Information Quality and Options

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Microstructure researchers have long understood that information quality has an effect on price formation in the underlying asset market. However, option researchers have largely ignored the fact that information quality might also impact the options market. This article characterizes the nature of the impact by showing how option prices and implied volatility levels are related to the forward looking information quality path. This result follows from a noisy rational expectations model that abandons the normal distribution in favor of the gamma distribution, but maintains the standard assumption of exponential utility. Thus the new model bridges the gap between the microstructure literature that relies so heavily on the normal-exponential framework, and the options literature that relies exclusively on models that are consistent with the limited liability of stock prices. The model s tractability allows for a robustness check against the standard framework and provides a viable setting for analyzing the empirical implications of information quality for the options market. (JEL D8, G1)

A continuing puzzle in most option markets is the existence and time-varying nature of an implied volatility term structure and a volatility skew. Over the past three decades, researchers have accommodated these empirical features by abandoning the lognormal assumption of Black and Scholes (1973) in favor of alternative price processes that attempt to better capture the dynamics of the underlying asset. While this type of partial equilibrium analysis has proliferated, it sometimes falls short of reconciling the data (Das and Sundaram 1999) and it does very little to enhance our understanding of how the economy s primitives, such as information quality, affect the options market. Since information quality

Table 1
Basic Features of the Gamma-Exponential and Normal-Exponential Models

Item	Gamma-exponential model	Normal-exponential model
Final payoff prior beliefs	X is Gamma	X

if investors have heterogeneous normally distributed beliefs and constant absolute risk aversion (CARA). This coincides with the Brennan and Cao (1996) result that a quadratic derivative effectively completes the market in the normal-exponential model. However, if CARA investors have heterogeneous gamma-distributed beliefs, Wilson (1968) shows that the appropriate side bets are logarithmic instead of quadratic. This fact is exploited below by introducing a logarithmic payoff derivative that effectively completes the market in the gamma-exponential model. Each investor therefore has the same probability-weighted marginal rate of substitution (MRS), which Brennan (1979) refers to as the economy s risk-neutral valuation relationship (RNVR). Although the investors have asymmetric information, the RNVR is not investor dependent and very sharp predictions can be made about the relationship between information quality and options.

The article is organized as follows. Section 1 analyzes the equilibrium without derivatives trading. Since no derivatives are traded, the main difference between this section and the normal-exponential model is the distributional assumptions. Section 2 extends the equilibrium by adding a logarithmic derivative that effectively completes the market. This allows for precise predictions concerning the information quality level, option prices, and implied volatility levels. Section 3 analyzes endogenous information acquisition, the role of short-lived information, and a limiting version of the gamma-exponential model that converges to the Black and Scholes (1973) model. Section 4 discusses the article s empirical implications and Section 5 offers concluding remarks.

1. The Economy without Derivatives

The economy is a noisy rational expectations equilibrium that is similar in concept to the normal-exponential framework. There are $N + 1$ dates indexed by $n = 0, 1, \ldots, N$ and there is a continuum of rational investors indexed by $i \in [0, 1]$. The ith investor s date n wealth is denoted by $W_{n,i}$ and thus $W_{0,i}$ is the investor s endowed wealth level. The ith investor s utility function is $-\exp\left[-\tau_i W_{N,i}\right]$, where τ_i is the investor s absolute risk aversion parameter. There is no intermediate consumption, which coincides with the utility function being de ned over $W_{N,i}$. Finally there are no wealth constraints, short sale constraints, or other market frictions in the model.

The model departs from the normal-exponential framework by relying on the gamma distribution instead of the normal distribution. A random variable V has a gamma distribution if its probability density takes the form

$$g(v) = \frac{\alpha^{\delta}}{\Gamma(\delta)} v^{\delta-1} e^{-\alpha v}, \qquad v > 0 \tag{1}$$

where $\alpha > 0$, $\delta > 0$, and $\Gamma(\delta) \equiv \int_0^\infty z^{\delta-1}e^{-z}dz$. The parameters α and δ are the scale and shape parameters, respectively. The shorthand notation $V \stackrel{d}{=} Gamma(\alpha, \delta)$ is used where $\stackrel{d}{=}$ denotes equivalence in probability distribution. The mean of V is $\delta\alpha^{-1}$ and the variance of V is $\delta\alpha^{-2}$.

At dates $0, 1, \ldots, N-1$ the investors trade a riskless asset and a risky asset. The riskless asset has a perfectly elastic supply and a unit price (i.e., the interest rate is zero). The risky asset is competitively traded, does not pay intermediate dividends, has a price of S_n per share at date n, and provides a nal liquidating payoff of X per share at date N. For notational convenience, the prices are sometimes indexed up through date N, in which case S_N is identical to X. The investors have the common prior belief that $X \stackrel{d}{=} Gamma(\eta\zeta, \zeta)$, where $\eta > 0$ and $\zeta > 0$. The prior mean and variance of X are therefore $E(X) = \eta^{-1}$ and $Var(X) = \eta^{-2}\zeta^{-1}$. For a given mean, the parameter ζ is an indicator of the payoff s prior precision.

The ith investor at the nth trading date observes a costless private signal $Y_{n,i}$ that is correlated with the risky asset s payoff.[3] The signal has a multiplicative form $Y_{n,i} = \varepsilon_{n,i}X^{-1}$, where $\varepsilon_{n,i} \stackrel{d}{=} Gamma(\theta_{n,i}, \theta_{n,i})$. Note that $E(\varepsilon_{n,i}$

assumed that X, Z_n for all n, and $\varepsilon_{n,i}$ for all n and i are mutually independent random variables.

To close the model, it is assumed (as in the normal-exponential model) that noise traders supply a random quantity of shares to the market. The aggregate noise trader supply of the risky asset at date n is given by

$S_0, S_1, \ldots, S_{N-1}$, such that $\hat{D}_{n,i}$ solves

$$J_{n,i} = \max_{D_{n,i}} E\left[J_{n+1,i}\left(W_{n+1,i}\right)\middle|\mathcal{I}_{n,i}\right] \tag{4}$$

subject to the budget constraint $W_{n+1,i} = W_{n,i} + D_{n,i}(S_{n+1} - S_n)$ and S_n satisfies

$$\int_0^1 \hat{D}_{n,i}di = E^M\left[\int_0^1 \hat{D}_{n,i}di \,\middle|\, S_0, S_1, \ldots, S_n, Z_n\right] \tag{5}$$

which is the date n market clearing condition. The indirect utility function at date n, denoted by $J_{n,i}$ in Equation (4), satisfies the terminal value condition $J_{N,i} = -\exp\left[-\tau_i W_{N,i}\right]$. Thus the investor uses dynamic programming to determine an intertemporally consistent trading strategy that maximizes his expected utility at each date. The gamma-exponential equilibrium is characterized next.

Proposition 1. There exists a partially revealing rational expectations equilibrium in which the ith investor's optimal date n demand for the risky asset is

$$\hat{D}_{n,i} = \frac{\mu_{n,i}}{\tau_i}\left[\frac{1}{S_n} - \frac{1}{E(S_{n+1}|\mathcal{I}_{n,i})}\right] \tag{6}$$

for $n = 0, 1, \ldots, N-1$. The constant $\mu_{n,i}$ is

$$\mu_{n,i} = \zeta + \sum_{k=0}^{n}\theta_{k,i} + \sum_{k=0}^{n}\beta_k \tag{7}$$

and the equilibrium risky asset price at date n is given by Equation (2).

The proof is in the Appendix.[6] By rearranging Equation (6) it is easy to see that the ith investor at date n takes a long (short) position in the risky asset if and only if $E\left[\frac{S_{n+1}}{S_n}|\mathcal{I}_{n,i}\right]$ is greater (less) than 1. Furthermore, note that $\mu_{n,i}$ is a measure of the ith investor's cumulative information quality up through date n. Since $\hat{D}_{n,i}$ is proportional to $\mu_{n,i}$, investors with higher quality information take more aggressive positions. While $\hat{D}_{n,i}$ depends on the signal history via $\mathcal{I}_{n,i}$, the investor's optimal trade $\hat{D}_{n,i} - \hat{D}_{n-1,i}$ depends only on the investor's current period signal. Since investors receive private signals at each trading date, there is nontrivial trading volume in

[6] As discussed in the Appendix, a version of the Separation Principle holds in the gamma-exponential model. Although the price is nonlinearly related to the noise variable and the signal is nonlinearly related to the payoff, the conditional densities $p(s_n|x)$ and $p(y_{n,i}|x)$ have scale (shape) parameters that are linear in (independent of) x. Thus, the ith agent's posterior scale and shape parameters are propagated linearly through time.

every period. In the special case of $N = 1$, the equilibrium in Proposition 1 reduces to that shown in Table 1.

1.2 Properties of the equilibrium

Since the above equilibrium is new, it is worthwhile to explore its properties. These properties are shown in Table 2 and a sketch of their proof is in the Appendix. Property 1 shows that price changes and the price level have a negative covariance. A similar result holds in the normal-exponential framework (Brennan and Cao 1996). However, Property 2 shows that successive price changes negatively covary, a result that contradicts the normal-exponential framework (Brennan and Cao 1996). Thus, the statistical features of the two frameworks prices are different. Property 3 shows that the conditional volatility tends to be higher (lower) at low (high) price levels, a phenomenon that has been observed empirically at least since

losses. Odean (1998a) provides empirical evidence that is consistent with this behavior. Comparative statics reveal that investors with higher $\mu_{n,i}$ exhibit stronger negative correlations between their optimal trades and price changes. This can be contrasted with the normal-exponential model (Brennan and Cao 1996, Corollary 1) in which rational investors with less than average signal precisions exhibit positive correlations between their optimal trades and price changes. Investor behavior in the gamma-exponential model is therefore different than in the standard framework.

Property 6 reveals a positive covariance between expected short selling and stock returns, which is consistent with the empirical evidence.[7] However, as shown in Property 7, investors that are short at date n expect to cover their short positions during the next trading round. This is due to the asymmetry of the gamma distribution. Since the price in Equation (2) exhibits limited liability, short selling involves taking a position with limited upside but potentially unlimited downside, while going long is just the opposite situation. To determine which investors are more likely to be short sellers, the quantity

$$\frac{E\left[\left|\hat{D}_{n,i}\right| 1_{\left\{\hat{D}_{n,i}<0\right\}}\right]}{E\left[\left|\hat{D}_{n,i}\right|\right]}$$

is examined, which measures the likelihood that the ith investor optimally goes short at date n. Comparative statics reveal that this quantity is increasing in $\mu_{n,i}$. Investors with poor information quality are more likely to "play it safe" and take long positions in the risky asset, while investors with higher information quality are more likely to undertake short positions.

Lastly, Properties 8 and 9 show that both the expected risk premium and the posterior variance are path dependent.[8] Fixing S_n, both quantities are lower (higher) if the market runs up (slides down) to the level S_n. All else equal, investors expect a lower risk premium and a lower variance following a market rally and expect a higher risk premium and a higher variance after a market downturn. The path dependence implies that the variance is heteroskedastic, a feature that also shows up in Foster and Viswanathan (1993, Proposition 2). In Property 8, a positive (negative) risk premium is consistent with the ith investor holding a long (short) position in the risky asset. Furthermore, note that the risk premium explicitly depends on information quality. Comparative statics reveal that the risk

[7] Woolridge and Dickinson (1994) find a positive relationship between monthly short selling activity and stock returns. Brent, Morse, and Stice (1990) find that months with an increase in short interest have security returns that are 3–4% higher than months with a drop in short interest.

[8] Taking limits as $\theta_{k,i} \to 0$ for all k gives the expected risk premium and the posterior variance conditional on the public information set that includes only prices.

premium is decreasing in β_n if the ith investor is long the risky asset. This is consistent with Li (2005) and Brevik and d Addona (2005), who nd that low quality public information can increase the risk premium. However, the relationship between the risk premium and β_k for $k < n$ depends on both the sign of $\hat{D}_{n,i}$ and the sign of $S_n - S_k$. If the investor is long the risky asset and the stock market has increased from date k to date n, the risk premium is decreasing in β_k. But the opposite result may hold if the market has suf ciently decreased between dates k and n. Thus, under certain conditions the equilibrium is consistent with Veronesi s (2000) result that higher quality public information increases the risk premium.

2. The Economy with Derivatives

In order to motivate derivatives trading in the gamma-exponential model, it is useful to return to Proposition 1 and examine the ith investor s probability-weighted MRS at date n. A direct calculation reveals that the MRS is a gamma density with scale parameter $\mu_{n,i} S_n^{-1}$ and shape parameter $\mu_{n,i}$, i.e.,

$$\phi(S_{n+1}|\mathcal{I}_{n,i}) = \frac{1}{\Gamma(\mu_{n,i})} \left(\frac{\mu_{n,i}}{S_n} \right)^{\mu_{n,i}} (S_{n+1})^{\mu_{n,i}-1} e^{-S_{n+1}\left(\frac{\mu_{n,i}}{S_n}\right)}. \qquad (8)$$

Since $E^{\phi}\left[S_{n+1}|\mathcal{I}_{n,i}\right] = S_n$ for all i, it is clear that all investors in Proposition 1 agree on the price of the traded risky asset. However, since the scale and shape parameters in Equation (8) depend on i there is generally disagreement about the values of other nontraded payoffs such as options. By opening the right derivatives market, the investors can optimally trade away the differences in their scale and shape parameters and thus coordinate their private valuations. This turns out to be important since it implies that the equilibrium relationship between information quality and options is not investor dependent if the right derivative contract is traded.

The right derivative contract is determined by examining the results in Wilson (1968). In the case of heterogeneous gamma-distributed beliefs, Wilson (1968) shows that Pareto ef ciency is achieved via logarithmic side bets. The equilibrium in Proposition 1 is therefore extended by introducing a zero-net-supply derivative contract whose payoff at date N is $\log(X)$, where $\log(\cdot)$ denotes the natural logarithm. The log derivative is available for trading at each date (i.e., it is long-lived) and its price at date n is denoted by L_n. Although there is a continuum of states and only three assets are available for trading, the log derivative effectively completes the market. Thus the log derivative contract in the gamma-exponential model plays the same role as the quadratic derivative contract in the normal-exponential model (Brennan and Cao 1996).

When the log derivative market is open, the ith investor s information set includes not only his private signals and the risky asset s price history

but also the log derivative s price history. Since the latter price history can potentially alter the ith investor s posterior beliefs, it is useful to examine the informativeness of the log derivative. Before the log derivative market is open, the ith investor s marginal valuation at date n is $E^{\phi}\left[\log(X)|\mathcal{I}_{n,i}\right]$, which can be evaluated by iterating the expectation and successively applying Equation (8) at dates $N-1, N-2, \ldots, n$. Thus the marginal valuation is a function of S_n. It therefore seems reasonable to conjecture that the equilibrium log derivative price is also a function of S_n. Since the investors observe S_n directly, the log derivative provides no new information about X. This conjecture is veri ed in equilibrium and it parallels Brennan and Cao s (1996) result that their quadratic derivative is uninformative. The next proposition presents the equilibrium with a zero-net-supply log derivative. With the exception of adding the log derivative, the model structure is the same as in Section 1.

Proposition 2. There exists a partially revealing rational expectations equilibrium in which the ith investor s optimal risky asset demand is

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by using the spanning result of Bakshi and Madan (2000) and Carr and Madan (2001). They show that any twice continuously differentiable function $h(x)$ can be represented as

$$h(x) = h(b) + (x - b)h_x(b) + \int_b^\infty h_{xx}(k) \max[0, x - k]\, dk + \int_0^b h_{xx}(k) \max[0, k - x]\, dk \tag{13}$$

where the subscripts on h denote differentiation. The payoff $h(x)$ is equivalent to a portfolio that contains $h(b) - bh_x(b)$ dollars in the riskless asset, $h_x(b)$ shares of the risky asset, and a continuum of European call and put options.[10] If b is equal to the current underlying asset price, the portfolio includes all out-of-the-money calls and puts. Since $h(x) = \log(x)$ is concave, $h_{xx}(k) < 0$. This implies that buyers (writers) of the log derivative are writing (buying) calls and puts. On the other hand, since Brennan and Cao s (1996) quadratic derivative is convex, $h_{xx}(k) > 0$. This implies that buyers (writers) of their quadratic derivative are buying (writing) calls and puts. Thus in both the normal-exponential and gamma-exponential frameworks, investors with higher (lower) than average information quality undertake short (long) call and put positions. The two frameworks agree that better informed investors are the ones that write options.

The fact that the optimal log derivative demand in Equation (10) is so similar to the optimal quadratic derivative demand in Brennan and Cao (1996) is remarkable. In the normal-exponential framework, the ith investor s posterior variance is a constant and the quadratic derivative allows the investors to coordinate these variances under the RNVR. However, in the gamma-exponential framework, the ith investor s posterior variance at date n is heteroskedastic (see Property 9 in Table 2). The posterior variance depends on both the investor s information quality and the path of prices and signals, but the demand in Equation (10) depends only on information quality. This highlights the fact that the log derivative s main role is to allow investors to coordinate their posterior shape parameters rather than their posterior variances. A nonzero trading volume in the log derivative arises precisely because investors have heterogeneous shape parameters in Equation (8).

While investors coordinate their posterior shape parameters by trading the log derivative, they coordinate their posterior scale parameters by

[10] Equation (13) connects the log derivative contract to the types that are actually observed in practice (i.e., calls and puts). The agents do not trade the calls and puts individually, but instead they trade a portfolio that spans the log function. Thus, Proposition 2 can be interpreted in terms of three-fund separation in which one of the three funds includes standard calls and puts. See Cass and Stiglitz (1970) for early work on portfolio separation.

altering their optimal demands for the risky asset. This can be seen by comparing Equations (6) and (9). Investors that go long (short) the log derivative in Proposition 2 find it optimal to decrease (increase) their risky asset demand relative to the economy in Proposition 1. This offset arises because the equilibrium price of the log derivative at date n is increasing in S_n, i.e., the log derivative has a positive delta.

In Proposition 2, the ith investor's probability-weighted MRS at date n is

$$\phi(S_{n+1}|\mathcal{I}_{n,i}) = \frac{1}{\Gamma(\mu_n)}\left(\frac{\mu_n}{S_n}\right)^{\mu_n}(S_{n+1})^{\mu_n-1}e^{-S_{n+1}\left(\frac{\mu_n}{S_n}\right)} \tag{14}$$

where μ_n is from Equation (11). Unlike Equation (8), the right-hand side of Equation (14) does not depend on i.[11] Since the log derivative price in Equation (12) is linear in $\log(S_n)$ at date n, a single long-lived log derivative contract effectively completes the market for the entire sequence of trading dates $n = 0, 1, \ldots, N-1$. Every nontraded uninformative asset is now viewed by all investors as being redundant, and any asset can be valued by taking an expectation of the asset's payoff using the RNVR in Equation (14). This fact is exploited to derive a relationship between option prices and the forward looking information quality path.

2.1 The relationship between option prices and information quality

Since Equation (14) depends on the information quality parameter μ_n, European option values will also depend on information quality. For example, a one-period call option with strike price K has a date n value equal to

$$C_n = E^{\phi}[\max(S_{n+1}-K,0)|S_n] = S_n\frac{\Gamma\left(1+\mu_n,\frac{K}{S_n}\mu_n\right)}{\Gamma\left(1+\mu_n\right)} - K\frac{\Gamma\left(\mu_n,\frac{K}{S_n}\mu_n\right)}{\Gamma(\mu_n)} \tag{15}$$

where $\Gamma(\delta,d) \equiv \int_d^{\infty} z^{\delta-1}e^{-z}dz$ is the incomplete gamma function. Likewise, a one-period put option with strike price K has a date n value equal to

$$P_n = K\frac{\gamma\left(\mu_n,\frac{K}{S_n}\mu_n\right)}{\Gamma\left(\mu_n\right)} - S_n\frac{\gamma\left(1+\mu_n,\frac{K}{S_n}\mu_n\right)}{\Gamma(1+\mu_n)} \tag{16}$$

[11] Using the terminology of Duffie and Kan (2002), Equation (14) is a universal equivalent martingale measure (EMM). Although Duffie and Kan (2002) show robust nonexistence of a universal EMM when agents have asymmetric information, this article, like that of Brennan and Cao (1996), shows how derivatives trading can produce a universal EMM even though such an item does not exist in the absence of derivatives trading (see Equation (8)).

where $\gamma(\delta, d) \equiv \Gamma(\delta) - \Gamma(\delta, d)$. Since μ_n affects both δ and d in the incomplete gamma functions, it is not immediately obvious whether information quality is positively or negatively related to the option values in Equations (15) and (16). This issue is resolved next.

Proposition 3. The option values in Equations (15) and (16) are decreasing in μ_n.

The proof relies on two results from the literature. Consider two different information quality levels, μ_{high} and μ_{low}, where $\mu_{\text{high}} > \mu_{\text{low}}$. Substituting each of these levels into Equation (14) for μ_n produces a pair of gamma distributions for the RNVR. As Ali (1975, Theorem 3(iv)) shows, the gamma distribution that uses μ_{high} second-order stochastically dominates the gamma distribution that uses μ_{low}. Since the two distributions have the same mean, the results in Jagannathan (1984) can be used to deduce that the option values in Equations (15) and (16) are lower (higher) using μ_{high} (μ_{low}). The second-order dominating distribution, which is less risky in this case, produces lower option values. This suggests that information quality, as captured by μ_n, is a measure of option market risk. When information quality is extremely high, Proposition 3 tells us that options will be priced very near their intrinsic values, resulting in very low time values.

While Equation (14) speci es the conditional single-period RNVR, the conditional multiperiod RNVR is needed to study the impact of information quality on long-lived options. Letting $R_{n+1} = S_{n+1}/S_n$, Equation (14) implies that $R_{n+1} \stackrel{d}{=} Gamma\left(\mu_n, \mu_n\right)$. The conditional returns under the RNVR are therefore independent gamma-distributed random variables with nonstationary shape and scale parameters. The multiperiod return is thus a product of independent gamma-distributed conditional returns, and the probability density for this product can be calculated using standard mathematical methods. The multiperiod RNVR can then be used to value long-lived European options. These results are summarized in the next proposition, whose proof is in the Appendix.

Proposition 4. Let $0 \leq m < n \leq N$. The RNVR for S_n conditional on S_m is

$$\phi(S_n|S_m) = \frac{(a_{mn}S_n)^{\mu_{n-1}}}{b_{mn}S_n\,(S_m)^{\mu_{n-1}}}\,\mathsf{G}^{n-m,0}_{0,n-m}\left[\frac{a_{mn}S_n}{S_m}\,\middle|\,\mu_m - \mu_{n-1}, \mu_{m+1} - \mu_{n-1}, \ldots, 0\right] \tag{17}$$

de ned for $S_n > 0$. The parameters a_{mn} and b_{mn} are

$$a_{mn} \equiv \prod_{k=m}^{n-1} \mu_k \qquad b_{mn} \equiv \prod_{k=m}^{n-1} \Gamma(\mu_k) \tag{18}$$

and $G^{n-m,0}_{0,n-m}[\cdot|\cdot,\ldots,\cdot]$ is Meijer s G-function, which is described in the Appendix. Using Equation (17), the value of a European put option at date m with strike price K and date n payoff $\max(K - S_n, 0)$ is

$$P_m = \binom{K}{b_{mn}} G^{n-m,1}_{1,n-m+1}\left[\begin{matrix} a_{mn}K \\ S_m \end{matrix}\middle|\begin{matrix} 1 & & & \\ \mu_m, & \ldots, & \mu_{n-1}, & 0 \end{matrix}\right] \tag{19}$$
$$- \binom{S_m}{a_{mn}b_{mn}} G^{n-m,1}_{1,n-m+1}\left[\begin{matrix} a_{mn}K \\ S_m \end{matrix}\middle|\begin{matrix} 1 & & & \\ 1+\mu_m, & \ldots, & 1+\mu_{n-1}, & 0 \end{matrix}\right].$$

The value of an otherwise similar call option is $C_m = S_m + P_m - K$.

Proposition 4 shows that the forward looking path of the weighted average cumulative information quality between dates m and n plays an important role in determining option values. The intuition follows from the fact that the date n stock price can be written as $S_n = S_m R_{m+1} \cdots R_n$, where $R_{k+1} = S_{k+1}/S_k$ for $k = m, m+1, \ldots, n-1$. Since Equation (14) implies that the mean of R_{k+1} is equal to 1 and the variance is equal to $1/\mu_k$, each of the parameters $\mu_m, \mu_{m+1}, \ldots, \mu_{n-1}$ has an impact on the risk-neutral distribution in Equation (17). This explains why Equation (12) depends on the forward looking information quality path. It also indicates that the information quality path is a key determinant of the volatility skew and the implied volatility term structure, a fact that is investigated later in the article.[12]

Since μ_m is the cumulative information quality, the order in which the investors receive their private signals can have a profound impact on option values. Let $S_0 = 50$ and consider a two-period economy with parameters $\zeta = 1$, $\beta_0 = \beta_1 = 3$, $\theta_0 = 0$, and $\theta_1 = 1$. In this case $\mu_0 = 4$, $\mu_1 = 8$, and the value of an at-the-money (ATM) call option with two periods until expiration is 11.96. Now suppose the order of the investors private signals is reversed so that $\theta_0 = 1$ and $\theta_1 = 0$. In this case $\mu_0 = 5$, $\mu_1 = 8$, and the value of an ATM call option with two periods until expiration is 11.17. The option price falls more than 6.5% by simply reversing the signal order. Thus if high quality information is going to be received by investors, receiving this information early in the option s life results in a lower risk-neutral variance and a lower option value. More generally, all else equal, an increase in any one of the parameters $\mu_m, \mu_{m+1}, \ldots, \mu_{n-1}$ in Equation (19) will produce a lower option value. The reasoning follows from Proposition 3 and its proof.

[12] Das and Sundaram (1999) show that conventional option pricing models, such as jump diffusion and stochastic volatility models, do not admit risk-neutral distributions that are suf ciently exible to match the data. Proposition 4 suggests that the information quality path is an important omitted variable in these conventional models. The information quality path should also be relevant for explaining the ne structure of asset returns (Carr, Gemen, Madan, and Yor 2002) and the tail behavior of the risk-neutral distribution (Wu 2006).

Since the RNVR in Equation (17) is not a lognormal distribution (Black and Scholes 1973), the option values in Proposition 4 are consistent with both a volatility skew and an implied volatility term structure.[13] Owing to the model s structure and the cumulative nature of investor information, it is clear that $\mu_m < \mu_{m+1} < \cdots < \mu_{n-1}$. Short-dated options are priced with higher conditional risk neutral variances relative to longer-dated options, and this results in a nonincreasing term structure of implied volatility. The term structure s downward slope is directly related to the change in the weighted average cumulative information quality from period to period,

(21) is the appropriate measure in the normal-exponential model. Like the conditional RNVR in Equation (14), the right-hand side of Equation (20) does not depend on i. This implies that the relationship between option prices and information quality is not investor-dependent.

Consider the date m value of a European call option expiring at date $n > m$. Using Equation (20), the option s value is $C_m = E_m^{\phi}[\max(S_n - K, 0)]$ where K is the strike price. To see that C_m depends on the information quality path, rewrite the payoff in terms of the future price increments to get

$$\max[(S_n - S_{n-1}) + (S_{n-1} - S_{n-2}) + \cdots + (S_{m+1} - S_m) + S_m - K, 0].$$

Note that $E_m^{\phi}[(S_k - S_{k-1})] = 0$ for any $k > m$ and $E_m^{\phi}[(S_k - S_{k-1})(S_j - S_{j-1})] = 0$ for any $j > m$ and $k > m$ such that $j \neq k$. Thus the future price increments under the RNVR in Equation (20) have zero means and are uncorrelated. Since the variance of a sum of uncorrelated random variables is equal to the sum of the variances, the multiperiod RNVR for S_n conditional on S_m is normally distributed with mean S_m and variance $\xi_m^{-1} + \xi_{m+1}^{-1} + \cdots + \xi_{n-1}^{-1}$. Thus C_m explicitly depends on the information quality path $\xi_m, \xi_{m+1}, \ldots, \xi_{n-1}$ via $\xi_m^{-1} + \xi_{m+1}^{-1} + \cdots + \xi_{n-1}^{-1}$. So regardless of whether the gamma-exponential or normal-exponential framework is used, option prices are driven by the forward looking information quality path.

3. What Determines the Information Quality Path?

The relationship between option prices and information quality can be sharpened by considering the ith investor s information acquisition problem. By endogenizing the ith investor s private signal quality $\theta_{n,i}$ at each date n, the weighted average cumulative information quality μ_n is also endogenized. Thus, the option prices in Equation (19), the volatility skew, and the implied volatility term structure are all driven by information acquisition costs.

Suppose that the cost of a private signal at date n with quality θ is given by the function $c(\theta)$, where $c(0) = 0$, $c'(\theta) > 0$, and $c''(\theta) > 0$. The cost function is increasing and convex, as in Verrecchia (1982). Letting $\hat{\theta}_{n,i}$ denote the ith investor s optimal signal quality at date n, the next proposition characterizes the equilibrium effects of endogenous information acquisition in the gamma-exponential model.

Proposition 5. Suppose that all investors have access to costly private information with cost function $c(\theta)$ at date n. Then the following statements are true:

(i) If the log derivative is traded as in Proposition 2, the ith investor s optimal signal quality satis es $\hat{\theta}_{n+1,i} < \hat{\theta}_{n,i}$ for all n;

(ii) All else equal, if the ith investor is long (short) the log derivative at each date n, the quality of information acquired by the ith investor in the presence of the log derivative is higher (lower) than that acquired in the economy without the log derivative;

(iii) A shift in the cost function $c(\theta)$ can alter option prices even if the underlying asset price does not change.

The proof is in the Appendix. Property (i) shows that relatively higher (lower) quality information is acquired when the horizon is distant (near). All else equal, this behavior implies that the path of the weighted average cumulative information quality is increasing at a decreasing rate. Thus the term structure of implied volatility not only slopes downward as in Section 2.1, but the graph of the implied volatility versus the expiration date is convex. This sharpens the model s predictions when private information is long-lived.

Property (ii) shows how information quality is altered when derivatives are traded. From Equation (13), being long (short) the log derivative is equivalent to being short (long) the call and put options that comprise the log payoff portfolio. Thus, call and put writers (buyers) optimally acquire more (less) precise information relative to what they would have acquired without derivatives trading. This result is only partially consistent with those in Cao (1999, p. 139) and Massa (2002, p. 946).

Lastly, property (iii) reveals that time-varying information acquisition costs are potentially important for explaining the observed variation in option prices and implied volatilities. Option values may move in response to time-varying information acquisition costs even if the underlying asset price does not move. This provides one possible economic explanation for the observations in Bakshi, Cao, and Chen (2000). Furthermore, since it is typical not to accommodate information acquisition costs when calculating an option s delta, option positions that are delta-hedged may not have full protection. If costs are uctuating through time, full protection would require that the sensitivity to changes in information quality also be hedged.[14]

3.1 The role of short-lived private information

Section 2.1 reveals that the type of private information that is available to investors is important for explaining observable option market features. If investors have access to only long-lived private information as in Proposition 2, the cumulative nature of the information suggests that the conditional return variance under the RNVR is always decreasing

[14] Bakshi and Kapadia (2003) show that delta-hedged S&P 500 index options tend to have negative returns. They attribute their empirical results to a negative risk premium for latent volatility. The gamma-exponential model suggests that uctuating information quality might also be a contributing factor.

through time. This implies that the implied volatility term structure slopes downward, which is inconsistent with the empirical results that are documented in Bates (1996). To reconcile this issue within the gamma-exponential framework, an alternative economy is examined that includes short-lived private information.

In the economy with short-lived private information, investors not only receive private signals on the final payoff X, but also on the intermediate prices. Suppose the ith investor at date n receives $N - n$ private signals given by $Y_{n,k,i} = \varepsilon_{n,k,i} S_k^{-1}$ for $k = n + 1, n + 2, \ldots, N$. The noise terms $\varepsilon_{n,k,i}$ are independent of the other variables in the model and satisfy $E(\varepsilon_{n,k,i}) = 1$ and $Var(\varepsilon_{n,k,i}) = \theta_{n,k,i}^{-1}$. If $\theta_{n,k,i} \to 0$, the ith investor's date n signal on the future price S_k is uninformative. Thus the model in Proposition 2 is nested within this more general framework.

To see the implications of short-lived private information, note that once date n arrives and S_n is observed, all of the signals on the date n price that were observed prior to date n are uninformative. Some of the investor's information expires at date n and this is the sense in which the information is short-lived. Thus, unlike the economy with only long-lived private information, the parameters $\mu_m, \mu_{m+1}, \ldots, \mu_{n-1}$ in Equations (17) and (19) need not be an increasing sequence. The exact intertemporal pattern of information quality that prevails in equilibrium will depend on the average quality of the expiring information relative to the average quality of any new information that is observed on a given date. This intertemporal pattern gets embedded in the term structure of implied volatility, a fact that is outlined next.

Proposition 6. In the economy with short-lived private information, the following are true:

(i) an increasing term structure of implied volatility is consistent with a period of time during which the average quality of expiring information is greater than the average quality of newly acquired information, i.e., the parameters in Equations (17) and (19) satisfy $\mu_m > \mu_{m+1} > \cdots > \mu_{n-1}$;

(ii) an anomalous bulge (dip) in the term structure of implied volatility is consistent with a time period in which the average quality of expiring information is greater (less) than the average quality of newly acquired information, followed by a time period in which the average quality of expiring information is less (greater) than the average quality of newly acquired information.

Property (i) says that the term structure slopes upward if information expires persistently faster than it is acquired, while property (ii) says that bulges and dips are caused by information quality reversals. While the information quality path has a strong impact on the term structure of implied volatility, its effect on the implied volatility skew is more muted. To see this, note that a_{mn} and b_{mn} in Equation (18) and the

G-functions in Equation (19) are all symmetric[15] with respect to the parameters $\mu_m, \mu_{m+1}, \ldots, \mu_{n-1}$. Thus, a permutation of these parameters has no effect on the prices of options that expire at date n. In turn, there is no impact on the implied volatility skew for date n. This has empirical implications since the precise ordering of the parameter set $\mu_m, \mu_{m+1}, \ldots, \mu_{n-1}$, which is necessary to ascertain the average quality of the investors future information ow, can only be inferred from the term structure of implied volatility.

For S&P 500 index options, the empirical slope of the implied volatility term structure can be positive or negative. For example, Derman and Kani (1994) note that the S&P 500 term structure was upward sloping during May 1993, while Das and Sundaram (1999) note that the term structure was downward sloping during October/November 1997. While conventional option pricing models such as the jump-diffusion model are unable to explain this type of time variation (Das and Sundaram 1999), the gamma-exponential model offers a rational explanation. Downward sloping implied volatility term structures are due to the cumulative nature of long-lived information (Section 2.1), while upward sloping implied volatility term structures, as well as bulges and dips, are driven by expiring information (Proposition 6).

3.2 Black and Scholes (1973) limiting version of the model

Although Equation (19) is based on the gamma distribution, the Black and Scholes (1973) model is obtained in the limit as the trading interval decreases and the average information quality simultaneously increases. This limiting result is useful for two reasons. First, it shows that the Black and Scholes model arises as the equilibrium option pricing model only under a very restrictive set of conditions on trading frequency and information quality.[16] To the extent that these conditions do not mirror actual nancial markets, the Black and Scholes model will not be a very accurate description of option prices. Second, even if these conditions do hold, it reveals that the dependence of option values on information quality does not vanish in the limit. The option price in Equation (19) depends on information quality and this property is inherited by the limiting Black and Scholes model. As shown below, the volatility parameter of the limiting lognormal distribution

[15] The symmetry of the G-function can be seen by examining expressions (A24) (A26) in the Appendix. A similar type of symmetry shows up in the RNVR of the normal-exponential framework (see Section 2.2).

[16] Other discrete time equilibrium derivations of the Black and Scholes (1973) model exist. For early work see Rubinstein (1976), Brennan (1979), and Stapleton and Subrahmanyam (1984), while for more recent work see Camara (2003, 2005). Unlike the current article, none of these derivations relies on information quality.

explicitly depends on the average information quality over the life of the option.

Fix a time interval $[t, T]$ and assume that the N periods of the gamma-exponential model are equally spaced within $[t, T]$. The step size is $(T - t)/N$ and trading takes place at t, $t + (T - t)/N$, $t + 2(T - t)/N$, and so on. Letting $R_n = S_n/S_{n-1}$, the risky asset s payoff at time T can be written as $S_0 R_1 R_2 \cdots R_N = S_0 \exp\left[\sum_{n=1}^{N} \log(R_n)\right] = S_0 \exp\left[\sqrt{\frac{1}{N}} \sum_{n=1}^{N} \log(R_n) \sqrt{N}\right]$. Since the denominator in the exponent is N and the sum involves independent random variables, it should be possible to let $N \to \infty$ and scale the parameter μ_n in such a way that the central limit theorem can be applied. In this case $\sqrt{\frac{1}{N}} \sum_{n=1}^{N} \log(R_n) \sqrt{N}$ will have a limiting normal distribution and $S_0 \exp\left[\sqrt{\frac{1}{N}} \sum_{n=1}^{N} \log(R_n) \sqrt{N}\right]$ will have a limiting lognormal distribution, which coincides with Black and Scholes (1973). This fact is formally stated in the next proposition.

Proposition 7. Let $R_n = S_n/S_{n-1}$ and let μ_n in (14) be equal to $k_n N$, where $k_n \in [k_l, k_u]$ for all n and $0 < k_l < k_u < \infty$. Then the time T payoff, which is given by the product $S_0 R_1 R_2 \cdots R_N$, tends to the lognormal distribution under the RNVR as $N \to \infty$. The limiting distribution is equal to $S_0 \exp\left[-\frac{1}{2} A + \sqrt{A} U\right]$, where $A = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \frac{1}{k_{n-1}}$ and U is a standard normal random variable. The Black and Scholes (1973) volatility parameter is given by

$$\sigma = \sqrt{\frac{A}{T - t}} = \sqrt{\frac{1}{T - t}} \left[\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \frac{1}{k_{n-1}}\right]^{\frac{1}{2}}. \tag{22}$$

The proof is in the Appendix. Although the variance of R_n under the RNVR tends to zero as N becomes large, the number of conditional returns in the product $S_0 R_1 R_2 \cdots R_N$ tends to in nity. Owing to the choice of scaling ($\mu_n = k_n N$), the variance tends to the nonzero nite quantity A. Since A is the variance over $[t, T]$, the appropriate volatility input to the Black and Scholes (1973) model is given by Equation (22).

Equation (22) shows that the volatility depends on the information quality path over the given horizon. To the extent that information quality is time varying, different horizons may produce different values for σ and thus a term structure of implied volatility may persist in equilibrium. However, since all options expiring at a given T use the same σ, the volatility skew disappears in the limiting version of the model.

4. Empirical Implications

This section investigates the connection between information quality and observed option market phenomena.[17] As shown below, information quality can potentially explain why option prices sometimes move even if the underlying asset price does not move (Bakshi, Cao, and Chen 2000), why option markets sometimes overreact (Stein 1989, Poteshman 2001), and why implied volatility is a biased predictor of future realized volatility (Canina and Figlewski 1993, Christensen and Prabhala 1998).

4.1 Information quality and option price movements

According to Bakshi, Cao, and Chen (2000), S&P 500 index options are not redundant securities. Depending on the sampling interval, call option prices and the underlying index move in opposite directions between 7.3% and 16.2% of the time. Likewise, put options and the underlying index move in the same direction a similar proportion of the time.[18] For both puts and calls, the effect is more pronounced for longer term options. These empirical results suggest that a variable other than the underlying index is partially responsible for option price movements.

A potential explanation is that shifts in information quality may more than offset movements in the underlying index.[19] Table 3 illustrates how joint movements in information quality and the underlying asset affect call option values in the gamma-exponential model. As Panel A shows, there is about a 3.5% increase (decrease) in the value of a one-period ATM call option if the underlying asset increases (decreases) by 1% and information quality does not move. The increase (decrease) in the call option value is exacerbated if the underlying asset increases (decreases) and the information quality simultaneously decreases (increases). For example, a 1% increase in the underlying asset and a 10% decrease in information quality produces an 8.78% increase in the value of a one-period ATM call option. On the contrary, option values may decrease (increase) if the underlying asset and the information quality simultaneously increase (decrease). For example, a 1% increase in the underlying asset and a 10% increase in information quality produces a 1% decrease in the one-period ATM option value. Thus, shifts in information quality may cause call option values to move in the opposite direction of the underlying asset. This is a robust feature of the model.

[17] An excellent survey of the empirical options literature can be found in Bates (1996, 2003). The survey is silent with respect to how information quality affects option prices.

[18] These occurrence rates are for the Type I violations in Bakshi, Cao, and Chen (2000). Since the Type I violations are unassociated with microstructure effects and minimum tick size restrictions, they are the most appropriate type to analyze in the context of the gamma-exponential model.

[19] Buraschi and Jiltsov (2006) offer an alternative explanation that is based on differences in prior beliefs. Unlike the gamma-exponential model, their explanation does not rely on information quality.

Table 3
Sensitivity of call option values to joint movements in information quality and the underlying asset price

Panel A: one period options

		1% Increase in S			1% Decrease in S		
	Change in μ_0	−10%	0%	+10%	−10%	0%	+10%
Moneyness	ITM	7.86	3.34	−0.56	1.18	−3.30	−7.17
	ATM	8.78	3.54	−1.00	1.69	−3.49	−7.96
	OTM	9.77	3.73	−1.49	2.26	−3.67	−8.80

Panel B: two-period options

		1% Increase in S			1% Decrease in S		
	Change in μ_0	−10%	0%	+10%	−10%	0%	+10%
Moneyness	ITM	7.44	2.48	−1.78	2.46	−2.46	−6.68
	ATM	8.07	2.56	−2.16	2.90	−2.54	−7.21
	OTM	8.71	2.65	−2.56	3.36	−2.62	−7.75

The table shows the percent (%) change in the call option value for a given change in the underlying asset price and the information quality parameter. The one-period call option values are from Equation (15) whereas the two-period option values are from Proposition 4. The initial parameters are $S_0 = 100$, $\mu_0 = 5$, and $\mu_1 = \mu_0 - 0.5$. ATM denotes at the money. ITM and OTM are 5% in-the-money and 5% out-of-the-money, respectively.

Panel B shows the percent changes in value for two-period call options. Although the percent changes in option value in Panel B are not always larger than those in Panel A, the changes in Panel B are larger if the underlying asset and the information quality both simultaneously increase or decrease. For example, if S increases by 1% and μ_0 increases by 10%, the one-period ATM call option falls by 1% but the two-period ATM call option falls by 2.16%. The larger change for the two-period option is partially driven by the fact that the information quality shift in Table 3 is cumulative, affecting both μ_0 and μ_1. Panels A and B show that longer-dated options are more sensitive than shorter-dated options precisely when the joint movements in S and μ_0 are consistent with option values and the underlying asset moving in opposite directions. For a given sequence of information quality shifts, longer-dated options will therefore be the ones that appear to have higher violation rates, as in Bakshi, Cao, and Chen (2000).

4.2 Information quality and option market overreactions

The gamma-exponential model offers a rational-based explanation for the empirical results in Stein (1989) and Poteshman (2001).[20] In particular, it shows that changes in information quality can generate movements in the nearby and distant implied volatilities that are consistent with the overreaction hypothesis. However, these movements are driven by the

[20] Other explanations for option market overreaction can be found in Diz and Finucane (1993) and Heynen, Kemna, and Vorst (1994). Unlike the explanation given here, these papers do not rely on information quality.

Table 4
Relationship of nearby and distant implied volatilities

Panel A: upward sloping term structure of implied volatility

Scenario	μ_0	μ_1	$i_1^n - i_0^n - 2\left(i_0^d - i_0^n\right)$	$i_0^d - i_0^n$	$\frac{\Delta i_0^d}{\Delta i_0^n}$
1a	2.0	1.5	−0.059	0.205	1.53
1b	1.0	0.5	−0.204	0.693	
2a	5.0	4.5	−0.014	0.048	1.11
2b	4.0	3.5	−0.019	0.067	
3a	10.0	9.5	−0.005	0.016	1.05
3b	9.0	8.5	−0.006	0.019	

Panel B: downward sloping term structure of implied volatility

Scenario	μ_0	μ_1	$i_1^n - i_0^n - 2\left(i_0^d - i_0^n\right)$	$i_0^d - i_0^n$	$\frac{\Delta i_0^d}{\Delta i_0^n}$
4a	2.0	2.5	−0.041	−0.102	0.85
4b	1.0	1.5	−0.096	−0.237	
5a	5.0	5.5	−0.012	−0.029	0.94
5b	4.0	4.5	−0.016	−0.040	
6a	10.0	10.5	−0.004	−0.011	0.97
6b	9.0	9.5	−0.005	−0.013	

The table shows the relationship between the nearby and distant implied volatility (IV) in the gamma-exponential model. For each scenario, i_0^n is the IV at date 0 of a single-period call option expiring at date 1, i_1^n is the IV at date 1 of a single-period call option expiring at date 2, and i_0^d is the IV at date 0 of a two-period call option expiring at date 2. All options are at-the-money ($K = 100$) and the time step between dates is one month. The single-period options are valued using Equation (15) while the two-period options are valued using Equation (19). The IVs are inferred from the Black-Scholes model using a zero interest rate.

cumulative nature of the information quality shift and are not related to true overreaction.

To see this, let i_0^n (respectively, i_1^n) denote the implied volatility at date 0 (date 1) for a one-period ATM call option expiring at date 1 (date 2). Furthermore, let i_0^d denote the implied volatility at date 0 for a two-period ATM call option. As in Stein (1989), i_0^n, i_0^d, and i_1^n are the nearby implied volatility, the distant implied volatility, and the leading nearby implied volatility, respectively. Table 4 shows how these three implied volatilities are related for six different information quality scenarios. Panel A illustrates the case of $\mu_0 > \mu_1$ while Panel B illustrates $\mu_0 < \mu_1$. Each numbered scenario contains two sub-scenarios, *a* and *b*, where *b* s information quality path is a downward parallel shift of *a* s. The implied volatility levels therefore increase as the sub-scenario moves from *a* to *b*.

As the information quality moves from 1*a* to 1*b*, the change in the distant implied volatility is 1.53 times the change in the nearby implied volatility. Across all of the scenarios, $\Delta i_0^d / \Delta i_0^n$ ranges from a low 0.85 to a high of 1.53. These values are all higher than the highest plausible theoretical coef cient in Stein (1989, Table III) and are suf ciently high

to explain the empirical results in Stein (1989, Table IV). The quantity $i_1^n - i_0^n - 2\left(i_0^d - i_0^n\right)$, which is related to the linear endpoint approximation in Stein (1989), is also examined in Table 4. As the information quality moves from 1*a* to 1*b*, the implied volatility level increases and the quantity $i_1^n - i_0^n - 2\left(i_0^d - i_0^n\right)$ decreases. This also occurs for the other scenarios and is consistent with the results in Stein (1989, Table V). Lastly, the column labeled $i_0^d - i_0^n$ shows that the gamma-exponential model is partially consistent with the results in Poteshman (2001, p. 861). As the information quality moves from 1*a* to 1*b*, the volatility level increases and the quantity $i_0^d - i_0^n$ also increases. This also occurs for the other scenarios in Panel A, which is consistent with Poteshman (2001). However, the effect is reversed for the downward sloping term structures in Panel B.

4.3 Information quality and the bias of implied volatility

The literature review of Poon and Granger (2003) summarizes 22 studies that use index option implied volatilities to forecast future stock market volatility. With the exception of Canina and Figlewski (1993), all the studies conclude that the Black-Scholes implied volatility contains useful information about future volatility. However, the majority of studies also conclude that the Black-Scholes implied volatility is biased. While some of the bias can be explained by the issues discussed in Christensen and Prabhala (1998), the gamma-exponential model offers a new explanation. The information quality parameters that drive the realized returns in the gamma-exponential model are a subset of those that drive the implied volatilities.[21] If the common variation between the two sets of parameters is low, the gamma-exponential model can generate implied volatilities that are biased forecasts of future volatility.

To see this, note that when η and ζ are held fixed, the realized volatility of the date n equilibrium price is driven solely by β_n. This follows from Equation (2) and the fact that β_n and $Var(Z_n^{-1})$ are inversely related. However, the implied volatility for a short-term option is driven by μ_n, which is only partially determined by β_n. Since short-term options are the ones that are typically used in empirical work, the gamma-exponential model suggests that the implied volatility is an imperfect predictor of future realized volatility. The implied volatility will be a good predictor if the period to period variation in μ_n is mostly due to changes in β_n, while the implied volatility will be a poor predictor if there is low common variation between μ_n and β_n. The latter case can occur if private information quality, θ_n, is fluctuating through time.

[21] This shows how noise in financial markets, as captured by Equation (3), can drive a wedge between the information quality that impacts the underlying asset market and the information quality that impacts the option market.

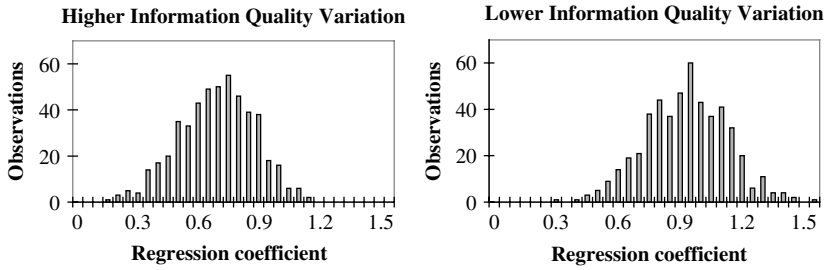

Figure 1
Regression coefficient histograms
To generate each graph, the regression equation $RLZ_n = \alpha_0 + \alpha_1 IV_n + \epsilon_n$ is estimated 500 times using independent simulated daily data from the gamma-exponential model, where IV_n is the log implied volatility at time n and RLZ_n is the log realized return volatility from n to $n + 30$. The 500 estimates of α_1 are then plotted as a histogram. The average $\hat{\alpha}_1$ is 0.668 (0.904) for the left (right) graph and every $\hat{\alpha}_1$ is statistically signi cant at the 1% level. Each estimation uses 6,000 simulated days whose returns are generated from Equation (2) with $X = 100$. For the rst 3,000 days $\beta_n = 250 + u_n$ and for the last 3,000 days $\beta_n = 200 + u_n$, where u_n is i.i.d. uniform on [0, 50]. The left (right) graph has higher (lower) variation in μ_n. For the left graph $\mu_n = \beta_n + 25 + v_n$ where v_n is i.i.d. uniform on [0, 25], while for the right graph $\mu_n = \beta_n + 25$. The implied volatilities are from 30-day ATM options.

The above remarks are illustrated with a simulation, as shown in Figure 1. To construct each graph in the gure, the regression equation $RLZ_n = \alpha_0 + \alpha_1 IV_n + \epsilon_n$ is estimated 500 times using independently generated daily data from the gamma-exponential model.[22] The log implied volatility at time n for a 30-day ATM option is IV_n and the log realized return volatility from n to $n + 30$ is RLZ_n. The 500 estimates of α_1 are plotted as histograms. The left (right) graph of Figure 1 has a higher (lower) total variation in μ_n and a lower (higher) correlation between μ_n and β_n. This fact shows up in the average value for $\hat{\alpha}_1$, which is 0.668 (0.904) for the left (right) graph. As discussed in Poon and Granger (2003), $\hat{\alpha}_1$ should be close to 1 if implied volatility is unbiased. On average this is more likely to occur in the right-hand graph, when there is lower total variation in μ_n and a higher correlation between μ_n and β_n. Furthermore, to the extent that the correlation between μ_n and β_n is time-varying, the gamma-exponential model can potentially explain why the magnitude of the bias might change over time.

4.4 A recipe for estimating and testing the model

A formal evaluation of the model can be carried out by estimating the model s parameters and testing the model s predictions out-of-sample. This section provides a recipe for conducting such an evaluation. In the interest of brevity, the recipe is discussed but not carried out on actual

[22] Christensen and Prabhala (1998) show that there is a volatility regime shift around the October 1987 stock market crash. The simulations accommodate such a regime shift by using a lower average value for β_n (and thus a higher average volatility) during the second half of the simulation. The difference between the two histograms in Figure 1 is driven by the difference in the intertemporal covariation between β_n and μ_n.

data. Since the gamma-exponential model involves only a single risky asset, options on a broad stock market index should be used to test the model. A popular choice for empirical work is the S&P 500 index option contract (SPX). SPX options are European exercise style and are heavily traded, thus reducing the incidence of stale prices. Since put options can be converted to otherwise similar call options using put-call parity, the ensuing discussion involves only call options.

Fix a time step such as 1 week. On a given trading day within week m, collect the market prices, strike prices, and expiration dates for N_m SPX options. Guidolin and Timmerman (2003) use Wednesdays, since it minimizes the number of holidays during their sample period. Let $C_{m,mkt}(S_m, K_{mj}, T_{mj})$ denote the market price of the jth call option contract where $j = 1, 2, \ldots, N_m$. The strike price K_{mj} and the expiration date T_{mj} are doubly indexed to indicate that they vary cross-sectionally for a fixed m and from week to week. Similarly, let $C_m(S_m, K_{mj}, T_{mj}, \boldsymbol{\mu}_m)$ denote the model price of the jth call option contract. The model price is from Proposition 4 and $\boldsymbol{\mu}_m$ represents a vector of information quality parameters that is applicable to the time period that starts in week m and ends in the week that includes the furthest expiration date. Each element of $\boldsymbol{\mu}_m$ is the information quality parameter for a particular week, starting with week m and looking forward in time.

An estimate of $\boldsymbol{\mu}_m$, denoted by $\hat{\boldsymbol{\mu}}_m$, can be obtained by solving the following program[23]

$$\min_{\boldsymbol{\mu}_m} \sum_{j=1}^{N_m} \left[C_m(S_m, K_{mj}, T_{mj}, \boldsymbol{\mu}_m) - C_{m,mkt}(S_m, K_{mj}, T_{mj}) \right]^2 \qquad (23)$$

which minimizes the sum of the squared pricing errors for a cross-section of options. A typical implementation of Equation (23) might involve the two nearest-term monthly expirations and all strikes within ±5% of the underlying index level. Using only the two nearest-term expirations implies that for any given week, the number of elements of $\hat{\boldsymbol{\mu}}_m$ would be about 7 or 8. As long as N_m is greater than this number, which is typically the case for SPX options, Equation (23) is well-specified.

While Equation (23) ignores the model's intertemporal restrictions on the information quality path, it is simple to implement and provides several time series of estimated weekly information quality. For example, extracting the first element of the vector $\hat{\boldsymbol{\mu}}_m$ as m varies provides a time series of current information quality, extracting the second element of the

[23] Dividends can be obtained from the S&P 500 Information Bulletin. These should be discounted and subtracted from the index level to obtain the S_m that is used in Equation (23). Since SPX options expire on the Saturday following the third Friday of the expiration month, the last time step before expiration may be a partial week. This can be accommodated by scaling the information quality parameter (e.g., use $\frac{k}{7}\mu_n$ where $k < 7$). This type of scaling might also be necessary if American options, such as the OEX contract, are used instead of European options.

vector $\boldsymbol{\mu}_m^{\square}$ as m varies provides a time series of 1-week forward information quality, and so forth. From Proposition 6, these time series should be useful for explaining the shape of the implied volatility term structure. Likewise, the difference between the rst two elements of $\boldsymbol{\mu}_m^{\square}$ as m varies provides a time series of information quality changes. This time series provides empirical evidence on the rate of information acquisition and information expiration. It should be useful for estimating the path of β_m, which controls the realized volatility.

Information acquisition costs are also relevant for explaining the shape of the implied volatility term structure. If short-lived private information is relatively expensive, investors may substitute long-lived private information, which will tend to produce a downward sloping term structure. On the other hand, if long-lived information is relatively expensive, investors may instead buy short-lived private information that will periodically expire and produce an upward sloping term structure. Thus the cost differential between short- and long-lived private information should be a determinant of the sign of the implied volatility term structure s slope.

The empirical relevance of information quality for option pricing can be assessed by examining how well the gamma-exponential model predicts out-of-sample option prices. Taking the estimated vector $\boldsymbol{\mu}_m^{\square}$ for week m as given, one could move forward in time by one week and calculate the squared pricing error for the jth contract in week $m + 1$ as

$$\left[C_{m+1}(S_{m+1}, K_{mj}, T_{mj}, \boldsymbol{\mu}_m^{\square}) - C_{m+1,mkt}(S_{m+1}, K_{mj}, T_{mj})\right]^2. \qquad (24)$$

The error in Equation (24) can be averaged across all weeks and option contracts and then compared to other competing pricing models. If the gamma-exponential model is speci ed correctly, the average squared pricing error should be relatively small. This type of out-of-sample test closely follows Bakshi, Cao, and Chen (1997, Table 5) and Guidolin and Timmerman (2003, Table 4).

A different out-of-sample test is used by Buraschi and Jiltsov (2006, Table 3) and David and Veronesi (2002, Table 4). Instead of estimating $\boldsymbol{\mu}_m$ in Equation (23) by using all of the options whose strikes are within $\pm 5\%$ of the underlying index, only the options whose strikes are within, say, $\pm 3\%$ of the index could be used. The options that are not used to estimate $\boldsymbol{\mu}_m$ are then used to construct an out-of-sample cross-sectional pricing error for week m. The pricing errors are then averaged across time and the generalized method of moments is used to assess the model s performance statistically. Regardless of which test method is used, the estimated information quality parameters $\boldsymbol{\mu}_m^{\square}$ play a key role in the analysis. The same would be true if the normal-exponential framework were used instead of the gamma-exponential framework. More generally, information quality is likely to be an important variable for assessing

the out-of-sample performance of any option pricing model that relies on fundamentals.[24]

5. Concluding Remarks

This article studies the connection between option prices and information quality in a new multiperiod noisy rational expectations framework. It shows that the forward looking information quality path is the main determinant of the risk-neutral distribution of asset returns. This implies that information quality is a key driver of option values and implied volatility dynamics. Information quality can potentially explain why option prices sometimes move even if the underlying asset does not move, why option markets overreact, and why implied volatility is a biased predictor of future volatility. Easley, O Hara, and Srinivas (1998) argue that more complete option pricing models are needed. This article takes a step in that direction.

As discussed in Jiang and Tian (2005), the model-free implied volatility of Britten-Jones and Neuberger (2000) is a better volatility predictor than the Black Scholes implied volatility. Jiang and Tian (2005) attribute the superior performance to the fact that the model-free implied volatility relies on a collection of options with varying strike prices, whereas the Black Scholes implied volatility uses only the ATM option contract. The gamma-exponential model provides theoretical support for this view since a collection of options with varying strikes should be better than a single option for estimating the information quality path (Section 4.4).

Although the gamma-exponential model can generate implied volatility movements that are consistent with option market overreaction, this is not to say that behavioral phenomena do not impact the options market. Indeed, overreaction and information quality might jointly impact the market. Empirically disentangling these two effects could be interesting. Furthermore, although the gamma-exponential model does not include overcon dent investors, the results presented here tend to support the view that overcon dence should matter for option pricing. As discussed in Odean (1998b) and Daniel, Hirshleifer, and Subrahmanyam (2001), overcon dent investors believe that the precision of their private information is higher than its true precision. In the gamma-exponential model, $\theta_{n,i}$ measures the ith investor s date n signal precision. If overcon dent investors were added to the gamma-exponential model at date n, θ_n would increase. In turn, μ_n would increase and short-term option values and implied volatility levels would both decrease. Using

[24] Although this section focuses on asset pricing tests, an alternative test that is based on investors optimal option demands might also be possible. This sort of test could provide evidence on how close investors actual option portfolios are to the log (quadratic) derivative that is predicted by the gamma- (normal-) exponential model.

the results of Proposition 6, this suggests that overcon dence might be partially responsible for the shape and dynamics of the implied volatility term structure. This view tends to support the claims in Shefrin (1999), who argues that option markets are particularly vulnerable to behavioral phenomena.

Appendix A:

Proof of Proposition 1. Dynamic programming is used to solve the *i* th investor s intertemporal optimization problem in Equation (4). As a preliminary step, de ne the quantity $\lambda_{n,i}$ as

$$\lambda_{n,i} = \eta\zeta + \sum_{k=0}^{n} \theta_{k,i} Y_{k,i} + \sum_{k=0}^{n} \frac{(\beta_k - 1)}{S_k}. \tag{A1}$$

This expression is used shortly. At date $N-1$ the *i* th investor s problem is

$$\max_{D_{N-1,i}} E\left[-e^{-\tau_i \left(W_{N-1,i} - D_{N-1,i} S_{N-1} \right) - \tau_i D_{N-1,i} X} | \mathcal{I}_{N-1,i} \right]. \tag{A2}$$

To evaluate the above expectation, the density for X conditional on $\mathcal{I}_{N-1,i}$ is needed. Using Bayes rule this density is

$$\begin{aligned} p\left(x|\mathcal{I}_{N-1,i}\right) &= \frac{p(s_0, \ldots, s_{N-1}, y_{0,i}, \ldots, y_{N-1,i}|x)\, p(x)}{p(s_0, \ldots, s_{N-1}, y_{0,i}, \ldots, y_{N-1,i})} \\ &= \frac{p(x) \prod_{k=0}^{N-1} \left[p(s_k|x)\, p(y_{k,i}|x) \right]}{\int_0^\infty p(x) \prod_{k=0}^{N-1} \left[p(s_k|x)\, p(y_{k,i}|x) \right] dx} \end{aligned} \tag{A3}$$

where the second equality follows from the fact that the prices and signals are conditionally independent given X. Recall that the prior distribution of X is $Gamma\,(\eta\zeta, \zeta)$ and the distribution of $Y_{n,i}$ given X is $Gamma\left(x\theta_{n,i}, \theta_{n,i}\right)$. Using Equation (2), note that S_n given X has an inverted gamma distribution with density

$$p(s_n|x) = \frac{[x(\beta_n - 1)]^{\beta_n}}{\Gamma(\beta_n)} \left(\frac{1}{s_n} \right)^{1+\beta_n} e^{-x(\beta_n - 1)\frac{1}{s_n}}.$$

The right-hand side of Equation (A3) can therefore be evaluated to show that X conditional on $\mathcal{I}_{N-1,i}$ is $Gamma(\lambda_{N-1,i}, \mu_{N-1,i})$, where $\mu_{N-1,i}$ is from Equation (7) and $\lambda_{N-1,i}$ is from Equation (A1). The expectation in Equation (A2) is therefore equal to

$$-e^{-\tau_i \left(W_{N-1,i} - D_{N-1,i} S_{N-1} \right)} \left[\frac{\lambda_{N-1,i}}{\tau_i D_{N-1,i} + \lambda_{N-1,i}} \right]^{\mu_{N-1,i}} \tag{A4}$$

and the optimal demand is given by the first-order condition[25]

$$\tau_i \hat{D}_{N-1,i} + \lambda_{N-1,i} = \frac{\mu_{N-1,i}}{S_{N-1}}. \tag{A5}$$

Noting that $E(X|\mathcal{I}_{N-1,i}) = \mu_{N-1,i}\lambda_{N-1,i}^{-1}$, Equation (A5) can be rearranged to get

$$\hat{D}_{N-1,i} = \frac{\mu_{N-1,i}}{\tau_i}\left[\frac{1}{S_{N-1}} - \frac{1}{E(X|\mathcal{I}_{N-1,i})}\right].$$

To clear the markets, equate aggregate demand to the noisy supply to get

$$\int_0^1 \hat{D}_{N-1,i}di = E^M\left[\int_0^1 \hat{D}_{N-1,i}di \,\middle|\, S_0, S_1, \ldots, S_{N-1}, Z_{N-1}\right]. \tag{A6}$$

Using Equation (A5), the left-hand side of Equation (A6) is

$$\frac{\int_0^1 \frac{\mu_{N-1,i}}{\tau_i}di}{S_{N-1}} - X\sum_{k=0}^{N-1}\left(\int_0^1 \frac{\theta_{k,i}}{\tau_i}di\right) - \left(\int_0^1 \frac{1}{\tau_i}di\right)\left[\eta\zeta + \sum_{k=0}^{N-1}\frac{(\beta_k - 1)}{S_k}\right] \tag{A7}$$

where the law of large numbers is used to obtain the second term.[26] Since $E^M[\cdot]$ is evaluated at date $N-1$ using $X \stackrel{d}{=} Gamma\left(S_{N-1}^{-1}, 1 + Z_{N-1}\right)$, $E^M[X^{-1}] = S_{N-1}^{-1}Z_{N-1}^{-1}$. Thus, the right-hand side of Equation (A6) is similar to Equation (A7) with the exception that the second term is equal to

$$-S_{N-1}^{-1}Z_{N-1}^{-1}\sum_{k=0}^{N-1}\left(\int_0^1 \frac{\theta_{k,i}}{\tau_i}di\right). \tag{A8}$$

The market clearing price is therefore $S_{N-1} = XZ_{N-1}^{-1}$. Now fix an arbitrary trading date n and consider the induction hypothesis that the ith investor's indirect utility given $\mathcal{I}_{n+1,i}$ is

$$-B_{n+1,i}\left[\frac{\lambda_{n+1,i}}{\tau_i\hat{D}_{n+1,i} + \lambda_{n+1,i}}\right]^{\mu_{n+1,i}} e^{-\tau_i\left(W_{n+1,i} - \hat{D}_{n+1,i}S_{n+1}\right)} \tag{A9}$$

where $B_{n+1,i}$ is a constant and $\hat{D}_{n+1,i}$ is given by (6). The ith investor's date n problem is

$$\max_{D_{n,i}} E\left[-e^{-\tau_i\left(W_{n,i} - D_{n,i}S_n\right)}e^{-S_{n+1}\left(\tau_i D_{n,i} + \lambda_{n+1,i}\right)}\left(\lambda_{n+1,i}S_{n+1}\right)^{\mu_{n+1,i}}|\mathcal{I}_{n,i}\right]$$

where some of the constants that do not affect the problem have been omitted. To evaluate the above expectation, iterate the expectation and integrate over $y_{n+1,i}$ first. The conditional

[25] The optimal demand can also be obtained by letting $X \stackrel{d}{=} Gamma(\lambda, \mu)$ for constant λ and μ and then substituting the posterior estimates $\lambda_{N-1,i}$ and $\mu_{N-1,i}$ for λ and μ, respectively. This is an application of the Separation Principle since the optimal policy is determined separately from the posterior estimates. In the normal-exponential framework, the mean and variance are the relevant posterior estimates that are propagated through time. In the gamma-exponential framework, the scale and shape parameters are propagated through time according to the recursive equations $\lambda_{n,i} = \lambda_{n-1,i} + \theta_{n,i}Y_{n,i} + (\beta_n - 1)/S_n$ and $\mu_{n,i} = \mu_{n-1,i} + \theta_{n,i} + \beta_n$. The linearity of these recursive equations arises because $p(s_n|x)$ and $p(y_{n,i}|x)$ have scale (shape) parameters that are linear in (independent of) x.

[26] It is assumed that the law of large numbers can be invoked in this manner even though there is a continuum of random variables. See Judd (1985) for a discussion of the relevant issues.

density of $Y_{n+1,i}$ given both $\mathcal{I}_{n,i}$ and S_{n+1} takes the form

$$p(y_{n+1,i}|\mathcal{I}_{n,i}, S_{n+1}) = \frac{\Gamma(\mu_{n+1,i})\theta_{n+1,i}^{\theta_{n+1,i}}}{\Gamma(\theta_{n+1,i})\Gamma(\mu_{n,i}+\beta_{n+1})} \frac{y_{n+1,i}^{\theta_{n+1,i}-1}\left(\lambda_{n,i}+S_{n+1}^{\beta_{n+1}-1}\right)^{\mu_{n,i}+\beta_{n+1}}}{(\lambda_{n+1,i})^{\mu_{n+1,i}}} \tag{A10}$$

which facilitates the calculation since the term $\lambda_{n+1,i}^{\mu_{n+1,i}}$ in the denominator of Equation (A10) cancels the same term in the numerator of Equation (A9).[27] Integrate next over s_{n+1} using the conditional density of S_{n+1} given $\mathcal{I}_{n,i}$

$$p(s_{n+1}|\mathcal{I}_{n,i}) = \frac{\Gamma(\mu_{n,i}+\beta_{n+1})}{\Gamma(\beta_{n+1})\Gamma(\mu_{n,i})}\begin{pmatrix}1\\ s_{n+1}\end{pmatrix}^{1+\beta_{n+1}} \frac{(\beta_{n+1}-1)^{\beta_{n+1}}(\lambda_{n,i})^{\mu_{n,i}}}{\left(\lambda_{n,i}+s_{n+1}^{\beta_{n+1}-1}\right)^{\mu_{n,i}+\beta_{n+1}}}. \tag{A11}$$

The investor s indirect utility at date n is therefore

$$-B_{n,i}\left[\frac{\lambda_{n,i}}{\tau_i D_{n,i}+\lambda_{n,i}}\right]^{\mu_{n,i}} e^{-\tau_i\left(W_{n,i}-D_{n,i}S_n\right)} \tag{A12}$$

where $B_{n,i}$ is a constant. The rst-order condition produces

$$\hat{D}_{n,i} = \frac{1}{\tau_i}\left[\frac{\mu_{n,i}}{S_n}-\lambda_{n,i}\right] = \frac{\mu_{n,i}}{\tau_i}\left(\frac{1}{S_n}-\frac{1}{E(S_{n+1}|\mathcal{I}_{n,i})}\right) \tag{A13}$$

where $E(S_{n+1}|\mathcal{I}_{n,i}) = \mu_{n,i}\lambda_{n,i}^{-1}$ follows from Equation (A11). Aggregating the optimal demands as in Equations (A6) (A8) and solving for the date n market clearing price gives $S_n = XZ_n^{-1}$. Comparing Equation (A12) to (A9) and comparing Equation (A13) to (6) shows that the induction hypothesis is correct. □

Equilibrium Properties in Table 2: Using the model s distributional properties, note that $E\left(XS_n^{-1}\right) > 1$ for all n, $E(S_n|X) = X$ for all n, and the prices $S_0, S_1, \ldots, S_{N-1}$ are conditionally independent given X. Thus, a direct calculation reveals that

$$Cov(S_{n+1}-S_n, S_n) = -E\left[E(S_n^2|X)-(E(S_n|X))^2\right] = -E[Var(S_n|X)] < 0$$

and

$$Cov(S_{n+1}-S_n, S_n-S_{n-1}) = -E\left[(X-S_n)^2\right] < 0$$

which proves properties 1 and 2. Since $Var(S_{n+1}S_n^{-1}|S_n)$ is proportional to $\left(\eta\zeta S_n+\beta_n-1\right)^{-2}$ and $E(X|S_n)$ is proportional to $S_n\left(\eta\zeta S_n+\beta_n-1\right)^{-1}$, it is easy to show that

$$Cov\left[S_n, \sqrt{Var\left(S_{n+1}S_n^{-1}\middle|S_n\right)}\right] \propto E(X)\left[1-E\left(XS_n^{-1}\right)\right] < 0$$

[27] Each of the densities in Equations (A10) (A11) is an inverted Beta distribution, which is a member of the Liouville family of distributions (Fang, Kotz, and Ng 1990, Chapter 6). The joint unconditional distribution of $S_0, S_1, \ldots, S_{N-1}$ is an inverted Dirichlet distribution, which is also a member of this family. The analytic properties of this family, in conjunction with the CARA utility function and the fact that the random variables in $\mathcal{I}_{n,i}$ are conditionally independent given X, are what allow the investor s dynamic optimization problem to be solved in a simple way.

which proves property 3. Next, using the optimal demand in (6), a direct calculation reveals that

$$Cov\left(\hat{D}_{n,i}, S_{n+1} - S_n\right) = \tau_i^{-1}(1 + \theta_{n,i} + \mu_{n-1,i})\left[E\left(XS_n^{-1}\right) - 1\right] > 0.$$

Furthermore, $Cov(\hat{D}_{n+1,i} - \hat{D}_{n,i}, S_{n+1} - S_n)$ is equal to

$$\tau_i^{-1}(1 + \mu_{n+1,i} - \beta_{n+1})\left[1 - E(XS_{n+1}^{-1})\right] + \tau_i^{-1}\mu_{n,i}\left[1 - E(XS_n^{-1})\right] < 0.$$

For property 6, note that $E\left[\left|\hat{D}_{n+1,i}\right| 1_{\left\{\hat{D}_{n+1,i}<0\right\}}\middle| S_0, \ldots, S_n\right] \square \eta\zeta + \sum_{k=0}^{n}(\beta_k - 1)S_k^{-1}$ so that

$$Cov\left(E\left[\left|\hat{D}_{n+1,i}\right| 1_{\left\{\hat{D}_{n+1,i}<0\right\}}\middle| S_0, \ldots, S_n\right], \frac{S_{n+1}}{S_n}\right) \square Cov\left(\eta\zeta + \sum_{k=0}^{n}\frac{(\beta_k - 1)}{S_k}, \frac{S_{n+1}}{S_n}\right)$$
$$= Cov\left((\beta_n - 1)S_n^{-1}, S_{n+1}S_n^{-1}\right) > 0.$$

Lastly, properties 7 through 9 follow from the conditional densities in Equations (A10) and (A11) and the fact that X given $\mathcal{I}_{n,i}$ is distributed $Gamma(\lambda_{n,i}, \mu_{n,i})$. □

Proof of Proposition 2. The ith investor s problem at date $N - 1$ is

$$\max_{D_{N-1,i}, A_{N-1,i}} E\left[-e^{-\tau_i\left(W_{N-1,i} + D_{N-1,i}\left(X - S_{N-1}\right) + A_{N-1,i}\left(\log(X) - L_{N-1}\right)\right)}\middle|\mathcal{I}_{N-1,i}\right]. \quad \text{(A14)}$$

Since X given $\mathcal{I}_{N-1,i}$ is $Gamma(\lambda_{N-1,i}, \mu_{N-1,i})$, the above expectation is equal to

$$-\left(\lambda_{N-1,i}\right)^{\mu_{N-1,i}}\Gamma(\mu_{N-1,i} - \tau_i A_{N-1,i})$$
$$\left(\tau_i D_{N-1,i} + \lambda_{N-1,i}\right)^{\mu_{N-1,i} - \tau_i A_{N-1,i}}\Gamma(\mu_{N-1,i})\, e^{-\tau_i\left(W_{N-1,i} - D_{N-1,i}S_{N-1} - A_{N-1,i}L_{N-1}\right)}.$$

Differentiating with respect to $D_{N-1,i}$ and $A_{N-1,i}$ gives the pair of first-order conditions

$$\tau_i \hat{D}_{N-1,i} + \lambda_{N-1,i} = \left[\mu_{N-1,i} - \tau_i \hat{A}_{N-1,i}\right] S_{N-1}^{-1} \quad \text{(A15)}$$

$$\psi(\mu_{N-1,i} - \tau_i \hat{A}_{N-1,i}) = L_{N-1} + \log(\tau_i \hat{D}_{N-1,i} + \lambda_{N-1,i}) \quad \text{(A16)}$$

where $\psi(z) \equiv \frac{\Gamma'(z)}{\Gamma(z)}$ is the psi function (Ross 1978). De ning $f(v) \equiv \psi(v) - \log(v)$ for $v > 0$, Gradshteyn and Ryzhik (1994, §8.36) show that $f(v)$ has the integral representation

$$f(v) = \int_0^\infty e^{-kv}\left(\frac{1}{k} - \frac{1}{1 - e^{-k}}\right) dk. \quad \text{(A17)}$$

Differentiating Equation (A17) with respect to v shows that $f'(v) > 0$ and thus f^{-1} exists. Using Equation (A15), this fact allows Equation (A16) to be written as

$$\hat{A}_{N-1,i} = \tau_i^{-1}\left[\mu_{N-1,i} - f^{-1}(L_{N-1} - \log(S_{N-1}))\right].$$

Equating the aggregate derivative demand to the zero-net-supply and solving for L_{N-1} gives

$$L_{N-1} = \log(S_{N-1}) + \psi(\mu_{N-1}) - \log(\mu_{N-1})$$

where μ_{N-1} is de ned in Equation (11). Thus, $\hat{A}_{N-1,i}$ is equal to $\tau_i^{-1}\left[\mu_{N-1,i} - \mu_{N-1}\right]$ and $\hat{D}_{N-1,i}$ is

$$\hat{D}_{N-1,i} = \frac{1}{\tau_i}\begin{bmatrix}\mu_{N-1}\\ S_{N-1}\end{bmatrix} - \lambda_{N-1,i}\Bigg] = \frac{1}{\tau_i}\begin{bmatrix}\mu_{N-1} - \mu_{N-1,i}\\ S_{N-1} - E(X|\mathcal{I}_{N-1,i})\end{bmatrix}.$$

Aggregating $\hat{D}_{N-1,i}$ over i and equating to the noisy supply of shares as in Equations (A6) (A8) veri es that the equilibrium price is $S_{N-1} = XZ_{N-1}^{-1}$. Now x an arbitrary trading date n and consider the induction hypothesis that the ith investor s indirect utility given $\mathcal{I}_{n+1,i}$ is

$$\begin{aligned} &-B_{n+1,i}\left(\lambda_{n+1,i}\right)^{\mu_{n+1,i}}\Gamma(\mu_{n+1,i} - \tau_i\hat{A}_{n+1,i})\Delta_{n+1,i}\\ &\left(\tau_i\hat{D}_{n+1,i} + \lambda_{n+1,i}\right)^{\mu_{n+1,i}-\tau_i\hat{A}_{n+1,i}}\Gamma(\mu_{n+1,i}) \end{aligned} \tag{A18}$$

where $B_{n+1,i}$ is a constant and $\Delta_{n+1,i}$ is de ned as

$$\begin{aligned}\Delta_{n+1,i} = \exp\Bigg[&-\tau_i\left(W_{n+1,i} - \hat{D}_{n+1,i}S_{n+1} - \hat{A}_{n+1,i}L_{n+1}\right) - \tau_i\hat{A}_{n+1,i}\\ &\times\sum_{k=n+2}^{N-1}\left[\psi(\mu_k) - \log(\mu_k)\right]\Bigg].\end{aligned}$$

Furthermore, suppose that the investor s optimal date $n+1$ demands are given by Equations (9) and (10) and suppose that the equilibrium date $n+1$ prices are given by Equations (2) and (12). Using the densities in Equations (A10) and (A11), the expectation of Equation (A18) conditional on $\mathcal{I}_{n,i}$ can be evaluated. Upon doing this, the ith investor s date n maximization problem can be written as

$$\max_{D_{n,i},\,A_{n,i}} \begin{aligned} &-B_{n,i}\left(\lambda_{n,i}\right)^{\mu_{n,i}}\Gamma(\mu_{n,i} - \tau_i A_{n,i})\Delta_{n,i}\\ &\left(\tau_i D_{n,i} + \lambda_{n,i}\right)^{\mu_{n,i}-\tau_i A_{n,i}}\Gamma(\mu_{n,i}) \end{aligned} \tag{A19}$$

where $B_{n,i}$ is a constant. The rst-order conditions at date n are

$$\tau_i\hat{D}_{n,i} + \lambda_{n,i} = \left[\mu_{n,i} - \tau_i\hat{A}_{n,i}\right]S_n^{-1}$$

$$\psi(\mu_{n,i} - \tau_i\hat{A}_{n,i}) = L_n + \log\left(\tau_i\hat{D}_{n,i} + \lambda_{n,i}\right) - \sum_{k=n+1}^{N-1}\left[\psi(\mu_k) - \log(\mu_k)\right].$$

Solving these equations veri es the optimal demands in Proposition 2. Equating supply and aggregate demand veri es that the equilibrium prices at date n are given by Equations (2) and (12). Comparing Equations (A18) and (A19) veri es that the induction hypothesis is correct. Lastly, letting $\hat{J}_{n+1,i}$ denote the quantity in Equation (A18), the ith investor s probability-weighted MRS at date n is

$$p(s_{n+1}|\mathcal{I}_{n,i})\,\frac{E\left[\hat{J}_{n+1,i}\middle|\mathcal{I}_{n,i}, S_{n+1}\right]}{E\left[\hat{J}_{n+1,i}\middle|\mathcal{I}_{n,i}\right]}. \tag{A20}$$

The expectation in the numerator is evaluated using Equation (A10) and $p(s_{n+1}|\mathcal{I}_{n,i})$ is given in Equation (A11). Since Equation (A20) integrates to 1, it is a probability measure for S_{n+1}. Under this measure $S_{n+1} \stackrel{d}{=} Gamma(\mu_n S_n^{-1}, \mu_n)$, which coincides with Equation (14). Since the measure does not depend on i, the market is effectively complete and the allocation is Pareto ef cient. □

Meijer's G-function: The G-function arises when deriving the probability density for particular products of independent random variables, such as products of independent gamma-distributed random variables. The typical route for analyzing such products is to use the Mellin transform. The Mellin transform of a random variable $V > 0$ with density $g(v)$ is de ned as

$$M_\omega[g(v)] \equiv \int_0^\infty v^{\omega-1} g(v) dv = E\left[V^{\omega-1}\right] \tag{A21}$$

where ω is a complex number. Given the transform $M_\omega[g(v)]$, the probability density $g(v)$ is uniquely recovered by evaluating the inverse Mellin transform

$$g(v) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} v^{-\omega} M_\omega[g(v)] d\omega \tag{A22}$$

where $j \equiv \sqrt{-1}$ and c is a real number. The existence and uniqueness of the inverse transform is due to the fact that the Mellin transform and the Fourier transform are related by a simple logarithmic transformation, as discussed in Springer (1979).

To see how the Mellin transform pair in Equations (A21) (A22) is used to calculate the density for a product of returns, suppose that the single-period conditional return R_n has density $g_n(r_n)$. Let $R = R_1 R_2 \cdots R_N$ denote the gross return over N trading periods where R has density $g(r)$. Since the single-period conditional returns are independent, Equation (A21) implies that

$$M_\omega[g(r)] = M_\omega[g_1(r_1)] M_\omega[g_2(r_2)] \cdots M_\omega[g_N(r_N)]. \tag{A23}$$

Thus, the Mellin transform of a product of independent random variables is the product of the individual Mellin transforms. Under quite general conditions (see Springer 1979), the inverse transform of the product in (A23) can be written as

$$H_{r,s}^{p,q}(z) \equiv \frac{1}{2\pi j} \int_C \frac{\left[\prod_{k=1}^{p} \Gamma(b_k - \rho_k \omega)\right]\left[\prod_{k=1}^{q} \Gamma(1 - a_k + \alpha_k \omega)\right]}{\left[\prod_{k=p+1}^{s} \Gamma(1 - b_k + \rho_k \omega)\right]\left[\prod_{k=q+1}^{r} \Gamma(a_k - \alpha_k \omega)\right]} z^\omega d\omega \tag{A24}$$

which is a contour integral in the complex plane that is commonly known as the H-function.[28] The parameters p, q, r, and s are integers and empty products are set equal to 1. The parameters satisfy $0 \le p \le s$, $0 \le q \le r$, $\alpha_k > 0$ for $k = 1, 2, \ldots, r$, and $\rho_k > 0$ for $k = 1, 2, \ldots, s$ such that no pole of $\Gamma(b_k - \rho_k \omega)$ for $k = 1, 2, \ldots, p$ coincides with any pole of $\Gamma(1 - a_k + \alpha_k \omega)$ for $k = 1, 2, \ldots, q$. A common notation that suppresses the integral but shows the parameter dependence is

$$H_{r,s}^{p,q}(z) = H_{r,s}^{p,q}\left[z \,\middle|\, \begin{matrix} (a_1, \alpha_1), & \ldots, & (a_r, \alpha_r) \\ (b_1, \rho_1), & \ldots, & (b_s, \rho_s) \end{matrix}\right]. \tag{A25}$$

When $\alpha_k = 1$ for $k = 1, 2, \ldots, r$ and $\rho_k = 1$ for $k = 1, 2, \ldots, s$, the H-function in Equation (A24) reduces to Meijer s G-function (Gradshteyn and Ryzhik 1994, §9.3), i.e.,

$$\mathsf{G}_{r,s}^{p,q}\left[z \,\middle|\, \begin{matrix} a_1, & \ldots, & a_r \\ b_1, & \ldots, & b_s \end{matrix}\right] = H_{r,s}^{p,q}\left[z \,\middle|\, \begin{matrix} (a_1, 1), & \ldots, & (a_r, 1) \\ (b_1, 1), & \ldots, & (b_s, 1) \end{matrix}\right]. \tag{A26}$$

[28] See Mathai and Saxena (1978) and Srivastava, Gupta, and Goyal (1982) for more details.

The G-function is available in popular mathematical software products such as Mathematica and Maple.

Proof of Proposition 4. The Mellin transform pair in Equations (A21) and (A22) is used to derive $\phi(S_n|S_m)$. The Mellin transform of $\phi(S_n|S_m)$ is

$$M_\omega[\phi(S_n|S_m)] = E^\phi\left[S_n^{\omega-1}|S_m\right] = E^\phi\left[(S_m R_{m+1} R_{m+2} \cdots R_n)^{\omega-1}|S_m\right]$$

where $R_k = S_k/S_{k-1}$. Owing to Equation (14), the conditional returns under the RNVR are independent gamma-distributed random variables with $R_k \stackrel{d}{=} Gamma\left(\mu_{k-1}, \mu_{k-1}\right)$. Thus, the last expectation above can be directly evaluated to get

$$M_\omega[\phi(S_n|S_m)] = S_m^{\omega-1}\left[\prod_{k=m}^{n-1}\left(\mu_k\right)^{1-\omega}\frac{\Gamma(\mu_k+\omega-1)}{\Gamma(\mu_k)}\right]. \tag{A27}$$

Substituting the right-hand side of Equation (A27) into Equation (A22) and applying Equations 6.2.2 and 6.2.4 of Springer (1979, p. 196), which are H-function relationships, completes the derivation of $\phi(S_n|S_m)$. With $\phi(S_n|S_m)$ in hand, the value of a multiperiod option is obtained by integration. Using a fact from Srivastava, Gupta, and Goyal (1982, §5.2) gives the preliminary result

$$\int_0^K v^{\mu_n-1-1}\,\mathsf{G}_{0,n-m}^{n-m,0}\left[\begin{matrix}a_{mn}v\\ S_m\end{matrix}\middle|\mu_m-\mu\right.$$

utility before observing the private signal $Y_{n,i}$. The first-order condition is

$$\tau_i c'(\hat{\theta}_{n,i}) = \sum_{k=n}^{N-1} [\log(\mu_k) - \psi(\mu_k)] \tag{A29}$$

where $\hat{\theta}_{n,i}$ is the optimal signal quality. Equation (A29) holds for all n and thus represents a system of equations whose solution characterizes the ith investor's optimal information acquisition policy. Since the right-hand side of Equation (A29) is decreasing in n, it is immediately clear that the optimal signal qualities satisfy $\hat{\theta}_{n+1,i} < \hat{\theta}_{n,i}$ for all n. When the log derivative is not traded as in Proposition 1, the ith investor's information acquisition problem at date n can be written as

$$\max_{\theta_{n,i}} \; -\exp\left[\tau_i c(\theta_{n,i}) + \sum_{k=n}^{N-1} [\mu_{k,i} + \log\Gamma(\mu_{k,i}) - \mu_{k,i}\log(\mu_{k,i})]\right]$$

where again the terms in the indirect utility function that do not depend on $\theta_{n,i}$ have been omitted. In this case, the first-order condition is

$$\tau_i c'(\hat{\theta}_{n,i}) = \sum_{k=n}^{N-1} [\log(\hat{\mu}_{k,i}) - \psi(\hat{\mu}_{k,i})] \tag{A30}$$

where $\hat{\mu}_{k,i}$ is given by Equation (7) evaluated along the optimal signal quality path. Like Equation (A29), (A30) holds for all n and represents a system of equations whose solution characterizes the ith investor's optimal policy. The economies with and without the log derivative can be compared by noting that $\log(z) - \psi(z)$ is strictly decreasing in z. Thus, using the right-hand sides of Equations (A29) (A30) and using the optimal log derivative demand in Equation (10), it is straightforward to deduce that if the ith investor is long (short) the log derivative at each date n, the quality of information acquired by the ith investor in the presence of the log derivative is higher (lower) than that acquired in the economy without the log derivative. Lastly, note that a shift in $c(\theta)$ will alter $\hat{\theta}_{n,i}$ for all i. In turn, μ_n is altered and so are option prices, even if S_n does not change. This completes the proof. □

Proof of Proposition 7. Recall that the conditional returns under the RNVR in Equation (14) are independent random variables. The unconditional return $S_N/S_0 = R_1 R_2 \cdots R_N$ is therefore a product of N independent random variables where $R_{n+1} \overset{d}{=} Gamma(k_n N, k_n N)$. This product can be rewritten as $(Q_1 Q_2 \cdots Q_N)^{\frac{1}{N}}$, where $Q_n = (R_n)^{\frac{1}{N}}$. Note that $E[\log Q_{n+1}] = \frac{1}{N}[\psi(k_n N) - \log(k_n N)]$ and $Var[\log Q_{n+1}] = \frac{1}{N}\psi'(k_n N)$, where ψ is Euler's psi function (Ross 1978) and ψ' is its first derivative. Using known properties of the psi function (Abramowitz and Stegun 1972, pp. 259–60), it can be shown that $E[\log Q_{n+1}] \approx -1/(2k_n N)$ and $Var[\log Q_{n+1}] \approx 1/k_n$ as $N \to \infty$. The expression $(Q_1 Q_2 \cdots Q_N)^{\frac{1}{N}}$ can further be written as

$$\exp\left[\frac{1}{N}\sum_{n=1}^{N} \xi ET\xi q\ \Delta\Lambda\Phi\Lambda\ \Pi Re\ o.\Lambda i)\ \Theta\Lambda t)\ \Sigma\Sigma\Lambda c)\ \Gamma\Psi an)\ \Sigma\Gamma\Phi be)\ \Sigma\Sigma\Lambda\Xi show)\ \Gamma\Sigma\right.$$

in distribution to a standard normal random variable. The first term in the exponent converges to $-\frac{1}{2}A$, which completes the proof. □

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