Sudden Change in a Model of Small Firm Growth

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ABSTRACT. A "catastrophe theory" model of small firm growth allows for an abrupt change in small firm size. The model permits only *smooth* changes in the *endogenous* independent variables, turnover and profitability, yet allows *sudden* changes in the level of assets, the size variable. But not all growth paths suggested by the mathematics are likely to be met in the real world. Nevertheless, there are feasible phases of steady growth as well as feasible phases during which "jumps" occur.

I. Introduction

The "life cycle" (or "product life cycle") model of the small firm recognises that the firm can go through various stages (Casson, 1982, Chapter 16; Churchill and Lewis, 1983). Simplistically, the growth path of the firm is said to reflect changes in the entrepreneurial fortunes/abilities of the firm's founder or family[1] or the process whereby one significant product or service ultimately exhausts its market. Such models embrace, as a crucial assumption, the belief that the growth path of the firm in one instance of time is a lagged function of the growth path of the firm in previous instances of time. While the time scale on the horizontal axis is unspecified it is understood that changes in the slope of the curve representing the relationship between firm size and time can indicate changes in the rate at which the small firm is growing. However, it does so *smoothly*, in that there is no sudden change in size in the sense that firm size changes quite abruptly without any appreciable change in the passage of time. This rules out any abrupt metamorphosis in the nature of the small firm. There are no instantaneous changes of fortune as may come from the unexpected opening up of a new market, from the unforeseen access to a new line of credit or from unanticipated financial difficulties. Such events would have to be explained in terms of this model as involving extraordinary changes in the firm growth rate. How might such be modelled?

One point that might be made is that an ability to account for "jumps" in a small firm's history can be rather useful. The author has argued elsewhere (Aislabie, 1987, pp. 168—169) that:

> There are two critical points in the growth of any small firm. The first occurs when the entrepreneur is able to look beyond the need to earn an immediate income to considerations of a medium to long term nature. The second occurs when the pressure of administering the business means that considerations of a medium to long term nature are again pushed into the background.

Each of these critical points represents situations in which access to external finance can be extremely important. What appears to be unlikely is that adequate access to external finance is likely to lead to *smooth* changes in significant firm variables.

Returning to the analysis one possible, and perfectly acceptable explanation, is that the relevant influences are completely *exogenous*.[2] For example, government policy can change in a beneficial or harmful way for reasons quite beyond the control of the firm. While such influences can be allowed for in firm growth models, for example, by the use of dummy variables in multiple regression equations, such models are not concerned with the determinants of changes in these exogenous variables even if they are not stochastic events.

The problem is how to include influences on firm growth which lead to sudden or, at least, more dramatic, changes in firm size without appearing

Final version accepted on October 7, 1990

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Small Business Economics **4**: 307—314, 1992.

to specifically include as a determinant what one is trying to derive as an outcome. For example, little is achieved by assuming that possession of a particular kind of production function will lead, in due course, to a sudden change in firm size because that only changes the nature of the question being asked to one of why that firm, and not others, had that particular kind of production function.

Another problem is how to allow for the possibility of changes in firm size without having to introduce arbitrary limits to the postulated process to create an implicit upper or lower bound to firm size (as discussed by the author in Aislabie, 1984). Of course, there are modelling devices that can be used to represent non-exponential growth processes but in *qualitative* models, whose plausibility is not going to be appraised by empirical estimation and test, it becomes extremely important that they are subjected to the closest scrutiny.

This paper explores the possibility of using a "catastrophe theory" model of small firm growth to deal with these problems. The model allows only *smooth* changes in the two variables that drive small firm growth yet with the capability of *sudden* changes in the variable that correlates with firm size.

The remainder of the paper is divided into four sections. In the next is to be found a more comprehensive discussion of the issues involved in modelling small firm growth based on review of the relevant literature. The cusp catastrophe model is discussed in the third section while a model of small firm growth is presented in the fourth section. The implications of the argument are briefly considered in the Conclusion.

II. Literature review[3]

It should be noted that some discussions of the firm growth process, either implicitly or explicitly, are not relevant to the analysis of the process throughout the entire firm size range. This review is concerned with covering the issues pertaining to the growth process of the small firm while paying limited attention to that part of the literature on the firm growth process which does not distinguish out the small firm as an institutional reality in the way that it is in the small firm literature.

While Aislabie (1984) has canvassed some of the problems that arise in attempting small firm models consistent with the available data there are several underlying problems which are not really being addressed in such approaches. These are that there are no quantifiable models of the (entire) firm growth process[4] and it is doubtful that any data base (accessible to academic researchers) exists that provides adequate time series information on the variables that researchers would seek to incorporate in their models.[5]

While the number of contributions that *explicitly* consider the determinants of small firm growth are limited there is considerable *implicit* treatment of the topic in the literature on small firm performance and failure.[6] As a broad generalisation most non-stochastic models of small firm growth do not consider the small firm birth process and that literature, besides containing attempts to gather basic statistics on small firm growth, tends to concentrate on obstacles to growth.

It was Stanworth and Curran (1976), in surveying the literature on small firm growth, who noted that a "stage" model of growth was the dominant theme with the so-called "s-curve hypothesis" a minor theme. Although the tone of these papers is positivist Stanworth and Curran note that they lack precision in modelling, are not tested against appropriate data and appear to be reluctant to consider the realities represented by the actual size distribution of firms.[7] A similarly critical appraisal is to be found in Perry (1982).

A simple but effective model of financial obstacles to small firm growth is to be found in Binks (1979; 1980). He argues that "increases in demand for the firm's product occur in discrete steps of considerable size relative to turnover" because of the importance of a few large customers but that "increases in the supply of the firm's product often require positive net investment which is large in relation to the capital base."[8] However, institutional arrangements in the markets for finance do not suit this pattern of growth.[9] Some empirical support for the importance of structural barriers in small firm growth is to be found in Acs and Audretsch (1987)[10] while Boardman, Bartley and Ratliff (1981) found that ". . . high growth rate in sales for . . . small firms appeared to be dependent on relatively large size . . . heavy investment in operational assets thereby decreasing liquidity and utilizing an increasing amount of debt (especially in case of retail firms)".

Any review of this literature suffers from the weakness, that except for the research on small firm failure, there are few major contributors that deal with many of the areas touched upon. This reflects the fact that there few stylised "facts" about small firm growth that the model builder can draw upon. However, there are some pretty well-established negative statements that can be made with considerable confidence about small firm growth. Relatively few firms grow and even a smaller number find growth easy. Some apparently successful small firms become quite large, while others fail and others, again, stagnate. There appears to be some consensus as to the impediments to about small firm growth but it must be stressed that there is *no* properly estimated model of the determinants of about small firm growth that draws on a satisfactory data base. It is even possible that small firm growth is generated by a stochastic process (a question considered by Waring (1980)). While this does not preclude quantitative research of both a deterministic and a stochastic nature in this general area it does underline the importance of appropriate modelling.

III. The cusp catastrophe model

There are a variety of economic applications of the cusp catastrophic model. Besides those mentioned elsewhere in this paper reference can be particularly made to Brown (1979), Burghes and Wood (1984), Chidley, Lewis and Walker (1978), George (1981), Harris (1979), Isnard and Zeeman (1978) and Zeeman (1974). The basic idea behind the mathematics of this mdoel is summarised by Scapens, Ryan and Fletcher (1981):[11]

> Catastrophe Theory is concerned with the analytical explanation of *sudden* changes in the behaviour of a system arising as a result of *smooth* changes in the factors which determine the attainable equilibrium states of that system.

The application of the cusp catastrophic model closest in spirit to the one presented in this paper is to be found in Scapens, Ryan and Fletcher. They give an example in which credit extension is a function of return on funds and the company's operating risk. The example shows how an apparent continuous relationship between credit extension and return on funds will change once the company's operating risk is taken into account. The effect of the additional variable (operating risk) in the model is that while a small variation in return by a successful company may not change the credit worthiness class to which the firm is assigned a change in return that exceeds some limit will cause a sizeable "jump" (positive or negative) to appear in the relationship between credit extension and return on funds (as well as operating risk).

This story can be handled by a catastrophic manifold which embodies one state variable and two control variables, the normal factor and the splitting factor which is responsible for the "jump" conditions in the model.[12] Note that although there is a "jump" in the state variable both the other two variables are smooth. The jump in the state variable has not resulted from an assumption of the existence of a "jump" in either of these control variables.

Like all comparable models the geometric outcome arises because of the algebraic and topological relationship between the variables. The model is useful and challenging because the causes of the "jump" in the state variable are *endogenous*. Without a knowledge of the possibility of this kind of relationship an initial observation of the data gained, say, by plotting the state variable against each of the two control variables in turn might lead to a fruitless search for dummy or other kind of *exogenous* variables to explain an apparent obvious lack of regularity in the data.

Before giving the model any economic content it is useful to pursue further how the "jump" in the state variable is handled. This will be done with the aid of Figure 1, which draws upon Isnard and Zeeman (1976, p. 71). In the Figure the control variables are represented by the base of the diagram. The exposition first considers the various outcomes of the process which are to be found on the surface and then the changes in the control variables at the base that brought about these outcomes.

The state (or response) variable is given by the vertical axis and the (folded) surface in the Figure shows that response achieved for differing combinations of the control variables. Of course, an inspection of the Figure reveals that that since

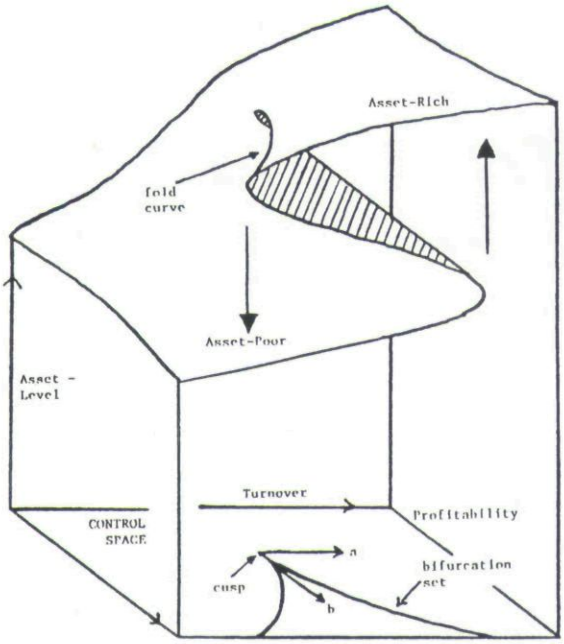

Fig. 1. The catastrophe theory model of small firm growth.

"jumps" (whether upwards or downwards) occur only at the edges of a fold then they are obviously not occurring elsewhere and, indeed, it is possible to trace out paths on the response surface that do not lead to a simple "jump". There are three possibilities.

The first is called bi-modality and means that where the fold occurs the path is on the top sheet or the bottom depending on its direction of approach. The second is called hysteresis and comes about where a fall would occur at a different point from an upward leap.[13] Finally, there is divergence in that two paths may start from slightly different initial conditions but lead to quite differnt values of the state variable because one path ends up on the top sheet but the other on the bottom sheet.[14] This will be clearer when the base of the diagram is discussed and to this we now turn.

Each point on the base diagram represents some combination of the control variables and each path shows how particular combinations of these variables may changes. Above each point is one or more points on the response surface (depending on where the point on the base is in relation to the fold in the surface).

The Figure indicates that the "jumps" occur on one of the two boundaries of what is termed the bifurcation set (the "jump" actually occurs when a boundary, that represents the edge of the fold in the response surface above, is reached by one of the variables which is called the splitting factor). These boundaries meet at a point from which the name cusp is derived. It will be noted that the area of the bifurcation set corresponds to the fold curve vertically above it.

While a "jump" will occur on one of the two boundaries of the bifurcation set whether it is the first or second boundary met by the splitting factor depends on whether or not the boundary is immediately below the fold in the response surface above. Where the first boundary is not the occasion of the "jump" a delay is said to occur. If the path of the splitting factor is followed, at the base and on the response surface, first in one direction and then in the other it will be seen that hysteresis has occurred in that a fall would occur at a different point from an upward leap.

Consideration has been given to the case where the splitting factor has traversed the bifurcation set from one boundary to the other. What happens if the path entered the bifurcation in the vicinity of the cusp and proceeded in the direction of the base of the bifurcation set? If the path is traced out on the response surface above it will be seen that the outcome depends on whether the path on the base below passes to the left or right of the cusp. But in neither case is there any "jump". In tracing out possible paths it should be noted that the shaded area in the Figure is inaccessible, points on the edge are semi-stable while those on the underside represent unstable maxima.

It will be noticed that the boundaries of the bifurcation set in Figure 1 do not meet at the origin of the two control variables. If they did it would mean that catastrophes could occur for all values of one control variable if the other changes suitably. Therefore, the Figure is drawn to imply that some values of the control variables cannot generate catastrophes. Also the boundaries can be drawn so that the bifurcation set is skewed towards the axis of the control variable whose changes are of less relative significance.

All the possibilities considered in the discussion can be given economic content although whether all paths that are mathematically feasible make good economic sense will depend on the precise contexts in which the model is used. This question will be taken up below.

IV. A model of small firm growth

The small firm is visualised as starting with self-employment (Casson (Chapter 16, 1982)). This provides inputs to the production function of the new economic unit including human capital which must involve some ability in dealing with potential customers (a matter that may have been taken care of by previous employers).

Since small firm growth involves more than substituting one income source for another Casson attributes the ability to achieve small firm growth to superior judgment. The firm may begin to grow because of the ability of its founder.

While increments to turnover may be large as suggested by Binks (1979; 1980) there is no data available on the size of the increments across all firms. There does, however, appear to be reason to believe that some small firms (the growing ones) can increase their sales beyond whatever the initial level may have been. This increasing turnover generates a need to obtain improved human and physical capital. Access to this human and physical capital depends on financial capital. At this point obstacles to small firm growth arise,[15] especially those relating to inadequate access to capital because the ability of the firm founder is not regarded as self-evident or yet proven. Despite these obstacles the record indicated that typically there is increasing reliance on financial capital provided by sources from without the firm. Access to outside funds will depend on the small firm's financial record.

If the assets of the firm are used as a measure of firm size then firm growth can be envisaged as a change in the amount of assets under the control of the firm. The assets of the firm depend, among other things, on firm turnover and firm profitability (as a indicator of the firm's ability to attract outside funds). Turnover will be used as the normal variable. It is expected that firm growth is driven by changes in turnover but with the precise relationship strongly affected by the presence of the splitting factor control variable, profitability, expressed in terms of profitability per unit.

While any change in assets is regarded as primarily driven by a change in turnover the ranges of turnover that can be achieved depend on the level of assets held at a particular point of time. A small firm can be regarded simplistically as being asset-rich or asset-poor (in terms of the assets needed to maintain its current level of turnover) because its profitability record reflects the perceptions credit providers have of the achieved turnover. An asset-poor firm can squeeze additional turnover out of its current assets.[16] In doing so, or as the result of simultaneous action, it will seek to improve its profitability per unit by containing and lowering costs. This can allow it to reach the position where its turnover and profitability per unit interact to create perceptions by credit-providers so that more financial resources are forthcoming.

The circumstances of the asset-poor firm changes when new investment allows it to "jump" to an asset-rich status. Of course, the situation will be reversed if an asset-rich firm suffers a decline in turnover. Disinvestment will only occur but only after a delay. That is, in terms of the above analysis, it is being suggested that hysteresis is more likely than bi-modality in that it is unlikely that any particular combination of base and surface points are likely to be found where the small firm instantaneously switches between an asset-rich and an asset-poor status. It is worthwhile exploring the underlying economic reasons why this should be so.

While the ability to achieve small firm growth may be attributed to superior judgment this can be a commodity which it is difficult to demonstrate to others. By the same token it is difficult to know whether any adverse experience of the firm represents a hurdle that will be overcome or a decline in judgment previously displayed. Consequently, there is a tendency for responses to the firm's experience — its changes in turnover — to lag behind the actual event with changes in access to assets representing point revisions of expectations as to the future of the firm.

The model allows that some firms do not fit into this pattern. However, in bringing to attention the alternatives inherent in the model, it is possible to reject some on the grounds of economic implausibility. For example, there is no expectation that once a firm possesses some (lesser) amount of assets it will immediately "jump" to a higher level of assets which, if it ever falls below, will signal an immediate fall to the earlier lower level of assets. Nor it is expected that there is some critical level of assets from which the small firm is likely to keep growing or keep declining.

The discussion suggests that it is highly unlikely that "jumps" could occur for all values of one control variable if the other changes suitably. As indicated above this is reflected in the fact that Figure 1 is drawn so that the boundaries of the bifurcation set do not meet at the origin of the two control variables.

Events, or the firm's reactions to them, can lead to a more plausible outcome where there is no "jump" at all. The technology may be such that it is not possible to squeeze additional turnover out of current assets in a situation where turnover expands hand-in-hand with a smooth increase in assets. This absence of significant indivisibilities can be associated with a slow asset growth pattern, slow growth in turnover and poor profitability.

V. Conclusion

The model presented above does not purport to give either a descriptive picture or a quantifiable framework. It does provide a guide to some patterns of small firm growth as well as, if only by inference, pointing to other patterns that are very unlikely. Perhaps its most interesting feature is that it can allow for growing firms which go through phases of steady growth as well as phases during which "jumps" occur. Furthermore, the current phase is determined by the relationship between the control variables, turnover and profitability.

While our present state of knowledge does not allow the relationship between the control variables and the likelihood of "jumps" to be discussed in other than a probabilistic sense it can be suggested that behind the stochastics can lurk an economic rationale. Clearly it is important to have some idea as to whether the firm is likely to be subject to "jumps" or not. As the author has discussed elsewhere,[17] most attention appears to have been paid to the significant downward "jump" which leads to a failure of the firm. A particular interesting, and useful, area of research would be to examine the conditions under which the small firm is likely to be subject to any kind of "jump" at all.

The relationship between the concepts outlined above and the "life cycle" model of the small firm needs to be explored. A tentative hypothesis is that the small firm that does not experience some kind of "jump" may go through whatever stages of development that are distinguished to stagnate until it fails, is taken over or is re-juvenated.

Notes

[1] Note that Penrose (1952) argues that the model appears to be hardly applicable to a firm that grows to be a publicly held corporation.

[2] For a textbook discussion of this distinction see Elliott (1973, Chapter 14).

[3] This literature review is restricted to contributions not explicitly considered elsewhere in the paper.

[4] The most satisfactory model of the firm growth process, that to be found in Marris (1963, 1964), is *qualitative*, in character.

[5] An exception to this generalisation, of course, are the United States Establishment Microdata files of the U.S. Small Business Administration (see Phillips and Kirchhoff (1989)). It should be noted that empirical studies referred to in this paper, and indeed most to be found in the literature, are usually based on very small samples and cannot purport to be representative of small firms in general.

[6] Such as is represented by Cragg and King (1988) and Berryman (1983).

[7] Among examples of the use of these models is Peterson and Shulman (1987) who show for their sample "... that a life cycle of capital structure among small growing firms depends on age, size and economic development of the host country" and, particularly (Figure 1), that recourse to bank credit depends on the stage of the life cycle.

[8] But note that Storey (1985) argues that it is useful to distinguish between the problems facing *new* firms and the problems facing *small* firms.

[9] Various government enquiries have acknowledged that there is a problem without coming to very explicit conclusions as to whether the government can, or should, do much about it. See Committee to Review the Functioning of Financial Institutions (1980, Appendix 2) and Australian Financing System (1981, Chapter 38). Perry (1983) has argued that concentration on the "finance gap" hides more serious structural problems facing the small firm. Ulrich and Arlow (1980) have developed a normative (financial) model of small firm growth.

[10] See also Vozikis (1984).

[11] Their Appendix provides a useful intuitive insight into the appropriate mathematics.

[12] In the Scapens, Ryan and Fletcher model these are designated as credit policy, measured return and operating risk respectively. Note that if a model with one state variable and two control variables displays catastrophic behaviour it must be a cusp catastrophe and possess the mathematical properties discussed in this section.

[13] In terms of Figure 1 the edge labelled "fold curve" is

important. Note that it continues up to the cusp and then returns on the inside obscured section at the right hand extremity of the shaded area (the topside of the shaded area is unattainable).

[14] That is, if the path starts to the "northwest" of the cusp and proceeds close to the left hand of the side edge labelled "fold curve" it is assumed that assets are being lost as one variable adjusts less than the other while if the path proceeds close to the right hand of the side edge labelled "fold curve" it is assumed that assets are being gained as, again, one variable changes less than the other.

[15] Another attempt to model these obstacles is to be found in Reid and Jacobsen (1988, Chapter 2). Shaffer (1986) has looked at the growth problems of tiny firms while others (for example, Weston and Brigham (1981, Chapter 25)) envisage the small firm seriously considering "going public" at the end of the second of four stages. There is also, as pointed out by Acs and Audretsch (1989), the literature on barriers to entry although much of this has in mind new firms very much larger than those contemplated in this paper.

[16] Scapens, Ryan and Fletcher (1981) make the same point in a different context.

[17] This paper is a complete recasting of the technical part of Aislabie (1990) which drew heavily on Scapens, Ryan and Fletcher (1981).

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