Modeling the Timing of Business Firm Exits

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ABSTRACT. The previous theoretical studies of the timing of business exits have employed two different approaches. They have either developed sequential decision-making models or models based on gambler's ruin. The purpose of this paper is to evaluate both approaches. For this task some simple models of the exit process are developed and it is shown that both types of models are capable of generating the observed patterns of firm mortality. The reasonableness of the assumptions and the importance of key variables in the two models are tested using a case study of restaurant exits. We find that the sequential decision-making model would have to be rather complex to serve as a realistic descriptive model of the exit process. Gambler's ruin models, on the other hand, provide a realistic descriptive model not only of the environment in which a large portion of this industry operates, but of the decision process as well. Models based on gambler's ruin may, therefore, provide a potentially useful approach for modeling exit process in other industries dominated by small firms.

1. Introduction

The exit of business firms from industry is a relatively frequent occurrence, particularly in industries dominated by small firms. Firms may exit the industry by filing for bankruptcy followed by liquidation.[1] Alternatively, firms may exit by simply discontinuing operations, discharging employees, and selling its assets. As shown in Table I, bankruptcies account for a relatively small percentage of total exits. Therefore, exits via bankruptcy/liquidation should count for even a smaller percentage. However, this way of exit is relatively more important for large corporations.[2] Small firms are more likely to exit by simply discontinuing operations.

The available theoretical analyses on the timing of business exits have followed two different approaches: (1) developed models based on the sequential decision-making approach to entrepreneurial action; or (2) developed models based on gambler's ruin. Sequential decision-making models assume that at any given moment the decision to continue or quit is based on the sum of past experiences and currently perceived alternatives. Gambler's ruin models assume that a firm (i.e., the gambler) starts off with a specified wealth position and then each period either wins (gains wealth) or loses (diminishes wealth) through some process involving random events. The gambler's ruin occurs when the firm's wealth position is reduced to zero.

A formal model of sequential decision-making is Jovanovic (1982). In his 'selection by learning' model, firms are uncertain about their own efficiency and can resolve this uncertainty only by entering and producing. Costs are random and different among firms and consist of a firm-specific effect plus a serially uncorrelated shock. Firms are assumed to know the distribution of costs among firms, but no firm knows its true costs. Efficient firms grow and survive; inefficient firms decline and fail. For new entrants, a few unfavorable (high cost) periods will lead to the conclusion that the firm specific cost effect is high and will induce exit. As the firm's experience base broadens, the marginal influence of an additional observation on the firm's belief about their efficiency lessens, and the probability that the current period's cost realization will induce exit declines. The model generally agrees with observed patterns of entry, growth, and exit behavior.

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TABLE I
Number of discontinued and bankrupt business per 10,000 operating concerns by industry, 1951—1955 annual average

Industry	Number of discontinued businesses	Number of business bankruptcies	Ratio of bankruptcies to total discontinuances
Manufacturing:			
Food and kindred products	450	57	0.13
Textile mill products	910	76	0.08
Apparel	1,080	116	0.11
Leather and leather goods	780	146	0.19
Lumber and timber products	1,820	17	0.01
Furniture and fixtures	660	164	0.26
Paper and allied products	350	51	0.15
Printing and publishing	330	21	0.06
Chemicals and allied products	590	56	0.09
Stone, clay, and glass products	640	38	0.06
Metals, primary and fabricated	500	63	0.13
Machinery, except electrical	690	57	0.08
Electrical machinery	630	114	0.18
Transport equipment	550	85	0.15
Subtotal	9,980	1,061	0.11
Retail:			
Groceries	560	19	0.03
Motor vehicles	1,080	26	0.02
Automotive parts and accessories	600	17	0.03
Shoes	540	47	0.09
Lumber and building products	550	44	0.08
Appliances, radios, and televisions	830	89	0.11
Home furnishings	700	74	0.11
Eating and drinking places	950	24	0.03
Drugs	270	24	0.09
Subtotal	6,080	364	0.06
Total	16,160	1,425	0.09

Source: Miklius, Casavant and Huang (1976).

In a recent paper, Frank (1988) presents a descriptive intertemporal model of industrial exit that is closely related to the model of Jovanovic. In this model, returns are a function of luck, effort and talent. Initially, the level of talent is unknown and can only be discovered through experience. The rate of entrepreneurial learning is assumed to be positively related to the scale of operation and the scale of initial operation (entry size) is assumed to offer evidence on the beliefs held by the entrepreneur (the larger the entry size, the more optimistic the entrepreneur). As sunk costs in Frank's model provide a measure of the entrants' optimism, there is a positive relationship between the magnitude of sunk costs and the life of the firm. Another assumption, resulting from the specified interaction between scale and entrepreneurial learning, is that new entrepreneurs will tend to work harder, while near retirement entrepreneurs will tend to work less. Frank's model predicts that exit rates are initially low, then increase to a peak, and then decline with the age of the firm.

Gambler's ruin models are based on initial work by Feller (1968) and have been generally used as predictive models. Wilcox (1976), analyzed the financial data for a large set of large firms and used a gambler's ruin model to predict which firms were likely to fail. Similarly, Santomero and Vinso (1977) and Vinso (1979) applied gambler's ruin models to predict bankruptcies of individual firms.[3]

The two types of models have been used for different purposes. The sequential decision-making models have been developed to provide theoretical explanations for empirical observations that smaller firms have higher and more variable growth rates and are less likely to survive. They have also been used to provide an explanation of the observed correlation patterns between firm size, concentration, and rates of return at a point in time and over time. Gambler's ruin models, as mentioned previously, have generally been used for prediction of firm exits.

The criticisms of the models, however, have tended to disregard the different purposes for which the models were developed. For example, Scott (1981) criticized gambler's ruin models for their simplistic assumptions about management behavior even though the realism of the assumptions is irrelevant in the case of predictive models and the models should only be tested by their predictive accuracy. Similarly, Frank (1988) rejects theoretical analyses based on gambler's ruin because the model is inconsistent with access to capital markets. Again, this criticism is not relevant since the model has been used for predictive purposes.[4]

Furthermore, most of the previous studies have focused on bankruptcy of firms rather than their exit. Even Frank (1988), whose study deals with firm exits, uses data provided by Altman (1983) to test the implications of his model. Altman's data, however, pertain to bankruptcies rather than exits. As discussed previously, bankruptcy does not necessarily result in exits and the time pattern of exits via bankruptcies is likely to be different than the time pattern of exits via discontinuing operations.[5]

Comparison of results between studies is further hindered by the use of different definitions of firm exit. The basic difference in the definitions is whether a change in ownership of a firm is to be counted as an exit or a continuance. Theoretically, only discontinuance should be counted as an exit as change of ownership could occur for many reasons, including positive factors. In empirical studies, it is sometimes not possible to distinguish between change of ownership and exit.

The purpose of this note is to examine the usefulness of the two approaches in explaining the timing of business exits rather than their bankruptcies. The first step is to investigate if the models are capable of reproducing the exit patterns actually observed in the real world. For this purpose, the available data on business firm exits are assembled from previous studies and analyzed in Section 2. These data clearly establish that the mortality of business firms in industries dominated by small firms exhibit something analogous to 'infant mortality' in biological populations, i.e., the conditional probability of a firm's exit decreases with its age. In Sections 3 and 4, some simple 'generic' models of the exit decision process are presented and it is shown that they are all capable of generating the observed shapes of the mortality functions. Section 5 reports on the results of a survey of 21 former Honolulu restaurant owners. While it obviously is hazardous to generalize from such a small survey, the results are interesting. They suggest that the economic environment in which most of the restaurants operate resembles that envisioned in the gambler's ruin model. It is hardly surprising, therefore, that the gambler's ruin model explains more than half of all restaurant exits in Honolulu. For the remaining restaurant exits the sequential decision-making model appears to be more appropriate. However, this model would have to be much more complex than the models presented in this paper and would prove difficult to obtain information. A brief summary of the conclusions is presented in the last section.

2. Previous studies of firm mortality functions

Several empirical studies of business mortality could be found in the literature. The first one is Hutchinson *et al.* (1938). This study covered the experience of all business enterprises, except financial concerns and public utilities, in the city of Poughkeepsie, New York, for a period of 94 years, 1844—1936 inclusive. Two definitions of mortality were used: (1) A change in ownership was assumed not to terminate the business as long as the firm's identity could be traced; and (2) a change of ownership was assumed to terminate the business. The total number of observations (firms) were 11,222 and 10,033 respectively.

The second available study, Churchill (1955), provided estimates of business survival for the US based on the experience of firms in the immediate post-WWII period (1947—54). Change of ownership was considered to be a termination of business. The estimates were based on 6.3 million business records available for the study. More recently, Star and Massel (1981) estimated the survival rates for various types of 17,252 retail businesses started in Illinois in one single year, 1974. Transfer of ownership was defined as a 'discontinuance' by Star and Massel. The most recent data available are found in Aungtrakul's (1988) study of the survival of restaurants in Honolulu, Hawaii. In this study, the change in business name was defined as a discontinuance. The data, shown in Table II along with the data from the other cited studies, indicate that restaurant survival in Honolulu roughly parallels survival in the retail and service industries reported in the other studies.

Lomax (1954) used the data compiled by Hutchinson *et al.*, to experiment with various possible mathematical forms for business conditional mortality functions ($CM(t)$), where conditional mortality is defined as the probability that a firm exits in period t given that it has survived through period $t-1$,[6] and concluded that the hyperbola

$$CM(t) = \beta/(\alpha + t), \tag{1}$$

where t is the age of the firm, gives the best fit for the retail and service industries while the exponential function

$$CM(t) = \alpha e^{-\beta t} \tag{2}$$

gives the better fit for firms in the manufacturing

TABLE II
Cumulative mortality rates

Age	*Poughkeepsie, N.Y.*[a] Manufact.		Retail		Services		*U.S. 1947*[b] Man.	Ret.	Serv.	*Illinois*[c]	*Hawaii*[d]
(years)	(1)[e]	(2)[f]	(1)[e]	(2)[f]	(1)[e] (percent)	(2)[f]					
0.5							18	26	23		
1.0	23.1	24.0	29.6	32.5	32.7	32.9				18.6	28.3
1.5							40	51	47		
2.0	34.6	37.1	43.8	45.8	45.7	47.2				39.5	43.0
2.5							54	64	60		
3.0	46.9	49.8	53.2	55.0	55.1	56.8				51.3	53.0
3.5							62	71	67		
4.0	54.7	57.8	59.4	61.6	61.8	63.7				59.3	58.2
4.5							67	75	72		
5.0	60.2	63.2	64.3	66.2	66.9	66.0				64.9	63.6
5.5							71	78	75		
6.0	65.5	67.7	68.4	70.4	70.8	73.0				66.8	66.6
6.5							74	80	77		
7.0	67.8	70.4	71.5	73.5	74.3	76.3				NA	69.3
7.5							76	81	78		
8.0	70.2	73.3	74.1	76.3	76.9	79.3				NA	71.4
8.5							77	82	79		
9.0	72.6	76.0	76.2	78.6	79.2	81.5				NA	73.8
9.5							78	83	80		
10.0	74.6	78.7	78.2	80.8	81.2	83.4				NA	76.1
10.5							80	84	81		

NA = Not Available
[a] Source: Hutchinson, Hutchinson and Newcomer (1938).
[b] Source: Churchill (1955).
[c] Source: Star and Massel (1981).
[d] Source: Aungtrakul (1988).
[e] Not counting change in proprietorship as a new business.
[f] Counting change in proprietorship as a new business.

industries. The coefficients of correlation were 0.99, 0.98, and 0.96 for the retail, service, and manufacturing industries respectively. Both specifications conform to the desirable boundary conditions and monotonic behavior exhibited by the data. However, the hyperbola has the advantage that simple expressions can be obtained for the cumulative mortality (M) and survival functions (S), i.e.,

$$S(t) = 1 - M(t) = [\alpha/(\alpha + t)]^{\beta}. \qquad (3)$$

Hyperbolas were fitted to all the available data sets and the estimated parameters are reported in Table III along with the estimated median life expectancy. The fits were quite good and all the estimated parameters were statistically significant at the 99 percent level with the exception of the Illinois data for which there are very few data points. The results show that while there are differences in the mortality of firms in different types of businesses and differences in the mortality of firms in the same type of business among available data sets, the form of the mortality function appears to fit the data in all sets and for every type of business. More important, they lead to the conclusion that the conditional probability

TABLE III
Estimated mortality function parameters using hyperbolas, by type of business

Item	Estimated parameters[a] α	β	Stnd. error of Y	Median life expectancy (years)
Poughkeepsie, 1844—1936[b]				
Manufacturing	3.81 (0.386)	1.08 (0.0729)	0.00880	3.43
Retail trade	2.01 (0.0923)	0.84 (0.0229)	0.00437	2.58
Services	2.03 (0.189)	0.91 (0.0496)	0.00949	2.32
Poughkeepsie, 1844—1936[c]				
Manufacturing	4.20 (0.318)	1.26 (0.0659)	0.00685	3.08
Retail trade	1.91 (0.169)	0.87 (0.0444)	0.00891	2.33
Services	2.32 (0.316)	1.03 (0.0865)	0.01346	2.23
U.S. 1947—1954				
Manufacturing	1.73 (0.141)	0.85 (0.0394)	0.08275	2.18
Retail trade	0.98 (0.641)	0.78 (0.0351)	0.09437	1.40
Services	1.05 (0.104)	0.73 (0.0377)	0.01196	1.66
Illinois: businesses started in 1974				
Retail trade	6.97 (2.75)	1.91 (0.608)	0.02128	2.76
Honolulu, Hawaii 1971—1985				
Restaurants	1.68 (0.119)	0.76 (0.117)	0.01017	2.51

[a] Asymptotic standard errors in parentheses.
[b] Not counting change in proprietorship as a new business.
[c] Counting change in proprietorship as a new business.

of a firm's survival indeed increases with the firm's age. However, use of the hyperbola as a functional form to model mortality is restrictive, once specified, the conditional mortality is necessarily monotonically decreasing and hypotheses about the form of the conditional mortality function cannot be tested.

In the economic duration literature, a class of functions derived from probability models known as hazard functions are utilized to model phenomena involving time spans (Kiefer, 1988). In the context of the lifespan of a firm, a hazard function is simply a conditional mortality function. The use of two simple hazard functions, one based on the Weibull distribution and the second on the log-logistic distribution, allow for simple tests of constant conditional mortality (independent of time) and of monotonic conditional mortality to be carried out. The conditional mortality (or hazard) function corresponding to the Weibull distribution is:

$$CM(t) = \alpha\beta t^{(\beta-1)}, \tag{4}$$

which is linear in logs. A simple test of a hypothesis of constant mortality is a test of $\beta = 1$. Acceptance of this hypothesis implies that $CM(t) = \alpha$ which is the conditional mortality or hazard function derived from the exponential distribution.

The conditional mortality function corresponding to the log-logistic probability model is[7]

$$CM(t) = \alpha\beta t^{(\beta-1)}/(1 + \alpha t^{\beta}). \tag{5}$$

When $\beta > 1$, conditional mortality first increases with t and then decreases. When $0 < \beta \leqslant 1$, conditional mortality decreases with age. A test of the monotonicity of conditional mortality is then equivalent to testing whether $\beta > 1$. After fitting (4), the hypothesis of constant conditional mortality was rejected (at the 99 percent significance level) for every data set. The results of fitting log-logistic functions to the available mortality data are given in Table IV. The estimated parameters for four of the data sets indicated the possible presence of a non-monotonic conditional mortality. However, in each of these cases, the peak conditional mortality would have occurred at less than one year of age, and conditional mortality was monotonically decreasing afterwards. As this peak occurred in range for which there were no data and β was only significantly larger than 1 in two cases (manufacturing in New York and retailing in Illinois), the data can be said to support a hypothesis of monotonically decreasing conditional mortality, especially after the first year. Thus, based on the available evidence, a hypothesis of a hyperbolic firm mortality functions could not be rejected.

3. Exit models based on gambler's ruin

We assume the following basic scenario. As in the case of an infant, a new firm may be born with birth defects. These 'birth defects' pertain to errors made by the entrepreneur. For example, he/she may have overestimated the demand for the firm's product or underestimated production costs, i.e., had he/she made the correct estimates, the firm would not have entered the industry. However, once the firm had entered it would take time to discover the errors. The signals on whether errors were made are provided by the firm's realized revenues and costs. However, there is a lot of noise in this information due, in part, to the stochastic fluctuations in demand. Given this basic scenario the main task is to determine the timing of exit which, in turn, requires specification of the 'decision rule', i.e., the definition of the event(s) that will cause the firm to leave the industry. One such rule is provided by economic theory which states that a firm will exit the industry if it cannot cover its variable costs. This rule, however, has to be modified to account for variations among periods, i.e., the firm may more than cover its variable costs during one period but not in the next one. Thus the modified exit decision rule is specified as follows: the firm will exit the industry in the first post entry period when its cumulative cash outflow exceeds its cumulative cash inflow. Therefore, the exit rule supplied by economic theory leads one to models based on gambler's ruin. The question remains whether models based on gambler's ruin are capable of generating the shapes of empirical mortality functions.

To simplify the discussion, initially let's assume that the entering firm has no cash reserves and no fixed assets or that the assets it owns have no salvage value. Let x_i represent revenues in period i and assume that the relationship between operating costs, which are defined to be equal to cash outlays, and revenues are given by $c_0 + cx_i$. The constant term (c_0) represents the fixed cash outlay per period and c the marginal cost per dollar of

TABLE IV
Estimated mortality function parameters based on the log-logistic probability model

Item	Estimated parameters[a] α	β	R^2	Age at max *CM* (years)	t-Stat Ho: $\beta > 1$
Poughkeepsie, 1844–1936[b]					
Manufacturing	0.228	1.018 (0.0214)	0.996	0.060	0.830
Retail trade	0.411	0.931 (0.00735)	1.000	0	
Services	0.468	0.937 (0.0235)	0.995	0	
Poughkeepsie, 1844–1936[c]					
Manufacturing	0.303	1.071 (0.0214)	0.998	0.217	3.305
Retail trade	0.450	0.940 (0.0191)	0.997	0	
Services	0.448	1.014 (0.0362)	0.990	0.030	0.375
U.S. 1947–1954					
Manufacturing	0.459	0.951 (0.234)	0.995	0	
Retail trade	0.727	0.886 (0.0300)	0.990	0	
Services	0.619	0.874 (0.0331)	0.987	0	
Illinois: businesses started in 1974					
Retail trade	0.253	1.230 (0.0710)	0.987	0.741	3.244
Honolulu, Hawaii 1971–1985					
Restaurants	0.406	0.889 (0.0117)	0.999	0	

[a] Standared errors in parentheses.
[b] Not counting change in proprietorship as a new business.
[c] Counting change in proprietorship as a new business.

revenue (which must be less than 1 for the firm to be profitable). Given these definitions, the firm will survive the first period if $x_1 \geqslant c_0/(1 - c)$ or $x_1 \geqslant \Theta$ where Θ is the revenue required to achieve a breakeven cash flow. Therefore, the probability that the firm will exit in the first period is simply the probability that revenue (x) is less than Θ or

$$P[x_1 < \Theta].$$

Similarly, the probability that the firm exits during the second period would be

$$P[x_1 + x_2 < 2\Theta \mid x_1 > \Theta].$$

That is, for the firm to fail in the second period, the firm has to have survived the first period, and the sum of revenues in excess of Θ in the first period plus the revenues in the second period have to be less than Θ.

In general, the probability of a firm failing in period n would be

$$P[X_n < n\Theta \mid X_{n-1} > (n-1)\Theta], \tag{6}$$

where

$$X_k = \sum_{i=1}^{k} x_i$$

or the firm has to have survived period $n - 1$ and the revenues in period n combined with the excess of revenues over the breakeven levels from the $n - 1$ previous periods has to be less than $n\Theta$.

Clearly, the probability of exiting in the first period is the highest, and the probability of exit in a given period decreases as n increases, i.e., the conditional probability of surviving given the firm has survived the previous period increases as n increases. This simple model is similar, but less complex, than most gambler's ruin models appearing in the literature and is quite similar to Tinsley's (1970) multiperiod model.

The actual shape of the mortality functions will depend on the distribution of revenues (x) and the break-even value (Θ). Generally, the smaller the ratio of the breakeven level of revenues (Θ) to expected revenues (μ) the greater the likelihood of the firm's survival. Thus, anything that would increase the breakeven level, such as increased debt service, would shift the mortality function upward and decrease the probability of the firm's surviving.

The affect of variation of revenues also depends on the ratio of Θ to μ. When this ratio exceeds 1, any increase in variation will increase the likelihood of survival, i.e., shifts the survival function upward. When the ratio is less than 1, the impact is reversed, as variation in revenues increases, the likelihood of survival decreases.

Applications of gambler's ruin models typically assume that the firm starts with an initial cash or capital position. A similar set of mortality functions is generated if the assumption that an entering firm has no cash reserve is relaxed. Suppose that a firm has initial cash reserves of F. The firm will exit in period n if

$$X_n < n\Theta - F/(1-c).$$

The corresponding conditional probability statement is

$$P[X_n < n\Theta - F/(1-c) | X_{n-1} > (n-1)\Theta - F/(1-c)]. \tag{7}$$

This probability statement generates a set of curves with the same shapes as those generated by (6), but in this case the probability of survival (for the exponential case) increases by a factor of $e^{F/[\mu(1-c)]}$ if $F/(1 - c) < n\Theta$ and equals 1 otherwise.

So far we have assumed that the firm starts at full speed, i.e., the distribution of revenues is the same in the first and subsequent periods. A more realistic assumption would be that a firm starting business would have expectations that revenues would start at some level and then increase to a higher level as the firm becomes established. That is, the firm will expect to have low or negative cash flow initially and that the cash flow will increase over time, eventually leveling off at a (hopefully) positive level. In this scenario, to be viable, the firm must have cash reserves to cover the initial cash outflow.

For example, suppose expected revenues in period i are given by $(1 - \alpha^i)\,\tau$, $0 < \alpha < 1$. That is, τ is the maximum expected level of revenues for the mature firm and revenues of the young firm approaches this level from below. Actual revenues in any given period (x_i) will be stochastically distributed around $(1 - \alpha^i)\,\tau$. In this specification, the smaller α, the faster revenues approach τ. In this scenario, assuming the same simple cost function as above, i.e., Cost $= c_0 + cx_i$, the conditional probability of failing is

$$P[X_n < (nc_0 - F)/(1-c) | X_{n-1} > ((n-1)c_0 - F)/(1-c)].$$

Thus the faster the growth in revenues (the smaller α), the larger the cash reserves (F), and the larger τ, the less likely the firm is to exit. The mortality function will be flat and equal to zero for the first $n = F/c_0$ periods and will start to increase thereafter.

All of the above models could be implemented using hazard functions and are capable of generating the shapes of observed mortality functions. That is, models based on gambler's ruin cannot be rejected on empirical grounds.

4. Sequential decision-making exit models

Suppose a firm starts with some revenue and cost expectations, and as the firm matures, management becomes aware (learns), that the actual performance is less than the expected performance. If actual performance continues to fall below expectations, the firm may decide to cease operations. That is, the firm has a prior estimate of the distribution of revenues and uses observed revenues to revise its estimates, and goes out of business when its posterior estimate falls below a certain level.

For most realistic distributions the derivation of a revised probability density function becomes very complex. One exception, however, is the normal distribution. If the firm had prior expectations of average revenues μ_0 with expected variance σ^2/n_0, and after n periods observes total revenues of X_n, and if revenues are distributed normally, then according to Bayes theorem, the posterior distribution will be normal with mean[8] equal to $(\mu_0 n_0 + X_n)/(n_0 + n)$ and variance $\sigma^2/(n_0 + n)$. If the firm has a decision rule such that it will cease operations when expected revenues fall below β, the probability that a firm will cease in period n is

$$P[X_n < \phi(n)|X_{n-1} > \phi(n-1)],$$

where

$$\phi(n) = n_0(\beta - \mu_0) + n\beta. \quad (8)$$

This exit rule is similar to the one employed by Jovanovic (1982) and is of the same general form as the conditional probability statements (6) and (7) above and will generate curves of the same general shape. The larger β, the smaller the prior estimate of the mean (μ_0), and the smaller the true mean of the population (μ), the more likely the firm is to receive a signal indicating that it should cease operations. Thus if the firm underestimates revenues, it is more likely to receive a negative signal. This has the expected impact of decreasing the probability of survival in any period.

When the firm has no strong prior beliefs regarding the expected distribution of revenues and costs, the decision to continue or not to continue operations can be modeled as a test of composite hypotheses. That is, the firm will continue in business if some measure of performance is greater than a critical value ϕ and will cease if the measure is below ϕ. Suppose the measure of performance is total revenues to date (X_n). The probability of the firm receiving or observing a signal that would cause it to cease operations in period n is then

$$P[X_n < n\phi|X_{n-1} > (n-1)\phi]. \quad (9)$$

This probability statement will also generate curves of the same general shape as those generated by (6). The larger ϕ and the smaller μ, the greater the probability of receiving a signal to quit in period n. The impact of σ again depends on the relationship of the criterion ϕ to the mean. Also, the smaller n, i.e., the younger the firm, the greater the likelihood of receiving a negative signal.

Another factor that may affect the mortality function are the costs of exiting from the industry. Suppose the assets of the firm have a positive salvage value (SV). At a given point in time, the firm faces the choice of staying in business for one more period, in which case the net benefits to the firm would be

$$CP + E[\pi] + \delta SV,$$

where $E[\pi]$ is the expected present value of profits from continued operations, CP is the firm's current net cash position and δ is the appropriate discount factor ($0 < \delta < 1$), or to sell the firm now and receive net benefits of

$$CP + SV.$$

The decision on whether to continue business or sell out then depends on the ratio $E[\pi]/(1 - \delta)SV$. The firm will be better off continuing whenever expected profits from continued operations exceed the loss in current wealth incurred by postponing the receipt of the salvage value by one period. Thus, firms that have no salvage value (i.e., those firms whose start-up, establishment, and capital costs are truly sunk costs) will be less likely to go out of business than firms with assets with salvage value.

The above leads to the conclusion that exit models based on sequential decision-making are also capable of generating the observed shapes of mortality functions.

5. Case study: restaurant exists

The two approaches are used descriptively to model the processes underlying exit decisions. The realism of the economic environment in which firms operate and the factors they actually consider in making exit decisions are important in this type of modeling effort. Therefore, as the next task we focused on a case study of restaurant exits in Honolulu. The information for this task was supplied by interviews with 21 owner/managers of restaurants that have failed.

The restaurant industry was selected because of the relative availability of respondents as well as its historically high entry and exit rates. The sample was not a random one since it was rather difficult to locate former restaurant owners/managers. Rather than drawing from a sampling frame, we relied on leads from those being interviewed to locate others. The interviews were unstructured (i.e., no survey instrument was used) although the attention of respondents was directed toward factors identified in the preceding section both by asking questions of what happened in the case of a particular restaurant with which the respondent was involved and by use of hypothetical examples.

The twenty-one interviewees can be readily separated into two groups: (1) Thirteen owners/managers of restaurants that are no longer in business; and (2) Eight current owners/managers of restaurants who in the past had started another restaurant which subsequently failed. The first group's responses clearly indicates the importance of cash flow problems in the exit decisions. That is, in 12 of the 13 cases the restaurant closed when the owners ran out of money to pay bills. In the thirteenth case, there were additional complications but the restaurant finally closed because the owners ran out of cash. Thus, over 60 percent of the failed restaurants in the sample failed because they ran out of cash. Since the sample was not a random one, there is no way to statistically infer the proportion of restaurants in the population which failed due to cash flow problems. Nor are there data on the proportion of single- vs. multi-unit restaurants in the population. However, the consensus of industry representatives is that between 60 to 70 percent of all restaurants are single unit restaurants. If we make an assumption that the probability of failure is the same for the two groups of restaurants, then about 60 percent of all restaurant exits are attributable to cash flow problems since the sample approximates this proportion.

That a restaurant fails because of a cash shortage is not surprising. All the respondents emphasized that the restaurant business necessitates a long-run commitment. None of them expected to earn a profit on their investment for the first 3-to-5 years. The primary concern during these years is survival and the main measure of performance is cash flow. If the restaurant survives longer than this period it is expected to start to show a profit, particularly, after the initial loans are paid off.[9] Furthermore, because of the risk involved, financial institutions, as a rule, do not lend to restaurants without an established track record unless the owner pledges his own personal assets as collateral. Thus, the firm's access to capital markets is rather limited.

The respondents attributed their 'running out of cash' to 'undercapitalization', which is consistent with the importance of the cash reserve position discussed above.[10] All respondents agreed that insufficient funding is the main cause of restaurant failure. Sufficient cash reserves to cover six to twelve months of operating expenses was suggested as a rule of thumb for the required cash reserve for starting a new restaurant.

It appears, therefore, that for a majority of the restaurants the economic environment in which they operate resembles the gambler's ruin models rather closely. In contrast, none of the restaurants with which the second group was involved failed because of a cash shortage. At the time these restaurants went out of business, their owners had at least one other successful restaurant in business and apparently could have continued. It was not possible from the interviews to derive a simple exit rule. Expectations, however, definitely played a role. The gross revenues of a new restaurant are expected to continue to increase due to the development of repeat patronage and a new restaurant is expected to start generating a positive cash flow by the end of the first year. However, none of the restaurants, where a shortage of cash was not the immediate cause of business failure, ceased operation during the first months when gross revenue failed to increase or when cash flow failed to become positive after being in business for a year. Instead, the owners tried to remedy the perceived problems by changes in the menu, changing prices, increased advertising, etc. Thus, the timing of exit depends on the persistence in trying and the time required to observe the effects of these changes. In some cases, it took up to three years before the restaurant was finally closed.

Further complications are introduced by owners attempts to sell the restaurant and the belief, shared by some of the respondents, that it is easier to sell an ongoing restaurant than it is to sell an out-of-business establishment. Such a belief encourages a restaurant owner to keep operating in the hope of finding a buyer and/or securing a better price.[11]

The exit decision is even further complicated by legal implications of the restaurant leases. A typical restaurant lease specifies a minimum rent or a percentage of gross sales, whichever is higher, and generally runs for 15 to 20 years. The lease obligates the restaurant owner to make these payments for the duration of the lease. If the restaurant fails, the owner has several options. One is simply 'to walk away from the business'. In this case the lessor takes over and sells the restaurant's equipment. A second option is for the restaurant owner to declare bankruptcy. In this situation, the lessor becomes one of the creditors and gets compensated, if at all, from the proceeds of sale of the restaurant's assets. To protect themselves against these actions, the lessor may ask the potential restaurant owner to guarantee lease payments with personal assets. The third option is to sell the restaurant and seek the lessor's permission to assign the lease to the new owner. Alternatively, the new owner may merely sublease the restaurant space and the original owner may still be liable for payment of the lease rent in case of default by the new owner.

Even if the owner of a failed restaurant continues to pay the minimum rent, the lessor may declare the lease to be in default since most standard leases require the restaurant to be open for business for a specified minimum time period. In practice, many of the issues pertaining to leases of discontinued restaurants are settled in negotiations between lessees and lessors. Nevertheless, the available opportunities to get out of leases at a minimal cost undoubtedly affects the timing of exit. For example, appearance of a potential purchaser may induce the owner to decide to leave the industry immediately if the future profitability of the restaurant is questionable. However, he would have continued operating in the absence of the appearance of a potential buyer.

The main fixed asset of restaurants is equipment. It accounts from over three-fourths to practically 100 percent of all fixed assets. Although there is an active second-hand market in restaurant equipment the liquidation value of these assets is relatively low. For example, the respondents estimated that one-year old restaurant equipment upon liquidation would return only 10 to 20 percent of its original value. The salvage value of assets, therefore, does not appear to be an important variable in exit decisions in the restaurant industry. The long-term restaurant lease is another asset, although in the case of failure, it becomes a liability. The effect of the lease on the decision process would be indirect, i.e., the more desirable the location of the restaurant the easier it may be to locate a potential tenant to take over the lease.

In short, it seems that in the case of restaurants where the cause of exit was not a cash shortage, the exit decision process is quite complex. While in these cases the sequential decision-making model appears to be the more capable of providing a description of the process, the model would have to be quite a bit more complicated than those discussed above.

6. Conclusion

The previous theoretical studies of the timing of business failures followed two approaches, i.e., either developed models based on sequential decision-making, or developed models based on gambler's ruin. A number of these studies, particularly those using gambler's ruin models, were specifically designed for bankruptcies rather than explaining the timing of business exits.

The first criteria used to evaluate the different models was their relative ability to reproduce observed patterns of mortality. On the basis of the results of previous studies we concluded that these patterns exhibit a conditional mortality function that declines with the age of the firm. We find that this criteria is not very restrictive since exit models based on either of the two approaches are capable of generating the empirical mortality functions.

For the purpose at hand, the realism of the assumptions is important. Therefore, we focused on restaurant exits in Honolulu as a case study and interviewed a small number of owner/managers of failed restaurants. Their responses suggest the usefulness of models based on gambler's ruin. According to the survey results more than half the restaurants failed because they ran out of cash and the access of owners to capital markets is very limited because financial institutions do not lend to firms without an established track record. Indeed it appears that the environment in which restaurants operate resembles the gambler's ruin scenario rather closely.

The remaining restaurants in the sample did not exit because of a cash shortage. For these restaurants the sequential decision-making models appear to be more appropriate. The survey results, however, suggest that the exit model is much more complex than the models developed in Section 4. For example, the model, in addition to the usual expectations with respect to revenues and costs, must include the timing of arrival of potential buyers and other opportunities to get out of the long-term lease as well as opportunities in other lines of business. The information needed for this type of model is notoriously difficult to obtain since it is not revealed in any of the firms transactions.

In summary, we conclude that gambler's ruin provides a potentially more productive approach to modeling the exit process, at least for firms in industries dominated by small firms. These would include industries in the retail sector, the services sector, and a number of product lines in manufacturing.

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Notes

[1] Declaring bankruptcy only does not necessarily result in the exit of the firm. There are two bankruptcy processes: reorganization and liquidation. Only liquidation involves the dismantling of the firm and the selling of its assets. Reorganization is a rehabilitation procedure which allows firms to continue operating while making a financial settlement with their creditors. Most bankrupt firms attempt to reorganize first and liquidate only when unsuccessful.

[2] For example, all the cases of exit via bankruptcy/liquidation cited by Platt (1985) involved large corporations.

[3] A large number of studies dealing with the prediction of firm bankruptcies followed the seminal paper by Altman (1968). Unlike the studies cited in this paper, they focused on the development and testing of a methodology for predicting bankruptcies rather than on the underlying theory of firm exits.

[4] In fact, one may argue that while the assumption of no access to capital markets is clearly unrealistic for large firms (particularly corporations), for industries dominated by small firms it may be more realistic than an assumption of unlimited access to such markets.

[5] There is some evidence supporting assertions of different exit behavior in different types or sizes of firms. Londregan (1988) in his study of plant closures in the petroleum refining industry found distinctly different exit patterns for small and large plants, the probability of failure for small operations decreased with age while that of large plants increased with age.

[6] For continuous mortality functions $M(t)$, conditional mortality $CM(t)$ is defined as $\delta M(t)/M(t)$. For discrete mortality functions, $CM(t) = [M(t)\text{-}M(t-1)]/M(t-1)$.

[7] It is generally simplest to estimate the survivor function, $S(t) = 1/(1 + \alpha t^{\beta})$, for the logistic distribution, as this function can be expressed as $1/S(t) - 1 = \alpha t^{\beta}$, which is linear in logs.

[8] Similar definitions can be found in most intermediate and advanced level business statistics textbooks. For example, see Theil (1971).

[9] These statements regarding the long-term nature of restaurant business are supported by other evidence. For example, typical restaurant leases are for 15 to 20 years as compared to a typical 5 year lease for retail stores.

[10] Some respondents referred to an unfavorable debt-to-equity ratio as undercapitalization. The reduction in the probability of survival due to this factor, however, is indirect. That is, the size of the loan repayment obligation increases the amount of sales necessary to achieve a positive or breakeven cash flow.

[11] This belief, however, was not universally held. The respondents generally agreed that it was easier to sell an ongoing restaurant that is doing well, but some argued that this is not true if one is trying to sell a failing restaurant.

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