Factor Substitution in Small and Large U.S. Manufacturing Establishments: 1977–82*

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ABSTRACT. The objective of this article is to assess the possibility of differences in factor substitution in the production of small and large U.S. manufacturing establishments. For empirical implementation, we specify and estimate separate three-factor translog production factors, using confidential plant-level data for five four-digit industries extracted from the U.S. Bureau of the Census Longitudinal Research Database. Our primary finding is that for most industries under study, small establishments appear to be at least as flexible as large establishments in factor substitution. This finding, together with our previous results on economies of scale (Nguyen and Reznek, 1991), suggests that for the five industries under examination, large size is not a necessary condition for efficient production.

I. Introduction

In recent years, there is a growing body of evidence indicating that small businesses play a significant role in the U.S. economy, and that a large portion of economic growth and change comes from them.[1] Most notably, the Small Business Administration (1987) reviewed recent research and concluded that "small is not necessarily inefficient and that small firms make contributions to overall efficiency" (p. 107). To explain the efficiency of small firms, the Report cited the following factors, among others:

> Many small firms act as market demand "shock absorbers." By employing flexible production technologies (emphasizing the use of labor and less capitalized capital goods), small firms have greater flexibility than large firms in adjusting their relative production levels and are thus better able to accommodate random, short-term fluctuations in demand . . . (pp. 108–109)

A key question, therefore, is whether small firms have greater flexibility in substituting one input for another (in particular labor for capital).[2] Technically, this question is equivalent to asking whether the elasticity of substitution between capital and labor in small firms is greater than in large firms. This question is also relevant for economic policy. For example, an answer of yes supports the argument that policies to promote small business do not necessarily involve sacrificing economic efficiency.

The objective of this paper is to directly test the above hypothesis. To do so, we develop a transcendental logarithmic (translog) model to estimate and compare the substitution elasticities for both small and large production units. An advantage of this model is that it can provide estimates of elasticities of substitution without imposing *a priori* restrictions on the substitution relationship among factor inputs.

Our work differs from others in that previous empirical studies of factor substitution have often been based on published aggregate data. Such aggregate data do not accurately reflect the activities of production units, and results based on these data are subject to aggregation bias.[3] In this paper, we apply microdata at the plant level to our production analysis. These confidential data are extracted from the Census Bureau's Longitudinal Research Database (LRD), which is perhaps the most comprehensive microdata base available for the study of production. It contains data for all manufacturing establishments, both "small" and "large" by any definition.[4]

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We classify the study as preliminary for two reasons: First, we examined only five 4-digit SIC industries. Our experiments with the data led us to select the following five industry groups: (1) SIC 2335: Women's, Misses's, and Juniors' Dresses, (2) SIC 2511: Wood Household Furniture, Except Upholstered, (3) SIC 2711: Newspapers, (4) SIC 3573: Electronic computing Equipment, and (5) SIC 3662: Radio and Television Transmission, Signaling, and Detection Equipment. Second, this study did not fully utilize the LRD, which contains annual panel data beginning in 1972. Instead, for simplicity we used cross section data for only two census years, 1977 and 1982.

We selected these data and industries mainly because we wanted to maximize the number of establishments (including both small and large) so that we could obtain robust parameter estimates of production models and hence robust elasticity estimates of small and large establishments. While the number of industries being studied is limited, and the data employed are far from perfect, use of these microdata can reduce aggregation bias to allow generation of some meaningful results, and more important, provide good direction for future research.

Our primary finding is that for the five industries under study small establishments are no less flexible than large establishments in factor substitution — in fact, some of the evidence suggests they may be more flexible. Further, the small establishments do not fare worse than the large establishments in factor substitution during the 1982 recession — in fact, some of the evidence suggests they fare better. Together with our previous results on economies of scale (Nguyen and Reznek, 1991) this finding suggests that, for the five industries under examination, large size is not a necessary condition for efficient production. The finding is also consistent with the view that since small producers cannot vary their scale of operations as much as large producers can, the small producers need to be able to substitute inputs readily so they can survive, especially over the business cycle or during periods of relatively high fluctuations in demand.

The remainder of the paper is organized as follows: Section II presents the factor substitution model. Section III briefly discusses the data and estimation procedures. The empirical results are discussed in section IV. Section V summarizes and concludes the paper.

II. Measuring factor substitution

Conventionally, the partial Allen elasticity of substitution (AES) and the cross-price elasticity of factor demand (CPE) are used to measure the substitution relationships among inputs in production. The AES between input X_i and X_j is defined as [5]

$$\sigma_{ij}^A = \frac{|\bar{F}_{ij}|}{|\bar{F}|} \frac{\sum_{i=1}^{n} F_i X_i}{X_i X_j} = \sigma_{ij}^A \tag{1}$$

where X_i is the i^{th} input (e.g. labor), F_i is the partial derivative of the production function $Q = F(X_1, X_2, \ldots, X_n)$, with respect to X_i, F_{ij} is the second partial derivative of F with respect to X_i and X_j, and $|\bar{F}|$ is the determinant of the bordered Hessian matrix F, and $|\bar{F}_{ij}|$ is the cofactor of F_{ij} in $|\bar{F}|$. The price elasticities of factor demand are defined as[6]

$$\eta_{ii} = \frac{\partial X_i}{\partial w_i} \cdot \frac{w_i}{X_i} = S_i \sigma_{ii}^A, \tag{2.a}$$

$$\eta_{ij} = \frac{\partial X_i}{\partial w_j} \cdot \frac{w_j}{X_i} = S_j \sigma_{ij}^A, \text{ and} \tag{2.b}$$

$$\eta_{ji} = \frac{\partial X_j}{\partial w_i} \cdot \frac{w_i}{X_j} = S_i \sigma_{ij}^A, \tag{2.c}$$

where η_{ii} and η_{ij} and η_{ji} are the own and cross-price elasticities of factor demand, and w_j and S_j are the price and the cost share of input j.

Before proceeding, some important properties of the above elasticities should be noted,

(i) Both the AES and CPE are one-input-one-price elasticities. That is, both elasticities measure the responsiveness of one input (input i) to changes in the price of another input (input j).

(ii) The cross AESs are symmetric (i.e., $\sigma_{ij}^M = \sigma_{ji}^M$), but the CPEs are generally non-symmetric (i.e., $\eta_{ij} \neq \eta_{ji}$).

(iii) Both the AES and CPE must have the same sign because the AES is simply equal to the CPE divided by a factor share which is always positive. A positive value of σ_{ij}^{A} or η_{ij} indicates that an increase in the price of input j will lead to an increase in the demand for input i, suggesting the inputs i and j are substitutes. Conversely, a negative value of the elasticities indicates that the two inputs are complements.

In comparing the AES and the CPE, it is clear that the AES is inferior to the CPE for two reasons. First, the AES is more restrictive than the CPE [due to property (ii)]. The second reason is that the CPE is an exact measure of the responsiveness of one input to a change in the price of another input, whereas the AES is not. For example, a cross-price elasticity between capital and labor, η_{KL}, having a value of 2 indicates that an increase of one percent in the price of labor will cause an increase of two percent in the demand for capital. In contrast, the AES is simply a disguised CPE. That is the AES is a result of dividing the CPE by a factor share. Thus, the AES is harder to interpret than the CPE.

In spite of the its shortcomings, the AES has been used as a standard statistic reported in empirical studies of factor substitution in production (e.g., see Berndt and Wood, 1975, Magnus 1979, and Solow, 1987). Recently, Blackorby and Russell (1989) formally show that (among other things) the AES does not measure the ease of substitution and provides no new information about factor shares, beyond the information already contained in the price elasticities. More importantly, they show that an alternative, the Morishima elasticity of substitution (MES), is an exact measure of factor substitution (or complementarity) because it is an exact measure of the curvature of the isoquant (or the ease of substitution). The MES, σ_{ij}^{M} and σ_{ji}^{M}, is defined as follows:

$$\sigma_{ij}^{M} = \partial \ln[X_i(w, Q)/X_j(w, Q)]/\partial \ln w_j = \eta_{ij} - \eta_{jj}, \text{ and} \tag{3.a}$$

$$\sigma_{ji}^{M} = \partial \ln[X_j(w, Q)/X_i(w, Q)]/\partial \ln w_i = \eta_{ji} - \eta_{ii}. \tag{3.b}$$

where Q denotes output, w is s vector of input prices. Other variables are already defined.

As with the AES, a positive value of σ_{ij}^{M} indicates that inputs i and j are substitutes, while a negative value indicates that the two inputs are complements. But there are important differences between the AES and the MES[7]: (1) the AES is sign symmetric (i.e., $\sigma_{ij}^{A} = \sigma_{ji}^{A}$) but the MES is sign asymmetric (i.e., σ_{ij}^{M} is not necessarily equal to σ_{ji}^{M}). (2) It is possible for two inputs to be Allen complements but Morishima substitutes (if the inputs are Allen substitutes, they must also be Morishima substitutes). (3) With more than two inputs, it is possible that one input may be a substitute for the other if one input price changes, but a complement if the other input price changes.

These differences arise because the AES is a one-input-one-price elasticity, while the MES is a two-input-one-price elasticity. That is, the AES measures the percent change in the demand for one input given a small percent change in the price of another input (where the change is divided by the cost share of the factor whose price changes), whereas the MES measures the percent change in the ratio of the demand for factor i to the demand for factor j, for a small percentage change in the price of factor j.

In this study, we use elasticities as indicators of the "flexibility" of the production unit in utilizing factors of production. A relatively more "flexible" production technology allows higher degrees of substitution among pairs of factor inputs. In contrast, a relatively "inflexible" production technology permits relatively less input substitution for the same pairs of inputs. On an absolute scale, we can say that an a production function is inflexible for a pair of factor inputs if the two inputs have elasticities of substitution that are zero or negative (complementary).[8]

Thus, elasticities of substitution reflect technical relationships among factor inputs that are important to entrepreneurs and economic policy makers. The above discussion indicates that MES is superior to the AES for measuring factor substitution.[9] But because the AES has been a standard statistic reported in the empirical literature of factor substitution, we estimate and report both the MES and the AES. We also calculate and report the CPE. This provides an opportunity for an empirical comparison of the three competing elasticities.

To estimate σ^A, σ^M, and η we need to specify and estimate the production function F with a specific functional form. Because we are interested in comparing factor substitution in the production process of small and large establishments, it is most appropriate to use a functional form that does not impose unnecessary restrictions on the substitution relationship among factor inputs.

Among the currently available flexible functional forms,[10] we choose to specify and estimate the following three-factor translog production function[11]

$$\begin{aligned}\ln(Q) = {} & \alpha_0 + \alpha_{DT}DT + \alpha_K \ln(K) + \alpha_L \ln(L) \\ & + \alpha_M \ln(M) + 0.5\alpha_{KK}(\ln K)^2 \\ & + 0.5\alpha_{LL}(\ln L)^2 + 0.5\alpha_{MM}(\ln M)^2 \\ & + \alpha_{KL}\ln(K)\ln(L) + \alpha_{KM}\ln(K)\ln(M) \\ & + \alpha_{LM}\ln(L)\ln(M), \end{aligned} \tag{4}$$

where ln is the natural logarithm; Q, K, L, M are output, capital, labor, and materials; DT is a dummy variable having value equal 1 if the establishment is owned by a multi-plant firm; otherwise $DT = 0$. This variable is included to capture the firm type effect, if any, on the production of the establishment.

For further generality, we do not assume constant returns to scale in production. For a complete discussion, see Nguyen and Reznek (1991). Imposing homogeneity of degree λ results in the following restrictions on the parameters in (4):

$$\alpha_K + \alpha_L + \alpha_M = \lambda \tag{4a}$$

$$\begin{aligned} & \alpha_{KK} + \alpha_{KL} + \alpha_{KM} = 0 \\ & \alpha_{LL} + \alpha_{KL} + \alpha_{LM} = 0 \\ & \alpha_{MM} + \alpha_{KM} + \alpha_{LM} = 0 \end{aligned} \tag{4b}$$

The returns to scale parameter can be estimated directly by substituting into (4a) into (4) before estimation.

Within the context of a three-factor translog production function, it can be shown that the AES between each pair of inputs can be estimated using the formula[12]

$$\sigma^A_{ij} = |G_{ij}|/|G|, \tag{5}$$

where $|G|$ is the determinant of the bordered Hessian matrix of the three-factor translog function (4) which is

$$G = \begin{bmatrix} 0 & S_K & S_L & S_M \\ S_K & (\hat{\alpha}_{KK} + S_K^2 - S_K) & (\hat{\alpha}_{KL} + S_K S_L) & (\hat{\alpha}_{KM} + S_K S_M) \\ S_L & (\hat{\alpha}_{KL} + S_K S_L) & (\hat{\alpha}_{LL} + S_L^2 - S_L) & (\hat{\alpha}_{LM} + S_L S_M) \\ S_M & (\hat{\alpha}_{KM} + S_K S_M) & (\hat{\alpha}_{LM} + S_L S_M) & (\hat{\alpha}_{MM} + S_M^2 - S_M) \end{bmatrix} \tag{6}$$

and $|G_{ij}|$ is the cofactor of G_{ij} in G; S_i is the actual cost share of input i ($i = K, L, M$); and $\hat{\alpha}_i \hat{\alpha}_{ij}$ are the estimated parameters of the production function (4). After we obtain the estimated σ^A_{ij}'s, we use them in equations (2) to estimate price elasticities of demand, η_{ij}. We then use formulas (3) to calculate the MES, σ^M_{ij}.

III. Data and estimation procedures

1. *Data*

For each of the five industries under study, we use cross-section establishment level data extracted from the Census Bureau's LRD for 1977 and 1982. While cross-section data are subject to certain limitations, they have some advantages for the purpose of this pilot study. In particular, for the most part, cross-section data reflect technology at a single time and thereby allow us to avoid the problem of separating the effects of factor substitution from those of technological change on production. Also, use of data from the two Censuses permits a preliminary comparison of changes in factor substitution between the two Census years. A more thorough comparison would require a great deal of additional work; see Section VI for an explanation.

The details on industry selection and data construction are fully discussed in Nguyen and Reznek (1989, 1991).

2. *Estimation procedures*

As stated earlier, the main objective of this study is to compare factor substitution in the production process of small and large establishments. Therefore, it is important to clarify the terms "small" and "large." Here, we want to emphasize that these terms are relative, and it is impossible to offer a universally accepted definition for small and large establishments. Whether an establishment is small or large in a particular industry depends on the type of industry. For example, an automobile manufacturing plant that has 600 employees is small, whereas a dress manufacturer having 500 employees can be rightly considered as large. Therefore, instead of drawing a definite line between small and large, for each industry we classify production units into five employment size classes,

- Size class 1: establishments with 5 to 19 employees.
- Size class 2: establishments with 20 to 49 employees.
- Size class 3: establishments with 50 to 99 employees.
- Size class 4: establishments with 100 to 199 employees.
- Size class 5: establishments with 200 or more employees.

Note that we do not include establishments with less than 5 employees because the data for those establishments in the LRD are largely imputed from the data for larger establishments.

For each industry under study, we estimate five separate production functions of the form (4), one for each of the five size classes; and we then use the estimated parameters to calculate elasticities of substitution and elasticities of demand for factor inputs. Theoretically, the translog production function (4) provides all the parameters needed for the estimation of the elasticities and hence it can be estimated using a single-equation estimation method. However, single-equation estimation methods neglect the additional information contained in the share-equations. Therefore, we simultaneously estimate the production function and n−1 share equations as a multivariate regression system.[13]

Using equation (4) and invoking cost minimization conditions we derive the following input demand equation system by equating cost shares to the logarithmic marginal products (i.e., first derivatives of the translog production function with respect to individual inputs),

$$
\begin{aligned}
S_K &= \frac{1}{\lambda}\,[\alpha_K + \alpha_{KK}\ln(K) + \alpha_{KL}\ln(L) \\
&\quad + \alpha_{KM}\ln(M)] \\
S_L &= \frac{1}{\lambda}\,[\alpha_L + \alpha_{LL}\ln(L) + \alpha_{KL}\ln(K) \\
&\quad + \alpha_{LM}\ln(M)] \\
S_M &= \frac{1}{\lambda}\,[\alpha_M + \alpha_{MM}\ln(M) + \alpha_{KM}\ln(K) \\
&\quad + \alpha_{LM}\ln(L)] \qquad (7)
\end{aligned}
$$

We use the Zellner's seemingly unrelated regression method (1962) as implemented in the SAS statistical package,[14] to jointly estimate the KLM production (4) and, two of the three share equations (7). We choose to delete the capital share equation from the estimation (see the discussion above). But we can derive the parameter estimates in the capital share equation using the parameter estimates from the production function and the other two share-equations, also using our assumption that production function is homogeneous of degree λ:

$$
\begin{aligned}
\alpha_K &= \lambda - \alpha_L - \alpha_M \\
\alpha_{KL} &= -\alpha_{LL} - \alpha_{LM} \\
\alpha_{KM} &= -\alpha_{MM} - \alpha_{LM} \\
\alpha_{KK} &= \alpha_{LL} + \alpha_{MM} + 2\alpha_{LM} \qquad (8)
\end{aligned}
$$

Kmenta and Gilbert (1968) showed that the Zellner estimates will converge to the efficient maximum likelihood estimates. For the small sample properties, Zellner (1963) also showed that his estimates are unbiased and efficient relative to ordinary least squares estimates.

IV. Empirical results

1. *The estimated production functions*[15]

Our primary focus in this paper is on the estimated elasticities; however, because these elasticities are calculated based on the parameter estimates of the production function, it is important to know how well the model performs. First, we find that all 50 estimated production functions reported in Tables I through V appear to be "well-behaved." A "well-

TABLE I
Parameter Estimates of the Translog KLM-Production Functions
(Asymptotic standard errors in parentheses)
Women's, Misses', and Juniors' Dresses (SIC 2335)

Parameters	Size Class 1 1977	1982	Size Class 2 1977	1982	Size Class 3 1977	1982	Size Class 4 1977	1982	Size Class 5 1977	1982
α_0	1.927*	1.590*	2.033*	1.495*	1.967*	1.549*	1.790*	1.715*	1.556*	2.480*
	(0.036)	(0.041)	(0.056)	(0.67)	(0.098)	(0.146)	(0.185)	(0.284)	(0.253)	(0.581)
α_L	0.587*	0.707*	0.590*	0.703*	0.591*	0.707*	0.600*	0.658*	0.613*	0.536*
	(0.007)	(0.010)	(0.009)	(0.012)	(0.014)	(0.022)	(0.022)	(0.036)	(0.025)	(0.063)
α_M	0.242*	0.237*	0.234*	0.247*	0.256*	0.230*	0.256*	0.242*	0.278*	0.228*
	(0.004)	(0.003)	(0.004)	(0.004)	(0.006)	(0.007)	(0.009)	(0.012)	(0.012)	(0.024)
α_{LL}	0.184*	0.140*	0.171*	0.170*	0.158*	0.155*	0.179*	0.166*	0.171*	0.048*
	(0.003)	(0.006)	(0.002)	(0.006)	(0.003)	(0.007)	(0.006)	(0.012)	(0.010)	(0.029)
α_{MM}	0.162*	0.162*	0.154*	0.140*	0.146*	0.142*	0.150*	0.127*	0.153*	0.120*
	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.005)	(0.006)	(0.007)	(0.008)	(0.013)
α_{LM}	−0.139*	−0.121*	−0.136*	−0.113*	−0.132*	−0.111*	−0.137*	−0.103*	−0.119*	−0.077*
	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.004)	(0.005)	(0.006)	(0.006)	(0.013)
λ	0.976*	0.130*	0.958*	1.111*	0.968*	1.082*	0.996*	1.043*	1.037*	0.937*
	(0.010)	(0.013)	(0.014)	(0.017)	(0.021)	(0.030)	(0.034)	(0.052)	(0.039)	(0.090)
α_T	0.140*	0.193*	0.009	0.051	0.052	0.168*	0.047	0.070	−0.112*	−0.092
	(0.037)	(0.061)	(0.031)	(0.043)	(0.031)	(0.044)	(0.038)	(0.056)	(0.51)	(0.141)
					Adjusted R^2					
ln Q	0.914	0.833	0.804	0.819	0.782	0.776	0.821	0.758	0.881	0.691
SL	0.931	0.776	0.909	0.824	0.889	0.850	0.945	0.789	0.908	0.672
SM	0.950	0.866	0.926	0.873	0.910	0.888	0.956	0.844	0.915	0.901
					Numbers of observations (N)					
	515	1080	787	942	423	361	172	137	79	40

* Denote "Statistically significant" (different from zero) at the five (or less) percent level.

behaved" production function requires that output increases monotonically with all inputs and its isoquants are convex. Monotonicity implies that all the estimated cost shares of inputs are non-negative. The convexity condition is satisfied if the bordered Hessian matrix of the first and second derivatives is negative. We find that for all 50 estimated productions there is no statistically significant violation of these conditions when evaluated at the means of the variables.[16] Second, we find that the data fit the model well. For all equations, the conventional measure of goodness of fit, R^2, is high. Third, all estimates for the first and second order coefficients ($\hat{\alpha}_i$, $\hat{\alpha}_j$) as well as the scale parameter (λ) are highly significant throughout 50 regressions.

The foregoing suggests that the estimates are robust and hence the elasticities derived from them are reliable. These estimated elasticities are discussed next.

2. *Factor substitution*

In this section, we first provide a brief discussion and general interpretation of the estimated elasticities. We next empirically compare the three types of elasticities. Finally, we discuss the results in the context of the flexibility of small and large establishments in factor substitution.

Tables VI–X show the values of the MES, σ^M, AES, σ^A, and the CPE, η, estimated by size class for the five industries under studies. All elasticities

TABLE II
Parameter Estimates of the Translog KLM-Production Functions
(Asymptotic standard errors in parentheses)
Wood Household Furniture Industry (SIC 2511)

Parameters	Size Class 1 1977	1982	Size Class 2 1977	1982	Size Class 3 1977	1982	Size Class 4 1977	1982	Size Class 5 1977	1982
α_0	1.868*	1.538*	1.201*	1.505*	2.327*	2.244*	2.969*	1.979*	1.732*	2.592*
	(0.112)	(0.079)	(0.129)	(0.120)	(0.244)	(0.270)	(0.329)	(0.294)	(0.142)	(0.194)
α_L	0.488*	0.553*	0.562*	0.577*	0.506*	0.558*	0.488*	0.608*	0.588*	0.597*
	(0.017)	(0.014)	(0.017)	(0.017)	(0.026)	(0.029)	(0.031)	(0.028)	(0.017)	(0.025)
α_M	0.207*	0.202*	0.225*	0.190*	0.156*	0.184*	0.105*	0.102*	0.144*	0.077*
	(0.009)	(0.009)	(0.010)	(0.011)	(0.011)	(0.016)	(0.013)	(0.018)	(0.011)	(0.017)
α_{LL}	0.128*	0.110*	0.137*	0.128*	0.147*	0.140*	0.141*	0.148*	0.173*	0.153*
	(0.007)	(0.006)	(0.009)	(0.007)	(0.010)	(0.010)	(0.011)	(0.010)	(0.008)	(0.011)
α_{MM}	0.156*	0.200*	0.214*	0.205*	0.182*	0.157*	0.195*	0.207*	0.242*	0.230*
	(0.006)	(0.005)	(0.007)	(0.006)	(0.010)	(0.010)	(0.013)	(0.010)	(0.009)	(0.010)
α_{LM}	−0.096*	−0.094*	−0.104*	−0.098*	−0.107*	−0.083*	−0.113*	−0.129*	−0.127*	−0.119*
	(0.005)	(0.003)	(0.005)	(0.005)	(0.007)	(0.007)	(0.008)	(0.008)	(0.007)	(0.008)
λ	0.939*	1.054*	1.079	1.051*	0.885*	0.930*	0.807*	0.986*	0.995*	0.912*
	(0.027)	(0.018)	(0.026)	(0.023)	(0.042)	(0.045)	(0.051)	(0.043)	(0.019)	(0.025)
α_T	0.187*	−0.042	0.119*	0.050	0.152*	0.134*	0.042	0.020	−0.009	−0.003
	(0.050)	(0.041)	(0.031)	(0.034)	(0.035)	(0.042)	(0.038)	(0.034)	(0.027)	(0.037)
					Adjusted R^2					
ln Q	0.827	0.879	0.844	0.851	0.728	0.700	0.678	0.784	0.913	0.837
SL	0.660	0.562	0.644	0.684	0.748	0.677	0.734	0.614	0.659	0.513
SM	0.856	0.826	0.891	0.871	0.934	0.749	0.928	0.849	0.901	0.896
					Numbers of observations (N)					
	262	397	275	340	179	166	118	119	174	148

* Denote "Statistically significant" (different from zero) at the five (or less) percent level.

are evaluated at the means of actual shares.[17] First, we find that the estimates of the own elasticities (σ^A_{ii} and η_{ii}), are negative, as expected. (The own MES is 0 by definition.) In addition, the absolute values of the own elasticities of materials and of labor are generally larger than that of capital, implying that the demands for materials and labor are more responsive to their own prices than the demand for capital is.

All the cross elasticities between capital and materials (σ^A_{KM}, σ^M_{KM}, σ^M_{MK}, η_{KM}, η_{MK}) and between labor and materials (σ^A_{LM}, σ^M_{LM}, σ^M_{ML}, η_{LM}, η_{ML}) are positive, indicating that capital and materials, and materials and labor behave as substitutes. (This confirms the theoretical result that if two goods are Allen substitutes, they must also be Morishima substitutes.) But the cross elasticities between labor and materials are generally greater than those between capital and materials. This result suggests that it is easier to substitute labor for materials than it is to substitute capital for materials.

Upon comparing the three types of elasticity, two major results emerge. First, the estimated MES and CPE show strong non-symmetric substitution relationships between capital and labor, and capital and materials. For example, for size class 5–19 in SIC 2335 industry (Table VI), the 1977 estimates for σ^M_{KL} and σ^M_{KM} equal 1.74 and 1.92, while those for σ^M_{LK} and σ^M_{MK} equal 0.35 and

TABLE III
Parameter Estimates of the Translog KLM-Production Functions
(Asymptotic standard errors in parentheses)
Newspapers Industry (SIC 2711)

Parameters	Size Class 1 1977	1982	Size Class 2 1977	1982	Size Class 3 1977	1982	Size Class 4 1977	1982	Size Class 5 1977	1982
α_0	1.776*	1.733*	1.513*	1.276*	2.369*	1.717*	2.066*	1.469*	2.127*	1.873*
	(0.051)	(0.060)	(0.083)	(0.079)	(0.198)	(0.140)	(0.276)	(0.228)	(0.104)	(0.111)
α_L	0.551*	0.540*	0.575*	0.647*	0.475*	0.547*	0.532*	0.647*	0.616*	0.619*
	(0.009)	(0.012)	(0.011)	(0.011)	(0.020)	(0.017)	(0.027)	(0.026)	(0.014)	(0.017)
α_M	0.222*	0.227*	0.244*	0.231*	0.211*	0.218*	0.238*	0.220*	0.142*	0.221*
	(0.004)	(0.006)	(0.005)	(0.006)	(0.010)	(0.008)	(0.014)	(0.011)	(0.010)	(0.013)
α_{LL}	0.117*	0.101*	0.116*	0.124*	0.087*	0.092*	0.098*	0.120*	0.129*	0.109*
	(0.004)	(0.005)	(0.005)	(0.004)	(0.006)	(0.006)	(0.009)	(0.009)	(0.006)	(0.007)
α_{MM}	0.138*	0.136*	0.123*	0.136*	0.106*	0.154*	0.123*	0.178*	0.152*	0.165*
	(0.002)	(0.003)	(0.003)	(0.003)	(0.005)	(0.005)	(0.007)	(0.006)	(0.006)	(0.007)
α_{LM}	−0.056*	−0.054*	−0.038*	−0.058*	−0.029*	−0.052*	−0.027*	−0.059*	−0.077*	−0.045*
	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.003)	(0.005)	(0.004)	(0.005)	(0.005)
λ	1.006*	1.031*	1.045*	1.128*	0.887*	1.032*	0.956*	1.090*	0.988*	1.024*
	(0.013)	(0.014)	(0.017)	(0.016)	(0.035)	(0.024)	(0.043)	(0.035)	(0.013)	(0.013)
α_T	0.014	0.011	0.036*	0.072*	0.104*	0.095*	0.151*	0.076*	0.079*	0.133*
	(0.015)	(0.017)	(0.016)	(0.014)	(0.028)	(0.019)	(0.038)	(0.029)	(0.030)	(0.038)
					Adjusted R^2					
ln Q	0.785	0.797	0.763	0.780	0.528	0.692	0.586	0.696	0.937	0.940
SL	0.542	0.300	0.374	0.465	0.382	0.335	0.344	0.384	0.433	0.358
SM	0.845	0.755	0.829	0.759	0.813	0.822	0.850	0.863	0.760	0.858
					Numbers of observations (N)					
	1319	964	910	1126	478	545	264	310	290	320

* Denote "Statistically significant" (different from zero) at the five (or less) percent level.

0.32, respectively. These estimates suggest that capital and labor are stronger substitutes when the price of labor varies than they are when the price of capital varies; and a similar result holds for capital and materials.

For this industry, we do observe a relatively symmetric Morishima substitution relationship between labor and materials. For example, the first row of Table VI shows an estimate of σ^M_{LM} equal to 3.31 which is approximately equal to an estimate of 3.35 for σ^M_{ML}. This means that labor and materials always behave as strong substitutes regardless of which price of two inputs changes. Further, these elasticities are larger than the corresponding Morishima elasticities involving capital. Again, this result is consistent with the observation that it is easier for plants to substitute labor and materials for each other than it is to substitute capital for either.

Second, for 24 of the 50 cases, the estimated Allen elasticity σ^A_{KL} is negative (admittedly many of these elasticities are close to zero), indicating that capital and labor are complements; but (with only one exception), the two corresponding estimated Morishima elasticities σ^M_{KL} and σ^M_{LK}, are positive, indicating that capital and labor are substitutes. For example, for size class 5—19 in the SIC 2335, the 1982 estimates for σ^M_{KL} and σ^M_{LK} are 1.37 and 0.36, whereas that for σ^A_{KL} ($= \sigma^A_{LK}$) equals −0.27.

TABLE IV
Parameter Estimates of the Translog KLM-Production Functions
(Asymptotic standard errors in parentheses)
Electronic Computing Equipment Industry (SIC 3573)

Parameters	Size Class 1 1977	1982	Size Class 2 1977	1982	Size Class 3 1977	1982	Size Class 4 1977	1982	Size Class 5 1977	1982
α_0	2.307*	1.452*	1.862*	1.920*	1.557*	0.548	1.222*	1.314*	1.715*	2.531*
	(0.220)	(0.149)	(0.322)	(0.291)	(0.394)	(0.346)	(0.398)	(0.405)	(0.266)	(0.191)
α_L	0.630*	0.620*	0.688*	0.709*	0.602*	0.759*	0.650*	0.742*	0.619*	0.674*
	(0.047)	(0.035)	(0.041)	(0.042)	(0.043)	(0.050)	(0.040)	(0.041)	(0.025)	(0.021)
α_M	0.102*	0.220*	0.124*	0.167*	0.202*	0.191*	0.178*	0.145*	0.159*	0.082*
	(0.029)	(0.019)	(0.025)	(0.018)	(0.025)	(0.020)	(0.019	(0.020)	(0.016)	(0.013)
α_{LL}	0.178*	0.114*	0.165*	0.158*	0.120*	0.144*	0.145*	0.161*	0.142*	0.153*
	(0.020)	(0.014)	(0.013)	(0.013)	(0.015)	(0.018)	(0.016)	(0.011)	(0.010)	(0.007)
α_{MM}	0.210*	0.203*	0.203*	0.173*	0.196*	0.220*	0.221*	0.207*	0.217*	0.206*
	(0.023)	(0.012)	(0.022)	(0.011)	(0.015)	(0.013)	(0.013)	(0.014)	(0.011)	(0.007)
α_{LM}	−0.120*	−0.80*	−0.137*	−0.095*	−0.92*	−0.111*	−0.116*	−0.120*	−0.108*	−0.110*
	(0.015)	(0.008)	(0.012)	(0.008)	(0.011)	(0.010)	(0.010)	(0.009)	(0.008)	(0.005)
λ	0.937*	1.138*	1.038*	1.054*	1.069*	1.252*	1.108	1.125*	1.037*	0.969*
	(0.052)	(0.033)	(0.061)	(0.054)	(0.064)	(0.056)	(0.058)	(0.058)	(0.032)	(0.023)
α_T	0.098	0.094*	0.048	0.065	−0.063	0.038	0.046	0.0.72	−0.033	0.053
	(0.066)	(0.048)	(0.060)	(0.062)	(0.053)	(0.058)	(0.054)	(0.059)	(0.094)	(0.065)
					Adjusted R^2					
ln Q	0.850	0.882	0.575	0.606	0.734	0.727	0.799	0.757	0.847	0.857
SL	0.706	0.500	0.684	0.600	0.468	0.585	0.703	0.749	0.628	0.692
SM	0.826	0.734	0.803	0.784	0.841	0.787	0.912	0.861	0.924	0.923
					Numbers of observations (N)					
	56	147	79	220	84	161	76	116	159	284

* Denote "Statistically significant" (different from zero) at the five (or less) percent level.

These results confirm that an increase in a factor price could have a negative effect on the demand of individual inputs, but a positive effect on the *ratio* of the demands for the two inputs. Furthermore these effects can differ depending on which input price changes. When the price of labor rises, the demand for capital falls but the capital-labor ratio rises — which means that the demand for labor declines even more than the demand for capital. Similarly, when the price of capital rises, the demand for labor falls, but the labor-capital ratio rises — which means that the demand for capital falls even more than the demand for labor. But the effect on the labor-capital ratio of a change in the price of capital is less strong than the effect on the capital-labor ratio of a change in the price of labor.

These results, while not surprising, do confirm that the Allen elasticity does not tell the whole story about input substitutability. The Allen elasticities suggest that, in many cases, capital and labor are two (symmetrically) complementary inputs. But the Morishima elasticities suggest that the two inputs are substitutable and that the substitution relationship is non-symmetric. So, the Morishima elasticity is preferred when the goal is to measure ease of substitution, where substitution is measured by changes in input ratios.

Next, we compare the estimated cross elasticities between labor and materials and capital

TABLE V
Parameter Estimates of the Translog KLM-Production Functions
(Asymptotic standard errors in parentheses)
Radio & TV Trans., Signal., & Det. Equip Industry (SIC 3662)

Parameters	Size Class 1 1977	1982	Size Class 2 1977	1982	Size Class 3 1977	1982	Size Class 4 1977	1982	Size Class 5 1977	1982
α_0	1.459*	1.376*	1.203*	1.826*	1.464*	1.034	1.403*	1.818*	2.104*	2.399*
	(0.162)	(0.146)	(0.150)	(0.153)	(0.258)	(0.268)	(0.357)	(0.335)	(0.135)	(0.121)
α_L	0.685*	0.682*	0.615*	0.665*	0.661*	0.759*	0.629*	0.716*	0.675*	0.669*
	(0.028)	(0.029)	(0.023)	(0.022)	(0.029)	(0.031)	(0.033)	(0.036)	(0.019)	(0.018)
α_M	0.169*	0.202*	0.218*	0.153*	0.174*	0.187*	0.172*	0.134*	0.095*	0.122*
	(0.016)	(0.019)	(0.011)	(0.011)	(0.015)	(0.017)	(0.018)	(0.016)	(0.013)	(0.013)
α_{LL}	0.169*	0.136*	0.118*	0.134*	0.157*	0.160*	0.143*	0.155*	0.162*	0.134
	(0.011)	(0.011)	(0.012)	(0.009)	(0.011)	(0.010)	(0.011)	(0.011)	(0.009)	(0.007)
α_{MM}	0.212*	0.199*	0.201*	0.195*	0.201*	0.205*	0.217*	0.198*	0.213*	0.182*
	(0.010)	(0.012)	(0.007)	(0.006)	(0.011)	(0.010)	(0.012)	(0.010)	(0.008)	(0.007)
α_{LM}	−0.129*	−0.095*	−0.100*	−0.095*	−0.117*	−0.102*	−0.116*	−0.099*	−0.124*	−0.089*
	(0.008)	(0.008)	(0.006)	(0.005)	(0.008)	(0.007)	(0.008)	(0.007)	(0.007)	(0.006)
λ	1.125*	1.164*	1.136*	1.054*	1.085*	1.182*	1.070*	1.054*	0.993*	0.986*
	(0.040)	(0.033)	(0.029)	(0.028)	(0.044)	(0.044)	(0.054)	(0.049)	(0.017)	(0.015)
α_T	0.037	−0.141*	0.014	0.069*	0.39	0.019	0.085*	0.048	0.093	0.027
	(0.051)	(0.051)	(0.031)	(0.30)	(0.035)	(0.039)	(0.042)	(0.040)	(0.058)	(0.048)
					Adjusted R^2					
$\ln Q$	0.822	0.822	0.808	0.764	0.677	0.710	0.688	0.669	0.924	0.928
SL	0.630	0.593	0.589	0.556	0.582	0.564	0.615	0.568	0.516	0.420
SM	0.864	0.789	0.836	0.833	0.822	0.822	0.860	0.862	0.820	0.818
					Numbers of observations (N)					
	157	223	289	363	204	260	158	189	254	307

* Denote "Statistically significant" (different from zero) at the five (or less) percent level.

and materials, in two ways: across size classes within the same year, and within size classes across years. In this discussion, we do not use the Allen elasticities because they give the same information as the price elasticities.

Examining the 1977 cross elasticities between materials and labor, we find that the furniture industry (SIC 2511) is the only industry in which the largest establishments (200 employees or more) show estimated σ^M_{LM}, σ^M_{ML}, η_{LM}, and η_{ML} that are substantially larger than those for smaller establishments. For the remaining four industries, establishments that show the largest estimates are found in smaller size classes. For example, in the dress industry (SIC 2335), the 1977 estimates of σ^M_{LM}, σ^M_{ML}, η_{LM}, and η_{ML} for establishments with 50–59 employees are 4.93, 4.90, 1.57 and 4.04, whereas the corresponding estimates for establishments with 200 employees or more are 3.87, 1.81, 1.23, and 2.31, respectively. We also observe a similar pattern in the cross elasticities between capital and materials.

Comparing elasticities within the same size classes across years, we see that the newspaper industry is the only one in which the estimated 1982 cross elasticities between materials and labor for larger establishments remained as high as the 1977 elasticities. For the remaining four industries, the estimates of the elasticities between materials and labor and between materials and

TABLE VI
Elasticities of Factor Inputs
Women's, Misses', and Juniors' Dresses (SIC 2335)

Size Class	Year	Elasticity Type[a]	KK	LL	MM	KL	LK	KM	MK	LM	ML
5–19	1977	σ_{ij}^M	0	0	0	1.74	0.35	1.92	0.32	3.31	3.35
		σ_{ij}^A	−3.07	−2.94	−4.73	0.49		0.14		3.66	
		η_{ij}	−0.30	−1.49	−1.87	0.25	0.05	0.05	0.01	1.44	1.85
	1982	σ_{ij}^M	0	0	0	1.37	0.36	4.89	0.68	5.90	5.58
		σ_{ij}^A	−3.05	−2.45	−17.69	−0.27		2.29		6.43	
		η_{ij}	−0.39	−1.54	−4.33	−0.17	−0.03	0.56	0.29	1.57	4.04
20–49	1977	σ_{ij}^M	0	0	0	1.41	0.28	2.64	0.28	3.77	3.77
		σ_{ij}^A	−3.14	−2.02	−8.46	0.26		0.32		4.08	
		η_{ij}	−0.26	−1.25	−2.54	0.16	0.02	0.10	0.03	1.23	2.52
	1982	σ_{ij}^M	0	0	0	1.81	0.46	6.35	0.65	7.70	7.51
		σ_{ij}^A	−3.52	−2.50	−30.65	0.16		1.70		8.56	
		η_{ij}	−0.44	−1.70	−6.02	0.11	0.02	0.33	0.21	1.68	5.81
50–99	1977	σ_{ij}^M	0	0	0	1.43	0.27	3.77	0.30	4.93	4.90
		σ_{ij}^A	−3.13	−1.92	−15.07	0.19		0.52		3.52	
		η_{ij}	−0.25	−1.30	−3.64	0.13	0.02	0.13	0.04	1.29	3.60
	1982	σ_{ij}^M	0	0	0	2.28	0.38	12.63	1.12	14.54	13.79
		σ_{ij}^A	−3.74	−3.98	−64.82	−0.70		5.29		15.85	
		η_{ij}	−0.47	−2.77	−11.68	−0.49	−0.09	0.95	0.66	2.86	11.02
100–199	1977	σ_{ij}^M	0	0	0	1.81	0.33	4.16	0.31	5.64	5.66
		σ_{ij}^A	−3.08	−2.37	−16.78	0.38		0.18		6.22	
		η_{ij}	−0.29	−1.56	−4.12	0.25	0.04	0.04	0.02	1.52	4.10
	1982	σ_{ij}^M	0	0	0	1.44	0.54	4.30	0.63	5.20	5.11
		σ_{ij}^A	−3.81	−1.71	−21.32	0.41		1.09		5.80	
		η_{ij}	−0.49	−1.16	−4.09	0.28	0.05	0.21	0.14	0.11	3.95
200+	1977	σ_{ij}^M	0	0	0	1.50	0.50	2.87	0.56	3.87	3.81
		σ_{ij}^A	−3.15	−2.22	−9.41	0.37		0.81		4.37	
		η_{ij}	−0.44	−1.28	−2.64	0.22	0.05	0.23	0.11	1.23	2.53
	1982	σ_{ij}^M	0	0	0	0.59	0.42	2.40	0.68	2.57	2.31
		σ_{ij}^A	−3.35	−0.99	−8.10	−0.05		1.95		2.66	
		η_{ij}	−0.43	−0.63	−1.93	−0.03	−0.01	0.47	0.25	0.64	1.68

[a] σ_{ij}^M = Morishima Elasticity of Substitution
σ_{ij}^A = Allen Elasticity of Substitution
η_{ij} = Price Elasticity of Factor Demand
Each *i*, *j* can take on a value of K (capital), L (labor), and M (materials).

capital for the largest establishments decline between 1977 and 1982, whereas the elasticities for smaller establishments either remain relatively stable or increase substantially. Again, using the dresses industry as an example, we find that the 1977 estimates of σ_{KM}^M, σ_{MK}^M, σ_{LM}^M, and σ_{ML}^M for establishments with 5–19 employees are 1.92, 0.32, 3.31, and 3.35, whereas the corresponding estimates in 1982 are 4.89, 0.68, 5.90, and 5.58. In contrast, the 1982 estimates of the same elasticities for establishments with 200 employees or more (2.40, 0.68, 2.57, 2.31) are (with one slight exception) a bit smaller than those for the small establishments in 1982 (2.87, 0.56, 3.87, 3.81).

The above results suggest that more often than

TABLE VII
Elasticities of Factor Inputs
Wood Household Furniture Industry (SIC 2511)

Size Class	Year	Elasticity Type[a]	KK	LL	MM	KL	LK	KM	MK	LM	ML
5–19	1977	σ_{ij}^M	0	0	0	2.03	0.79	1.85	0.88	3.09	3.00
		σ_{ij}^A	−2.19	−6.11	−3.40	0.67		0.97		3.90	
		η_{ij}	−0.61	−1.83	−1.43	0.20	0.19	0.41	0.27	1.65	1.17
	1982	σ_{ij}^M	0	0	0	1.81	0.66	2.60	1.19	3.74	3.21
		σ_{ij}^A	−2.53	−6.08	−4.44	−0.13		1.77		4.50	
		η_{ij}	−0.70	−1.89	−1.86	−0.04	−0.04	0.74	0.49	1.88	1.37
20–49	1977	σ_{ij}^M	0	0	0	2.10	0.61	3.15	1.21	4.64	4.04
		σ_{ij}^A	−2.68	−6.77	−5.67	−3.5		1.96		5.59	
		η_{ij}	−0.70	−2.21	−2.34	−0.11	−0.09	0.81	0.51	2.30	1.83
	1982	σ_{ij}^M	0	0	0	2.00	0.65	2.89	1.22	4.24	3.66
		σ_{ij}^A	−2.53	−6.63	−5.17	−0.20		1.87		5.15	
		η_{ij}	−0.71	−2.06	−2.12	−0.06	−0.06	0.77	0.52	2.12	1.60
50–99	1977	σ_{ij}^M	0	0	0	2.41	0.73	2.27	0.88	3.95	3.80
		σ_{ij}^A	−2.35	−7.23	−3.22	0.49		1.05		4.95	
		η_{ij}	−0.61	−2.26	−1.82	0.15	0.13	0.45	0.27	2.13	1.55
	1982	σ_{ij}^M	0	0	0	1.99	1.02	1.82	1.03	2.79	2.78
		σ_{ij}^A	−2.73	−5.16	−3.46	1.00		1.05		3.45	
		η_{ij}	−0.75	−1.67	−1.40	0.32	0.27	0.42	0.29	1.39	1.11
100–199	1977	σ_{ij}^M	0	0	0	2.30	0.59	2.65	0.92	4.35	4.02
		σ_{ij}^A	−2.28	−7.07	−4.92	0.04		1.34		5.38	
		η_{ij}	−0.58	−2.28	−2.08	0.01	0.01	0.57	0.34	2.27	1.74
	1982	σ_{ij}^M	0	0	0	3.49	0.39	4.54	1.13	7.64	6.90
		σ_{ij}^A	−2.01	−11.43	−9.60	−0.63		1.98		9.98	
		η_{ij}	−0.57	−3.70	−3.76	−0.20	−0.18	0.77	0.56	3.88	3.20
200+	1977	σ_{ij}^M	0	0	0	10.25	−0.41	13.62	2.43	24.28	21.43
		σ_{ij}^A	−2.88	−37.67	−28.68	−4.29		5.42		32.12	
		η_{ij}	−0.84	−11.57	−11.46	−1.32	−1.26	2.17	1.59	12.83	9.86
	1982	σ_{ij}^M	0	0	0	5.00	0.09	8.03	2.06	12.94	10.97
		σ_{ij}^A	−2.60	−18.47	−17.13	−2.32		4.02		16.97	
		η_{ij}	−0.81	−5.72	−6.50	−0.72	−0.72	1.52	1.25	6.44	5.25

[a] σ_{ij}^M = Morishima Elasticity of Substitution
σ_{ij}^A = Allen Elasticity of Substitution
η_{ij} = Price Elasticity of Factor Demand
Each *i*, *j* can take on a value of K (capital), L (labor), and M (materials).

not large establishments (with 200 employees or more) are less flexible than smaller establishments in both materials-labor and materials-capital substitution. Moreover, in 1982, large establishments in most industries appear to become less flexible in labor-materials and materials-capital substitution, whereas smaller establishments become more flexible. At the very least, we cannot conclude that the large establishments are more flexible in input substitution than the small establishments.

Finally, we turn to the estimated elasticities between capital and labor. Examining the 1977 estimated CPE, we find that, except for the Radio and T.V. Equipment industry (SIC 3662), all the

TABLE VIII
Elasticities of Factor Inputs
Newspapers Industry (SIC 2711)

Size Class	Year	Elasticity Type[a]	KK	LL	MM	KL	LK	KM	MK	LM	ML
5—19	1977	σ^M_{ij}	0	0	0	1.48	1.23	2.70	1.78	2.94	2.39
		σ^A_{ij}	−2.55	−3.13	−7.90	0.83		2.35		3.29	
		η_{ij}	−0.93	−1.17	−2.08	0.31	0.30	0.62	0.85	0.87	1.23
	1982	σ^M_{ij}	0	0	0	1.30	1.13	2.52	1.68	2.69	2.14
		σ^A_{ij}	−2.56	−2.72	−6.93	0.69		2.29		2.90	
		η_{ij}	−0.89	−1.03	−1.90	0.26	0.24	0.63	0.79	0.79	1.10
20—49	1977	σ^M_{ij}	0	0	0	1.41	1.62	3.44	2.68	3.24	2.17
		σ^A_{ij}	−3.21	−2.43	−12.56	1.03		3.83		2.87	
		η_{ij}	−1.22	−0.99	−2.64	0.42	0.39	0.80	1.46	0.60	1.17
	1982	σ^M_{ij}	0	0	0	1.44	1.23	3.16	1.96	3.37	2.64
		σ^A_{ij}	−2.77	−2.76	−10.48	0.74		2.80		3.67	
		η_{ij}	−0.97	−1.13	−2.49	0.30	0.26	0.67	0.99	0.87	1.51
50—99	1977	σ^M_{ij}	0	0	0	1.26	1.44	2.74	2.26	2.56	1.74
		σ^A_{ij}	−2.62	−2.12	−10.38	1.01		3.09		2.21	
		η_{ij}	−1.04	−0.85	−2.11	0.41	0.40	0.63	1.22	0.45	0.89
	1982	σ^M_{ij}	0	0	0	1.13	1.18	5.24	3.23	5.19	3.14
		σ^A_{ij}	−3.17	−2.95	−17.44	−0.06		5.33		5.10	
		η_{ij}	−1.20	−1.15	−4.01	−0.02	−0.02	1.23	2.02	1.17	1.99
100—199	1977	σ^M_{ij}	0	0	0	1.36	1.80	3.63	3.16	3.18	1.83
		σ^A_{ij}	−3.33	−2.48	−12.63	1.06		4.42		2.33	
		η_{ij}	−1.36	−0.94	−2.69	0.42	0.45	0.94	1.80	0.50	0.89
	1982	σ^M_{ij}	0	0	0	1.20	1.46	13.20	7.52	12.94	6.88
		σ^A_{ij}	−6.06	−5.37	−42.90	−2.25		13.62		12.52	
		η_{ij}	−2.32	−2.06	−10.02	−0.86	−0.86	3.18	5.21	2.92	4.81
200+	1977	σ^M_{ij}	0	0	0	1.79	0.94	4.22	1.92	5.07	4.09
		σ^A_{ij}	−2.17	−4.40	−14.54	0.32		2.90		6.39	
		η_{ij}	−0.82	−1.67	−3.52	0.12	0.12	0.70	1.10	1.55	2.43
	1982	σ^M_{ij}	0	0	0	1.25	1.67	5.93	4.21	5.51	2.97
		σ^A_{ij}	−4.14	−3.39	−17.62	0.08		6.53		4.85	
		η_{ij}	−1.63	−1.22	−4.32	0.03	0.03	1.60	2.57	1.19	1.75

[a] σ^M_{ij} = Morishima Elasticity of Substitution
σ^A_{ij} = Allen Elasticity of Substitution
η_{ij} = Price Elasticity of Factor Demand
Each *i, j* can take on a value of K (capital), L (labor), and M (materials).

estimates for η_{KL} for the smallest size class (5—19 employees) are consistently positive, while those for the largest size class (200 employees or more) are either smaller or negative. We find a similar pattern in the estimates for η_{LK}. Thus, in 1977, it appears that, in most industries under study, small establishments' utilization of labor and capital is more price responsive than that of large establishments. For the Radio & T.V. (SIC 3662), the 1977 estimates for η_{KL} and η_{LK} across all size classes are negative, indicating capital and labor behave as Allen complements in this industry.

Examining the 1977 estimated Morishima elasticities between capital and labor, we found

TABLE IX
Elasticities of Factor Inputs
Electronic Computing Equipment Industry (SIC 3573)

Size Class	Year	Elasticity Type[a]	KK	LL	MM	KL	LK	KM	MK	LM	ML
5–19	1977	σ_{ij}^M	0	0	0	3.27	0.70	3.06	0.85	5.62	5.47
		σ_{ij}^A	−2.69	−9.25	−5.94	0.42		1.07		6.95	
		η_{ij}	−0.61	−3.12	−2.59	0.14	0.09	0.47	0.24	3.03	2.35
	1982	σ_{ij}^M	0	0	0	1.33	0.77	2.64	1.34	3.19	2.63
		σ_{ij}^A	−3.44	−3.97	−4.26	−0.20		2.17		3.51	
		η_{ij}	−0.82	−1.40	−1.75	−0.07	−0.05	0.89	0.52	1.44	1.23
20–49	1977	σ_{ij}^M	0	0	0	2.85	0.44	4.11	0.86	6.51	6.10
		σ_{ij}^A	−2.20	−7.43	−9.31	−0.24		1.58		7.95	
		η_{ij}	−0.50	−2.94	−3.51	−0.10	−0.05	0.60	0.36	3.00	3.15
	1982	σ_{ij}^M	0	0	0	1.86	0.85	2.14	0.96	3.16	3.05
		σ_{ij}^A	−3.04	−4.19	−4.41	0.67		1.14		3.76	
		η_{ij}	−0.70	−1.61	−1.70	0.26	0.15	0.44	0.26	1.45	1.44
50–99	1977	σ_{ij}^M	0	0	0	1.42	0.66	2.45	1.09	3.21	2.78
		σ_{ij}^A	−3.03	−4.02	−4.19	−0.12		1.78		3.63	
		η_{ij}	−0.69	−1.46	−1.72	−0.04	−0.03	0.73	0.40	1.49	1.32
	1982	σ_{ij}^M	0	0	0	1.87	0.51	3.89	1.25	5.24	4.50
		σ_{ij}^A	−2.99	−5.54	−7.61	−0.73		2.52		6.04	
		η_{ij}	−0.68	−2.15	−2.92	−0.28	−0.17	0.97	0.57	2.32	2.35
100–199	1977	σ_{ij}^M	0	0	0	1.95	0.48	3.15	0.98	4.62	4.12
		σ_{ij}^A	−2.84	−5.83	−5.59	−0.54		1.87		5.35	
		η_{ij}	−0.59	−2.15	−2.36	−0.26	−0.11	0.79	0.39	2.26	1.97
	1982	σ_{ij}^M	0	0	0	2.37	0.56	2.63	0.79	4.44	4.21
		σ_{ij}^A	−2.67	−6.67	−4.73	0.05		1.19		5.27	
		η_{ij}	−0.54	−2.35	−2.10	0.02	0.01	0.53	0.24	2.34	1.86
200+	1977	σ_{ij}^M	0	0	0	2.17	0.58	3.14	1.14	4.74	4.17
		σ_{ij}^A	−2.71	−6.96	−5.56	−0.38		1.89		5.68	
		η_{ij}	−0.67	−2.30	−2.34	−0.13	−0.09	0.80	0.47	2.39	1.88
	1982	σ_{ij}^M	0	0	0	2.54	0.66	2.80	0.99	4.68	4.35
		σ_{ij}^A	−2.57	−7.82	−5.09	0.09		1.41		5.76	
		η_{ij}	−0.64	−2.46	−2.17	0.03	0.02	0.61	0.35	2.45	1.82

[a] σ_{ij}^M = Morishima Elasticity of Substitution
σ_{ij}^A = Allen Elasticity of Substitution
η_{ij} = Price Elasticity of Factor Demand
Each i, j can take on a value of K (capital), L (labor), and M (materials).

that, except in only one case (SIC 2511, 1977, 200+ employees), all the estimates of σ_{KL}^M and σ_{LK}^M are positive across all size classes in all five industries. These results indicate that (with the one exception) capital and labor are Morishima substitutes in response to changes in both labor and capital prices. However, comparing the estimates across size classes, we find that the estimated σ_{LK}^M for the smallest establishments are greater than those for the largest establishments in four of the five industries (all except SIC 2335). Similarly, the estimates of σ_{KL}^M for the smallest establishments (5–19 employees) are greater than those for the largest establishments (200 employees or more) in

TABLE X
Elasticities of Factor Inputs
Ratio & TV Trans., Signal., & Det. Equip. Industry (SIC 3662)

Size Class	Year	Elasticity Type[a]	KK	LL	MM	KL	LK	KM	MK	LM	ML
5–19	1977	σ_{ij}^M	0	0	0	3.06	0.50	4.14	0.98	6.68	6.21
		σ_{ij}^A	−2.38	−8.54	−8.84	−0.27		1.71		8.24	
		η_{ij}	−0.57	−3.16	−3.45	−0.10	−0.07	0.67	0.41	3.22	3.05
	1982	σ_{ij}^M	0	0	0	1.73	0.72	3.03	1.28	4.04	3.47
		σ_{ij}^A	−2.82	−4.88	−5.99	−0.09		2.06		4.75	
		η_{ij}	−0.74	−1.76	−2.25	−0.03	−0.02	0.77	0.54	1.79	1.71
20–49	1977	σ_{ij}^M	0	0	0	1.38	0.5	2.87	1.10	3.69	3.14
		σ_{ij}^A	−2.89	−3.89	−5.40	−0.37		2.05		4.16	
		η_{ij}	−0.65	−1.52	−2.08	−0.15	−0.08	0.79	0.46	1.60	1.62
	1982	σ_{ij}^M	0	0	0	1.52	0.69	3.06	1.25	3.90	3.33
		σ_{ij}^A	−2.96	−4.03	−6.29	−0.14		2.16		4.46	
		η_{ij}	−0.73	−1.58	−2.28	−0.06	−0.04	0.78	0.53	1.62	1.75
50–99	1977	σ_{ij}^M	0	0	0	2.13	0.58	3.30	0.99	4.86	4.44
		σ_{ij}^A	−2.58	−5.56	−7.08	−0.09		1.69		5.82	
		η_{ij}	−0.60	−2.17	−2.67	−0.04	−0.02	0.64	0.39	2.19	2.27
	1982	σ_{ij}^M	0	0	0	1.92	0.72	3.30	1.22	4.49	4.00
		σ_{ij}^A	−3.12	−4.95	−6.83	−0.06		2.04		5.25	
		η_{ij}	−0.73	−1.94	−2.53	−0.02	−0.01	0.76	0.48	1.95	2.06
100–199	1977	σ_{ij}^M	0	0	0	1.06	0.49	3.09	0.98	4.56	4.07
		σ_{ij}^A	2.74	−5.86	−5.53	−0.47		1.81		5.31	
		η_{ij}	−0.59	−2.13	−2.32	−0.17	−0.10	0.76	0.39	2.23	1.93
	1982	σ_{ij}^M	0	0	0	1.82	0.75	3.11	1.21	4.18	3.72
		σ_{ij}^A	−3.06	−4.57	−6.53	0.05		1.96		3.88	
		η_{ij}	−0.74	−1.80	−2.39	0.02	0.01	0.72	0.47	1.79	1.92
200+	1977	σ_{ij}^M	0	0	0	2.37	0.46	4.54	1.13	6.45	5.78
		σ_{ij}^A	−2.67	−6.31	−10.24	−0.60		2.35		7.63	
		η_{ij}	−0.60	−2.62	−3.69	−0.25	−0.14	0.85	0.53	2.75	3.17
	1982	σ_{ij}^M	0	0	0	1.42	0.77	2.67	1.19	3.33	2.90
		σ_{ij}^A	−3.10	−3.37	−5.58	0.13		1.92		3.76	
		η_{ij}	−0.74	−1.37	−1.99	0.05	0.03	0.68	0.46	1.34	1.53

[a] σ_{ij}^M = Morishima Elasticity of Substitution
σ_{ij}^A = Allen Elasticity of Substitution
η_{ij} = Price Elasticity of Factor Demand
Each i, j can take on a value of K (capital), L (labor), and M (materials).

three of the five industries (SICs 2335, 3573, and 3662). For the remaining two industries (SICs 2511 and 2711), though the estimates for σ_{KL}^M for largest establishments are greater than those for smallest establishments, the opposite is true for the estimated σ_{LK}^M. Thus, in these two industries large establishments appear to be more flexible in capital-labor substitution in response to changes in the wage rate, whereas small establishments are more flexible in capital-labor substitution in response to changes in the price of capital. But in general, we can conclude that, with exceptions as noted, small establishments in 1977 appeared to be more flexible than large establishments in substituting labor for capital.

In 1982, the estimates of the price elasticities

η_{KL} and η_{KL}, for establishments of all size classes, either decline or become negative across all five industries. Thus, based on the estimates of one-input-one-price such as the CPE and AES, it appears that, during the 1982 recession, small establishments lost their advantage over large establishments in capital-labor substitution.

But when we consider the two-input-one-price Morishima elasticities, we find that the 1982 estimates of σ_{LK}^{M} for establishments across all size classes remain positive and relatively stable. In contrast, the 1982 estimates of σ_{KL}^{M} decline in most industries, for most size classes. But the declines are for the most part not great, and the elasticities remain positive. Thus, based on the MES, it appears that for the industries under study in both 1977 and 1982, capital and labor are substitutes when the price of either capital or labor changes, even though both capital and labor, each alone, became less responsive to changes in the respective cross price in the 1982 recession. Further, the results suggest that during the 1982 recession, smaller establishments are no less able than large establishments to adjust their mix of capital and labor.

Overall, the estimates of the CPE and MES suggest that, in most industries under study, small establishments are at least as flexible as large establishments in factor substitution. In particular, small establishments in several industries became even more flexible during the 1982 recession in adjusting their mixes of labor and materials and capital and materials, while most large establishments became less flexible. Further, small establishments appear to have no disadvantage over large establishments in adjusting their mixes of capital and labor. So, our findings seem to indicate that, at the very least, large establishments have no advantage over large establishments in factor substitution; and some of the evidence suggests that in fact small establishments may have the advantage.

V. Concluding remarks and areas of future research

The purpose of this study is to assess the possibility of differences in factor substitution in the production processes of small and large establishments. For empirical implementation, we specify and estimate separate three-factor translog production functions, using confidential plant-level data for five four-digit industries extracted from the Census Bureau's Longitudinal Research Database. These data are unique in that they provide the most comprehensive information available on outputs and inputs of "small" and "large" establishments, by any definition of "small" and "large."

Based on our classification of plant size, the overall results seem to support the hypothesis that small production units tend to employ production technologies that are at least as flexible in factor substitution as those chosen by large producers. Together with our previous finding regarding economies of scale (Nguyen and Reznek 1991), this result appears to support the view that a large establishment size is not a necessary condition for efficient production.

But we want to emphasize that while the above conclusions are drawn with certain degree of confidence, they are by no means definite, for (at least) two reasons. First, we may have a type of selectivity bias. We deliberately selected five industries that have a large number of small establishments. It may be that the technologies and market conditions may naturally allow small establishments to survive in these industries.

Second, although the data set is valuable for empirical research such as ours, it is far from perfect. One limitation is that it does not contain information on input and output prices. As a result, real inputs and outputs cannot be measured accurately. Most importantly, our data set does not provide sufficient data necessary for constructing an accurate and theoretically sound measure of capital services. Instead, it provides data on capital stocks based on book values, which are generally not an appropriate proxy for capital input. This inaccurate measure of capital may cause biases in the model parameter estimates.

The above limitations suggest much additional research, the key to which lies in improving our data set. In particular, we plan to construct a panel (cross-section time-series) data set, using information from the LRD to link establishment data for all the years covered by the LRD — currently 1972—1986. Panel data would permit several improvements, including the following: (1) It would allow us to construct an improved measure of capital stock that, together with measures of capacity utilization, could be used to obtain estimates of capital services. It may also be possible to at least account partially for the vintage of capital by using data on establishment history. (2) It would allow us to incorporate into our models certain factors that we could not properly account for here — technical change, business cycle effects, firm effects, and perhaps others.

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Notes

[1] For example, Brock and Evans (1986) found that "Most of the 16.8 million businesses that filed tax returns in 1980 are small businesses by any standard. Eighty percent of 12.7 million are sole proprietorships, 60 percent of 1.4 million are partnerships, and 90 percent of the 2.7 million corporations filing in that year had annual business receipts of under $50,000" (p. 8). They also found that, in many industries, firms that had highest profits per dollars of sales in 1978 are those with 20—99 employees (see Brock and Evans, Table 2.1, p. 10). Brock and Evans' study appears to suggest that "large is not necessarily better."

[2] Our use of the term "flexibility" is different from that in much of the economic literature, but it seems natural to us. We examine flexibility within the context of input substitution possibilities — the more readily a firm or plant can substitute inputs for one another, the more flexible it is in input substitution.

The use of the term "flexibility" has not been consistent in the literature, although it has usually meant a way of measuring how readily firms or plants can respond to changes in demand. Stigler (1939) first used the term "flexibility" to mean those attributes of a production technology that accommodate greater output variations. Marschak and Nelson (1962) define flexibility as the reciprocal of the second derivative of total cost with respect to output. Mills and Schumann (1985) show that flexibility can be represented by the elasticity of supply.

Still another use of the term "flexible" is in the context of the flexible functional forms, which (loosely speaking) refers to linear functions that impose as few restrictions as possible on the technology. See Chambers (1988, chapter 5) for a more complete discussion.

[3] John Solow (1987) argues convincingly that it is impossible to determine whether two inputs are substitutes or complements using aggregate data such as those for 2-digit SIC industries.

[4] For detailed information about the LRD, see McGuckin and Pascoe (1988).

[5] See Allen, 1938, p. 504.

[6] For derivations of these elasticities, see Chambers, 1988, pp. 94—95.

[7] Chambers (1988) gives an accessible derivation of the following results in the text. The MES is $\sigma_{ij}^{M} = \eta_{ij} - \eta_{jj}$. Note that $\eta_{jj} < 0$ always, for a well behaved production function. So, if goods i and j are Allen substitutes (so that $\eta_{ij} > 0$), then they must always be Morishima substitutes. But if the two goods are Allen complements ($\eta_{ij} < 0$), they can be either Morishima complements or Morishima substitutes. Furthermore, if $|\eta_{ii}| > |\eta_{ij}| > |\eta_{jj}|$, the goods are Morishima substitutes when the price of input j changes, but they are Morishima complements when the price of input i changes.

[8] An example of an inflexible technology is the Leontief production function in which inputs are not substitutable because input ratios are fixed.

[9] For example, if capital and labor are complementary, then other things being equal, The Allen elasticities show that higher wage rates will dampen the demand for labor and the demand for new plant and equipment. The Morishima elasticities give extra information about which demand declines more.

[10] Once again, the word "flexible" here has a different meaning here than in the context of elasticities.

[11] The translog function was developed by Christensen, Jorgenson, and Lau (1971). Applied studies using this form include those by Berndt and Wood (1975), Burgess (1975), Christensen and Green (1979), Kokkelenberg and Nguyen (1989), and others. Other flexible functional forms include the extended generalized Cobb-Douglas form (Magnus, 1979), the symmetric generalized McFadden form (McFadden, 1978, and Diewert and Wales (1987). We choose the Translog function because it is the most widely used in empirical studies. Also it is the simplest flexible functional form and has the smallest number of parameters to be estimated. Moreover, Kokkelenberg and Nguyen (1987) found that the translog function is the easiest to work with in terms of achieving convergence, in producing plausible results, and in overall forecasting ability. Guilky, Lovells, and Sickles (1983) also found similar results.

[12] See Berndt and Christensen (1973).

[13] Because the three cost shares add to one, estimating all three share equations together results in a singular covariance matrix. The problem can be avoided by deleting one share-equation from the estimation procedure. We dropped the capital share equation and derived its coefficient estimates using the restriction in (4a). Recently Dhrymes (1989) has suggested using the generalized inverse (g-inverse) of the singular covariance matrix in the Aiken minimand, rather than disposing of one equation. We did not use this method because it has not been incorporated into the econometric software we used.

[14] See SAS Institute (1984) Chapter 20, which describes the SAS/ETS SYSNLIN procedure. We used SAS version 5.18

on Digital Equipment Corporation Microvax II microcomputer.

[15] The parameter estimates in this paper are very slightly different from those in Nguyen and Reznek (1991). The differences result from a small programming error that has been corrected for this paper. We wish to emphasize that all of the interpretations and conclusions in the first paper remain valid. For example, the scale parameter estimates (λ) in tables I through V change very little.

[16] The monotonicity condition is satisfied at all data points (i.e., there are no negative cost shares). For the convexity condition, the hessian bordered matrix is negative definite when evaluated at the means.

[17] Most previous studies evaluated elasticities of substitution at the means of fitted shares. However, recently Anderson and Thursby (1986) showed that it is more appropriate to use actual shares. In particular, they found that "a normal distribution for the AES (Allen elasticity of substitution) is appropriate if the estimator uses the *means* of the *actual* factor shares."

References

Allen, R. D. G., 1938, *Mathematical Analysis For Economists*, London: MacMillan.

Anderson, R. G. and J. G. Thursby, 1986, 'Confidence Intervals for Elasticity Estimators in Translog Models', *Review of Economics and Statistics* **3**, 647–656.

Berndt, E. R. and L. R. Christensen, 1973, 'The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing 1929–68', *Journal of Econometrics* **1**, 81–114.

Berndt, E. R. and D. O. Wood, 1975, 'Technology, Process, and the Derived Demand for Energy', *The Review of Economics and Statistics* **68**, 647–656.

Blackorby, C. and R. R. Russell, 1989, 'Will the Real Elasticity of Substitution Please Stand Up? (A Comparison of the Allen/Uzawa and Morishima Elasticities)', *American Economic Review* **79**, 882–888.

Brock, W. A. and D. S. Evans, 1986, *The Economics of Small Business: Their Role and Regulation in the U.S. Economy*, New York: Holmes and Meier Publishers.

Burgess D. F., 1975, 'Duality Theory and Pitfalls in the Specification of Technologies', *Journal of Econometrics* **3**, 105–121.

Chambers, R. G., 1988, *Applied Production Analysis: A Dual Approach*, New York: Cambridge University Press.

Christensen, L. R. and W. H. Green, 1976, 'Economies of Scale in U.S. Electric Power Generation', *Journal of Political Economy* **84**, 655–676.

Christensen, L. R., D. W. Jorgenson, and L. J. Lau, 1971, 'Conjugate Duality and the Transcendental Logarithmic Production Function', *Econometrica* **39**, 255–266.

Dhrymes, P. J., 1989, 'Autoregressive Errors in Singular Systems of Equations', unpublished paper presented at the Center for Economic Studies Seminar on April 13.

Diewert, W. E. and T. J. Wales, 1987, 'Flexible Functional Forms and Global Curvature Conditions', *Econometrica* **55**, 43–68.

Guilkey, D. K., C. A. K. Lovell, and R. C. Sickles, 1983, 'A Comparison of the Performance of Three Flexible Functional Forms', *International Economic Review* **24**, 591–616.

Kmenta J. and R. F. Gilbert, 1968, 'Small Sample Properties of Alternatives Estimators of Seemingly Unrelated Regressions', *Journal of the American Statistical Association* **63**, 1180–1200.

Kokkelenberg, E. C. and S. V. Nguyen, 1987, 'A Comparison of Three Flexible Functional Cost Forms Using Plant Level Data', Paper presented at the 1987 American Economic Association Conference in Chicago, December.

Kokkelenberg, E. C. and S. V. Nguyen, 1989, 'Modelling Technical Progress and Multifactor Productivity: A Plant Level Example', *Journal of Productivity Analysis* **1**, 69–90.

McFadden, D., 1978, 'The General Linear Profit Function', in M. Fuss and D. McFadden (eds.), *Production Economics: A Dual Approach to Theory and Application*, Vol. 1, Amsterdam: North-Holland, pp. 269–286.

McGuckin, R. and G. Pascoe, 1988, 'The Longitudinal Research Database: Status and Research Possibilities', *Survey of Current Business*, 30–37.

Magnus, J. R., 1979, 'Substitution Between Energy and Non-Energy in the Netherlands 1950–1976', *International Economic Review* **20**, 465–484.

Marschak, T. and R. R. Nelson, 1962, 'Flexibility, Uncertainty, and Economic Theory', *Metroeconomica* **14**, 42–60.

Mills, D. E. and L. Schumann, 1985, 'Industry Structure with Fluctuating Demand', *American Economic Review* **4**, 758–67.

Nguyen, S. V. and A. Reznek, 1989, 'Production Technologies, Economies of Scale, and Factor Substitution in Large and Small U. S. Manufacturing Establishment: A Pilot Study', A *Report* submitted to the U.S. Small Business Administration, October.

Nguyen, S. V. and A. Reznek, 1991, 'Returns to Scale in Small and Large U.S. Manufacturing Establishments', *Small Business Economics* **3**, 197–214.

SAS Institute Inc., 1984, *SAS Users Guide Version 5 Edition*, Cary, NC: SAS Institute, Inc.

SAS Institute Inc., 1985, SAS Users Guide: Basics, Version 5 Edition, Cary, NC: SAS Institute Inc.

Solow, J. L., 1987, 'The Capital-Energy Complementarity Debate Revisited', *American Economic Review* **77**, 605–614.

Stigler, G. J. 1939, 'Production and Distribution in the Short-Run', *Journal of Political Economy* **47**, 305–27.

U.S. Small Business Administration, 1987, *The State of Small Business: A Report of the President to the Congress 1987*, Washington D.C.: U.S. Government Printing Office.

Zellner, A., 1963, 'Estimators of Seemingly Unrelated Regressions: Some Exact Finite Samples Results', *Journal of American Statistical Association* **58**, 977–92.

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