Life-Cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk

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This article explores the implications of additive and endogenous habit formation preferences in the context of a life-cycle model of an investor who has stochastic uninsurable labor income. To solve the model, I analytically derive the habit-wealth feasibility constraints and show that they depend on the worst possible path of future labor income and on the habit strength, but not on the probability of the worst income. When there is only a slim chance of a severe income shock, the model implies much more conservative portfolios. The model also predicts that for some low to moderately wealthy households, the portfolio share allocated to stocks increases with wealth. Because of this feature, the model can generate more conservative portfolios for younger than for middle-aged households. The effects of habits on portfolio choice are robust to income smoothing through borrowing or flexible labor supply. One controversial finding is that for high values of the habit strength parameter, usually required for the resolution of asset pricing puzzles in general equilibrium, the life-cycle model predicts counterfactually high wealth accumulation. (*JEL*: G11, G12)

Additive habit formation preferences play an important role in the efforts to build a plausible general equilibrium model of asset prices.[1] Because tractable general equilibrium models with additive habit utility are confined to the representative agent setting, their implications for the behavior of individual agents are not well understood. In this article, I consider additive habit utility in the context of a life-cycle portfolio choice problem. The analysis provides new insights into the determinants and

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[1] A large body of literature explores additive habits in the models of asset pricing and aggregate consumption. See, for example, Sundaresan (1989), Constantinides (1990), Detemple and Zapatero (1991), Heaton (1995), Chapman (1998), and Campbell and Cochrane (1999).

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dynamics of the portfolio allocation over the life cycle and views the results obtained in a representative agent setting with additive habit preferences from a new angle. In addition, the solution method used here should prove useful in a general equilibrium model with heterogeneous agents.

Life-cycle models, including the one studied here, traditionally are used to examine the determinants of individual portfolio allocation and wealth accumulation and their dynamics with age. Understanding how households choose their savings and exposure to risk is important in asset pricing, macroeconomics, and public finance. Existing life-cycle portfolio models have some implications that are at odds with empirical observations.[2] Portfolio choice of the younger households presents a particularly challenging and common problem, because it represents a partial equilibrium counterpart to the equity premium puzzle. Standard life-cycle models, when reasonably parameterized, typically predict that young households should invest exclusively in equity. Not only is this prediction largely at odds with the data, but it also presents a challenge to the aggregate macro finance models that use the life-cycle model as a foundation. This article shows that an additive habit model implies that young investors should hold more conservative portfolios than middle-aged investors; it also has other empirically plausible and distinct implications.

In this article the model is set in partial equilibrium and is concerned with life-cycle choice of consumption and portfolio allocation for an investor with additive and endogenous (internal) habit formation preferences. I assume a typical economic environment for a life-cycle model. The investor receives labor income, which is subject to uninsurable transitory and permanent shocks during the working life and is constant during retirement periods. There are two tradable securities available to the investor: (i) a bond with constant return and (ii) a stock index with a stochastic return. Borrowing is constrained and short sales are not allowed. I assume that current habit is a fraction of consumption in the previous period and that exogenous variables follow a finite-state Markov process. Under these assumptions I analytically characterize the admissible habit-wealth region for every age and then solve the problem numerically within this region. The boundary of the admissible region depends on the *worst possible* future labor income path and on the habit strength parameter but does not depend on the probability of the worst-case income path. The optimal savings and portfolio choice policies

[2] For example, see a survey in Haliassos and Michaelides (2002) discussing standard CRRA and recursive utility implications. The ratio-habit model of Abel (1990) was studied in the life-cycle context by Gomes and Michaelides (2003) who showed that it shares many of the same difficulties with models based on CRRA utility. Later in the article, I summarize the main points made in Haliassos and Michaelides (2002) and make comparisons with the implications of additive habit model.

depend on wealth accumulated in excess of the admissible boundary and on the current habit level.

One of the main distinct implications of the habit model is that portfolio policies, in general, are not monotone in wealth. There is a region from low to moderate wealth (in excess of the admissible boundary) where the portfolio share allocated to stocks *increases* with financial wealth.[3] This policy function generates an empirically plausible positive relationship between wealth and portfolio allocation to equities for some households in the model, particularly the younger or not "too" wealthy households. Intuitively, when wealth is low, to insure that habit can be safely sustained under any scenario for income and asset returns, the households in the model resort to safer portfolios. As wealth builds, sustaining habits in adverse circumstances becomes less of a concern. Between moderate and high wealth (again, in excess of the admissible boundary), portfolio policies become qualitatively similar to those under expected utility (EU): the portfolio share in stocks is *decreasing* with wealth. This effect is attributable to the similarity between human capital and bonds in the model, which implies that as financial wealth grows relative to human capital, households compensate by increasing their exposure to bonds.[4]

Because there is a range of low to medium wealth in which the portfolio share of stocks is increasing, the model implies interesting life-cycle dynamics in portfolio allocation. In the model, younger households have relatively more conservative portfolios than the middle-aged households: the young investors have not yet accumulated enough wealth to sustain consumption sufficiently above habit. This pattern of portfolio allocation between young and middle-aged households is consistent with some empirical evidence, for example, as in Heaton and Lucas (2000) or Faig and Shum (2002).[5] Later in the life cycle, portfolio allocation to stocks declines, partly because of the substitution between labor income and bonds and partly because of the need to maintain habits with lower wealth. In contrast, I find that an identically parameterized EU (CRRA) model does not generate conservative portfolios for the young and that the decline of the portfolio share in stocks with age in the second half of the life cycle is less pronounced, even in simulations with a low- or zero-income state. Comparative statics experiments also introduce the bequest motive that slows down wealth decumulation after retirement. Bequests

[3] The references to wealth here and throughout the introduction are only relative and will be made more precise quantitatively later in the discussion of the policy functions in Section 3. The dollar values of various wealth thresholds in policy functions depend on age and on the level of permanent income that is used as a normalization variable in the model.

[4] A detailed discussion of this effect is provided, for example, in Jagannathan and Kocherlakota (1996).

[5] Faig and Shum (2002) suggested yet another explanation for lower allocation to stocks for younger households. In their model, people save for acquiring illiquid assets, such as houses or private businesses, which introduces extra demand for safer liquid assets earlier in the life cycle.

have a more significant effect on portfolio choice and the wealth accumulation of the CRRA agents: without a bequest motive, they accumulate much less wealth than the habit agents.

Motivated by the structure of the feasibility constraints of the model, I consider calibrations that demonstrate the effect of the worst labor income outcome on portfolio allocation. In the benchmark case, I follow the standard assumptions and calibrate transitory income shocks to a symmetric distribution matching the variance in the data. In this calibration, the average portfolio allocation to stocks is at or near 100% for most of the life cycle. In this case, it takes relatively high habit strength, or a utility curvature coefficient, to generate conservative (interior) portfolios. In the alternative calibrations, I augment the transitory shock component by introducing a state in which income is low (or even zero) with a small probability calibrated from the distributions reported in Carroll (1992). The mere possibility of such a state generates a much more conservative portfolio allocation, especially for the young households with little wealth. For the older working households with sufficient wealth, low-income states have little effect on portfolio choice, because their substantial wealth guarantees that consumption can be sustained comfortably above habit in all circumstances.[6]

To examine the robustness of the results obtained in the basic habit model, I consider two extensions. One extension allows for borrowing against future labor income and another introduces flexibility in labor supply.[7] I find that the main implications of the model are still valid in both extensions. I also find that both extensions mitigate the effects of habit by increasing the demand for stocks. However, quantitative results in both extensions are sensitive to the assumed income process. In particular, if there is a zero-income state, then the borrowing limit for young agents is low and portfolio choice in the models with and without borrowing is similar. On the contrary, if labor income has a "floor," then portfolios become more aggressive when borrowing is allowed. The possibility of borrowing primarily matters early in the life cycle when assets are low. Similarly, when there is a possibility of a zero wage (unemployment without insurance), the agents cannot increase their income by varying labor supply in bad states and must use savings to finance consumption. As a result, portfolio allocation during working life is similar to the original model. Consistent with the results of Bodie, Merton, and Samuelson (1992), flexible labor supply implies higher allocation to

[6] Low-income states may have greater significance for the portfolio choice of wealthier middle-aged households if asset liquidity is modeled in more detail, so that part of savings is allocated to illiquid assets, for example, a house as in Yao and Zhang (2004) and Cocco (2005).

[7] I assume that the agent must be solvent in any circumstances. Therefore, borrowing is allowed up to the present value of the lowest life-time labor income. Because pension income is nonzero, the borrowing limit is always nonzero, even if there is a zero-income state during working life.

stocks when labor income process is assumed to have a nonzero lower bound. In this case, agents can hedge against bad stock returns and income shocks by adjusting labor supply. Further details of these calibrations are provided in Section 3.4.

In relation to general equilibrium asset pricing models with additive habit preferences, an important and controversial issue remains to be reconciled with the results obtained in this article. In a representative agent model, it is typically necessary to assume relatively high habit strength to resolve the asset pricing puzzles.[8] Higher habit strength implies higher risk aversion that is necessary to reconcile asset prices with smooth aggregate consumption. In the life-cycle model, high habit strength implies conservative portfolio choice, advancing the model toward a resolution of the equity premium puzzle in a partial equilibrium setting. However, high habit strength creates a tension, because the model predicts counterfactually high wealth accumulation throughout the life cycle, well beyond what is observed in the data. This problem most likely reflects a failure of the representative agent assumption and should be investigated further in the context of a general equilibrium model with heterogeneous agents. One interesting question is whether it is possible to resolve the asset pricing puzzles and match stylized facts at the micro level at the same time.[9]

The analysis in this article builds on several strands of the portfolio choice literature and combines labor income uncertainty, finite horizon, and additive habit formation preferences. Stochastic (and uninsurable) labor income has become a standard element in many recent studies that explored portfolio choice with EU CRRA or recursive preferences.[10] Models with infinitely lived agents, such as in Heaton and Lucas (1997) and Viceira (2001), show that introducing labor income implies more aggressive portfolio choice because human capital, despite its risk, resembles bonds more than stocks. Because of this effect, studies with finite-horizon models—such as Bodie, Merton, and Samuelson (1992), Jagannathan and Kocherlakota (1996), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005)—found that portfolio share in stocks, in general, declines with age. Additive habit utility, on the contrary, implies that the typical young households found it optimal to invest more

[8] Equivalently, in models with exogenous habit formation [e.g., Campbell and Cochrane (1999)], it is necessary to have a very low consumption surplus ratio (consumption in excess of habit relative to consumption).

[9] Pijoan-Mas (2003) made an important first step in this direction. He considered such an equilibrium model with the "catching-up-with-the-Joneses" ratio-habit preferences of Abel (1990). Most of the representative agent asset pricing models with habit utility require additive habit specification to achieve resolution of the asset pricing puzzles. And the additive habit specification awaits further analysis in a heterogeneous-agent general equilibrium model.

[10] Earlier studies considered consumption and savings with income uncertainty but did not study portfolio choice, for example, Deaton (1991), Carroll (1992), and Gourinchas and Parker (2002).

conservatively than middle-aged households. Several other studies focused on portfolio choice with habit formation preferences in various settings. Hindy, Huang, and Zhu (1997) used an infinite-horizon complete markets model to investigate the impact of habits and consumption durability on portfolio allocation. Bodie et al. (2004) considered a finite-horizon model with complete markets and additive habit; they also incorporated labor supply flexibility and retirement decisions. A related model, but with constant labor income, is studied by Lupton (2001), who represented habit as "consumption liability" and simplified the problem to obtain the same representation as in Merton (1971). Heaton and Lucas (1997) and Gomes and Michaelides (2003) included nontradable risky labor income in an infinite- and a finite-horizon setting respectively. Gomes and Michaelides (2003) considered in detail a life-cycle model with ratio-habit preferences of Abel (1990). They concluded that ratio habit helps generate more conservative portfolio selection but that it also implies faster wealth accumulation and makes it more difficult to explain limited stock market participation. This article advances the existing literature by considering both feasibility constraints and the effects of low/zero-income realizations on portfolio allocation. One of the main differences with previous results obtained in the models based on the EU, recursive, and ratio-habit preferences is that portfolio policies have a range where allocation to stocks is increasing in wealth.

The rest of this article is organized as follows. In Section 1, I set up the model with additive habit formation utility and derive the feasibility constraints. The calibrations are described in Section 2. The results of the simulations and the extensions for borrowing and flexible labor supply are discussed in Section 3. In Section 3.5, I compare the habit model with stylized facts and with CRRA utility models; Section 3.6 presents the comparative statics analysis. Section 4 concludes. Appendix A includes a formal derivation of the feasibility constraints, and Appendix B describes the numerical method used to solve the model.

1. Life-Cycle Model

1.1 Household problem

There is one agent (household) in the model who lives a finite life with the deterministic horizon from t_0 to T and is characterized by the following habit formation utility function defined over the admissible consumption sequences $\{C_t\}_{t=t_0}^{T}$, s.t. $C_t > H_t \geq 0$:

$$U_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{T} \beta^{t-t_0}\frac{(C_t - H_t)^{1-\gamma}}{1-\gamma} + \beta^{T+1-t_0}B(W_{T+1})\right\}, \tag{1}$$

where $\beta < 1$ is the subjective discount factor, $\gamma > 0$ is the curvature parameter, H_t denotes time t habit, $B(W_{T+1})$ is a bequest function, and W_{T+1} is bequested wealth. Consumption paths such that $C_t \leq H_t$ for some t in some state with nonzero probability are assigned infinitely negative utility. I assume that habit is internal and depends on one lag of consumption with the strength parameter δ:[11]

$$H_t = \delta C_{t-1},\ H_{t_0} = H^0. \tag{2}$$

The household receives stochastic labor income Y_t, which is subject to permanent (P_t) and transitory (η_t) shocks. Income is the product of these shocks and the deterministic age-dependent component $f(t)$:

$$\begin{aligned} Y_t &= f(t)P_t\eta_t \\ P_t &= P_{t-1}\kappa_t \end{aligned} \tag{3}$$

It is assumed that $\log(\kappa_t)$ and $\log(\eta_t)$ are i.i.d. normal with means such that their exponent has a mean of one. In the calibrations, it will be assumed that during the retirement years Y_t becomes constant and equal to a fraction ϕ (replacement ratio) of the permanent income component just prior to retirement, that is, $Y_t = \phi f(65)P_{65}$, for all $t > 65$. This simplifying assumption allows us to reduce the dimensionality of the problem in the post-retirement years.[12]

There are two securities available for saving: a risk-free bond and a risky stock. The bond pays constant interest rate, r_b, and the stock represents a market index and pays stochastic return $r_{s,t}$. Neither security can be sold short.[13] Let the purchases of the bond and stock at time t be denoted B_t and S_t, respectively. The total cash on hand X_t available to the household at time t is defined as:

$$X_t = S_{t-1}(1 + r_{s,t}) + B_{t-1}(1 + r_b) + Y_t \tag{4}$$

At every period $t = t_0,\ldots,T$ the household chooses consumption and purchases of bond and stock subject to the following constraints:

[11] This particular functional form is assumed for simplicity. All qualitative results derived in this article also apply in a more general case when current habit is a weighted average of past habit and lagged consumption, which is a more general case considered by Constantinides (1990) in continuous time.

[12] Although there is less labor income risk (or none at all) after retirement, there are certainly other risks such as medical expenses and disability. However, computational burden and lack of empirical data prevent calibration of such risks, and they are left out of the model.

[13] For stocks, this is not a binding constraint, and for bonds, it depends on the calibration. Such liquidity constraints are commonly assumed in the life-cycle literature to capture limited ability of households to contract on future labor income to obtain uncollateralized loans.

$$
\begin{aligned}
& X_t = C_t + B_t + S_t \\
& B_t \geq 0,\ S_t \geq 0 \\
& C_t > H_t,\ t = t_0,\ldots,T
\end{aligned} \tag{5}
$$

The first three constraints are familiar budget, borrowing, and short-sales constraints. The last system of inequality constraints on consumption is specific to the additive habit preferences. Consumption cannot fall below habit at any time, regardless of the random realizations of the exogenous variables. In the next section, and more formally in Appendix A, I discuss how these constraints may be rewritten to make their economic intuition more transparent.

The homotheticity of the utility function allows us to reduce the dimensionality of the problem by normalizing all level variables by their contemporaneous permanent income, P_t. Let the lowercase letters denote normalized variables; one can then rewrite normalized cash on hand as follows:

$$
\frac{X_t}{P_t} \equiv x_t = [s_{t-1}(1 + r_{s,t}) + b_{t-1}(1 + r_b)] \times \frac{P_{t-1}}{P_t} + y_t \tag{6}
$$

Habits are normalized, too, which results in the following equation for habit evolution:

$$
\frac{H_t}{P_t} \equiv h_t = \delta \frac{C_{t-1}}{P_t} = \delta c_{t-1} \times \frac{P_{t-1}}{P_t} \tag{7}
$$

After the normalization, the state vector is defined as $z_t = [h_t, x_t, t]$. The household problem is to find the sequences of consumption $\{c_t(z_t)\}$, bondholdings $\{b_t(z_t)\}$, and stockholdings $\{s_t(z_t)\}$ as functions of the state z_t for $t = t_0,\ldots,T$ subject to the constraints (5) to maximize the utility in (1). For numerical solution, it is more convenient to write the household problem as a sequence of stochastic Bellman equations. Let $V_t(z_t)$ denote the value function at time t and state z_t. Then, the household problem can be written as this sequence of maximizations:

$$
\begin{aligned}
& V_t(z_t) = \max_{c_t, s_t, b_t} \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} + \beta E_t \left[V_{t+1}(z_{t+1}) \left(\frac{P_{t+1}}{P_t} \right)^{1-\gamma} \right], \\
& \qquad \text{for } t = t_0,\ldots,T, \text{ s.t. (7), (6), and (5)} \\
& V_{T+1} \equiv \lambda \frac{\left(\frac{w_{T+1}}{\lambda} \right)^{1-\gamma}}{1-\gamma} \\
& w_{T+1} = s_T(1 + r_{s,T+1}) + b_T(1 + r_b)
\end{aligned} \tag{8}
$$

The bequest function V_{T+1} in (8) is similar to the one considered in recent studies of portfolio choice. Its functional form is maintained for compatibility with the existing literature.[14] The solutions to the stochastic Bellman equations (8) are computed on the grid in endogenous variables space, using bi-cubic spline approximations for the points that do not fall on the grid. Appendix B provides further details of the computational procedure.

1.2 Characterizing a feasible set of endogenous variables

The constraints on consumption and habit in (5) restrict the feasible pairs of the endogenous variables (X_t, H_t) at time t and implicitly impose constraints on the control variables (consumption and portfolio choice) that are feasible in the previous period $t-1$. In the context of the present model, it is easier and more intuitive to characterize the restrictions on (X_t, H_t) and then use them to choose both feasible and optimal controls.[15] In addition, constraints on feasible pairs (X_t, H_t) contain important information about the economic structure of the problem and help to better understand the results from the model. It is also important to analyze these constraints to develop an efficient numerical method. In Appendix A, I formally derive the restrictions on the admissible cash-habit pairs when the exogenous variables follow a finite-state Markov process. In this section, I describe the intuition of the derivation and the economic interpretation of the constraints.

The constraints (5) imply that $X_t > H_t$ for all t. In other words, there must be sufficient total cash strictly exceeding the current habit. It should be noted that at any time, t, the constraints imply more than just $C_t > H_t$. It is not enough to restrict the household to consume today above today's habit. Consuming too much today may generate infeasible habits in the future. For example, consider a (budget-constraint-feasible) policy to consume all resources today and set $C_t = X_t > H_t$ $(t < T)$. If the following inequality holds:

$$\min_{\eta_{t+1},\kappa_{t+1}} Y_{t+1} \leq H_{t+1} = \delta X_t, \tag{9}$$

then the household does not have enough resources to consume above habit in the worst income state at time $t+1$. When (9) is satisfied with equality, the marginal utility is infinite. Thus, increasing consumption too much today may be inadmissible in the worst-case scenario

[14] This function can be thought of as providing utility of bequest during λ periods. For analyses using similar bequest functions, see, for example, Cocco, Gomes, and Maenhout (2005), Cagetti (2003), Gomes and Michaelides (2005), and Dynan, Skinner, and Zeldes (2004).

[15] The procedure of selecting optimal from among feasible controls is described in detail in Appendix B.

tomorrow. By this backward recursion, the feasibility constraints from $t+1$ impose a restriction on time t admissible consumption policy.[16]

To characterize the admissible endogenous variables, I start at time T and proceed backward. At time T, there is no need to look forward anymore and the only constraint is to have cash above habit. The admissible set for the endogenous variables is $X_T > H_T$. At time $T-1$, the household must have enough resources for the current habit. Thus, the admissible set is partly restricted by $X_{T-1} > H_{T-1}$. Another restriction emerges from the requirement that there should exist at least one feasible policy at $T-1$ such that in the next period, under any realizations of the exogenous variables, it guarantees cash above habit. Appendix A shows that this requirement imposes the following constraint on (X_{T-1}, H_{T-1}):

$$\frac{\underline{Y}_T}{1+r_b} + X_{T-1} - \left(1 + \frac{\delta}{1+r_b}\right) H_{T-1} > 0, \tag{10}$$

where $\underline{Y}_T$ is the lowest possible labor income at T. In deriving Equation (10), I assume that the worst possible outcome for cash on hand is when labor income has the lowest possible value and stock return is negative. If habit and cash at time $T-1$ satisfy (10) with equality, the marginal utility is infinite. This is because even if consumption is set equal to habit at $T-1$, leaving the *maximum possible* savings for tomorrow, there would not be enough cash in the worst case at time T to consume above habit, no matter what portfolio is chosen at $T-1$. To see this, note that if $C_{T-1} = H_{T-1}$, then the household saves $X_{T-1} - H_{T-1}$. Assuming that in the worst state the stock return at T is negative, then to *maximize* cash on hand in the worst case the household should not buy any stock and should invest only in bonds. Thus, the maximum cash in tomorrow's worst case is given by $\underline{X}_T = (1+r_b)(X_{T-1} - H_{T-1}) + \underline{Y}_T$ and tomorrow's habit is $H_T = \delta H_{T-1}$. Note that we have $\underline{X}_T - H_T = 0$ if we assume that (10) holds with equality. Thus, even if the household saves the maximum possible and in the safest possible portfolio, there will not be enough resources in the worst-case outcome to exceed the habit at time T.

Constraints similar to (10) are derived in Appendix A for a general time t. At every time t, there are $T-t+1$ constraints indexed by $\tau = 0,\ldots,T-t$ as follows:

[16] Portfolio policies are also constrained in a similar manner. For example, if the agent saves, portfolio must be allocated so that in the worst outcome for portfolio return and income, there is still enough cash on hand to finance consumption above habit. This recursive verification of feasibility is built into the solution algorithm described in Appendix B.

$$0 < \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k} + X_t - H_t \left[\sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k \right], \ \tau = 0,\ldots,T-t, \tag{11}$$

where the convention is to set the first sum equal to zero for $\tau = 0$. For future reference, it is convenient to adopt the following notation:

$$Y_{t,t+\tau} = \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k}, \ Y_{t,t} \equiv 0$$

$$\Delta_\tau = \sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k = \frac{1 - \left(\frac{\delta}{1+r_b} \right)^{\tau+1}}{1 - \frac{\delta}{1+r_b}}.$$

It is also convenient to normalize $Y_{t,t+\tau}$ by the time t permanent income component and use the lowercase notation. Note that $Y_{t,t+\tau}$ is a multiple of today's permanent income P_t for every τ and can be normalized as follows:

$$Y_{t,t+\tau} \equiv \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k} = P_t \sum_{k=1}^{\tau} f(t+k) \times \frac{\underline{\eta}_{t+k} \Pi_{i=1}^{k} \underline{\kappa}_{t+i}}{(1+r_b)^k},$$

$$1 \le \tau \le T - t + 1, \ Y_{t,t} \equiv 0,$$

$$y_{t,t+\tau} \equiv \frac{Y_{t,t+\tau}}{P_t} = \sum_{k=1}^{\tau} f(t+k) \times \frac{\underline{\eta}_{t+k} \Pi_{i=1}^{k} \underline{\kappa}_{t+i}}{(1+r_b)^k}, \ 1 \le \tau \le T-t+1, \ y_{t,t} \equiv 0,$$

where $\underline{\kappa}_t$ and $\underline{\eta}_t$ denote, respectively, the minimum possible permanent and transitory shock values at time t.[17] Using normalized notation, the constraints in (11) can be written compactly as:

$$0 < y_{t,t+\tau} + x_t - h_t \Delta_\tau, \ \tau = 0,\ldots,T-t. \tag{12}$$

The interpretation of inequalities in (12) is very intuitive. Note that $y_{t,t+\tau}$ is the time t present value of the *minimum possible* labor income received between t and $t+\tau$. Thus, the first two terms in the equation above correspond to the minimum possible present value of available resources for a given time horizon τ. On the contrary, $h_t \Delta_\tau$ is the *infimum* of the present value of the future habits up to time $t+\tau$ when the current habit is h_t. To see this, note that if the household consumes $h_{t+\tau} + \epsilon$ in each period between t and $t+\tau$,

[17] Note that in the above expressions, the worst permanent income shocks "cumulate," whereas the transitory shocks do not. The above normalization is exact and does not rely on any particular values of $\underline{\kappa}$ and $\underline{\eta}$. Therefore, it is preserved in any Markov chain discretization of the labor income process.

where $\epsilon > 0$ is a very small number, then the limit, as ϵ goes to zero, of the present value of the habits generated by this consumption sequence is $h_t \Delta_\tau$. Thus, consumption cannot be decided based on the budget constraint alone, and the household must consider what future habit it will generate. Note, too, that while the worst-case labor income enters the constraints, its probability does not: what matters is only that this probability is positive.

1.3 When do the feasibility constraints matter?

Two recent articles, Heaton and Lucas (1997) and Gomes and Michaelides (2003), computed solutions to the additive habit formation models with labor income risk and endogenous additive habits but without referring to the constraints discussed here. Thus, it is instructive to examine under what conditions these constraints become important for the solution of the problem.

Any two consecutive constraints from (12) for τ and $\tau + 1$ intersect at the point given by:

$$h^*_{t,\tau} = \frac{\underline{y}_{t+\tau+1}}{\delta^{\tau+1}}$$
$$x^*_{t,\tau} = \frac{\underline{y}_{t+\tau+1}}{\delta^{\tau+1}} \Delta_\tau - y_{t,t+\tau}.$$

The first intersection for $\tau = 0$ occurs at a point given by $x^*_{t,0} = h^*_{t,0} = \frac{\underline{y}_{t+1}}{\delta}$. All subsequent intersections lie to the northwest of this point. If the lower bound on labor income is high, and/or the habit strength is low, this intersection point will be far away from the actual habit level occurring in simulations. In this case, the only relevant constraint to keep track of is $x_t > h_t$. To see this, consider a numerical example shown in Figure 1, where cash and habit are normalized by the expected permanent income component (denoted Ey) and the risk-free interest rate, r_b, is assumed to be 1%. The parameters for the example on panel A were chosen to be within the range of calibrations reported by the two articles mentioned above. On panel A, the strength parameter δ is 0.4 and the worst labor income outcome equals $0.7 \times Ey$. In this case, the first intersection of the constraints occurs at the point $(x^*_{t,0}, h^*_{t,0}) = (1.75, 1.75) = (0.7/0.4, 0.7/0.4)$. In simulations consumption (normalized by the permanent income Ey) is given by $c_t \approx 1.0$.[18] Therefore, the habit is approximately given by $h_t = 0.4 \times c_t = 0.4$. Thus, even the first intersection of the constraints on panel A at the point (1.75, 1.75) is quite far away from the range of realizations of the endogenous variables. For such parameter values it is not necessary to pay attention to all the constraints in (12). Because the

[18] This is the approximate average across simulations of a stationary consumption distribution. See, for example, Heaton and Lucas (1997).

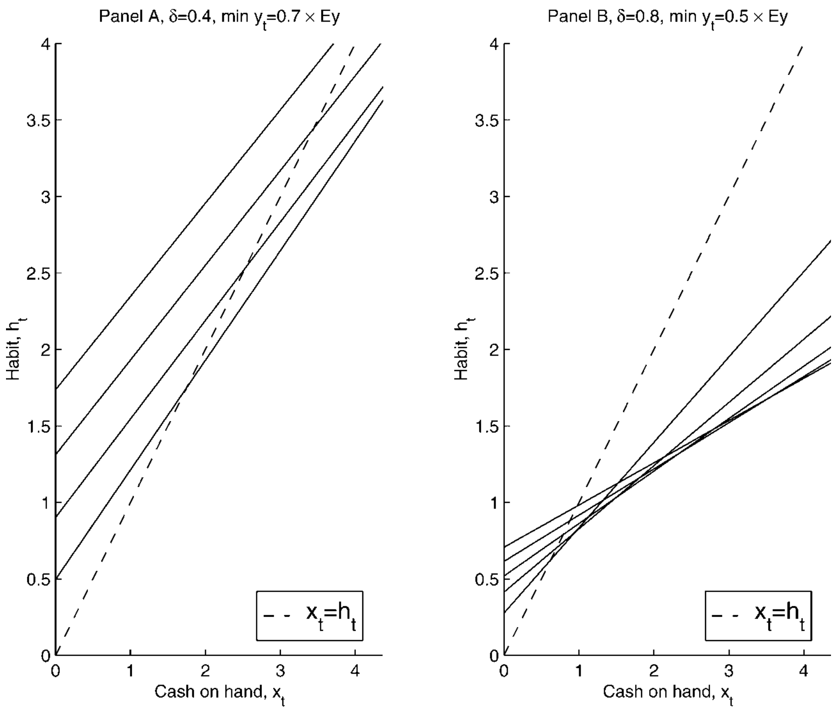

Figure 1
Constraints on endogenous state variables
The region below the low envelope of the lines defines admissible states.

endogenous variables in the computations usually are confined to a rectangle $[x_l, x_h] \times [h_l, h_h]$, if h_h is well below the first intersection, the only relevant constraint in this rectangle would be $x_t > h_t$. As the lowest labor income realization falls, and/or the habit strength increases, more constraints will be within a given rectangle. Figure 1, panel B, shows another example with a higher habit strength, $\delta = 0.8$, and a lower worst income, $0.5 \times Ey$. In this case, constraints move downward considerably and become tighter.

Because only some values of cash on hand and habit are admissible, to solve the problem, one may restrict attention to the points in the state space to the right of the *lower envelope* of the constraints as shown in Figure 2. To rule out inadmissible states, it is convenient to transform cash on hand into cash on hand *in excess* of the minimum admissible level. This minimum level depends on the current habit. To proceed, it is convenient to define a variable τ^* as follows:

$$\tau^* = \arg \max_{\tau \in [0,\ldots,T-t+1]} \{h_t \Delta_\tau - y_{t,t+\tau}\}. \tag{13}$$

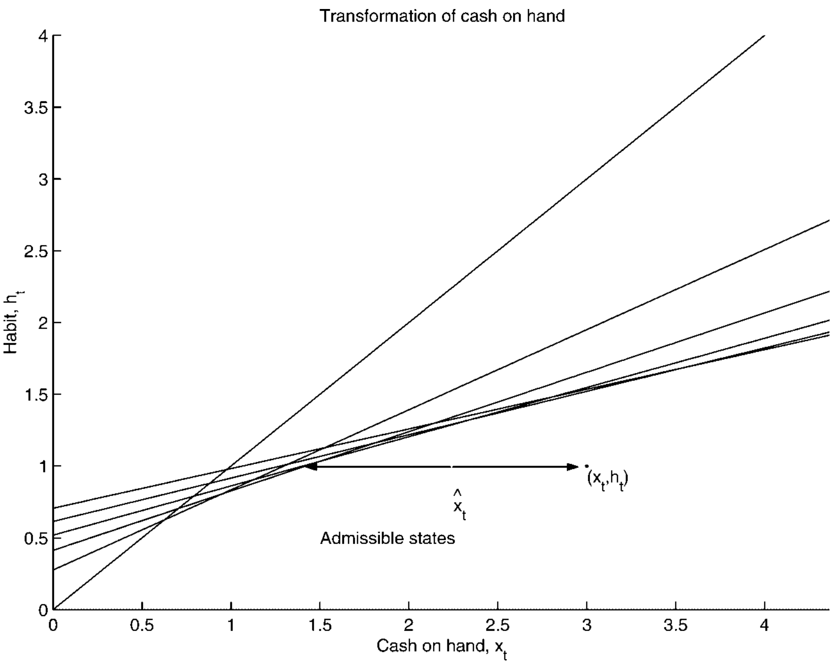

Figure 2
Cash-on-hand variable is transformed to cash on hand in excess of admissible boundary

In the above expression, τ^* simply identifies the constraint from (12) corresponding to the lower envelope segment at the level of habit h_t. This is the constraint closest to being binding. Geometrically, this corresponds to selecting the constraint line closest to the point (x_t, h_t) moving horizontally to the left from (x_t, h_t) until reaching the lower envelope of the constraints lines, as shown in Figure 2. Note that if $x_t = (h_t\Delta_{\tau^*} - y_{t,t+\tau^*})$, then the distance from (x_t, h_t) to the admissible boundary would be zero, and the strict inequality constraint corresponding to τ^* in (12) would be violated. Thus, the amount $(h_t\Delta_{\tau^*} - y_{t,t+\tau^*})$ is equal to the minimum admissible cash on hand for a given habit. Using the definition of τ^*, the transformed cash on hand in excess of the minimum admissible level is defined as follows:

$$\hat{x}_t = x_t - (h_t\Delta_{\tau^*} - y_{t,t+\tau^*}),\ \hat{x}_t > 0. \tag{14}$$

Thus, $\hat{x}_t$ corresponds to the horizontal distance to the feasible boundary as shown by the arrows in Figure 2. Note that $\hat{x}_t > 0$ guarantees corresponding x_t to be feasible.[19] Besides automatically ruling out inadmissible endogenous states, there are other numerical advantages to using $\hat{x}_t$. Appendix B explains further how the transformed $\hat{x}_t$ is used in the numerical procedure.

2. Calibration

Time period is assumed to be one year. The household is assumed to live from age 25 until age 75 and begins retirement after age 65. The age-income profile $f(t)$ is taken from Cocco, Gomes, and Maenhout (2005). They estimated age-income profiles from the Panel Study of Income Dynamics (PSID) data using cubic polynomials and they reported the estimated coefficients for three cohorts divided by education. I use the age-income profile for high-school graduates in the calibrations that follow. The utility function parameters are set as follows: $\beta = 0.95$ and $\gamma = 5$. The habit strength parameter δ is considered in the range of $[0, 0.8]$. In the baseline case, there is no bequest motive ($\lambda = 0$), primarily for the sake of transparency. In comparative static analysis, I relax this assumption, introduce a bequest motive, and compare its impact on the behavior of habit versus EU investors.

From the earlier discussion, it is clear that the distribution of income shock is crucial for the feasible set of endogenous variables. Typically, income shock η_t is modeled as a finite-state Markov process with several symmetric outcomes around the mean of 1 matching the estimated volatility of income.[20] For the problem with habit formation, it is important to calibrate the worst realizations of labor income. I assume that shock η_t is i.i.d. and consider two different calibrations: symmetric and asymmetric.[21] Transitory shock η_t in the symmetric distribution has two values around the mean of 1 matching the standard deviation of 27%. Permanent income shocks are calibrated to have standard deviation of

[19] This resembles Lupton's (2001) transformation to account for habit liability, but there are two important differences. First, the transformation here involves the lowest possible present value of future income. Second, the transformation does not substitute out the habit from the model. In the case of nonstochastic income considered by Lupton, the problem simplifies to one-variable state [as in Merton (1971)], so that wealth net of habit liability is a sufficient endogenous state. The simplification does not happen here because when income is stochastic, one still needs to track habit h_t to make sure that x_t satisfies feasibility constraints.

[20] See, for example, Storesletten, Telmer, and Yaron (1998), Cocco, Gomes, and Maenhout (2005), and Heaton and Lucas (1997).

[21] The assumptions of i.i.d. shocks to income and returns are made primarily for transparency and compatibility with existing literature. Nothing in the model relies on these assumptions, and, if anything, the portfolio allocations implied by the model would be more conservative for positively correlated returns and income shocks. The available empirical evidence shows that income shocks and stock returns are not highly correlated. For example, Davis and Willen (2000) reported correlations close to zero and statistically insignificant for most occupational and educational cohorts, and similar empirical evidence is presented by Cocco, Gomes, and Maenhout (2005).

10%.[22] In another calibration, I use Carroll's (1992) results: he estimated that at an annual frequency with a very small probability, labor income may fall substantially. To reflect that, I augment the symmetric transitory income shock calibration by introducing a third state with either zero or low income. The probability of this state is very low. I calibrate it using the histograms reported in Carroll's article. For brevity, I only report two cases: zero income with 0.5% probability, and low income—equal to 10% of the permanent level—with 1.3% probability.[23]

Stock returns are calibrated to be an i.i.d. Markov process with two symmetric outcomes around the mean. The mean return is set at 6%, and the standard deviation is 15%. The bond return is assumed to be constant 1%. Although the equity premium here is 5%, which is lower than historic average value of 6%, one could argue for even a lower premium given that transactions costs of holding stocks and taxes are ignored in the model. Jagannathan, McGrattan, and Scherbina (2000) and McGrattan and Prescott (2003) have argued that the equity premium is much lower than 6% after accounting for diversification costs, taxes, and liquidity premium of the short-term government bonds.

For every calibration, I simulate a cross section of 10,000 households to compute the distributions of the variables of interest: wealth, consumption, and portfolio fraction invested in stocks. Portfolio fraction in stocks reported is the average of individual portfolio shares in stocks across households with positive assets. The initial values for habit for all calibrations is set to $0.5 \times f(t_0) \times P_{t_0}\delta$. Thus, the consumption just before entering working life is assumed to be half of the permanent labor income component in the first period of the model. Initial normalized habit will be $0.5 \times \delta$ as δ changes across calibrations. In all calibrations with symmetric income shocks, the initial asset holdings are zero. For initial habit to be admissible in the calibrations with zero/low-income state, the initial wealth is set to $0.8 \times f(t_0) \times P_{t_0}$, that is, equal to 80% of the permanent income in the first period.[24]

Often throughout the discussion of results wealth is expressed in terms of permanent income. Some of the graphs already show the results in dollar terms, but those that use normalized scales can be converted to dollar scale by using the graph of the expected permanent income shown as a dash-dot line on the upper panel of Figure 5. This income profile

[22] For consistency, I use shock volatilities from the same source as income profiles. Heaton and Lucas (1996, 1997) and Storesletten, Telmer, and Yaron (1998) estimated somewhat higher volatility of the transitory income shock.

[23] Other low-income states considered were between 15 and 30% of permanent income level. Qualitatively, the results are similar to those reported and are available on request from the author.

[24] This initial wealth is chosen to be admissible for the initial habit level and to produce comparable consumption profiles in a chosen range of habit strengths. The alternative would be to set the initial habit at zero, so that the usual assumption of zero starting financial wealth is in the admissible range. However, zero initial habit implies unrealistically low consumption early in life for higher habit strength.

corresponds to the average in simulations. For example, for a 25-year-old agent, the relevant permanent income is just above \$20,000 (in 1992 dollars). Income reaches a high plateau at about \$30,000 for 40–50-year-old agents and then declines to about \$27,500 just before retirement. At retirement income remains constant at approximately \$18,500.

3. Simulation Results

3.1 Savings and portfolio policies

To better understand the intuition for the results obtained in simulations, it helps examine optimal savings and portfolio policies. Figure 3 shows portfolio allocation and savings policies in the first period for the asymmetric income shock calibration with zero income. Qualitatively, the policies shown are similar to the other calibrations. There are a few intuitive properties of the policies that are present throughout the life cycle.

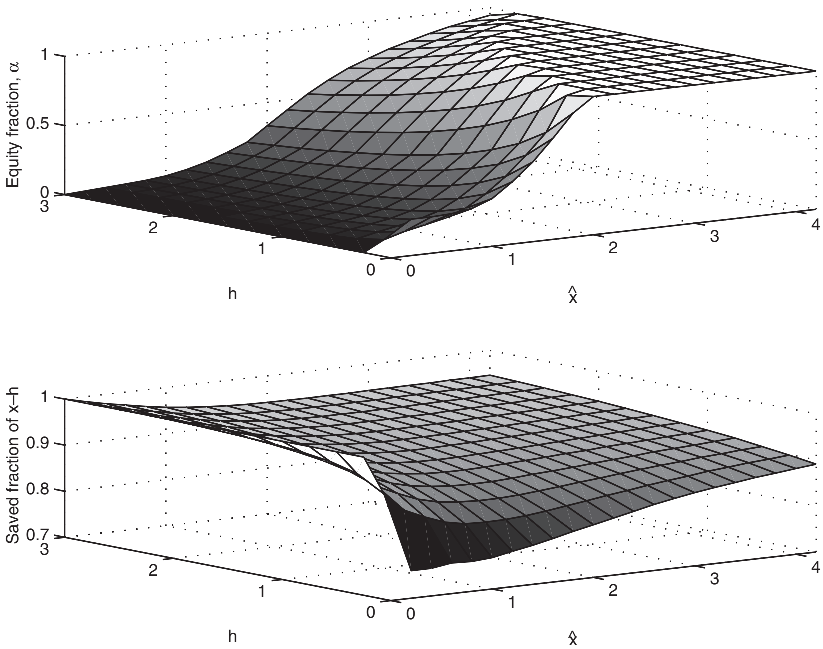

Figure 3
Policy functions for asymmetric distribution of income shock (with zero income), γ=5, δ=0.6 for the first period of the model
Top panel shows portfolio fraction invested in stock and bottom panel the fraction of available resources $x_1 - h_1$ to be saved. Transformed wealth $\hat{x}$ is defined in Equation (14) as cash in excess of admissible boundary for a given habit h_t. Both transformed wealth and habit are normalized by the permanent income.

Figure 3, lower panel, shows that the savings rate (out of discretionary resources, $x - h$) increases when the household is closer to the boundary of the admissible region (i.e., $\hat{x}$ closer to zero). As $\hat{x}$ goes to zero, the savings rate approaches 1. This is because the expected marginal utility is higher near the admissible boundary and it approaches infinity. Holding $\hat{x}$ constant, the savings rate increases with habit because the same lifetime resources will have to be used to sustain the higher habit.

The upper panel shows the portfolio fraction allocated to stocks. The allocation to stocks is decreasing approaching the admissible boundary and converges to zero as $\hat{x}$ goes to zero. This is because stock is a risky asset, and its presence in the portfolio decreases the *lower bound* on future wealth. Therefore, when wealth is lower, the portfolio must become more conservative to guarantee that consumption stays above habit in the worst-case scenario. Holding $\hat{x}$ constant, the allocation to stocks declines with habit because higher habit is harder to maintain. Therefore, portfolio choice is more influenced by the worst possible outcomes.

Compared with the expected and recursive utilities studied earlier in the literature, habit utility generates new implications for the relationship between wealth and portfolio share in stocks. This deserves a special discussion. In many life-cycle models of portfolio choice, including the one with habit utility, there is a well-known effect of substitutability between bond income and labor income. Because of this effect, households with substantial human capital but low financial capital (i.e., young households) have an incentive to tilt their portfolios toward stocks. On the contrary, households with relatively low human capital but high financial wealth (i.e., older households) would want to allocate their portfolios more conservatively. In the models with expected/recursive utility and moderate risk aversion, this effect counterfactually implies complete portfolio specialization in stocks for young (or not very wealthy) households.

Although the same mechanism applies in the model with habit, there is a new effect in portfolio choice that helps resolve the problem. Figure 4 shows two policy functions at ages 25 (first period) and 64 (one period before retirement), both evaluated for habit level $h = 0.3$.[25] The constraint equations at this habit (both for younger and for older households) imply that the minimum admissible cash on hand is just under 0.73. Thus, the correspondence between $\hat{x}$ shown in the figure and cash on hand x is given by $x \approx \hat{x} + 0.73$ with values of $x \leq 0.73$ being inadmissible. The substitutability of human and financial capital is apparent

[25] For $\delta = 0.6$ the habit of 0.3 corresponds to the initial habit at age 25. Recall that consumption before entering working life was assumed to be 0.5 times the initial permanent income level, so the initial normalized habit is $0.5 \times 0.6 = 0.3$. Portfolio policies for nearby habit levels are similar.

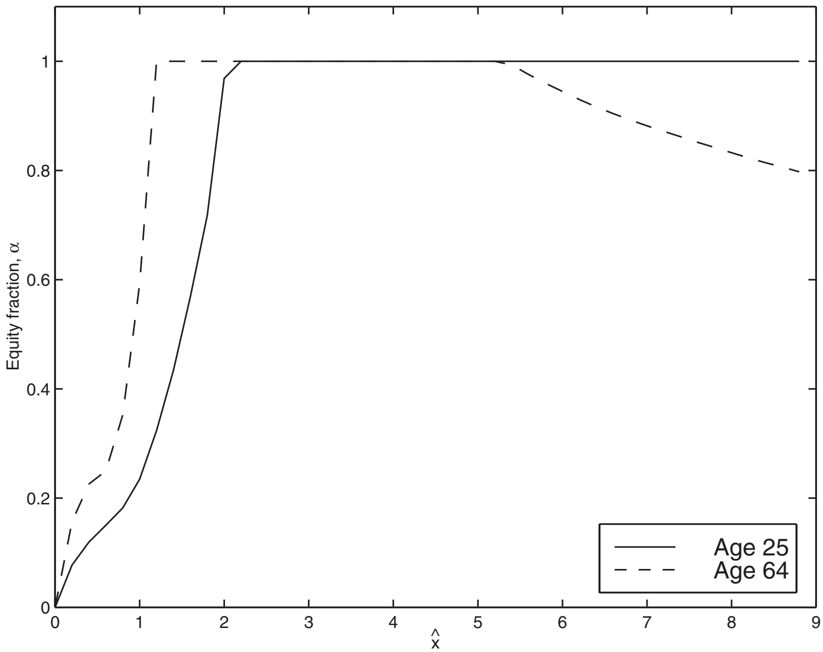

Figure 4
Portfolio policy functions for asymmetric income shock (zero income), γ=5, δ=0.6 at ages 25 and 64 and evaluated at h=0.3
For this level of habit, the correspondence between cash on hand x and its transformed value $\hat{x}$ shown on the graph is given by: $x \approx \hat{x} + 0.73$.

in Figure 4: from medium cash on hand and higher, a young investor's policy is to hold only stocks; for older investors, it is optimal to reduce exposure to stocks at higher cash on hand.[26] However, near the minimum admissible level of cash on hand, both types of investors behave similarly and rapidly decrease their exposure to stocks. When cash on hand approaches the admissible boundary, both old and young investors must reduce their exposure to stocks: stocks aggravate the worst-case scenario for cash on hand. Note that for the lower wealth and younger investor, the portfolio is more conservative than that for the older investor with the same wealth. This is because the young investor has to sustain

[26] To illustrate how the scale on the figure translates into dollars, we have to use the appropriate permanent income for each age. Also, because the expected value of the transitory shock η is equal to 1 and the current income is included into cash on hand, we must subtract 1 from cash on hand to obtain assets threshold for the *average* agent. For example, for a 25-year-old investor, the expected permanent income is about \$20,000. Her portfolio fraction in stocks reaches 1 at cash on hand equal to 2.73 (0.73 added due to transformation). Subtracting expected income (1.0) from cash on hand gives assets equal to 1.73 times permanent income, which corresponds to \$34,600. For a 64-year-old agent, we obtain by similar calculation that her portfolio policy reaches 1 at \$25,600 [$\approx (1.2 + 0.73 - 1) \times \$27,500$] and it declines starting from \$130,000 [$\approx (5 + 0.73 - 1) \times \$27,500$].

the same habit under income uncertainty for many periods, whereas the older investor's income soon will become riskless. Therefore, for the younger who are not "too rich," the model predicts quite conservative portfolios. Although this effect is present in all calibrations, it is particularly apparent in the simulations with asymmetric income shock distribution with low/zero-income state.

3.2 Symmetric income shock distribution

Figure 5 shows the means of cross-sectional distributions of consumption, financial wealth, and portfolio share invested in stocks. The results are reported for three values of the habit strength parameter, $\delta = 0.8$, 0.6, and 0 (EU CRRA case). The effect of increasing δ is to increase wealth accumulation throughout the life cycle. Other things equal, a higher δ narrows the admissible region for endogenous states by tightening the feasibility constraints. Therefore, consistent with the properties of

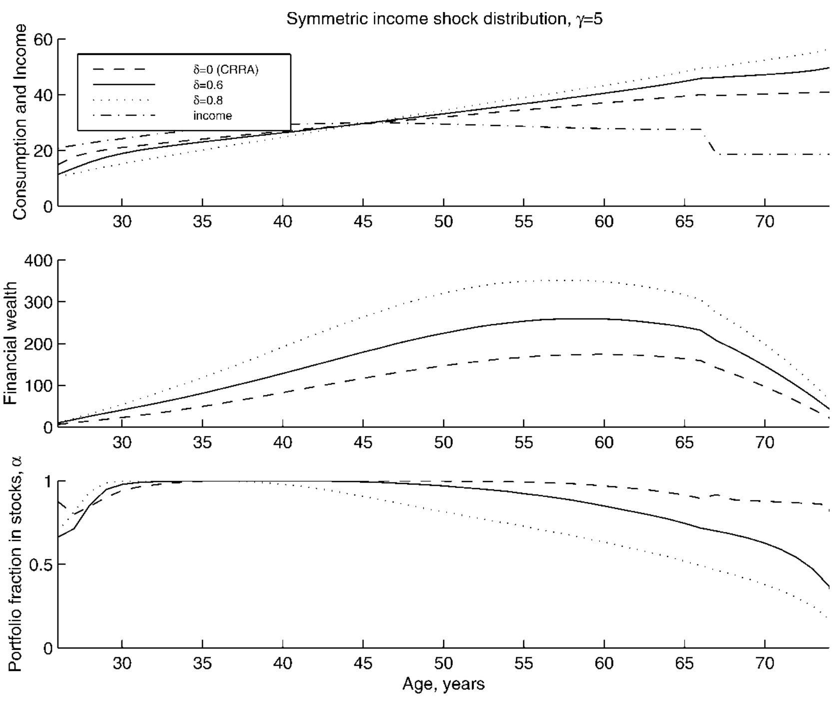

Figure 5
Symmetric distribution of income shock for various values of δ, γ=5
Financial wealth is the total holdings of bond and stock. Consumption, income, and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).

savings policies, a higher δ has the effect of increasing savings out of discretionary resources $(x_t - h_t)$ and leads to faster wealth accumulation.

As a result of higher savings under stronger habit, the consumption is lower early in life for higher values of δ. The consumption profiles are uneventful, except that two habit cases show a slight increase toward the end of life cycle. This is driven by the anticipated change in the utility function in the last period, T, where the future habit does not matter anymore because the agent dies. Note that consumption for the CRRA case does not have this feature, and consumption is almost flat after retirement.[27]

The bottom panel of Figure 5 shows the portfolio allocation in stocks throughout the life cycle. As expected from the earlier overview of the portfolio policies, higher habit strength reduces stockholdings. The figure shows a common problem of the life-cycle portfolio choice models: the predicted share invested in stocks is at, or close to, 100% throughout life for the CRRA case.[28] Essentially, this problem is a manifestation of the equity premium puzzle in a partial equilibrium setting. This effect is a well-known feature of the portfolio choice models with nonfinancial income and was shown first by Merton (1971) in a model where constant labor income substitutes for bonds in the financial portfolio.[29]

The predicted portfolio choice is not supported by the empirical evidence, though. For example, Poterba and Samwick (2002) and Ameriks and Zeldes (2001) summarized the data on portfolio allocation of U.S. households. Their findings suggested that, even conditional on nonzero stockholdings, household portfolios are far from 100% equity. Similar conclusions can be made from the findings of Heaton and Lucas (2000) and from a collection of international portfolio studies in Guiso, Haliassos and Jappelli (2002). As to the life-cycle shape of the portfolio share in equities, the evidence from all of these studies is mixed and depends on how the estimation procedures control for age, time, or cohort effects. Later in the article, I will revisit the implications of habit and EU models in view of the empirical evidence.

Figure 5 shows that simulated portfolios are interior very early and later in life so that portfolio allocation is hump-shaped with a plateau. There are two effects at play in the portfolio choice of the older investors. First, as human capital decreases, households compensate by shifting

[27] There is a very small growth of consumption in this case, which is driven by the assumed relative values of asset returns and the subjective discount factor in the utility function, β. Consumption in the CRRA case does not decline because the age of death is deterministic.

[28] This prediction depends on utility parameters and correlation of stock returns and permanent income. For example, in Cocco, Gomes, and Maenhout (2005) portfolios become interior in the later stage of life cycle for CRRA preferences with $\gamma = 10$ and correlation of permanent income and stock returns of 0.15.

[29] Merton did not consider a case with borrowing constraints. However, if one applies his result to a model in which labor income is constant but borrowing against it is not allowed, then the implied portfolio allocation would be constrained at 100% in equities earlier in the life cycle.

their portfolio allocation toward bonds which were crowded out earlier by human capital. The second effect is specific to the habit model. As households decumulate assets but maintain consumption levels (hence the habit levels), the wealth must be invested more safely to insure that habit can be maintained in any circumstances. Younger households have not yet accumulated savings, and their wealth is closer to the admissible boundary. However, their portfolios are not as conservative as those of the retired investors, because they own substantial human capital with a generous lower bound on realizations. Also, note that even for the higher levels of habit strength, the model is unable to generate conservative portfolios throughout the entire life cycle. However, the calibrations attempted so far do not exploit one of the key properties of the additive habit formation preferences: that the lower bound on labor income affects savings and portfolio choice.

One interesting and important observation from the simulations shown in Figure 5 is that, for high values of habit strength, $\delta = 0.8$, there is a substantial accumulation of wealth. Although in the representative agent general equilibrium models the higher habit strength helps achieve a low risk-free interest rate and a high equity premium, in partial equilibrium this presents a problem. For the utility assumed in the present model, the levels of habit strength typically required to resolve the asset pricing puzzles would be in a range above 0.8 [Constantinides (1990)].[30] For such strong habits, the life-cycle model predicts dramatic wealth accumulation relative to the cases of low habit strength and CRRA utility. Empirically, wealth accumulation is not even close to what the model predicts for high habit strength. For example, Gentry and Hubbard (2001) computed wealth-income ratios conditional on age and entrepreneurial status. For people without businesses, they found that median (mean) wealth-income ratios before retirement range from 0.3 (1.2) for people younger than 35 to 1.4 (2.9) for people between 35 and 55. Although the model with low-to-moderate habit strength has plausible wealth-income ratios, the high-habit strength calibration ($\delta = 0.8$) has median and mean wealth-income ratios on the order of six to eight for the middle-aged households.[31] It is quite likely that this problem is due to a failure of the aggregation assumptions required by the representative agent models and that one need not assume high habit strength when explicitly modeling heterogeneous individual consumption. Aggregate

[30] Alternatively, for exogenous habit [Campbell and Cochrane (1999)], one must have consumption surplus ratio as low a 0.05. Even in the case of calibration for $\delta = 0.8$, the simulated consumption surplus ratio $(\frac{C_t - H_t}{C_t})$ is only about 0.3–0.25 for the larger part of the life cycle.

[31] The model in this article is too stylized to formally investigate the determinants of cross section of life cycle wealth accumulation, so this analysis is not attempted. Such an exercise would have to be primarily empirical and would require explicit modeling of household heterogeneity along factors such as education, demographic status, risk preferences, bequest motive, entrepreneurial/occupational status, and real-estate ownership.

consumption is quite smooth, and for the resolution of asset pricing puzzles, the representative agent models require high risk aversion (habit strength).[32] To confront the problem of excessive wealth accumulation, it would be necessary to construct a model with heterogeneous agents. Such a model would not aggregate analytically. Future research can determine whether heterogeneous-agent model can match asset returns and still have correct micro-level implications. Pijoan-Mas (2003) considered such a model with ratio-habit preferences. It seems feasible to combine his approach to solving the model with the techniques specific to the additive habit utility used in this article.

3.3 Asymmetric income shock distribution

Typically, in the life-cycle literature it is assumed that uncertainty in the individual labor income stems from unemployment. The parameters for the labor income process usually are estimated from the panel data on earnings, and the empirical procedures use averages across individuals to obtain stable estimates.[33] Such averaging may underestimate the lower bound on nonfinancial income received during the periods of unemployment. There are also other risks and nondiscretionary expenses that may significantly reduce household's disposable income. For example, out-of-pocket expenses on health care are especially likely to occur later in the life cycle. Although public and private health insurance may cover such expenses, not all households have access to insurance and there are often substantial copays. Another major income shock is disability. For reasons of moral hazard, disability is only partially insurable. It may entail a permanent and significant reduction of income, even if disability benefits are awarded, which is far from guaranteed.

All of these risks reduce nonfinancial income in the worst-case scenario. Therefore, they may have a significant impact in the model with habit formation preferences. Although it is hard to calibrate the probabilities of specific events leading to major income shocks, the distribution of income constructed by Carroll (1992) using PSID data showed the evidence of significant income shocks faced by households. His distribution shows a non-negligible mass with zero- or near-zero-income realizations. Carroll argued that these observations are unlikely to be driven simply by measurement error, because the households with zero or near-zero income also report experiencing "bad" events, such as

[32] In the case with exogenous habit, as in Campbell and Cochrane (1999), the relative-risk aversion is equal to the utility curvature coefficient divided by the excess consumption ratio, that is, to $\gamma \frac{C_t}{C_t - H_t}$. In the case of endogenous habit, as in Constantinides (1990), the risk aversion is not expressed easily analytically, but the same intuition applies: the lower the excess consumption ratio, the higher the risk aversion.

[33] Such calibrations of the labor income shock distribution parameters are abundant in the literature. See, for example, Heaton and Lucas (1996), Storesletten, Telmer, and Yaron (1998), Cocco, Gomes, and Maenhout (2005), and many others. One of the data sets commonly used for this purpose is the PSID.

unemployment, injury, health problems, and divorce. In this section, I use the figures from Carroll (1992) and assume that income could experience an unusually bad state with a small probability. Several calibrations using Carroll's data were attempted; here I report the results of two of them. In one case, households may have zero income with probability equal to 0.5%; in the other case, the low-income state is 10% of permanent income with a probability of 1.3%. These shocks are assumed to be uncorrelated with stock returns and may occur at any period during the working years.[34]

In the calibrations with low/zero-income state, I use habit strength of 0.6 and compare it to the case of a symmetric income shock considered earlier. I also compute a case with EU (CRRA) and a zero-income state for comparison. A reminder: in these calibrations, the initial asset holdings are assumed to be equal to 80% of the initial permanent income. This is done to make sure that initial habit is admissible in all compared calibrations and is the same as in the earlier experiments. For the sake of brevity, I only discuss portfolio allocation profiles in these cases shown in Figure 6. Consumption and wealth accumulation are qualitatively similar to the cases presented earlier for symmetric income shock distribution.

Portfolio choice profiles for the cases of low and zero income are qualitatively similar and have a distinct hump shape. Portfolios are somewhat more conservative when income can fall all the way to zero rather than when it is bounded at a minimum of 10% of permanent income. For the latter case, the feasibility constraints are relaxed, which is increasing portfolio exposure to stocks (Section 1.3 and Figures 1 and 2). When sufficient wealth is accumulated in mid-life, and when there are no income shocks at retirement, the profiles with the zero and the low-income state nearly coincide. In the case of habit utility with symmetric income shock, portfolios are aggressive earlier in life because of the extra "cushion" provided by labor income. In mid-life and retirement, portfolio choice is close to the two asymmetric distribution cases. This last implication of the model is likely to change if asset liquidity is considered in more detail, because liquid wealth is critical to maintaining consumption above habit under adverse circumstances. Liquid wealth may be low even for middle-aged households, because the bulk of their assets may be tied up in housing that is not very liquid.

[34] The empirical case for correlated bad transitory income shocks and stock returns is weak. Empirical evidence such as in Davis and Willen (1999) and in Cocco, Gomes, and Maenhout (2005) supported zero correlation of transitory income shocks and stock returns. If anything, these studies found only a small correlation between permanent income shocks and stock returns. Moreover, most of the events that potentially lead to bad transitory income shock, such as divorce, exceeding insurance limits, and liability payments [see Carroll (1992)], are unlikely to be correlated with stock market performance.

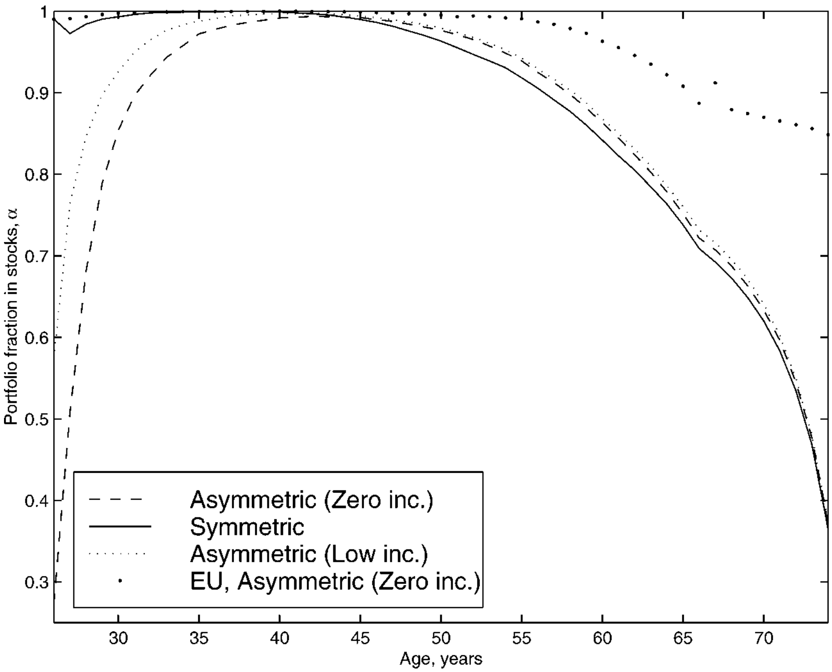

Figure 6
Portfolio choice with asymmetric distribution of income shock, γ=5
Solid line ("Symmetric") corresponds to benchmark. Zero income $\eta = 0$ occurs with probability 0.5%. Low income $\eta = 0.1$ occurs with probability of 1.3%. "EU" denotes expected utility (CRRA) case with $\delta = 0$ and zero-income state.

The case of EU is shown last. Because all calibrations in Figure 6 start with nonzero wealth, the EU investor is virtually unaffected by the possibility of zero income. In fact, because initial wealth is higher here than in the case shown earlier in Figure 5, the EU investor has more stocks early in life. Later in life and at retirement, EU investors reduce their exposure to stocks, but this reduction is not as strong as for habit investors.

This calibration shows that there is a significant effect of introducing low-labor-income realizations in the model with habit formation. Even though the expected present value of labor income is virtually unchanged by the assumption of a zero/low-income state, there is a substantial effect on portfolio choice. The main difference is that portfolios early in life are much more conservative. EU preferences, under similar conditions, show more aggressive portfolios than habit preferences throughout the life cycle.

3.4 The extensions for borrowing and flexible labor supply

The previous discussion demonstrates the importance of interaction of uninsurable labor income risk with habits. Therefore, it is interesting to consider how portfolio choice may be affected by income hedging mechanisms. I consider two channels for insuring against income fluctuations: one allows for borrowing against future income, and another allows for labor supply flexibility. I now describe the results obtained in these two extensions.

3.4.1 Borrowing against future labor income. I assume that households can borrow up to the present value of the lowest possible life-time labor income. This is an upper bound on borrowing such that it is feasible to repay debt in any circumstances without default. Therefore, this calibration demonstrates full potential of borrowing constraints to affect portfolio choice. Tighter borrowing constraints would generate portfolio choice profiles between what is shown in this section and those obtained earlier. Note that, because retirement income is always nonzero, the present value of life-time income is nonzero, even when a zero income shock is possible during working life.

The feasibility constraints and the transformation of cash on hand have to be modified in this case, and Section A.1 in appendix provides further details. The rest of the problem (state variables, etc.) is not affected by relaxing borrowing constraint. The only additional parameter needed in this extension is the borrowing rate r_d that is set at 10%, comparable to some of the lower rates available on uncollateralized credit card loans.[35] I consider the same two cases with a low/zero-income state as in the previous section. One case with zero-income state that occurs with probability of 0.5% and another with low-income state equal 10% of the permanent component with probability of 1.3%.

Figure 7, panel A, shows portfolio allocation in stocks for the two cases. The average portfolio profiles still retain their hump-shaped form. The case with zero income is quantitatively similar to the model without borrowing (see Figure 6 for comparison). The intuition for this result is as follows. In this case an agent can borrow only against retirement income. However, at the beginning of life cycle the present value of retirement income is small. As a result, the young agents have to rely on their own savings to ensure that consumption stays above habit in adverse circumstances. The borrowing limit is increasing as retirement approaches, but households also accumulate sufficient wealth over time and do not need to borrow. Therefore, relaxing borrowing limit does not affect portfolio

[35] Lower borrowing rate (8% in some experiments) does not change any implications of the model because, given other parameters, households prefer to save but in extreme circumstances. Although some "teaser" rates on credit cards may be lower, they only apply for a short period of time.

Figure 7
Portfolio allocation for two extensions of the model which allow borrowing against future labor income and flexibility in labor supply
Portfolios are shown for two calibrations of income process with low-income state equal to either zero or 10% of permanent level. Zero income $\eta = 0$ occurs with probability 0.5%. Low income $\eta = 0.1$ occurs with probability of 1.3%.

choice of middle-aged and old agents. When the worst income state is 10% of the permanent level, the borrowing limit is significant from the outstart. The dashed line on Figure 7, panel A, shows that portfolio allocation is more aggressive for young investors compared with the same case without borrowing shown in Figure 6 (labeled as "asymmetric, low income"). In this case borrowing is allowed exactly when it matters most—when assets are low at the beginning of life cycle. The effect of relaxing borrowing constraint is similar to that of assuming a nonzero "floor" on labor income. When borrowing constraint is relaxed, the feasibility boundary shifts to the left which implies more aggressive portfolio for the same cash-habit state pairs. In the middle and late stages of life cycle, borrowing opportunity does not affect portfolio allocation because households accumulate sufficient wealth.

It is also interesting to note that in both cases no borrowing occurred in simulations. For compatibility with the previous section, households were assumed to have a nonzero initial habit and wealth.[36] To see whether the model can generate any borrowing at all, I repeated the simulations setting the initial habit and wealth to zero. For such initial conditions there were a few observations of borrowing in the first two periods. As expected, the agents who borrowed received a bad-income shock. However, as soon as their bad luck is over, they begin to save. Because bad-income outcomes are infrequent, after two-three periods in the simulations there are no households who borrow.[37]

The above calibrations show that the effect of borrowing on portfolio choice is most pronounced at the beginning of life cycle and that quantitatively it depends on the assumed income process. Therefore, the economic mechanisms that limit unsecured borrowing by young households are potentially important determinants of their portfolio choice. Although these considerations are outside the scope of this article, it seems reasonable to assume a small borrowing limit at the beginning of life cycle. It may be argued that young households lack credit and income histories necessary to obtain sizable loans. Furthermore, if some loans are requested during low income spells, credit checks would prevent such requests from being granted. Of course, one can obtain a credit line in advance, but its limit may be insufficient and assets would remain the main source of maintaining consumption in adverse circumstances.

3.4.2 Flexible labor supply. In this section I consider an extension of the model which allows for labor supply flexibility. The utility function is modified to include preferences for leasure. The utility is assumed to be a CRRA in leasure and has additive habits in consumption:

$$u(c_t, h_t, l_t) = \frac{(c_t - h_t)^{1-\gamma} l_t^{\sigma}}{1-\gamma}, \ \sigma < 0, \ \underline{l} \leq l_t \leq 1.$$

The curvature of leasure in the utility is set to $\sigma = -4$, to match the curvature of (excess) consumption. The income earned during working life depends on the amount of leasure chosen (l_t) and on the wage realization (Y_t) so that the total income is given by $I_t = Y_t \frac{1-l_t}{1-\underline{l}}$ and I_t replaces Y_t in the budget constraint. The level of leasure $\underline{l}$ at which full wage is paid is set at 0.4 so that in simulations the average leasure choice is interior and corresponds to devoting about 45–30% of time endowment

[36] Recall that habits are set assuming consumption before entering working life if equal to half of the initial permanent income level: $h_{t_0} = \delta \times 0.5 \times f(t_0)$ and initial cash is set $0.8 \times f(t_0)$ so that initial habit is feasible throughout all calibrations and in particular in the calibration with zero-income shock.

[37] Undoubtedly if the cross section is made larger than 10,000, there may be more incidents of borrowing in the later periods consisting of households who have drawn low income for several periods. However, the frequency of such outcomes would be too small to affect any of the results. Portfolio allocation profiles in the simulations with zero initial habit and wealth remain similar to what is already reported.

to work.[38] The feasibility constraints remain the same in this case. During retirement it is assumed that $l_t = 1$ and that pension income is independent from leasure. The transitory shock component η_t becomes part of the state vector during working life, because it determines the wage rate (the price of leasure).[39] Other parts of the problem and solution method are unaffected by this extension.

The results of this calibration are shown in Figure 7, panel B, for the two cases of labor income process. In the case with zero-income state, portfolio allocation for young and middle-aged agents is similar to the one shown earlier for the case of fixed labor supply. Because there is a possibility of a zero-income state, regardless of labor supply choice, young households must invest cautiously and rely on savings to finance consumption in bad-income states. The differences from the base case appear a few years before and during retirement when portfolio allocation to stocks is higher. Households anticipate that at retirement the price of leasure will drop to zero and that leasure will increase and remain constant throughout retirement. It is optimal then to reduce consumption so that consumption-leasure composite in the utility function stays smooth. The anticipated consumption decline implies habit decline, and households can choose riskier portfolios. After retirement, the usual mechanism of depleting human capital takes over and the average portfolio fraction in stocks is decreasing with age. The other case (with low income of 10% of the permanent component) is qualitatively similar. When compared with the case with the same low-income state but without flexible labor (see Figure 6, dotted line), portfolio allocation at the beginning of life cycle is slightly more aggressive with flexible labor. This is consistent with Bodie, Merton, and Samuelson (1992) who showed that with flexible labor supply households can hedge income and stock returns fluctuations.[40]

In summary, the results of the two extensions demonstrate that both borrowing and flexibility of labor supply can mitigate the effects of habits and generate higher demand for stocks. The main implication that portfolio choice is more conservative for young agents than for middle-aged agents is robust to both extensions. Furthermore, quantitative results depend critically on the assumed income process. In particular, when

[38] None of the reported results are affected by this parameter; it is only important that a typical household's optimal leasure choice is in the interior of the interval $[\underline{l}, 1]$.

[39] The computational time required for this extension is significantly longer because policy and value functions have to be computed for every value of transitory shock η. In addition, one extra optimal control for optimal leasure search has to be added during working periods.

[40] In this article, the differences between portfolios implied by the models with fixed and flexible labor supply narrow and disappear later in the life cycle, because households are restricted from holding leveraged stock and portfolio share in stocks cannot exceed 1 in both cases. In Bodie, Merton, and Samuelson (1992), leverage is allowed and the differences persist throughout the life cycle.

income process has a zero outcome, portfolio choice closely resembles that in the basic habit model.

3.5 Further discussion: habits, expected utilities, and stylized facts

In light of the empirical evidence, it is helpful to compare the habit model to the standard life-cycle models based on EU.[41] In a recent survey, Haliassos and Michaelides (2002) identified three main challenges to the existing portfolio choice models. The first challenge is the limited stock market participation of (predominantly) households with low wealth, an issue not addressed in the model presented here.[42] Second is the positive relationship between the equity portfolio share and wealth (or income) in the data, which is in contrast to the models in which the portfolio share of stocks declines with cash on hand. Third, the share of portfolio invested in stocks declines with age in the models, but the data suggest anything but this pattern: empirically, the share of stocks is either hump-shaped, increasing and then flattening out, or simply increasing, depending on the empirical procedure, data set (country), and asset definitions.

As already mentioned, the present model can help explain the positive relationship between portfolio equity share and wealth, as well as some aspects of the age dynamics of portfolio allocation to stocks. At the intersection of the second and third challenges just described is perhaps one of the most economically interesting implications of the habit model: the portfolio allocation of young households.[43] Empirically, young households who typically have lower wealth and a lower fraction of portfolios invested in equity contribute both to the positive relationship between wealth and allocation to stocks and to the hump-shaped or increasing age profile of the equity portfolio share. In the EU models, young investors usually specialize in equities because their human capital is not significantly correlated with stock returns and, for low risk aversion, a high equity premium makes stocks very attractive. Habit formation provides both a theoretical and an economic rationale for why the portfolios of young households may be conservative. In the usual economic interpretation of habit (standard of living), the desire to maintain one's standard of living dictates that when wealth is low, one must carefully manage resources so as not to fall behind the minimum acceptable level of consumption. Theoretically, this corresponds to higher risk

[41] The formal empirical tests are outside the scope of this article, and the discussion here is intended only as a broad assessment of the model's predictions relative to the data and as a comparison to the predictions of the existing theoretical models with expected utility.

[42] Modeling limited participation usually requires fixed costs and is not attempted here to keep the focus on the properties related to habit formation utility. Gomes and Michaelides (2005) showed that in a setting with relatively small fixed participation cost, Epstein–Zin utility, and heterogeneously risk-averse and impatient agents, it is possible to match the observed limited stock market participation.

[43] More generally, the discussion here is also applicable to all other investors who have relatively high human capital compared with their financial wealth accumulated at any given age.

aversion for the investor near the boundary of admissible resources. In general equilibrium asset pricing models, the variation of risk aversion creates time-varying risk premia. In partial equilibrium, the endogenous adjustment in risk aversion with wealth helps generate empirically plausible portfolio choice patterns.

The empirical evidence mainly suggested that portfolio allocation to stocks is either increasing or hump-shaped with age, although the literature has not yet reached consensus. However, the comparative statics for portfolio allocation between young and middle-aged households seem to be more coherent across studies. For the most liquid financial assets, Heaton and Lucas (2000) found that the share of equities is hump-shaped over a life time. Consistent with their results, Faig and Shum (2002) found that the fraction of *cash* in household liquid assets is U-shaped over a life time. However, Heaton and Lucas (2000) also demonstrated that equity allocation is sensitive to the measure of wealth and may be increasing with age.[44] Also, the findings of Ameriks and Zeldes (2001) suggested that all empirical analyses of equity allocation patterns over the life cycle should be taken with a grain of salt. They showed that there is fundamental lack of identification of the effects of time, age, and birth cohort on portfolio allocation because they are linearly dependent.[45] Also, controlling either for time or for cohort effects produces surprisingly different results. Controlling for time effect, the predicted portfolio share is slightly hump-shaped. When cohort effect is accounted for, the predicted share in stocks is increasing over the life cycle. Several studies in a collection edited by Guiso, Haliassos, and Jappelli (2002) examined individual portfolio allocation in select European countries (Germany, Italy, the Netherlands, and the United Kingdom) and reconfirmed most of the key findings from the U.S. data. International evidence suggests that, conditional on stock ownership, the portfolio allocation to stocks is increasing slightly with age, but this effect is not always significant and is sensitive to the definition of total wealth. Both the U.S. and the international data are consistent with the view that wealthier or high-income households, conditional on participation, invest more in stocks.

Although Haliassos and Michaelides (2002) mainly discussed the results from the EU-based models, it is also worth noting here the main differences between additive and ratio-habit models. An in-depth analysis of a life-cycle portfolio choice problem with a ratio-habit utility is undertaken by Gomes and Michaelides (2003), and the interested reader is referred to their paper for details. In the ratio-habit preferences

[44] Additionally, tax status of the portfolio matters, for example, tax deferred investments, may have different allocations than taxable ones [see, e.g., Poterba and Samwick (2002) and Gomes, Michaelides, and Polkovnichenko (2004)].

[45] Specifically: age=calendar year−birth year. Calendar year is used to control for time effects and birth year for cohort effects.

[Abel (1990)], consumption is replaced with the ratio of consumption to habit so that per-period utility is defined as $u(C, .H) = \frac{1}{1-\gamma}\left(\frac{C}{H}\right)^{1-\gamma}$. Gomes and Michaelides (2003) concluded that ratio habit makes it harder to jointly match patterns in portfolio choice and stock market participation. The problem is that habit increases wealth accumulation and makes it harder to explain limited stock market participation with fixed costs. The main difference from the additive habit utility is that ratio habit implies portfolio choice qualitatively similar to the CRRA EU: portfolio share invested in stocks is decreasing in wealth. As a result, the ratio-habit model implies higher stock allocation for younger than for middle-aged investors.

Although the additive habit model compares favorably to its EU counterpart overall, it is not without deficiencies. The most important one was already mentioned and is related to the counterfactually high wealth accumulation observed for high habit strength. Ultimately, this issue can only be resolved in a general equilibrium model with heterogeneous agents. Another challenge is that the level of portfolio allocation to stocks is still somewhat high as compared with the actual data. This question will be addressed in the comparative statics by considering the model's predictions for a lower level of the equity premium. Finally, realistic wealth accumulation profiles require a bequest motive. This is also addressed in the next section.

3.6 Some comparative statics

In this section, I consider several perturbations of the model to understand the sensitivity of the results to the assumed parameters. I already discussed the comparative statics with respect to habit strength, δ. The effects of changing utility curvature, γ, are somewhat similar to habit strength and for the sake of brevity are not reported in a separate figure.[46] Lowering the utility curvature lowers the demand for precautionary savings.[47] As a result of lower precautionary savings, consumption profile shifts upward early in life and downward later in life. The portfolio choice profile shifts upward, indicating that households invest entirely or almost entirely in equities until the age of 65 with a minor decline afterward. These comparative statics patterns occur in calibrations with or without a zero/low-income state.

Previous simulations demonstrated that, without a bequest motive, the agents in any finite-horizon model rationally dissave all their wealth by the end of life, which contradicts the empirical evidence. Prior studies of life cycle and portfolio choice showed that bequests are crucial in matching wealth accumulation profiles [e.g., Cocco, Gomes, and Maenhout

[46] These results are available on request from the author.

[47] The coefficient γ was set to 3 for this experiment.

(2005), Gomes Michaelides (2005)]. To explore the role of bequests in the habit formation model, and to contrast it to the EU-based model, I modify the problem to include a terminal bequest function in the Bellman equation (8). Cocco, Gomes, and Maenhout (2005) reported experiments with a bequest function identical to the one used here, and they calibrated bequest strength, λ, in a range from 1 to 5. They found that only for higher λ's does the bequest motive has any significant effect. Motivated by their results, I set the bequest parameter to 5.

Figure 8 shows wealth accumulation and portfolio choice for calibrations with and without bequest for habit formation utility with $\delta = 0.6$ and for CRRA utility. As expected, bequest motive increases savings. Also, later in life, wealth is not decumulated completely. Bequest motive has relatively more impact on wealth accumulation in the case of EU. Habit utility investors must accumulate substantial wealth during working periods to guarantee that habit can be

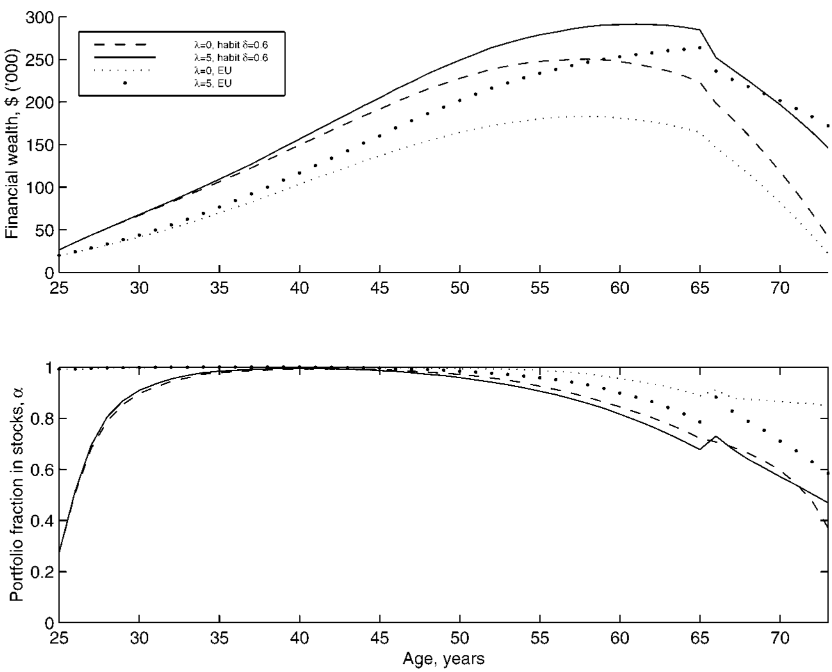

Figure 8
Bequest motive effect for asymmetric distribution of income shock for habit (δ=0.6) or EU cases (both with γ=5)
Financial wealth is the total holdings of bond and stock. Financial wealth is in thousands of dollars (inflation adjusted with 1992 as a base year). The bottom panel shows α, the fraction of financial wealth invested in stock.

maintained safely. For them, bequest operates mostly by slowing down wealth decumulation after retirement. On the contrary, EU agents without a bequest motive do not maintain as significant wealth as the habit agents. For them, bequest motive increases savings more significantly during working life. The terminal wealth left after bequest is somewhat higher for EU agents. This is understandable, because habit at the end of life is nonzero. Other things equal, this increases the marginal utility of consumption for habit agents relative to the EU agents.

In both habit utility and EU cases, the effect of bequest is to decrease portfolio allocation to stocks by a small amount for a few years before retirement. After retirement, there is a slight shift back to stocks because retirement income is constant. There is a larger effect of bequest on portfolio allocation in the case of EU agents. This is related to the argument for wealth just presented. For EU agents, bequest increases wealth accumulation significantly, and higher financial wealth leads to more conservative portfolios. There is a similar but smaller effect in the case of habit utility, because habit agents accumulate substantial wealth with or without a bequest motive. These results suggest that issues other than the strength of the bequest motive may be important for portfolio allocation later in the life cycle. These may include capital income taxes and health care expenses risks.

For the moderate habit strength, δ, required for realistic (not too high) wealth accumulation profiles, the level of portfolio share invested in stocks that is predicted by the habit formation model is still somewhat high.[48] Empirical studies cited earlier found that the share of wealth invested in stocks for market participants is in the range of 30–70%, whereas the model sometimes predicts a share close to 100%. In calibrations considered so far, bonds earn 1%, like Treasury bills, and holding a diversified portfolio of stocks is assumed to be costless. Because of these assumptions, equities may be unrealistically attractive in the model. In reality, household savings in risk-free assets usually are not only short-term bonds. Treasury bills can be a proxy for cash, as well as for the most liquid transactions accounts, such as money market, checking, and savings accounts. Other savings in safe assets usually are high-grade corporate bonds or long-term government bonds or mutual funds that hold such bonds. The yields of these assets are typically higher than the yields on Treasury bills. Also, it is costly to hold a diversified portfolio of stocks, either directly or through a mutual fund. Recognizing these issues, McGrattan and Prescott (2003) estimated the returns on equities and bonds that households would be able to earn after accounting for

[48] The same problem of increasing wealth accumulation prevents using a higher curvature coefficient, γ, to generate more conservative portfolio choice.

liquidity and transactions costs.[49] They argued that in the postwar U.S. data, the long-term bond real return was 4% and the equity index real return was slightly less than 5%, so the equity premium is just under 1%. For the comparative statics exercise here, I assume that the equity premium is 3% and raise the bond return to a constant 3%, keeping the expected stock return at 6%. This is motivated by the fact that transaction costs of holding equity funds were relatively high in the periods used by McGrattan and Prescott, and there was a considerable decline in mutual funds fees and expenses by the late 1980s and throughout the 1990s when most of the micro-level portfolio data were collected. The labor income process is assumed to have a zero-income state, as in the previously reported calibrations. Figure 9 shows the portfolio allocation profiles from this calibration for habit and EU cases and, for comparison, the previously reported calibrations with a high equity premium. With lower

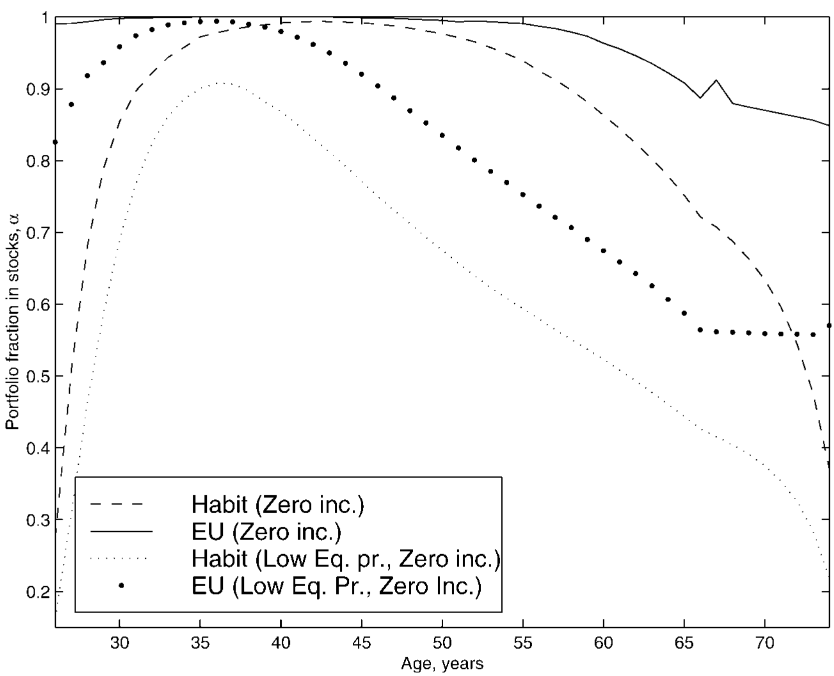

Figure 9
Asymmetric distribution of income shock, γ=5, habit (δ=0.6) or EU for high and low equity premium

[49] Additionally, McGrattan and Prescott considered the effect of capital income taxation on the yields of various investments. In a related article, Jagannathan, McGrattan, and Scherbina (2000) have also argued that the equity premium has become lower than a historic average computed by Mehra and Prescott (1985).

equity premium, portfolios become more conservative and remain interior throughout life. Note that EU still predicts a higher portfolio allocation to stocks than habit utility. Also, for the younger investors aged between 25 and 35, the average EU portfolio allocation is still close to 100% in stock.

4. Conclusion

This article solves a life-cycle model of consumption and portfolio choice with additive habit formation utility for a household with uninsurable risky labor income. I show that additive habit imposes feasibility constraints on the model which depend on the worst possible scenario for future labor income and on the habit strength parameter, but not on the probability of the worst possible labor income. As a consequence, even the slightest possibility of a "disastrously low" income implies more conservative portfolios and higher savings rates.

The model with habit formation compares favorably to several stylized facts about life-cycle portfolio allocation, particularly that it implies for some households a positive relationship between the portfolio share in stocks and wealth/income. Because of this relationship, the model also predicts relatively more conservative portfolios for younger investors with lower savings, which is consistent with the empirical facts. The main implications of the model are robust to income smoothing through borrowing or flexible labor supply. Although a high curvature coefficient or high habit strength helps bring the allocation to equities into the range observed in the data, they also imply counterfactually high wealth accumulation. This connection, between habit strength and savings rates, can be related to wealth distribution and asset market participation in a general equilibrium setting. It would be interesting to explore a model with habit formation and heterogeneous agents to see whether it is capable of generating plausible results jointly at the aggregate and micro levels.

Appendix A: Feasibility Constraints on Cash and Habits

The best way to derive the constraints on endogenous variables is to begin from the last period, T, where there is only one constraint that $X_T > H_T$. Going back one period to $T-1$, we have to ensure that not only $X_{T-1} > H_{T-1}$ but also that there is at least one policy function such that there will be enough cash on hand for the last period no matter what happens to the exogenous variables (income shocks and stock return). We will look for the policy function that delivers maximum possible cash on hand under the worst possible shock realizations. If such a policy delivers cash on hand above the habit for the next period T, then a pair (X_{T-1}, H_{T-1}) is feasible. If even such a policy cannot guarantee that $X_T > H_T$, there clearly is no other policy that can do that in all realizations of the exogenous shocks. Also, strictly speaking, maximum cash on hand is not going to be achievable because the constraint on current consumption is strict $(C_{T-1} > H_{T-1})$. Thus, we have to make sure that:

$$\sup_{S_{T-1},B_{T-1}} \{ \min_{\eta_T,P_T,r_{s,T}} X_T \} > H_T. \tag{A1}$$

It is also clear that habit H_T cannot fall below δH_{T-1} because of the consumption restriction at $T-1$: $C_{T-1} > H_{T-1}$.

Because the exogenous shocks are assumed to be i.i.d., there is a state where, with nonzero probability, the stock return is negative and takes the lowest possible value while labor income shocks are lowest, too.[50] Thus, the policy that maximizes cash on hand under the worst-case scenario is not going to call for buying any stocks because doing so only aggravates the worst-case outcome. Therefore, we can restrict our attention to bonds only. To find the supremum in the left-hand side of Equation (A1), suppose that the household consumes the amount: $C_{T-1} = H_{T-1} + \epsilon$. This consumption generates habit for time T given by $H_T = \delta(H_{T-1} + \epsilon)$. The amount left for savings is given by $(X_{T-1} - H_{T-1} - \epsilon)$, which, invested in bonds, gives the maximum possible cash on hand in the worst case at time T. Then feasibility requires that:

$$X_T = (1 + r_b)(X_{T-1} - H_{T-1} - \epsilon) + \underline{Y}_T > H_T = \delta(H_{T-1} + \epsilon),$$

where $\underline{Y}_T$ denotes the lowest possible income realization at T. A pair (X_{T-1}, H_{T-1}) is feasible if there is an ϵ such that the above inequality is satisfied. To find the lowest admissible cash on hand X_{T-1} for a habit H_{T-1}, we send ϵ to zero and obtain the constraint on time $T-1$ habit and cash:

$$(1 + r_b)(X_{T-1} - H_{T-1}) + \underline{Y}_T > \delta H_{T-1}.$$

Rearranging this constraint in a more convenient present value form yields:

$$\frac{\underline{Y}_T}{1 + r_b} + X_{T-1} - \left(1 + \frac{\delta}{1 + r_b}\right) H_{T-1} > 0. \tag{A2}$$

Consider going back one more period to $T-2$. For a pair (X_{T-1}, H_{T-1}) to be feasible, it must be the case that $X_{T-2} > H_{T-2}$, and there should exist a portfolio policy that guarantees under any realizations of shocks that the next period's feasibility constraints will be satisfied. There are two feasibility constraints at time $T-1$. The first constraint is $X_{T-1} > H_{T-1}$. From the above analysis, it is clear that to satisfy this constraint it must be the case that:

$$\frac{\underline{Y}_{T-1}}{1 + r_b} + X_{T-2} - \left(1 + \frac{\delta}{1 + r_b}\right) H_{T-2} > 0.$$

Also, substituting into (A2), the expressions $X_{T-1} = (1 + r_b)(X_{T-2} - H_{T-2} - \epsilon) + \underline{Y}_{T-1}$ and $H_{T-1} = \delta(H_{T-2} + \epsilon)$, and taking the limit as ϵ goes to zero, after rearrangement, yields:

[50] Note that i.i.d. is not essential here. It is enough for the transition probability matrix to be such that in the states where the income shocks have the lowest possible value it is also possible for the stock return to be negative.

$$\left[\frac{\underline{Y}_{T-1}}{1+r_b}+\frac{\underline{Y}_T}{(1+r_b)^2}\right]+X_{T-2}-\left[1+\frac{\delta}{1+r_b}+\frac{\delta^2}{(1+r_b)^2}\right]H_{T-2}>0.$$

The process of iterating backward continues, each time substituting $X_{T-k}=(1+r_b)(X_{T-k-1}-H_{T-k-1}-\epsilon)+\underline{Y}_{T-k}$ and $H_{T-k}=\delta(H_{T-k-1}+\epsilon)$ into all constraints derived for the previous period and taking limit as ϵ goes to zero. Following this procedure, one derives $T-t+1$ constraints for a generic time t. These constraints have the same familiar form as follows:

$$\sum_{k=1}^{\tau}\frac{\underline{Y}_{t+k}}{(1+r_b)^k}+X_t-\left[\sum_{k=0}^{\tau}\left(\frac{\delta}{1+r_b}\right)^k\right]H_t>0,\ \tau=0,\ldots,T-t.$$

The economic interpretation of these constraints is explained in detail in the main text of the article.

A.1 Allowing for borrowing against future income

When borrowing against future labor income is allowed, feasibility constraints have to be modified. Feasibility requires that the present value of the worst-case life-time resources must strictly exceed the present value of the lowest possible sequence of habits. As before, assume that stocks would only aggravate the worst-case scenario for cash on hand, and therefore to establish the existence of admissible portfolio policies, we need to consider only risk-free lending and borrowing. Also, for the model to be self-consistent, borrowing cannot exceed the present value of life-time income. Because, in general, borrowing and lending rates are different, the present value of both habits and the available life-time resources depend on whether and at what times household is going to borrow. Let us denote r_d as the borrowing rate, and it is reasonable to assume that $r_d \geq r_f$. Consider a situation at time t with cash on hand X_t and habit H_t. Clearly if $X_t \leq H_t$ household has to borrow right now, at time t, for consumption to stay above habit. One (extreme) possibility is that *some* nonzero debt will have to be maintained till the end of life, and the future income together with X_t will be "barely" enough to finance consumption above habits in the remaining periods and pay off the loan. In this case, the relevant (marginal) discount rate for habits and income would be r_d and the present-value constraint becomes:

$$\sum_{k=1}^{T-t}\frac{\underline{Y}_{t+k}}{(1+r_d)^k}+X_t-\left[\sum_{k=0}^{T-t}\left(\frac{\delta}{1+r_d}\right)^k\right]H_t>0. \tag{A3}$$

On the contrary, if the resources are sufficiently high so that household never borrows in the worst-case scenario for income, the relevant discount rate is r_f and the relevant constraint is:

$$\sum_{k=1}^{T-t}\frac{\underline{Y}_{t+k}}{(1+r_f)^k}+X_t-\left[\sum_{k=0}^{T-t}\left(\frac{\delta}{1+r_f}\right)^k\right]H_t>0. \tag{A4}$$

There are also intermediate levels of cash on hand X_t when borrowing would be necessary in a few periods along the worst-case income path, but not immediately from time t. In these cases, because $r_d \geq r_f$, borrowing would be deferred until all resources are exhausted so that for a few periods (denoted as τ), the relevant discount rate will be r_f and then it will become r_d. In this case, the constraint is a mixture of the two border cases given above. In general, there are $T-t+1$ possible values of the horizon τ from 0 to $T-t$ which index all feasibility constraints. Note that the case in (A3) corresponds to $\tau=0$ and that in (A4) to $\tau=T-t$ so that all constraints can be folded into a general notation and written as:

$$\sum_{k=1}^{\tau}\frac{\underline{Y}_{t+k}}{(1+r_f)^k}+\frac{1}{(1+r_f)^{\tau}}\sum_{k=\tau+1}^{T-t}\frac{\underline{Y}_{t+k}}{(1+r_d)^{k-\tau}}+X_t-$$
$$\left[\sum_{k=0}^{\tau}\frac{\delta^k}{(1+r_f)^k}+\frac{1}{(1+r_f)^{\tau}}\sum_{k=\tau+1}^{T-t}\frac{\delta^k}{(1+r_d)^{k-\tau}}\right]H_t>0,\ \tau=0,\ldots,T-t. \tag{A5}$$

where it is understood that the sums in the cases of $\sum_{k=1}^{k=0}$ or $\sum_{k=T-t+1}^{k=T-t}$ are simply set to 0. The normalization of these constraints by the permanent income is the same as in the case without borrowing. The transformation of cash on hand here follows the same procedure as in the case without borrowing, except the terms $\underline{Y}_{t,t+\tau}$ and $\Delta_{t,\tau}$ have to be replaced respectively by the terms defined below:

$$\underline{Y}_{t,\tau}=\sum_{k=1}^{\tau}\frac{\underline{Y}_{t+k}}{(1+r_f)^k}+\frac{1}{(1+r_f)^{\tau}}\sum_{k=\tau+1}^{T-t}\frac{\underline{Y}_{t+k}}{(1+r_d)^{k-\tau}}$$
$$\Delta_{\tau}=\left[\sum_{k=0}^{\tau}\frac{\delta^k}{(1+r_f)^k}+\frac{1}{(1+r_f)^{\tau}}\sum_{k=\tau+1}^{T-t}\frac{\delta^k}{(1+r_d)^{k-\tau}}\right]H_t,\ \tau=0,\ldots,T-t.$$

Appendix B: Numerical Method

This section first describes the general steps of the numerical method and then explains how the transformed cash on hand is used in the numerical procedure. The model is solved recursively backward starting from the last period. In the last period ($t=T$), the policy functions are trivial, and without the bequest motive, the agent consumes all available wealth ($c_T=x_T$). As a result, the value function is $V_T(\hat{x}_T,\, h_T)=\frac{(x_T(\hat{x}_{T,T})-h_T)^{1-\gamma}}{1-\gamma}$.[51] For the problem with bequest, the solutions start at $T+1$ when all available wealth is bequested and $V_{T+1}(w_{T+1})=B(w_{T+1})$. There are two control variables to be solved for in every period: the savings rate out of cash on hand in excess of the current habit and the fraction of savings allocated to stocks. For every age t before T, and for each point in the state space, the optimization is performed using a grid search over policy functions. I compute the value associated with each admissible savings rate and the share of savings invested in stocks. In deciding admissibility, the algorithm is using the transformed cash on hand. This procedure is explained below. From the Bellman equation, the value function is current utility plus the discounted expected continuation value $[\beta E_t V_{t+1}(.)]$, computed using V_{t+1} from the previous iteration. I perform all of the numerical integrations using Gaussian quadrature to approximate the distributions of labor income shocks and the risky asset return. To evaluate the value function, for points that do not lie on the state space grid, I use bi-cubic spline interpolation. Once the value of all alternative admissible policies is computed, I select the policy associated with the maximum of the value function. The corresponding value function is stored as this period's value function $[V_t(.)]$ and then used to solve the previous period's maximization problem. This process is iterated until $t=1$.

[51] In the last period, the only constraint is $x_T>h_T$, and the transformation of cash on hand has a particularly simple form: $\hat{x}_T=x_T-h_T$. So, the value function is $V_T(\hat{x}_T,h_T)=\frac{\hat{x}_T^{1-\gamma}}{1-\gamma}$.

As already mentioned, instead of using cash on hand as a state variable, I use a transformed variable $\hat{x}_t$ which is defined in (14) as follows:

$$\hat{x}_t = x_t - (h_t\Delta_{\tau^*} - y_{t,t+\tau^*}),$$

where τ^* is defined in (13) so that $(h_t\Delta_{\tau^*} - y_{t,t+\tau^*})$ represents the minimum feasible cash on hand for habit h_t (see Figure 2 and Section 1.3 for more details). The state space is discretized so that $(\hat{x}_t, h_t)$ lie in a rectangle $[0.1, 20] \times [0, 4]$, and the grid has (uniform) steps in the range from 0.2 to 0.05 on $\hat{x}_t$ and from 0.5 to 0.1 on h_t with smaller steps chosen for calibrations with higher habit strength or utility curvature.

There is no need to define explicitly the evolution for $\hat{x}_t$ because it is not used in the computations. Only Equation (6) and the transformation equation are used. The procedure is as follows. For a given grid point $(\hat{x}_t, h_t)$, I compute x_t corresponding to this point using the transformation equation. Then for every savings and portfolio policy, I use the Equation (6) to compute x_{t+1} in all possible future exogenous states. Also, every savings policy corresponds to a given level of consumption today and allows me to compute future habit h_{t+1}. Using the obtained values of (x_{t+1}, h_{t+1}) for every exogenous state tomorrow, the algorithm transforms x_{t+1} into $\hat{x}_{t+1}$ and checks that $\hat{x}_{t+1} > 0$ in every possible exogenous state. This verifies the admissibility of the considered policy. Savings and portfolio policies that generate $\hat{x}_{t+1} \leq 0$ in any of the future states are ruled out because they imply infinitely negative utility. For feasible policies, the expected value function is computed by interpolating on the grid in $(\hat{x}_{t+1}, h_{t+1})$ coordinates using bi-cubic splines.

There are two advantages of using $\hat{x}_t$ instead of the usual x_t, both related to the computational properties of the problem. First, the transformation allows me to use standard fast interpolation routines for the rectangular regions rather than the slower routines for the irregularly shaped areas, as shown in Figure 2. Second, if needed, the transformation can reduce the dimensionality of the problem. Usually one can reduce the number of grid points by using a nonuniform grid and by concentrating the grid points in the areas where the gradient of the value function is high. For this problem, the gradient of the value function is high everywhere in the vicinity of the lower envelope of the constraints shown in Figure 2. Thus, if one were to use an untransformed x_t for the computations, the grid on x_t would have to be dense throughout the chosen domain for x_t. This is because the area with high gradient of the value function depends jointly on cash and habit and stretches from low to high values of x_t along the lower envelope of the feasibility constraints. On the contrary, the transformation maps all points that are close to the admissible border to the points with the low values of $\hat{x}_t$, allowing for the grid on $\hat{x}_t$ to be more dense near zero and less dense away from zero. However, for the specific software used to solve this problem (MATLAB), it turned out that the interpolation algorithm is much faster when the grids have uniform rather than variable density (because uniform grids simplify the search for the location of interpolant on the grid). Thus, the main benefit of using the transformation in this article comes from the fast interpolation over the rectangular areas for state variables.

References

Abel, A. B., 1990, "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80, 38–42.

Ameriks, J., and S. Zeldes, 2001, "How Do Household Portfolio Shares Very With Age?," working paper, Columbia University.

Bodie, Z., J. B. Detemple, S. Otruba, and S. Walter, 2004, "Optimal Consumption Portfolio Choices and Retirement Planning," *Journal of Economic Dynamics and Control*, 28(6), 1115–1148.

Bodie, Z., R. C. Merton, and W. F. Samuelson, 1992, "Labor Supply Flexibility and Portfolio Choice in a Life Cycle Model," *Journal of Economic Dynamics and Control*, 16(3–4), 427–449.

Cagetti, M., 2003, "Wealth Accumulation over the Life Cycle and Precautionary Savings," *Journal of Business and Economic Statistics*, 21(3), 339–353.

Campbell, J. Y., and J. H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–251.

Carroll, C. D., 1992, "The Buffer-Stock Theory of Savings: Some Macroeconomic Evidence," *Brooking Papers on Economic Activity*, 2, 61–156.

Chapman, D., 1998, "Habit Formation and Aggregate Consumption," *Econometrica*, 66, 1223–1230.

Cocco, J., 2005, "Portfolio Choice in the Presense of Housing," *The Review of Financial Studies*, 18, 535–567.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout, 2005, "Consumption and Portfolio Choice over the Life-Cycle," *The Review of Financial Studies*, 18, 491–533.

Constantinides, G. M., 1990, "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98, 519–543.

Davis, S., and W. Paul, 2000, "Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implication for Portfolio Choice," NBER Working Paper 7905.

Deaton, A., 1991, "Saving and Liquidity Constraints," *Econometrica*, 59, 1221–1248.

Detemple, J. B., and F. Zapatero, 1991, "Asset Prices in an Exchange Economy with Habit Formation," *Econometrica*, 59, 1633–1657.

Dynan, K. E., J. Skinner, and S. P. Zeldes, 2004, "Do the Rich Save More?," *Journal of Political Economy*, 112(2), 397–444.

Faig, M., and P. Shum, 2002, "Portfolio Choice in the Presence of Personal Illiquid Projects," *Journal of Finance*. 57(1), 303–328.

Gentry, W. M., and G. R. Hubbard, 2001, "Entrepreneurship and Household Savings," NBER Working Paper.

Gomes, F., and A. Michaelides, 2003, "Portfolio Choice with Habit Formation: A Life-Cycle Model with Uninsurable Labor Income Risk," *Review of Economic Dynamics*, 6(4), 729–766.

Gomes, F., and A. Michaelides, 2005, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, 60, 867–904.

Gomes, F., A. Michaelides, and V. Polkovnichenko, 2004, "Wealth Accumulation and Portfolio Choice with Taxable and Tax-Deferred Accounts," Working Paper, Financial Markets Group, London School of Economics.

Gourinchas, P., and J. Parker, 2002, "Consumption over the Life Cycle," *Econometrica*, 71(1), 47–89.

Guiso, L., M. Haliassos, and T. Japelli, 2002, "Household Portfolios: An International Comparison," in Luigi Guiso, Michael Haliassos, and Tullio Jappelli (eds.), *Household Portfolios*, MIT Press, Cambridge, MA.

Haliassos, M., and A. Michaelides, 2002, "Computing Solutions to Life Cycle Portfolio Choice Models," in Luigi Guiso, Michael Haliassos, and Tullio Jappelli (eds.), *Household Portfolios*, MIT Press Cambridge, MA.

Heaton, J., 1995, "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specifications," *Econometrica*, 63, 681–717.

Heaton, J., and D. J. Lucas, 1996, "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, 104, 443–487.

Heaton, J., and D. J. Lucas, 1997, "Market Frictions, Savings Behavior, and Portfolio Choice," *Macroeconomic Dynamics*, 1, 76–101.

Heaton, J., and D. J. Lucas, 2000, "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk," *Journal of Finance*, 55, 1163–1198.

Hindy, A., C.-F. Huang, and S. H. Zhu, 1997, "Optimal Consumption and Portfolio Rules with Durability and Habit Formation," *Journal of Economic Dynamics and Control*, 21, 525–550.

Jagannathan, R., and N. R. Kocherlakota, 1996, "Why Should Older People Invest Less in Stocks Than Younger People," *Minneapolis Federal Reserve Quarterly Review*, 20, 11–23.

Jagannathan, R., E. McGrattan, and A. Scherbina, 2000, "The Declining US Equity Premium," *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall, 3–19.

Lupton, J. P., 2001, "The Habit Liability in Life-Cycle Consumption and Portfolio Choice," Working Paper, University of Michigan.

McGrattan, E., and E. Prescott, 2003, "Average Debt and Equity Returns: Puzzling?," *American Economic Review Papers and Proceedings*, 93, 392–397.

Mehra, R., and E. C. Prescott, 1985, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15, 145–161.

Merton, R. C., 1971, "Optimum Consumption and Portfolio Choice Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373–413.

Pijoan-Mas, J., 2003, "Pricing Risk in Economies with Heterogeneous Agents and Incomplete Markets," CEMFI Working Paper No. 0305.

Poterba, J., and A. Samwick, 2002, "Taxation and Household Portfolio Composition: US Evidence from the 1980's and 1990's," *Journal of Public Economics*, 87, 5–38.

Storesletten, K., C. Telmer, and A. Yaron, 1998, "Persistent Idiosyncratic Shocks and Incomplete Markets," Manuscript, Carnegie Mellon University.

Sundaresan, S. M., 1989, "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," *The Review of Financial Studies*, 2, 73–89.

Viceira, L. M., 2001, "Optimal Portfolio Choice for Long Horizon Investors with Nontradable Labor Income," *Journal of Finance*, 55, 1163–1198.

Yao, R., and H. H. Zhang, 2004, "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints," *The Review of Financial Studies*, 18, 197–239.