Inflation Uncertainty, Asset Valuations, and the Credit Spreads Puzzle

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Investors' learning of the state of future real fundamentals from current inflation leads to macroeconomic state dependence of asset valuations and solvency ratios of firms within given rating categories. Since credit spreads are convex functions of solvency ratios, average spreads are higher than spreads at average solvency ratios. Macroeconomic shocks carry risk premiums so that expected default losses are more sensitive to changes in the price of risk than are credit spreads. By incorporating state dependence and increasing the price of risk, the econometrician obtains high credit spreads while maintaining average default losses at historical levels—the credit spreads puzzle. (*JEL* G12, G13, G14, C3, C5)

Intuition developed in pricing options led to the development of the structural form approach for the pricing of risky corporate debt. The approach has been useful in understanding the implications of a firm's capital structure, and the value and volatility of its asset value process on the pricing of its risky corporate debt.[1] In summary, the approach assumes that a firm defaults on all its liabilities when the value of its assets at maturity is below a threshold. The approach has led to a recent revolution in measuring and managing credit risk with the focus shifting from traditional accounting-based measures of credit risk to market-based measures that use the information on asset values and their volatilities.

I am grateful to an anonymous referee, Yacine Ait-Sahalia, the Editor at RFS, Kerry Back, Michael Brandt, Michael Brennan, Mark Carey, Darrell Duffie, Greg Duffee, Pierre Collin-Dufresne, Phil Dybvig, Bob Goldstein, Alfred Lehar, Vadim Linetsky, Francis Longstaff, Mark Lowenstein, Gordon Sick, Pietro Veronesi, and Frank Zhang for helpful comments, and to seminar participants at the Mathematical Finance Conference in honor of Robert J. Elliott, the Credit Risk and Asset Pricing Conference at the Wharton School of the University of Pennsylvania, the 16th Annual Conference on Financial Economics and Accounting at UNC Chapel Hill, HEC-Montreal Finance Department, the McGill-IFM Risk Management Conference, the Western Finance Association Meetings at Keystone, Colorado, the Third Annual Empirical Asset Pricing Retreat at the University of Amsterdam, and the Gutmann Center Symposium on Credit Risk and the Management of Fixed Income Portfolios at the University of Vienna. I am grateful for a research grant from the SSHRC. Send correspondence to Alexander David, 2500 University Drive NW, Calgary, Alberta T2N 1N4, Canada. Appendices are available at: http://homepages.ucalgary.ca/~adavid. E-mail: adavid@ucalgary.ca.

[1] Since the seminal work of Merton (1974) and Black and Cox (1976), several important contributions have been made within the generalized structural form framework, including Brennan and Schwartz (1984), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). There is also an extensive developing literature on the reduced form approach, which has offered more flexibility for empirical work on credit risk starting with Duffie and Singleton (1999), and more recently some hybrid models that have features of both approaches (see, e.g., Mamaysky 2002).

doi:10.1093/rfs/hhm041 Advance Access publication August 31, 2007

However, recent empirical testing has cast doubts on the ability of the entire structural form family of models to simultaneously generate both default probabilities and credit spreads that match historical experience.[2] Elton et al. (2001) argue that measures of expected default losses based on empirical frequencies of default implicit in credit ratings and historical recovery rates are too low to be consistent with the high observed credit spreads. Using related calibration methodologies, Delianedis and Geske (2001) and Amato and Remolona (2003) reach a similar conclusion. Extending this logic, Huang and Huang (2003) (henceforth HH) calibrate various structural form models to match the *average* solvency ratios of firms of different rating categories, and in addition choose free parameters to match the historical default probabilities and recovery rates on defaulted bonds, and find that the model-generated spread accounts for less than a third of the total spread over Treasuries on investment-grade corporate bonds. By constraining the models to match historical default losses, the calibration methodology of these authors is seemingly robust to the specification of the asset value process or the default threshold since changing either assumption leads to roughly equivalent variation in expected default losses and credit spreads. This problem has been called the *credit spreads puzzle*.

Additionally, Chen et al. (2006) (henceforth CCG) have contributed to a deepening of the puzzle. These authors argue that the credit spreads and equity premium puzzles are related since they measure risk premiums of different liabilities on the same underlying asset value process. It is therefore natural to consider models in the literature that have been successful in generating a large equity premium and assess their ability in addressing the credit spreads puzzle. Intuitively, and as implied by standard asset pricing theory, it is fairly clear that for a given level of expected default losses, a model's bond prices will be lower (and thus credit spreads higher) if there is a large positive covariance between the losses from default and investors' pricing kernel (marginal utility of consumption). Similarly, the equity premium is higher in a model when the covariance between the pricing kernel and stock returns is strongly negative. However, quite surprisingly, by strictly applying the calibration methodology of HH, CCG find that two well-established models in the literature with high prices of risk (volatility of the marginal utility of consumption), the Campbell and Cochrane (1999) (henceforth CC) habit formation model and the Bansal and Yaron (2004) (henceforth BY) long-term risks model, can only generate small credit spreads after controlling for the expected loss from default. In

[2] Assessing the ability of structural form models in fitting spreads while constraining them to match historical default experience is a relatively recent exercise. An earlier set of articles starting with Jones et al. (1984) (for a survey and some recent results see Eom et al. 2004) studies the conditional performance of these models solely in fitting credit spreads by varying the inputs of the model over time. These studies find mixed support for the validity of the family of models.

addition, the CC model counterfactually predicts that conditional default probabilities of firms decline in recessions. In this sense, the credit spreads puzzle is more puzzling than the equity premium puzzle. CCG point out that besides being unable to match the level of credit spreads, the standard structural form model also has difficulty in explaining the high observed volatility in time series of credit spreads, which these authors call the *credit spreads volatility puzzle.* We will discuss in Section 4.4 how CCG are able to improve the performance of the CC model in both respects by incorporating systematic variation in firms' default boundaries.

To address these puzzling phenomena, we construct a generalization of Merton's basic structural form model, with three distinctive features. The first key aspect of our model is its unobserved regime-switching structure for fundamentals. Drifts of inflation and earnings growth jump erratically, and investors learn about these drifts over time. Second, the asset value process, which is normally exogenously specified, is endogenously determined in a classic asset pricing framework.[3] Asset valuation ratios (which are constant in most existing credit risk models) and asset volatilities vary over time as investors update their beliefs about the hidden states of fundamentals and reassess the prospects of future real growth of fundamentals. In periods when investors are less optimistic about strong future earnings growth, asset valuations fall. When they are more uncertain about the current state of fundamentals, they revise their beliefs more rapidly, leading to greater volatility of asset values (the discounted value of future fundamentals). David (1997) shows that such a Bayesian learning process can generate several stylized facts about volatility processes in the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) literature. Third, we model the joint shifting distribution of inflation and real fundamentals and find that states of strong real fundamental growth are more persistent when inflation is low so that inflation predicts real growth rates. This is the signaling role of inflation (also called the "proxy hypothesis" in Fama 1981). In macroeconomics, there is robust empirical support that the credit quality of firms weakens in periods after increases in short nominal rates (for a survey of this literature, see Bernanke and Gertler 1995). Consistent with this literature, as seen in the top panel of Figure 1, most periods of enhanced spreads in the past forty years have been preceded by bouts of rising short rates.[4]

[3] Our model is related to but distinct from the seminal work on incomplete information and credit risk of Duffie and Lando (2001), in which the asset value process is observed imperfectly by agents due to accounting noise. In our model, the asset value process is formulated by investors conditional on their imperfect information of fundamentals.

[4] A notable exception is the increase in spreads in 1998 following the Russian government default and the LTCM crisis. While various contagion channels have been suggested for the increase in credit spreads from Russia to spill over to the US market, the default in Russia itself was preceded by a resurgence in inflation in 1997 and early 1998.

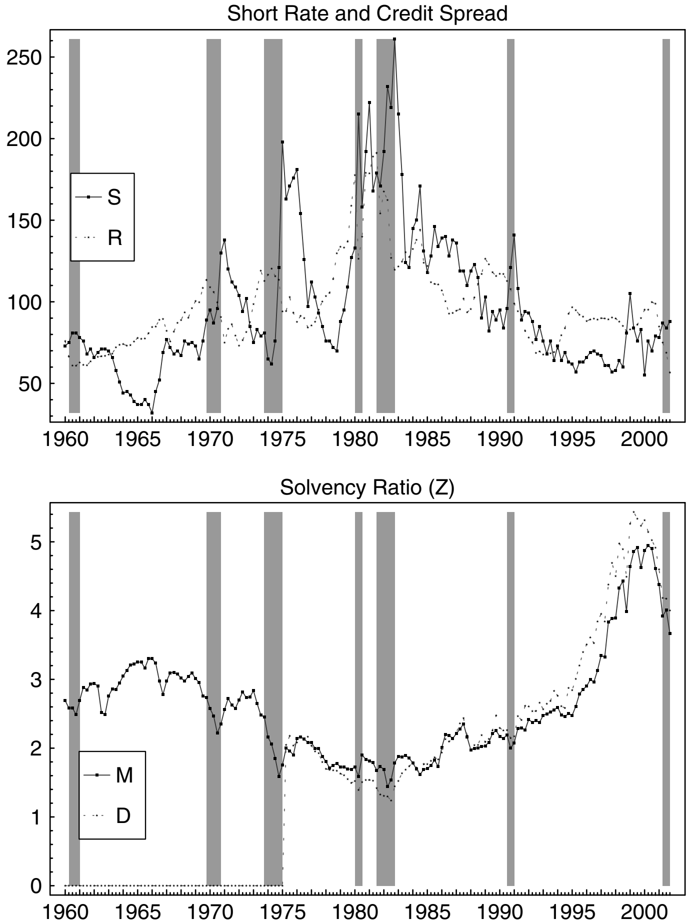

Figure 1
Credit risk and the macroeconomy
The top panel shows the time series of Moody's Baa–Aaa yield spread and the three-month Treasury Bill rate. The latter has been rescaled. The bottom panel shows time series of the solvency ratio, Z_t, for a firm with an average solvency ratio of 2.5. The solvency ratio is defined as the value of a "representative" Baa firm's assets divided by the face value of its debt outstanding, $Z_t = (P_t^E + D_t P_t)/D_t$. The value of equity is approximated as P_t^E, the value of the S&P 500, and the price of debt is $P_t = e^{-y_t^{Baa} 10}$, where y_t^{Baa} is Moody's Baa average bond yield, and D_t is an approximation of the liabilities of Baa-rated firms. The growth of D_t each quarter is the average growth rate of liabilities of all nonfinancial firms in Compustat from 1975: to 2001 (prior data are not reliable). The debt level at the first date is adjusted to match the solvency ratio implied from our analysis. A series in which debt growth and bond prices are from our calibrated model, but with market equity valuations for the sample from 1960 to 2001, is also shown, which is backed out from equity and Treasury market valuations using a generalization of Moody's KMV methodology described in Section 3.1. Shaded areas denote NBER dated recessions.

Using the options logic, we provide a formula for the defaultable bond price in this model as a function of investors' beliefs about the hidden states with a combination of Fourier Transform and projection methods. An attractive feature of our model is the availability of closed-form solutions for the term structure of interest rates and asset values.[5] This enables us to calibrate our model parameters and investors' beliefs to observed data on both fundamental and financial variables, the latter consistent with a rich information set for agents.

If an empirical asset value series were available, we could adapt standard econometric methods such as the Generalized Methods of Moments (GMM) or the Simulated Method of Moments (SMM) to fit the parameters of the regime switching model. However, we face the added complexity (confronting all implementations of structural form models) of not having available such an observable process. We resolve the problem by treating the asset value process as "missing data" and adapting the well-known recursive Expectations Minimization (EM) algorithm to the framework. It is a generalization of the method used by Moody's KMV to iteratively back out a series of asset values and volatilities from observed equity data, which uses the Black–Scholes–Merton (BSM) formula, and so is only consistent with constant volatility and interest rates. The equivalence of the KMV methodology with the EM algorithm and a maximum likelihood procedure to fit the BSM model has been shown by Duan et al. (2004).

To assess the ability of our model to generate empirically plausible credit spreads, we make the assumption that the rating agencies adopt the "through-the-business-cycle" approach to assigning firms to a given rating category. That is, each firm is rated not on the basis of its current conditions but rather on its average probability of default through both good and bad years of the business cycle. A brief description of this method of rating is in Carey and Treacy (1998):

> Under this philosophy, the migration of borrowers' rating up and down the scale as the overall economic cycle progresses will be muted: Ratings will change mainly for those firms that experience good or bad shocks that affect long-term condition or financial strategy and for those whose original downside scenario was too optimistic. (box on page 899)

In keeping with this tradition, the solvency ratio for our "representative firm" will fluctuate through the business cycle with investors' expectations

[5] In recent years, several authors have written regime-switching models of the term structure (see, e.g., Bansal and Zhou 2002, Ang and Bekaert 2002, Dai et al. 2003). Our work is different in two respects: (i) we do not model regimes of interest rates, but instead fundamental regimes (both real and inflationary), and (ii) the regimes are unobserved, so that volatility of interest rates is endogenously determined in contrast to the above articles. David and Veronesi (2006) pursue a similar direction in discrete time, while Ang and Piazzesi (2003) construct a term-structure model with fundamentals in an affine setting.

of fundamental growth rates; however, we will assign it to a given rating category based on its average solvency ratio. For example, HH point out that the average solvency ratio for Baa-rated firms is 2.5. The bottom panel of Figure 1 shows the time series of our model-based solvency ratio and its empirical counterpart constructed from aggregated series of liabilities of firms and aggregate prices for equity and debt. Indeed, the ratio has shown considerable variation in the sample, falling as low as 1.4 and rising at its peak to almost 5. The ratio fluctuates so much because asset valuations affecting the numerator are far more variable than debt in the denominator.[6]

For addressing the credit spreads puzzle, we make two key departures in our calibration methodology relative to the analysis in HH. First, consistent with the through-the-cycle rating approach described above, we do not assume that within a rating category firms' credit risk is constant over time. This is implicitly the approach taken by these authors when they assume constant solvency ratios for such firms. Using a result in Bergman et al. (1996), we show that credit spreads are convex functions of firms' solvency ratios, and hence average spreads are larger than the spread evaluated at the average solvency ratio. By taking into account this *convexity effect* from time-varying solvency ratios, we are able to justify credit spreads that are 60% higher than those reported by HH, who report the spread at the average solvency ratio for alternative models. Our second departure from HH is to build in explicitly the effect of stochastic asset volatility, a feature that these authors abstract from for reasons of tractability. In our model, asset volatility is inversely related to asset valuations, which makes the impact of credit risk convexity even larger, providing another 50% increase over their benchmark. Overall, our analysis justifies a default-related spread of Baa bonds of 106 basis points, which is close to the average historical spread of Baa bonds over Aaa bonds. We attribute the remaining portion of the spread of Baa bonds over Treasuries, which is the spread of Aaa bonds over Treasuries, to issues such as illiquidity and taxes of corporate bonds that are not modeled in this article.

It is relevant to point out why the convexity effect described above has a nonequivalent impact on credit spreads and default probabilities. Indeed, explaining this differential impact is the key to our resolution of the credit spreads puzzle, which requires models to generate large spreads while holding expected default losses at historically observed levels. In a nutshell, our explanation is as follows: an econometrician who takes into account the state dependence in the credit risk of firms will, by the convexity effect, obtain higher expected default losses and credit spreads for given levels of the prices of risk relative to the case where he ignores

[6] The growth rate of aggregated liabilities has an annual volatility of only about 4%. The volatility of the implied asset value process from equity data is significantly higher at about 22%.

the state dependence. Increasing the prices of risk in our model leads to a simultaneous decline in asset volatility and an increase in the asset risk premium. Both effects lead to a decline in the expected default losses, which are computed under investors' objective measure, but only the former lowers credit spreads. Therefore, the expected default losses are more sensitive than credit spreads are to changes in the prices of risk. By incorporating the state dependence and increasing the prices of risk, the econometrician can maintain expected default losses at historical levels and still have spreads at a higher level relative to the case of no state dependence in credit risk.

As further support for our approach, we show that an econometrician who takes into account the state dependence of solvency ratios and asset volatilities is able to reconcile the high credit spreads and the high equity premium, while one who ignores the state dependence would require too low an equity premium to calibrate the model to match historical expected default losses. We also show that the state dependence is able to rationalize the volatility of credit spreads. Finally, we show that our model-implied credit spread explains nearly two-thirds of the variation in the historical Moody's Baa–Aaa spread, and is a very significant predictor of business cycles.

The plan for the remainder of this article is as follows: In Section 1, we present the basic structure of the model and provide closed-form solutions for asset values and the term structure. In Section 2, we formulate the structural form model for credit risk in our setup. In Section 3, we provide a calibration of investors' beliefs of the underlying states that uses information in both fundamentals and financial variables. Section 4 provides an analysis of the credit spreads puzzle and its synthesis with the equity premium puzzle. Section 5 presents concluding remarks. Proofs of all propositions (unless explicitly stated) are in the Appendix.[7]

1. Structure of the Model

We consider one of the simplest defaultable corporate bond structural-form models. We assume that the firm continuously issues a single class of discount debt with maturity T, there is no chance of default prior to maturity of the debt, there are no bankruptcy costs, and finally, there are no taxes. These assumptions simplify the exposition, but are not responsible for the main results. Under these assumptions, one can use results similar to Merton (1974) and use option pricing methods to price defaultable bonds.

[7] Two additional appendices will be made available to readers. Appendix B contains the SMM methodology for estimating the asset value process and the parameters of the model, and Appendix C provides a detailed description of the projection method used to solve the partial differential equation (PDE) for corporate bond prices.

Assumption 1. *The price of the single homogeneous good in the economy, Q_t, evolves according to the log-normal process:*

$$\frac{dQ_t}{Q_t} = \beta_t dt + \sigma_Q dW_t, \tag{1}$$

where the process followed by β_t is described below and $\sigma_Q = (\sigma_{Q,1}, \sigma_{Q,2}, 0)$ is a 1×3 constant vector known by investors.

Assumption 2. *Real earnings, E_t, evolves according to the log-normal process:*

$$\frac{dE_t}{E_t} = \theta_t dt + \sigma_E dW_t, \tag{2}$$

where $W_t = (W_{1t}, W_{2t}, W_{3t})'$ is a three-dimensional vector of independent Weiner processes, the 1×3 constant vector $\sigma_E = (0, \sigma_{E,2}, 0)$ is assumed known and constant. The process for θ_t is described below.

We use these fundamentals to price all assets in our model. In order to do so, we need to specify a stochastic discount factor to be used to discount real payoffs. We make the following assumption:

Assumption 3. *There exists a real pricing kernel M_t taking the form*

$$\frac{dM_t}{M_t} = -k_t dt - \sigma_M dW_t, \tag{3}$$

where $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3})$ is a 1×3 constant vector of the market prices of risk, and $k_t = \alpha_0 + \alpha_\theta \theta_t + \alpha_\beta \beta_t$ is the real short rate of interest. M is used to price real claims and determine the expected real returns of all securities. We restrict the real rate of interest to be a linear function of the two (hidden) state variables of the model.

The kernel is observed by investors but not by the econometrician. Using it, agents price a generic real payout flow of $\{d_t\}$ as

$$M_t P_t = E\left[\int_t^\infty M_s d_s ds | \mathcal{F}_t\right], \tag{4}$$

where $\mathcal{F}_t$ is investors' filtration to be described in Assumption 5. As in several recent papers (e.g., Berk et al. 1999 and Brennan and Xia 2002), we have specified an exogenous pricing kernel process to formulate equilibrium relationships among endogenous financial variables. The linear dependence of the real rate on the drifts of fundamentals has a theoretical basis in general equilibrium models. For example, in a Lucas (1978) economy where investors have power utility $U(C, t) = e^{-\phi t} \frac{C^{1-\gamma}}{1-\gamma}$, we would have $C_t = E_t$, $M_t = U'(E_t)$, and hence $k_i = \phi + \gamma \theta_i + \frac{1}{2}\gamma (1 - \gamma) \sigma_E \sigma_E'$ and $\sigma_M = \gamma \sigma_E$. In this case, the real rate is not affected by the inflation drift, that is, $\alpha_\beta = 0$. Our specification generalizes the real rate process to economies in which expected inflation affects the real cost of borrowing. The specified real rate can be supported in an equilibrium if we allow a storage technology that can be bought or sold, which is in perfectly elastic supply, and offers a safe instantaneous return of k_t. We provide further motivation for its dependence on expected inflation next.

The literature on the effect of expected inflation on the real rate of return is extensive, and it is beyond the scope of this article to build in all the effects without significantly complicating our analysis. At the micro level there is a fairly large literature on various tax and accounting channels through which expected inflation affects the real return on capital (Feldstein 1980a, Feldstein 1980b, Auerbach 1979, and Cohen and Hassett 1999).[8] Among the several monetary channels that lead to the non-neutrality of money, of particular relevance are cash-in-advance models in which expected inflation is a tax on money balances and raises the real cost of transactions, thereby affecting the real interest rate, capital accumulation, and business cycle variation (e.g., Svensson 1985 and Lucas and Stokey 1987). Cooley and Hansen (1989) calibrate such a model to the U.S. economy and report a positive relationship between the expected inflation and the marginal product of capital (the real rate).[9]

Given the multiplicity of channels that affect the relationship between the expected inflation and the real short rate, we treat the sign and size of this relationship as an empirical question that can be estimated from the joint time-series variation of fundamentals and asset prices. Our chosen functional form of the real rate can be seen as a reduced form for the above noted effects.[10]

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[8] The articles by Feldstein point out that the typical depreciation allowance is based on the original or "historic" cost of the asset rather than its current value. When prices rise, this historic cost method of depreciation causes the real value of depreciation to fall and the real value of taxable profits to rise, increasing the real return on invested capital. This effect can be countered by the effect of expected inflation on the value of inventories depending on the method of accounting used. If firms use the first-in-first-out method of accounting, then the cost of maintaining inventory levels is understated in the presence of inflation, hence lowering the real return on capital. The opposite effect holds with the last-in-first-out accounting method. Auerbach (1979) and Cohen and Hassett (1999) extend this analysis by studying further distortions on the choice of maturity of real assets and investment.

[9] The cash-in-advance constraint applies only to the consumption good while leisure and investment are modeled as credit goods. These authors use a constant return to scale production function of the form $f(K, N) = K^\theta N^{1-\theta}$, and calibrate the response of a representative agent to increases in the expected rate of inflation. They report that both capital (K) and labor (N) choices decrease as an optimal response to an increase in the expected rate of inflation, which could lead to either an increase or a decrease in the marginal product of capital. For the calibrated economy, the net result is positive (see Table 2 of Cooley and Hansen 1989).

[10] Using related but different empirical methodologies from ours, Geske and Roll (1983) and more recently Brennan and Xia (2002) both find a positive relationship. (Geske and Roll 1983, p. 12) write:

> The recent experience from a period (1981–1982) characterized by large deficits and a low rate of Federal Reserve monetization seems to be that real rates have increased dramatically. At least, journalists, investment bankers, and foreign government leaders seem to think so.

For notational convenience, we stack the fundamental processes (1) and (2). Let $Y_t = (Q_t, E_t)'$, so that

$$\frac{dY_t}{Y_t} = \varrho_t dt + \Sigma_2 dW_t, \tag{5}$$

where $\frac{dY_t}{Y_t}$ is to be interpreted as "element-by-element" division, $\varrho_t = \left(\beta_t, \theta_t\right)'$, and $\Sigma_2 = (\sigma'_E, \sigma'_Q)'$ Similarly, we find it useful to add the kernel process to the stacked vector and write $X_t = (Q_t, E_t, M_t)'$, which has the drift vector $\nu_t = \left(\beta_t, \theta_t, -k_t\right)'$, and volatility matrix $\Sigma = (\sigma'_Q, \sigma'_E, -\sigma_M)'$.

Assumption 4. *ν_t follows an N-state, continuous-time finite state Markov chain with generator matrix Λ,*[11] *that is, over the infinitesimal time interval of length dt,*

$$\lambda_{ij} dt = prob\left(\nu_{t+dt} = \nu_j | \nu_t = \nu_i\right), for \neq j, \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}.$$

Assumption 5. *Investors do not observe the realizations of ν_t but know all the parameters of the model.*

Since investors do not observe ν_t, they need to infer it from the observations of past earnings, inflation, and the pricing kernel. This will generate a distribution on the possible states $\nu_1, \ldots, \nu_N$ that in turn generates changes in "uncertainty" as they learn about the current state. At time t, investors' distribution over hidden states is summarized by the posterior probabilities:

$$\pi_{it} = \text{prob}(\nu_t = \nu_i | \mathcal{F}_t),$$

where $\mathcal{F}_t$ is the filtration generated by observing the entire path $(X_s)_{0 \leq s \leq t}$. Let $\pi_t = (\pi_{1t}, ..., \pi_{Nt})$ be the vector of beliefs.

Lemma 1. *Given an initial condition $\pi_0 = \hat{\pi}$ with $\sum_{i=1}^N \hat{\pi}_i = 1$ and $0 \leq \hat{\pi}_i \leq 1$ for all i, the probabilities π_{it} satisfy the N-dimensional system of stochastic differential equations:*

$$d\pi_{it} = \mu_i(\pi_t)dt + \sigma_i(\pi_t)d\tilde{W}_t, \tag{6}$$

$$\textit{in which} \quad \mu_i(\pi_t) = [\pi_t \Lambda]_i, \quad \sigma_i(\pi_t) = \pi_{it}[\nu_i - \overline{\nu}(\pi_t)]' \Sigma'^{-1}, \tag{7}$$

[11] The transition matrix over states in a finite interval of time, s, is $\exp(\Lambda s)$ (see, e.g., Karlin and Taylor 1982).

$$\overline{\nu}(\pi_t) = \sum_{i=1}^{N} \pi_{it}\, \nu_i = E_t\left(\frac{dX_t}{X_t}\right), \quad \textit{and}$$

$$d\tilde{W}_t = \Sigma^{-1}\left(\frac{dX_t}{X_t} - E\left(\frac{dX_t}{X_t}\right)|\mathcal{F}_t\right) = \Sigma^{-1}\left(\nu_t - \overline{\nu}(\pi_t)\right)dt \; + \; dW_t. \quad (8)$$

Moreover, for every $t > 0$, $\sum_{i=1}^{N} \pi_t = 1$.

Proof. See Wonham (1964) or David (1993).

The filtering theorem for jumps in the underlying drift was first derived (to the best of our knowledge) in Wonham (1964). David (1993) provides a proof using the limit of Bayes' rule in discrete time. The first application of this theorem in financial economics, as well as several properties of the filtering process, are derived in David (1997).

We make the following summary comments about the updating process (6): (i) The elements of the generator matrix, Λ, completely capture the transition probabilities between states. Absent any new information, beliefs tend to mean-revert to the unconditional stationary probabilities that are completely determined by Λ. For example, if there are only two states with $\lambda_{12} = 2$ and $\lambda_{21} = 1$, then the belief that the economy is in state 1 mean-reverts to $1/3$. (ii) The diffusion term describes the change in beliefs due to new information. When they have high uncertainty about the current state vector, they react more to news, and the volatility of their beliefs increases. (iii) Conditional on a given level of uncertainty, the volatility of beliefs is proportional to the parameters determining the ratio of signal (difference in drifts across states) to noise (Σ). (iv) Agents update their beliefs about underlying states by observing not only the path of fundamentals, but also their pricing kernel. This is analogous to the Lucas economy mentioned above in which the agent would learn about the drift rate of the marginal utility of consumption.

It is useful to note that investors' beliefs change with their inferred shocks, $d\tilde{W}$, in Equation (8) as opposed to the true shocks, dW, that affect fundamentals. Similarly, they infer that the process $\{X_t\}$ follows: $dX_t/X_t = \overline{\nu}(\pi_t)dt + \Sigma d\tilde{W}_t$. At each point in time, substituting the definition of $d\tilde{W}_t$, we have $dX_t/X_t = \overline{\nu}(\pi_t)dt + \Sigma\left[\Sigma^{-1}(\nu_t - \overline{\nu}(\pi_t))dt \; + \; dW_t\right] = \nu_t dt + \Sigma dW$, which is the same process as in Assumptions 1–3. In the filtering literature, $d\tilde{W}_t$ is an "innovations" process under the investors' filtration and under the separation principle it can be used for dynamic optimization. See David (1997) for a discussion. As a special case we will write:

$$\frac{dY_t}{Y_t} = \overline{\varrho}(\pi_t)dt + \Sigma_2 d\tilde{W}_t = \varrho(\pi_t)dt + \Sigma_2 dW_t, \quad (9)$$

and the kernel under investors' filtration as $dM_t/M_t = -\overline{k}(\pi_t)dt - \sigma_M d\tilde{W}_t$, where the real rate in the economy, $\overline{k}(\pi_t)$, is its expected value conditional on investors' filtration.

1.1 Asset value process and the term structure of interest rates

For evaluating nominal claims and nominal risk premiums we will also use the nominal pricing kernel, $N_t = M_t/Q_t$, which follows:

$$\frac{dN_t}{N_t} = -r_{it}dt - \sigma_N dW_t, \tag{10}$$

where $r_{it} = k_{it} + \beta_{it} - \sigma_N \sigma_Q'$ and $\sigma_N = \sigma_M + \sigma_Q$. The nominal rate differs from the real rate by the sum of expected inflation and the inflation risk premium, which is the covariance between inflation and the nominal pricing kernel. With unobserved states, the projected nominal interest rate at time t is $\sum_{i=1}^{N} r_i \pi_{it}$. The following proposition provides expressions for the value-to-earnings (henceforth V/E) ratio and the nominal bond price:

Proposition 1. *(a) The V/E ratio at time t is*

$$\frac{V_t}{E_t}(\pi_t) = \sum_{j=1}^{N} C_j \pi_{jt} \equiv C \cdot \pi_t, \tag{11}$$

where the vector $C = (C_1, \ldots, C_N)$ *satisfies* $C = A^{-1} \cdot \mathbf{1}_N$,

$$\begin{aligned} A \;=\; & Diag(k_1 - \theta_1 + \sigma_M \sigma_E', k_2 - \theta_2 \\ & + \sigma_M \sigma_E', \cdots, k_N - \theta_N + \sigma_M \sigma_E') - \Lambda. \end{aligned} \tag{12}$$

(b) The price of a nominal zero-coupon bond with maturity τ *is*

$$B(\pi_t, t, T) = \sum_{i=1}^{N} \pi_{it} B_i(t, t+\tau), \tag{13}$$

$$B_i(t, t+\tau) = E\left(\frac{M_T}{M_t} \cdot \frac{Q_T}{Q_t} | \nu_t = \nu_i\right) = \left[\sum_{i=1}^{N} \Omega_i e^{\omega_i \tau}\right] \cdot (\Omega^{-1} \boldsymbol{1}_N), \tag{14}$$

where Ω_i *and* ω_i, $i = 1, \ldots, N$, *are the ith eigenvector and ith eigenvalue, respectively, of the matrix* $\hat{\Lambda} = \Lambda - Diag(r_1, r_2, \ldots, r_N)$.

Notice that an "averaging" result holds for linear securities: prices conditional on investors' filtration are the averages of prices that would hold in the N regimes if they were observed by investors. This "averaging" property does not hold for nonlinear securities prices such as defaultable bonds that are derived in the next section. The proof follows from minor modifications to the discrete-time proof in David and Veronesi (2006). Also see Veronesi (2000), Veronesi and Yared (1999), and David and Veronesi (2002) for derivations in continuous time with similar assumptions.

In (a), each constant C_i represents investors' expectation of future earnings growth conditional on the state being ν_i today, discounted using the process for the pricing kernel process $\{M_t\}$, and normalized to make it independent of the current payout and time. Hence, a high C_i implies that investors assess a high value relative to current earnings in state ν_i. Since they do not actually observe the state ν_i, they weight each C_i by its conditional probability π_i, thereby obtaining (11). Notice in particular that the form of the constant vector C suggests that: (i) if $\alpha_\theta < 1$, a higher growth rate of earnings implies a higher V/E; (ii) if $\alpha_\beta > 0$, a higher inflation state implies a lower V/E, which is the *real rate effect* of inflation; and (iii) in addition, the V/E ratio in a given state of growth depends on the future sustainability of the growth rate and is determined by the transition probabilities λ_{ij} as shown in the solution to the N-equations in (a).

Similarly, the bond price is a weighted average of the nominal bond prices that would prevail in each state ν_i. Since investors do not actually observe the current state, they price the bond as a weighted average. Both higher inflation and higher growth rate of earnings lead to lower long-term bond prices when $\alpha_\beta > -1$ and $\alpha_\theta > 0$. It is useful to notice that all asset prices follow continuous paths even though the drift rates for earnings and inflation jump between a discrete set of states. This results from the continuous updating process.

Using the characterization of inflation and earnings, processes under the observed filtration, we are now able to specify the processes for the *real* and *nominal* value of the firm's asset value.

Proposition 2. *(a) The nominal asset return process under the investor's filtration is given by*

$$\frac{dV_t^Q}{V_t^Q}(\pi_t) = (\mu_V^Q(\pi_t) - \delta(\pi_t))dt + \sigma_V^Q(\pi_t)d\tilde{W}_t,$$

where $\delta(\pi_t) = \frac{1}{C.\pi_t}$, *the expression for* $\mu_V^Q(\pi_t)$ *is in the Appendix, its volatility is*

$$\sigma_V^Q(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^N C_i \pi_{it}(\nu_i - \overline{\nu}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^N C_i \pi_{it}}, \qquad (15)$$

and the vector C is determined in Proposition 1. The volatility of the real asset process satisfies $\sigma_V(\pi_t) = \sigma_V^Q(\pi_t) - \sigma_Q$.
(b) The real and nominal risk premiums satisfy $\mu_V(\pi_t) - k(\pi_t) = \sigma_V(\pi_t)\sigma_M'$, *and* $\mu_V^Q(\pi_t) - r(\pi_t) = \sigma_V^Q(\pi_t)\sigma_N'$, *respectively.*

This proposition shows that asset volatilities have an exogenous component due to noise in the fundamental process and a learning-based endogenous component. The latter is larger when investors are more uncertain about the state of underlying fundamentals because in such times investors react more to incoming fundamental news. Nominal (real) risk premiums are determined by investors as the negative of the covariances of the nominal (real) asset value with the nominal (real) pricing kernel.

The following proposition states conditions under which the volatility of the firm's asset value with respect to the first shock (inflation) is mostly negative, while the volatility with respect to the second shock (earnings) is positive. We will say state vectors x and y are (weakly) positively related when $x_i \geq x_j$ iff $y_i \geq y_j$.

Proposition 3. *Suppose the inflation state vector, β, and the asset V/E state vector, C, are negatively related, and the earnings state vector, θ, and C are positively related, and all volatilities, $\sigma_{Q,1}$, $\sigma_{Q,2}$, and $\sigma_{E,2}$, are all positive. Then, $\sigma_{V,1} \leq 0$, $\sigma^{Q}_{V,1} \leq \sigma_{Q,1}$, $\sigma_{V,2} \geq 0$, and $\sigma^{Q}_{V,2} \geq 0$.*

The proposition is intuitive: if the V/E ratio is lower in states when the inflation drift is higher, then if realized inflation is higher than expected, investors' conditional probability of higher inflation increases and asset values decline. Such a negative correlation between *realized* real asset returns and *realized* inflation has been noted by several authors. In our empirical section we examine the estimated values of the fundamentals states and V/E ratios in the hidden states and verify that the stated conditions in the proposition are satisfied for the estimated parameters. In addition, we see that the exogenous component of inflation volatility, $\sigma_{Q,1}$, is "small"—of the order of 1% at an annual rate—so that in most periods $\sigma^{Q}_{V,1} < 0$. The sign of $\sigma_{V,3}$ is in general indeterminate; however, we will find that for our estimated parameters, this volatility is negative in each period in our sample.

The risk of regime switches in the model is systematic and affects the prices of defaultable bonds in the next section. As for all nontraded assets in equilibrium models, for risk-neutral valuation, the drifts of the belief processes must be reduced by their market prices of risk (e.g., Cox et al. 1985 or Hull 1993). As in these frameworks, the market price of risk of π_i is the negative of its covariance with the pricing kernel of the economy. The analytical expression is given for these market prices in the following proposition.

Proposition 4. *The nominal market price of risk of the state variable π_{it}, for each $i \in \{1, \ldots, N\}$, investors' conditional probabilities of the economy being in state i, is time-varying and is given by*

$$\rho_i(\pi_t) = \pi_{it} \cdot ((\beta_{it} - \overline{\beta}(\pi_t)) - (k_{it} - \overline{k}(\pi_t))). \tag{16}$$

It is evident that even though the market prices of risks associated with the two shocks are constant, the market prices of risks for the beliefs are time varying, and increase in the fundamental uncertainties of earnings and inflation. The risk adjustments in a sense distort the mean reversion of the belief processes and effectively change the generator elements in Assumption 4.[12]

2. Price of Defaultable Debt

The firm issues a single class of zero coupon debt. If the real face value of debt remains constant, credit spreads in structural form models tend to zero as maturity increases (Collin-Dufresne and Goldstein 2001, Ericsson and Reneby 1998). We will instead assume that the firm issues nominal debt at a pace that ensures that there is no trend in the distance between the nominal asset value and the nominal face value of outstanding debt. Black and Cox (1976) motivate the same assumption as a safety covenant for bondholders. In support of this assumption, the empirical work in Eom et al. (2004) suggests no additional role for maturity (besides very short maturity bonds) in fitting spreads beyond the other inputs in structural form models.

Assumption 6. *The nominal face value of debt at time t is given by*

$$D_t^Q = D_0^Q \cdot \exp[\int_{s=0}^{t} (r(\pi_s) - \delta(\pi_s))ds], \tag{17}$$

where $r(\pi_t)$ is the nominal rate of interest defined in Proposition 1(b), D_0^Q is the initial face value of the debt of the firm, and $\delta(\pi_t)$ is the earnings yield in Proposition 2. The net growth rate of the debt, $r(\pi_t) - \delta(\pi_t)$, can result from gross issuance at the riskless rate and retirement at the earnings yield, or any such combination.

Assumption 7. *Similar to Merton (1974), we assume the firm defaults at maturity, T, if the market value of the firm's asset in nominal dollars at maturity is below the face value of its debt in nominal dollars, D_T^Q.*

Assumption 8. *The nominal asset value of the firm is a traded security.*

The assumption and Proposition 2 imply that the nominal asset value of the firm can be written as $dV_t^Q/V_t^Q = (r(\pi_t) - \delta(\pi_t))dt + \sigma_V^Q d\tilde{W}_t^*$, where

[12] The point is made best in the 2-state case. In this case, π_1 mean-reverts to $\pi_1^* = \lambda_{21}/(\lambda_{12} + \lambda_{21})$ under the objective measure. However, under the risk-neutral measure, π_1 mean-reverts to the positive root of $(\lambda_{12} + \lambda_{21})(\pi_1^* - \pi_1) - ((\beta_1 - \beta_2) - (k_1 - k2))\pi_1(1 - \pi_1)$, which is smaller than π_1^* when $\beta_1 - k_1 > \beta_2 - k_2$. Such an effective change in the generator element under the risk-neutral measure is also evident in the analysis of Pan (2002), who prices systematic jump risk in derivative securities, and the ratings-based credit risk model of Jarrow et al. (1997a). An advantage of our approach is that the functional form of these risk adjustments is endogenously derived.

$d\tilde{W}_t^* = d\tilde{W}_t + \sigma_N dt$, and $\sigma_N = \sigma_M + \sigma_Q$ are nominal market prices of risk. Define the solvency ratio of the firm as $Z_t = V_t^Q / D_t^Q$, and $z_t = \log Z_t$. Under the nominal risk-neutral measure, the solvency ratio follows:

$$\frac{dZ_t}{Z_T} = \sigma_V^Q(\pi_t) d\tilde{W}_t^*. \tag{18}$$

By standard no-arbitrage pricing (e.g., Chapter 5.G in Duffie 1996), every contingent claim $f(t, z_t, \pi_t)$ must satisfy the fundamental PDE:

$$\begin{aligned} r(\pi_t) f = {} & \frac{\partial f}{\partial t} - \frac{1}{2} \frac{\partial f}{\partial z} \sigma_V^Q(\pi_t) \sigma_V^Q(\pi_t)' \\ & + \sum_{j=1}^{N} \frac{\partial f}{\partial \pi_j} \left(\mu_j(\pi_t) - \rho_j(\pi_t) \right) \\ & + \frac{1}{2} \frac{\partial^2 f}{\partial z^2} \sigma_V^Q(\pi_t) \sigma_V^Q(\pi_t)' + \sum_{j=1}^{N} \frac{\partial^2 f}{\partial z \partial \pi_j} \sigma_V^Q(\pi_t)\, \sigma_j'(\pi) \\ & + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 f}{\partial \pi_k \partial \pi_j} \sigma_k(\pi_t)\, \sigma_j'(\pi_t), \end{aligned} \tag{19}$$

where $\mu_i(\pi)$ and $\sigma_j(\pi)$ are given in Equation (7), $\sigma_V^q(\pi_t)$ is in Equation (15), and $\rho_i(\pi_t)$ is given in Equation (16). In addition, natural boundary conditions at 0 and 1 for each belief process and terminal conditions determined by the final payoff of the contingent claim are imposed.

At maturity, the payoff for each dollar of face value of outstanding debt is $\mathrm{Min}(Z_T, 1)$. That is, in default, the recovery on each dollar of defaultable debt equals the solvency ratio. We extend the Fourier Transform methodology that has so far been used in the affine framework (e.g., Heston 1993) to solve for defaultable bond prices in the following proposition.

Proposition 5. *The defaultable bond price for a firm with nominal asset value V_t^Q, nominal debt outstanding D_t^q, and hence $Z_t = V_t^Q / D_t^q$, and when investors' beliefs are π_t satisfies*

$$\begin{aligned} P(Z_t, \pi_t, t, T) = {} & B(\pi_t) \cdot \Pi_2(Z_t, \pi_t, t, T) \\ & + G(\pi_t, z_t) \cdot (1 - \Pi_1(Z_t, \pi_t, t, T)), \end{aligned} \tag{20}$$

where $\Pi_1(\cdot, \cdot)$ and $\Pi_2(\cdot, \cdot)$ are the time-t prices of Arrow-Debreu securities that pay a dollar if the firm is solvent at T $(Z_T > 1)$ under two different measures. These two probabilities can be found by fast Fourier Inversion of the characteristic function of the discounted log solvency ratio under the *risk-neutral measure:*

$$f(z_t, \omega_1, \pi_t, T-t) = E[e^{-\int_t^T r(\pi_s)ds} e^{i\omega_1 z_T} | z_t, \pi_t]. \quad (21)$$

$B(\pi_t) = f(z_t, 0, \pi_t, T-t)$ *and* $G(\pi_t, z_t) = f(z_t, 1/i, \pi_t, T-t)$ *are obtained by simply evaluating* $f(\cdot,\cdot)$ *at two points, and are the bond price and forward solvency ratio, respectively, under the risk-neutral measure.*

Proof: In Appendix C.

The characterization of the risky-bond price is analogous to that in Merton (1974), with the extension to the case of the stochastically growing debt of the firm. The call option value (equity of the firm) for the case where π_t was 1-dimensional was solved in David and Veronesi (2002). In this article, we extend the solution to the general case where π_t is N-dimensional, and in addition, interest rates and the default boundary are both stochastic. An $M = 220$ term polynomial approximation of $f(\cdot,\cdot)$ is provided in Equation (C11) in the proof.

For convenience, we also write the T-period spread on the defaultable bond as

$$s(Z_t, \pi_t, t, T) = -\frac{1}{T}\log\left[\Pi_2(\cdot,\cdot) + \frac{G(\pi_t, z_t)}{B(\pi_t)} \cdot (1 - \Pi_1(\cdot,\cdot))\right]. \quad (22)$$

We see in the following corollary that the credit spread is a decreasing and convex function of the solvency ratio, Z_t, of the firm. As we will see in Section 4, the convexity property will partially resolve the *credit spreads puzzle.*

Corollary 1. *(i) The defaultable bond price* $P(Z_t, \pi_t, t, T)$ *is an increasing and concave function of the solvency ratio,* Z_t. *(ii) The credit spread* $s(Z_t, \pi_t, t, T)$ *is a decreasing and convex function of the solvency ratio.*

We end this section with some comments on the various channels through which unexpected inflation (as measured by inflation volatility) and expected inflation (as measured by the filtered beliefs) will affect the firm's probability of default under the *risk-neutral measure* in our model. As seen from Proposition 2, the volatility of firms' nominal asset value equals the sum of its real volatility and inflation volatility, while by Assumption 6, debt growth is locally smooth. Its direct implication is that credit risk for firms increases with inflation volatility as the defaultable bond price is the value of the safe bond price less a put option on the *nominal* asset value of the firm written by bondholders and purchased by equity holders. An increase in inflation volatility increases the value of this put option.

By Corollary 1(ii), credit spreads are decreasing in $Z_t(\pi_t) = \frac{(\sum_{i=1}^N C_i \cdot \pi_{it}) E_t Q_t}{D_t^Q}$. Therefore, the comments on the V/E ratios in the N

states, C, below Proposition 1 apply. Their implications are that *ex ante*, the following effects hold:

1. For $\alpha_\beta > 0$, an increase in expected inflation increases the real rate of interest. This lowers Z_t and increases spreads.
2. If the transition probabilities to lower earnings states are higher in higher earnings states, then an increase in expected inflation lowers Z_t and raises spreads.
3. In addition, if the transitions between earnings states are more likely in higher inflation states, higher expected inflation raises investors' forecast of the endogenous component of future asset volatility, and since spreads are convex in Z_t, causes them to increase. David and Veronesi (2006) examine the volatility forecasting power of inflation-based uncertainty measures.

3. Calibration

In this section we provide a brief description of the calibration methodology and the parameter estimates of our model.

3.1 Calibration methodology

We calibrate the model by using information in both fundamental and financial variables to obtain the time series of investors' beliefs over fundamental states, as well as the underlying parameters. Suppose the V/E series is observable: then, using it, the data on fundamentals, and the Treasury yields, we estimate the parameters using a SMM method as described below.

Let Ψ denote the set of structural parameters in the fundamental processes of earnings growth and inflation in Equations (1) and (2), and the pricing kernel in Equation (3). Let the likelihood function for the fundamentals data observed at discrete points of time (quarterly) be $\mathcal{L}$. The simulation-based procedure used to compute $\mathcal{L}$ also generates time series of filtered probabilities that investors have of each of the underlying states. To extract information about investors' beliefs from fundamentals, as well as equity prices and Treasury bond yields, we add moments of asset prices and use the pricing formulae for the V/E ratio and Treasury bond prices in Proposition 1 to generate model-determined moments. Let $\{e(t)\}$ denote the errors of the pricing variables, $\epsilon(t) = \left(e(t)', \frac{\partial \pounds}{\partial \Psi}(t)'\right)'$, where the second term is the score of the likelihood function of fundamentals with respect to Ψ. We now form the SMM objective:

$$c = \left(\frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\epsilon}_{\mathbf{t}}\right)' \cdot \Omega^{-1} \cdot \left(\sum_{t=1}^{T}\frac{1}{T} \cdot \boldsymbol{\epsilon}_{\mathbf{t}}\right). \tag{23}$$

The details of the procedure are in Appendix B.

Since the asset value series is in fact not observed, our estimation problem falls under the class of "missing data" problems, for which the EM procedure is applicable. The procedure is recursive. In brief, we use estimates of bond prices, the parameter estimates, and beliefs at the nth stage to estimate the asset value at time t as $V_t^{Q(n)} = P_t^E + P_t^{(n)} \cdot D_t^Q$, where P_t^E is the market value of the S&P 500 at each date, D_t^Q is formulated using Assumption 6 with D_0^Q set so that the initial solvency ratio of the firm is k, when the initial V/E ratio of the firm is its unconditional value of $C \cdot \pi^*$, and k is the long-run solvency ratio of a given rating category. The estimated stage n V/E ratio at date t is simply $V_t^{Q(n)}/E_t$, and the solvency ratio is $Z_t^{(n)} = V_t^{Q(n)}/D_t^Q$. Now the SMM estimation procedure is used to minimize c in Equation (23) conditional on the nth stage bond prices, and to find the optimal parameter values of stage $n+1$. New estimates of bond prices are obtained using Proposition 5. The steps are repeated until convergence. Further details are provided in Appendix B.

The first formal description of the general EM methodology is in Dempster et al. (1977). More recently, Duan et al. (2004) provide an equivalence between the EM procedure used in industry by Moody's KMV (see Crosbie and Bohn 2002) and the maximum likelihood (ML) procedure to estimate structural credit risk models in Duan (1994). Ericsson and Reneby (2005) extend the ML approach to allow for stochastic interest rates and more general default boundaries. Our method generalizes the EM procedure in addition to account for both a stochastic term structure and volatility. We note that upon convergence, our empirical procedure for fitting the asset value process is *internally consistent* in that bond prices at each date are computed using a formula that incorporates both features.

The simultaneous joint estimation of the unobserved asset value and asset volatility has posed a conundrum to both academics and practitioners, especially when asset volatility varies stochastically over time. The most popular approach has been the two-equation-and-two-unknown problem: (i) equity is a call option on the asset value, $E = \text{Call Option}(V, \sigma_V)$, and (ii) equity and asset volatilities are related by $\sigma_E = \sigma_V\, E_V\, V/E$ (see, e.g., Jones et al. 1984). Both relationships hold only when the volatilities are constant. In their implementation, however, researchers back out time-varying volatilities. More recently some authors have used historically inferred asset values from the call option inversion using (i) (Vassalou and Xing 2004), or have used historical volatilities of equities and transformed to asset volatilities using (ii) (Eom et al. 2004), to obtain current estimates of asset volatility. In addition to being internally inconsistent, these methods are necessarily *backward looking*. In contrast, our method uses forward-looking information in asset prices to back out investors' current beliefs, formulates stochastic asset volatility as in Equation (15) in Proposition 2, and uses the bond pricing formula in Proposition 5 that is consistent with this stochastic volatility. Finally, our calibration method is similarly consistent with the valuation effects of a stochastic term structure of interest rates.

3.2 Estimation results for the regime switching model

In this subsection, we briefly describe the results of the estimation of the regime switching model for the fundamental variables—earnings and inflation. We start with the description of the data series used. Equity values are estimated using the S&P 500 operating earnings per share, and the price-to-operating earning ratio series from 1960–2001, which are obtained from Standard and Poor's. The time series of nominal earnings is deflated using the Consumer Price Index series, which is also used to compute the time series of inflation levels. The time series of constant maturity zero coupon yields are from the Federal Reserve Board.

We estimate a model with three regimes for inflation, $\beta_1 < \beta_2 < \beta_3$, and two regimes for earnings growth, $\theta_1 < \theta_2$, which lead us to six composite states overall. Some further comments on the selection of number of regimes in each series are in Appendix B. In our estimation, we found that the unconstrained estimates of the transition matrix led to several zero elements, leading to a more parsimonious four-state model $\{(\beta_1, \theta_2), (\beta_2, \theta_1), (\beta_2, \theta_2), (\beta_3, \theta_1)\}$. That is, we found that the (β_1, θ_1) and (β_3, θ_2) states had close to zero probability of occurring in the sample. Overall, the four-state and six-state model lead to almost the same value for the SMM objective function. Gray (1996) and Bansal and Zhou (2002) use a similar criterion for the choice among alternative regime specifications.

The top panel of Table 1 reports the means and volatilities of fundamentals, as well as the pricing kernel parameters. The middle panel reports the transition probability matrix, as well as the asset prices in the four states. We estimate that inflation averages 1.3, 4.3, and 9.3% in the three states, while earnings growth averages −9% and 5.1% in the low- and high-growth states, respectively. A few comments are in order: First, while all the states have statistically significant estimates, the standard errors for the inflation states are much smaller than for the earnings growth states. The volatility parameter estimates and the top panels of Figure 2 show that inflation is much less noisy than the earnings series, leading to better estimates for the former series. Second, the high-growth rate of earnings is far more persistent in the low-inflation state: from the (LI-HG) state, there is a 98% chance of returning to this state, and a 2% chance of transitioning to the (MI-HG) state in a year; therefore, there is almost a zero chance of growth slowing in a quarter in which there is also low inflation. From the (MI-HG) state, on the other hand, there is an 8.2% chance of a transition to the (MI-LG) state in the following quarter. This is the signaling role of inflation—it provides an early warning signal of an unsustainable high-growth rate of fundamentals. The overall SMM objective function value,

Table 1
Four-state model calibration

States	β_1	β_2	β_3	θ_1	θ_2	
	0.0133	0.043	0.093	−0.090	0.051	
	(0.003)	(0.004)	(0.005)	(0.014)	(0.020)	
Volatilities	$\sigma_{Q,1}$	$\sigma_{Q,2}$	$\sigma_{E,2}$			
	0.017	0.002	0.112			
	(0.001)	(0.000)	(0.009)			
Generator Elements	λ_{13}	λ_{21}	λ_{23}	λ_{24}	λ_{32}	λ_{42}
	0.039	0.111	0.114	0.285	0.110	0.300
	(0.003)	(0.004)	(0.003)	(0.007)	(0.012)	(0.018)
Pricing Kernel	α_0	α_β	α_θ	$\sigma_{M,1}$	$\sigma_{M,2}$	$\sigma_{M,3}$
	−0.206	0.826	0.351	0.076	0.536	−0.137
	(0.001)	(0.023)	(0.046)	(.040)	(0.100)	(0.021)
GMM Error Value ($\chi^2(6)$):	9.23	p-Value: 0.161				

Implied quarterly transition probability matrix

	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)
(LI-HG)	0.995	0.000	0.005	0.000
(MI-LG)	0.026	0.883	0.027	0.064
(MI-HG)	0.000	0.023	0.977	0.000
(HI-LG)	0.000	0.072	0.000	0.928

Implied annual transition probability matrix

	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)
(LI-HG)	0.981	0.000	0.019	0.000
(MI-LG)	0.087	0.632,	0.089	0.192
(MI-HG)	0.003	0.074	0.914	0.007
(HI-LG)	0.009	0.215	0.010	0.765

Implied stationary probabilities, V/E ratio, 3-month treasury yield
5-year treasury yield, slope of term-structure, and credit spreads

State	π^*	C_i	$i_{.25}$	i_5	$i_5 - i_{.25}$	$s(Z_t = 2.5)$
(LI-HG)	0.493	25.449	0.031	0.034	0.030	0.003
(MI-LG)	0.101	11.827	0.046	0.070	0.024	0.020
(MI-HG)	0.337	13.333	0.090	0.085	−0.005	0.015
(HI-LG)	0.067	8.964	0.141	0.105	−0.036	0.025

The top panel reports SMM estimates of the following model for CPI, Q_t, real earnings, E_t, and the real pricing kernel, M_t :

$$\frac{dQ_t}{Q_t} = \beta_t dt + \sigma_Q dW_t,$$

$$\frac{dE_t}{E_t} = \theta_t dt + \sigma_E dW_t,$$

$$\frac{dM_t}{M_t} = -k_t dt - \sigma_M dW_t,$$

where $\sigma_Q = (\sigma_{Q,1}, \sigma_{Q,2}, 0)$, $\sigma_E = (0, \sigma_{E,2}, 0)$, $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3})$, $k_t = \alpha_0 + \alpha_\theta \theta_t + \alpha_\beta \beta_t$, and the vector $v_t = (\beta_t, \theta_t, -k_t)'$ follows a four-state regime switching model with the generator matrix Λ whose nonzero diagonal elements are shown as $\lambda_{i,j}$. The pricing kernel is observed by investors but not by the econometrician. Assets are priced using the formulas in Proposition 1. Estimates are obtained from data on the fundamentals as well as six financial variables using the EM-SMM methodology described in Appendix B. Standard errors are in parentheses. The middle panels report the quarterly and annual *implied* transition probability matrix between the four states. Rows may not sum to one due to rounding. The bottom panel reports implied asset prices for the calibrated. The V/E ratio and bond yields are computed as shown in Proposition 1. Spreads for defaultable bonds of ten-year maturity are computed using Equation (22), using the conditional solvency ratio in state i as $Z_i = (C_i)/(C \cdot \pi^*) k$ (see Footnote 13 for an explanation).

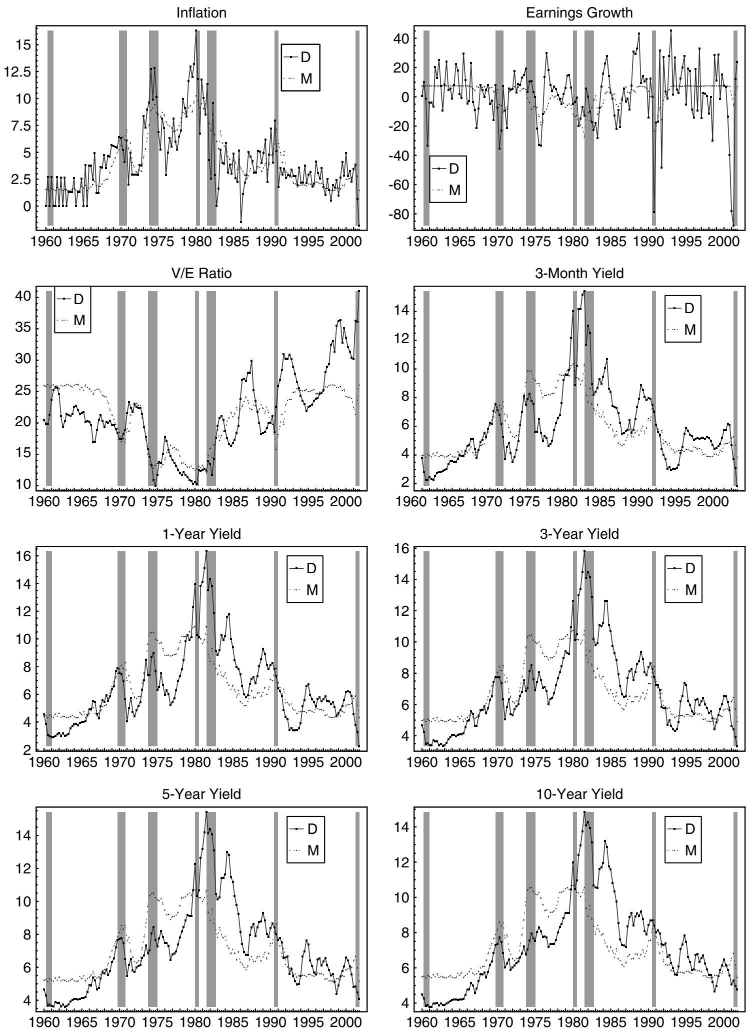

Figure 2
Fundamental and financial variables: empirical and model fitted (1957–2001)
We display the time series of filtered probabilities of investors of the four possible underlying states obtained using the SMM methodology in Appendix B. The states are numbered as (β_1, θ_2), (β_2, θ_1), (β_2, θ_2), and (β_3, θ_1); the parameter estimates are in Table 1.

which has a chi-squared distribution with six degrees of freedom, is 9.23, implying a p-value larger than 16%, so we fail to reject our model.

We next turn to the kernel parameter estimates, the fourth set of parameters in Table 1. We notice immediately that α_θ and α_β are both significantly positive, which means that the real rate is higher in states of faster growth of real fundamentals, as well as states of higher inflation.

Ceteris paribus, we will have therefore lower V/E ratios and higher Treasury yields in such periods. These observations are confirmed in the bottom panel, which reports the implicit parameters for the V/E and bond yields across the states using the pricing formulas for stocks and bonds in Equations (11) and (13), respectively.

As we will see in the next section, the market prices of risk in our model play an important role in determining the relation between the credit spreads and the equity premium puzzles. Notice that $\sigma_{M,1}$ and $\sigma_{M,2}$ are both positive. By Proposition 3 we know that the volatility of the asset value with respect to the inflation shock is negative, but with respect to earning shock is positive (it is straightforward to verify from Table 1 that the inflation and V/E state vectors are negatively related as is needed in this proposition), which implies that the risk premium associated with inflation shocks is *negative,* and with earnings shocks is *positive.* The negative covariance between expected inflation and expected returns for stocks has been noted by several authors (see, e.g., Fama 1981 or Stulz 1986). While our model is not consumption based, our interpretation is that positive shocks to inflation occur during periods of weak earnings growth and therefore investors are willing to accept a negative risk premium to bear them. Positive earnings shocks occur in periods of strong real fundamentals and investors therefore require a positive risk premium to bear these. Finally, the price of risk of the kernel shock and its volatility are both negative, leading to a positive (but small) risk premium.

Besides the kernel parameters, the transition probabilities also have a large effect on V/Es. Consider the two states of medium inflation growth. If there were no regime switches, the V/E computed using Proposition 1 would be around 6.6 and 16.7 in the low- and high-growth rate states, respectively. However, with the estimated switching frequencies, the V/Es are 11.8 and 13.3; that is, the differences in V/E are to a great extent eliminated. This is of course due to the instability of high growth in the medium inflation state. The implied parameters also show why the slope of the term structure is another proxy for this signal. There is an extensive literature documenting the forecasting power of the slope of the term structure (see, e.g., Estrella and Mishkin 1998). In our calibrated model, the slope is positive in the LI-HG and MI-LG states, but is negative in the MI-HG and MI-HG states, in each of which the three-month rate is in double digits. From the transition probability matrix we see that in each of these latter states there is a large probability of low growth in the following quarter. Therefore, in the model the slope is a proxy for investors' assessed risk of the economy shifting to a low-growth state.

The time series of the filtered probabilities are shown in Figure 3. The 1960s and 1990s were decades when the low-inflation and high-earnings states prevailed. There were several switches within the medium inflation states in the 1970s, with two bouts of high inflation and low

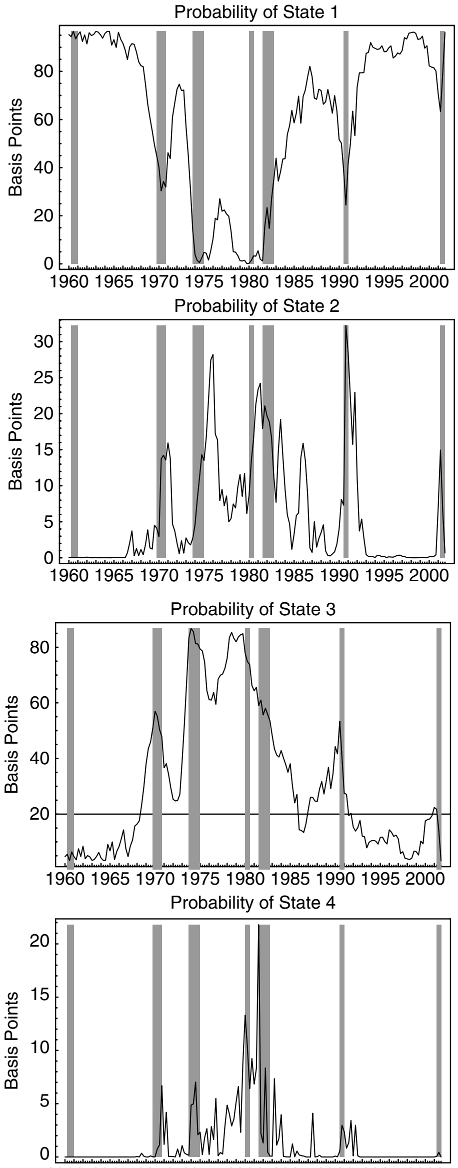

Figure 3
Conditional probabilities of four states (1957–2001)
We display the time series of financial and fundamental variables series used in the SMM estimation of the model, and their fitted values. The calibrated values of the parameters are shown in Table 1. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 3. As a measure of the fit of our model to the data, we provide the R^2 of the following regressions: $y(t) = \alpha + \beta x(t) + \epsilon(t)$, where y is the variable under consideration and x is its expected value based on model: Inflation −68.9%, Earnings −17.5%, V/E Ratio −49.8%, 3-Month Treasury −50.4%, 1-Year Treasury 51.0%, 3-Year Treasury 47.6%, 5-Year Treasury—45.4%, and 10-Year Treasury 43.6%.

growth in the mid-1970s and early 1980s. The mid- to late 1980s were characterized by high probabilities of medium inflation and high growth, before the low-inflation boom years of the 1990s. Among more recent dates, by late 1999 the state of medium inflation and high growth had become a likely possibility, which was followed by a spike of the medium-inflation and low-growth probability in the spring of 2000, causing a severe drop in the model-implied V/E. These filtered beliefs and the above implied parameters are used to generate time series of model-implied time series of fundamentals and financial variables in Figure 2. We report the explanatory power of our model for these variables in the footnote to this figure. As suggested earlier, we are much more successful in measuring the state of inflation (R^2 of 69%) than real earnings growth (R^2 of 16%). Among financial variables, the fit for the V/E ratio is about 50%, although notably, the model failed to fit the high valuations of the late 1990s. The fits for historical yield series have R^2s of between 44% and 51%, with the better fits for the short maturities. Despite the changes in regimes, the model fails to match the high yields during the early 1980s when the Federal Reserve experimented with monetary policy, although the fits are quite accurate outside this period. The β coefficients of each variable are significant at the 1% level and are all close to one (not reported).

4. The Credit Risk Convexity Effect and the Credit Spreads Puzzle

For constructing model-based spreads of a given rating category, we take the approach that the rating agencies adopt the "through-the-business-cycle" approach, which was described in the introduction, to assign firms to a given rating category based on their average solvency ratio through good and bad times. The time path of the solvency ratio is constructed as described in Section 3.1. The spread at time t is generated using the calibrated π_t series as $s(Z_t, \pi_t, t, T)$ in Equation (22).

As seen in Figure 4, default probabilities under the *risk-neutral measure* change quite significantly with the state of fundamentals. We plot the ten-year default probabilities for various levels of the long-run solvency ratio, k, conditional on having perfect knowledge of fundamentals being in each of the four possible composite states. In the plot, we set $Z_i = C_i/(C \cdot \pi^*)\, k$.[13] Default probabilities for each given level of the solvency ratio shift dramatically with the state of inflation, and only slightly with the state of earning growth. Take the case where $k = 2.5$: in the LI-HG state, the default

[13] Implicit in this calculation is that the firm's value at time t is $V_t^Q = C_i E_t Q_t$. Its debt level at t to be in a rating category with a long-run solvency ratio of k is determined by $\frac{(C \cdot \pi^*) E_t Q_t}{D_t^Q} = k$. Now substituting for $Z_t = V_t^Q / D_t^Q$ gives the stated level of the conditional solvency ratio. Alternatively, this is the level of the solvency ratio that the firm would have at t starting with $Z_0 = k$ and the processes for the asset value and debt as specified.

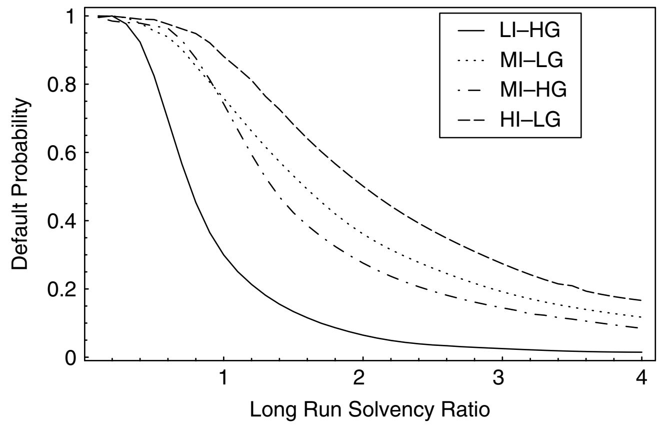

Figure 4
Conditional risk-neutral probability of default in the four states (1957–2001)
This figure shows the risk-neutral probabilities of default at the ten-year horizon for different values of the firm's long-run solvency ratio, k, and conditional on the investors knowing with certainty each of the four states of fundamentals: State 1, low inflation–high growth, State 2, medium inflation–low growth, State 3, medium inflation–high growth, and State 4, high inflation–low growth. The default probabilities are computed as in $\Pi_2(Z_t, \pi_t, t, T)$ in Proposition 5, where the solvency ratio in state i is $Z_i = (C_i)/(C \cdot \pi^*)\, k$ (see Footnote 13 for an explanation).

probability is about 3.5%; in the MI-LG and MI-HG states it is around 26% and 19%, respectively; and in the (HI-LG) state it is about 37%. Using the unconditional probabilities of the four states, π^* as shown in Table 1, we obtain an unconditional average of the default rate of a firm with a long-run solvency of 2.5 to be about 13.5%. The average default probability under the *objective measure* is lower at 4.6% due to the effect of the positive asset risk premium of about 7.3% that increases the drift of the solvency ratio.

For our calibration exercises in the following section, we generate a model-implied Baa–Aaa spread series using the firm's average solvency ratio of $k = 2.5$ in each state. The choice is based on the average leverage of BBB (equivalent to Moody's Baa rating) firms as reported in Standard and Poor's 1999 publication *Corporate Ratings Criteria.* For bonds with ten-year maturities we generate a credit spread of 31 bps in the LI-HG state, which balloons to over 257 bps in the stagflation state. In the "normal" recession MI-LG state, it averages 205 bps, while in the state of medium inflation but still solid growth it is 157 bps. Using the stationary distribution over states we obtain an unconditional average of the credit spread of 106 bps. By comparison, the Baa–Aaa spread in the sample from 1960–2001 was about 97 bps, while the Baa less ten-year Treasury spread is larger at 172 bps.

A substantial part of the spread is attributed to a *credit risk convexity* effect, which will arise in any other model with a time-varying solvency ratio. To illustrate the credit risk convexity effect in a simple and easily verifiable way, we adapt the BSM model to our structural assumptions of debt growth at the rate $r - \delta$. We incorporate a proportional bankruptcy cost parameter, α, to enable the model to fit historical recovery rates as well; building this feature into our model is straightforward.[14] For example, fixing the asset value-to-debt ratio to 2.5, choosing the asset-volatility parameter to equal 21.9%, which is the average volatility endogenously generated as in Equation (15) using the calibrated belief process, we obtain a credit spread for a bond with a ten-year maturity of 46 bps for the case $\alpha = 0$. Next, if we use our calibrated stationary probabilities and asset V/E ratios in the four states in the bottom panel of Table 1, then using the formula $Z_i = C_i/(C \cdot \pi^*)2.5$ we obtain solvency ratios of 3.34, 1.60, 1.80, and 1.24. Calculating the spread using these initial ratios, and then averaging using the stationary probabilities, we obtain an average spread of 73 bps, which is 59% higher than the spread using the average solvency ratio. Finally, if we use a market price of risk of 0.36 (this is the Sharpe ratio obtained for the asset value process in our model), we obtain an average default probability in the four states under the *objective measure* of 3.6%, which is 0.9% lower than the average ten-year default probability of BBB bonds as reported by Standard and Poor's (and used in the calibration by HH).

The intuition behind the result is straightforward: the credit spread is *convex* in the solvency ratio. The top panel of Figure 5 illustrates the point.

[14] The bond price for this case is given by

$$P_t = e^{-r\cdot(T-t)} \cdot \left[N(d_{2t}) + (1-\alpha)Z_t(1 - N(d_{1t}))\right],$$

$$d_{1t} = \frac{\log Z_t + \sigma^2/2 \cdot (T-t)}{\sigma \cdot \sqrt{T-t}}, \quad d_{2t} = d_{1t} - \sigma \cdot \sqrt{T-t},$$

and the credit spread is $s_t = -1/T \log(P_t) - r$. Since $\mu - r = \sigma_m \sigma$, where μ is the drift of the asset value process under the objective measure, and σ_m is the market price of risk of the single shock with positive volatility, the default probability under the objective measure is $1 - N(d_2^{OM})$, where $d_{jt}^{OM} = d_{jt}^{OM} + \sigma_m \cdot \sqrt{T-t}$, for $j = 1, 2$. The recovery rate conditional on default and the expected loss per dollar of debt under the objective measure are given by

$$R_t^{OM} = (1-\alpha)Z_t e^{\sigma_m \sigma (T-t)}(1 - N(d_{1t}^{OM}))/(1 - N(d_{2t}^{OM})), \text{ and}$$

$$L_t^{OM} = (1 - N(d_{2t}^{OM})) \cdot (1 - R_t^{OM}).$$

For computing risk premiums, the volatilities of bonds and equities are

$$\sigma_{P,t} = \frac{\partial P_t}{\partial Z_t}\frac{Z_t}{P_t}\sigma \quad \text{and} \quad \sigma_{E,t} = \frac{1 - \frac{\partial P_t}{\partial Z_t}}{1 - \frac{P_t}{Z_t}}\sigma,$$

where

$$\frac{\partial P_t}{\partial Z_t} = e^{-r\cdot(T-t)}\left((1-\alpha)(1 - N(d_{1t})) + \frac{\alpha}{Z_t \sigma \sqrt{T}}\right).$$

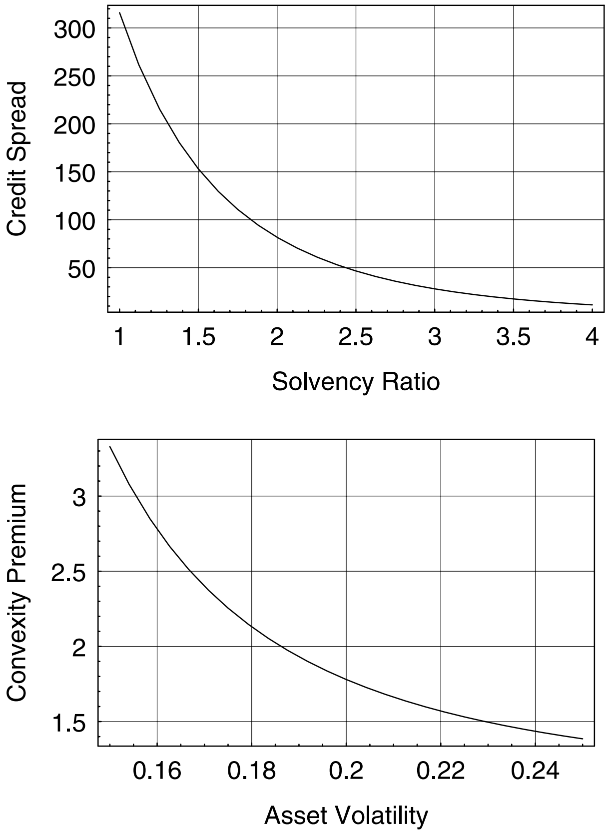

Figure 5
The convexity effect of solvency ratios and the credit spreads puzzle
The top panel shows credit spreads for bonds with ten years to maturity assuming an asset volatility parameter of 21.9% and bankruptcy costs of zero in the Black–Scholes–Merton model, as shown in Footnote 14. The bottom panel shows the *convexity premium* defined as the ratio of the average credit spread in the four states to the credit spread at the average solvency ratio of 2.5. The average spread is calculated using the solvency ratios in the four states defined as $Z_i = C_i/(C \cdot \pi^*)2.5$ (see Footnote 13), where the V/E ratios, C, and stationary probabilities, π^*, are in in the bottom panel of Table 1. Using the formula, we obtain solvency ratios of 3.34, 1.60, 1.80, and 1.24 in the four states, respectively.

When solvency ratios vary over time, the average spread is higher than the spread at the average solvency ratio. HH in all their calibrations always report the latter. As such, the amount of the spread they justify by credit risk is too low. In our model, debt grows at a steady pace while asset valuations vary more with changing investors' beliefs of the state of the economy, which cause the solvency ratio to fluctuate. The "through-the-ratings" cycle approach implies that rating agencies do not change companies' ratings based on such fluctuations. In calibrating structural form models for bonds within a given rating category, therefore, the econometrician too must account for changes in the solvency ratio over time.

The convexity effect of changing asset valuations and solvency ratios is inversely proportional to asset volatility because for higher levels of volatility, spreads are less sensitive to the initial solvency ratio. We note that in their calibrations HH use higher levels of asset volatilities, of about 26% for BBB-rated firms. In this case, the increased spread due to the convexity effect is lower, at 42%. In the right panel of Figure 5, we calculate the ratio of the average spread to the spread at the average solvency ratio—which we call the *convexity premium*—for the case where the solvency ratios and their probabilities are given as in our calibrated model above. As another example, a firm with asset volatility of 16%, and the same asset valuation ratios as above, would have an average spread 2.7 times higher than the spread at the average solvency ratio.

In computing model-implied spreads, HH not only matched the historical default probability under the objective measure, but also recovery rates. By having nonzero bankruptcy costs, we are able to match both estimates. Computing again the average spread with a time-varying solvency ratio as calibrated above, we find that with a market price of risk of $\sigma_m = 0.322$, volatility of 21.7%, and bankruptcy costs of $\alpha = 0.35$, we match the historical ten-year default probability of 0.045, the recovery rate of 0.51 cents per dollar, and obtain a credit spread of 131 bps, more than twice as high as HH found in most models. Eom et al. (2004) use estimates of α in this range and point to recent empirical evidence supporting such values. Finally, incorporating bankruptcy costs (assumed zero in our model) will further increase the spread by another 50–70 bps.

It is important to point out that the convexity effect approximated from the BSM model with changing solvency ratios was able to boost the credit spread to 73 bps, while our average model spread is higher, at 106 bps. A large proportion of the remainder is due to the fact that asset volatility is stochastic and negatively covaries with asset valuations (the former is higher and the latter is lower in higher inflation states). This further boosts credit spreads in low valuation states and lowers them in higher valuation states, enhancing the convexity effect. To quantify this effect, we first find that the average volatility forecast at the ten-year horizon, which is relevant for pricing, in the four states is 0.178, 0.318, 0.232, and 0.299, respectively.[15] Using the BSM model with the solvency ratios specified earlier and these volatility forecasts, and averaging using the stationary probabilities, provides a spread of 96 bps. That is, this effect explains an incremental 23 bps. This effect also increases the average default probability under the risk-neutral measure to 4.6%, at about the observed level for BBB bonds. It is also important to note that the effect of stochastic

[15] Let $\overline{v}(t, T, \pi_t) = E[\int_t^T ||\sigma_V^Q(\pi_s)||ds]$ be the forecast of volatility to the maturity of the bond. Then $\overline{v}(t, T, \pi_t)$ follows the PDE: $0 = -\overline{v}_t + ||\sigma_V^Q(\pi)|| + \overline{v}_\pi\,(\mu(\pi) - \rho(\pi)) + \frac{1}{2}\sigma(\pi)\sigma(\pi)'$. We solve this PDE with projection methods similar to the solution of the fundamental PDE in Equation (19).

volatility on its own is small, raising spreads to only 54 bps when used with a constant solvency ratio of 2.5. Also notice from the bottom panel of Figure 1 that estimated solvency ratios in our model vary in a slightly wider range than in the approximation here, which uses only the four mean states so that the convexity effect is larger in our model by the remaining 10 bps.

It is also of interest to separate the impact of the real rate and signaling effects of inflation on average credit spreads. To achieve this, we fit the model by constraining $\alpha_\beta = 0$, and in this case find that the fitted V/E ratios display smaller variation than the general case. Solvency ratios in the model are also less volatile, thereby lowering the convexity effect. By repeating the analysis above, we find that the average spread for Baa bonds is 16 bps lower (90 bps rather than 106 bps). This implies that most of the impact of changes in expected inflation on average spreads is through the signaling role of inflation as opposed to its real rate effect.

4.1 Nonequivalent variation in expected default losses and credit spreads

In the analysis so far we have shown that the convexity effect that arises from the state-dependence of solvency ratios and asset volatilities in our model is consistent with expected losses at historical levels and credit spreads at empirically observed levels. In this section, we analyze the three following issues that shed further light on how this convexity effect helps to reconcile the credit spreads and equity premium puzzles. First, what inputs is the econometrician free to choose in matching low default losses and high spreads? Second, what is the role of state dependence of the inputs of the structural form model? Finally, are models that lead to a high equity premium also consistent with high credit spreads?

To facilitate the discussion, following CCG, we find it fruitful to write the bond price in our model as

$$\begin{aligned} P_t &= E_t\left[\frac{N_T}{N_t}\ \left(1 - \mathbf{1}_{Z_T<1}(1 - Z_T)\right)\right] \\ &= E_t\left[\frac{N_T}{N_t}]E_t[\left(1 - \mathbf{1}_{Z_T<1}(1 - Z_T)\right)\right] \\ &\quad + \mathrm{Cov}\left[\frac{N_T}{N_t}, 1 - \mathbf{1}_{Z_T<1}(1 - Z_T)\right]. \end{aligned} \tag{24}$$

The first term is the value of discounted expected losses, which are pinned down from data on historical defaults in model calibration. The second term shows that the bond price will be lower (and hence spreads higher) in models where the covariance of the pricing kernel and the solvency ratio is smaller at the ten-year horizon. Therefore, the discussion of raising spreads while controlling for default losses can be focused around this covariance term.

We first discuss the degrees of freedom the econometrician has in matching both historical default losses and credit spreads. Consider the case where the econometrician holds the firm's solvency ratio and asset volatility fixed for a given rating category. By changing the market price of risk, the econometrician can control the model's expected default losses by altering the drift of the asset value process under the objective measure. However, having matched its historical average, the econometrician has no further room to alter the covariance term. We immediately notice this difference between the credit spreads and equity premium puzzles: in the latter, the market price or risk is unconstrained, but in the former it is used to pin down the model's expected losses from default under the objective measure. In formulating the credit spreads puzzle, HH have correctly constrained the average solvency ratio and asset volatility by using historical data. If, for example, the constraint on a fixed level of average asset volatility is removed, obtaining higher credit spreads is no longer as challenging: by increasing the volatility parameter, credit spreads in the model increase, and by simultaneously increasing the price of risk, expected default losses are maintained at historical levels. High spreads can similarly be obtained by lowering the average solvency ratio and increasing the price of risk. In our analysis, the only variable we vary freely is the price of risk, while solvency ratios and asset volatilities are endogenous functions of the price of risk. With this single degree of freedom, we study the ability of the model to match historical averages of solvency ratios, asset volatilities, historical default losses, and credit spreads.

Our strategy of varying the price of risk to match moments of financial and real variables is analogous to changing investors' preference parameters in general equilibrium complete market models. In such models, $\sigma_m = \gamma\sigma_x$, where γ is the coefficient of relative risk aversion (CRRA), and σ_x is the volatility of aggregate consumption. Starting with the classic paper of Mehra and Prescott (1985), various papers have changed the market prices of risk by keeping σ_x fixed, but varying the CRRA. By raising the CRRA, the econometrician can match the observed equity premium. While we have chosen to not restrict our analysis to be within the complete markets-general equilibrium framework, we can back out the level of the CRRA to be consistent with our calibrated market price of risk in this framework.[16]

[16] In our analysis, the vector σ_M is calibrated to match the valuation ratios of equities and bonds and leads to an average Sharpe ratio on equities of 0.32. In a single shock general equilibrium framework with aggregate consumption volatility of around 2%, we would require investors to have a CRRA of 16 to obtain a Sharpe ratio of this magnitude. One other way to obtain a high Sharpe ratio is to assume different preferences such as external habit formation that conditionally raises investors' CRRA for a low level of γ as in Campbell and Cochrane (1999). With these preferences, the average CRRA that justifies the Sharpe ratio on equities is above 45. David (2008) shows that another way to obtain a high Sharpe ratio is to model speculation among agents that leads to high per capita consumption volatility with a CRRA of only 0.5.

In attempting to calibrate to higher spreads, the econometrician can choose to ignore the state dependence of solvency ratios by evaluating spreads at their historical average or choose to estimate their average values in N different states. To isolate the impact of the convexity effect that arises from the state dependence, we will continue to use the BSM pricing formulas for spreads and expected losses in Footnote 14 with the assumption that there are N observable states with distinct solvency ratios $Z_i = C_i/(C \cdot \pi^*)k$ (see Footnote 13) and asset volatility, which arise from investors' learning in our model. Let $\overline{Z} = \sum_{i=1}^{N} \pi_i^* Z_i$ and L_H be the average historical expected default losses per dollar of debt. Expected default losses combine the effects of default probability and loss conditional on default. We also denote the functions $s = s(\overline{Z})$, $\overline{s} = \sum_{i=1}^{N} \pi_i^* s(Z_i)$, $L^{OM} = L^{OM}(\overline{Z})$, and $\overline{L}^{OM} = \sum_{i=1}^{N} \pi_i^* L^{OM}(Z_i)$.

To illustrate the impact of solely changing the prices of risk, we hold all other parameters constant at their estimated values. Firms' solvency ratios and asset volatilities are formulated endogenously as functions of the prices of risk, as shown in Propositions 1 and 2. In the remainder of this section we simply refer to these two functional relationships as "endogeneity." As in our model, we now use a three-dimensional vector of market prices of risk, σ_M. However, owing to our chosen orthogonalization in Assumption 2, the V/E ratio in Proposition 1 only depends on $\sigma_{M,2}$, the price or risk of the earnings shock, and we will only consider its variations. We also assume in this approximation that the variations in $\sigma_{M,2}$ have no influence on the time series of investors' beliefs. For the large values of $\sigma_{M,2}$ considered, the signal-to-noise ratio in investors' kernel is low and we find this to be a reasonable assumption. Average asset volatility across the four states is approximated as $0.45 \cdot ||\sigma_V^Q(\pi^*)||$, where π^* is the vector of stationary probabilities in Table 1 and σ_V^Q is in Equation (15). The factor 0.45 is used because σ_V^Q is a concave function of beliefs, and hence average volatility (from our fitted model) is lower than the volatility at the stationary probabilities. Forecasted volatility in the four states is then approximated by scaling this average volatility using the optimal forecasts for our calibrated model described in Footnote 15. The scaling factors for volatility in the four states are 0.81, 1.45, 1.06, and 1.37, respectively. The drift of the solvency ratio under the objective measure is $\sigma_V^Q(\sigma_M + \sigma_Q)'$. Given these relationships, the econometrician attempts to find a $\sigma_{M,2}$ to match average historical default losses.

There are three effects of an increase in $\sigma_{M,2}$ on s, $\overline{s}$, L, and $\overline{L}$.

1. Asset volatility with respect to the earnings shock (which contributes to a positive risk premium) is nearly insensitive to $\sigma_{M,2}$, while asset volatilities with respect to the inflation and kernel shocks are both

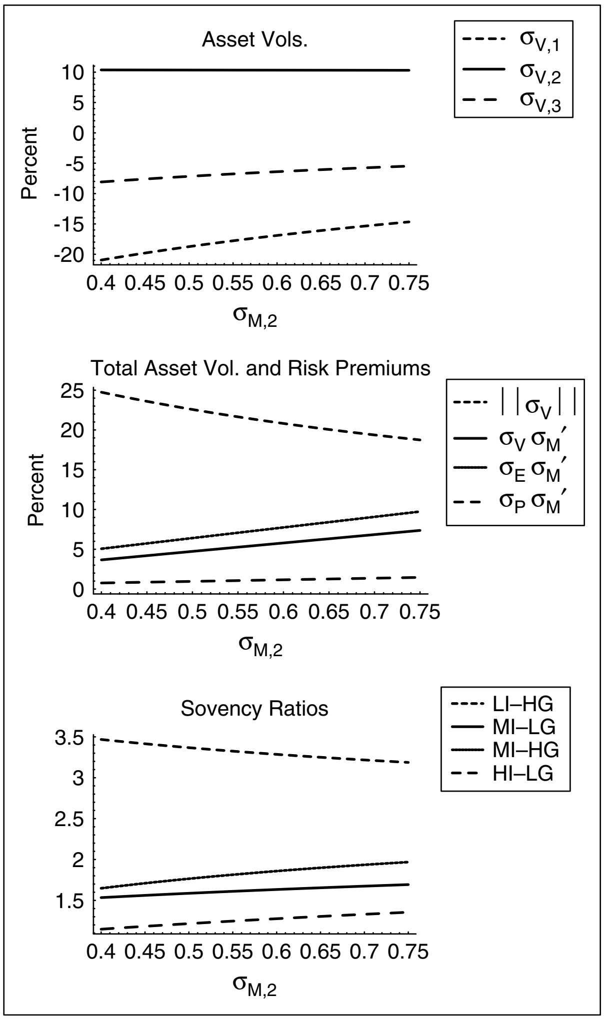

Figure 6
Asset volatility, risk premiums, and solvency ratios as functions of $\sigma_{M,2}$
Asset volatilities are plotted as functions of $\sigma_{M,2}$ as in Equation (15). We plot their averages across the four states as approximated by $0.45 \cdot \sigma_V^Q(\pi^*)$, where $\sigma_V(\pi_t)$ is in (15) in the top panel. The middle panel plots $||\sigma_V(\pi^*)||$ as well as the risk premiums on the asset value, equities, and bonds, as shown in Footnote 14. The bottom panel shows the solvency ratios in the four states given by $Z_i = C_i/(C \cdot \pi^*)2.5, i = 1, \ldots, 4$ (see Footnote 13) as functions of $\sigma_{M,2}$. The V/E ratios, C, are in Proposition 1(a). All parameter estimates (besides $\sigma_{M,2}$) and the stationary probabilities, π^*, are in Table 1.

negative and decline in absolute value as $\sigma_{M,2}$ increases (Figure 6, top panel). With a higher price of risk, investors discount the future more heavily, leading to less volatile asset valuations. Proposition 3 provides intuition for the signs of these volatilities. The cumulative effect is a decline in total asset volatility (Figure 6, center panel) that lowers s, $\overline{s}$, L, and $\overline{L}$.

2. A mean-preserving decline in spread of the solvency ratios in the S states (Figure 6, bottom panel) that lowers $\overline{L}$ and $\overline{s}$ but leaves L and s unchanged. Therefore, the convexity effect on default losses and credit spreads both decline when the price of risk is raised.
3. The drift of the nominal asset value under the objective measure, $\sigma_V^Q(\sigma_M + \sigma_Q)$, *increases.* This happens because the negative risk premium from the inflation shock declines more rapidly than the positive risk premium from the earnings shock as $\sigma_{M,2}$ increases (see Section 3.2 for the signs of the prices of risk). This lowers L^{OM} and $\overline{L}^{OM}$, but leaves s and $\overline{s}$ unchanged. In addition, the real risk premium on the asset value, $\sigma_V \sigma'_M$, equities, $\sigma_E \sigma'_M$, and defaultable bonds, $\sigma_B \sigma'_M$, all increase (Figure 6, middle panel).

We obtain two major results using these inputs: First, strikingly, credit spreads are decreasing in the price of risk of the earnings shock, while all premiums are increasing. Therefore, the credit spreads puzzle cannot be resolved simply by raising the market price of risk. Second, the expected (under the objective measure) default losses are more sensitive to changes in $\sigma_{M,2}$ than credit spreads—that is, there is nonequivalent variation in the two measures. This ensures that the convexity effect remains positive even with the endogeneity described. Intuition for the result can be obtained from Figure 7. As can be seen, L^{OM}, $\overline{L}^{OM}$, s, and $\overline{s}$ are all decreasing functions of $\sigma_{M,2}$. However, since the third effect only affects the functions computed under the objective measure, these decline faster than the two credit spread functions. Consider an increase in $\sigma_{M,2}$ from $\sigma^\star_{M,2}$ to $\overline{\sigma}^\star_{M,2}$. This increase in the price of risk is just sufficient to ensure that $\overline{L}(\overline{\sigma}^\star_{M,2}) = L(\sigma^\star_{M,2}) = L_H$. However, $\overline{s}(\overline{\sigma}^\star_{M,2}) > s(\sigma^\star_{M,2})$, since the decline in the credit spread is less rapid. The implication is that if the econometrician accounts for the state dependence of the solvency ratio, then calibrated credit spreads will be higher, even though the econometrician raises the market price of risk sufficiently to lower expected default losses to their historical average level. The amount $\overline{s}(\overline{\sigma}^\star_{M,2}) - s(\sigma^\star_{M,2})$ is the convexity effect with the endogeneity. It is smaller than the convexity effect $\overline{s}(\overline{\sigma}^\star_{M,2}) - s(\overline{\sigma}^\star_{M,2})$, which would result if the econometrician ignores the endogeneity. For example, using the calibrated parameters for Baa-rated bonds for our model, the former effect accounts

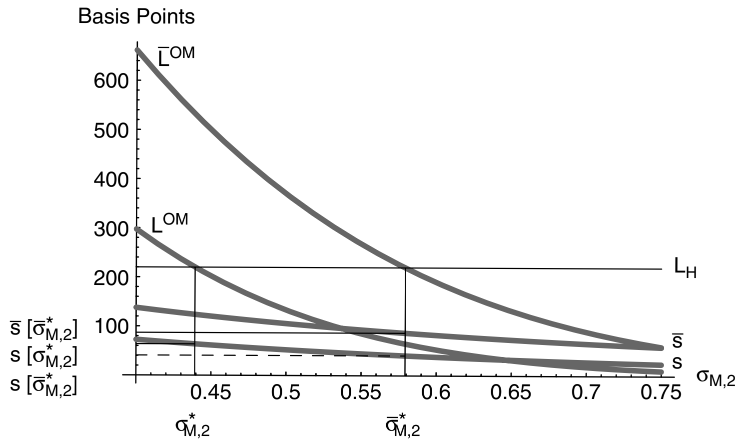

Figure 7
The convexity effect with endogenous solvency ratios and asset volatilities
Credit spreads and expected default losses under the objective measure are computed for bonds with ten years to maturity and long-run solvency ratios of 2.5 using the Black–Scholes–Merton model, as shown in Footnote 14. The average spread is calculated using the solvency ratios in the four states, asset risk premiums, and average volatilities, as described in Figure 6. Forecasted volatility in the four states is approximated by scaling this average volatility using the optimal forecasts for our calibrated model, described in Footnote 15. The scaling factors for volatility in the four states are 0.81, 1.45, 1.06, and 1.37, respectively. Average historical expected default losses per dollar of debt is L_H. s and L^{OM} are computed at the average solvency ratio, while $\bar{s} = \sum_{i=1}^{N} \pi_i^* s(Z_i)$ and $\overline{L}^{OM} = \sum_{i=1}^{N} \pi_i^* L^{OM}(Z_i)$. The drift of the nominal asset value under the objective measure is $\sigma_V^Q(\sigma_M + \sigma_Q)'$. An econometrician who accounts for the endogeneity of solvency ratios and asset volatilities would find a convexity effect of $\bar{s}(\bar{\sigma}^{\star}_{M,2}) - s(\sigma^{\star}_{M,2})$, while one who ignores the endogeneity would find a convexity effect of $\bar{s}(\bar{\sigma}^{\star}_{M,2}) - s(\bar{\sigma}^{\star}_{M,2})$.

for 29 bps, while the latter for 46 bps. To put these into perspective, note that the historical Baa–Aaa spread in our sample is 96 bps.[17]

Returning to the relation between the credit spreads and equity premium puzzles, we note that $\sigma_{M,1}$ and $\sigma_{M,3}$ have been held fixed in our analysis in this subsection, and using the implied $\sigma_{M,2}$ and equity volatilities in this section, we obtain equity premia of 7.3 and 5.1% and Sharpe ratios of 0.32 and 0.22 for the cases with and without state dependence. Empirical estimates of the Sharpe ratio for equity indices range between 0.3 and 0.5 (see, e.g., Cochrane 2001). Therefore, allowing for state dependence of solvency ratios and asset volatilities has an added advantage of providing a reconciliation of the high Sharpe ratios on equities and high credit spreads.

[17] While we did not reestimate our model for alternative rating categories, we used the parameter estimates for the Baa case above and calibrated the solvency ratios to match their average values in HH. We then approximate the convexity effects with exogenous and with endogenous solvency ratios and volatilities at 32 and 16 bps points for the A rating, and 21 and 10 bps for the Aa rating, respectively. The historical spreads over Aaa bonds are 60 and 28 bps, as reported in HH.

4.2 The time series of credit spreads and the credit spreads volatility puzzle

Returning to our model's results, one naturally questions if movements in the aggregate solvency ratio and asset volatility lead to spread movements that are consistent with time series of observed credit spreads. Using the calibrated solvency ratio series, $\{Z_t\}$ and beliefs of the states, $\{\pi_t\}$, we use Equation (22) to generate the time series of model spreads, $\mathrm{CSM}(t)$. The historical Baa–Aaa spread obtained from Moody's and the fitted series are in Figure 8 and are used in the regressions below:

$$\begin{aligned} \mathrm{CS}(t) &= \underset{(0.535)}{0.0306} + \underset{(17.537)}{0.887}\mathrm{CSM}(t-1) \qquad R^2 = 0.642, \end{aligned} \tag{25}$$

where t-statistics are in parentheses. We make the following summary comments: (i) The model spreads explain about 64% of the variation in the historical spread. While some readers have questioned the role of inflation in recent years, the fits in the first and second halves of the samples explain 54 and 72% of the variation, respectively (regressions not shown). (ii) The insignificant intercept term follows from the fact that in our model there is no credit spreads puzzle. (iii) We have used the one-quarter lag of the model spread. Highly significant regressions are also obtained with the contemporaneous regression; however, the fit one quarter ahead is better, suggesting that investors do not fully adjust to all incoming news on inflation instantaneously. (iv) Several authors in the past have presented the regressions in changes of spreads, due to the possible nonstationarity of historical spreads. We find that the evidence for nonstationarity of the spread is not strong.[18] While we present only level regressions here, we have also obtained very significant results for returns on corporate bond indexes, which are more tightly linked to the theory than changes in spreads (these are available upon request). In addition, there has been a recent resurgence in understanding the level of spreads (see Eom et al. 2004 and Huang and Huang 2003).

The credit risk convexity effect outlined above is related to the volatility of credit spreads, since more volatile spreads enhance this convexity. We find that the standard deviation of the historical Baa–Aaa spread is about 45 bps, and the model-fitted spread in Figure 8 has a standard deviation of 43 bps. Therefore, the changing solvency ratio and asset volatility in our model are able to address the credit spreads volatility puzzle raised in CCG, who point out that for several credit risk models in which the solvency ratio is held constant in the calibration exercise, the predicted volatility of spreads is too low (around 10 bps). In addition, using our

[18] The estimate for the autoregressive parameter using the Augmented Dickey–Fuller procedure (Dickey and Fuller 1979) with no time trend is 0.914, with a t-statistic of -2.667. The 5 and 10% critical values of the t-statistic are -2.876 and -2.568, respectively. The estimate for the autoregressive parameter using the Perron procedure (Perron 1989) is 0.909, with a t-statistic of -2.810. The 10% critical value is -3.130.

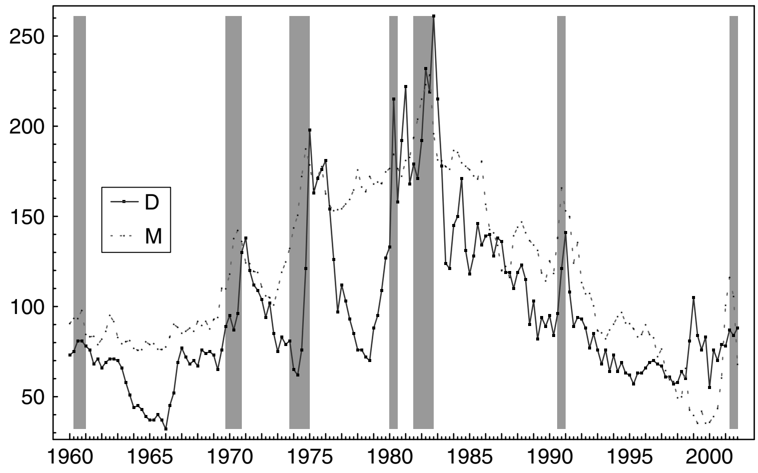

Figure 8
Moody's Baa–Aaa yield spread and its model based value (1960–2001)
The plot shows the time series of Moody's Baa–Aaa yield spread and its model-based counterpart. Model spreads are calculated using the "through-the-business-cycle" approach: The spread at time t is generated using the calibrated π_t series as $s(Z_t, \pi_t, t, T)$ in Equation (22), where the calibrated solvency ratio series, $\{Z_t\}$, and belief series, $\{\pi_t\}$, are shown in Figures 1 and 3, respectively. The model series explains 65% of the variation in the data series for the full sample. It also explains 54 and 72% of the variation in the first and second halves of the sample, respectively.

bond pricing formula and calibrated beliefs but always using the solvency ratio of 2.5 leads to a standard deviation of only 7 bps.

Since both the level and volatility of credit spreads are related to the volatility of the solvency ratio, it is important to ascertain if empirical solvency ratios have been as volatile as required by our model to generate a convexity effect of the required magnitude. Our estimated solvency ratio series is shown in the bottom panel of Figure 1. We compare its variation to that of a similarly constructed series in CCG for the inverse of the solvency ratio, the leverage ratio. CCG report that the variation of the annual book leverage ratio series from 1975 to 1998 is 9%. Restricting to this subsample we find the volatility of our implied leverage ratio is similar at 9.2%. For the full sample, the volatility is 10.1%. The autocorrelation coefficient of the leverage ratio is 0.86, so that as for the credit spreads series, it does not have a unit root. When decomposing the convexity effect in our model above, we noted that the statistics of leverage ratios are not sufficient to determine the moments of credit spreads. In addition, the variation of forecasted asset volatility and its covariation with the leverage ratio are also needed. When the covariation is positive, as in our model, the convexity effect is larger. To shed further light on the roles of time variation of the leverage ratio and asset volatility, we examine the empirical distributions of credit spreads and the relevant state variables below.

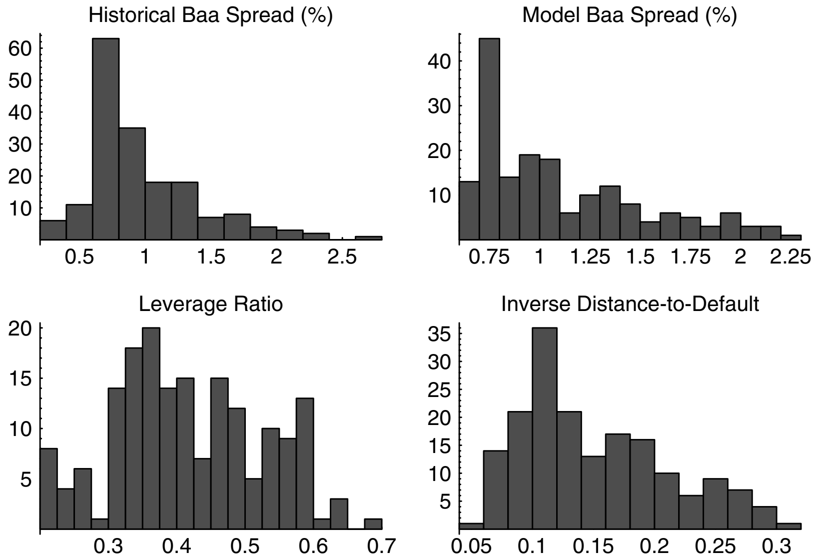

Figure 9
Empirical distributions of credit spreads, leverage ratio, and the inverse distance-to-default (1960–2001)
The top panels show the empirical distributions of Moody's Baa–Aaa yield spread and its model-based counterpart that is generated as described in the note to Figure 8. The bottom left panel shows the empirical distribution of the implied leverage ratio (the inverse of the solvency ratio) that is displayed in the bottom panel of Figure 1. The bottom right panel shows the distribution of the Inverse Distance-to-Default, which equals the forecasted asset volatility divided by the solvency ratio. Forecasted asset volatility for our calibrated model is calculated as described in Footnote 15.

The top panels of Figure 9 show that the empirical distributions of the historical credit spread and our model-fitted series are both positively skewed with skewness coefficients of about 1.3. The positive skewness arises from the widening of spreads in medium- and high-inflation states. The bottom left panel shows that the leverage ratio displays a smaller level of skewness (skewness coefficient of only 0.11). The right panel shows that the distribution of the inverse distance-to-default (forecasted asset volatility divided by the solvency ratio) in our model has a level of positive skewness comparable to that of credit spreads. The distance-to-default is a widely used statistic in the industry to measure the credit risk of firms (see, e.g., Crosbie and Bohn 2002) as it incorporates the effects of declining solvency ratios, as well as increasing volatility during bad times. This double impact plays a role in boosting our model average spread to empirical levels, as reported earlier.

4.3 Credit spreads and macroeconomic recessions

As can be seen in Figure 8, credit spreads increase during economic recessions (see also Duffie and Singleton 2003). This might suggest that a two-state observable regime switching model for credit spreads, such as in

Hackbarth et al. (2006), might be adequate to capture most of the variation in credit spreads.[19] However, such a model provides limited success in fitting historical spreads: the regression of the historical Baa–Aaa spread on the NBER recession indicator yields an R^2 of only 9.6%, which might suggest that macroeconomic factors overall play a small role in explaining the time variation in credit spreads. In contrast, once we explicitly build the forecasting effect of inflation into investors' information sets, the fit improves to 64%, as noted in the previous subsection.

An additional point of interest is to test whether credit spreads can be used to forecast recessions or the reverse, whether recessions forecast spreads. We find that the historical spread does have limited forecasting power, but its effect is insignificant once we include the model-based spread. A standard Granger Causality test of the null hypothesis of our model spread not causing NBER recessions with four lags yields an F-statistic of over 11 with a p-value of the order 10^{-8}, which means the null of no causality is strongly rejected. The p-value of the reverse test is 0.69. In this sense, our model spread leads recessions.

We attribute the forecasting power of the model-based spread to the signaling role of inflation, since the slope of the term structure, which is another proxy for this role, provides similar results. High-growth-rate states are less stable in higher inflation states, causing asset valuations to fall and asset volatility to rise in periods of rising inflation expectations. Using a similar model, David and Veronesi (2006) find that measures of inflation uncertainty forecast stock and Treasury bond volatilities up to two years ahead. The latter are known to be strongly contemporaneously related to recessions (Schwert 1989).

4.4 Related work on the credit spreads puzzle

As seen above, our key argument in addressing the credit spreads puzzle is to explicitly incorporate the cyclical variation of credit risk into the calibration exercise. This feature was ignored by authors such as HH and Delianedis and Geske (2001). We now look at the work of other authors in addressing the credit spreads puzzle and determine the role of this cyclical variation in these alternative explanations.

We start with CCG's analysis of the CC habit-formation model. In their calibration exercise, CCG assume that the solvency ratio of a typical Baa-rated firm is constant at each period in their sample. Therefore, they do not account for the impact of cyclical variation in asset valuations

[19] Their model provides some key insights into financing and default policies of firms through the business cycle. However, it is well known that market participants are unsure in real time of the state of the economy. Therefore, it would be hard for firms to adjust leverage and default policies in real time as advocated by HMM. The current article explicitly builds in investors' uncertainty of the state of the economy. Moreover, higher uncertainty itself can affect the level of spreads due to higher volatility in these times.

on solvency ratios. However, they build in alternative channels that cause time variation in the credit risk of firms. In particular, in the CC model, the market price of risk is countercyclical due to variations in the surplus consumption ratio. In addition, the countercyclicality of the volatility of the surplus consumption ratio causes asset volatility to be countercyclical. Both channels lower the covariance term between the pricing kernel and solvency ratios in Equation (24); that is, they make it more likely that default losses occur during periods of high marginal utility and hence increase credit spreads. However, CCG find that while the CC model spread is higher and more volatile than in a simple benchmark model that holds the price of risk and asset volatility constant, it fails to match both moments of historical data. In addition, the habit-formation model implies that risk premiums are countercyclical, and hence, counterfactually, that the probability of default under the objective measure is lowest in recessions. CCG are able to substantially improve the performance of the CC model in both respects by postulating a default boundary that increases more rapidly than debt in recessions for all firms. The incorporation of this countercyclical time-varying boundary in effect causes larger systematic time variation in the credit risk of firms within the rating category and leads to more defaults in bad times.

On a similar note, CCG find that, owing to a relatively low duration of corporate bonds relative to equities, neither the low-frequency time variation in the growth rate to fundamentals nor the stochastic consumption growth volatility in the long-term risks model of BY can themselves lead to higher credit spreads. However, the BY model in addition has a countercyclical price of risk, which when combined with the other features leads to some improvement. Therefore, the BY model shares with our model and the CC model the common feature that the cyclical variation of the asset risk premium plays a role in increasing credit spreads, particularly in bad times.

Without questioning the validity of the various channels that are required to provide large credit spreads in the models analyzed in CCG, we summarize below how our analysis is different.

1. The CC model requires larger variation in the default boundary than the leverage ratio. While volatility and risk premiums are higher in recessions in our model as well, solvency ratios are endogenous and fall in these periods, thereby avoiding the procyclicality of default probabilities.
2. The economic mechanism that gives rise to countercyclical asset volatility in our model is quite different from that in CC. Volatility in the CC model is countercyclical and is driven by the volatility of the surplus–consumption ratio that is specified *exogenously*. CC explain that the model is "reverse-engineered" to obtain a countercyclical

stock volatility and risk premium.[20] In our work, the countercyclical asset volatility arises endogenously from investors' learning about the state of fundamentals.

3. Both the BY and CC model require a time-varying price of risk. Although we have used constant market prices of risk, we generate countercyclical asset risk premiums from the variation in asset volatility.

Cremers et al. (2006) generalize the class of models that are tested in HH by allowing for both firm-specific and systematic jumps in the asset values of firms. The novelty of their calibration exercise is to infer the size of the jump risk premia from the prices of S&P 100 options. The large systematic jump mean under the risk-neutral measure as opposed to the objective measure is able to reconcile the high spreads and the low expected default losses, while incorporating idiosyncratic jumps can control the clustering of defaults of individual firms. While the paper does not conduct a time-series analysis of credit spreads, it admits to some difficulty in matching the stylized fact that the jump risk premium implicit in options prices increases sharply after the stock market crash of 1987 without a corresponding increase in credit spreads.

In other work, Tang and Yan (2006) are able to justify large default-related credit spreads in a flow-based structural form model. Defining default as a shortfall of cash flow places different constraints on the empirical exercise from value-based structural form models. Longstaff et al. (2006) calibrate a reduced form model to spreads in the credit default swaps market and support the view that default risk accounts for the predominant component of spreads.

5. Conclusion

We study the credit spreads puzzle: the inability of structural form credit risk models to reconcile low historical averages of losses from default and their high credit spreads for bonds *within specified rating categories.* Since both sets of averages are conditioned on ratings, this article suggests that the rating methodology followed by the rating agencies should affect the way an econometrician evaluates the performance of structural form models in their ability to match historical data. An implication of the "through-the-cycle" rating procedure used by rating agencies is that credit risk for firms within a given rating category will fluctuate in

[20] In fact, if the volatility of the surplus consumption ratio is held constant, then Li (2007) shows that the asset volatility in the CC model is counterfactually procyclical.

response to macroeconomic shocks. Large firm-specific shocks, on the other hand, will cause the agencies to move the firm into an alternative rating category so that such shocks do not affect the time-series averages for the specified rating category. Looking at historical data with this perspective, the econometrician must recognize that the credit quality of all firms within a rating category weakens at certain stages of the business cycle and in these periods their credit spreads increase. Similarly, during periods preceding strong periods of growth, credit spreads decrease. Since credit spreads are convex in the solvency ratio, this state dependence of credit risk leads to higher average spreads than the spreads if the solvency ratio and asset volatility were held constant at their historical averages. In earlier work, authors such as HH ignore this cyclical variation of credit risk while some others, such as CCG, only account for it partially.

In addition to causing state dependence, macroeconomic shocks carry risk premiums so that they lead to nonequivalent variation in the averages of expected default losses, which are computed using objective probabilities, and credit spreads, which are computed under risk-neutral probabilities. Firm-specific shocks cause equivalent variation in credit spreads and expected default losses and therefore cannot increase credit spreads once we control for expected default losses. Our modeling choice in this article of having only macroeconomic shocks is predicated on our goal of characterizing the state dependence in credit risk of the representative firm of a given rating category.

We estimate the size of the effects of the state dependence on historical averages using an unobservable regime-switching framework where investors' changing expectations of the state of future real fundamental growth causes endogenous state dependence of solvency ratios of firms and their asset volatilities. High real growth is unstable in higher inflation states, causing asset valuations and solvency ratios to fall and asset volatility to rise rapidly. The average spread produced by the model is about 60% larger than the spread computed at the average solvency ratio by several authors—the credit risk convexity effect. The negative relation between asset valuations and volatility enhances this convexity, causing spreads to increase a further 50%. An econometrician who ignores the state dependence of the solvency ratios requires too low a risk premium to keep expected default losses at their historical levels, making it more difficult to match the high equity premium and high credit spreads simultaneously despite varying the market prices of risk.

A natural extension of this article is to take the implications for credit spreads to individual-firm-level data. It holds promise since recent work by Duffie et al. (2007) has revealed that interest rates are highly significant in predicting defaults even in the presence of several key explanatory variables such as leverage ratios.

Appendix A.:

Proof of Proposition 2. Let $i_Q = (1,0,0)$, $i_E = (0,1,0)$, and $i_M = (0,0,1)$. These are useful since we can write $\sigma_Q = i_Q \Sigma$, for example. Notice that the following covariances hold:

$$\begin{aligned}
dE\, dQ' &= E\, Q\, \sigma_Q \sigma_E',\\
d\pi_i d\pi_j' &= \pi_i \pi_j (\nu_i - \overline{\nu})' \left(\Sigma\Sigma'\right)^{-1} (\nu_j - \overline{\nu}) dt,\\
dE d\pi_i' &= E i_E \Sigma \Sigma^{-1} (\nu_j - \overline{\nu}) \pi_i\, dt = (\theta_i - \overline{\theta}) \pi_i E,\\
dQ\, d\pi_i' &= Q i_Q \Sigma \Sigma^{-1} (\nu_j - \overline{\nu}) \pi_i\, dt = (\beta_i - \overline{\beta}) \pi_i Q.
\end{aligned}$$

(a) Recall that the real asset value is $V_t = (\sum_{i=1}^N C_i \pi_i)\, E_t$. Hence, from Ito's lemma:

$$\begin{aligned}
dV &= \sum_{i=1}^N C_i \pi_i dE + E \sum_{i=1}^N C_i d\pi_i + \sum_{i=1}^N C_i dE d\pi_i'\\
&= V \left(\overline{\theta} dt + \sigma_E\, d\widetilde{W} + \frac{\sum_{i=1}^N C_i [\pi\Lambda]_i}{\sum_{i=1}^N C_i \pi_i} dt + \frac{\sum_{i=1}^N C_i \pi_i (\nu_i - \overline{\nu})' (\Sigma')^{-1}\, d\widetilde{W}}{\sum_{i=1}^N C_i \pi_i} \right.\\
&\left. + \; \frac{\sum_{i=1}^N C_i \pi_i (\theta_i - \overline{\theta})}{\sum_{i=1}^N C_i \pi_i} dt \right) = V \left(\overline{\theta} + \frac{\pi \Lambda C}{\sum_{i=1}^N C_i \pi_i} + \frac{\sum_{i=1}^N C_i \pi_i (\theta_i - \overline{\theta})}{\sum_{i=1}^N C_i \pi_i} \right) dt\\
&+ \; V \left(\sigma_E + \frac{\sum_{i=1}^N C_i \pi_i (\nu_i - \overline{\nu})' (\Sigma')^{-1}}{\sum_{i=1}^N C_i \pi_i} \right) d\widetilde{W}. \qquad \text{(A.1)}
\end{aligned}$$

Using Equation (12), we can write $\pi\Lambda C = -1 + \pi\, \text{diag}(k - \theta)\, C + \pi C \sigma_E \sigma_M$. Therefore, the drift equals

$$\begin{aligned}
\mu_V - \delta(\pi_t) &= \overline{\theta} + \frac{\sum_{i=1}^N C_i\, \pi_i (k_i - \theta_i)}{\sum_{i=1}^N C_i\, \pi_i} + \sigma_M \sigma_E' + \frac{\sum_{i=1}^N C_i \pi_i (\theta_i - \overline{\theta})}{\sum_{i=1}^N C_i \pi_i} \qquad \text{(A.2)}\\
&= \frac{\sum_{i=1}^N C_i\, \pi_i k_i}{\sum_{i=1}^N C_i\, \pi_i} + \sigma_M \sigma_E' - \frac{1}{C.\pi_t}.
\end{aligned}$$

Turning now to the nominal value process, recall that the nominal asset value $V_t^Q = (\sum_{i=1}^N C_i \pi_i)\, Q_t\, E_t$. Hence, from Ito's lemma and the characterization of the fundamental processes under the observed filtration (9),

$$\begin{aligned}
dV^Q &=\\
&= \sum_{i=1}^N C_i \pi_i Q dE + \sum_{i=1}^N C_i \pi_i E dQ + EQ \sum_{i=1}^N C_i d\pi_i + \sum_{i=1}^N C_i \pi_i\, dE dQ'\\
&+ \; E \sum_{i=1}^N C_i d\pi_i dQ' + Q \sum_{i=1}^N C_i d\pi_i dE'\\
&= V^Q \left[\overline{\theta} dt \;+\; \sigma_E d\widetilde{W} \;+\; \overline{\beta} dt + \sigma_Q\, d\widetilde{W} \;+\; \frac{\sum_{i=1}^N\; C_i [\pi\Lambda]_i}{\sum_{i=1}^N C_i \pi_i} dt \right]\\
&+ \left[\frac{\sum_{i=1}^N\; C_i \pi_i (\nu_i - \overline{\nu})' (\Sigma')^{-1}\, d\widetilde{W}}{\sum_{i=1}^N C_i \pi_i} + \sigma_E \sigma_Q' + \frac{\sum_{i=1}^N\; C_i\, \pi_i (\theta_i - \overline{\theta})}{\sum_{i=1}^N\; C_i\, \pi_i} dt \right.
\end{aligned}$$

$$
\begin{aligned}
&+ \left.\frac{\sum_{i=1}^N C_i\,\pi_i(\beta_i-\overline{\beta})}{\sum_{i=1}^N C_i\,\pi_i}dt\right] \\
&= V^Q\left(\overline{\theta}+\overline{\beta}+\sigma_E\sigma'_Q+\frac{\pi\,\Lambda C}{\sum_{i=1}^N C_i\pi_i}+\frac{\sum_{i=1}^N C_i\,\pi_i\,(\theta_i-\overline{\theta})}{\sum_{i=1}^N C_i\,\pi_i}+\frac{\sum_{i=1}^N C_i\pi_i(\beta_i-\overline{\beta})}{\sum_{i=1}^N C_i\pi_i}\right)dt \\
&+ V^Q\left(\sigma_E+\sigma_Q+\frac{\sum_{i=1}^N C_i\,\pi_i\,(\nu_i-\overline{\nu})'(\Sigma')^{-1}}{\sum_{i=1}^N C_i\,\pi_i}\right)d\widetilde{W}.
\end{aligned}
$$

Now using the definition of $\mu_V(\pi_t)$ from (a), and the fact that $(\Sigma')^{-1}\,\sigma'_Q=(\Sigma')^{-1}\,\Sigma'\,i'_Q=i'_Q$, the drift equals

$$
\mu_V^Q(\pi_t)-\delta(\pi_t)=\overline{\beta}(\pi_t)+\mu_V(\pi_t)-\delta(\pi_t)+\sigma_E\sigma'_Q+\frac{\sum_{i=1}^N C_i\,\pi_i(\beta_i-\overline{\beta}(\pi_t))}{\sum_{i=1}^N C_i\,\pi_i}, \qquad (A.3)
$$

and clearly $\sigma_V^Q(\pi_t)=\sigma_V(\pi_t)+\sigma_Q$, which completes the proof.

(b) By Equation (A.2) we have

$$
\mu_V(\pi_t)-k(\pi_t)=\frac{\sum_{i=1}^N C_i\,\pi_i(k_i-\overline{k}(\pi_t)}{\sum_{i=1}^n C_i\,\pi_i}+\sigma_M\sigma'_E.
$$

From Equation (A.1) we have

$$
\begin{aligned}
\sigma_V(\pi_t)\,\sigma'_M &= \sigma_E\sigma'_M+\frac{\sum_{i=1}^N C_i\,\pi_{it}\,(\nu_i-\overline{\nu}(\pi_t))'(\Sigma')^{-1}}{\sum_{i=1}^N C_i\,\pi_{it}}\,\Sigma' i'_M \\
&= \sigma_E\,\sigma'_M+\frac{\sum_{i=1}^N C_i\,\pi_{it}\,(k_i-\overline{k}(\pi_t))}{\sum_{i=1}^N C_i\,\pi_{it}},
\end{aligned}
$$

which proves that the relationship for the real risk premium holds. Using it and Equation (A.3), we have $\mu_V^Q(\pi_t)-r(\pi_t)$

$$
\begin{aligned}
&= \mu_V(\pi_t)-\overline{k}(\pi_t)+\overline{k}(\pi_t)-\overline{r}(\pi_t)+\overline{\beta}_t(\pi_t)+\sigma_E\,\sigma'_Q+\frac{\sum_{i=1}^N C_i\,\pi_i(\beta_i-\overline{\beta}(\pi_t))}{\sum_{i=1}^N C_i\,\pi_i} \\
&= \sigma_V\,\sigma'_M+\sigma_N\sigma'_Q+\sigma_V\,\sigma'_Q=(\sigma_Q+\sigma_V)(\sigma_M+\sigma_Q)'=\sigma_V^Q\,\sigma'_N,
\end{aligned}
$$

which establishes the relationship for the nominal risk premium. Note that in the second equality we used $\overline{r}(\pi_t)=\overline{k}(\pi_t)+\overline{\beta}(\pi_t)-\sigma_N\sigma'_Q$, and $\sigma_V\sigma'_Q=\sigma_E\sigma'_Q+\frac{\sum_{i=1}^N C_i\,\pi_i(\beta_i-\overline{\beta}(\pi_t))}{\sum_{i=1}^N C_i\,\pi_i}$. ■

Proof of Proposition 3. Let $\pi_i^\circ=(\pi_i\quad C_i)/(\sum_{j=1}^N\pi_j C_j)$. Notice that $0\le\pi_i^\circ\le 1$ and $\sum_{j=1}^N\pi_j^\circ=1$. Further, let $\overline{\beta}^\circ=\sum_{j=1}^N\beta_j\pi_j^\circ$, and similarly define $\overline{\theta}^\circ$. Then, the asset volatilities in Equation (15) can be written as

$$
\begin{aligned}
\sigma_{V,1}^Q &= \sigma_{Q,1}+\frac{\overline{\beta}^\circ-\overline{\beta}}{\sigma_{Q,1}}-\frac{(\overline{\theta}^\circ-\overline{\theta})\,\sigma_{Q,2}}{\sigma_{Q,1}\,\sigma_{E,2}} \\
\sigma_{V,2}^Q &= (\sigma_{Q,2}+\sigma_{E,2})+\frac{\overline{\theta}^\circ-\overline{\theta}}{\sigma_{E,2}}
\end{aligned}
$$

$$\sigma^{Q}_{V,3} = -\frac{(\overline{\beta}^{\circ} - \overline{\beta})\,\sigma_{M,1}\,\sigma_{E,2} + (\overline{k}^{\circ} - \overline{k})\,\sigma_{Q,1}\,\sigma_{E,2} + (\overline{\theta}^{\circ} - \overline{\theta})\,(\sigma_{M,2}\,\sigma_{Q,1} - \sigma_{M,1}\,\sigma_{Q,2})}{\sigma_{M,3}\,\sigma_{Q,1}\,\sigma_{E,2}}.$$

Now, given the stated conditions on the drifts and the vector C, $\overline{\beta}^{\circ} \leq \overline{\beta}$, and $\overline{\theta}^{\circ} \geq \overline{\theta}$, and if all the volatilities are positive (note that $\sigma_{E,1} = 0$), then $\sigma_{V,1} \leq \sigma_{Q,1}$, $\sigma_{V,2} \geq 0$, and by $\sigma^{Q}_{V} = \sigma_{V} + \sigma_{Q}$, we get the signs on the real volatilities as claimed. ■

Proof of Proposition 4. The nominal market price of risk for any state variable is given by its covariance with the nominal pricing kernel of the economy. Therefore, using Equations (7) and (10) we obtain $\rho_i(\pi_t) = \mathrm{Cov}(d\pi_{it}, \frac{dN_t}{N_t}) = \pi_{it}(\nu_{it} - \overline{\nu}_t)' \cdot (\Sigma')^{-1} \cdot \sigma'_N$. Using the fact that $\sigma_N = (i_Q + i_M)\,\Sigma$, where $i_M = (0, 0, 1)'$, $i_Q = (1, 0, 0)'$, and $r_i = k_i + \beta_i - \sigma_N\,\sigma'_Q$, provides the statement in the proposition. ■

Proof of Corollary 1. (i) We appeal to Theorems 1 and 2 in Bergman et al. (1996). The solvency ratio $dZ_t = (\mu^{Q}_{V}(\pi_t) - r(\pi_t))dt + \sigma^{Q}_{V} d\tilde{W}_t$ and beliefs in Equation (6) have the same structure as Equations (2a) and (2b) in their paper. Notably different, though, the beliefs are N-dimensional, while their general state variable y was of single dimension. However, the authors explain their results are valid for the case when y is a vector. The crux of the proof is that, subject to certain regularity conditions verified below, for given realizations of the innovations $\{\tilde{W}_s\}$, and hence $\{\pi_s\}$, for $s \in (t, T]$, the risk-neutralized solvency ratio process in Equation (18) satisfies the "no-crossing" property, that is, starting at a higher level for Z_t would yield a higher solvency ratio at maturity.

The payoff at maturity of the defaultable bond is an increasing and concave function of Z_T. The bond price will be an increasing and concave function of Z_t if (a) $\mu_i(\pi_t)$ and $\sigma_i(\pi_t)$, the drift and volatility for the conditional probability of state i for $i = 1 \cdots N$ do not depend on the level of Z_t, and (b) the covariance between percentage changes in Z_t and each conditional probability process, $\sigma^{Q}_{V}\,\sigma_i(\pi_t)'$, does not depend on the level of Z_t. These sufficiency conditions are easily verified for each updating process in Equation (6), and the volatility of Z_t in Equation (15).

(ii) To establish the convexity of the spread, let $g(x, y) = \log(f(x, y)^{-1/T})$, where $f(x, y)$ is a positive function of scalar x and a vector y of dimension N, with $\frac{\partial^2 f(x)}{dx^2} < 0$. Then, $\frac{\partial^2 g(x)}{dx^2} = \frac{1}{T f^2}\left[\left(\frac{\partial f}{dx}\right)^2 - f\,\frac{\partial^2 f}{dx^2}\right] > 0$. It follows from (i) that the credit spread which is convex, is Z_t. ■

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