

Does Anonymity Matter in Electronic Limit Order Markets?

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We develop a model in which limit order traders possess volatility information. We show that in this case the size of the bid–ask spread is informative about future volatility. Moreover, if volatility information is in part private, we establish that (i) the size of the bid–ask spread and (ii) its informativeness about future volatility should change in the same direction when limit order traders' identifiers stop being disclosed. We test these predictions using data from the Paris Bourse. As expected, we find that the average quoted spread and its informativeness are significantly smaller when limit order traders' identifiers are concealed. These findings suggest that the limit order book is a channel for volatility information. (*JEL* G10, G14, G24)

“Broker bids are an additional piece of information that can, in some circumstances, be useful in predicting future market activity. It is apparent that some traders attempt to second-guess future price movements based on trading by particular brokers.”—“ASX Market Reforms—Enhancing the Liquidity of the Australian Equity Markets”, Consultation Paper of the Australian Stock Exchange (2003).

In the past decade, the security industry has witnessed a proliferation of electronic trading systems. These systems (e.g., INET, ArcaEx, or Reuters D2000-2) are predominantly limit order markets—that is, markets in which traders can either post quotes (submit limit orders) or hit posted

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quotes (submit market orders).¹ Yet they still differ in many ways because market organizers experiment with various trading rules. This evolution raises intriguing questions about the effects of market design on market quality.

A case in point is the amount of information provided on traders' identities. Some markets (e.g., the Hong Kong Stock Exchange or the Australian Stock Exchange) disclose, for each limit order, the issuing broker's identification code. In other markets (e.g., INET, Euronext, or the NYSE), brokers' identifiers are concealed. Does it matter? How is market liquidity affected by the disclosure of limit order traders' identities? Is the informational content of the limit order book altered by anonymity? We take advantage of a change in the organization of the Paris Bourse to study these questions empirically. The analysis shows that the limit order book contains information about the likelihood or the magnitude of future price changes—that is, volatility information.

Intuitively, the limit order book is a conduit for volatility information because limit orders have option-like features. A sell (resp. buy) limit order is similar to a (free) call (resp. put) option with a strike price equal to the price of the limit order [Copeland and Galai (1983)]. Speculators exercise these options (“pick off limit orders”) when limit orders become stale after the arrival of new information. As option values depend on volatility, limit-order traders should use volatility information to price their orders. For instance, in anticipation of increased volatility, they should bid less aggressively to reduce their exposure to the risk of being picked off.

We formalize this intuition in a simple model. We contrast two different trading mechanisms: a *non-anonymous* market (limit-order traders' IDs are visible) and an *anonymous* market (limit-order traders' IDs are concealed). When volatility information is *symmetric*, we find that anonymity does *not* matter. A wide bid–ask spread *signals* that limit order traders expect high future volatility, but concealing traders' identifiers does not alter market liquidity and the informativeness of the bid–ask spread on future volatility (i.e., its correlation with future volatility).

This suppress result breaks down when limit order traders (“dealers” for brevity) have asymmetric information about future volatility.² In this case, uninformed dealers learn volatility information from the limit order book, as the latter contains offers posted by better-informed dealers. Now, the information conveyed by the limit order book is less precise when trading is anonymous, because anonymity prevents uninformed dealers from distinguishing informed offers from uninformed offers.

¹ Bloomfield et al. (2005), Section 2, provide an excellent discussion of the literature on limit order markets.

² Ni et al. (2006) provide evidence of trading on volatility information in option markets. Our analysis shows how volatility information can also be exploited through the placement of limit orders in cash markets.

If informed dealers' participation rate is small, uncompetitive quotes constitute a weak signal that the risk of being picked off is large, as these quotes most likely come from uninformed dealers. Thus, these quotes invite competition from other uninformed dealers who instead would stay put if they knew that these quotes come from informed dealers. In this case, we show that (i) the average size of the quoted spread and (ii) its informativeness are smaller in the anonymous system because uninformed dealers are (i) more aggressive and thereby (ii) set the best quotes more frequently in this system. The first effect works to decrease the bid–ask spread while the second reduces its correlation with future volatility. Opposite findings are obtained when the proportion of informed traders is large. In this case, a wide bid–ask spread constitutes a strong warning that the risk of being picked off is large and thereby leads uninformed dealers to behave less aggressively than when trading is non-anonymous.

These results yield two crisp predictions about the effects of switching from non-anonymous to anonymous trading. First, a switch to anonymity should alter the size of the bid–ask spread and its informativeness about future volatility. In particular, when informed dealers' participation rate is small, a switch to anonymity should lead to a decline in (i) the correlation between the bid–ask spread and future volatility and (ii) the size of the bid–ask spread. Second, for a given participation rate of informed dealers, the size of the bid–ask spread and its informativeness should evolve in the *same direction* after a switch to anonymity. We test these predictions using data from Euronext (the French Stock Exchange). In this market, identifiers for broker-dealers' limit orders were disclosed until April 23, 2001. Since this date, the limit order book of Euronext is anonymous.³

After controlling for changes in market conditions in a multivariate setting, we find that the quoted spread and the effective spread for the stocks in our sample are significantly smaller after the switch to anonymity. Moreover, we divide each trading day into intervals of thirty minutes and we find that the bid–ask spread in one period is a statistically significant predictor of volatility in the subsequent interval (after controlling for variables that forecast future volatility). But the strength of the association between the bid–ask spread and future volatility is significantly smaller after the switch to anonymity. The results are robust when we model time variations in conditional returns volatility using a Generalized Autoregressive Conditional Heteroskedasticity GARCH(1,1) framework. These findings support our main testable hypotheses and corroborate the interpretation provided by the model. They also suggest that volatility information is not entirely public, as if it were, the model indicates that a

³ Euronext is of particular interest as its design is close to that of other markets (e.g., the Toronto Stock Exchange, the Stockholm Stock Exchange, or INET).

switch to anonymity would have no effect. Thus, limit order books reflect both public and private volatility information.

Researchers have shown that concealing pre-trade information about *liquidity demanders'* identities (e.g., block traders) impairs market liquidity.⁴ In contrast, we focus on the effects of pre-trade information about *liquidity suppliers'* identities, and our findings establish that concealing this type of information can *improve* market liquidity. Waisburd (2003) analyzes the effect of revealing traders' identities *post-trade*, using data from Euronext. He considers a sample of stocks trading in two different regimes: one in which brokers' identities are revealed post-trade and one in which these identities are concealed. He finds that liquidity is smaller in the post-trade anonymous regime. Our empirical findings go in the opposite direction. Thus, taken together, these results demonstrate that various facets of anonymity (e.g., pre-trade vs. post-trade anonymity) have different effects.

Few articles analyze the effects of providing pre-trade information on liquidity suppliers' identities. Rindi (2002) studies the effect of pre-trade disclosure of informed traders' demand. Volatility information plays no role in her model, however. Simaan et al. (2003) argue that non-anonymous trading facilitates collusion among liquidity suppliers. They find that dealers post more aggressive quotes in Electronic Communication Networks (ECNs) than in Nasdaq, as predicted by the collusion hypothesis. The collusion hypothesis, however, does not predict that a switch to anonymity should affect the informativeness of bid-ask spreads on future volatility, as we find empirically.⁵

Our findings also contribute to the recent literature on the informational content of the limit order book [Irvine et al. (2000), Kalay and Wohl (2002), Harris and Panchapagesan (2005), Cao et al. (2003)]. This literature has analyzed whether the limit order book (e.g., order imbalances) predicts the *direction* of future price changes. Our results show that limit order books can also convey information on the *magnitude* of future price changes. In this way, we complement papers identifying specific variables [e.g., earnings in Seppi (1992) or credit ratings in Odders-White and Ready (2006)] about which trading is informative.

The article is organized as follows. Section 1 describes the model. Section 2 derives the testable hypotheses that we test in Section 3. Section 4 concludes. The proofs are collected in the Appendix.

⁴ Papers on this topic include Seppi (1990), Forster and George (1992), Benveniste et al. (1992), Madhavan and Cheng (1997), Garfinkel and Nimalendran (2003), Reiss and Werner (2004), and Theissen (2003). This line of research contributes to the broader debate on the effects of transparency in security markets (see O'Hara (1995) for a review).

⁵ Comerton-Forde et al. (2005) extend our empirical analysis for liquidity to a different sample of stocks listed on the Paris Bourse, the Tokyo Stock Exchange, and the Korea Stock Exchange. They also find that liquidity is larger in the anonymous environment, for the three markets considered in their study.

1. The Model

In this section we present the model that guides our empirical analysis. In this model, some traders are informed about the likelihood of a change in the asset value. They use this information to price their limit orders and, thereby, the limit order book conveys information on future price volatility. This signaling role for the limit order book is key for our testable implications.

1.1 Market participants

We consider the market for a risky security. There are three dates, $t = 0, 1, 2$. At date 2, the final value of the security, $\tilde{V}_2$, is realized:

$$\tilde{V}_2 = v_0 + \tilde{I} * \tilde{\epsilon}_1, \quad (1)$$

where $\tilde{\epsilon}_1$ is equal to, $+\sigma$ or $-\sigma$, with identical probabilities. Variable $\tilde{I}$ is equal to 1 if an information event occurs at date 1 and zero otherwise. An information event occurs with probability θ_0 ($0 < \theta_0 < 1$).⁶ Hence, at date 0, the *expected* volatility of the security is:

$$Var(\tilde{V}_2) = E((\tilde{V}_2 - v_0)^2) = \theta_0 \sigma^2. \quad (2)$$

The *realized* volatility is known at date 1. It is either large (equal to σ^2) or small (zero), depending on whether an information event occurs or not.

1.1.1 Speculators and liquidity traders. At date 1, a trader arrives and submits a market order that executes against limit orders posted at date 0 (see below). Figure 1 depicts the trading process at date 1.

If an information event occurs, a speculator arrives with probability α and observes the innovation, ϵ_1 . If ϵ_1 is positive (negative), he picks off all sell (buy) limit orders with a price below $v_0 + \sigma$ (resp. above $(v_0 - \sigma)$). Otherwise, a liquidity trader arrives and submits a buy or a sell market order with equal probabilities. The liquidity trader's order size, $\tilde{q}_1$, is either "small" (equal to one round lot) or "large" (equal to two round lots) with equal probabilities.

1.1.2 Limit order traders. Limit orders are posted at date 0. They specify a price and the maximum quantity a trader is willing to sell or buy at this price. There are two kinds of limit order traders: (a) *value traders* and (b) *pre-committed traders* [see Harris and Hasbrouck (1996)]. Pre-committed

⁶ An information event can be seen, for instance, as the arrival of public information (corporate announcements, price movements in related stocks, headline news and so on). Uncertainty on information events is a feature of other models—e.g., Easley and O'Hara (1992). The implications of the model are identical when (i) an information event occurs with certainty but (ii) the price impact of the event, σ , is uncertain. Thus, the model also applies to scheduled corporate announcements (e.g., earnings announcements).

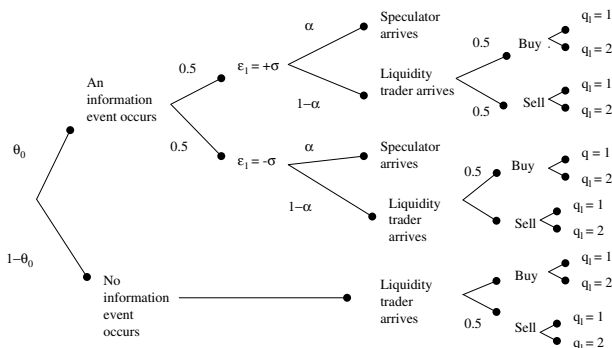


Figure 1
Tree diagram of the trading process

At date 1, an information event occurs with probability θ_0 . If an information event occurs, a speculator arrives with probability α and observes the innovation, ε_1 . This innovation is either $+\sigma$ or $-\sigma$ with equal probabilities. Otherwise, a liquidity trader arrives and submits a buy or a sell market order with equal probabilities. The liquidity trader's order size, q_1 , is either equal to one round lot or two round lots with equal probabilities.

traders must buy or sell a given number of shares. Value traders (henceforth “dealers”) do not need to trade per se but submit limit orders if this is profitable.

Dealers can be either informed or uninformed. *Informed dealers* have private information on future volatility—that is, they know whether or not an information event takes place at date 1. *Uninformed dealers* do not have this knowledge. Observe that informed and uninformed dealers have the same valuation for the security as $E(\tilde{V}_2 | I = 1) = E(\tilde{V}_2 | I = 0) = v_0$.⁷ Hence, bid–ask quotes bracket v_0 and it cannot be optimal for dealers, whatever their type, to trade against the book. Yet volatility information is useful because it enables dealers to adjust their order submission strategy to the level of the risk of being picked off.

1.2 Timing and market structure

At date 0, dealers post their limit orders *sequentially*, in stages L (first stage) and F (second stage). With probability β (resp. $(1 - \beta)$), the trader acting in stage L is an informed dealer (resp. pre-committed trader). Then, in stage F , an uninformed dealer arrives, observes the limit order book, and either

⁷ In other words, informed dealers have no information on the direction of future price movements. Corporate events (spin-offs, earnings and dividends announcements, take-overs and so on) are often associated with a substantial increase in volatility [see Boehmer et al. (1991)]. Moreover, Kim and Verrechia (1997) show that volatility increases with the precision of the information released by the announcement. Thus, traders with information on the occurrence of corporate announcements or their precision have volatility information, but not necessarily directional information. Consider the case of merger announcements. Numerous empirical studies have shown that this type of announcement has no impact on the price of the acquiring firm, on average. Thus, a dealer learning that a merger announcement is pending can correctly anticipate that it will trigger a price reaction for the acquiring firm without being able to predict its direction.

submits limit orders or stays put. We call the trader acting in stage L (resp. F): the *Leader* (resp. the *Follower*).⁸ In the *non-anonymous* limit order market, the follower observes the leader's identity (informed/precommitted). In the *anonymous* market, this information is not available. *Time priority* is enforced—that is, the limit order placed by the leader at a given price is executed before the limit order placed by the follower.

The buy and sell sides of the limit order book are *segmented*. That is, traders intervening on each side are different and do *not* observe offers on the opposite side (e.g., sell limit order traders do not observe buy limit orders). In this way, we can analyze the buy and sell sides of the limit order book separately and, from now on, we focus only on the sell side. Without segmentation, the follower's inferences depend on offers on each side. The exposition is then more involved without delivering additional insights.

Sell limit orders can be posted at prices A_1 and A_2 , $A_1 < A_2$, such that:

$$A_2 - A_1 = A_1 - v_0 = \Delta, \quad (3)$$

with $\Delta < \sigma < 2\Delta$ (Δ is the tick size). Hence, limit orders at price A_1 *only* are exposed to the risk of being picked off, as $A_1 < v_0 + \sigma < A_2$. The price schedule ("limit order book") set by the leader is a vector giving the number of shares supplied by the leader at each price. For instance, $(0, 2)$ means that the leader has a sell limit order for two round lots at price A_2 and none at price A_1 . The leader chooses one limit order book among three different possibilities: (a) a "*Thin*" book [$T \equiv (0, 2)$], (b) a "*Shallow*" book [$S \equiv (1, 2)$] or (c) a "*Deep*" book [$D \equiv (2, 2)$]. After observing the leader's price schedule, the follower submits a limit order at price A_1 or stays put. Dealers choose their order submission strategy to maximize their expected profits. Pre-committed traders' decisions are exogenous: they choose price schedule $K \in \{T, S, D\}$ with probability Φ_K ($0 < \Phi_K < 1$).

Our goal is to compare the liquidity and the informativeness of the limit order book in the anonymous and non-anonymous environments. We use two different measures of market liquidity: (a) the *small trade spread* and (b) the *large trade spread*. The small trade spread, $\tilde{S}_{\text{small}}$, is the quoted (half) spread at the end of the bidding stage. Hence, the *expected* small trade spread is:

$$\begin{aligned} E(\tilde{S}_{\text{small}}) &= \text{prob}(\tilde{Q}_1 \geq 1)A_1 + \text{prob}(\tilde{Q}_1 = 0)A_2 - v_0 \\ &= \Delta(1 + \text{prob}(\tilde{Q}_1 = 0)), \end{aligned} \quad (4)$$

⁸ In our model, the informed dealer always submits his limit orders before the uninformed dealer. The sequence of moves could be random but this formulation would needlessly complicate the presentation. Actually, the follower's order submission strategy depends on the leader's type only when (i) the leader has a chance to be informed and (ii) the follower is uninformed. Hence, this configuration is the only case in which concealing the leader's identity has an effect, if any.

where $\tilde{Q}_1$ is the number of shares supplied at price A_1 at the end of stage F . The large trade spread, $\tilde{S}_{\text{large}}$, is the difference between the marginal execution price of a large market order and the unconditional expected value of the security. It is conceptually similar to the *effective spread* in our empirical analysis. A large market order walks up the limit order book if the quoted depth is insufficient (i.e., $\tilde{Q}_1 < 2$). Thus, the expected large trade spread is:

$$\begin{aligned} E(\tilde{S}_{\text{large}}) &= \text{prob}(\tilde{Q}_1 = 2)A_1 + (1 - \text{prob}(\tilde{Q}_1 = 2))A_2 - v_0 \\ &= \Delta(2 - \text{prob}(\tilde{Q}_1 = 2)). \end{aligned} \quad (5)$$

Last, we measure the *informativeness* of the bid–ask spread on future price volatility by the covariance between the magnitude of the price movement between dates 0 and 2 and the size of the small trade spread—that is, $\text{Cov}(|\tilde{V}_2 - v_0|, \tilde{S}_{\text{small}})$.

2. Anonymity, Liquidity, and Bid–Ask Spread Informativeness

In this section, we analyze the equilibria of the limit-order market and we derive implications that we test in the next section. We focus on Perfect Bayesian equilibria, which means that (a) the follower’s belief about the likelihood of an information event must be consistent with the leader’s order submission strategy (i.e., determined by Bayes’ Rule whenever possible) and (b) dealers’ order submission strategies maximize their expected profit given other traders’ strategies. For brevity, we focus on the case $A_1 < v_0 + \alpha\sigma$ —that is, $\Delta < \alpha\sigma$. In this way, the risk and cost of being picked off are large enough to make limit orders at price A_1 unprofitable *when* there is an information event. This condition is not key for the findings (see the discussion at the end of Section 2.4). It just reduces the number of cases to consider when we describe the equilibria of the limit order market.

2.1 The follower’s optimal reaction

We first study the follower’s optimal strategy, given her beliefs about the occurrence of an information event. We denote this belief by θ_K , as the follower can learn volatility information by observing the limit order book.

Let $\Pi(n; K, \theta_K)$ be the follower’s expected profit, conditional on the arrival of a buy order, when she submits a sell limit order (at price A_1) for n round lots. The follower’s optimal order is the number of round lots, $n^*(\theta_K, K)$ that maximizes $\Pi(n; K, \theta_K)$. We have $n^*(\theta_K, K) \leq 2$ because it is not optimal to post a limit order whose size exceeds liquidity traders’ maximal demand. If the leader sets a thin book, the bid–ask spread is wide (equal to $A_2 - v_0 = 2\Delta$). The follower can then undercut the leader’s offer with a *small limit order* ($n = 1$) or a *large limit order* ($n = 2$). Alternatively,

she can decide to stay put ($n = 0$). A shallow or a deep book leaves no room for price improvement. The follower then decides to expand the quoted depth at price A_1 or stays put ($n = 1$ or $n = 0$). The next lemma describes her optimal order submission strategy for each state of the limit order book.

Lemma 1. *The follower's optimal order submission strategy is as follows.*

1. *When the follower observes a thin book, she submits, at price A_1 , (i) a large limit order if $\frac{2\theta_T\alpha\sigma}{\theta_T\alpha+1} \leq \Delta$ and (ii) a small limit order if $\theta_T\alpha\sigma < \Delta < \frac{2\theta_T\alpha\sigma}{\theta_T\alpha+1}$. If $\Delta = \theta_T\alpha\sigma$, she submits a small limit order with probability u_T and stays put otherwise. If $\Delta < \theta_T\alpha\sigma$, she stays put.*
2. *When the follower observes a shallow book, she submits a small limit order at price A_1 if $\frac{2\theta_S\alpha\sigma}{\theta_S\alpha+1} \leq \Delta$ and stays put otherwise.*
3. *When the follower observes a deep book, she stays put.*

To understand this result, consider the case in which the follower faces a thin book (the analysis is similar when the follower observes a shallow or a deep book). If she submits a small limit order, her expected profit is:

$$\begin{aligned}\Pi(1; T, \theta_T) &= \theta_T[\alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)(A_1 - v_0)] + (1 - \theta_T)(A_1 - v_0), \\ &= A_1 - (v_0 + \theta_T\alpha\sigma) = \Delta - \theta_T\alpha\sigma.\end{aligned}\tag{6}$$

If, instead, she submits a large limit order, her expected profit is:

$$\begin{aligned}\Pi(2; T, \theta_T) &= \theta_T[2\alpha(A_1 - (v_0 + \sigma)) + E(\tilde{q}_l)(1 - \alpha)(A_1 - v_0)] \\ &\quad + (1 - \theta_T)E(\tilde{q}_l)(A_1 - v_0),\end{aligned}$$

because (a) a speculator exhausts the depth available at price A_1 and (b) a liquidity trader submits a market order with size $E(\tilde{q}_l)$, on average. As $E(\tilde{q}_l) = \frac{3}{2}$, we obtain (after some manipulations):

$$\begin{aligned}\Pi(2; T, \theta_T) &= \underbrace{\Pi(1; T, \theta_T)}_{\text{expected profit on the first round lot}} \\ &\quad + \underbrace{\left(\frac{\alpha\theta_T + 1}{2}\right)(A_1 - v_0 - \left(\frac{2\theta_T}{\alpha\theta_T + 1}\right)\alpha\sigma)}_{\text{expected profit on the second round lot}},\end{aligned}\tag{7}$$

The expected profit on a large limit order is smaller than that on a small limit order if $\Delta < \left(\frac{2\theta_T}{\theta_T\alpha+1}\right)\alpha\sigma$. Speculators exhaust the depth available at price A_1 , unlike liquidity traders. Hence, the second round lot of a large limit order is relatively more exposed to the risk of being picked off. For this reason, when the follower assigns a sufficiently large posterior

probability to the occurrence of an information event, she is better off restricting the size of her limit order. When $\Delta < \theta_T \alpha \sigma$, even a small limit order loses money [see Equation (6)] and staying put is optimal. When $\Delta = \theta_T \alpha \sigma$, the follower is indifferent between staying put and submitting a small limit order at A_1 . Thus, she plays a mixed strategy: she submits a small limit order with some probability denoted u_T .

When the limit order book is uninformative, the follower's belief about an information event is not affected by the leaders' offers. Thus, it is equal to her prior belief, θ_0 . To fix things, we assume that

$$\frac{2\theta_0\alpha\sigma}{\theta_0\alpha + 1} < \Delta. \quad (8)$$

This condition guarantees that the leader faces maximal competition from the follower because, *if the limit order book is uninformative*, the follower acts in such a way that two round lots are offered at the end of the bidding stage (see Lemma 1).⁹

2.2 A benchmark: volatility information is symmetric

When volatility information is public, the state of the limit order book does not convey new information, because all dealers have the same information. The follower's belief is either (i) $\theta_K = 1$ if there is an information event or (ii) $\theta_K = 0$ if there is no information event. As the follower does not learn information from the offers posted by the leader, she behaves in the same way in the anonymous and non-anonymous trading systems. For this reason, anonymity has no effect: market liquidity and the informativeness of the limit order book are identical in each regime. These claims are summarized in the following proposition.

Proposition 1. *(benchmark) Suppose dealers have symmetric information on future volatility.*

1. *In the anonymous and non-anonymous trading mechanisms, the unique subgame perfect equilibrium of the limit order market is as follows: (a) the dealer acting in stage L chooses schedule T if there is an information event and schedule D otherwise; (b) the follower acts as described in Lemma 1 with (b.1) $\theta_K = 1$ if there is an information event and (b.2) $\theta_K = 0$ if there is no information event.*
2. *The average small (resp. large) trade spread is identical in the anonymous and non-anonymous trading mechanisms.*
3. *The bid–ask spread is informative—that is, $\text{Cov}(|\tilde{V}_2 - v_0|, \tilde{S}_{\text{small}}) > 0$, and its informativeness is identical in the anonymous and non-anonymous trading mechanisms.*

⁹ Also, observe that the follower always fills the book in such a way that submitting another limit order is not profitable. Hence, increasing the number of followers cannot make the outcome “more competitive.”

When an information event is impending, posting a limit order at price A_1 is not profitable (as $A_1 < v_0 + \alpha\sigma$). Hence, the dealer acting in stage L establishes a thin book and the follower does not undercut his offer. Conversely, in absence of an information event, limit orders at price A_1 are profitable. Hence, the dealer acting in stage L posts a large limit order at price A_1 , anticipating that otherwise the follower would do so. These strategies imply that the bid–ask spread is more likely to be large when dealers expect an information event, which implies:

$$Cov(|\tilde{V}_2 - v_0|, \tilde{S}_{small}) = \sigma Cov(\tilde{I}, \tilde{S}_{small}) > 0. \quad (9)$$

2.3 Equilibria with asymmetric volatility information

Now, we consider the case in which volatility information is asymmetric. In this case, the follower learns volatility information from the limit order book, and this information is more precise when she observes the type of the leader. For this reason, order submission strategies depend on whether trading is anonymous or not. We first study the case in which trading is anonymous.

2.3.1 The anonymous limit order market. When an information event is impending, the informed dealer knows that submitting a limit order at price A_1 is not profitable. Hence, he sets a thin limit order book (schedule T). When there is *no* information event, the risk of being picked off is nil. The informed dealer can then post the competitive schedule D (which results in a small bid–ask spread) or he can attempt to reap a larger profit by posting a wide bid–ask spread (the less competitive schedule T), at the risk of being undercut by the follower. Formally, let λ (resp. $(1 - \lambda)$) be the probability with which the informed dealer chooses the competitive schedule D (resp. schedule T) when there is no information event.

The follower’s posterior belief conditional on observing a thin book, denoted $\theta_T(\lambda, \beta)$, is:

$$\begin{aligned} \theta_T(\lambda, \beta) &\equiv \text{prob}(I = 1 \mid K = T) \\ &= \left[\frac{(1 - \beta)\Phi_T + \beta}{(1 - \beta)\Phi_T + \beta(\theta_0 + (1 - \theta_0)(1 - \lambda))} \right] \theta_0. \end{aligned} \quad (10)$$

Observe that:

$$\theta_T(\lambda, \beta) \geq \theta_0. \quad (11)$$

Thus, when she observes a wide bid–ask spread, the follower marks up the probability of an information event and thereby her incentive to submit additional limit orders is weakened (the follower’s order size decreases in θ_T ; see Lemma 1). We call this effect of a wide spread the *deterrence effect*. Intuitively, a wide spread acts as a warning for the follower as the book is more likely to be thin when the informed dealer knows that an

information event is pending.¹⁰ When β is large enough, the deterrence effect is so strong that, in equilibrium, the follower can choose not to improve upon a wide bid–ask spread, as shown in the next proposition.

Proposition 2. *Let $\beta^* \equiv \frac{\Phi_T(r-\theta_0)}{(1-r)\theta_0 + \Phi_T(r-\theta_0)}$, $r \equiv \frac{\Delta}{\alpha\sigma}$, and $\lambda^*(\beta) \equiv (\frac{(\beta+(1-\beta)\Phi_T)}{\beta})(\frac{r-\theta_0}{r(1-\theta_0)})$. When $\beta > \beta^*$, the following order submission strategies constitute a perfect bayesian equilibrium in the anonymous market:*

1. *Given an information event, the informed dealer posts schedule T. Given no information event, the informed dealer posts schedule D with probability $\lambda^*(\beta)$ and schedule T with probability $(1 - \lambda^*(\beta))$.*
2. *When the book is thin, the follower submits a small limit order with probability $u_T^* = \frac{3}{4}$ and otherwise does nothing. When the book is shallow, the follower submits a small limit order. When the book is deep, the follower stays put.*

In equilibrium, the follower’s posterior belief when the leader sets a wide bid–ask spread is $\theta_T(\lambda^*(\beta), \beta)$. It turns out that:

$$\alpha\sigma\theta_T(\lambda^*(\beta), \beta) = \Delta. \quad (12)$$

Hence, in equilibrium, the follower is indifferent between undercutting the wide spread or staying put (see Lemma 1). For this reason, she follows a mixed strategy: she improves upon the wide spread sometimes but not always. Uncertainty on the follower’s behavior confronts the informed dealer with the following trade-off. If he posts a thin book, he obtains a large expected profit *in case of execution* but he takes the risk of being undercut. If he posts a deep book, the expected profit in case of execution is smaller but execution is guaranteed. In equilibrium, the probability of being undercut is such that these two actions yield the same expected payoff to the informed dealer when there is no information event. Thus, randomly choosing one of the two actions ($0 < \lambda^*(\beta) < 1$) is optimal for the informed dealer.

Intuitively, other things equal, a decrease in the informed dealer’s “participation rate,” β , reduces the informativeness of the limit order book and thereby weakens the deterrence effect. For this reason, when β is small enough ($\beta \leq \beta^*$), the follower always undercuts a wide bid–ask spread, as shown by the next proposition.

Proposition 3. *When $\beta \leq \beta^*$, the following order submission strategies constitute a perfect bayesian equilibrium in the anonymous market:*

¹⁰ A wide spread deters uninformed dealers from entering more competitive orders in the limit order book as it signals that the risk of being picked off is large. Hence, posting a wide spread is a form of limit pricing [see Milgrom and Roberts (1982) and Harrington (1987)].

1. *Given an information event, the informed dealer chooses schedule T . When there is no information event, the informed dealer chooses schedule D —that is, $\lambda = 1$.*
2. *When the book is thin, the follower submits, at price A_1 , a small limit order if $\beta^{**} < \beta \leq \beta^*$ and a large limit order if $\beta \leq \beta^{**}$ with $\beta^{**} \equiv \frac{\Phi_T(r(\alpha\theta_0+1)-2\theta_0)}{\theta_0(2-r(1+\alpha))+\Phi_T(r(\alpha\theta_0+1)-2\theta_0)} > 0$. For other states of the limit order book, the follower behaves as described in Proposition 2.*

In this case, a thin or a shallow book attract competition from the follower. For this reason, the informed dealer always posts a deep book whenever this is profitable. Given the informed dealer's order submission strategy, the follower's posterior belief about the likelihood of an information event, after observing a thin book, is $\theta_T(1, \beta)$. It is easily checked that:

$$\theta_T(1, \beta)\alpha\sigma \leq \Delta < \left(\frac{2\theta_T(1, \beta)}{\theta_T(1, \beta)\alpha + 1}\right)\alpha\sigma, \text{ for } \beta^{**} < \beta \leq \beta^*, \quad (13)$$

which implies that for $\beta \in (\beta^{**}, \beta^*]$, the follower optimally submits a small limit order when she observes a thin book (see Lemma 1).¹¹ Intuitively, the deterrence effect is too weak to deter the follower from undercutting the leader's offer but strong enough to deter her from posting a large limit order. If $\beta \leq \beta^{**}$, the deterrence effect is so weak that the follower submits a large limit order when the leader establishes a thin book, as she would when the limit order book is uninformative.

2.3.2 The non-anonymous limit order market. The equilibrium strategies in the non-anonymous market can be obtained by considering special cases of the analysis in the previous section. When $\beta = 0$ or $\beta = 1$, there is no uncertainty on the leader's type, even if trading is anonymous. Consequently, equilibrium strategies for the dealers when (a) $\beta = 1$ or (b) $\beta = 0$ in the anonymous market must be identical to those followed in the non-anonymous market when the leader is (a) an informed dealer or (b) a precommitted trader.¹² This remark yields the next result.

Proposition 4. *The following order submission strategies form a perfect bayesian equilibrium in the non-anonymous market:*

1. *When the leader is informed, the dealers behave as described in Proposition 2 when $\beta = 1$.*

¹¹ Note that $\beta^* < 1$ because $r < 1$ and $\beta^{**} > 0$ under Condition (8). Last, it is easily checked that $\beta^{**} < \beta^*$.

¹² The informed dealer never chooses a shallow book in the equilibria described in Section 2.3.1. Thus, when $\beta = 1$, the follower's belief conditional on observing a shallow book cannot be determined by Bayes' rule because a shallow book is out-of-the equilibrium path (an observation with a zero probability of occurrence). The equilibrium obtained by taking β to 1 in Proposition 2 is sustained by the following specification for the follower's belief after observing a shallow book: $\theta_S(\lambda, 1) = \theta_0$. This is natural as, in equilibrium, Bayes rule implies that $\theta_S(\lambda^*, \beta) = \theta_0$ for any value of β strictly less than 1.

2. When the leader is a pre-committed trader, the follower behaves as described in Proposition 3 when $\beta = 0$.

In the next section, we use the previous findings to analyze how liquidity and the informativeness of the bid–ask spread change following a switch from a non-anonymous to an anonymous limit order market (“a switch to anonymity”).

2.4 Testable predictions

First, we compare the expected small and large trade spreads [Equations (4) and (5)] in the anonymous market and in the non-anonymous market. We obtain the following result.

Corollary 1. *When traders have asymmetric information on future volatility, a switch to anonymity (a) reduces the expected small and large trade spreads when $\beta \leq \beta^{**}$, (b) reduces the expected small trade spread but increases the expected large trade spread when $\beta^{**} < \beta \leq \beta^*$, and (c) enlarges the expected small and large trade spreads when $\beta > \beta^*$.*

The intuition for this result is as follows. Anonymity prevents the follower from observing the leader’s type. Hence, her order submission strategy in the anonymous market is determined by the *likelihood*, β , that an informed dealer is active. If informed dealers’ participation rate is small enough ($\beta \leq \beta^{**}$), a wide bid–ask spread constitutes a weak signal that the risk of being of picked off is large because most likely quotes are posted by uninformed limit order traders. Accordingly, the follower submits a large limit order when she observes a wide bid–ask spread. Thus, she bids more aggressively than in the non-anonymous market when the leader posts a wide spread and turns out to be *informed* (if she knew that the leader was informed, the follower would indeed behave much more cautiously). As the informed dealer faces more intense competition, he also behaves more competitively than in the non-anonymous market. For these reasons, a switch to anonymity improves liquidity when $\beta \leq \beta^{**}$.

As β increases, it becomes more likely that quotes have been set by an informed dealer. Hence, a wide bid–ask spread constitutes a stronger warning for the follower. For this reason, if $\beta > \beta^{**}$, she only submits a small limit order, with probability 1 if $\beta \in (\beta^{**}, \beta^*]$ and probability $u_T^* = 3/4$ if $\beta > \beta^*$. Accordingly, the follower is *less* aggressive than in the non-anonymous market when the leader turns out to be *uninformed*. Thus, for $\beta > \beta^{**}$, the switch to anonymity increases the large trade spread on average because it reduces the frequency with which large limit orders are submitted. For $\beta > \beta^*$, it also reduces the probability of a small, but

aggressively priced, limit order. For this reason, in this case, the switch to anonymity enlarges the small trade spread as well.¹³

Corollary 2. *Assume that volatility information is asymmetric. In the non-anonymous market, the bid–ask spread is informative about future volatility ($\text{Cov}(|\tilde{V}_2 - v_0|, \tilde{S}_{\text{small}}) > 0$). In the anonymous market, the informativeness of the bid–ask spread (a) is nil when $\beta \leq \beta^*$ and (b) is strictly larger than its level in the non-anonymous market when $\beta > \beta^*$.*

Intuitively, the informativeness of the limit order book depends on the extent to which uninformed dealers contribute to its liquidity. Specifically, as uninformed dealers’ “participation rate” increases, quotes become less informative. When $\beta \leq \beta^*$, the follower intervenes *more* frequently in the anonymous system. Actually, in this case, she always undercuts the wide bid–ask spread in the anonymous trading system while she sometimes stays put in the non-anonymous system (when the wide bid–ask spread is posted by an informed dealer). As a result, the informativeness of the bid–ask spread is smaller in the anonymous trading system. In contrast, when $\beta > \beta^*$, the follower intervenes *less* frequently in the anonymous system (see the discussion following Corollary 1). Consequently, quotes in this system are more informative.

Corollaries 1 and 2 yield our main testable implications. First, Corollary 2 implies that, *in the non-anonymous market*, the size of the spread in a given period should be positively correlated with future price volatility. Second, Corollary 2 implies that the strength of this association should be altered by the switch to anonymity. In particular, when $\beta \leq \beta^*$, the switch to anonymity should result in a significant drop in the informativeness of the bid–ask spread.¹⁴ Last, taken together, Corollaries 1 and 2 yield a joint restriction on the evolution of bid–ask spreads and their information content after a switch to anonymity. Actually, for a fixed value of β , the *quoted* spread and its informativeness in the anonymous market are either both smaller or both larger than in the non-anonymous market. Thus, following a switch to anonymity, the size of the bid–ask spread and its informativeness should change in the *same* direction.

The positive association between the bid–ask spread and future volatility obtains when volatility information is public or private. When volatility information is public, however, a switch to anonymity should have no effect on the informativeness of the bid–ask spread or its size (Proposition 1).

¹³ A switch to anonymity makes the informed dealer more aggressive (that is, he posts the small spread more frequently). However, this effect is not sufficient to counterweight the decrease in the follower’s aggressiveness when $\beta > \beta^*$.

¹⁴ The informativeness of the bid–ask spread is nil in the anonymous market when $\beta \leq \beta^*$ because the limit order book is the only public source of information for uninformed dealers. When these dealers receive other public signals on future volatility, the informativeness of the bid–ask spread is smaller in the anonymous market when $\beta \leq \beta^*$, but not nil.

Thus, a test of the previous implications enables us to check whether the bid–ask spread contains information beyond that available from other public signals.

2.4.1 Other parameter values. We have analyzed in detail the case in which $\Delta < \alpha\sigma$. We obtain similar conclusions for other parameter values. In particular, consider the case in which $\alpha\sigma \leq \Delta < \frac{2\alpha\sigma}{\alpha+1}$. In this case, it is profitable to submit a small limit order at price A_1 when there is an information event. Thus, the informed dealer posts a shallow book when there is an information event. In this case, it is the shallow book (rather than the thin book) that signals that an information event is pending. But the implications are qualitatively identical to those derived when $\Delta < \alpha\sigma$. In particular, a lack of liquidity (manifested by an increase in the large trade spread) foreshadows an informational event. Furthermore, a switch to anonymity decreases the size and the informativeness of the large trade spread if β is small enough.¹⁵

3. Empirical Analysis

3.1 Institutional background and dataset

The Amsterdam Stock Exchange, the Brussels Stock Exchange, and the Paris Bourse merged in September 2000, giving birth to Euronext. The three exchanges decided then to harmonize their trading rules. This decision provided the impetus for the switch to anonymity of the Paris Bourse (Euronext Paris).

Euronext Paris uses a trading platform, called “Nouveau Système de Cotation” (NSC). NSC is an electronic limit order market, operating continuously for most of the stocks. Limit orders are submitted through brokers who trade for their own account or on behalf of other investors. They specify a limit price and a quantity to buy or to sell at the limit price. NSC also enables traders to submit hidden orders—that is, orders that display only a portion of their total size. Limit orders are stored in the limit order book and are executed in sequence according to price and time priority. If a limit order is marketable (i.e., if its price crosses a limit on the opposite side of the book), then it is immediately executed. If the size of a buy (resp. sell) marketable order exceeds the depth available at the best ask (resp. bid) price, then the order walks up (resp. down) the book until it is filled (entirely or partially, depending on its limit price and size).

¹⁵ When $\alpha\sigma \leq \Delta < \frac{2\alpha\sigma}{\alpha+1}$, the small trade spread is not affected by the switch to anonymity. We have focused on the case $\Delta < \alpha\sigma$ to show that a switch to anonymity affects, in general, both the small trade spread and the large trade spread. When $\frac{2\alpha\sigma}{\alpha+1} \leq \Delta$, the tick size is so large that it is always profitable to submit a large limit order at price A_1 , even if an information event occurs *with certainty*. In this situation, the deterrence effect has no bite.

Broker-dealers observe (on their computer terminals) all visible limit orders (price and associated depth) standing in the limit order book. Until April 23, 2001, the issuing broker's identifier was also displayed for each order. On this date, Euronext Paris ceased to disclose these identifiers. The switch to anonymity applied to all stocks listed on Euronext Paris and was the only major change in trading organization for the constituent stocks of the CAC40 index over our sample period. Thus, our study focuses on these stocks.¹⁶

The data (trades, quotes, and orders) are provided by Euronext Paris ("BDM database"). We use a time-stamped record of all transactions (prices and quantities), best bid and ask quotes, and quoted depth for the constituent stocks of the CAC40 index. We drop one stock from the sample because it was delisted from the index during the sample period. Our final data set comprises 39 stocks.

We use a 14 trading day pre-event sample (March 26 to April 12, 2001) and a 14 trading day post-event sample (April 30 to May 20, 2001). The two weeks of observations around April 23, 2001, are dropped to avoid contamination of our findings due to the proximity of the event date. The market may not have reached its new equilibrium one week after the structural change. We therefore repeat our analysis using a second post-event sample, also containing 14 trading days and extending from July 2 to July 19, 2001.

In all our treatments, we exclude observations collected during the first and last five minutes of the continuous trading period to avoid capturing effects due to the proximity of the opening and closing times. Some marketable limit orders are larger than the quoted depth and walk up or down the limit order book. In our dataset these orders are reported as multiple trades occurring at the same time but at different prices. Following Biais et al. (1995), we aggregate these multiple trades to a single transaction at the weighted average price.

Table 1 presents summary statistics. The figures reveal a high level of trading activity for the stocks in our sample. The average daily number of transactions is slightly lower after the switch to anonymity but exceeds 1,200 in all three sample periods. The share trading volume and the average trade size are higher in the post-event periods. The trading volume

¹⁶ For other stocks, counterparty IDs used to be disclosed immediately *after* completion of a transaction until April 23, 2001, but not after this date. Thus, other stocks have experienced a change in both pre-trade and post-trade anonymity. In contrast, post-trade anonymity has always been in force for CAC40 stocks. Thus, to better isolate the effects due to the change in pre-trade anonymity, we exclusively focus on CAC40 stocks. Minor additional changes in trading rules took place on April 23, 2001, for the stocks in our sample. The most important, maybe, is a change in the treatment of orders triggering a trading halt. Trading halts occur when price changes exceed pre-specified thresholds. Before April 23, 2001, traders had the possibility to submit marketable limit orders resulting in a halt *without* execution of their order. In contrast, as of April 23, 2001, marketable limit orders triggering a halt are partially executed up to the threshold price. As this change applies to all stocks, there is no obvious way to control for its possible effects.

Table 1
Sample summary statistics

	Pre-event	Post-event 1				Post-event 2		
	Mean	Mean	t-value	z-value		Mean	t-value	z-value
Number of trades	1,435	1,371	0.28	0.15		1,248	0.84	0.75
Trade price (€)	85,30	89,80	0.34	0.61		67,85	1.54	1.46
Trading volume (shares)	1,323,177	1,433,757	0.26	0.91		1,532,290	0.48	1.26
Trading volume (€ mio)	83	99	0.73	1.00		83	0.01	0.24
Average trade size (shares)	718	834	1.13	1.16		998	2.42	2.38
Daily return volatility	0.0063	0.0047	5.21	4.46		0.0048	4.38	4.23
Market capitalization (€ mio)	26,431	33,843	1.00	0.64		25,977	0.08	0.06

Table 1 reports cross-sectional daily averages for the variables listed in the first column. For each variable, we first calculate averages for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively, for both post-event periods. The pre-event period includes data from March 26, 2001, to April 12, 2001. The two post-event periods include data from April 30, 2001, to May 18, 2001, and from July 2, 2001, to July 19, 2001, respectively. In order to compute the number of trades, the trade price, and the average trade size, we treat transactions occurring at the same time as a single trade. The trade price is thus the volume-weighted price of all transactions occurring at the same time. Volatility is measured by the standard deviation of 30-minute midquote returns. For each post-event period, the last two columns report the test statistics (a *t*-test and a *z*-value for the Wilcoxon test) of the null hypothesis that the pre- and post-period differences in means and medians, respectively, are zero.

(in euros) increases between the pre-event period and the first post-event period but subsequently decreases. All differences are insignificant, however, with the exception of the average trade size (significantly larger in the second post-event period) and return volatility (significantly lower in the post-event periods).

3.2 Empirical findings

The model predicts that the switch to anonymity has changed (a) the size of quoted and effective spreads (Corollary 1) and (b) the informativeness of the quoted spread (Corollary 2). Moreover, these changes should have the same sign (see the discussion at the end of Section 3.4). We first study the effect of the switch to anonymity on the size of bid–ask spreads and then its effect on the informativeness of the quoted spread.

3.2.1 Anonymity and market liquidity.

3.2.1.1 Univariate analysis. Table 2 reports summary statistics on measures of bid–ask spreads before and after the switch to anonymity.

Quoted spreads (in euros and relative to the stock price) are lower in the post-event periods.¹⁷ The quoted spread decreases from 0.177 euros to 0.146 euros on average from the pre-event period to the first post-event

¹⁷ We sample the bid–ask spread each time there is a change in the size of the inside spread or in the quantities offered at the best quotes. We compute both equally weighted daily and time-weighted averages of the quoted spread. As the results for the two weighting schemes are virtually identical, we restrict the presentation to the equally weighted spread measures.

Table 2
Changes in spread and quoted depth (univariate analysis)

	Pre-event	Post-event 1				Post-event 2		
	Mean	Mean	t-value	z-value		Mean	t-value	z-value
Quoted spread €, equally weighted	0.177 (0.12)	0.146 (0.10)	1.36	1.34		0.112 (0.07)	3.12	3.16
Quoted percentage spread, equally weighted	0.22% (0.08%)	0.17% (0.06%)	3.67	3.35		0.17% (0.05%)	3.63	3.15
Effective spread, equally weighted	0.154 (0.11)	0.129 (0.08)	1.27	1.10		0.097 (0.06)	3.13	3.30
Depth (shares)	1,016 (759)	1,211 (850)	1.16	1.41		1,680 (6,803)	2.05	2.45
Depth (€)	74,176 (55,165)	93,556 (71,810)	1.45	1.50		98,342 (327,850)	1.33	1.04

Table 2 reports cross-sectional daily averages for the variables listed in the first column. We first calculate averages for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively, for both post-event periods. The pre-event period includes data from March 26, 2001, to April 12, 2001. The two post-event periods include data from April 30, 2001, to May 18, 2001, and from July 2, 2001, to July 19, 2001, respectively. Standard deviations of each variable (dispersion of the daily spread and depth across days and stocks) are given in parentheses. For each post-event period, the last two columns report the test statistics (respectively a *t*-test and a *z*-value for the Wilcoxon test) of the null hypothesis that the pre/post periods differences in means and medians, respectively, are zero.

period (a decline of 17.5%), and it decreases further to 0.112 euros in the second post-event period (the decline is significant only in the second post-event period). Percentage spreads decrease significantly between the pre-event period and each post-event period by about five basis points.

For each transaction, we compute the effective spread defined as:

$$Effective\ Spread = 2 * | P - m |,$$

where *m* is the midquote (the midpoint of the best bid and the best ask price) five seconds prior to the transaction and *P* is the transaction price. The effective spread differs from the quoted spread when a marketable order executes at multiple prices because the quoted depth is insufficient to fill the order in full. This variable is a proxy for the large trade spread in our model. In theory, effective spreads and quoted spreads could evolve in the same direction ($\beta \leq \beta^{**}$ or $\beta > \beta^*$) or in opposite directions after the switch to anonymity ($\beta^{**} < \beta \leq \beta^*$). Table 2 shows that on average, the effective spread decreases from 0.154 euros to 0.129 euros in the first post-event period and decreases further to 0.097 euros in the second post-event period.¹⁸ The difference between the pre-event period and the second post-event period is statistically significant.

¹⁸ The average effective spread is smaller than the average quoted spread as traders submit their market orders when the quoted spread is smaller than average. This observation is not due to trades occurring within the quoted spread (there are no price improvements in Euronext).

In our model, the effective spread is affected by a switch to anonymity because this switch affects the quoted depth. In reality, a decrease in the effective spread can be due both to (a) an increase in cumulative depth and (b) a decrease in trade size. To control for the impact of trade size, we sort the transactions by size, form deciles, and then calculate the cross-sectional average effective spread for each decile, before and after the switch to anonymity. The results are presented in Figure 2. Overall, the effective spread has decreased for each trade size decile and in each post-event period. The decrease is statistically significant only in the second post-event period. Thus, consistent with the model, the reduction in the effective spread appears to be driven by an increase in depth. In particular, we observe (last two lines of Table 2) that the quoted depth (measured

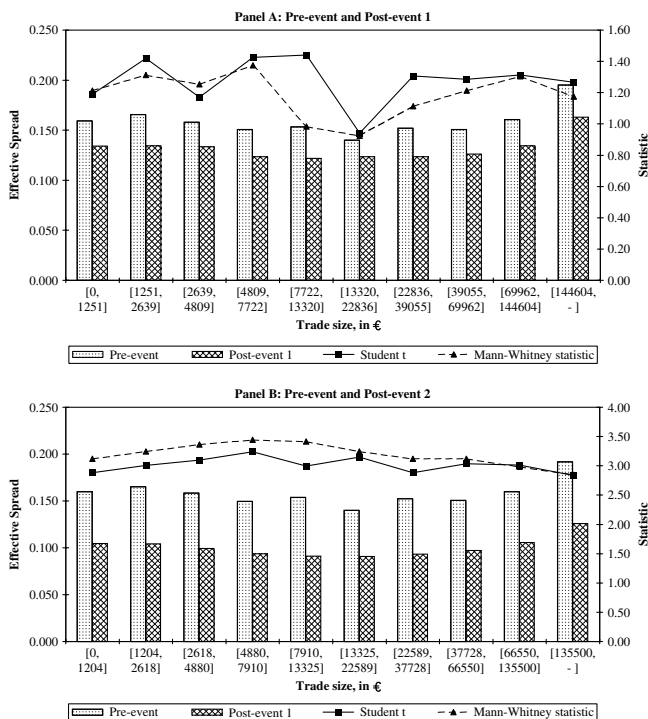


Figure 2
Effective spread

Figure 2 reports the cross-sectional daily average effective spread by trade size decile (trade size is measured in euros). We first calculate the average effective spread for each stock and each day. Then, we average over the 14 days of the pre-event period and the post-event period, respectively. The pre-event period includes data from March 26, 2001, to April 12, 2001. For Panel A, the post-event 1 period includes data from April 30, 2001, to May 18, 2001. For Panel B, the post-event 2 period includes data from July 2, 2001, to July 19, 2001. We also report the test statistics (a t -value and z -value for a Wilcoxon test) of the null hypothesis that the pre/post periods differences in means and medians, respectively, are zero.

in number of shares and in euros) has increased (not always significantly) after the switch to anonymity.

3.2.1.2 Regression analysis. Quoted and effective spreads decline after the switch to anonymity. This decline could be due to other factors than the switch to anonymity per se. Hence, we use a regression framework to measure the contribution of the switch to anonymity to the improvement in liquidity. Our regression model is:

$$s_{i,t} = \gamma_0 + \gamma_1 \log(Volu_{i,t}) + \gamma_2 TS_{i,t} + \gamma_3 P_{i,t} + \gamma_4 \sigma_{i,t} + \gamma_5 D_t^{post} + \varepsilon_{i,t}, \quad (14)$$

where $s_{i,t}$ is a measure of the spread, $Volu_{i,t}$ is the trading volume (in euros), $TS_{i,t}$ is the average tick size, $P_{i,t}$ is the price level, and $\sigma_{i,t}$ is the standard deviation of 30-minute midquote returns. All variables are calculated for each stock and each day (indices i and t identify the stock and the trading day, respectively). D_t^{post} is a dummy variable that captures the effect of the switch to anonymity on the bid–ask spread (it takes on the value 1 for the observations in the anonymous regime). We control for the effects of trading volume, the price level, and return volatility because several studies document the importance of these variables for bid–ask spreads [see Stoll (2000)]. As the tick size potentially affects the size of the spread, we also include the effective average tick size for stock i as explanatory variable.¹⁹

We estimate separate regressions for the two post-event periods and for the three spread measures described above. The results are reported in Table 3 (under the label “Regression 1”). To account for potential autocorrelation in the residuals, we compute t -statistics using Newey–West standard errors. The independent variables explain a large part of the variation in bid–ask spreads, as evidenced by R^2 s ranging from 0.63 to 0.87. All explanatory variables are significant and signed as expected. The coefficient on the post-event dummy is negative and significant in each case. The magnitude of this coefficient indicates that the switch to anonymity has reduced the quoted spread and the effective spread by about 0.02 euros in each post-event period. When compared to the average pre-event quoted and effective spread of 0.177 euros and 0.154 euros, respectively, the reduction in spreads is economically significant.

¹⁹ The minimum tick size is a function of the price level in Euronext. For instance, at the time of our study, the tick size was 0.01 euros for prices below 50 euros and 0.05 euros for prices between 50.05 and 100 euros. This implies that the tick size changes whenever the price of a stock crosses a threshold level. If the bid and the ask straddle a threshold price, the minimum tick size is different on the bid and the ask side of the book. Our effective tick size measure takes this into account as it is the average minimum tick size calculated from all bid and ask quotations for stock i on day t .

Table 3
Effect of anonymity on spreads

	Panel A: Pre-event and Post-event 1			
	Regression 1 : Baseline regression		Regression 2: Fixed effects	
	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Effective spread in €, equally weighted
Constant	0.101* (16.85)	0.343* (38.50)	0.053* (7.26)	0.128* (11.19)
Log(volume)	-0.031* (-23.39)	-0.047* (-28.06)	-0.019* (-13.77)	-0.020* (-8.21)
Tick Size	0.561* (4.58)	0.563* (5.50)	0.681* (5.27)	1.152* (6.17)
Trade Price	0.0015* (15.74)	-0.0003* (-5.21)	0.0011* (14.98)	0.0004* (2.28)
Volatility	7.295* (13.74)	9.680* (13.04)	7.008* (10.27)	5.471* (10.12)
Post-event 1 (median of the daily dummies for Specification 3)	-0.024* (-7.81)	-0.027* (-8.52)	-0.018* (-5.60)	-0.025* (-10.56)
Number of negative daily dummies				(-12.28)
Number of significantly negative daily dummies at 5%				(-5.49)
Adj. R2	0.86	0.64	0.71	0.89
				0.73
				0.75
				0.90
				0.74
				0.75
				14
				14
				14
				11
				14
				0.74
				0.75

Table 3
(Continued)

Panel B: Pre-event and Post-event 2									
Regression 1 : Baseline regression			Regression 2: Fixed effects			Regression 3: Fixed effects and day dummies for the post-sample period			
Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	
Constant	0.092* (14.45)	0.345* (36.09)	0.050* (7.36)	0.089* (9.42)	0.304* (22.97)	0.040* (3.01)	0.091* (8.70)	0.303* (20.50)	0.039* (2.66)
Log(volume)	-0.030* (-21.44)	-0.047* (-24.81)	-0.020* (-16.39)	-0.020* (-8.55)	-0.030* (-8.72)	-0.009* (-2.86)	-0.021* (-7.74)	-0.030* (-7.52)	-0.009* (-2.40)
Tick Size	0.308* (2.33)	0.528* (4.87)	0.476* (3.51)	-0.040* (-0.25)	0.387* (3.56)	0.218* (1.23)	-0.044* (-0.27)	0.379* (3.38)	0.211* (1.19)
Trade Price	0.0017* (16.06)	-0.0003* (-4.97)	0.0013* (16.39)	0.0018* (14.28)	-0.0003* (-0.07)	0.0014* (14.05)	0.0018* (14.17)	-0.0003* (-3.74)	0.0014* (13.98)
Volatility	7.029* (14.45)	10.071* (14.06)	6.756* (9.68)	5.815* (11.16)	8.771* (11.42)	4.498* (8.30)	5.825* (10.91)	8.721* (11.10)	4.453* (7.96)
Post-event 2 (median of the daily dummies for Specification 3)	-0.023* (-9.10)	-0.033* (-10.49)	-0.019* (-8.01)	-0.026* (-11.74)	-0.035* (-12.04)	-0.024* (-9.62)	-0.026* (-9.62)	-0.035* (-9.62)	-0.023* (-9.62)

Table 3
(Continued)

Panel B: Pre-event and Post-event 2											
Regression 1 : Baseline regression				Regression 2: Fixed effects				Regression 3: Fixed effects and day dummies for the post-sample period			
Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Quoted spread in €, equally weighted	Quoted percentage spread, equally weighted (in %)	Effective spread in €, equally weighted	Effective spread in €, equally weighted
0.87	0.63	0.73	0.90	0.70	0.76	0.90	0.71	0.90	0.71	0.77	0.77
Number of negative daily dummies											
Number of significantly negative daily dummies at 5%											
Adj. R2											

Table 3 presents the estimates of the regression model defined in Equation (14) and reported below:

$$s_{i,t} = \gamma_0 + \gamma_1 \log(\text{Vol}_{i,t}) + \gamma_2 \text{TS}_{i,t} + \gamma_3 \text{P}_{i,t} + \gamma_4 \sigma_{i,t} + \gamma_5 D_t^{\text{post}} + \varepsilon_{i,t}$$

where $s_{i,t}$ is a measure of the spread, $\text{Vol}_{i,t}$ is the trading volume (in euro), $\text{TS}_{i,t}$ is the average tick size, $\text{P}_{i,t}$ is the price level, and $\sigma_{i,t}$ is the standard deviation of 30-minute midquote returns. All variables are calculated for each stock and each day (indices i and t identify the stock and the trading day, respectively). D_t^{post} is a dummy variable that captures the effect of the switch to anonymity on the bid-ask spread (it takes on the value 1 for the observations in the anonymous regime). Panel A [B] reports the results of the regressions for the pre-event period and the post-event 1 [post-event 2] period. The pre-event period includes data from March 26, 2001, to April 12, 2001. The two post-event periods include data from April 30, 2001, to May 18, 2001, and from July 2, 2001, to July 19, 2001, respectively.

In Regression 1, we report the results of an OLS regression of each spread measure on a set of control variables. In Regression 2, we allow for stock-specific intercepts. In Regression 3, we control for cross-correlation by introducing 14 dummy variables T_t that equal 1 if the day is t (in the post-event period) and 0 otherwise. We omit the estimates of the intraday dummies and the fixed effects. However, in Regression 3, we report the median of the day dummy variables. We compute Newey-West standard errors with lag 2 to control for heteroskedasticity and autocorrelation. Corresponding t -statistics are reported in parentheses. “*” denotes significance at the 5% level.

The presence of a fixed stock effect can be a source of correlation in the residuals of a given stock. We therefore allow for fixed stock effects in our regression by including stock-specific dummy variables. The results are also presented in Table 3 (“Regression 2”; we omit the coefficients on the stock-specific dummy variables to conserve space). The coefficient on the dummy variable capturing the impact of a switch to anonymity remains significantly negative and equal to about -0.02 euros for the quoted spread and the effective spread.

The residuals in our regressions may also be contemporaneously correlated across stocks because the switch to anonymity affects all stocks at the same time. To address that concern, we include separate dummy variables for each day of the post-event period, as in Boehmer et al. (2005). We also allow for stock fixed effects by including a dummy variable for each stock. Testing the median of the 14 post-event dummy variables against zero provides a robust test of the hypothesis that spreads are lower in the post-event period.²⁰ We show the results in the last three columns of Table 3 (“Regression 3”). Each of the post-event dummy variables is negative. Furthermore, the median of these post-event dummy variables is negative and significantly different from zero. Again, the value of the median of the post-event dummy variables indicates that the switch to anonymity has reduced quoted and effective spreads by about 0.02 euros.

To sum up, the multivariate analysis indicates that the switch to anonymity has reduced *both* quoted spreads and effective spreads by about 0.02 euros. This reduction is consistent with our model when $\beta \leq \beta^{**}$ (see Corollary 1). Thus, we expect a decline in the informativeness of the bid–ask spread on future volatility (Corollary 2).

3.2.2 The spread as a signal of future price changes. We now test this prediction and more generally those pertaining to the informativeness of the bid–ask spread about future volatility. We use the following methodology. We partition each trading day into fifteen 30-minute intervals and two 25-minute intervals (the first and last interval). As in our model, we measure the magnitude of the price change in interval τ for stock i by $Vol_{i,\tau} = |m_{i,\tau} - m_{i,\tau-1}|$ where $m_{i,\tau}$ is the midquote at the

²⁰ Another way to control for contemporaneous correlation [also proposed by Boehmer et al. (2005)] is to aggregate the data across stocks. This results in a time-series regression with 28 observations, one for each trading day. We estimate this model and find the post-event dummy to be negative and significant. Results are omitted for brevity.

end of interval τ .²¹ We then estimate the following regression model:

$$Vol_{i,\tau+1} = a_0 + a_1 Vol_{M,\tau} + a_2 Vol_{i,\tau} + a_3 N_{i,\tau} + a_4 ATr_{i,\tau} + (a_5 + a_6 D^{post})s_{i,\tau} + \sum_{k=2}^{k=16} b_k T_{k,\tau} + \sum_{i=2}^{i=39} c_i D_i + \varepsilon_{i,\tau}, \quad (15)$$

where, for stock i and interval τ , $N_{i,\tau}$ is the number of transactions, $ATr_{i,\tau}$ is the average trade size, and $s_{i,\tau}$ is the quoted bid–ask spread in euros. $Vol_{M,\tau}$ is the market volatility, defined as the absolute change in the value of an equally weighted index of the sample stocks (calculated using midquotes). D^{post} is a dummy variable equal to 1 in the post-event period and zero in the pre-event period. $T_{k,\tau}$ is a trading interval dummy equal to 1 if $k = \tau$, and the D_i are stock-specific dummy variables allowing for stock fixed effects. We include lagged variables in the set of explanatory variables only to avoid a simultaneity bias.

$Vol_{i,\tau+1}$ is a measure of *ex-post* volatility in interval $\tau + 1$. Many other studies develop measures of *ex-post* volatility based on absolute returns [e.g., Jones et al. (1994) or Ahn et al. (2001)]. It is well known that there are systematic intraday patterns and clustering in volatility. We include the trading interval dummies, $T_{k,\tau}$, and the lagged volatility, $Vol_{i,\tau}$, in the set of independent variables to control for these effects. Serial dependence in the news arrival process could induce correlation between volatility and trading volume [see Bollerslev et al. (1994)] because both variables are determined by the rate of arrival of new information. Hence, several authors have used measures of trading activity to forecast future price volatility [for instance, Bollerslev and Domowitz (1993)]. Here, we use the number of trades and the average trade size as measures of trading activity because Jones et al. (1994) suggest these variables have different informational content for future volatility. Finally, we include a measure of market volatility, $Vol_{M,\tau}$, in the set of explanatory variables to control for commonalities in volatility changes across stocks (Black (1976)).

The last explanatory variable—that is, the lagged quoted spread in a given period ($s_{i,\tau}$)—is the main focus of this section.²² The model has three predictions regarding the effect of this variable. First, in the non-anonymous market, the bid–ask spread is informative about future volatility (Corollary 2). Thus, we expect $a_5 > 0$. Second, we expect the strength of this relationship to be smaller after the switch to anonymity.

²¹ We exclude the overnight return from the sample. Thus, for the first interval of each trading day, the change in midquote over this interval is calculated as the difference between the last midquote of the interval and the *first* midquote of the interval. We have also used 15-minute intervals for all the tests reported in this section. Results are qualitatively similar to those reported for 30-minute intervals and are omitted for brevity.

²² Our empirical findings are similar when we use the effective spread instead of the quoted spread.

To test this prediction, we interact the coefficient on the quoted spread with a dummy variable (D_i^{post}) equal to 1 after the switch to anonymity and we test whether the coefficient on this variable is negative—that is, $a_6 < 0$. Last, in the model, the bid–ask spread cannot be negatively related to future volatility. Thus, we expect $a_5 + a_6$ to be non-negative. We do not exclude the possibility that in the *anonymous market*, the bid–ask spread is *not* informative about future volatility (i.e., $a_5 + a_6 = 0$). In fact, this occurs in the model when $\beta < \beta^*$ (see Corollary 2).

Table 4 (“Regression 1”) reports the results for each post-event period. The coefficients for the trading intervals and stock specific dummy variables are jointly significant. We do not report their estimates to save space.

Consistent with our first hypothesis, we find that the size of the spread in the pre-event period is positively and significantly related to future volatility (e.g., $a_5 = 0.64$ for the second post-event period). In fact, the R^2 of the regression falls when the bid–ask spread is not used as an explanatory variable (see the two last lines of Table 4). As predicted by our second hypothesis, the sensitivity of future price volatility to the size of the spread is significantly smaller in the anonymous regime (e.g., $a_6 = -0.57$ for the second post-event period). Last, we cannot reject the null hypothesis $a_5 + a_6 \geq 0$ in favor of the alternative $a_5 + a_6 < 0$. In fact, the results suggest that the bid–ask spread loses its informativeness after the switch to anonymity because $a_5 + a_6$ is not different from zero for each post-event period.

The pooled regression analysis restricts the effect of anonymity to be uniform across stocks. The model indicates that cross-sectional variations in β can in principle generate cross-sectional variations in the sign of the effect of anonymity on the informativeness of the bid–ask spread (a_6). To explore this possibility, we also estimate individual regressions for all sample stocks. The findings are summarized in Columns 3 and 4 of Table 4.

Our main result is confirmed in these individual regressions. When comparing the pre-event period to the first [second] post-event period, the coefficient on the lagged spread, a_5 , is positive in 39 [38] out of 39 cases and significant at the 10% level or better in 22 [23] cases. The mean of the coefficient values is 0.63 [0.63]. The coefficient on the interaction term, a_6 , is negative in 37 [37] cases and significantly so in 22 [31] cases. The mean value is -0.36 [-0.80].²³ Consistent with the model and the results of the pooled regression, we do not reject the null hypothesis $a_5 + a_6 \geq 0$. The sum of the coefficients is significantly positive in the first post-event period and not significantly different from zero in the second (the t -values from a cross-sectional t -test are 3.89 and 1.02, respectively).

²³ A possible concern is that there may be contemporaneous correlation among the residuals for different stocks. To address this issue we analyze the residuals from the separate regressions for each stock. The mean of the 741 pairwise correlations is 0.059 [0.057], suggesting that contemporaneous correlation of the residuals does not pose problems.

Table 4
Bid-ask spreads, future volatility, and anonymity

	Regression 1: Volatility in $[\tau - 1, \tau]$ defined as $ m_{i, \tau} - m_{i, \tau-1} $				Regression 2: Volatility in $[\tau - 1, \tau]$ defined as $ \mu_{i, \tau} $			
	Pooled regression		Summary of individual regressions		Pooled regression		Pooled regression	
	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2
Volatility in $[\tau, \tau + 1]$								
Constant	0.19* (9.05)	0.14* (6.87)	0.21	0.24	0.11* (6.45)	0.22* (11.01)		
Volatility in $[\tau - 1, \tau]$	0.11* (7.23)	0.13* (7.70)	0.07	0.08	0.10* (6.18)	0.11* (6.04)		
Average spread in $[\tau - 1, \tau]$	0.37* (6.84)	0.64* (11.95)	0.63 [39/22]	0.63 [38/23]	0.44* (7.43)	0.76* (13.13)		
Average spread in $[\tau - 1, \tau]$ Dummy Post	-0.38* (-9.30)	-0.57* (-13.73)	-0.36 [37/22]	-0.80 [37/31]	-0.38* (-8.48)	-0.59* (-13.10)		
Number of trades in 1,000 in $[\tau - 1, \tau]$	0.237* (3.75)	0.287* (4.51)	0.597	0.359	0.208* (3.29)	0.266* (4.05)		
Average trade size in 1,000 shares in $[\tau - 1, \tau]$	0.005 (1.67)	-0.004* (-3.67)	-0.000	-0.000	0.005 (1.79)	-0.003* (-3.26)		
Market volatility in $[\tau - 1, \tau]$	0.07* (3.44)	0.04 (1.50)	0.083	0.062	0.07* (3.44)	0.04 (1.71)		
Average spread in $[\tau - 1, \tau]$ (1 + Dummy Post)	-0.01 (-0.20)	0.07 (1.16)	0.28 (3.89)	-0.18 (-1.02)	0.07 (1.12)	0.17 (2.46)		

Table 4
(Continued)

Volatility in $[\tau, \tau + 1]$	Regression 1: Volatility in $[\tau - 1, \tau]$ defined as $ m_{i,\tau} - m_{i,\tau-1} $				Regression 2: Volatility in $[\tau - 1, \tau]$ defined as $ u_{i,\tau} $			
	Pooled regression		Summary of individual regressions		Pooled regression		Pooled regression	
	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2	Panel A: Pre-event and Post-event 1	Panel B: Pre-event and Post-event 2
R ² of the regression with spread and interaction term	0.2596	0.2554	0.1255	0.1490	0.2581	0.2527		
R ² of the regression without spread and interaction term	0.2465	0.2186			0.2451	0.2112		

For each stock in our sample, we partition each trading day into fifteen 30-minute intervals and two 25-minute intervals. Using two measures of volatility, we estimate the regression model defined in Equation (15) and reported below:

$$Vol_{i,\tau+1} = a_0 + a_1 Vol_{M,\tau} + a_2 Vol_{i,\tau} + a_3 N_{i,\tau} + a_4 ATr_{i,\tau} + (a_5 + a_6 D^{post})s_{i,\tau} + \sum_{k=2}^{16} b_k T_{k,\tau} + \sum_{i=2}^{39} c_i D_i + \varepsilon_{i,\tau}$$

where $s_{i,\tau}$ is the average quoted spread in interval τ , D^{post} is a dummy variable equal to 1 in the post event-period and zero in the pre-event period, $N_{i,\tau}$ is the number of transactions in interval τ , $ATr_{i,\tau}$ is the average trade size in interval τ , and $Vol_{M,\tau}$ is a proxy for the market volatility in interval τ defined as:

$$Vol_{M,\tau} = \frac{1}{39} \sum_{i=1}^{39} |(m_{i,\tau} - m_{i,\tau-1})|$$

D_i is a dummy variable equal to 1 when the stock is i and 0 otherwise, and $T_{k,\tau}$ is a dummy variable that is 1 when the interval is k and 0 otherwise. Panel A [B] reports the results of the regressions for the pre-event period and the post-event 1 [post-event 2] period. The pre-event period includes data from March 26, 2001, to April 12, 2001. The two post-event periods include data from April 30, 2001, to May 18, 2001, and from July 2, 2001, to July 19, 2001, respectively. In Regression 1, we measure price volatility in any interval $[\tau - 1, \tau]$ for stock i by $Vol_{i,\tau} = |m_{i,\tau} - m_{i,\tau-1}|$ where $m_{i,\tau}$ is the midquote at the end of interval τ . Columns 1 and 2 show the results of the pooled regression. Columns 3 and 4 summarize the results obtained when estimating the model separately for each stock. Regression coefficients are cross-sectional averages of the coefficients across the 39 stocks. For the bid-ask spread and the spread interacted with the dummy post, we report in brackets first the number of coefficients whose signs are as expected (positive for the spread, negative for the interaction term), and second the number of coefficients whose signs are as expected and which are significant at the 10% level or better. In Regression 2, the dependent variable is the absolute value of the residual of a regression of the changes in midquotes on its lagged value (see Equation (16)). Otherwise the specification is as in columns 1 and 2. t -statistics are reported in parentheses. “*” denotes significance at the 5% level. To save space, we omit the estimates of the intraday dummies and the fixed effects.

Last, we also estimate the cross-sectional correlation between (i) the change in the size of the bid–ask spread and (ii) the change in its informativeness following the switch to anonymity. We expect this correlation to be positive since, for a given value of β , the switch to anonymity should affect the size of the bid–ask spread and its informativeness in the same direction. Specifically, we estimate the impact of the switch to anonymity on the bid–ask spread (γ_5 in Equation (14)) for each stock separately and combine the results with those for a_6 (Columns 3 and 4 of Table 4). We find that *both* the size and the informativeness of the bid–ask spread decrease for 31 [32] of the 39 sample stocks when considering the first [second] post-event period. As predicted, the cross-sectional correlation between coefficients γ_5 and a_6 is positive (0.21 and 0.74 when considering the first and the second post-event period, respectively).²⁴

Overall, the findings support our main predictions: (i) the size of the spread is positively related to the magnitude of future price changes in the non-anonymous trading system and (ii) the strength of this association is significantly smaller after the switch to anonymity. Moreover, as expected, liquidity and the informativeness of the bid–ask spread have evolved in the same direction following the switch to anonymity. As discussed at the end of Section 3.4, these findings are consistent with a scenario in which the limit order book contains volatility information, beyond that available from other public sources.

3.2.3 Robustness tests.

3.2.3.1 Alternative measures of volatility. Various market microstructure effects can induce transient deviations of midquotes from the fair value of the security. At high frequency, these transient deviations result in a negative correlation in midquote returns [see Hasbrouck (1993)], implying that mean changes in midquotes are predictable. To account for this possibility, we modify our baseline methodology as follows. We run the following regression for each stock:

$$\Delta m_{i,\tau+1} = a_i + b_i \Delta m_{i,\tau} + u_{i,\tau+1}, \quad (16)$$

where $\Delta m_{i,\tau}$ is the change in midquotes for stock i in interval τ . We then use the absolute value of the residuals in each interval ($|u_{i\tau}|$) as our measure of ex-post volatility in interval τ . Using this alternative measure of volatility we repeat the previous analysis. The results are provided in Table 4 (“Regression 2”). Clearly, they are very similar to those presented

²⁴ We eliminate one outlier when calculating the correlation for the second post-event period. One stock has a large positive coefficient γ_5 . This coefficient has the largest absolute value of all 39 coefficients but is not significantly different from 0. Including this observation reduces the correlation from 0.74 to -0.01 .

earlier (“Regression 1”). We also repeat our baseline analysis with *ex-post* volatility measured by (a) the squared changes in midquotes or (b) the absolute change in the logarithm of midquotes. The results are qualitatively unchanged but the regressions R^2 are smaller. For brevity, we do not report the results in these cases.

3.2.3.2 Transitory vs. permanent volatility. Our model predicts that the bid–ask spread contains information on the volatility of the “efficient price.” Empirically, we observe changes in midquotes, not changes in the efficient price. This raises the possibility that the bid–ask spread contains information on transitory price changes rather than permanent price changes. This is a concern if transitory price changes largely contribute to the volatility of midquote changes. To gauge this contribution, consider the following simple model for the changes in midquotes [in the spirit of Hasbrouck (1993)].²⁵ Let $\tilde{m}_{i,\tau}$ and $\tilde{v}_{i,\tau}$ be respectively the midquote and the efficient price at the end of interval τ for stock i . We have $\tilde{m}_{i,\tau} = \tilde{v}_{i,\tau} + \tilde{d}_{i,\tau}$ where $\tilde{d}_{i,\tau}$ is the deviation between the midquote and the efficient price due to market microstructure effects. The efficient price, $\tilde{v}_{i,\tau}$, follows a random walk and the $\tilde{d}_{i,\tau}$ are i.i.d with variance σ_{id}^2 . Innovations in the efficient price, $\tilde{\omega}_{i,\tau} \equiv \Delta \tilde{v}_{i,\tau}$, and the $\tilde{d}_{i,\tau}$ are independent. In this case, the volatility of midquote changes is given by $Var(\Delta m_{i,\tau}) = \sigma_{i,\omega}^2 + 2\sigma_{i,d}^2$. The first-order autocorrelation between midquote changes is:

$$corr(\Delta m_{i,\tau}, \Delta m_{i,\tau-1}) = -\frac{\sigma_{id}^2}{Var(\Delta m_{i,\tau})}. \quad (17)$$

Thus, $2 * |corr(\Delta m_{i,\tau}, \Delta m_{i,\tau-1})|$ is equal to the fraction of total volatility due to transitory volatility. We estimate $corr(\Delta m_{i,\tau}, \Delta m_{i,\tau-1})$ in our sample for each stock separately. For the sample defined over the pre-event period and the first post-event period, we find that even at a 10% level of significance only 9 out of a total of 39 correlations are significant. We obtain the same result for the sample defined over the pre-event and the second post-event period. Thus, at the data frequency we use, transitory volatility accounts for only a small fraction of the volatility of midquote changes.

3.2.3.3 GARCH specification. Many empirical studies model time-varying conditional variances of returns using the (GARCH) framework (see Bollerslev et al. (1994) for a survey). It is of interest to check whether our findings are robust within this framework. To this end, we estimate

²⁵ Hasbrouck (1993) considers transaction prices instead of midquotes. He does not require pricing errors to be independent from innovations in the efficient price as we do here.

the following GARCH(1,1) model:

$$\Delta q_{i,\tau+1}^a = \mu_i + \theta_i \Delta q_{i,\tau}^a + \eta_{i,\tau+1} \quad (18)$$

$$\sigma_{i,\tau+1}^2 = \omega_i + \lambda_i \eta_{i,\tau}^2 + \gamma_i \sigma_{i,\tau}^2 + \delta_1 Vol_{M,\tau}^a + \delta_2 N_{i,\tau} + \delta_3 ATr_{i,\tau} + (\delta_4 + \delta_5 D^{post}) s_{i,\tau} \quad (19)$$

$$\eta_{i,\tau+1} | \Psi_\tau \sim N(0, \sigma_{i,\tau+1}^2).$$

The formulation for the conditional mean equation accounts for first-order autocorrelation in the changes in midquotes. Equation (19) models movements in conditional volatility using a GARCH(1,1) with exogenous explanatory variables (e.g., $s_{i,\tau}$) that are identical to those in our baseline regression model. As for the effect of the bid–ask spread, we expect to find that $\delta_4 > 0$, $\delta_5 < 0$ and $\delta_4 + \delta_5 \geq 0$. There are some differences to the baseline model. First, to be closer to the standard specification in GARCH modeling, we focus on percentage returns for the midquotes by taking a logarithmic transformation of the midquotes series [i.e., $\Delta q_{i,\tau+1} \equiv \log(m_{i,\tau+1}) - \log(m_{i,\tau})$]. Second, we control for intraday seasonalities by using *adjusted returns* (as suggested by Engle (2000)). That is, we regress the midquote returns on a set of time-of-day dummies and retain the fitted values of the regression. Then, in each interval, we divide the actual midquote return by the fitted value to obtain the adjusted midquote return, $\Delta q_{i,\tau}^a$. Market volatility, $Vol_{M,\tau}^a$, is also measured using adjusted returns.

We estimate the model for each post-event period and for each stock separately. We summarize the main findings in Table 5. When comparing the pre-event period to the first [second] post-event period, the coefficient on the lagged spread, δ_4 , is positive in 26 [31] out of 39 cases and significant at the 10% level or better in 16 [20] cases. The mean of the coefficient values is 2.86 [2.56]. The coefficient on the interaction term, δ_5 , is negative in 36 [34] cases and significantly so in 17 [19] cases. The mean value is -1.96 [-1.77]. These results confirm the conclusions of the baseline regressions. The lagged bid–ask spread is positively related to future price volatility, but the strength of this relationship is smaller in the anonymous trading environment.²⁶ Consistent with the model and our previous results, the null hypothesis $\delta_4 + \delta_5 \geq 0$ is not rejected in favor of the alternative $\delta_4 + \delta_5 < 0$. In the first post-event period the sum of the coefficients is significantly positive (t -value: 2.38) whereas in the second post-event period it is not significantly different from zero (t -value: 1.41). These results are similar to those of the stock-specific OLS regressions.

²⁶ Bollerslev and Domowitz (1993) estimate a GARCH(1,1) model in the deutsche mark–dollar market that is conceptually close to our Equation (19). Interestingly, they also find a positive and significant contribution of the bid–ask spread to movements in conditional volatility.

Table 5
Bid-ask spreads, future volatility, and anonymity: GARCH results

		Panel A : Pre-event and Post-event 1	Panel B : Pre-event and Post-event 2
Average spread in $[\tau - 1, \tau]$	Mean	2.86	2.56
	Median	1.15	1.24
	# >0	26	31
	#>0 and significant 10%	16	20
	# <0	13	8
	#<0 and significant 10%	0	1
Average spread in $[\tau - 1, \tau]^*$ Dummy Post	mean	-1.97	-1.77
	median	-0.90	-0.60
	# <0	36	34
	#<0 and significant 10%	17	19
	# >0	3	5
	#>0 and significant 10%	2	1
Average spread in $[\tau - 1, \tau]^*$ (1 + Dummy Post)	t-values	2.38	1.41

For each stock in our sample, we partition each trading day into fifteen 30-minute intervals and two 25-minute intervals. We estimate the GARCH(1,1) model defined in Equations (18) and (19) and reported below for individual stocks:

- (1) $\Delta q_{i,\tau+1}^a = \mu_i + \theta_i \Delta q_{i,\tau}^a + \eta_{i,\tau+1}$
- (2) $\sigma_{i,\tau+1}^2 = \omega_i + \lambda_i \eta_{i,\tau}^2 + \gamma_i \sigma_{i,\tau}^2 + \delta_1 \text{Vol}_{M,\tau}^a + \delta_2 N_{i,\tau} + \delta_3 \text{ATr}_{1,\tau} + \delta_4 + \delta_5 \text{DPost}_{s_{i,\tau}}$
- (3) $\eta_{i,\tau+1} | \psi_\tau \sim N(0, \sigma_{i,\tau+1}^2)$

The dependent variable is the (adjusted) midquote return. To compute it, we first take a logarithmic transformation of the midquotes series. Second, to control for intraday seasonalities we regress the midquote returns on a set of time-of-day dummies and we retain the fitted values of the regression. Then, in each interval, we divide the actual midquote return by the fitted value to obtain the adjusted midquote return. The mean equation (1) includes the lagged midquote return. The variance equation (2) includes the same explanatory variables as our baseline model -i.e., the variables of interest (the lagged quoted spread and an interaction term) and a set of control variables (lagged market volatility, measured by the adjusted midquote return of an equally weighted portfolio of the sample stocks, the lagged number of trades, and the lagged average trade size).

Panel A [B] reports the results of the regressions for the pre-event period and the post-event 1 [post-event 2] period. The pre-event period includes data from March 26, 2001, to April 12, 2001. The two post-event periods include data from April 30, 2001, to May 18, 2001, and from July 2, 2001, to July 19, 2001, respectively.

The table presents summary statistics for the variables of interest.

To sum up, the robustness tests confirm the findings in the baseline analysis: (i) the bid–ask spread predicts future volatility, and (ii) the forecasting power of the spread is lower in the anonymous regime.

4. Conclusions

Cautious bidding by limit order traders with volatility information signals that they expect an increase in volatility. In turn, it induces less-informed limit order traders to shade their own bids. In this framework, we show that a switch to anonymity can increase or decrease the frequency with which uninformed traders decide to improve upon non-aggressive quotes (a “wide bid–ask spread”). For this reason, a switch to anonymity alters (i) the size of the quoted spread and (ii) the informativeness of the quoted spread on future volatility (i.e., the correlation between these two

variables). The direction of the impact of anonymity on these variables depends on informed traders' participation rate. However, other things equal, this direction should be *identical* for both variables.

We exploit the decision of Euronext Paris to conceal limit order traders' identifiers to test these predictions. For a sample of 39 actively traded stocks, a pooled regression analysis reveals that:

1. Quoted and effective spreads are significantly smaller after Euronext's switch to anonymity.
2. There is a positive and significant relationship between price volatility and the *lagged* bid–ask spread when trading is non-anonymous.
3. The strength of this relationship is significantly weaker after the switch to anonymity. Hence, the informativeness of the bid–ask spread and its size have changed in the same direction after the switch to anonymity.

These empirical findings are in line with the predictions of the model and are not easily explained by alternative theories. When information on future volatility is symmetric, the limit order book is informative because limit orders reflect publicly available volatility information. However, in this case, we show that a switch to anonymity should have no impact on market liquidity and the informativeness of the bid–ask spread, an implication at odds with our empirical findings. Rather, these findings lend support to the hypothesis that volatility information is asymmetric. In this case, the limit order book contains volatility information beyond that publicly available.

There are several interesting venues for future research. Our model is based on a simple intuition: a lack of liquidity in the limit order book foreshadows an information event. This lack of liquidity manifests itself by a large spread but more generally by a steeper book. This suggests that the slope of the book, in addition to the size of the spread, may also contain information on future price volatility.²⁷ This could be tested with more detailed data. On another front, the analysis raises intriguing questions about the relationships between changes in option prices and the liquidity of the underlying securities. Options contain information on the price volatility of the underlying security [see, for instance, Lamoureux and Lastrapes (1993)]. How does this information affect limit order prices in the market for the underlying security? Conversely, how does volatility information contained in the limit order book affect option prices?

²⁷ Naes and Skjeltorp (2006) find empirically a negative relationship between volatility and the slope of the book. Their results, however, are not directly comparable to ours because they analyze the contemporaneous (instead of the lagged) relationship between volatility and the slope of the book at the daily frequency.

A: Appendix

Proof of Lemma 1 The case in which the follower observes a thin book is analyzed in the text. The follower's optimal reaction when she observes a shallow book is the value of n that maximizes $\Pi(n; S, \theta_S)$. We obtain:

$$\begin{aligned}\Pi(n; S, \theta_S) &= \theta_S[n\alpha(A_1 - (v_0 + \sigma)) + \frac{1}{2}(1 - \alpha)(A_1 - v_0)] + \frac{1}{2}(1 - \theta_S)(A_1 - v_0) \\ &= \Pi(1; S, \theta_S) + (n - 1)\theta_S\alpha(A_1 - (v_0 + \sigma)).\end{aligned}$$

As $A_1 < (v_0 + \sigma)$, we deduce that $\Pi(1; S, \theta_S) > \Pi(n; S, \theta_S)$ for $n \geq 2$. Moreover $\Pi(1; S, \theta_S) > 0$ if and only if $\Delta > (\frac{2\theta_S}{\theta_S\alpha+1})\alpha\sigma$. Thus, $n^*(S) = 1$ if $\Delta > (\frac{2\theta_S}{\theta_S\alpha+1})\alpha\sigma$ and $n^*(S) = 0$, otherwise. Now consider the case in which the book is deep at the end of stage L . The follower's expected profit if she offers n round lots at price A_1 is:

$$\Pi(n; D, \theta_D) = \theta_D[n\alpha(A_1 - (v_0 + \sigma))].$$

This is negative if $n > 0$ because $A_1 < (v_0 + \sigma)$. Thus, $n^*(D) = 0$.

Proof of Proposition 1

Part 1. Dealers' order submission strategies. Observe that (a) $\theta_S = \theta_T = 1$ when there is an information event and (b) $\theta_S = \theta_T = 0$ when there is no information event (since dealers have perfect information). Then, the follower's optimal reaction in each state of the book follows from Lemma 1 and the condition $\Delta < \alpha\sigma$.

Now consider the best response for the dealer acting in stage L given the follower's bidding strategy. When there is an information event, the follower does not undercut a thin book. The informed dealer is then better off setting a thin book as, in this way, he executes all market orders at price A_2 . When there is no information event, the follower always fills the book so that eventually two round lots are offered at price A_1 . We deduce that:

$$\Pi_{I=0}^L(T) = 0, \quad \Pi_{I=0}^L(S) = A_1 - v_0, \quad \Pi_{I=0}^L(D) = \frac{3}{2}(A_1 - v_0),$$

where $\Pi_{I=0}^L(K)$ is the leader's expected profit if he posts schedule K when $I = 0$ (no information event). It follows that $\Pi_{I=0}^L(D) > \Pi_{I=0}^L(S) > \Pi_{I=0}^L(T)$. Hence, the dealer acting in stage L chooses schedule D when there is no information event.

Part 2. Liquidity. As dealers' order submission strategies are identical in the anonymous and the non-anonymous trading mechanism, we deduce that $\text{prob}(\tilde{Q}_1 = 0)$ and $\text{prob}(\tilde{Q}_1 = 2)$ are also identical in both mechanisms. This implies [see Equations (4) and (5)] that the expected large trade spread and the expected small trade spread are identical in each trading system when volatility information is public.

Part 3. Informativeness. By definition:

$$\text{Cov}(|\tilde{V}_2 - v_0|, \tilde{S}_{small}) = \sigma \text{Cov}(\tilde{I}, \tilde{S}_{small}) = \sigma[E(\tilde{I}\tilde{S}_{small}) - E(\tilde{I})E(\tilde{S}_{small})].$$

We deduce, after straightforward manipulations, that

$$\text{Cov}(|\tilde{V}_2 - v_0|, \tilde{S}_{small}) = \sigma\theta_0(1 - \theta_0)[E(\tilde{S}_{small} | \tilde{I} = 1) - E(\tilde{S}_{small} | \tilde{I} = 0)].$$

As $\tilde{S}_{small}$ is either equal to Δ or 2Δ , we obtain that

$$\begin{aligned} \text{Cov}(\tilde{V}_2 - v_0, \tilde{S}_{small}) &= \sigma\theta_0(1 - \theta_0)\Delta[\text{prob}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 1) \\ &\quad - \text{prob}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 0)]. \end{aligned} \quad (\text{A1})$$

Now, given the order submission strategies described in the first part of Proposition 1, we deduce that

$$\begin{aligned} \text{prob}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 1) &= (1 - \beta)\Phi_T + \beta, \\ \text{and } \text{prob}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 0) &= 0, \end{aligned}$$

in *both* trading systems. This implies:

$$\text{Cov}(\tilde{V}_2 - v_0, \tilde{S}_{small}) = \sigma\theta_0(1 - \theta_0)\Delta[(1 - \beta)\Phi_T + \beta] > 0,$$

in *both* trading systems.

Proof of Proposition 2.

Part 1. We show that the follower's order submission strategy is a best response to the informed dealer's strategy. First, consider the case in which the book is thin at the end of the first stage. Substituting $\lambda^*(\beta)$ by its expression in $\theta_T(\lambda, \beta)$ [given by Equation (10)], it is easily checked that

$$\Delta = \theta_T(\lambda^*(\beta), \beta)\alpha\sigma \text{ and } \Delta < \left(\frac{2\theta_T(\lambda^*(\beta), \beta)}{\theta_T(\lambda^*(\beta), \beta)\alpha + 1}\right)\alpha\sigma.$$

Using Lemma 1, we conclude that when she observes a thin book, the follower is indifferent between submitting a limit order for one round lot or staying put. The mixed strategy given in the proposition is then a best response for the follower. In equilibrium, the informed dealer never chooses a shallow book (whether $I = 1$ or not). Thus, a shallow book does not contain information, which implies $\theta_S = \theta_0$. Hence, from condition (8), we deduce that:

$$\left(\frac{2\theta_S}{\theta_S\alpha + 1}\right)\alpha\sigma < \Delta.$$

Using Lemma 1, we conclude that the follower's optimal reaction when she observes a shallow book is as described in Proposition 2. Last, it is optimal for the follower (whatever her posterior belief about the occurrence of an information event) to stay put when she observes a deep book (Lemma 1).

Part 2. We show that the informed dealer's order submission strategy is a best response. We denote by $\Pi_{I=i}^L(K)$ the informed dealer's expected profit if he posts schedule K , when $I = i$, $i \in \{0, 1\}$. When $I = 0$, given the follower's reaction in each state, we have:

$$\begin{aligned} \Pi_{I=0}^L(T) &= (1 - u_T^*)E(\tilde{Q}_u)(A_2 - v_0) + \frac{u_T^*}{2}(A_2 - v_0) \\ &= \frac{3}{2}(1 - u_T^*)(A_2 - v_0) + \frac{u_T^*}{2}(A_2 - v_0), \\ \text{and } \Pi_{I=0}^L(S) &= A_1 - v_0, \\ \text{and } \Pi_{I=0}^L(D) &= E(\tilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0). \end{aligned}$$

As $u_T^* = \frac{3}{4}$, we obtain $\Pi_{I=0}^L(D) = \Pi_{I=0}^L(T) > \Pi_{I=0}^L(S)$. Thus, when $I = 0$, the leader optimally chooses schedule D or schedule T . As she is indifferent between these two

schedules, choosing schedule D with probability $\lambda^*(\beta)$ and schedule T with probability $(1 - \lambda^*(\beta))$ is a best response. When $I = 1$, given the follower's reaction, we have (using $\Delta < \alpha\sigma$):

$$\begin{aligned}\Pi_{I=1}^L(T) &= (1 - \alpha)\left[\frac{3}{2}(1 - u_T^*)(A_2 - v_0) + \frac{u_T^*}{2}(A_2 - v_0)\right] > 0, \\ \Pi_{I=1}^L(S) &= \alpha(A_1 - (v_0 + \sigma)) + (1 - \alpha)(A_1 - v_0) = A_1 - v_0 - \alpha\sigma < 0, \\ \Pi_{I=1}^L(D) &= \Pi_{I=1}^L(S) + \frac{\alpha + 1}{2}(A_1 - (v_0 + \frac{2\alpha\sigma}{\alpha + 1})) < 0.\end{aligned}$$

We deduce that:

$$\Pi_{I=1}^L(T) > 0 > \max\{\Pi_{I=1}^L(S), \Pi_{I=1}^L(D)\}.$$

Thus, when $I = 1$, the leader optimally chooses schedule T .

Proof of Proposition 3.

Part 1. We first show that the follower's order submission strategy is a best response. First consider the case in which the book is thin. From Equation (10), we obtain:

$$\theta_T(1, \beta) = \left\lceil \frac{(1 - \beta)\Phi_T + \beta}{(1 - \beta)\Phi_T + \beta\theta_0} \right\rceil \theta_0.$$

It is easily checked that $\alpha\theta_T(1, \beta)\sigma \leq \Delta$ when $\beta \leq \beta^*$. Moreover

$$\begin{aligned}\Delta &< \left(\frac{2\theta_T(1, \beta)}{\theta_T(1, \beta)\alpha + 1}\right)\alpha\sigma \text{ when } \beta > \beta^*, \\ \Delta &\geq \left(\frac{2\theta_T(1, \beta)}{\theta_T(1, \beta)\alpha + 1}\right)\alpha\sigma, \text{ when } \beta \leq \beta^*.\end{aligned}$$

The follower's best response when she observes a thin book derives from these remarks and Lemma 1. In other possible states of the book, the follower's optimal reaction is derived as in Part 1 of the proof of Proposition 2.

Part 2. We show that the informed dealer's order submission strategy is a best response. When $I = 1$, the argument is identical to that in the proof of Proposition 2 (with $u_T^* = 1$). When $I = 0$, given the follower's reaction, straightforward computations yield:

$$\begin{aligned}\Pi_{I=0}^L(T) &\leq \Delta, \\ \Pi_{I=0}^L(S) &= A_1 - v_0 = \Delta, \\ \Pi_{I=0}^L(D) &= E(\tilde{Q}_u)(A_1 - v_0) = \frac{3}{2}(A_1 - v_0) = \frac{3}{2}\Delta.\end{aligned}$$

Thus, the informed dealer optimally chooses schedule D when there is no information event.

Proof of Proposition 4.

It follows immediately from the arguments in the text.

Proof of Corollary 1.

In what follows, a superscript “a” (resp. “na”) indexes the value of a variable in the anonymous (resp. non-anonymous) market. For instance, $\text{prob}^j(\tilde{Q}_1 = x)$ denotes the probability that the quoted depth is equal to x shares in system j .

Part 1. The Small Trade Spread. Using Equation (4), we obtain that

$$E(\tilde{S}_{small}^a) - E(\tilde{S}_{small}^{na}) = \Delta(prob^a(\tilde{Q}_1 = 0) - prob^{na}(\tilde{Q}_1 = 0)). \quad (A2)$$

We deduce from Proposition 4 that

$$prob^{na}(\tilde{Q}_1 = 0) = \beta(1 - u_T^*)(\theta_0 + (1 - \theta_0)(1 - \lambda^*(1))) > 0. \quad (A3)$$

Moreover, we deduce from Propositions 2 and 3 that

$$prob^a(\tilde{Q}_1 = 0) = \begin{cases} 0 & \text{when } 0 \leq \beta \leq \beta^*, \\ (1 - \beta)\Phi_T(1 - u_T^*) + \beta(1 - u_T^*)(\theta_0 + (1 - \theta_0)(1 - \lambda^*(\beta))) & \text{when } \beta > \beta^*. \end{cases}$$

Then, after substituting $\lambda^*(\beta)$ by its expression in the previous equation, we obtain from Equation (A2):

$$E(\tilde{S}_{small}^a) - E(\tilde{S}_{small}^{na}) = \begin{cases} -\Delta prob^{na}(\tilde{Q}_1 = 0) < 0 & \text{when } 0 \leq \beta \leq \beta^*, \\ \Delta[(1 - u_T^*)(1 - \beta)\Phi_T(1 - (1 - \theta_0)\lambda^*(1))] > 0 & \text{when } \beta > \beta^*. \end{cases}$$

Part 2. Using Equation (5), we obtain that

$$ES_{l_{arg e}}^a - ES_{l_{arg e}}^{na} = \Delta(prob^{na}(\tilde{Q}_1 = 2) - prob^a(\tilde{Q}_1 = 2)). \quad (A4)$$

Using Proposition 4, we obtain:

$$prob^{na}(\tilde{Q}_1 = 2) = (1 - \beta) + \beta(1 - \theta_0)\lambda^*(1) < 1. \quad (A5)$$

Moreover, we deduce from Propositions 2 and 3 that

$$prob^a(\tilde{Q}_1 = 2) = \begin{cases} 1 & \text{when } \beta \leq \beta^{**}, \\ (1 - \beta)(\Phi_S + \Phi_D) + \beta(1 - \theta_0) & \text{when } \beta^{**} < \beta \leq \beta^*, \\ (1 - \beta)(\Phi_S + \Phi_D) + \beta(1 - \theta_0)\lambda^*(\beta) & \text{when } \beta > \beta^*. \end{cases} \quad (A6)$$

Hence, we deduce (after some algebra) from Equations (A4), (A5), and (A6), and the expression for $\lambda^*(\beta)$ that

$$ES_{l_{arg e}}^a - ES_{l_{arg e}}^{na} = \begin{cases} \Delta(prob^{na}(\tilde{Q}_1 = 2) - 1) < 0 & \text{when } \beta \leq \beta^{**}, \\ \Delta[(1 - \beta)\Phi_T + \beta(1 - \theta_0)(\lambda^*(1) - 1)] > 0 & \text{when } \beta^{**} < \beta \leq \beta^*, \\ \beta\Phi_T(1 - (1 - \theta_0)\lambda^*(1)) > 0 & \text{when } \beta > \beta^*. \end{cases}$$

For the two last lines, the sign of this difference is obtained after substituting $\lambda^*(1)$ by its expression.

proof of corollary 2.

In what follows, a superscript “a” (resp. “na”) indexes the value of a variable in the anonymous (resp. non-anonymous) market. Recall (see Equation (A1) in the proof of Proposition 1) that

$$\begin{aligned} Cov(\tilde{V}_2 - v_0, \tilde{S}_{small}) &= \sigma\theta_0(1 - \theta_0)\Delta[prob(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 1) \\ &\quad - prob(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 0)]. \end{aligned} \quad (A7)$$

1) Informativeness of the bid–ask spread in the non-anonymous system. Using the order submission strategies described in Proposition 4, we obtain:

$$\begin{aligned} prob^{na}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 1) &= \beta(1 - u_T^*), \\ \text{and } prob^{na}(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 0) &= \beta(1 - u_T^*)(1 - \lambda^*(1)). \end{aligned}$$

We deduce that

$$Cov^{na}(\tilde{V}_2 - v_0, \tilde{S}_{small}) = \sigma\theta_0(1 - \theta_0)\beta(1 - u_T^*)\lambda^*(1) > 0 \text{ for } \beta > 0. \quad (A8)$$

2) Informativeness of the bid–ask spread in the anonymous system. Using the order submission strategies described in Propositions 2 and 3, we obtain:

$$prob^a(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 1) = \begin{cases} ((1 - \beta)\Phi_T + \beta)(1 - u_T^*) & \text{when } \beta > \beta^*, \\ 0 & \text{when } \beta \leq \beta^*, \end{cases}$$

and

$$prob^a(\tilde{S}_{small} = 2\Delta \mid \tilde{I} = 0) = \begin{cases} ((1 - \beta)\Phi_T + \beta(1 - \lambda^*(\beta)))(1 - u_T^*) & \text{when } \beta > \beta^*, \\ 0 & \text{when } \beta \leq \beta^*. \end{cases}$$

We deduce that

$$Cov^a(\tilde{V}_2 - v_0, \tilde{S}_{small}) = \begin{cases} \sigma\theta_0(1 - \theta_0)\beta(1 - u_T^*)\lambda^*(\beta) > 0 & \text{when } \beta > \beta^*, \\ 0 & \text{when } \beta \leq \beta^*. \end{cases} \quad (A9)$$

As, $\lambda^*(\beta) > \lambda^*(1)$, we deduce from Equations (A8) and (A9) that

$$Cov^a(\tilde{V}_2 - v_0, \tilde{S}_{small}) - Cov^{na}(\tilde{V}_2 - v_0, \tilde{S}_{small}) > 0 \text{ when } \beta > \beta^*.$$

Hence, for $\beta > \beta^*$, the bid–ask spread is more informative in the anonymous system. For $\beta \leq \beta^*$, the informativeness of the bid–ask spread is nil in the anonymous trading system (Equation (A9)) and therefore smaller than in the non-anonymous system [see Equation (A8)].

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