# Ambiguity and Nonparticipation: The Role of Regulation

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We investigate the implications of ambiguity aversion for performance and regulation of markets. In our model, agents' decision making may incorporate both risk and ambiguity, and we demonstrate that nonparticipation arises from the rational decision by some traders to avoid ambiguity. In equilibrium, these participation decisions affect the equilibrium risk premium, and distort market performance when viewed from the perspective of traditional asset pricing models. We demonstrate how regulation, particularly regulation of unlikely events, can moderate the effects of ambiguity, thereby increasing participation and generating welfare gains. Our analysis demonstrates how legal systems affect participation in financial markets through their influence on ambiguity. (*JEL* G1, G2, D8)

One of the more puzzling aspects of financial markets is the surprisingly large fraction of individuals who do not participate. For the period 1982–1995, the U.S. Consumer Expenditure Survey found that more than two-thirds of households held neither stocks nor bonds (cited in Paiella 2006). Campbell (2006), using data from the 2001 Survey of Consumer Finance, found that even at the eightieth percentile of wealth, almost 20% of households had no public equity even when their retirement savings are considered.[1] Looking across asset classes reveals similar nonparticipation issues, as does looking internationally, where participation rates are shown to vary substantially across countries.[2] Because such behavior is inconsistent with standard models of optimal portfolio

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[1] Mankiw and Zeldes (1991) provide similar evidence, finding that the equity market participation rate of individuals with $100,000 in liquid assets is only 47.7%.

[2] Participation in China's equity markets, for example, as measured by the number of brokerage accounts, is estimated to be in the 5–7% range of all households. See Guiso, Haliassos, and Jappelli (2003) for participation rates in Europe.


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choice and investment behavior (see Merton 1969; Samuelson 1969), there is now a large literature attempting to explain the participation puzzle.

Much of this literature has focused on the role played by participation costs (see Paiello 2007; Vissing-Jorgenson 2002; Guiso, Haliassos, and Jappelli 2003). While undoubtedly an important factor for some investors, the empirical evidence suggests that this can only be a partial explanation. Gouskova, Juster, and Stafford (2004), for example, conclude that "costs are not a major consideration to participate in the stock market (during the 1990s), and some behavioral or other explanation might be needed."[3] Hong, Kubik, and Stein (2004) argue that one such behavioral cause may be social interaction, finding that nonsocial households are less likely to participate in equity markets than are their more interconnected counterparts. In this research, we investigate another explanation, the role of ambiguity aversion.

Ambiguity aversion arises when individuals consider both risk and uncertainty (or ambiguity) in their decision making. Knight (1921) originally developed the notion of individuals making a distinction between known odds (risk) and ambiguous odds (uncertainty). This distinction has seen resurgence due to the work of Gilboa and Schmeidler (1989) and Schmeidler (1989), who provide an axiomatic foundation for decision making with ambiguity aversion.[4] Ambiguity aversion is often ascribed to naïve investors who do not have the requisite experience to form priors over the occurrence of particular states of the world. If these individuals perceive ambiguity to be too great, they simply choose not to participate, allowing nonparticipation to arise endogenously as an equilibrium outcome.

In this article, we investigate the implications of ambiguity aversion for the performance and regulation of markets. Our goal is not simply to show that ambiguity aversion can lead to nonparticipation or that it can affect asset prices; these results are well known (see, for example, Dow and Werlang 1992; Epstein and Wang 1994). Instead, our goal is to demonstrate that regulation, particularly regulation of unlikely events, can moderate the effects of ambiguity, thereby increasing participation in financial markets and generating welfare gains. We develop our results in a multiasset model that includes both standard expected utility maximizing agents and ambiguity-averse agents. This heterogeneity of investors, rather than the more standard representative trader approach, is consistent with extensive empirical and experimental evidence that some, but

[3] One such alternative is proposed by Merton (1987), who argued that incomplete information could explain limited participation. A variety of authors (see Basak and Cuoco 1998) have extended this research. Other explanations are given by Guiso, Sapienza, and Zingales (forthcoming), who focus on the role of trust, and Ang, Bekaert, and Liu (2005), who consider loss aversion.

[4] This distinction was noted by Savage (1954), but in his model of subjective expected utility it plays no role. The standard model of asset pricing is based on Savage's foundation for expected utility maximization. So the distinction between known odds and ambiguous ones plays no role in the standard asset pricing model. Particular applications of ambiguity aversion to asset pricing can be found in recent papers by Dow and Werlang (1992); Epstein and Wang (1994); Chen and Epstein (2002); Cao, Wang, and Zhang (2003); Liu, Longstaff, and Pan (2003); Liu, Pan, and Wang (2005); and Routledge and Zin (2004).

not all, subjects act as if they do not have a prior. More important is that the heterogeneity of investors leads to interesting insights into participation and the potential effects of regulation on asset markets.

In the economy we analyze there are safe assets and risky assets. The risky assets have normally distributed payoffs, but the mean and variance of these distributions are unknown to naïve traders. We show that there is positive alpha (expected excess return unexplained by correlation with the market) to holding assets avoided by naïve investors. We also show that reducing the perceived maximum variance of payoffs or increasing the perceived minimum mean payoff can (depending on the specification of the economy) increase participation by ambiguity-averse investors, which in turn can lower the alpha. We argue that changing these perceived minimum mean payoffs and maximum variances can be accomplished by particular regulations. Our analysis thus provides one explanation for why the legal structure can play an important role in financial market participation and performance.

We illustrate the importance of these effects by considering a range of applications including deposit insurance, suitability rules, insider "short-swing" trading profit restrictions [SEC Rule 16(b)], and mutual fund restrictions. A more general application is to explain the perplexing upside-down structure of securities regulations, in which firms with the most available public information are subject to the greatest regulatory scrutiny. As we show here, regulations can induce alternative equilibrium outcomes by either reducing or increasing perceived ambiguity. Our results suggest that, for some securities, limiting participation through greater ambiguity may be a more cost-effective means of investor protection than direct regulatory supervision of firms or markets. Thus, we demonstrate an intriguing role for the legal system to influence nonparticipation as well as participation in financial markets.

Apart from providing insights into the structure and impact of market regulation, our analysis of ambiguity aversion provides implications for broader concepts, such as contracting and the role of disclosure. Contracting cannot solve the participation problems induced by ambiguity aversion because the problem involves low probability specifications of the world (minimum possible mean and maximum possible variance in our model) whose very existence would make complete contracting prohibitively expensive. Similarly, disclosure cannot solve ambiguity-induced problems because the difficulty is the probability to attach to unlikely states, not their existence. Indeed, disclosure can exacerbate problems by scaring away potential market participants. Further, the palliative role of arbitrage is limited because ambiguity aversion induces nonparticipation, and not merely mispricing.[5] These difficulties underscore why market solutions may be incapable of removing the influences of ambiguity aversion, even when all market participants act rationally. Our work thus

[5] Although it does induce mispricing relative to the CAPM.

establishes a normative role for legal rules and structures to influence financial market participation.

This article is organized as follows. In Section 1, we discuss the distinction between expected utility and ambiguity aversion, and we motivate our use of ambiguity-averse investors. In Section 2, we develop a multiasset model involving naïve and sophisticated investors. We solve for individual demands and participation decisions, and we solve for market equilibria. Section 3 then investigates how changing perceptions of extreme models of payoffs influences investor participation, and how this in turn affects equilibrium outcomes. In Section 4, we discuss how welfare can be improved via regulation, and we illustrate the implications of our model by examining some specific legal rules and regulations designed to limit extreme events. Section 5 considers generalizations and extensions by looking at the role of market solutions, such as contracting, disclosure, and arbitrage in dealing with ambiguity. Section 5 also considers the role of learning and the extent to which it eventually weakens the case for regulation. Section 6 concludes by discussing the implications of our results for the role of law in affecting financial market behavior and participation.

## 1. Expected Utility and Ambiguity

In the von Neumann-Morgenstern expected utility theory, decision makers have preferences over, and make decisions between, objective distributions. Applications of this theory to asset markets assume that the distributions of payoffs to portfolios of assets are known to investors. This assumption is usually justified with the rational expectations hypothesis. For some assets and some investors this is a reasonable assumption. For others, it is surely not reasonable. Do unsophisticated investors know the distribution of payoffs to even simple portfolios? Do U.S. investors know the distribution of payoffs to all foreign stocks? Do any investors know the distribution of payoffs to new assets? Even for established assets, investors with common information often disagree about the distribution of payoffs.

The Savage generalization of expected utility theory provides a Bayesian approach to subjective uncertainty about payoff distributions. In this approach, individuals' subjective distributions of payoffs are derived from their preferences over stochastic consumption streams. This allows similarly informed investors to disagree about the predicted distribution of payoffs on portfolios. But it does imply that each investor acts as if he or she has some subjective distribution. In some cases, this seems reasonable; in others, such as with the example of a new asset, it is much less plausible.

We model a fraction of our investors as Savage expected utility maximizers. We assume that these sophisticated investors know the payoff distribution for each asset. This rational expectations assumption for expected utility traders is a strong, but standard, assumption. Allowing them to place a prior on the set of distributions the ambiguity-averse investors deem to be possible would greatly complicate our analysis, but it would not change our results in important ways. The other investors are aware of the set of possible payoff distributions, but they are unable or unwilling to place a prior on this set. These naïve investors are what is now termed in the literature ambiguity averse.

We have two motivations for considering ambiguity-averse investors. First, the expected utility approach yields predictions about individual portfolios that are inconsistent with actual portfolios for many investors. Most important, for us, is that expected utility traders should hold diversified portfolios. They should not be overweighted in stocks that they are familiar with because of geography, such as the stocks of local or national firms. They should hold at least small amounts, at least indirectly through funds, of foreign assets whose payoffs are not perfectly correlated with their portfolios.[6] Barberis and Thaler (2003) provide a concise survey of the evidence against expected utility, and they offer alternatives including ambiguity aversion to explain this observed behavior.

Second, there is direct experimental evidence that some individuals do not always act as if they have a prior. The most notable example is the Ellsberg Paradox (Ellsberg 1961). In a simple version of the Ellsberg thought experiment, an individual is given an opportunity to bet on the draw of a ball from one of two urns. Urn one has fifty red and fifty black balls. Urn two has one hundred balls, which are some mix of red and black. First, subjects are offered a choice between two gambles: \$1 if the ball drawn from urn one is red and nothing if it is black or \$1 if the ball drawn from urn two is red and nothing if it is black. If a subject chooses the first gamble and has a prior on urn two, the predicted probability of red in urn two is less than 0.5. Next, subjects are offered a choice between two new gambles: \$1 if the ball drawn from urn one is black and nothing if it is red or \$1 if the ball drawn from urn two is black and nothing if it is red. If a subject again chooses the first gamble and has a prior on urn two the predicted probability of black in urn two is less than 0.5. This cannot be, so they do not act as if they have only one prior on urn two.

Experiments like Ellsberg's (1961) have been repeated many times in many settings. This evidence led Gilboa and Schmeidler (1989) to weaken the Savage axioms in order to produce a decision theory consistent with the behavior observed by Ellsberg.[7] Their approach yields a utility function defined over payoffs as in Savage but rather than a single prior it yields a set of priors.

[6] Of course, if investors have private information that is not fully reflected in local or national stock prices, then they should over- or underweight these stocks. We find it implausible that information accounts for all of the local bias. Similarly, transaction costs can account for some of the lack of diversification. But, again, the transaction costs connected with mutual fund investments are low enough to make this explanation implausible as well.

[7] They actually begin with Anscombe and Aumann's (1963) framework, which is a standard alternative to Savage's approach. The axiom that they weaken is the Independence axiom.

The axioms also imply that the decision maker evaluates any act according to the minimum expected utility it yields.[8]

The Gilboa and Schmeidler (1989) model has itself been generalized to allow for the possibility that the decision maker is not so pessimistic as to select the act that maximizes the minimum expected utility. Two recent papers by Ghirardato, Maccheroni, and Marinacci (2004) and Klibanoff, Marinacci, and Mukerji (2005) provide alternative approaches to separating ambiguity and the decision maker's attitude toward ambiguity. We follow the Gilboa and Schmeidler model to illustrate our ideas, but the results could be generalized to allow for less ambiguity aversion, although at considerable loss of tractability. The important aspect of these models for our results about the effect of regulations is that naïve investors facing ambiguity are ambiguity averse, but exactly how ambiguity averse they are is not important for our qualitative results.

Our ambiguity-averse naïve investors face a set of payoff distributions, and they do not aggregate these distributions to produce a predicted payoff distribution. There are at least two other reasonable ways to view the decision problem faced by our naïve decision makers. First, they could be thought of as choosing robust portfolios. That is, they could search for portfolios that are robust to their uncertainty about the correct model for payoffs. Hansen and Sargent (2000) follow this approach to evaluating macroeconomic models. Garlappi, Uppal, and Wang (2004) and Maenhout (2004) use similar approaches to consider asset pricing issues. Second, they could be thought of as behavioral traders, who either have biased beliefs or who do not maximize expected, or minimum expected, utility.

## 2. The Model

We analyze an economy with three assets: there is one risk-free asset, money, which has a constant price of 1 and is in zero net supply, and there are two risky financial assets with independent, normally distributed payoffs $v^i$, $i = 1, 2$.[9] These risky assets can be interpreted as individual stocks, bonds, mutual funds, or even bank deposits. All investors know that payoffs are independent and normal. The set of possible mean payoffs for asset $i$ is $\{\bar{v}^i_1, \ldots, \bar{v}^i_N\}$; the set of possible variances is $\{\sigma^i_1, \ldots, \sigma^i_N\}$. All pairs of mean and variance are possible and we let $\Theta^i = \{\theta^i_1, \ldots, \theta^i_n\}$, with $n = N^2$ elements, be the set of possible parameters.[10]

[8] In the Ellsberg (1961) framework, this model implies that the individual acts as if he has a set of priors for urn two, which includes a prior in which the probability of red is less than 0.5 and a prior in which the probability of black is less than 0.5. Since he acts as if he evaluates each act according to its minimum expected utility, he will never choose urn 2, as in his pessimistic view it will be unlikely to pay off.

[9] We consider two risky assets in order to be able to discuss relative prices of risky assets. Our results generalize immediately to any number of assets.

[10] As will become apparent, only the minimum and maximum mean payoff and maximum variance affect decisions made by naïve traders. So changes to the set $\Theta^i$ that leave these values unchanged have no impact on the market. In particular, $\Theta^i$ can be a continuum.

There are $J$ investors indexed by $j = 1, \ldots, J$. All investors have CARA utility for wealth, with the risk aversion parameter set equal to 1:

$$u_j(w) = -\exp(-w). \tag{1}$$

There are two types of investors in the economy: sophisticated investors ($S$) and naïve investors ($U$). Sophisticated investors constitute a fraction $1 - \mu$ of all investors, while naïve investors constitute the remaining fraction $\mu$. The sophisticated investors are standard expected utility maximizers ($EU$) with rational expectations about payoff parameters. Let $(\hat{v}^i, \hat{\sigma}^i)$ denote the true mean payoffs and variance for asset $i$. Since our sophisticated traders have rational expectations, they know $(\hat{v}^i, \hat{\sigma}^i)$.

The naïve investors also care about means and variances, but they differ from sophisticated investors in that they do not know the parameters.[11] Instead, they consider each normal distribution of payoffs, $N(\theta^i)$, as a possible payoff distribution. Following Gilboa and Schmeidler's (1989) axiomatic foundation for ambiguity aversion, we model these naïve investors as choosing a portfolio to maximize their minimum expected utility over the set of possible distributions. To make our analysis of the equilibrium interaction between $S$ and $U$ traders interesting, we assume that naïve investors consider as possible mean payoffs above and below $\hat{v}^i$ and variances above and below $\hat{\sigma}^i$. That is, the true parameter values are convex combinations of the extreme values considered possible by the naïve traders.

The per capita endowments of assets are $(\bar{x}^1, \bar{x}^2)$. The exact distribution of endowments over investors does not affect their demands for risky assets because of the CARA-Normal structure, so we do not specify it.[12] We denote a typical investor's wealth by $w$. Where no confusion would occur, we will drop the investor index. The investor's budget constraint is

$$w = m + p^1 x^1 + p^2 x^2, \tag{2}$$

where $m$ is the quantity of money, $p^i$ is the price of asset $i$, and $x^i$ is the quantity of risky asset $i$. Investors are allowed to go long or short in each asset. If the investor chooses portfolio $(m, x^1, x^2)$, his random next period wealth will be

$$\tilde{w} = m + \tilde{v}^1 x^1 + \tilde{v}^2 x^2. \tag{3}$$

[11] These investors can be thought of as inexperienced potential investors, who do not have enough experience in financial markets to reliably access payoff distributions. Perhaps they have not yet participated in the asset market, and although they can imagine many possible payoff distributions, they are unable to place a prior on this set of distributions. They know that holding cash is safe, but are just not sure how to think about risky assets.

[12] For ambiguity-averse investors, the most natural interpretation is that they have no endowment of risky assets. But, regardless of their endowments, what they care about is their final asset position. So, in an equilibrium in which ambiguity-averse investors choose not to hold a risky asset, they will trade, if necessary, in order to achieve a zero asset position.

For a sophisticated investor, with CARA utility of wealth and payoff parameters $(\hat{v}^i, \hat{\sigma}^i)$, the expected utility of this random wealth is a strictly increasing transformation of

$$(\hat{v}^1 - p^1)x^1 + (\hat{v}^2 - p^2)x^2 - 1/2\hat{\sigma}^1(x^1)^2 - 1/2\hat{\sigma}^2(x^2)^2 + w. \tag{4}$$

Calculation shows that the sophisticated investor's demand function for asset $i$ is given by

$$x_S^{i*}(p^i) = \frac{\hat{v}^i - p^i}{\hat{\sigma}^i}. \tag{5}$$

A naïve investor evaluates the expected utility of wealth for each parameter vector and chooses the portfolio that maximizes the minimum of these expected utilities. In effect, the naïve investor tries to avoid the worst case outcomes, and so chooses a portfolio that explicitly limits exposure to such adverse outcomes. The expected utility of random wealth, given parameters $(\theta^1 = (\bar{v}^1, \sigma^1), \theta^2 = (\bar{v}^2, \sigma^2))$, is a strictly increasing transformation of

$$(\bar{v}^1 - p^1)x^1 + (\bar{v}^2 - p^2)x^2 - 1/2\sigma^1(x^1)^2 - 1/2\sigma^2(x^2)^2 + w. \tag{6}$$

Thus, the naïve investor's decision problem can be written as

$$\underset{(x^1,x^2)}{\text{Max}}\ \underset{(\theta^1,\theta^2)}{\text{Min}}((\bar{v}^1 - p^1)x^1 + (\bar{v}^2 - p^2)x^2 - 1/2\sigma^1(x^1)^2 - 1/2\sigma^2(x^2)^2 + w). \tag{7}$$

Examining the minimization problem reveals that for any portfolio the minimum occurs at the maximum possible variance for each asset. Denote these variances by $\sigma^i_{\text{max}}$. Consequently, what matters to the naïve investor is not the "expected" variance, but rather the largest variance. Whether the minimum occurs at the maximum or minimum mean payoff depends on whether the investor is long or short in the asset. The minimum occurs at minimum mean payoff for asset $i$ if the investor is long in asset $i$ and at maximum mean payoff for asset $i$ if the investor is short in asset $i$. Denote these mean payoffs by $\bar{v}^i_{\text{min}}$ and $\bar{v}^i_{\text{max}}$, respectively. Calculation shows that the naïve investor's demand function for asset $i$ is

$$x_U^{i*}(p^i) = \begin{bmatrix} \frac{\bar{v}^i_{\text{min}} - p^i}{\sigma^i_{\text{max}}} & \text{if} & \bar{v}^i_{\text{min}} > p^i \\ 0 & \text{if} & \bar{v}^i_{\text{min}} \le p^i \le \bar{v}^i_{\text{max}} \\ \frac{\bar{v}^i_{\text{max}} - p^i}{\sigma^i_{\text{max}}} & \text{if} & \bar{v}^i_{\text{max}} < p^i \end{bmatrix}. \tag{8}$$

There are several properties of this demand function that will be important for our analysis. First, note that if the price of asset $i$ is above the minimum possible mean payoff and below the maximum possible mean payoff, then the naïve investor will not participate in the market for asset $i$.[13] This occurs

[13] Here by not participating we mean that his final asset position will be zero. This interpretation is most natural if ambiguity-averse investors do not initially hold the risky asset.

because a naïve investor is heavily influenced by the worst possible state, and what is worst depends on the investor's asset position. If the investor holds a positive quantity of the asset, he evaluates it using the lowest possible mean payoff, $\bar{v}^i_{\text{min}}$, and the highest possible variance, $\sigma^i_{\text{max}}$. If the investor goes short, the worst possible mean switches to $\bar{v}^i_{\text{max}}$ and the worst variance stays at $\sigma^i_{\text{max}}$.[14] So unless the price of the asset is above $\bar{v}^i_{\text{max}}$ or below $\bar{v}^i_{\text{min}}$, a naïve investor will not participate in the asset market.

Second, note that the naïve investor's decision about whether to hold the asset is independent of the set of variances he believes to be possible. All that matters for the participation decision is the price, the minimum mean payoff, and the maximum mean payoff. If the naïve investor decides to hold the asset, then variance matters, just as it does for the sophisticated investor. But only the maximum possible variance affects the quantity to be held. The other variances the naïve investor believes to be possible do not affect his decision about whether to participate or his decision about how much to hold if he chooses to participate.

Third, note that the naïve investor's demand function is continuous in price but that it has kinks at $\bar{v}^i_{\text{min}}$ and $\bar{v}^i_{\text{max}}$. In particular, for any price between $\bar{v}^i_{\text{min}}$ and $\bar{v}^i_{\text{max}}$, the naïve investor does not hold the asset. This contrasts with sophisticated investors who hold a nonzero position in any asset as long as its price is not equal to its mean payoff.[15] Graphs of the demand for asset $i$ for naïve and sophisticated investors are shown in Figure 1. Note that the sophisticated investor always holds a larger amount (in absolute value) of the risky asset than does the naïve investor. This is because for any given parameters these investors evaluate the tradeoff between mean and variance equivalently. They both avoid risk and require compensation in expected payoff in order to hold risk. But the naïve investor also avoids ambiguity in the distribution of payoffs, and so as long as the set of possible means and variances is nondegenerate, he further reduces the size of his position in the risky asset.

In equilibrium, the per capita demand for asset $i$ must equal its per capita supply. Equating the demands from Equations (5) and (8) to this supply then yields

$$\mu x^{i*}_U(p^i) + (1-\mu)x^{i*}_S(p^i) = \bar{x}^i. \tag{9}$$

Depending on the parameters of the economy, there are two possible types of solutions to this equation.

First, if at a price between $\bar{v}^i_{\text{min}}$ and $\bar{v}^i_{\text{max}}$ the sophisticated investors are willing to hold the entire supply of the asset, then in equilibrium the naïve investors

[14] We will show that in fact the equilibrium price cannot be above the maximum mean payoff. So, in equilibrium, naïve investors never go short.

[15] More generally, if asset payoffs are correlated, sophisticated investors also hold assets in order to diversify their portfolios.

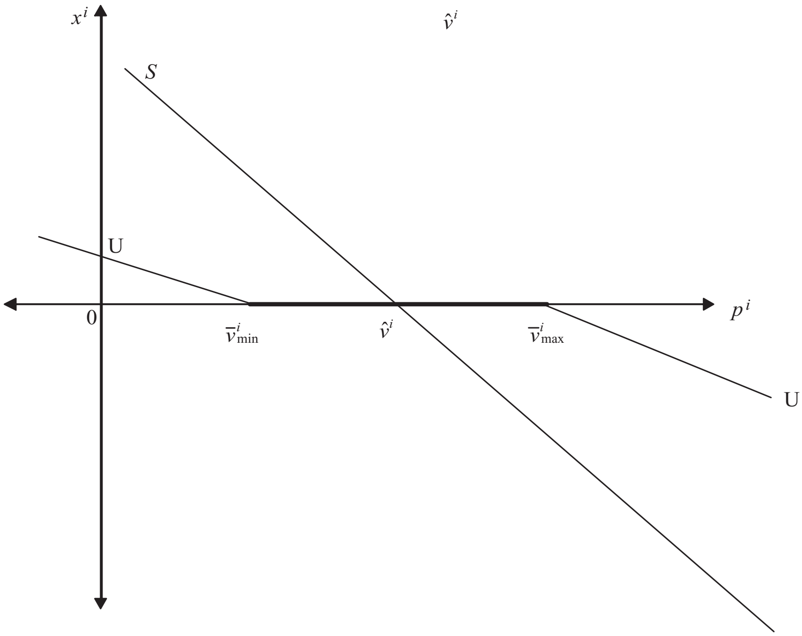


**Figure 1**
**Demand functions for asset *i***
This figure shows the demands ($x$) for asset $i$ by sophisticated investors ($S$) and naïve investors ($U$). The prior mean payoff is given by $\hat{v}^i$, and $\bar{v}^i_{\min}$ and $\bar{v}^i_{\max}$ are the minimum and maximum mean payoffs perceived by ambiguity-averse investors, respectively.

will not participate in the market. If only sophisticated investors participate in the market, the market clearing price must be

$$\hat{p}^i = \hat{v}^i - \tfrac{\hat{\sigma}^i \bar{x}^i}{1-\mu}. \tag{10}$$

Thus, $\hat{p}^i$ will be the market clearing price for asset $i$ if $\bar{v}^i_{\max} \geq \hat{p}^i \geq \bar{v}^i_{\min}$. Note that $\bar{v}^i_{\max} \geq \hat{p}^i$ as $\bar{v}^i_{\max} \geq \hat{v}^i \geq \hat{p}^i$, so the binding condition is $\hat{p}^i \geq \bar{v}^i_{\min}$.

Second, it is possible that both types of investors participate in the market for asset $i$. If we conjecture that both types of investors participate, then the market clearing price must be

$$p^{i^*} = \frac{\mu\hat{\sigma}^i \bar{v}^i_{\min} + (1-\mu)\sigma^i_{\max}\hat{v}^i - \bar{x}^i \sigma^i_{\max}\hat{\sigma}^i}{\mu\hat{\sigma}^i + (1-\mu)\sigma^i_{\max}}. \tag{11}$$

This can be an equilibrium price only if ambiguity-averse investors are willing to participate, i.e., only if $p^{i^*} < \bar{v}^i_{\min}$. Calculation shows that this constraint is met if and only if $\hat{p}^i < \bar{v}^i_{\min}$. In order to ensure that the price is sensible (greater than zero) even if there are only naïve investors in the market, we assume that $\bar{v}^i_{\min} - \bar{x}^i \sigma^i_{\max} > 0$.

As the binding condition for a nonparticipation equilibrium is $\hat{p}^i \geq \bar{v}^i_{\min}$, one and only one of these equilibria will prevail for any economy. Thus, there is a unique equilibrium. This equilibrium is either one in which ambiguity-averse investors do not participate, a nonparticipating equilibrium, or one in which they do participate, a participating equilibrium. These results are summarized in the proposition below.

**Proposition.** *There is a unique equilibrium in the market for asset i. It is one of two types:*

1. *Nonparticipating: if* $\hat{p}^i = \hat{v}^i - \frac{\hat{\sigma}^i \bar{x}^i}{1-\mu} \geq \bar{v}^i_{\min}$, *then in the equilibrium* $x^{i^*}_A = 0$, $x^{i^*}_U = \frac{\bar{x}^i}{1-\mu}$ *and* $\hat{p}^i$ *is the market clearing price.*
2. *Participating: if* $\hat{p}^i = \hat{v}^i - \frac{\hat{\sigma}^i \bar{x}^i}{1-\mu} < \bar{v}^i_{\min}$, *then in the equilibrium both* $x^{i^*}_A > 0$ *and* $x^{i^*}_U > 0$, *and* $p^{i^*}$ *is the market clearing price.*

There are two important aspects of the equilibrium to note. First, naïve investors will short asset $i$ only if the asset price is above $\bar{v}^i_{\max}$. This cannot occur in equilibrium, as at such prices all *EU* traders also want to short asset $i$. Second, naïve investors hold positive quantities of asset $i$ only if $\hat{p}^i < \bar{v}^i_{\min}$. So, if naïve investors consider a sufficiently low mean payoff to be possible, we have a nonparticipating equilibrium. A participating equilibrium can arise only if the set of models considered by the naïve investors is not too large.

To illustrate the pricing effects of nonparticipation by naïve investors, suppose that all investors have correct beliefs $(\hat{v}^1, \hat{\sigma}^1)$ about the payoff of asset one and that there is sufficient ambiguity about asset two that naïve investors do not participate in the market for asset two. For example, asset one may be stock in a large, established firm and asset two may be stock in a new, small company. Then from the proposition above we see that equilibrium prices are $p^1 = \hat{v}^1 - \bar{x}^1\hat{\sigma}^1$ and $p^2 = \hat{v}^2 - \hat{\sigma}^2\bar{x}^2/1 - \mu$.

These equilibrium prices are identical to the prices that would be observed in an equilibrium of a heterogeneous beliefs economy in which all traders are expected utility maximizers, all investors agree about the distribution of payoffs for asset one, and fraction $\mu$ of the investors act as if the variance of payoffs for asset two is infinite. Thus, as in Lintner (1969), there is a representative agent ($A$) for this economy with the common CARA utility function and beliefs (about the independently distributed payoffs to assets):

$$\sigma^{A1} = \hat{\sigma}^1, \quad \sigma^{A2} = \hat{\sigma}^2(1-\mu)^{-1}, \quad \bar{v}^{A1} = \hat{v}^1, \quad \bar{v}^{A2} = \hat{v}^2. \tag{12}$$

An economy with a single investor of type $A$ would have equilibrium asset prices that are identical to the equilibrium prices in our heterogeneous investor economy. The beliefs of this investor are sometimes called "market beliefs," as the assets are priced as if these are the beliefs that prevail in the market. But it is important to note that the beliefs of this representative investor are not the same as the beliefs of any investor in our actual economy.

From the point of view of this artificial agent, assets are, of course, priced correctly. To see the empirical implications of this pricing result, it is useful to consider returns to holding the assets rather than the asset prices. The return on asset $i$ is

$$\tilde{r}^i = \frac{\tilde{v}^i}{p^i} - 1 \tag{13}$$

and the return on the market (holding the entire supply of both risky assets) is

$$\tilde{r}^M = \frac{\tilde{v}^1 \bar{x}^1 + \tilde{v}^2 \bar{x}^2}{p^1 \bar{x}^1 + p^2 \bar{x}^2} - 1. \tag{14}$$

The representative agent has correct beliefs about mean payoffs, so he will also have correct beliefs mean returns: $\bar{r}^i$ for asset $i$ and $\bar{r}^M$ for the market. Further, as assets are priced correctly from the point of view of the representative agent, the Capital Asset Pricing Model (CAPM) must hold from his perspective. That is, according to his beliefs the expected excess return on each asset is linearly related to the expected excess return on the market and the coefficient in this linear relation is a measure of the sensitivity of the returns on the asset to returns on the market as a whole. Thus,

$$\bar{r}^i = \beta^{Ai} \bar{r}^M. \tag{15}$$

To see this, note that according to the representative agent the beta for asset $i$ is[16]

$$\beta^{Ai} = \frac{\mathrm{Cov}^A(\tilde{r}^M, \tilde{r}^i)}{\mathrm{Var}^A(\tilde{r}^M)} = \frac{(\bar{x}^1 p^1 + \bar{x}^2 p^2)\bar{r}^i}{(\bar{x}^1)^2 \sigma^{A1} + (\bar{x}^2)^2 \sigma^{A2}}. \tag{16}$$

In this expression, the covariance of the return on the market and the return on asset $i$ and the variance of the return on the market are calculated using the artificial beliefs of the representative agent, rather than correct beliefs.

The representative agent's beliefs about the variance of returns to asset two are incorrect as $\sigma^{A2} = \hat{\sigma}^2(1-\mu)^{-1}$, and so his beliefs about the variance of the market and the covariance of the market and returns to each asset are incorrect. Thus, the betas calculated from the representative agent's point of view are not the betas that would be computed from actual payoff data. The actual betas are

$$\beta^1 = \frac{\mathrm{Cov}(\tilde{r}^M, \tilde{r}^1)}{\mathrm{Var}(\tilde{r}^M)} = \frac{(\bar{x}^1 p^1 + \bar{x}^2 p^2)\bar{r}^1}{(\bar{x}^1)^2 \sigma^1 + (\bar{x}^2)^2 \sigma^2} \tag{17}$$

and

$$\beta^2 = \frac{\mathrm{Cov}(\tilde{r}^M, \tilde{r}^2)}{\mathrm{Var}(\tilde{r}^M)} = \frac{(\bar{x}^1 p^1 + \bar{x}^2 p^2)(1-\mu)\bar{r}^2}{(\bar{x}^1)^2 \sigma^1 + (\bar{x}^2)^2 \sigma^2}, \tag{18}$$

[16] This expression is simple because the risk-free rate in our economy is zero and returns to assets are independent.

where the variances and covariances are computed using the true distribution of equilibrium returns.

Both of the actual betas differ from the representative agent's betas and thus both assets are mispriced relative to the correct CAPM.[17] This mispricing can be captured in $\alpha^i$; the constant term in the regression of excess returns on the return to holding the market:

$$\bar{r}^i = \alpha^i + \beta^i \bar{r}^M.$$

Using the true betas from (17) and (18), calculation shows that

$$\alpha^1 = \frac{-\mu \bar{x}^2 \bar{r}^1 \bar{r}^2 p^2}{p^1[(\bar{x}^1)^2\sigma^1 + (\bar{x}^2)^2\sigma^2]} < 0 \tag{19}$$

and

$$\alpha^2 = \frac{\mu \bar{x}^1 \bar{r}^1 \bar{r}^2 p^1}{p^2[(\bar{x}^1)^2\sigma^1 + (\bar{x}^2)^2\sigma^2]} > 0. \tag{20}$$

There is a positive excess expected return to holding the asset avoided by naïve investors and a negative excess expected return to holding the other risky asset. This occurs because in order to induce the sophisticated investors to hold the supply of the asset with ambiguous returns (according to the naïve investors), its price must be low and, thus, its returns must be high. Conversely, the ambiguity-averse investors overweight their portfolios in the nonambiguous asset, thus increasing its price and lowering its returns.

## 3. Characterization of Equilibrium

In this section, we analyze how changes in the economy affect equilibrium asset prices.[18] We are particularly interested in changes in the fraction of naïve investors, and in the set of means and variances they consider possible, as these are variables that regulations can potentially affect. We want to analyze the effects of changes in the payoff distribution parameters while keeping the behavior of sophisticated investors constant. So, we hold the beliefs of sophisticated investors constant while we change the set of models the naïve investors consider.

### 3.1 Fraction of naïve investors

We first consider how the distribution of investor types influences the equilibrium outcome. In an economy with only sophisticated investors, the equilibrium price of asset $i$ is $\hat{v}^i - \hat{\sigma}^i \bar{x}^i$. It is straightforward to show that an increase in the fraction of naïve investors decreases the price of risky assets. At the critical level

[17] This follows from the fact that the representative agent has the correct mean payoffs.

[18] These effects can equivalently be expressed in terms of returns.

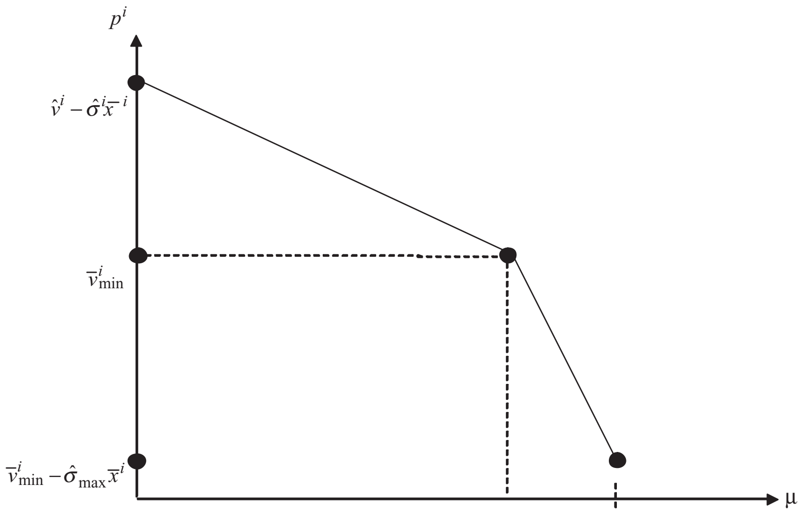


**Figure 2**
**Effect of $\mu$ on equilibrium price of asset *i* (holding $\hat{v}^i$ constant)**
This figure demonstrates the equilibrium price as a function of $\mu$, the fraction of naïve investors. The kink occurs at the switch point between the nonparticipating equilibrium and the participating equilibrium. In both equilibria, price is decreasing in the fraction of naïve investors.

$\mu = 1 - \frac{\hat{\sigma}^i \bar{x}^i}{\hat{v}^i - \bar{v}^i_{\min}}$, the economy switches from a nonparticipating to a participating equilibrium. In the limit with only naïve investors, the equilibrium price of asset $i$ is $\bar{v}^i_{\min} - \sigma^i_{\max} \bar{x}^i$. The equilibrium price is a continuous and decreasing function of the fraction of naïve investors, $\mu$, in both the nonparticipating and the participating equilibrium. This relationship is described in Figure 2.

To understand why these effects occur, consider first a market for asset *i* that is in a nonparticipating equilibrium. Suppose that an increase in the fraction of naïve investors leaves the economy in a nonparticipating equilibrium. More naïve investors means fewer sophisticated investors, so the price of asset *i* must decrease as the aggregate risk must be held by fewer investors. In order to induce investors to hold this increased per capita risk, they must be offered an increase in mean payoff, which requires a fall in asset prices. This effect is similar to Merton's (1987) incomplete information effect. In Merton's analysis, some investors are unaware of some assets. If there is a decrease in the number of investors who are aware of an asset, then the asset price must fall. The difference in our model is that all investors are aware of all assets, but naïve investors are unwilling to hold assets about which they have too little information to form beliefs.[19]

Conversely, in a participating equilibrium, the equilibrium price of the asset, $p^{i^*}$, is identical to the price that would occur in a CARA-Normal economy with

[19] Brav, Constantinidis, and Geczy (2002) and Vissing-Jorgenson (2002) provide empirical evidence linking nonparticipation to increased equity market premiums.

fraction $\mu$ of investors with beliefs $(\bar{v}^i_{\min}, \sigma^i_{\max})$ and fraction $1 - \mu$ of investors with beliefs $(\hat{v}^i, \hat{\sigma}^i)$. Since $\bar{v}^i_{\min} < \hat{v}^i$ and $\sigma^i_{\max} > \hat{\sigma}^i$, increasing the fraction of naïve investors has the same effect as increasing the fraction of pessimistic investors in a standard asset pricing framework. It reduces the equilibrium price of the asset and increases its return.

Finally, since increasing the fraction of naïve investors can decrease $\hat{p}^i$, substituting naïve investors for sophisticated investors can cause $\hat{p}^i$ to fall below $\bar{v}^i_{\min}$. If this happens, the equilibrium shifts from a nonparticipating to a participating equilibrium. In the new equilibrium, the asset price thus decreases from a price that was initially above $\bar{v}^i_{\min}$ to $p^{i*} < \bar{v}^i_{\min}$.

### 3.2 Maximum possible variance

Whether equilibrium prices are sensitive to changes in the set of variances considered by naïve investors depends on whether the economy is a participating or a nonparticipating equilibrium. First, note that only the maximum possible variance matters. Changes in the set of variances considered by naïve investors have no effect unless the maximum possible variance changes. Second, if the economy is a nonparticipating equilibrium, then changes in $\sigma^i_{\max}$ have no effect as naïve investors do not participate in the asset market. Further, changes in $\sigma^i_{\max}$ cannot move the economy from a nonparticipating equilibrium to a participating equilibrium, or vice versa. Finally, note that if the economy is a participating equilibrium, then increases in $\sigma^i_{\max}$ cause the price of asset $i$ to fall. This occurs because an increase in $\sigma^i_{\max}$ reduces the demand by naïve investors.[20]

### 3.3 Minimum mean payoff

Clearly, only the minimum possible mean payoff affects the demands of unsophisticated investors. If naïve investors do not participate in the asset market, then a change in $\bar{v}^i_{\min}$ has no affect on the price of asset $i$ as long as the market remains in a nonparticipating equilibrium after the change. However, if naïve investors do participate in the market for asset $i$, then increasing $\bar{v}^i_{\min}$ causes the price of asset $i$ to increase. This occurs as an increase in $\bar{v}^i_{\min}$ increases the demand for asset $i$ by naïve investors. Finally, an increase in $\bar{v}^i_{\min}$ can cause the market to switch from a nonparticipating to a participating equilibrium. The critical value of $\bar{v}^i_{\min}$ for this switch is $\hat{v}^i - \frac{\hat{\sigma}^i \bar{x}^i}{1-\mu}$. When $\bar{v}^i_{\min}$ equals this value, the prices in the nonparticipating and the participating equilibria are equal. As $\bar{v}^i_{\min}$ increases, the market moves to a participating equilibrium and the equilibrium price of asset $i$ increases smoothly. This effect is illustrated in Figure 3.

[20] That changes in the maximum variance could affect equilibrium was also noted by Spiegel and Subrahmanyam (2000). These authors show that in an asymmetric information environment, variances of informed trader information above some $\sigma^i_{\max}$ can result in a market breakdown as market makers are unable to quote break-even prices.

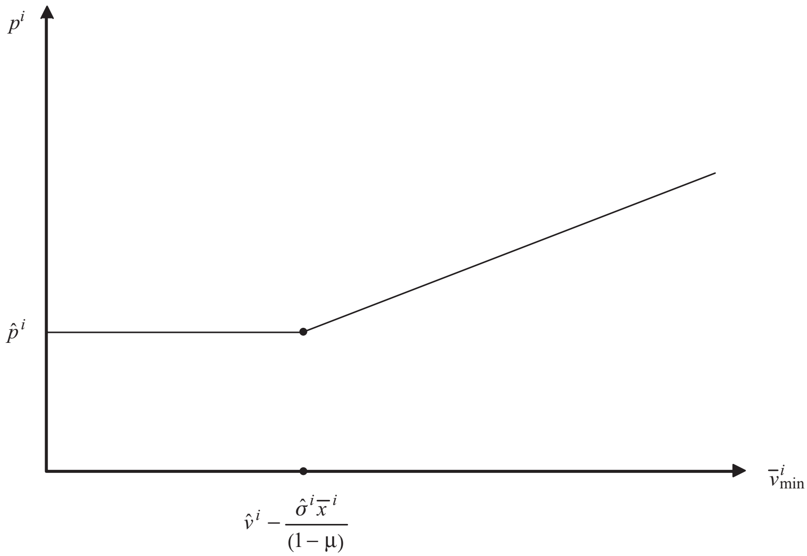


**Figure 3**
**Effect of $\bar{v}^i$ on equilibrium price of asset *i* (holding $\hat{v}^i$ constant)**
This figure demonstrates the equilibrium price as a function of the perceived minimum mean payoff. In the nonparticipating equilibrium, naïve investors hold none of the assets. The kink occurs at the switch point between the nonparticipating equilibrium and the participating equilibrium. In the participating equilibrium, increases in the perceived minimum payoff induce greater holdings by the naïve investors, causing prices to increase.

These comparative static results (summarized in Table 1) suggest that influencing the participation of naïve agents can potentially have large effects on equilibrium asset prices and risk premiums.[21] The analysis also suggests that participation is crucially affected by the parameters of the perceived "worst case" outcome. We now turn to the question of how regulation can affect the perceived worst case outcome, and thus affect the participation and welfare of investors.

## 4. Ambiguity and Regulation

We have shown that increasing the perceived minimum mean payoff of the asset or lowering the perceived maximum variance can affect naïve investors' demands, and thereby equilibrium asset prices. The welfare consequences of policies designed to influence the participation of naïve agents are more subtle. It is not just that such policies might reduce the cost of capital; forcing all individuals to save more and invest their savings in the stock market would also have

[21] The effects are large relative to those that could be expected in an economy with only sophisticated traders. Even if sophisticated traders do not have rational expectations and instead consider extreme models to be possible, the effect of extreme models on asset prices is multiplied by the prior on the model, which would most naturally be small. With unsophisticated investors, the effect of extreme models is direct; it is not multiplied by any prior belief.

**Table 1**
**Comparative static results**

| Variable | Price effect |
|---|---|
| $\mu \uparrow$ | $p^i \downarrow$ |
| $\sigma^i_{\max} \uparrow$ | $p^i \downarrow$ |
| $\bar{v}^i_{\min} \uparrow$ | $p^i \uparrow$ |

This table shows the effect on the equilibrium price of changing the fraction of naïve investors ($\mu$), the maximum variance ($\sigma^i_{\max}$), and the minimum mean payoff ($\bar{v}^i_{\min}$). A fall in price translates into an increased payoff for investors, and an increase in price corresponds to a decrease in investor payoff.

this effect, but it is far from obvious that such a policy would be socially desirable. The reason that intervention may be desirable in an economy with some naïve investors is that unsophisticated agents have beliefs, or act as if they have beliefs, that are wrong. If they knew more, they would have different beliefs, and they would make objectively better decisions. And these better decisions would benefit society more broadly through their effect on the cost of capital.

There are a variety of mechanisms that could be used to achieve these goals. An obvious one is investor education. The National Association of Securities Dealers (NASD), the NYSE, the SEC, the American Association of Individual Investors (AAII), and many other organizations engage in investor education. To the extent that these efforts reach unsophisticated investors who would not otherwise participate in financial markets, and change their beliefs about how the markets work, they can induce participation. However, reaching the right audience is not a simple task, and even if they can be reached, simply providing them with data may not do much. To turn data into knowledge as reflected in beliefs, and ultimately into actions, requires a sophistication that investors who do not participate in the markets may not have in the first place. Although education serves a useful role, we believe that there is a role for regulation designed to convince unsophisticated investors that perceived worst case models are, in fact, impossible.

In this section, we consider several applications of regulations designed specifically to apply to unlikely models. Our analysis shows why regulation designed to exclude some previously possible models can have a large effect on asset prices, and correspondingly why seemingly irrelevant rules can play an important role in the economy.

### 4.1 Deposit insurance and guarantees

Perhaps the most direct approach to raising the minimum mean payoff of an investment is to have a government guarantee that no matter what happens the investor will always receive at least their money back. Such governmental guarantees can be found in many settings, but few play a more important role than the government guarantee of bank deposits.[22] Numerous researchers have

[22] For example, the government guarantees the payment of principal and interest for GNMA securities, through the PBGC it guarantees the payment of minimum pension benefits, and through the SIPC it guarantees funds held in broker/dealer accounts.

shown (see, for example, Diamond and Dybvig 1983) that the introduction of deposit insurance dramatically enhanced the stability of the banking system by reducing the risk of bank runs. In a world with some unsophisticated investors, however, deposit insurance can play an augmented role by inducing these individuals to participate in the banking system in the first place.

Consider, for the moment, the decision problem confronting an unsophisticated investor who is considering whether to invest in a risky venture such as a deposit in an uninsured bank or an investment in a money market fund, or to hold cash. The payoff on investments is random and the distribution of these payoffs may be ambiguous for an unsophisticated investor. If the investor considers several models of how banks work and one of these models has bank runs (or money market fund defaults) occurring with high probability, then the naïve investor refuses to put any money in the bank, preferring instead to "keep it under the mattress." In the correct model of the economy, bank failures are extremely unlikely, but an unsophisticated individual's portfolio decisions are greatly influenced by the worst case scenario. Thus, while a sophisticated investor may be willing to make a bank deposit or, for a higher interest rate, invest in a money market fund (which holds very low risk assets, but is not insured), an unsophisticated investor would not do so even for implausibly high interest rates.

Suppose the government introduces a guarantee on deposits. Now losses to the investor from bank runs do not occur and the worst case scenario of payoffs disappears.[23] The naïve investor optimally deposits money in the bank (while still eschewing the potentially lethal money market fund). By inducing participation in the banking system, deposit insurance results in unsophisticated investors replacing a noninterest-bearing investment with a comparably safe but interest-bearing one. Perhaps more important, this regulatory approach expands the resources available to the banking system and facilitates greater risk sharing in the economy.

Is there any evidence that such an ambiguity effect ever arises? The market effects surrounding the "Too Big to Fail" doctrine suggest that it does. During testimony to Congress in April 1985, the Comptroller of the Currency announced that henceforth the eleven largest banks in the United States would not be allowed to fail, in effect extending 100% deposit insurance to this select group of banks. O'Hara and Shaw (1991) document that this announcement elicited a statistically significant market effect, with the prices of included banks rising an average of almost 2% relative to nonincluded banks. Yet, it was widely presumed that such banks would have been bailed out in any case (and only one large U.S. bank had actually failed in the previous fifty years). Such a large market reaction, however, is perfectly consistent with ambiguity

[23] We assume that the naïve investor believes the government guarantee. Otherwise, he might maintain the same set of models and still not make a deposit. Of course, our discussion also assumes that the naïve investors consider cash to be a safe asset. We also assume that the guarantee is not relevant for the sophisticated investor. That is, it does not change the actual distribution of payoffs.

aversion; removing even an unlikely event from consideration can induce large effects.

Conversely, recent events suggest that the absence of a presumed guarantee can have similarly large effects. The IMF's refusal to bail out Russia following its bond market default in 1998 set off a veritable "free fall" in emerging market debt prices as investors fled to higher quality issues. Intriguingly, this "flight to quality" affected virtually all emerging market debt, even though conditions in Russia were hardly representative of other emerging markets. Yet, such behavior is consistent with ambiguity aversion: investors now perceived that outcomes they previously believed precluded by IMF guarantees were possible. With this new lower $\bar{r}^i_{\min}$, investors opted not to participate in these markets even with very high payoffs.

IMF guarantees and the "Too Big to Fail" policy have both been criticized for creating moral hazard problems in markets. What may not have been appreciated, however, is that such absolute guarantees may also promote stability in markets by enhancing participation by investor groups. Whether such positive participation effects could be large enough to offset the negative moral hazard effects depends, in part, on the prevailing equilibrium. If the market remains in a nonparticipating equilibrium, then providing guarantees only exacerbates moral hazard and is surely the wrong policy.[24] But if there is a participatory equilibrium, or the guarantee moves the markets to such an equilibrium, then the guarantee may be welfare enhancing. This suggests a proactive role for regulatory guarantees in settings where ambiguity is likely to be important.

### 4.2 Suitability rules

Earlier we showed that greater participation by unsophisticated investors results in a lower risk premium. But how do you induce such investors to invest? Because an unsophisticated investor fears the worst, he might not entrust his assets to a financial advisor or broker/dealer for fear that the broker would simply steal the money, or extract it in other ways (such as excessive commissions or fees generated on excessively risky investments).

Suitability rules provide one solution to this problem. Suitability rules are imposed by all major U.S. securities industry self-regulatory organizations.[25] These rules require financial advisors to select only suitable investments for their clients; failure to do so can result in the advisor having to compensate the investor for all losses. Thus, while the investor is still subject to the risk of the underlying investment, suitability rules are designed to rule out losses

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[24] An interesting example of such a trade off is the government guarantees underlying the debt of Fannie Mae and Freddie Mac. Critics argue that the guarantees allowed Fannie and Freddie to reach gargantuan sizes and assume larger risks, all to the benefit of their shareholders. Proponents point to the need to make certain that these firms have access to ready capital. Recent suggested reforms suggest that the perils of moral hazard are seen as more problematic than those of ensuring capital availability.

[25] The NYSE's suitability requirement is Rule 405, also known as the "know your customer" rule. The NASD suitability requirements are found in Rule 2310 of the NASD Manual. See also the SEC website for a discussion of suitability requirements.

due to broker misbehavior. If unsophisticated investors are aware of these rules then they may affect both $\bar{v}^i_{\min}$ and $\sigma^i_{\max}$. Note that by having these rules in force, brokers are much less likely to act in unsuitable ways, and so the rules themselves may rarely come into play. Nonetheless, their existence can induce changes in participation by simply ruling out particular models of payoffs of assets.

### 4.3 Rule 16(b): Insider "short swing" trading profit rules

Another risk of concern to investors is posed by insider trading in a stock. Investors may be unwilling to buy stock for fear that insiders will profit at their expense. A particular concern is that insiders will take advantage of their access to information by engaging in short-term trading in the company's stock. An unsophisticated investor who is unsure of how payoffs are generated may consider possible a model of payoffs in which insiders frequently take advantage of him by engaging in short-term trading, and so may opt not to invest in the stock.

Rule 16(b) of the Securities Exchange Act is designed to remove this ambiguity by requiring an insider (defined as an officer, director, or 10% shareholder) to pay off any profits made on a trading position held for less than six months.[26] Note that unlike insider trading prohibitions in general, Rule 16(b) does not require showing that the insider actually traded on any specific information; simply earning a profit on a trade closed out within a six-month period is enough to trigger the forfeiture rules. But for a naïve investor, such an over-arching rule provides the exact reassurance needed to rule out a model of payoffs determined by such behavior. With the insiders precluded from short-term profits, the investor now perceives a higher $\bar{v}^i_{\min}$, and so may be willing to participate in the market.

### 4.4 Mutual fund diversification restrictions

The examples given above all concern regulations affecting the perceived minimum mean payoff, $\bar{v}^i_{\min}$. As demonstrated in the previous sections, however, an unsophisticated investor is also influenced by the perceived maximum variance, $\sigma^i_{\max}$. An example of a variance-related regulation can be found in the Investment Act of 1940. In particular, among other rules, this act requires that a diversified investment company have no asset holding greater than 25% of its portfolio, and the remaining holdings each can be no more than 5% of its portfolio. This rule necessitates diversification, and this in turn limits the variance of the fund's payoffs.

In a world with only sophisticated investors, it is hard to see why such a rule is necessary; investors seeking to reduce their risk could simply hold larger proportions of risk-free assets in their portfolios, and thereby reach their desired risk-payoff trade off. But in a world with unsophisticated investors, this

[26] See Securities Exchange Act of 1934, 15 USC.

may not occur. A potential investor may consider how the payoff to investment in a mutual fund is determined. If, in one of these models, the mutual fund places much of his investment in a very risky asset, the naïve investor may simply choose not to invest in the fund, fearing that a bad outcome could entail catastrophic loss. This nonparticipation can occur even if, in most worlds the investor considers possible, investing in the fund is advantageous. Because a naïve investor may not participate, the traditional diversification role envisioned in standard asset pricing models is ineffective because these traders will simply not hold the risky asset in his or her portfolio. Regulations requiring such diversification address this ambiguity-induced shortcoming.

### 4.5 Which assets should regulators focus on?

If naïve investors believe that some classes of asset have ambiguous returns, then these asset classes will have positive expected excess returns that cannot be accounted for by the covariation of their returns with the market; that is, they have positive alphas. This suggests that regulators could focus their education efforts on assets with positive alphas. Of course, there may be many reasons for assets to be mispriced, but among the assets with positive alphas are the assets that naïve investors choose to avoid. Education programs that inform investors about the true returns to holding these assets could increase participation and, thus, reduce mispricing. For example, naïve investors may concentrate their portfolio holdings in the stocks of large, well-established companies, and correspondingly avoid stocks in new, small companies. This would generate a positive alpha for young or small companies. If investor education programs could be used to convince naïve investors of the true distribution of returns to holding these stocks, the mispricing would disappear. Naïve investors would be better off and the misallocation of investment across firms would be corrected.

### 4.6 Unlisted securities regulation

The examples given above demonstrate how regulations that change the perceived minimum mean payoffs and maximum variances can induce greater participation by investors, thereby changing the resulting equilibrium in the market. Such a proactive role for regulation is consistent with traditional arguments for regulatory action, although to our knowledge our paper is the first to demonstrate a role for the regulation of unlikely events. The presence of unsophisticated investors, however, suggests another regulatory approach, one that purposely exploits the existence of ambiguity to restrict participation in risky assets. Such an approach can provide investor protection at virtually no explicit cost.

To see how this would work, consider an asset in which at least some investors face difficulty in determining means and variances. Such difficulties could arise from inexperience on the part of investors, or it may reflect a more fundamental problem, such as a paucity of information about the underlying asset. In our model, depending on the structure of regulations, two alternative equilibria could arise. If the ambiguity is severe enough, a nonparticipating equilibrium will occur in which the unsophisticated investors hold none of the asset. Alternatively, to try to reach a participatory equilibrium, the regulators could require extensive disclosure, impose various limitations on the assets' issuers, mandate various educational initiatives, and the like, all in an attempt to reduce the underlying ambiguity.

Which equilibrium yields the lowest cost of capital? We would argue that the answer depends, in part, on the cost of these regulatory initiatives. For high-enough cost levels, the reduction in the equilibrium equity premium may be overwhelmed by the expense needed to induce participation. In this case, the optimal regulatory approach would be to retain the ambiguity and protect investors through their nonparticipation.

We suggest that just such an outcome may explain the paradoxical world of unlisted securities regulation. An equity security listed and traded on a U.S. exchange or on the NASDAQ faces a plethora of SEC regulations, as well as listing requirements, disclosure rules, and even corporate governance restrictions imposed by the exchange. Yet, such listed firms are also among the most actively followed by analysts, investment banks, ratings agencies, the financial press, and even Internet chat rooms. In contrast, unlisted securities (generally defined as securities that trade on the Pink Sheets or on the OTCBB) face much lower regulatory requirements. The SEC requires substantially less disclosure, and there are few, if any, requirements imposed by their trading locales. Moreover, since unlisted firms tend to be small and tightly held, there is often little information available on these firms from other sources.[27] Thus, securities regulation appears to be upside down: firms with the most information face the greatest regulatory scrutiny, while firms with the least information face the least.

Such a regulatory structure makes sense, however, in a world in which ambiguity aversion plays a role. To induce participation by unsophisticated investors in markets for unlisted stocks, the regulators would have to require vast amounts of information, and even then such securities would remain very risky investments. It may be far better to foster participation in stocks with greater information, and leave the ambiguity to restrict participation in more unknown ventures.[28]

[27] Even Internet chat sites often restrict discussion of such stocks. Motley Fool, for example, prohibits chat rooms for stocks with prices below $5.00, a price level much more typical of unlisted stocks. For more discussion of unlisted stocks, see Macey, O'Hara, and Pompilio (forthcoming).

[28] A recent example of how such nonparticipation may be optimal is found in the reverse mergers some Chinese companies have used to access the U.S. capital markets. Unable to meet listing requirements, these Chinese companies arrange to be bought by a U.S. shell company. The newly merged company then trades in the U.S. OTC markets. Yet, as pointed out in a recent *Business Week* article, "Something few Chinese executives consider is that Americans generally shun such shells as they typically trade over the counter. Nobody follows them, there's no market in the shares. So, after having gone through a lot of time and expense and effort, the underlying purposes are not realized." See "Going Public, Chinese Style," *Business Week*, March 5, 2007.

## 5. Generalizations and Extensions

### 5.1 Market forces and ambiguity

Our analysis suggests that regulation can play an important role when at least some traders are unsophisticated. Such a proactive role for direct interference in markets is not generally advocated in the finance literature, where the ameliorative role of market forces is often presumed sufficient to ensure that assets are priced correctly.[29] As we discuss in this section, however, when some traders care both about risk and ambiguity, standard market solutions such as contracting, disclosure, or even arbitrage cannot operate as they do in the more standard paradigm. Consequently, the market may not be able to "solve" the problems induced by ambiguity aversion, underscoring the robustness of our results.

Consider first the role of contracting. Why can't rational traders simply contract between themselves to rule out undesirable outcomes? Certainly, for many specific issues, they can. But the problems here arise from unlikely outcomes, or events that occur with very low probability. Contracting is costly and writing contracts to cover every possible state of the world, let alone every possible model generating such states, is prohibitively expensive. This general difficulty led to the legal concept of fiduciary duties, which essentially "fill in" the holes in contracts by requiring agents to act in particular ways when unspecified events occur.[30] Yet, fiduciary duties are themselves subject to ambiguity, and this problem is particularly severe when the events in question are improbable in nature.[31] Thus, for an unsophisticated agent, the inability to contract on every possible state of the world amplifies the ambiguity, rather than resolves it.

A related difficulty attaches to the enforcement of contracts. Even if complete contracting is feasible, enforcement of contracts is not automatic. Tort enforcement is typically a civil action, and so enforcement relies upon bringing legal action for breach of contract. The ambiguity attached to such a legal process, both in terms of timing and outcome, may be enough to induce nonparticipation. Conversely, laws are enforced via criminal procedures, and the stricter enforcement of laws versus contracts is a particular benefit when ambiguity aversion is a factor. This distinction also underscores why laws may succeed where contracts would fail.

Alternatively, disclosure is often viewed as a market solution when investors face uncertain outcomes. When traders care simply about risk, providing them with information on the various potential outcomes can ameliorate many problems by allowing agents to quantify correctly the risks they face. But when

[29] For an excellent analysis of the role of regulation in markets, see Shleifer (2005).

[30] In particular, an agent owes a principal the duties of care, trust, and loyalty. For an insightful analysis of the economic role of fiduciary duties, see Hart (1994).

[31] This problem is exemplified by the role of fiduciary duties in banking. As discussed by Macey and O'Hara (2003), the courts have vacillated in their view toward the duty of the care of bank directors, with a greater standard enforced when instability problems in banking are high.

they also care about ambiguity, disclosure can actually exacerbate difficulties by making investors aware of obscure outcomes that they otherwise might not have thought possible. For unsophisticated traders, it is the possibility of a bad distribution of outcomes, and not its likelihood of occurrence, that is of greater relevance. Disclosure of risks may thus have the opposite effect of inducing nonparticipation of investors, rather than participation.

Of course, the specific effects of disclosure may depend upon whether the market is in a participatory or nonparticipatory equilibrium. If naïve investors are not participating, then disclosing the panoply of potential risks has no effect on these traders. However, by reducing the perceived risk faced by sophisticated agents, disclosure could reduce the equilibrium risk premium, thereby exhibiting the positive role often ascribed to disclosure remedies. The more interesting case is when unsophisticated investors are participating, but shift to nonparticipation upon learning of outcomes heretofore deemed infeasible. Now disclosure induces nonparticipation in the market and increases the equilibrium risk premium. Consequently, disclosure can exacerbate, rather than resolve, the difficulties induced by ambiguity aversion.

Finally, the role of arbitrage is generally viewed as a panacea for a wide variety of market difficulties. Certainly, if some agents trade irrationally, then rational traders can profit from their mistakes, and in the process push markets to price assets correctly. In our model, however, problems arise not because traders choose sub-optimal trades but rather because some traders choose not to trade at all. The traders remaining in the market will all trade optimally (given their beliefs), but they cannot completely offset the loss of the nonparticipating traders: the aggregate risk must still be borne, albeit now by a smaller number of traders. Thus, while arbitrage can ameliorate mispricing in markets, it cannot eliminate the problems caused by nonparticipation. This result highlights another limit to the role of arbitrage in markets.

### 5.2 Learning and ambiguity

In our analysis, traders are assumed to be either naïve or sophisticated. In a static environment, such a characterization seems sensible, but in a dynamic setting it is less obvious. Over time, one might expect naïve traders to become more knowledgeable, and consequently ambiguity aversion may play less of a role in their decision making. Epstein and Schneider (2007) provide an interesting analysis of learning by ambiguity-averse agents. Over time as their agents receive information, and update their beliefs, they look more and more like expected utility agents. To the extent that our naïve agents actually receive information about payoffs, and process it correctly, they, too, would seem less ambiguity averse over time. However, consider a naïve agent who chooses not to participate in the markets for the risky asset. Does this agent actually receive and process information about the payoffs to the risky asset? It seems plausible that such an agent might simply ignore the information generated by a market that he does not understand and has chosen not to participate in. In this case, he would remain constantly naïve.

Even if naïve agents do learn, it is unclear how long such learning would take in actual markets. Analyzing the speed of learning would require us to be precise about how time is measured (is a period a year, a quarter, a day?), but certainly anecdotal evidence suggests that for many individuals acquiring financial sophistication is a slow process.

## 6. Conclusions

Nonparticipation in markets is a large and puzzling problem. While clearly nonoptimal for individuals, it also imposes costs on the economy through its effect on the equity premium. In this research, we have investigated the implications of ambiguity aversion for the performance and regulation of markets. We developed a model in which all agents are rational, but their decision making may incorporate both risk and ambiguity. Our model demonstrates that nonparticipation can arise from the decision by some traders to avoid ambiguity. In equilibrium, these participation decisions can affect the equilibrium risk premium, and distort the performance of the market when viewed from the perspective of traditional asset pricing models.

The main result of our analysis is that regulation, particularly regulation of unlikely events, can moderate the effects of ambiguity, thereby increasing participation in financial markets, and generating welfare gains. These welfare gains arise because legal rules designed to limit "worst case" outcomes can succeed in fostering participation when more traditional market remedies such as disclosure will fail. An intriguing feature of these rules is that they are asymmetric; because ambiguity aversion is driven by extreme negative outcomes, effective regulation need only concentrate on these "left tail" events. As we have demonstrated here, both the absence and presence of legal rules can influence participation and equilibrium in markets.

Our focus on the positive effects of the legal structure in reducing ambiguity provides new, and we believe, important insights into the role that the legal system can play in markets. Recently, a large empirical literature has developed highlighting the interactions of law and finance (see, for example, La Porta et al. 1997, 2000). A variety of authors have shown that factors such as bankruptcy provisions, minority voting rules, corporate governance requirements, and judicial efficiency can influence market valuations. Indeed, La Porta et al. (2002) conjecture that "by limiting expropriation, the law raises the price that securities fetch in the market place."[32] Our analysis here demonstrates another reason why this occurs. Legal systems that more effectively deal with

[32] Building from these results, the literature then goes on to attempt to understand how legal systems affect growth. The view in this literature is that an improved legal system leads to greater growth. An important qualification to this view is offered in Allen, Qian, and Qian (2005), who argue that although China is rapidly growing, its legal system is not well developed. Instead, in China the private sector seems to be solving problems through reputation and relationships.

worst case events (such as bankruptcy) may be better able to foster participation and its consequent positive effects on the financial markets. By restraining ambiguity, legal systems can influence not only the risks of markets but also their uncertainty as well.

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