

Biases in Decomposing Holding-Period Portfolio Returns

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A growing number of studies in finance decompose multiperiod portfolio returns into a series of single-period returns, using these to test asset pricing models or market efficiency or to evaluate the returns to investment strategies such as those based on momentum, size, and value–growth. We provide a formal analysis of the decomposition method. Crucially, we argue and present empirical evidence that some methods researchers use involve portfolios that nobody would seriously consider *ex ante*, that transactions costs associated with such portfolios make them poor investment vehicles, and that they can lead to spurious statistical inferences. (*JEL* G10)

Calculating portfolio returns over an investment period is a fundamental requirement in finance. A basic precept of this calculation is that the returns should measure the wealth effect to an investor who holds the portfolio over the investment period. Inferences drawn from an inaccurate calculation have little economic relevance. Tests of market efficiency based on inaccurate returns are potentially misleading. Many studies in finance make inferences about multi-month holding-period portfolio returns, based on decomposed monthly portfolio returns.¹ In doing this, researchers often employ a simplified decomposition, involving rebalancing the portfolio at the start of every month to equal weights. This simplification is unrealistic, as rebalancing monthly is a strategy that no investor seriously considers *ex ante* and furthermore involves prohibitive transactions costs. Importantly, this approach is inaccurate in isolating the investor wealth effect of the investment strategy under consideration and can result in spurious statistical inferences, as we show below.

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¹ This is consistent with Fama and French (1998): “[a]nnual returns suffice for estimating expected returns, but tests of asset pricing models (which also require second moments) are hopelessly imprecise unless returns for shorter intervals are used” (p. 1977).

In this article, we present a straightforward calculation of decomposed portfolio returns that preserves their buy-and-hold property, minimizing transactions costs, that measures the investor wealth effect, and that largely avoids market microstructure biases as discussed in Roll (1983), Blume and Stambaugh (1983), and Conrad and Kaul (1993). We examine the calculation of monthly portfolio returns from a multi-month holding period; the procedure generalizes naturally to other intervals.² Based on the rationale that these monthly returns should be the returns earned by an investor who holds the portfolio, we show that the monthly portfolio return in each holding-period month is a weighted average with the weight attached to each stock in the portfolio depending upon the stock's performance over previous holding-period months. This ensures that compounding the decomposed monthly portfolio returns yields the multi-month buy-and-hold portfolio return.

In contrast to the decomposed buy-and-hold approach we discuss, studies in the literature commonly employ a simplified method to compute the monthly portfolio returns over a multi-month investment horizon. For example, the simplified decomposition calculates the monthly return on an equally weighted portfolio in each month as the arithmetic average of individual stock returns in that month. This simplification involves rebalancing the portfolio to equal weights at the start of every holding-period month. In the less popular case of value-weighting, the simplified approach calculates the monthly portfolio return in each holding-period month as the weighted average of all individual stock returns in that month with each weight held constant at the value determined at the beginning of the holding period by each stock's market capitalization. While investors undoubtedly do revise their investment weights over time, irregular new information flows, fund flows, or liquidity requirements are as likely to determine these revisions as are regular reviews of optimal weights. Revisions in practice are unlikely to restore initial weights, especially with prohibitive transactions costs. Theoretically, rebalancing confounds the return on an investment strategy under examination with the effect of, and underlying reason for, portfolio revision.

Crucially, we show that rebalanced methods can lead to spurious statistical inferences. In particular, rebalancing results in an upward bias to finding a premium associated with size—increasing the probability of making Type I errors—and a downward bias to finding a momentum premium—increasing the likelihood of making Type II errors. In the

² Examples of studies that report portfolio returns for intervals other than monthly are Lakonishok, Shleifer, and Vishny (1994), who use annual portfolio returns from a long-term investment horizon of five years to examine contrarian investment strategies, Wermers (1999), who investigates the relation between mutual fund herding and stock prices and reports herding-classified portfolio returns in each of four post-formation quarters, and Bollen and Busse (2005), who use daily returns of decile portfolios from a holding period of one quarter to measure persistence in mutual fund performance.

results over the period 1951–2003 that we report below, the difference in mean and risk-adjusted monthly returns between the rebalanced and the decomposed buy-and-hold methods is generally significant especially for low-price, small, and loser stocks. The bias in mean monthly returns over a multi-month holding period can be as high as 0.694% per month (over 8% per year) and can lead to spurious statistical inferences. For example, the rebalanced method gives a significant size effect over the period 1951 to 2003, while the decomposed buy-and-hold method shows no size effect, and while the decomposed buy-and-hold method reveals a significant momentum profit of 1.017% per month over January 1978 to December 2003, the rebalanced method reduces this by 44% to an insignificant 0.569% per month. Consistent with rebalancing requiring more trading, adjusting for transactions costs has the greater effect in reducing the profitability of investment strategies using the rebalanced method. Realistic levels of transactions costs can result in the rebalanced method turning the overestimate of the size effect into an underestimate and generating a significantly negative momentum effect.

To show that rebalancing to equal weights is a common practice and not a minor technical issue, we searched recent issues of the *Journal of Finance*, the *Journal of Financial Economics*, and the *Review of Financial Studies* for examples of articles that use this method.³ Because of somewhat different orientations, the *Journal of Finance* publishes more papers using decomposed portfolio returns than either the *Journal of Financial Economics* or the *Review of Financial Studies*. We found 16 papers in the *Journal of Finance* over the 10-year period 1996–2005 that clearly used the rebalanced method, and seven papers in the *Journal of Financial Economics* and four in the *Review of Financial Studies* also using the rebalanced method over the five-year period 2001–2005. Table 1 lists these papers. There are many other papers that we cannot include in Table 1 because they fail to clarify their method for calculating portfolio returns.⁴ Authors who use the rebalanced method either ignore or are unaware of the associated bias and the recommendation in early studies to use buy-and-hold returns (Roll (1983), Blume and Stambaugh (1983)). By formalizing the calculation of single-period portfolio returns over a multiperiod holding horizon, we remind researchers of the biases that result from using rebalanced methods. Our aim is that researchers in future will clarify how they decompose long-horizon portfolio returns and will avoid the biases we discuss in this article.

³ Some earlier, influential studies also use the rebalanced method (see, for example, Chan, Hamao, and Lakonishok (1991), Jegadeesh and Titman (1993), and Lakonishok, Shleifer, and Vishny (1994)).

⁴ A strength of the papers in Table 1, which enables us to include them, is the clarity of their exposition. It is infeasible to replicate all the studies whose methods are unclear, but in Section 2 on empirical evidence, we report on some studies where we have replicated results to show that the papers present misleading inferences.

Table 1
Recently published papers in the Journal of Finance, the Journal of Financial Economics, and the Review of Financial Studies that use the rebalanced method

Article	Evidence
Panel A: Journal of Finance papers from 1996 to 2005 that use the rebalanced method	
Fama and French (1996)	"At the end of June of each year t (1963–1993), the NYSE stocks on COMPUSTAT are allocated to ten portfolios . . . Equal-weight returns on the portfolios are calculated from July to the following June" (Table 2, p. 61). ". . . the high E/P return (HE/P) is the average of the top three E/P decile returns [of the LSV sorts]" (p. 72).
Carhart (1997)	"The portfolios are equally weighted monthly so the weights are readjusted whenever a fund disappears" (Table 3, p. 64).
La Porta, Lakonishok, Shleifer, and Vishny (1997)	". . . returns are reported for 5 years after formation . . . Annual portfolio returns are obtained by equally weighting the returns on all stocks that belong to the portfolio . . . Portfolios are rebalanced to equal weights at the end of each year" (pp. 861–862).
Daniel, Grinblatt, Titman, and Wermers (1997)	"All quintile portfolios were rebalanced monthly" (p. 1053).
Wermers (1999)	"Equal-weighted, size-adjusted, quarterly abnormal returns are calculated for each of these ten portfolios during . . . the following four quarters. . . . For example, the return shown for portfolio B1 in the first quarter (quarter +1) represents the hypothetical size-adjusted quarterly return that would accrue to investing, on April 1, 1975, in an equal-weighted portfolio of stocks the funds most strongly buy as a herd during the first quarter of 1975, holding this portfolio until June 30, 1975, and then rebalancing to hold an equal-weighted portfolio of stocks" (p. 607).
Zheng (1999)	"I construct all trading portfolios at the beginning of each quarter . . . I hold these portfolios for three months. . . . For example, in order to construct the returns for portfolio 5 for July through September 1970, I first select funds with positive new money in the quarter that ends in June 1970. The July through September monthly returns of these selected funds are then weighted by their corresponding new money measure. The three weighted average numbers are the monthly returns earned by portfolio 5 for the three months desired" (p. 906).
Moskowitz and Grinblatt (1999)	"Ranking the 20 industries based on their L -month lagged returns, we form portfolios of the highest and lowest past performing industries, hold them for H months, and rebalance monthly" (p. 1269).
Nofsinger and Sias (1999)	"Monthly abnormal returns are calculated as the difference between the raw return for firm i in month t and the cross-sectional average return for firms in the same capitalization decile in month t . Capitalization deciles (breakpoints based on firms included in our sample) are formed annually at the beginning of each October" (fn. 10, p. 2271).
Shumway and Warther (1999)	"Returns are calculated as equal-weighted averages of monthly returns for all the stocks in the portfolio" (Table 4, p. 2371).
Wermers (2000)	"Every fund existing during a given calendar quarter (and having a complete data record) is included in the computation of that quarter's average net returns. . . . (TNA weights are updated at the beginning of each quarter). These quarterly . . . returns are compounded to give the quarterly rebalanced annual returns" (Table 1, p. 1662).
Lee and Swaminathan (2000)	"The portfolios are rebalanced each month" (Table 3, p. 2032).
Lamont and Polk (2001)	"If a firm exits from the CRSP database, we drop it from the portfolio" (fn. 2, p. 1701). This requires rebalancing portfolios during their 12-month holding period.
Conrad, Cooper, and Kaul (2003)	"We compute the average cross-sectional differences in returns based on the predictive variables and methods that are commonly used in the literature . . . and the monthly returns of the portfolios are calculated from July to June of the following year" (p. 1972).
Korajczyk and Sadka (2004)	"The portfolio is rebalanced on a monthly basis" (Table 4, p. 1057).
Sapp and Tiwana (2004)	"We compute monthly returns for the two sets of new-money portfolios using two portfolio-weighting schemes. First, we compute cash-flow-weighted returns for the portfolio using the cash flows realized during the previous quarter by the funds within the portfolio. Additionally, we compute equally weighted returns for the new-money portfolios" (p. 2610).

Table 1
(continued)

Article	Evidence
Cohen, Coval, and Pastor (2005)	"All estimators are constructed using the past 12 months performance record of each fund. Fund returns are then averaged within each of the 25 portfolios over months 1, 2, and 3 following portfolio formation. The three-month return series are linked across quarters to form a monthly series of returns on each portfolio, and the alphas of the resulting 25 return series are reported" (Table VIII, p. 1087).
Panel B: Journal of Financial Economics papers from 2001 to 2005 that use the rebalanced method	
Cohen, Gompers, and Vuolteenaho (2002)	"On June 30 of year t , . . . we set the portfolio weight for each stock equal to the year $t - 1$ cash-flow news . . . We compute monthly returns for this cash-flow-news portfolio from July 1 to June 30 of the next year ($t + 1$) and rebalance the weights every month" (p. 433).
Chan (2003)	"I form monthly equal-weighted portfolios of the winner and loser stocks . . . As an example, suppose we want to look at how good news affects returns over four months. At the end of each calendar month, we calculate the abnormal return for all stocks . . . We then average the abnormal returns for the calendar month across stocks to get the abnormal return on a portfolio" (pp. 228–229).
Hogan, Jarrow, Teo, and Warachka (2004)	" . . . we long the top return decile, short the bottom return decile, and hold this portfolio for six months. The portfolio is rebalanced monthly to account for stocks that drop out of the database" (p. 543).
Teo and Woo (2004)	"We hold the portfolios for one year, then reform them . . . Stocks that disappear during the course of the year are included in the equally weighted average until they disappear, then the portfolio weights are readjusted appropriately. That is, the portfolio weights are rebalanced to equal at the end of every month" (p. 374).
Gebhardt, Hvidkjaer, and Swaminathan (2005)	"In computing future portfolio returns, we include every firm that has . . . return data available for that month. Similar to Jegadeesh and Titman (1993), holding period portfolio returns are calculated as the equal-weighted average" (p. 660).
Nagel (2005)	"For all variables, portfolio boundaries are defined by quintile breakpoints . . . Returns in each portfolio are equally weighted and they are reported in percent per month" (p. 289).
Hanna and Ready (2005)	"We examine simple portfolio strategies, similar to those reported in both Fama and French (1992) and Haugen and Baker (1996), that hold only the stocks in a particular decile. We also consider strategies that are similar to those reported by Jegadeesh and Titman (1993)" (p. 483). "The results in Fama and French (1992), Jegadeesh and Titman (1993), and Haugen and Baker (1996) reflect simple averages of the monthly returns of the stocks in each portfolio. This calculation implies a portfolio strategy that maintains equal weights on all of the component stocks, which in turn implies monthly rebalancing. We report results for both equally weighted and value-weighted portfolios" (p. 501).
Panel C: Review of Financial Studies papers from 2001 to 2005 that use the rebalanced method	
Lakonishok and Lee (2001)	"We calculate the portfolio returns by equally weighting the returns of individual stocks. We rebalance the portfolios annually so that each stock starts with the same weight at the beginning of the period" (p. 97).
Chan, Chen, and Lakonishok (2002)	"For each of the resulting nine portfolios, equally weighted returns are calculated over the subsequent 12 months, and the process is repeated" (p. 1425).
Ahn, Conrad, and Dittmar (2003)	"We follow the method outlined in Jegadeesh and Titman (1993) for the construction of the momentum strategy payoffs" (p. 467). ^a
Bollen and Busse (2005)	"We sort funds each quarter . . . and form deciles of funds. We then examine the performance of the deciles the following period" (p. 576). "Table 2 shows the results in the post-ranking quarter. Note that we calculate the average both across funds and across time" (p. 577).

^a Ahn, Conrad, and Dittmar (2003) report virtually no momentum profits for the 3×6 strategy (three-month ranking and six-month holding). We examine the 3×6 momentum strategy using the same sample and period of Ahn, Conrad, and Dittmar (2003) and find that the decomposed buy-and-hold method yields significant momentum profits of 0.585% ($t = 2.86$) per month, but insignificant profits using the rebalanced method.

We begin by describing the computation of monthly portfolio returns over a multi-month holding period and discuss the biases from using rebalanced methods. We also examine the calculation of the monthly profits from a long–short portfolio over a multi-month investment horizon. This calculation differs from the commonly used monthly return difference between long and short portfolios, which does not reflect the underlying investor wealth effect. Subsequently, we present empirical results and provide examples from the literature of spurious inferences based on the rebalanced method, using the papers of Barber and Lyon (1997) and Chordia and Shivakumar (2002) as illustrations. Finally, we analyze the impact of transactions costs.

1. Decomposing Multi-month Holding-period Returns

In this section, we explain the procedure for calculating individual month portfolio returns over a multi-month holding period. We present the corresponding rebalanced methods and discuss the biases that arise. Finally, we describe methods of calculating monthly profits of a long–short strategy. Although we focus on monthly portfolio returns over a multi-month holding period, the methods in this section apply to other frequencies such as daily, weekly, and annual.

1.1 Decomposed buy-and-hold methods

An individual stock i 's return in month t is

$$r_{it} = \frac{P_{it} + D_{it} - P_{i,t-1}}{P_{i,t-1}}, \quad (1)$$

where P_{it} is stock i 's price per share at the end of month t , and D_{it} is stock i 's dividend per share with the ex-dividend date falling in month t .

Suppose an investor holds a standard portfolio of N stocks with weight w_i invested in stock i at the start of the holding period and $\sum_{i=1}^N w_i = 1$. To begin the analysis we ask, what is the investor's m -month buy-and-hold portfolio return given the monthly returns for each stock in the portfolio? The answer is

$$r_{P, 1 \rightarrow m} = \sum_{i=1}^N w_i (1 + r_{i1})(1 + r_{i2}) \cdots (1 + r_{im}) - 1, \quad (2)$$

where r_{it} ($t = 1, \dots, m$) is stock i 's return in month t . Equation (2) gives the portfolio return that the investor actually earns over the m -month holding period.⁵

In empirical tests, researchers often decompose the m -month buy-and-hold portfolio return into m monthly portfolio returns. For example,

⁵ We ignore transactions costs and other microstructure effects at this stage.

studies usually estimate asset pricing models such as the capital asset pricing model (CAPM) and the Fama–French three-factor model using monthly observations. Similarly, studies of momentum strategies over intermediate holding periods typically base their statistical tests on average monthly returns. Based on the principle that the return should equal the amount the investor earns, the monthly portfolio return over an m -month holding period is

$$r_{P1} = \sum_{i=1}^N w_i r_{i1}.$$

$$r_{P\tau} = \sum_{i=1}^N \frac{w_i \prod_{t=1}^{\tau-1} (1 + r_{it})}{\sum_{j=1}^N w_j \prod_{t=1}^{\tau-1} (1 + r_{jt})} r_{i\tau}, \quad \tau = 2, \dots, m; \quad (3)$$

Equation (3) follows from $\prod_{t=1}^m (1 + r_{Pt}) - 1 = \sum_{i=1}^N w_i (1 + r_{i1})(1 + r_{i2}) \cdots (1 + r_{im}) - 1$, namely, compounding the decomposed m monthly portfolio returns produces the m -month buy-and-hold return on the portfolio.

We can identify two special cases of Equation (3): the monthly returns on an equally weighted portfolio and on a value-weighted portfolio over an m -month holding period.

(i) *An equally weighted portfolio*

Equation (3) with $w_i = 1/N$ gives the return on an equally weighted portfolio in an individual month τ from an m -month holding period as

$$r_{P1}^{ew} = \frac{1}{N} \sum_{i=1}^N r_{i1}.$$

$$r_{P\tau}^{ew} = \sum_{i=1}^N \frac{\prod_{t=1}^{\tau-1} (1 + r_{it})}{\sum_{j=1}^N \prod_{t=1}^{\tau-1} (1 + r_{jt})} r_{i\tau}, \quad \tau = 2, \dots, m; \quad (4)$$

Note that for the first month of the holding period, the equally weighted portfolio return equals the arithmetic average of individual stock returns, but this is not the case from month two onwards. Specifically, the month- τ ($\tau = 2, 3, \dots, m$) portfolio return on an equally weighted m -month ($m > 1$) buy-and-hold investment strategy is the weighted average of the month- τ stock returns with the weight depending upon return performance over the previous holding-period months.

(ii) *A value-weighted portfolio*

Equation (3) with $w_i = MV_{i,0} / \sum_{j=1}^N MV_{j,0}$, where $MV_{i,0}$ is stock i 's market value at the beginning of the holding period, gives

the return on a value-weighted portfolio in an individual month τ from an m -month holding period as

$$r_{P\tau}^{vw} = \sum_{i=1}^N \frac{MV_{i,\tau-1}}{\sum_{j=1}^N MV_{j,\tau-1}} r_{i\tau}, \quad \tau = 1, 2, \dots, m. \quad (5)$$

Equation (5) shows that the month- τ return on a value-weighted portfolio over an m -month holding period is a weighted average of the month- τ returns on all stocks in the portfolio with the weights determined by the market values at the start of the month. Note that Equation (5) assumes market values are adjusted for capitalizations. Otherwise, Equation (3) with initial weights equal to relative market values gives the correct calculation.

1.2 Rebalanced methods

Instead of the calculations given by Equations (3)–(5), some studies calculate returns for individual months within a multi-month holding period using an investment strategy that involves monthly rebalancing. These procedures calculate each monthly portfolio return as a weighted average of stock returns in that month with the weight in all holding-period months held constant at the value determined at the beginning of the holding period. This gives the portfolio return in each month over an m -month holding period as

$$r(rb)_{P\tau} = \sum_{i=1}^N w_i r_{i\tau}, \quad \tau = 1, 2, \dots, m, \quad (6)$$

where rb denotes rebalanced.

For an equally weighted portfolio, which is the most popular approach, the portfolio return in each holding-period month is the arithmetic average of all stock returns in that month,

$$r(rb)_{P\tau}^{ew} = \frac{1}{N} \sum_{i=1}^N r_{i\tau}, \quad \tau = 1, 2, \dots, m. \quad (7)$$

For a value-weighted portfolio the portfolio return in each holding-period month is the weighted average of all stock returns in that month with the weight in all holding-period months equal to a stock's market value relative to the total market value of all stocks at the beginning of the holding period,

$$r(rb)_{P\tau}^{vw} = \sum_{i=1}^N \frac{MV_{i0}}{\sum_{j=1}^N MV_{j0}} r_{i\tau}, \quad \tau = 1, 2, \dots, m. \quad (8)$$

These rebalanced methods are inaccurate in reflecting investor wealth in individual months over a multi-month holding period, unless investors rebalance their portfolios back to the initial weights at the beginning of each month. The standard approach to measuring investment performance is to assume a passive, buy-and-hold strategy over the investment holding period. This isolates the performance of the investment strategy. Assuming instead that investors rebalance at regular discrete intervals to keep their initial portfolio weights fixed is unrealistic in practice and confounds the theoretical hypothesis under examination. Crucially, rebalancing can result in different inferences compared to using the approach of Equation (3).

Taking the equally weighted approach as an example, Equations (4) and (7) show that the bias of the portfolio return in month τ from the rebalanced method is

$$\text{Bias}_\tau = \sum_{i=1}^N \left\{ \frac{1}{N} E[\tilde{r}_{i\tau}] - E \left[\frac{\prod_{t=1}^{\tau-1} (1 + \tilde{r}_{it})}{\sum_{j=1}^N \prod_{t=1}^{\tau-1} (1 + \tilde{r}_{jt})} \tilde{r}_{i\tau} \right] \right\}, \quad \tau = 2, \dots, m. \quad (9)$$

Clearly, there is no bias if the returns on stocks in the portfolio are independent and have identical expectations. However, this is not the case in practice, and expected returns vary across individual stocks due to their different risk exposures. In general, the bias can be either positive or negative depending on the time-series properties of both the portfolio and individual stock returns.⁶ For illustration, consider the bias in the second holding-period month. After some rearrangement and using the approximation $1/(1 + \tilde{r}_{\tau-1}) \approx 1 - \tilde{r}_{\tau-1}$, which ignores higher order moments from the Taylor's series expansion, we can rewrite Equation (9) with $\tau = 2$ as

$$\text{Bias}_2 \approx E[\tilde{r}_1 \tilde{r}_2] - \frac{1}{N} \sum_{i=1}^N E[(1 - \tilde{r}_1) \tilde{r}_{i,1} \tilde{r}_{i,2}],$$

where $\tilde{r}_\tau$ is the average return in month τ of stocks in the portfolio.

Assuming $\tilde{r}_{\tau-1}$ is uncorrelated with individual stock returns, the bias for $\tau = 2$ is

$$\begin{aligned} \text{Bias}_2 &\approx E(\tilde{r}_1)E(\tilde{r}_2) + \text{Cov}(\tilde{r}_1, \tilde{r}_2) \\ &\quad - \frac{1}{N} \sum_{i=1}^N E(1 - \tilde{r}_1) [E(\tilde{r}_{i,1})E(\tilde{r}_{i,2}) + \text{Cov}(\tilde{r}_{i,1}, \tilde{r}_{i,2})]. \end{aligned} \quad (10)$$

⁶ There is no bias if the holding period is a single month. However, this involves frequent portfolio revision, which makes a strategy based on a single month holding period unattractive due to transactions costs. We provide evidence on this in the next section.

The two covariance terms show that the sign of the bias depends on the autocovariance in portfolio returns and on the average autocovariance in individual stock returns. Empirical evidence shows that portfolio returns are positively autocorrelated (Lo and Mackinlay (1990)), and Mech (1993) shows that transactions costs cause portfolio return autocorrelation by delaying price adjustment. Hence, positive autocovariance in portfolio returns can contribute to a positive bias in the rebalanced method. Research also shows that individual stock returns are negatively autocorrelated because of nonsynchronous trading (Fisher (1966)) or transactions costs and bid-ask spreads (Roll (1984), Jegadeesh and Titman (1995)). Lo and Mackinlay (1990) report negative average autocovariances for individual stocks, with the effect most pronounced in small stocks. We therefore expect to observe this negative serial correlation particularly in small and low-price stocks. For example, Liu (2006) shows that infrequently traded stocks are highly correlated with size and bid-ask spread. Branch and Freed (1977) and Conrad and Kaul (1993) find a negative relation between price and bid-ask spread. Therefore, we are more likely to find a positive bias in small and low-price stock portfolios. On the other hand, Kaul and Nimalendran (1990) find that stock returns are positively autocorrelated once they extract the bid-ask effect. Therefore, we may observe a negative bias in large and high price stock portfolios. A negative bias can also arise if expected stock returns are constant over time but vary cross-sectionally, as higher (lower) expected returns translate into higher (lower) expected weights in the bracketed term in Equation (9), while rebalancing to equal weights undoes this effect. Finally, Equation (10) shows that even if returns are independent, the bias is nonzero.

As an intuitive illustration of the above discussion, suppose an investor invests \$10 at the start of a two-month holding period in an equally weighted portfolio comprising non-dividend-paying stocks A and B, which have the same price of \$5 at the portfolio initiation (month 0). Consider two scenarios of stock price evolution over the two-month holding period.

Scenario 1: prices at the end of months 1 and 2 are \$6 and \$3 for A, and \$4 and \$7 for B.

Scenario 2: prices at the end of months 1 and 2 are \$6 and \$9 for A, and \$4 and \$1 for B.

For both scenarios, the decomposed buy-and-hold method gives zero returns for holding-period months 1 and 2, which reflects the real wealth of the investor because there is no change in the portfolio value in either month. In contrast, the rebalanced method yields a portfolio return for the second holding-period month of 12.5% in the case of Scenario 1 and -12.5% in the case of Scenario 2. If we assume that both A and B have zero expected returns, the upward (downward) bias resulting from the rebalanced method for the case of Scenario 1 (2) is consistent with the negative (positive) autocorrelations in individual stock returns.

It is clear that how these biases affect the returns to specific investment strategies is ultimately an empirical issue. To the extent that investment strategies in the research literature typically consider a long–short strategy in portfolios polarized on some variable hypothesized to predict return, the negative bias due to cross-sectional variation in expected stock returns may be a less serious issue. In contrast, given the nature of the microstructure effects involved, the biases arising from autocovariances in portfolio and individual stock returns are likely to be serious in low-price, small-stock portfolios. It is a well-known fact that many return anomalies are related to small and low-price stocks. For example, in considering a significance test of the size effect calculated as the difference between a long position in small stocks and a short position in large stocks over an m -month holding period, we show in the following section that the long portfolio (small stocks) largely determines the bias. Rebalancing to maintain equal weights as in Equation (7) places more weight on stocks in the small stock portfolio that have higher observed returns in the holding period and less weight on stocks that have lower observed returns in the holding period compared with Equation (4). This results in an upward bias to the returns to the long side and consequently to the difference between the long and short sides, overestimating the true size effect. The following section also shows the bias of rebalancing methods when testing momentum effects. Sorting on the past six-month return, the short side (i.e., past losing stocks that tend to be small) causes rebalancing to impart a downward bias to the estimated momentum effect. Results using rebalancing methods therefore contaminate the true underlying investment returns with measurement error due to market microstructure effects. The error can be a substantial component of the estimated return. In contrast, using the decomposed buy-and-hold approach that we present above minimizes these microstructure effects, enabling the researcher to focus on the true returns to the underlying investment strategy.

1.3 Profits of a long–short strategy

Empirical tests also frequently examine the profits in a strategy involving buying and selling equal amounts of two assets or portfolios. We refer to this as a long–short portfolio. To test the momentum effect, for example, the researcher forms a long–short portfolio by buying one dollar of past intermediate-term winning stocks and shorting one dollar of past intermediate-term losing stocks, and holds this position for 3–12 months. If this strategy produces consistently positive profits, this is evidence of a momentum effect. The principle of calculating long–short portfolio profits also applies, in a straightforward fashion, to calculating benchmark-adjusted portfolio returns for single periods within a multiperiod test interval, where the test portfolio corresponds to the long side and the benchmark portfolio corresponds to the short side.

So how should we calculate the monthly profits of a long–short portfolio over a multi-month holding period? Suppose we go long \$1 in portfolio or stock L for m months, and at the same time we short \$1 of portfolio or stock S for m months. The monthly profits of this long–short strategy over the m -month holding period are

$$AP_1 = r_{L,1} - r_{S,1},$$

$$AP_\tau = r_{L,\tau} \prod_{t=1}^{\tau-1} (1 + r_{L,t}) - r_{S,\tau} \prod_{t=1}^{\tau-1} (1 + r_{S,t}), \quad \tau = 2, \dots, m; \quad (11)$$

where $r_{L,t}$ is the return of portfolio or stock L in month t and $r_{S,t}$ is the return of portfolio or stock S in month t . Equation (11) follows from the expression for the m -month profit,

$$\sum_{t=1}^m AP_t = \prod_{t=1}^m (1 + r_{L,t}) - \prod_{t=1}^m (1 + r_{S,t}).$$

Equation (11) corresponds to a natural investment strategy because the initial investment of \$1 in the long and short sides increases or decreases with changes in the market values of the long and short portfolios.

However, empirical studies commonly calculate the profit in each holding-period month as the simple return difference between the long and short portfolios in that month,

$$AP(rb)_\tau = r_{L,\tau} - r_{S,\tau}, \quad \tau = 1, 2, \dots, m. \quad (12)$$

As we report in the text below, the monthly return difference between the long and short portfolios held for a multi-month period (with the portfolio monthly returns calculated using the decomposed buy-and-hold method) generally underestimates the true profits if the phenomenon under examination holds. More importantly, this monthly return difference does not correspond to a natural investment strategy over a multi-month holding period because it requires the investor to restore investment in the long and short portfolios to an equal weighting every month.

2. Empirical Evidence

This section provides empirical evidence on the biases from using rebalanced methods by examining two well-known anomalies, namely size and price momentum. The data are from the CRSP and COMPUSTAT merged (CCM) database. Our sample includes all NYSE/AMEX/NASDAQ ordinary common stocks over the period 1951–2003.⁷ Data on the

⁷ An earlier version of the paper also examined the book-to-market (B/M) effect, and data on B/M is available on the CCM database from 1951. Results on B/M show similar magnitudes of bias to those we report here for size and momentum, but do not result in spurious inferences. These results are available on request.

Fama–French market, size, and book-to-market risk factors are from Kenneth French’s website.⁸

The analysis involves ranking stocks into decile portfolios based on market capitalization (MV) or prior six-month buy-and-hold return (r_{-6}), using NYSE breakpoints. We denote the extreme decile portfolios as *S*(hort) and *L*(ong), where this designation indicates the subsequent investment strategy. The symbol *S* stands for the largest- MV decile or lowest- r_{-6} decile, and *L* for the smallest- MV decile or highest- r_{-6} decile. We denote intermediate decile portfolios as *D2* to *D9*. In examining the size effect we reform deciles annually and hold them for 12 months, similar to Fama and French (1992). In examining momentum, we follow the standard approach in the literature and sort eligible stocks into portfolios at the beginning of each month and hold these portfolios for six months. Momentum portfolios therefore involve overlapping ranking and holding periods on a monthly basis. The nonoverlapping monthly portfolio returns over the full testing period correspond to the technique of Jegadeesh and Titman (1993). To be eligible, stocks must have their MV or six-month returns available immediately before portfolio formation when we examine the MV or momentum effect. Also, we require one monthly return immediately after portfolio formation. These requirements ensure that we can form stocks into portfolios and can hold eligible stocks for at least a month. For a stock delisted in the holding period, we assume its post-delisting monthly returns are zero over the remaining holding period months. We find similar results when substituting the risk-free rate for the post-delisting returns.

2.1 The size effect

Table 2 reports the 12-month holding-period results of MV -classified decile portfolios. Looking at the results over the full sample period 1951–2003, the bias in rebalanced portfolio returns is immediately apparent. Using the decomposed buy-and-hold method, Panel A shows no significant size effect: the return difference between the smallest- MV portfolio (*L*) and the largest- MV portfolio (*S*) is 0.313% ($t = 1.65$) per month. The Fama–French three-factor model performs well against the decomposed buy-and-hold returns of the MV portfolios with the sole exception that the largest- MV portfolio loses significantly after adjusting for the three-factor model. Similar to the raw return result, there is no size effect after adjusting for the three factors. Of course, this result may come as no surprise as the three-factor model includes a size factor designed to remove the size effect. Therefore, in untabulated results, we also examine CAPM alphas and Fama–French two-factor alphas excluding the size factor *SMB*. These results are very similar to those of the three-factor model. Jensen’s alpha or the Fama–French two-factor alpha are only significant

⁸ The website is <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

Table 2
Holding-period performance per month of portfolios classified by MV

	S	$D2$	$D3$	$D4$	$D5$	$D6$	$D7$	$D8$	$D9$	L	$L-S$
Panel A: Decomposed buy-and-hold method with equation (4)											
Testing period: July 1951–June 2003											
Raw (%)	0.922 (5.39)	1.045 (5.95)	1.067 (5.65)	1.095 (5.72)	1.067 (5.45)	1.148 (5.58)	1.138 (5.28)	1.168 (5.30)	1.164 (5.07)	1.235 (5.22)	0.313 (1.65)
$\hat{\alpha}_{3F}$ (%)	-0.070 (-2.79)	-0.008 (-0.20)	-0.031 (-0.69)	-0.008 (-0.18)	-0.076 (-1.72)	-0.022 (-0.54)	-0.037 (-0.89)	-0.039 (-0.99)	-0.049 (-0.95)	0.044 (0.48)	0.115 (1.14)
Testing period: July 1951–June 1977											
Raw (%)	0.771 (3.44)	0.935 (4.03)	0.971 (3.88)	0.949 (3.66)	1.017 (3.85)	1.049 (3.85)	1.055 (3.70)	1.107 (3.70)	1.064 (3.47)	1.179 (3.50)	0.408 (1.60)
$\hat{\alpha}_{3F}$ (%)	-0.076 (-2.49)	0.012 (0.22)	-0.010 (-0.17)	-0.040 (-0.77)	0.002 (0.05)	0.000 (0.00)	0.011 (0.24)	0.005 (0.11)	-0.050 (-0.91)	0.028 (0.30)	0.104 (0.99)
Testing period: July 1977–June 2003											
Raw (%)	1.073 (4.16)	1.154 (4.37)	1.163 (4.10)	1.240 (4.41)	1.117 (3.86)	1.247 (4.04)	1.221 (3.78)	1.228 (3.79)	1.265 (3.70)	1.292 (3.89)	0.218 (0.78)
$\hat{\alpha}_{3F}$ (%)	-0.033 (-0.90)	-0.017 (-0.29)	-0.062 (-0.90)	0.026 (0.38)	-0.155 (-2.19)	-0.069 (-1.05)	-0.111 (-1.63)	-0.113 (-1.87)	-0.097 (-1.17)	0.033 (0.22)	0.066 (0.41)

(negatively) for the largest- MV decile and the size effect is insignificant after adjusting for either the CAPM or the two Fama–French factors (market and book-to-market).

In contrast, the rebalanced method leads to a different inference. Panel B shows that with rebalancing the estimated raw return difference between the smallest- MV decile (L) and the largest- MV decile (S) is 43% higher at 0.448% ($t = 2.34$) per month over the full sample period 1951 to 2003, suggesting a significant size effect. The three-factor model is also less successful in explaining the rebalanced portfolio returns. Seven out of ten rebalanced decile portfolios lose significance after adjusting for the three Fama–French factors, and the adjusted return difference between L and S is 0.197% ($t = 1.90$) per month, a figure that is 71% higher than its insignificant estimate from the decomposed buy-and-hold method. Untabulated results show that the CAPM is unable to explain the performance of L and S , and Jensen's alpha for $L-S$ is 0.451% ($t = 2.34$) per month, which is 43.8% higher than its decomposed buy-and-hold counterpart of 0.315% ($t = 1.65$). These results indicate that rebalancing can prevent an asset pricing model such as the CAPM or the three-factor model from carrying out its intended role. The results also imply that a size effect induced by rebalancing can create spuriously significant CAPM or three-factor alphas when using variables correlated with size to examine cross-sectional patterns in returns.

Inspecting the individual decile returns shows that rebalancing typically gives the *lower* estimated raw and three-factor-adjusted returns for decile portfolios S and $D2-D9$. This is consistent with our previous conjecture that a negative bias is possible within larger stock portfolios. But for the smallest stocks in the long portfolio this difference is reversed, with rebalancing *overstating* the buy-and-hold return and the difference being an order of magnitude larger than for the other nine decile portfolios. It is this that drives the difference between the two estimated size effects. Characteristics of the MV portfolios reported in Panel C of Table 2 show that price is monotonically decreasing from the largest- MV decile (S) to the smallest- MV decile (L) with L having a substantially lower average stock price than S . This is why we observe a large positive bias in the smallest- MV portfolio; low-price stocks are very sensitive to microstructure effects, as Ball, Kothari, and Shanken (1995) show.

The patterns in the subperiod results in Table 2 mirror the full period patterns. Consistent with other studies, the significant size effect associated with the rebalanced method is primarily due to the earlier period. The subperiod results also confirm that the rebalanced method overstates the magnitude and significance of the size effect, although the bias over the recent subperiod does not result in any difference in statistical inference, since all estimates indicate an insignificant size effect.

Turning to the long–short profits of buying \$1 of the smallest- MV portfolio (L) and shorting \$1 of the largest- MV portfolio (S), the (untabulated) mean profits calculated according to Equation (11) are 0.412% per month for equally weighted portfolios over the full sample period, with corresponding estimates of 0.549% and 0.276% over the 1951–1977 and 1977–2003 subperiods. These estimates are up to one-third higher than the estimates for $L - S$ calculated using the decomposed buy-and-hold method.

2.2 The momentum effect

Table 3 presents the six-month holding-period results of decile portfolios classified by past six-month buy-and-hold returns (r_{-6}).⁹ The decomposed buy-and-hold results reported in Panel A of Table 3 show that the momentum effect is present over the full sample period and in both subperiods. The mean return difference between the equally weighted long and short portfolios is 0.875% per month ($t = 4.91$) over the full sample period, 0.736% per month ($t = 3.27$) over the earlier subperiod, and 1.017% per month ($t = 3.66$) over the later subperiod. The three-factor-adjusted returns on $L - S$ are also significantly positive. Panel B shows that rebalancing imparts a downward bias to the momentum effect, and can turn the significant return differences between L and S insignificant. For the rebalanced portfolios, the mean return difference between L and S is 0.583% per month ($t = 3.01$) over the full sample period, and 0.597% per month ($t = 2.58$) and 0.569% per month ($t = 1.82$) over the two subperiods. These figures are as much as 44.1% lower than the decomposed buy-and-hold estimates and in the case of the second subperiod using the rebalanced method gives a spuriously insignificant momentum effect.

Examining the individual portfolios shows that rebalancing tends to overstate the portfolio returns. However, this effect is most marked in the return to the short side of the momentum portfolio, which drives the above results. For example, over the recent 26-year subperiod from January 1978 to December 2003, rebalancing turns the insignificant loser portfolio's decomposed buy-and-hold return of 0.826% ($t = 1.84$) per month into a highly significant 1.277% ($t = 2.63$), resulting in a bias of 0.451% per month (over 5.5% per annum). The large positive bias in the loser portfolio return from using the rebalanced method is consistent with our earlier prediction, because this past six-month price losing portfolio contains lower priced and smaller capitalization stocks (Table 3, Panel C).¹⁰ Finally, untabulated results show that long–short profits are

⁹ Jegadeesh and Titman (1993) show that the results for the 6×6 strategy (six-month ranking and holding periods) are representative.

¹⁰ Examining decile portfolio performance within the one-third lowest price stocks and measuring bias as the mean difference between rebalanced and decomposed buy-and-hold portfolio returns, the bias is as high as 0.694% ($t = 10.7$) per month (over 8% per year) for the past six-month loser portfolio over the full sample period 1951 to 2003.

Table 3
Holding-period performance per month of portfolios classified by past six-month returns

Panel A: Decomposed buy-and-hold method with Equation (4)										
	S	D2	D3	D4	D5	D6	D7	D8	D9	L-S
Testing period: July 1951–December 2003										
Raw (%)	0.768 (2.60)	0.989 (4.39)	1.142 (5.70)	1.177 (6.28)	1.246 (6.90)	1.279 (7.27)	1.317 (7.44)	1.359 (7.47)	1.448 (7.48)	1.643 (6.86)
$\hat{\alpha}_{3F}$ (%)	-0.760 (-5.94)	-0.337 (-5.04)	-0.102 (-1.99)	-0.024 (-0.53)	0.078 (1.83)	0.130 (3.15)	0.192 (4.32)	0.238 (4.87)	0.342 (5.81)	0.540 (6.17)
Testing period: July 1951–December 1977										
Raw (%)	0.711 (1.84)	0.889 (2.77)	0.992 (3.42)	0.990 (3.66)	1.064 (4.13)	1.116 (4.47)	1.130 (4.57)	1.177 (4.72)	1.244 (4.82)	1.447 (4.86)
$\hat{\alpha}_{3F}$ (%)	-0.623 (-5.55)	-0.309 (-4.00)	-0.130 (-2.21)	-0.090 (-2.00)	0.014 (0.37)	0.085 (2.62)	0.116 (2.94)	0.170 (3.44)	0.250 (3.62)	0.457 (4.01)
Testing period: January 1978–December 2003										
Raw (%)	0.826 (1.84)	1.090 (3.45)	1.295 (4.68)	1.369 (5.28)	1.431 (5.66)	1.446 (5.83)	1.507 (5.95)	1.545 (5.82)	1.656 (5.72)	1.843 (4.89)
$\hat{\alpha}_{3F}$ (%)	-0.896 (-3.86)	-0.354 (-3.32)	-0.055 (-0.70)	0.061 (0.83)	0.153 (2.09)	0.185 (2.52)	0.267 (3.42)	0.293 (3.55)	0.405 (4.37)	0.551 (4.37)

Panel B: Rebalanced method with Equation (7)

Raw (%)	1.063	1.073	1.187	Testing period: July 1951–December 2003					
	(3.40)	(4.64)	(5.81)	1.208	1.263	1.291	1.328	1.363	1.453
	–0.541	–0.300	–0.093	(6.35)	(6.92)	(7.26)	(7.46)	(7.49)	(7.55)
$\hat{\alpha}_{3F}$ (%)	(–3.67)	(–4.11)	(–1.72)	–0.022	0.067	0.118	0.175	0.215	0.317
				(–0.47)	(1.58)	(2.96)	(4.18)	(4.76)	(5.92)
									(6.13)
Raw (%)	0.853	0.958	1.039	Testing period: July 1951–December 1977					
	(2.14)	(2.91)	(3.50)	1.024	1.086	1.137	1.151	1.192	1.261
	–0.513	–0.262	–0.101	(3.71)	(4.13)	(4.48)	(4.59)	(4.73)	(4.85)
$\hat{\alpha}_{3F}$ (%)	(–4.16)	(–3.08)	(–1.56)	–0.071	0.023	0.094	0.123	0.172	0.255
				(–1.40)	(0.55)	(2.81)	(3.25)	(3.73)	(3.96)
									(4.74)
Raw (%)	1.277	1.191	1.339	Testing period: January 1978–December 2003					
	(2.63)	(3.65)	(4.77)	1.396	1.445	1.449	1.508	1.538	1.649
	–0.554	–0.315	–0.054	(5.33)	(5.69)	(5.82)	(5.97)	(5.84)	(5.80)
$\hat{\alpha}_{3F}$ (%)	(–2.04)	(–2.68)	(–0.66)	0.055	0.135	0.160	0.236	0.256	0.364
				(0.76)	(1.89)	(2.31)	(3.23)	(3.35)	(4.30)
									(3.88)
Panel C: Price and MI' of portfolios over July 1951–June 2003									
UP	12.55	19.26	23.27	26.09	27.86	29.19	30.82	30.32	30.21
MI' (\$m)	163.5	400.3	560.5	674.9	726.5	779.8	808.7	812.7	736.9
									27.28
									444.4
									280.9

At the beginning of each month, we sort stocks into decile portfolios based on their past six-month buy-and-hold returns using NYSE breakpoints. These portfolios are equally weighted and the holding period is six months. We use the technique of Jegadeesh and Titman (1993) to calculate the nonoverlapping monthly portfolio returns over the entire testing period. The poorest past six-month price performing portfolio is $S(hort)$, the best past six-month price performing portfolio is $L(long)$, and $L-S$ denotes the difference between L and S . The notation $\hat{\alpha}_{3F}$ is the intercept estimate of the Fama–French three-factor model, UP is the unadjusted price per share at the beginning of the holding period, and MI' is the market capitalization at the beginning of the holding period. The sample includes all NYSE/AMEX/NASDAQ ordinary common stocks over the period 1951 to 2003. Numbers in parentheses are t -statistics.

higher than the $L-S$ differences for the full sample period and for both subperiods.

2.3 Examples of spurious inferences in the literature

The above analysis of size and momentum shows that the rebalanced method can lead to different conclusions compared with the decomposed buy-and-hold method. We now provide two cases from the published research literature where spurious inferences occur and where employing the decomposed buy-and-hold method overturns reported results.¹¹

Barber and Lyon (1997) provide a robustness check of Fama and French (1992) by reporting results on the relation between firm size, book-to-market ratios, and security returns for nonfinancial firms and a holdout sample of financial firms. They report a similar relation for financial and nonfinancial firms. In Table 4, Panel A we replicate the results of Barber and Lyon (1997) on the size premium. Barber and Lyon (1997) report “a significant size premium for both financial and nonfinancial firms. For nonfinancial firms, the size premium is 0.65 percent per month ($t = 2.02$)” (pp. 877–879). Our replication of this result gives an insignificant size premium of 0.407% per month ($t = 1.38$) after applying the decomposed buy-and-hold method.¹²

An example of spurious inferences about momentum in the literature appears in Chordia and Shivakumar (2002). Chordia and Shivakumar (2002) report that a set of macroeconomics variables related to the business cycle explains the profits to momentum strategies and argue that the momentum effect is a manifestation of rational time-varying expected returns. Fundamental to their results is evidence on raw momentum profits and on how these vary over business cycles. They document an insignificant average momentum payoff of 0.27% per month ($t = 1.10$) for the period 7/26–12/94 (Chordia and Shivakumar (2002), Table 1). Applying the decomposed buy-and-hold method, our replication of Chordia and Shivakumar (2002) in Table 4, Panel B shows a significant 0.518% ($t = 2.31$), almost double its rebalanced counterpart. Chordia and Shivakumar (2002) claim that momentum payoffs are related to the business cycle by showing that the difference in momentum payoffs between expansionary and contractionary periods is significant at 1.25% per month ($t = 2.10$) (Chordia and Shivakumar (2002), p. 993). Our replication with the decomposed buy-and-hold method shows that this difference is insignificant (0.774%, $t = 1.05$). This occurs because the

¹¹ We select the papers simply as examples.

¹² The portfolio returns of Barber and Lyon (1997) are generally slightly higher than our calculation using the rebalanced method. We conjecture that Barber and Lyon (1997) calculate arithmetic averages each month over surviving stocks, whereas we assume that the returns over the remaining holding period months are zero for delisting stocks. The same observation applies to our replication of Chordia and Shivakumar (2002) that we describe next.

Table 4
Examples of spurious inferences in the literature

Panel A: *MV*-based deciles with NYSE/AMEX/NASDAQ stocks over the testing period 7/1973–6/1994

	<i>S</i>	<i>D2</i>	<i>D3</i>	<i>D4</i>	<i>D5</i>	<i>D6</i>	<i>D7</i>	<i>D8</i>	<i>D9</i>	<i>L</i>	<i>L–S</i>
Replication of Barber and Lyon (1997)											
Mean (%)	0.990 (3.32)	1.141 (3.72)	1.264 (3.92)	1.227 (3.65)	Decomposed buy-and-hold method		1.329 (3.49)	1.326 (3.44)	1.311 (3.25)	1.396 (3.56)	0.407 (1.38)
					1.328 (3.91)	1.415 (3.89)					
Mean (%)	1.012 (3.37)	1.171 (3.75)	1.293 (3.99)	1.270 (3.73)	Rebalanced method		1.356 (3.50)	1.346 (3.40)	1.300 (3.15)	1.638 (4.03)	0.626 (2.04)
					1.351 (3.91)	1.425 (3.85)					
Mean (%)	1.03	1.15	1.33	1.30	Results of Barber and Lyon (1997)		1.41	1.38	1.32	1.68	0.65 (2.02)
					1.34	1.51					

Panel B: Past-6-month-return-based deciles with NYSE/AMEX stocks over the testing period 7/1976–12/1994

Decomposed buy-and-hold method						Rebalanced method					
<i>L</i>	<i>S</i>	<i>L-S</i>	<i>L</i>	<i>S</i>	<i>L-S</i>	<i>L</i>	<i>S</i>	<i>L-S</i>	<i>L</i>	<i>S</i>	<i>L-S</i>
Replication of Chordia and Shivakumar (2002), Table 1											
7/26-12/94	1.507 (5.87)	0.989 (2.66)	0.518 (2.31)	1.579 (6.07)	1.315 (3.39)	0.264 (1.12)	1.61 (6.06)	1.34 (3.39)	0.27 (1.10)	Results of Chordia and Shivakumar (2002)	

For the results in Panel A, we sort stocks at the beginning of July each year into *MV* decile portfolios using NYSE breakpoints and hold the portfolios for 12 months. The sample includes all nonfinancial NYSE/AMEX/NASDAQ ordinary common stocks over the period 1973 to 1994 with positive book-to-market ratios available, which is consistent with the nonfinancial sample of Barber and Lyon (1997). For the results in Panel B, we sort stocks at the beginning of each month into equally sized decile portfolios based on their past six-month buy-and-hold returns, and we hold the portfolios for six months. Consistent with Chordia and Shivakumar (2002), the sample includes all NYSE/AMEX ordinary common stocks over the period 1926 to 1994. The notation *L* stands for the past six-month price winning decile or the smallest-*MV* decile, *S* for the past six-month price losing decile or the largest-*MV* decile, and *L–S* for the difference between *L* and *S*. Portfolios are equally weighted, and the reported performance is in percentage per month. Numbers in parentheses are *t*-statistics.

rebalanced method underestimates momentum payoffs more heavily over the contractionary period, with the extent of underestimation being 0.566% per month, or about 6.8% per year (result not tabulated).

2.4 Transactions costs

Table 5 reports the effect of incorporating transactions costs into the analysis. For this purpose, we need to consider the amount of trading involved in rebalancing portfolios during each holding-period month and in revising portfolios from one holding period to the next. An investor incurs costs associated with rebalancing in the case of the rebalanced method but not in the case of the decomposed buy-and-hold method, while portfolio revision necessarily occurs with both methods. Panel A reports the average traded value per month involved in longing \$1 of portfolio *L* and shorting \$1 of portfolio *S* for the size and momentum investment strategies implemented over the period 1951–2003. Appendix A explains the details behind these calculations. We report these average traded values separately for the stock purchases and stock sales involved in each strategy and report results for the rebalanced and decomposed buy-and-hold methods and for the difference between them. The rebalanced method applied to a size strategy results in average traded values increasing by between 83% and 140% compared with the decomposed buy-and-hold method. This increase is proportionately smaller in the case of momentum strategies as these involve six-month rather than 12-month holding horizons and winning and losing stocks rarely retain their status in successive holding periods. Nevertheless, the difference in average monthly traded values associated with momentum strategies between rebalanced and decomposed buy-and-hold methods are similar to those for the size portfolios and all differences are significant. For the size (momentum) strategy, the average traded value of bought plus sold for a long–short portfolio of longing \$100 of *L* and shorting \$100 of *S* is over \$11 (\$15) per month more for the rebalanced method than for the decomposed buy-and-hold method.

Panel B of Table 5 reports the returns after transactions costs to portfolios *L*, *S*, and *L*–*S* for the size and momentum investment strategies (Appendix B explains the calculation of transactions-cost-adjusted returns). We consider four alternative estimates of transactions costs. The first uses the model of transactions costs and empirical results of Keim and Madhavan (1997) (Appendix C). The other transactions costs we assume are 0.5%, 1.0%, and 1.5% for both buying and selling. Consistent with the much greater trading that rebalancing requires, transactions costs clearly have a much greater effect on the performance of the rebalanced strategy than the decomposed buy-and-hold strategy. With the transactions costs adjustment using the results of Keim and Madhavan (1997), the size and momentum premia from the rebalanced strategy are each less than the corresponding premia from the decomposed

Table 5
Traded values associated with portfolios and transactions-cost-adjusted performance

Panel A: Average traded value per month in longing \$1 of L and shorting \$1 of S

	Decomposed buy-and-hold method		Rebalanced method		Rebalanced minus decomposed	
	Value bought	Value sold	Value bought	Value sold	Value bought	Value sold
<i>MV</i> -classified portfolio with 12-month holding period						
<i>L</i> (%)	2.710 (8.03)	4.098 (6.58)	6.403 (19.3)	7.771 (23.5)	3.693 (19.9)	3.673 (8.35)
<i>S</i> (%)	2.480 (6.50)	1.504 (6.69)	4.532 (19.5)	3.611 (15.1)	2.052 (7.93)	2.108 (19.5)
Past-six-month-return-classified portfolio with 6-month holding period						
<i>L</i> (%)	13.904 (11.2)	15.618 (11.0)	17.625 (14.0)	19.161 (15.9)	3.721 (27.0)	3.543 (9.47)
<i>S</i> (%)	15.360 (10.7)	14.335 (12.4)	19.385 (17.1)	18.285 (15.8)	4.026 (8.42)	3.950 (17.9)
Panel B: Transactions-cost-adjusted return per month						
Decomposed buy-and-hold method			Rebalanced method			
<i>MV</i> portfolio			<i>r</i> −6 portfolio		<i>MV</i> portfolio	
Transactions costs based on Keim and Madhavan (1997)						
<i>L</i> (%)	1.154 (4.88)		1.422 (5.95)	1.143 (4.73)		1.336 (5.66)
<i>S</i> (%)	0.923 (5.40)		1.163 (3.91)	0.923 (5.26)		1.613 (5.09)
<i>L</i> − <i>S</i> (%)	0.231 (1.22)		0.259 (1.44)	0.219 (1.14)		−0.277 (−1.39)
Transactions cost level: 0.5% for buy, 0.5% for sell						
<i>L</i> (%)	1.205 (5.10)		1.500 (6.28)	1.297 (5.38)		1.461 (6.20)
<i>S</i> (%)	0.940 (5.50)		0.914 (3.09)	0.961 (5.47)		1.253 (3.99)
<i>L</i> − <i>S</i> (%)	0.264 (1.39)		0.586 (3.28)	0.336 (1.75)		0.208 (1.07)
Transactions cost level: 1% for buy, 1% for sell						
<i>L</i> (%)	1.174 (4.96)		1.358 (5.69)	1.227 (5.09)		1.276 (5.43)
<i>S</i> (%)	0.959 (5.60)		1.061 (3.58)	1.002 (5.70)		1.444 (4.59)
<i>L</i> − <i>S</i> (%)	0.215 (1.14)		0.298 (1.66)	0.225 (1.17)		−0.168 (−0.86)
Transactions cost level: 1.5% for buy, 1.5% for sell						
<i>L</i> (%)	1.144 (4.83)		1.217 (5.10)	1.156 (4.80)		1.093 (4.65)
<i>S</i> (%)	0.978 (5.71)		1.208 (4.07)	1.043 (5.93)		1.636 (5.19)
<i>L</i> − <i>S</i> (%)	0.167 (0.88)		0.009 (0.05)	0.113 (0.59)		−0.543 (−2.76)

We form *MV*-based decile portfolios at the beginning of July each year and hold these portfolios for 12 months. We form r_{-6} -based decile portfolios each month and hold them for 6 months, where r_{-6} is the past six-month buy-and-hold return. We classify portfolios using NYSE breakpoints, and portfolios are equally weighted. The notation L stands for the smallest-*MV* decile or the highest- r_{-6} decile, S for the largest-*MV* decile or the lowest- r_{-6} decile, and $L-S$ for the difference between L and S . The figures for portfolio S in Panel B present the transactions-cost-adjusted negative returns corresponding to its short position. The sample includes all NYSE/AMEX/NASDAQ ordinary common stocks over the period 1951 to 2003. Numbers in parentheses are t -statistics.

buy-and-hold strategy. The reduction in the size premium using the rebalanced method from adjusting for transactions costs is 51.12% (from 0.448% to 0.219%), while the reduction is 26.20% (from 0.313% to 0.231%) using the decomposed buy-and-hold method. The corresponding reduction in the momentum premium is 147.5% using the rebalanced method and 70.4% using the decomposed buy-and-hold method. The dramatic decrease in momentum profits from both strategies is due to winning and losing stocks rarely retaining their positions in consecutive holding periods. Looking at other results, with a moderate level of transactions costs of 1% for both buying and selling, we do not observe a significant size premium even with the rebalanced method. Transactions costs again have the greatest effect on momentum: the rebalanced method gives insignificant momentum profits at all transactions cost levels. Even with the decomposed buy-and-hold method, a 1% level of transactions costs for buying and selling is high enough to eliminate any significant momentum premium. Overall, the transactions-cost-adjusted results serve to emphasize that the rebalancing strategy is a poor, impractical investment vehicle.

Finally, we examine the impact of transactions costs on the size and momentum effects when the holding period is one month, as some researchers use a one-month testing period (see, for example, Grundy and Martin (2001)), which necessarily involves no rebalancing within the holding period. As with the 6-, or 12-month holding period, we base this examination on NYSE/AMEX/NASDAQ ordinary common stocks over the full testing period July 1951–June 2003 using equally weighted decile portfolios with NYSE breakpoints. Untabulated results show that the size premium before transactions costs is higher at 0.493% ($t = 2.40$) per month with a one-month holding period than with a 12-month holding period. Because of the monthly revision of the portfolio composition, however, a transactions cost level as low as 0.5% for buying and selling transforms the significant size premium to an insignificant 0.334% ($t = 1.63$) per month. For the momentum strategy, the past six-month loser and winner portfolios do not show any momentum profits over a one-month holding period. In fact, the point estimate is negative (-0.166% , $t = -0.69$). Grundy and Martin (2001) find significantly positive momentum profits over a one-month holding period based on NYSE/AMEX stocks using equally sized and weighted decile portfolios with a one-month gap between ranking and holding periods. With this setting, we also find significant one-month holding-period momentum profits of 1.048% ($t = 4.28$) per month over the full testing period July 1951–June 2003. Again, transactions costs for buying and selling of 0.5% turns these momentum profits insignificant (0.205%, $t = 0.84$). All these results show that the one-month holding period strategy is unattractive after taking into account transactions costs. Similar to the multi-month holding period, momentum profits are particularly sensitive to the transactions costs adjustment, which

lends support to the findings of Lesmond, Schill, and Zhou (2004) that momentum profits are not sufficient to offset trading costs.

3. Conclusion

The criterion for calculating returns should be based on an investment perspective. The return should correspond to the amount entering an investor's account. Although we do not know exactly how investors manage their portfolios, the finance literature commonly assumes a multi-month holding period but adopts a rebalancing strategy to calculate monthly portfolio returns. However, a rebalancing strategy does not correspond to the returns that investors achieve under the hypothesized holding period investment strategy, unless they rebalance their portfolios back to the initial weights at the beginning of each discrete interval. In practice, new information flows and new fund flows determine revisions of portfolio weights and these are unlikely to occur at regular intervals. In addition, the transactions costs of regular rebalancing are prohibitive. In contrast, a buy-and-hold investment assumption focuses on the theoretical hypothesis under examination. As an increasing number of studies in finance base their statistical tests on individual period returns over a multiperiod holding horizon, in this article we present a formal analysis of the decomposition of multiperiod buy-and-hold returns into consistent individual period returns. By comparing rebalanced returns with decomposed buy-and-hold returns, we find and predict that rebalancing imparts an upward bias to the size premium and a downward bias to the momentum effect. Empirically, the two methods can produce a portfolio return difference of more than 8% per year, and can lead to different statistical inferences. Transactions-cost-adjusted results reinforce the conclusion that rebalancing is an impractical investment strategy. We believe that some conclusions, drawn from implementing the rebalanced method in past studies, deserve reexamination. The dangers of using rebalanced methods are relevant for tests of asset pricing models, of market efficiency, and for the analysis of investment strategies, such as size, momentum, and value—glamor implemented in real time. As a minimum requirement, researchers should report single-period return results based on the decomposed buy-and-hold approach we discuss in this article in addition to any results based on a rebalancing assumption.

Appendix A: Portfolio Rebalancing

n investment strategy potentially requires the investor to rebalance a portfolio within individual holding periods for a fixed set of stocks and to revise the stocks in the portfolio from one holding period to the next. To deal with transactions costs, we need to calculate for all stocks in a portfolio the degree of rebalancing in terms of the traded values bought and sold each month throughout the investment period. We explain below how we calculate the

traded values bought and sold using the rebalanced and decomposed buy-and-hold methods with reference to a long portfolio. The same calculations apply to a short portfolio with the traded values bought (sold) in the case of a short portfolio corresponding to the traded values sold (bought) in the case of a long position in the same portfolio.

(i) *Traded value bought: rebalanced method, long portfolio*

For each stock i in portfolio L during holding period h , we calculate the traded value bought in each month over the m -month holding period ($m > 1$). For month t ($t = 2, 3, \dots, m$), we calculate the value bought at the end of month t necessary to adjust to the appropriate weight for the following month. For the first holding-period month, in order to preserve the same number of trading amounts as months in the holding period, we sum the values bought at the beginning and end of the month.

In the first holding-period month, we have to consider whether a stock is part of the portfolio of the previous holding period. If so, the traded value of the stock at the end of the previous holding period includes the necessary revision to adjust to the new weight at the beginning of the first month of the new holding period and the value bought is zero. If not, the value bought at the start of the first month is the new portfolio weight. At the end of all holding-period months, there is a further adjustment to restore original weights. We achieve this by returning the amount invested in each stock to $\$w_{i,h}$ at the start of every month, where $w_{i,h}$ is the portfolio weight of stock i in holding period h . Therefore, with the rebalanced method, the traded value bought of stock i in portfolio L in the first holding-period month is

$$BUY(L, rb)_{i,h,1} = \begin{cases} w_{i,h} & + \max[-r_{i,1}w_{i,h}, 0] & \text{if } i \notin L \text{ in } h-1, \\ 0 & + \max[-r_{i,1}w_{i,h}, 0] & \text{if } i \in L \text{ in } h-1, \end{cases} \quad (A1)$$

where $r_{i,1}$ is stock i 's return in the first holding-period month (unadjusted for transactions costs).

The traded value bought of stock i at the end of month t is

$$BUY(L, rb)_{i,h,t} = \max[-r_{i,t}w_{i,h}, 0], \quad t = 2, \dots, m-1, \quad (A2)$$

where $r_{i,t}$ is the month- t return (unadjusted for transactions costs) of stock i in the m -month holding period.

Finally, the traded value bought of stock i at the end of the final holding-period month is

$$BUY(L, rb)_{i,h,m} = \max[w_{i,h+1} - (1 + r_{i,m})w_{i,h}, 0], \quad (A3)$$

where $r_{i,m}$ is the month- m return of stock i (unadjusted for transactions costs) and $w_{i,h+1}$ is stock i 's weight in portfolio L in holding period $h+1$, which is zero if L does not include stock i in holding period $h+1$.^{13,14}

(ii) *Traded value sold: rebalanced method, long portfolio*

The traded value sold of stock i in portfolio L at the end of month t is

$$SELL(L, rb)_{i,h,t} = \max[r_{i,t}w_{i,h}, 0], \quad t = 1, \dots, m-1. \quad (A4)$$

¹³ If the holding period is one month ($m = 1$), Equation (A1) with $\max[w_{i,h+1} - w_{i,h}(1 + r_{i,1}), 0]$ replacing the original second term gives the traded value bought of stock i for both the rebalanced and the decomposed buy-and-hold methods.

¹⁴ In this and subsequent equations, we do not repeat notation defined for earlier equations.

The traded value sold of stock i at the end of the final holding-period month, m , is

$$SELL(L, rb)_{i,h,m} = \max[(1 + r_{i,m})w_{i,h} - w_{i,h+1}, 0].^{15} \quad (A5)$$

(iii) *Traded value bought: decomposed buy-and-hold method, long portfolio*

With the decomposed buy-and-hold method, the only transactions are at the beginning and end of the holding period. Therefore, the traded value bought of stock i in portfolio L in holding-period month t is zero for $t = 2, 3, \dots, m - 1$. The traded value bought of stock i in the first holding-period month (at initiation) is

$$BUY(L, dc)_{i,h,1} = \begin{cases} w_{i,h} & \text{if } i \notin L \text{ in } h - 1, \\ 0 & \text{if } i \in L \text{ in } h - 1, \end{cases} \quad (A6)$$

where dc denotes decomposed buy-and-hold. The traded value bought of stock i at the end of the final holding-period month is

$$BUY(L, dc)_{i,h,m} = \max[w_{i,h+1} - w_{i,h}(1 + r_{i,1})(1 + r_{i,2}) \cdots (1 + r_{i,m}), 0]. \quad (A7)$$

(iv) *Traded value sold: decomposed buy-and-hold method, long portfolio*

With the decomposed buy-and-hold method, the traded value sold of stock i in portfolio L in holding-period month t is zero for $t = 1, 2, \dots, m - 1$. The traded value sold of stock i at the end of the final holding-period month m , is

$$SELL(L, dc)_{i,h,m} = \max[w_{i,h}(1 + r_{i,1})(1 + r_{i,2}) \cdots (1 + r_{i,m}) - w_{i,h+1}, 0]. \quad (A8)$$

On the basis of the monthly traded values of each stock in the portfolio, the traded value (either bought or sold) of the portfolio in each holding-period month is the sum of the traded values for the constituent stocks in that month.

Appendix B: Transactions-cost-adjusted Returns

o account for transactions costs (TC) in holding a portfolio for m months ($m \geq 1$), we calculate portfolio returns based on transactions-cost-adjusted returns of individual stocks in the portfolio.

(i) *Transactions-cost-adjusted returns: decomposed buy-and-hold method, long portfolio*

With the decomposed buy-and-hold method, we apply the transactions cost adjustment to the *first* and *last* holding-period month returns of each stock. In the first month, stock i 's transactions-cost-adjusted return is

$$r(L, TC, dc)_{i,1} = \frac{w_{i,h}(1 + r_{i,1})}{w_{i,h} + TC_{i,h,0}} - 1, \quad (B1)$$

where $TC_{i,h,0}$ is stock i 's estimated transactions cost for the traded value bought of $w_{i,h}$ at the beginning of the holding period. $TC_{i,h,0}$ equals zero if stock i is in portfolio L in holding period $h - 1$, since the transactions cost adjustment then occurs at the end of holding period $h - 1$.

Stock i 's final month transactions cost-adjusted return is

$$r(L, TC, dc)_{i,m} = r_{i,m} - \frac{TC_{i,h,m}}{w_{i,h}(1 + r_{i,1})(1 + r_{i,2}) \cdots (1 + r_{i,m-1})}, \quad (B2)$$

¹⁵ If the holding period is one month ($m = 1$), $\max[w_{i,h}(1 + r_{i,1}) - w_{i,h+1}, 0]$ gives the traded value sold of stock i for both the rebalanced and the decomposed buy-and-hold methods.

where $TC_{i,h,m}$ is stock i 's estimated transactions cost at the end of the holding period for the traded value of $|w_{i,h+1} - w_{i,h}(1 + r_{i,1}) \cdots (1 + r_{i,m})|$.¹⁶ If $w_{i,h+1} - w_{i,h}(1 + r_{i,1}) \cdots (1 + r_{i,m}) > 0$, the investor buys stock i , and sells otherwise.

Given the transactions-cost-adjusted monthly returns for individual stocks in Equations (B1) and (B2), Equation (3) gives the portfolio's transactions-cost-adjusted monthly returns over the m -month holding period.

(ii) *Transactions-cost-adjusted returns: rebalanced method, long portfolio*

The rebalanced method requires a transactions cost adjustment every month of the m -month holding period ($m > 1$). For stock i in portfolio L , its transactions-cost-adjusted return in the first holding-period month is

$$r(L, TC, rb)_{i,1} = \frac{w_{i,h}(1 + r_{i,1}) - TC_{i,h,1}}{w_{i,h} + TC_{i,h,0}} - 1, \quad (B3)$$

where $TC_{i,h,1}$ is the transactions cost at the end of the first holding-period month for the traded value of $|-w_{i,h}r_{i,1}|$ to rebalance the portfolio weight to $w_{i,h}$. If $-w_{i,h}r_{i,1} < 0$, the investor sells $|-w_{i,h}r_{i,1}|$ of stock i . If $-w_{i,h}r_{i,1} > 0$, the investor buys $|-w_{i,h}r_{i,1}|$ of stock i . The symbol $TC_{i,h,0}$ denotes the transactions cost at the beginning of the holding period. If stock i is in portfolio L in holding period $h - 1$, $TC_{i,h,0} = 0$. Otherwise, $TC_{i,h,0}$ is the transactions cost for the traded value bought of $w_{i,h}$.

For month t , the transactions-cost-adjusted return is

$$r(L, TC, rb)_{i,t} = \frac{w_{i,h}(1 + r_{i,t}) - TC_{i,h,t}}{w_{i,h}} - 1 = r_{i,t} - \frac{TC_{i,h,t}}{w_{i,h}}, \quad t = 2, 3, \dots, m - 1, \quad (B4)$$

where $TC_{i,h,t}$ is the transactions cost for the traded value of $|-w_{i,h}r_{i,t}|$. If $-w_{i,h}r_{i,t} < 0$, the investor sells $|-w_{i,h}r_{i,t}|$ of stock i . If $-w_{i,h}r_{i,t} > 0$, the investor buys $|-w_{i,h}r_{i,t}|$ of stock i .

For the final holding period month, stock i 's transactions-cost-adjusted return is

$$r(L, TC, rb)_{i,m} = \frac{w_{i,h}(1 + r_{i,m}) - TC_{i,h,m}}{w_{i,h}} - 1 = r_{i,m} - \frac{TC_{i,h,m}}{w_{i,h}}, \quad (B5)$$

where $TC_{i,h,m}$ is the transactions cost for the traded value of $|w_{i,h+1} - w_{i,h}(1 + r_{i,m})|$ at the end of holding period h . If $w_{i,h+1} - w_{i,h}(1 + r_{i,m}) > 0$, the investor buys stock i , and sells otherwise.

Using Equations (B3)–(B5), the portfolio's transactions-cost-adjusted monthly returns over the m -month holding period under the rebalanced method are

$$r(TC, rb)_{L,\tau} = \sum_{i=1}^N w_{i,h} r(L, TC, rb)_{i,\tau}, \quad \tau = 1, 2, \dots, m, \quad (B6)$$

where N is the number of stocks in the long portfolio L in holding period h .

(iii) *Transactions-cost-adjusted returns: decomposed buy-and-hold method, short portfolio*

Under the decomposed buy-and-hold method, the transactions cost adjustment again applies only to the *first* and the *final* holding-period monthly returns. For stock i in portfolio

¹⁶ If the holding period is one month ($m = 1$), the transactions-cost-adjusted return of stock i over the one-month period is $[w_{i,h}(1 + r_{i,1}) - TC_{i,h,m}]/(w_{i,h} + TC_{i,h,0}) - 1$, where $TC_{i,h,m}$ is stock i 's estimated transactions cost for the traded value of $|w_{i,h+1} - w_{i,h}(1 + r_{i,1})|$ at the end of the one-month holding period. If $w_{i,h+1} - w_{i,h}(1 + r_{i,1}) > 0$, the investor buys stock i , and sells otherwise.

S , its first month transactions-cost-adjusted (negative) return is¹⁷

$$r(S, TC, dc)_{i,1} = \frac{w_{i,h}(1 + r_{i,1})}{w_{i,h} - TC_{i,h,0}} - 1, \quad (\text{B7})$$

where $TC_{i,h,0}$ is stock i 's estimated transactions cost for the traded value sold of $w_{i,h}$ at the beginning of the holding period. Note that $TC_{i,h,0}$ equals zero if stock i is in portfolio S in holding period $h - 1$, since the transactions cost adjustment then occurs at the end of holding period $h - 1$.

The transactions-cost-adjusted (negative) return of stock i in the final holding-period month is

$$r(S, TC, dc)_{i,m} = r_{i,m} + \frac{TC_{i,h,m}}{w_{i,h}(1 + r_{i,1})(1 + r_{i,2}) \cdots (1 + r_{i,m-1})}, \quad (\text{B8})$$

where $TC_{i,h,m}$ is the transactions cost for the traded value of $|w_{i,h}(1 + r_{i,1}) \cdots (1 + r_{i,m}) - w_{i,h+1}|$ at the end of holding period h .¹⁸ If $w_{i,h}(1 + r_{i,1}) \cdots (1 + r_{i,m}) - w_{i,h+1} < 0$, the investor sells, and buys otherwise.

Given the transactions-cost-adjusted monthly returns in Equations (B7) and (B8), Equation (3) gives the transactions-cost-adjusted monthly (negative) returns of the short portfolio (S) over the m -month holding period.

(iv) *Transactions-cost-adjusted monthly returns: rebalanced method, short portfolio*

The rebalanced method again requires a transactions cost adjustment in every month of the m -month ($m > 1$) holding period. For the first month of the holding period, stock i 's transactions-cost-adjusted (negative) return is

$$r(S, TC, rb)_{i,1} = \frac{w_{i,h}(1 + r_{i,1}) + TC_{i,h,1}}{w_{i,h} - TC_{i,h,0}} - 1, \quad (\text{B9})$$

where $TC_{i,h,1}$ is the transactions cost for a traded value of $|w_{i,h}r_{i,1}|$ at the end of the first holding-period month. If $w_{i,h}r_{i,1} < 0$, the investor sells, and buys otherwise.

For holding-period month t , stock i 's transactions-cost-adjusted (negative) return is

$$\begin{aligned} r(S, TC, rb)_{i,t} &= -\frac{w_{i,h} - [w_{i,h}(1 + r_{i,t}) + TC_{i,h,t}]}{w_{i,h}} \\ &= r_{i,t} + \frac{TC_{i,h,t}}{w_{i,h}}, \quad t = 2, 3, \dots, m-1, \end{aligned} \quad (\text{B10})$$

where $TC_{i,h,t}$ is the transactions cost for a traded value of $|w_{i,h}r_{i,t}|$ at the end of month t . If $w_{i,h}r_{i,t} < 0$, the investor sells, and buys otherwise.

For the final holding-period month m , stock i 's transactions-cost-adjusted negative return is

$$r(S, TC, rb)_{i,m} = -\frac{w_{i,h} - [w_{i,h}(1 + r_{i,m}) + TC_{i,h,m}]}{w_{i,h}} = r_{i,m} + \frac{TC_{i,h,m}}{w_{i,h}}, \quad (\text{B11})$$

¹⁷ To be consistent with the common convention in the literature, we present the negative return from shorting a stock or portfolio.

¹⁸ If the holding period is one month ($m = 1$), the transactions-cost-adjusted (negative) return from shorting stock i for both the decomposed and rebalanced methods is $[w_{i,h}(1 + r_{i,1}) + TC_{i,h,m}]/(w_{i,h} - TC_{i,h,0}) - 1$, where $TC_{i,h,m}$ is stock i 's estimated transactions cost for the traded value of $|w_{i,h}(1 + r_{i,1}) - w_{i,h+1}|$ at the end of the one-month holding period h . If $w_{i,h}(1 + r_{i,1}) - w_{i,h+1} < 0$, the investor sells, and buys otherwise.

where $TC_{i,h,m}$ is the transactions cost of stock i for a traded value of $|w_{i,h}(1 + r_{i,m}) - w_{i,h+1}|$ at the end of the holding period h . If $w_{i,h}(1 + r_{i,m}) - w_{i,h+1} < 0$, the investor sells, and buys otherwise.

Given Equations (B9)–(B11), the short portfolio's rebalanced transactions-cost-adjusted monthly negative returns over the m -month holding period are

$$r(TC, rb)_{S,\tau} = \sum_{i=1}^N w_{i,h} r(S, TC, rb)_{i,\tau}, \quad \tau = 1, 2, \dots, m, \quad (B12)$$

where N is the number of stocks in the short portfolio S in holding period h .

Appendix C: Estimating Transactions Costs

part from assuming different transactions cost levels, we also estimate the transactions cost for an individual stock based on the results of Keim and Madhavan (1997) as follows,

$$TC(buy)_{it} = 0.767 + 0.336D_{NASDAQ} - 0.084 \ln(MV_{it}) + 13.807/P_{it}, \quad (C1)$$

$$TC(sell)_{it} = 0.505 + 0.058D_{NASDAQ} - 0.059 \ln(MV_{it}) + 6.537/P_{it}, \quad (C2)$$

where $TC(buy)_{it}$ is the transactions cost (as a percentage of traded value) of buying stock i in month t , $TC(sell)_{it}$ is the transactions cost of selling stock i in month t , $D_{NASDAQ} = 1$ for NASDAQ stocks and $D_{NASDAQ} = 0$ for NYSE/AMEX stocks, MV_{it} is the market capitalization of stock i at time t in thousands of dollars, and P_{it} is the price of stock i at t . Wermers (2000) and Cooper, Gutierrez Jr., and Marcum (2005) also use the results of Keim and Madhavan (1997) with a time-adjustment factor.

The above estimates ignore the effects of technical and indexing trades as well as trade size. In addition, we set upper and lower limits on the estimates from Equations (C1) and (C2) of zero and 2.5%. Barber and Odean (2000) find that the average round-trip trading cost for individual investors is 4.03% over the period 1991 to 1996, suggesting a half-trip cost of less than 2.5%. In addition, the results of Keim and Madhavan (1997) are for the period 1991–1993. Given the decline in transactions costs over time, it is likely that Equations (C1) and (C2) overestimate transactions costs over 1994–2003, and underestimate them over 1951–1990. Taken together, our procedures are more likely to be conservative in estimating transactions costs.

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