited attention and heterogeneous entrepreneurial ability is found in Holmes and Schmitz (1990). Their primary concern is the determination of business transfers. Their main results, the specialization in entrepreneurial activities and business transfers, are driven by the assumption of continual technological progress. However, this model does not address the question of the choice between self-employment and salaried employment; all individuals are self-employed. Nor does it allow an individual to maintain more than one project at a time or to maintain a current project while innovating a new one. In addition, no product improvement is possible to restore the profitability of a deteriorating current product. Therefore, this model does not address the questions of firm size or the choice between new product innovation and current product improvement.

One puzzling aspect of Holmes and Schmitz (1990) is the definition of entrepreneurship. The authors state that "entrepreneurs are those individuals who respond to the opportunities for creating new products (and the like) that arise because of technological breakthroughs" (p. 266). This would not be troubling if these opportunities were due to spillovers from technological breakthroughs of other firms' R&D efforts. However, Holmes and Schmitz also assume that "the developer of a new business can fully capture the benefits associated with the new product" (p. 267). Therefore, there are no spillovers. That raises the question of who, if not the entrepreneurs, creates the technological breakthroughs which drive the model.

The characterization of entrepreneurs as those who respond to profit opportunities is similar to the view in Kirzner (1979). Kirzner is concerned with the alertness of the entrepreneur to profit opportunities. The interesting question is what affects this entrepreneurial alertness. The analysis below models the alertness of the entrepreneur by showing that the amount of attention which the entrepreneur pays to evaluating potential profit opportunities depends on the opportunity cost of that attention. Thus, if an entrepreneur with high managerial ability has current operations whose profitability can therefore be maintained with high probability, then the opportunity cost of alertness to new opportunities is high. This opportunity cost of alertness may be large enough to divert the entrepreneur's attention away from perceiving new profit opportunities as they arise. Thus, entrepreneurs with high managerial ability will be less alert to new profit opportunities than those with low managerial ability.

The next section presents the model of allocating limited entrepreneurial attention among an endogenous number of projects. The optimal policy and its implications, derived in Gifford and Wilson (1991) and Gifford (1992a), are described. In Section 3, the optimal choice between a career as an innovative entrepreneur, a managerial entrepreneur or a salaried worker is derived. Concluding remarks follow.

2. The model

Following Kaldor (1934), a firm is defined as a "production combination possessing a given unit of coordinating ability" (p. 69). The model of Gifford and Wilson (1991) and Gifford (1992a) is adapted here to the analysis of the effect of entrepreneurial and managerial abilities on the optimal career choice. Entrepreneurial ability is measured by the probability of an entrepreneur correctly perceiving a profit opportunity in introducing a new product. Denote this probability for entrepreneur x by ε_x. If a new project is evaluated, then with probability ρ_0 it is a profitable endeavor and with probability ε_x, entrepreneur x will correctly perceive this. With probability $1 - \varepsilon_x$, the entrepreneur will mistakenly perceive the project to be a bad one. With probability $1 - \rho_0$ the new project is bad and with probability 1 the entrepreneur correctly perceives this.[3] Thus an evaluated new project will be good and adopted by entrepreneur x with probability $\rho_0\varepsilon_x$. The evaluated new project will be discarded with probability $1 - \rho_0\varepsilon_x$.

Managerial ability is defined as the ability to maintain current projects by evaluating and possibly improving them. A current project is either functioning, in which case it earns positive profits $g > 0$ and cannot be improved, or it is failed and generates losses $b > 0$. If a current project fails, then it can be restored to a profit making state only if evaluated by the entrepreneur. The probability that a failed current project can be restored upon evaluation by entrepreneur x is denoted by ρ_x, which indicates managerial ability.

A current project is denoted by the number of periods since it was last evaluated by the entrepreneur. Call this the "age" of the project. Let the sequence $a \equiv \{a_i\}_{i \in Z_+}$ denote the projects currently held by a firm, where $a_i = 1$ if there is a current project of age i and $a_i = 0$ otherwise and $a \in A$, the set of all such sequences. A project of age i is functioning with probability γ^i, where $(1 - \gamma) = \phi$ is the one period probability of failure. Therefore the current state of the firms is completely described by a. The current returns to a project of age i are $r_i \equiv \gamma^i g - (1 - \gamma^i)b$. Denote the number of current projects by $N(a) = \Sigma_{i=1}^{\infty} a_i$.

Time is discrete with a discount rate β, $0 < \beta < 1$, and at the beginning of any period the entrepreneur may evaluate a new project for possible adoption or evaluation a current project for possible improvement. If a current project i is evaluated, it is functioning in the next period with probability $\rho_{xi} = \gamma^i + (1 - \gamma^i)\rho_x$. If a new project is evaluated, it is adopted with probability $\rho_{x0} \equiv \rho_0\varepsilon_x$. Let $u \in Z$ denote the evaluation decision, where $u = 0$ if a new project is evaluated and $u = i > 0$ if current project i is evaluated. Note that $u = i$ implies $a_i = 1$.

Each period the entrepreneur may also freely discard any number of current projects. Let $q \equiv \{q_i\}_{i \in Z_+}$ denote the retained projects, where $q \in A$, $q_i \leqslant a_i$ and $q_u = 1$, for $u > 0$. Define the shift operator $S{:}A \rightarrow A$ such that $(Sq)_i = q_{i-1}$ and $(Sq)_1 = 0$. Let q_{-j} denote the sequence q' with $q'_i = q$ and $q_j = 0$. Thus q_{-j} denotes the deletion of project j from the sequence q of retained projects. Define $a + a' = \{\max(a_i, a'_i)\}_{i \in Z_+}$. Let e_1 denote the sequence with a one in the first position and zeros elsewhere. Then $e_1 + a$ is the addition of a project of age one to the sequence a of projects.

The current returns to the firm after discard in state q are $R(q) = \Sigma_{i \in Z_+} r_i q_i$. The transition of the state takes place after a project, new or current, has been evaluated and adopted or restored, current projects have been discarded or retained, retained projects have aged by one period and all current projects fail with probability ϕ. Thus the state in the next period is $e_1 + Sq_{-u}$ with probability ρ_{xu} and is Sq_{-u} with probability $1 - \rho_{xu}$. The optimality equation for entrepreneur x is

$$F_x(a) = \max_{q,u}\{R(q) + \beta[\rho_{xu}F_x(e_1 + Sq_{-u}) + (1 - \rho_{xu})F_x(Sq_{-u})]\}.$$

If R(q) is bounded then there exists a stationary optimal policy $\lambda_x(a)$ for entrepreneur x which prescribes the pair $\{u_{x\lambda}(a), q_{x\lambda}(a)\}$ of evaluation and discard rules.[4]

The optimal allocation of entrepreneurial attention for $\varepsilon_x = 1$ and $\rho_x = \rho$ is derived in Gifford and Wilson (1991). The model has been extended here to allow for the heterogeneity of entrepreneurial and managerial ability in order to analyze the optimal career choice based on these abilities. The optimal policy for heterogeneous entrepreneurs is a straight forward extension of the case of $\rho_{x0} = \rho_0$ and $\rho_{xi} = \rho_i$. The intuition of the optimal policy is that either current projects are never evaluated or they are evaluated according to some criteria. Since projects differ only by their ages, the criterion for discarding a project unevaluated is the lowest age at which current expected profits are negative. The criterion for evaluating a current project is the age at which the value of evaluating the current project is greater than the value of waiting another period to evaluate the project. Thus, if current projects are evaluated at all, then each current project is evaluated periodically.

To describe the optimal policy, let V_i denote the discounted expected returns to a project of age one which is discarded unevaluated at age i. Then $V_i = \Sigma_{v=1}^{i}\beta^{v-1}r_v$. Let d denote the value of i which maximizes V_i. Then d is the largest integer i for which $r_i \geqslant 0$. Note that r_i, V_i and d are independent of x. Let W_{xi} denote the incremental returns to entrepreneur x from a current project of age one which is evaluated every i periods instead of evaluating a new project. Then $W_{wi} = \Sigma_{s=0}^{\infty}\beta^{si}(\rho_{xi} - \rho_{x0})^s V_i = V_i/[1 - \beta^i(\rho_{xi} - \rho_{x0})]$. Let c_x denote the value of i which maximizes W_{xi}. Let A^s denote the set of all sequences $\{a_i\}_{i \in Z_+}$ such that $a_i = 0$ for i $> s$. To simplify notation, let $W_{xc} \equiv W_{xc_x}$. Then the optimal policy is as follows:

1) if $V_d > W_{xc}$, then follow an innovation rule for which, for all $a \in A^{d+1}$, $u(a) = 0$ and $q(a) = a_{-(d+1)}$;
2) if $V_d \leqslant W_{xc}$, then follow a managerial rule for which, for $c = c_x$ and all $a \in A^c$,

$$u(a) = \begin{cases} c_x & \text{if } a_c = 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$q(a) = a.$$

If the firm starts at period zero with no projects, then under the innovation rule, the state never leaves A^{d+1}. Similarly, under the managerial rule, the state never leaves A^c. In fact, this policy is fully defined for every attainable state under the policy if the firm begins in A^s.

The following statements summarize the implications, derived in Gifford (1992a), of the optimal policy for entrepreneur x.

For the innovative entrepreneur:

1) There is a ρ_{x0} large enough, for any $\rho_x < 1$, such that the innovation rule is optimal.
2) Firm size is bounded by $d < \infty$, where d is the smallest integer satisfying $r_{d+1} < 0$.
3) The rate of innovation is equal to $\varepsilon_x\rho_0$.
4) The rate of growth of the firm decreases with firm size.

For the managerial entrepreneur:

1) There is a ρ_x large enough such that the managerial rule is optimal.
2) Firm size is bounded by $c_x < \infty$, where c_x is the smallest integer c satisfying $r_{c+1} + \beta W_{cx}(\rho_{c+1,x} - \rho_{0x}) < W_{cx}(\rho_{cx} - \rho_{0x})$.
3) If the managerial rule is optimal then $\rho_{xc} > \rho_{x0}$ and $c_x < d$.
4) c_x is non-decreasing in ρ_{x0}, $(1 - \rho_x)$ and g/b.
5) The rate of innovation decreases with firm size and increases with monopoly profits.
6) The rate of growth decreases in firm size and with the age of the firm for young firms.
7) For any $a \in A^c$, there are unattainable states $a' \in A^d$ for which $N(a') > N(a)$ and $F_x(a') > F_x(a)$.
8) If $\rho_x = 1$, then $V_d < W_{xc}$ and firm size approaches the steady state $a^c \in A^c$, in which there are c_x projects of ages 1 to c_x, and $q_x(a^c) = a^c$ and $u_x(a^c) = c_x$.

These results may be interpreted as follows. For sufficiently high entrepreneurial ability (and technological opportunity) relative to managerial ability, the innovative rule is optimal and firm size is bounded by d, where d is the largest integer for which current expected returns are non-negative. Therefore the bound on firm size is increasing in monopoly profits, g/b, and decreasing in the frequency of failure, ϕ. The rate of new product innovation is constant and equal to $\varepsilon_x\rho_0$, the product of entrepreneurial ability, ε_x, and technological opportunity, ρ_0, and is independent of firm size and monopoly profits. The firm conducts no product improvement or process innovation.

For sufficiently high managerial ability relative to entrepreneurial ability, the managerial rule is optimal and firm size is bounded by $c_x < d$, where c_x implies that the value of keeping a project of age c_x unevaluated for one more period and then evaluating it at age $c_x + 1$ and every c_x periods thereafter is less than the value of evaluating the project now at age c_x. This bound on firm size is increasing (non-decreasing) in entrepreneurial ability, ε_x, technological opportunity, ρ_0, and monopoly, g/b, and decreasing in managerial ability, ρ_x. This last implication is due to the fact that high managerial ability implies a high opportunity cost of acquiring another project. The rate of new product innovation is less than $\varepsilon_x\rho_0$ and decreases with firm size. Since the bound on firm size, c_x, increases in monopoly profits, g/b, the rate of innovation increases in monopoly profits. The rate of product improvement is inversely related to the rate of new product innovation. Therefore, the rate of product improvement is increasing in firm size and decreasing in monopoly profits.

The last two implications of the managerial rule are perhaps the most interesting. If managerial ability is large relative to entrepreneurial ability, then firm size is bounded away from a larger firm size that would be more profitable if it could be attained costlessly. For example, a set of d current projects of ages 1 to d would have greater expected value than any set of projects of ages less than or equal to c_x, since $r_i \geqslant 0$ for $i \leqslant d$. However, the cost of attaining these additional projects embodied in the neglect of projects already acquired prevents these states from being optimal. Since $c_x < d$, the greatest age at which a project's expected returns are greater than or equal to zero, current projects are evaluated before their expected returns are zero. If managerial ability is equal to one, then there is a steady state of c projects, one of each age 1 to c_x, and project c_x is evaluated and restored with certainty every period. New product innovation ceases and the firm conducts only product improvement.

3. Optimal career choice

We next address the issue of the optimal choice of employment between being an innovative entrepreneur, a managerial entrepreneur or a salaried employee. Whether or not an individual x becomes an entrepreneur, that is, self-employed, in which case one of the two rules would be optimal, or works for wages depends on the expected profit as an entrepreneur. Assume that either job is full-time; that is, an individual may not divide her time between a salaried job and operating as an entrepreneur. The expected profit to an entrepreneur is the expected return to the firm. The expected profit of an innovative entrepreneur x starting with no projects is $\pi_{xd} \equiv \beta\rho_{x0}V_d/(1-\beta)$. Note that, in this case, the expected profit is increasing in technological opportunity, entrepreneurial ability and monopoly profits but is independent of managerial ability. The expected profit of a managerial entrepreneur x starting with no projects is $\pi_{xc} \equiv \beta\rho_{x0}W_{xc}/(1-\beta)$. In this case, expected profit depends not only on technological opportunity, entrepreneurial ability and monopoly profits but also on managerial ability. To determine the expected discounted lifetime earnings as a salaried employee, assume that the marginal revenue product from hiring an employee is constant and equal to ω.[5] Then, the expected life-time earnings of a salaried employee are $\omega/(1-\beta)$. Thus for individual x, max $\{\pi_{xd}, \pi_{xc}\}$ determines the reservation wage of entrepreneur x when considering working as a salaried employee.

To determine the optimal career, we will first consider the choice between a career as an innovative entrepreneur and a career as a salaried employee. Let $h(\varepsilon_x)$ denote the value of managerial ability ρ_x which makes the expected returns to being an innovative entrepreneur with entrepreneurial ability ε_x equal to the discounted returns from being a salaried employee. Then working as a salaried employee improves on being an innovative entrepreneur if and only if $\varepsilon_x < \omega/\beta\rho_0V_d$. This implies a fixed cut off value of ε_x below which the individual chooses to become a salaried employee over being an innovative entrepreneur, independent of managerial ability. Therefore, the graph of $h(\varepsilon_x)$ is a vertical line at $\omega/\beta\rho_0V_d$, as depicted in Figure 1.

The analysis of the two other pairs of career choices is a bit more complicated. To choose between the careers of managerial entrepreneur and innovative entrepreneur, the individual com-

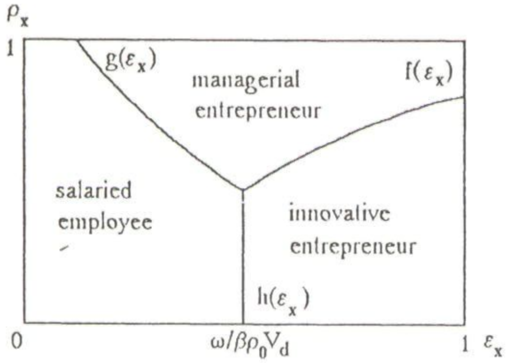

Fig. 1.

pares W_{xc} to V_d. To determine the effects of entrepreneurial and managerial ability on this choice requires the determination of the effects of ε_x and ρ_x on W_{xc}. Similarly, to evaluate the choice between a salaried employee and a managerial entrepreneur the individual compares ω to $\beta\varepsilon_x\rho_0 W_{xc}$. To determine the effects of entrepreneurial and managerial ability on this choice requires the determination of the effects of ε_x and ρ_x on $\beta\varepsilon_x\rho_0 W_{xc}$. However, in both of these comparisons, c_x may change as ε_x and ρ_x vary. Therefore, let $c(\varepsilon_x, \rho_x)$ denote the value of c_x for different values of entrepreneurial and managerial ability. By definition, c_x is the largest integer i satisfying

$$\frac{r_i}{V_{i-1}} \geqslant \frac{(\rho_{x,i-1} - \beta\rho_{xi}) - (1-\beta)\rho_{x0}}{1 - \beta^{i-1}[\rho_{x,1-1} - \rho_{x0}]}. \quad (3.1)$$

The first lemma establishes that it does not matter how $c(\varepsilon_x, \rho_x)$ changes in response to simultaneous changes in ε_x and ρ_x in order to establish the effects of these changes on W_{xc} and $\varepsilon_x\rho_0 W_{xc}$.

Lemma 1: For any c which satisfies (3.1), a) W_{xc} is decreasing in ε_x and increasing in ρ_x and b) $\varepsilon_x\rho_0 W_{xc}$ is increasing in both ε_x and ρ_x.

Proof: For any $i > 0$ and ε_x and ρ_x, let $W_{xi}(\varepsilon_x, \rho_x) \equiv V_i/[1 - \beta^i\rho_{xi} + \beta^i\varepsilon_x\rho_0]$. Then,

$$\frac{\partial W_{xi}}{\partial \varepsilon_x} = -\frac{\beta^i\rho_0 W_{xi}}{1 - \beta^i[\rho_{xi} - \rho_{x0}]} < 0. \quad (3.2)$$

Therefore, for any $i > 0$, $W_{xi}(\varepsilon_x, \rho_x)$ is decreasing in ε_x. Since $c(\varepsilon_x, \rho_x)$ is the value of i which maximizes $W_{xi}(\varepsilon_x, \rho_x)$, and an increase in ε_x can only decrease $W_{xi}(\varepsilon_x, \rho_x)$, for all i, it follows that for $\varepsilon'_x > \varepsilon_x$ and $c' = c(\varepsilon'_x, \rho_x)$, the largest integer satisfying (3.1) for ε'_x and ρ_x, $W_{xc}(\varepsilon'_x, \rho_x) < W_{xc}(\varepsilon_x, \rho_x) \leqslant W_{xc}(\varepsilon_x, \rho_x)$. That is, the new maximum return after an increase in ε_x cannot be greater than the previous maximum return, whether c increases of decreases.

Next note that

$$\frac{\partial W_{xi}(\varepsilon_x, \rho_x)}{\partial \rho_x} = \frac{\beta^i(1-\gamma^i)W_{xi}}{1 - \beta^i[\rho_{xi} - \rho_{x0}]} > 0. \quad (3.3)$$

Therefore, for any $i > 0$, an increase in ρ_x increases $W_{xi}(\varepsilon_x, \rho_x)$. For $\rho''_x > \rho_x$ and $c''(\varepsilon_x, \rho''_x)$, $W_{xc''}(\varepsilon_x, \rho''_x) \geqslant W_{xc}(\varepsilon_x, \rho''_x) > W_{xc}(\varepsilon_x, \rho_x)$. That is, an increase in ρ_x can only increase $W_{xi}(\varepsilon_x, \rho_x)$, for all i. Therefore, the maximum return after an increase in ρ_x can only be larger than the previous maximum return, whether $c(\varepsilon_x, \rho_x)$ increases or decreases. This establishes a).

To establish b), it is sufficient to establish that

$$\frac{\partial \varepsilon_x\rho_0 W_{xi}(\varepsilon_x, \rho_x)}{\partial \varepsilon_x} = \frac{\rho_0[1 - \beta^i\varepsilon_x]}{1 - \beta^i(\rho_{xi} - \rho_{x0})} W_{wi}(\varepsilon_x, \rho_x) > 0 \quad (3.4)$$

and that $\partial\varepsilon_x\rho_0 W_{xi}/\partial\rho_x > 0$. An argument similar to that for a) proves b). Q.E.D.

With the result of Lemma 1 we can proceed to evaluate the two other career choice pairs. Working as a salaried employee improves upon being a managerial entrepreneur if and only if $\omega > \beta\varepsilon_x\rho_0 W_{xc}$ which depends on ρ_x as well as ε_x. Let $g(\varepsilon_x)$ denote the value of ρ_x which maintains the equality $\omega = \beta\varepsilon_x\rho_0 W_{xc}$. That is, $g(\varepsilon_x)$ is the level of managerial ability for any level of entrepreneurial ability which makes the individual indifferent between being a managerial entrepreneur and being a salaried employee. The next lemma establishes that this critical level of managerial ability is decreasing in the level of entrepreneurial ability and this level managerial ability is equal to one for some level of entrepreneurial ability greater than zero.

Lemma 2: For the function $g(\varepsilon_x)$ defined above, a) $dg/d\varepsilon_x \leqslant 0$, b) $g(\varepsilon_x) = 1$ for $0 < \varepsilon_x < \omega(1-\beta^c)/\beta\rho_0[V_c - \beta^{c-1}\omega]$.

Proof: a) follows directly from Lemma 1 since an increase in ε_x increases $\beta\varepsilon_x\rho_0 W_{xc}$ and so requires a decrease in ρ_x to deep π_{xc} unchanged. More formally, substituting $g(\varepsilon_x)$ for ρ_x in W_{xi} and setting $\partial\beta\varepsilon_x\rho_0 W_{xi}/\partial\varepsilon_x = 0$ results in

$$\frac{dg(\varepsilon_x)}{d\varepsilon_x} = -\left[\frac{1-\beta^i\rho_{xi}}{\beta^i\varepsilon_x(1-\gamma^i)}\right] < 0.$$

The value of ε_x which gives $g(\varepsilon_x) = 1$ is derived by noting that if $\varepsilon_x = 0$, then no projects can be acquired by a managerial entrepreneur and so $\beta\varepsilon_x\rho_0 W_{xi}(0, \rho_x) = 0$ and being a salaried employee is preferred for some $\varepsilon_x > 0$. In fact, assuming $\rho_x = g(\varepsilon_x) = 1$ and solving $\omega = \beta\varepsilon_x\rho_0 W_{xc}$ for ε_x, we have that $\varepsilon_x = \omega(1-\beta^c)/\beta\rho_0[V_c - \omega\beta^{c-1}]$. This value of ε_x is less than or equal to zero only if $V_c \leqslant \beta^{c-1}\omega$, which contradicts $\beta^{c-1}\omega = \beta^c\varepsilon_x\rho_0 W_{xc} < W_c$. This establishes b). Q.E.D.

The value of ε_x which makes $g(\varepsilon_x) = 0$ is not derived as it will prove to be irrelevant to the choice of career. However, $g'' > 0$. The characteristics of $g(\varepsilon_x)$ are depicted in Figure 1. For pairs of (ε_x, ρ_x) above (below) this line, the expected profits from being a managerial entrepreneur are greater (less) than the discounted lifetime wage $\omega/(1-\beta)$.

The last step in determining the optimal career is to characterize all pairs (ε_x, ρ_x) of entrepreneurial and managerial ability for which the individual is indifferent between a career as an innovative entrepreneur and a career as a managerial entrepreneur. This choice is determined by the pairs of (ε_x, ρ_x) which maintains $V_d = W_{xc}$. Let $\rho_x = f(\varepsilon_x)$ define the mapping from entrepreneurial abilities to managerial abilities which maintains $V_d = W_{xc}$.

Lemma 3: For the function $f(\varepsilon_x)$ defined above, a) $df(\varepsilon_x)/d\varepsilon_x > 0$ and b) $f(1) < 1$.

Proof: V_d is independent of ε_x and ρ_x. From Lemma 1 we have that W_{xc} is decreasing in ε_x and increasing in ρ_x. Therefore, an increase in ε_x requires an increase in ρ_x to maintain $V_d = W_{xc}$. To see this, substitute $f(\varepsilon_x)$ for ρ_x in $\partial W_{xi}/\partial\varepsilon_x = 0$ and solve for $df(\varepsilon_x)/d\varepsilon_x = \rho_0(1-\gamma^i)$. This establishes a). From Gifford (1992a), we know that, even for $\varepsilon_x = 1$, $\rho_x = 1$ implies that $W_{xc} > V_d$. Therefore, for $\varepsilon_x = 1$, the value of ρ_x which makes $W_{xc} = V_d$ is $f(1) < 1$, which establishes b). Q.E.D.

Figure 1 depicts a positive relationship $\rho_x = f(\varepsilon_x)$ between ρ_x and ε_x which maintains $W_{xc} = V_d$, where $c = c(\varepsilon_x, \rho_x)$. For pairs of (ρ_x, ε_x) below (above) this line, $W_{xc} < (\geqslant) V_d$, and the innovative (managerial) entrepreneur rule is preferred. The value of $f(0)$ is irrelevant. At $\varepsilon_x = 0$, a salaried job is optimal.

The next lemma states that at $\varepsilon_x = \omega/\beta\rho_0 V_d$, and $\rho_x = g(\omega/\beta\rho_0 V_d)$, all three careers have the same discounted value. Therefore, at $\varepsilon_x = \omega/\beta\rho_0 V_d$, $g(\varepsilon_x) = f(\varepsilon_x) = h(\varepsilon_x)$.

Lemma 4: a) If $\varepsilon_x = \omega/\beta\rho_0 V_d$, and $\rho_x = g(\omega/\beta\rho_0 V_d)$, then $\pi_{xc} = \pi_{xd} = \omega/(1-\beta)$. b) $\rho_x = g(\varepsilon_x) = 1$ only if $\varepsilon_x < \omega/\beta\rho_0 V_d$ and $\rho_x = g(\varepsilon_x) = 0$ only if $\varepsilon_x > \omega/\beta\rho_0 V_d$.

Proof: If $\varepsilon_x = \omega/\beta\rho_0 V_d$, we have that $\omega = \beta\varepsilon_x\rho_0 V_d$ and for $\rho_x = g(\omega/\beta\rho_0 V_d)$, we have that $\omega = \beta\varepsilon_x\rho_0 W_{xc}$. Therefore, for $\varepsilon_x = \omega/\beta\rho_0 V_d$ and $\rho_x = g(\omega/\beta\rho_x W_d)$, $\omega = \beta\varepsilon_x\rho_0 V_d = \beta\varepsilon_x\rho_0 W_{xc}$, which establishes a). The negative slope of $g(\varepsilon_x)$ therefore implies that for $\rho_x = g(\varepsilon_x) = 1$, $\varepsilon_x < \omega/\beta\rho_0 V_d$ and for $\rho_x = g(\varepsilon_x) = 0$, $\varepsilon_x > \omega\beta\rho_0 V_d$. Q.E.D.

The following theorem combines the results of the four Lemmas to determine the optimal career choice and expected income.

Theorem 1:
a) If $\varepsilon_x > \omega/\beta\rho_0 V_d$ and $\rho_x < f(\varepsilon_x)$, then $\pi_{xd} > \max\{\omega/(1-\beta), \pi_{xc}\}$. b) If $\varepsilon_x < \omega/\beta\rho_0 V_d$ and $\rho_x < g(\varepsilon_x)$, then $\omega/(1-\beta) > \max\{\pi_{xd}, \pi_{xc}\}$. c) If, for any $\varepsilon_x, \rho_x > \max\{f(\varepsilon_x), g(\varepsilon_x)\}$, then $\pi_{xc} > \max\{\pi_{xd}, \omega/(1-\beta)\}$.

Proof: From Lemma 4, if $\varepsilon_x = \omega/\beta\rho_0 V_d$ and $\rho_x = f(\omega/\beta\rho_0 V_d)$, then $\pi_{xc} = \pi_{xd} = \omega/(1-\beta)$. Therefore, $f(\omega/\beta\rho_0 V_d) = g(\omega/\beta\rho_0 V_d)$. The rest follows directly from the definitions of $g(\varepsilon_x)$ and $f(\varepsilon_x)$. Q.E.D.

Theorem 1 states that, for individuals with sufficiently high entrepreneurial ability ($\varepsilon_x > \omega/\beta\rho_0 V_d$) and sufficiently low managerial ability $\rho_x < f(\varepsilon_x)$), the most profitable career is as an innovative entrepreneur. For individuals with sufficiently low entrepreneurial ability ($\varepsilon_x < \omega/\beta\rho_0 V_d$) and managerial ability ($\rho_x < g(\varepsilon_x)$), the most profitable career is as a salaried employee. For individuals with sufficiently high managerial ability given the entrepreneurial ability ($\rho_x > \max\{f(\varepsilon_x, g(\varepsilon_x)\}$), the most profitable career is as a managerial entrepreneur.

Note that an individual becomes an innovative entrepreneur only if entrepreneurial ability is sufficiently high ($\varepsilon_x > \omega/\rho_0 V_d$). However, even if this is not the case, but managerial ability is sufficiently high ($\rho_x \geqslant g(\varepsilon_x)$) and entrepreneurial ability is not too low ($\varepsilon_x > \omega(1 - \beta_c)/\beta\rho_0(V_c - \beta_{c-1}\omega)$), then that individual will become a managerial entrepreneur but not an innovative entrepreneur. This explains the appearance of small, owner-operated firms which are not considered innovative.

In addition, even if entrepreneurial ability is high ($\varepsilon_x > \omega/\rho_0 V_d$), the individual will choose to become a managerial entrepreneur rather than an innovative entrepreneur if managerial ability is sufficiently high ($\rho_x > g(\varepsilon_x)$). This implies that even entrepreneurs with very high innovative ability may not be very innovative because of the high opportunity cost implied by their high managerial skills. They would be initially highly innovative but as firm size grows they would apply their high managerial talents to maintaining their current operations.

If individuals are uniformly distributed over entrepreneurial and managerial ability then the size of these areas are the proportion of individuals who choose these three careers. Changes in the wage rate, that is the productivity of employed labor, have significant effects on the choice of career. Increased wages of course imply that fewer individuals become self-employed. However, as the next theorem states, if the wage rate increases sufficiently, then all innovative entrepreneurial firms will be eliminated but managerial entrepreneurs will continue to operate.

Theorem 2: As ω increases, the percentage of individuals becoming innovative entrepreneurs decreases and is zero for $\omega \geqslant \beta\rho_0 V_d$ and the percentage of individuals becoming managerial entrepreneurs decreases but is still positive at $\omega = \beta\rho_0 V_d$.

Proof: The vertical line at $\varepsilon_x = \omega/\beta\rho_0 V_d$ shifts to the right as ω increase and at $\omega = \beta\rho_0 V_d$, the expected value of a career as an innovative entrepreneur is no greater than that of a salaried employee, even if $\varepsilon_x = 1$, and the area to the right if this line disappears. As ω increases, assuming $\rho_x = g(\varepsilon_x)$, the value of ε_x which satisfies $\omega = \beta\varepsilon_x\rho_0 W_{xc}$ also increases, since this value of $\varepsilon_x = \omega(1 - \beta^c\rho_{xc})/\beta\rho_0[V_c - \omega\beta^{c-1}]$ is increasing in ω. The negative slope of $g(\varepsilon_x)$ and the result of Lemma 4 imply that $g(1) < 1$, which establishes b). Q.E.D.

Thus, as the productivity of employed labor increases, the number of innovative entrepreneurs falls to zero. The wage rate of $\omega = \beta\rho_0 V_d$ would be paid by the remaining managerial entrepreneurs. Thus, as the opportunity cost of being self-employed increases, fewer individuals become entrepreneurs. However, at a wage high enough to eliminate innovative entrepreneurs, those individuals with sufficiently high entrepreneurial *and* managerial abilities remain managerial entrepreneurs.

These results also imply a diversity of incomes and management training across individuals in the same industry. If firms are in the same industry, so that they face the same technological opportunity and monopoly profits, then expected profits depend only on their individual abilities. For sufficiently high managerial abilities, relative to entrepreneurial ability, so that $V_d \leqslant W_{xc}$, expected profits are increasing in both abilities. For individuals with sufficiently high entrepreneurial ability relative to managerial ability, so that $V_d > W_{xc}$, expected profit is increasing in entrepreneurial ability but independent of managerial ability, as long as it is sufficiently low. The implication is that, for high entrepreneurial ability, it does not pay to increase managerial ability through management training like an MBA program unless it is increased sufficiently to make $V_d \geqslant W_{xc}$, so that the entrepreneur chooses to become a managerial, rather than an innovative, entrepreneur.

Among the managerial entrepreneurs, firm size depends on their entrepreneurial ability. As pointed out in Section 2, the bound on firm size, c, increases within the region of managerial entrepreneurs as entrepreneurial ability, ε_x, increases. Individuals who become managerial entrepreneurs with low entrepreneurial ability will have smaller firms than those managerial entrepreneurs with higher entrepreneurial ability. In addition, the rate of innovation by managerial entrepreneurs will of course be less than that of innovative entrepreneurs, and the rate of innovation by managerial entrepreneurs declines with firm size and increases with monopoly profits. The rate of innovation by innovative entrepreneurs is independent of firm size and monopoly profits.

Finally, it bears repeating that the choice between a career as a managerial entrepreneur and an innovative entrepreneur also affects the composition and R&D conducted by the entrepreneur. A managerial entrepreneur conducts both new product innovation and product improvement. When evaluating a current project, the managerial entrepreneur may improve the profitability of the project through process innovations or product improvements. When not trying to improve a current product, the managerial entrepreneur conducts new product innovation. For given levels of managerial and entrepreneurial abilities, and thus a fixed bound on firm size, the more product lines the firm has, the more frequently the managerial entrepreneur conducts product improvement and the less time there is for new product innovation. However, an increase in monopoly profits increases the bound on firm size. Thus, for a given firm size, the higher the level of monopoly profits, the more new product innovation the firm can undertake. However, for an innovative entrepreneur, the rate of new product innovation is constant and no product improvement takes place.

4. Conclusion

This paper has provided a highly stylized model of the choice between three possible careers: as an innovate entrepreneur, as a managerial entrepreneur or as a salaried employee. The model focuses on the role of limited attention in determining the expected rewards to entrepreneurs. Because of this emphasis, many other characteristics of the innovation and managerial processes are ignored, such as optimal timing and funding of research, the sources of declining profits for neglected projects and the possibility of delegation of more responsibilities to employees. For example, employees could alert the entrepreneur to a declining performance before the project is scheduled to be evaluated. Or employees could be delegated the responsibility of product improvement themselves. This could be captured by a lower failure rate ϕ. Another limitation of the model is that entrepreneurs do not learn in any significant sense about their environment or their abilities. All of these possible extensions of the model seriously complicate the analysis and their treatment will have to wait until these complications can be addressed. However, the model does capture an opportunity cost of new product innovation and firm growth which has not been recognized before. This opportunity cost is embodied in the neglect of current product lines whose profitability could be increased with attention from the entrepreneur.

The analysis above also predicts the choice between three careers depending on the entrepreneurial and managerial abilities of the individual. Explorations of the determinants of the supply of entrepreneurial effort have been very limited. To test the hypotheses concerning the choice of employment generated by the analysis above, one would need some measure of entrepreneurial and managerial ability.[6] Managerial ability has been closely connected to the professionalization of management by Chandler (1977). More work along the lines of Evans and Leighton (1989) would be helpful. For example, they consider the effects of success as a wage earner among the determinants of self-employment. They find that people who switch from wage work to self-employment tend to be those receiving low wages. As they point out, this "disadvantage" effect may the economist's comparative advantage model. This is the effect we see in the analysis above. Those with a low opportunity cost of self-employment, their salaries as wage earners, tend to become entrepreneurs. However, the opportunity cost of becoming an innovative entrepreneur is endogenous. The highest foregone opportunity may not be as a wage earner, but as a managerial entrepreneur. The value of this alternative career depends on both managerial and entrepreneurial ability. In addition, if managerial skills can be acquired, then the value of acquiring these skills should be weighed against not only the alternative wage but also against the value of a career as an innovative entrepreneur.

Notes

1 See Casson (1982), Herbert and Link (1988), Reid and Jacobsen (1988) and Barreto (1989) for more extensive reviews of the role of the entrepreneur in economic theory.

2 The allocation of attention, or time, was first addressed in Becker (1965). See Gifford (1992b) for references to other literature.

[3] The assumption is that the entrepreneur can make only a type 1 error: reject a good project. The possibility of adopting bad new projects would imply eventual learning about whether a project is good or bad, not just whether a good project has failed. This seriously complicates the analysis and will be dealt with in future research.

[4] See Gifford and Wilson (1991).

[5] With a constant marginal productivity of non-entrepreneurial labor and an implied horizontal demand for new products in the product market, the optimal firm size is determined solely by the fixed input of entrepreneurial attention.

[6] See Gifford (1991) for alternative interpretations of the results of this model and their testability and for a discussion of product versus process innovation.

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