

International Capital Markets and Foreign Exchange Risk

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Relations between foreign exchange risk premia, exchange rate volatility, and the volatilities of the pricing kernels for the underlying currencies, are derived under the assumption of integrated capital markets. As predicted, the volatility of exchange rates is significantly associated with the estimated volatility of the relevant pricing kernels, and foreign exchange risk premia are significantly related to both the estimated volatility of the pricing kernels and the volatility of exchange rates. The estimated foreign exchange risk premia mostly satisfy Fama's (1984) necessary conditions for explaining the forward premium puzzle, but the puzzle remains in several cases even after taking account of the pricing kernel volatilities.

In an influential paper, Fama (1984) reports a negative association between the forward premium in the foreign exchange market and the subsequent change in the spot exchange rate. This unexpected finding, which has become known as the forward premium puzzle, prompted Fama to emphasize the importance of time variation in the foreign exchange risk premium. In particular, Fama (1984) points out that the forward premium puzzle implies that the foreign exchange risk premium must be more volatile than, and negatively correlated with, the expected currency depreciation rate.

In this article, we relate foreign exchange rate risk premia and foreign exchange rate volatility to general variation in risk premia for returns denominated in different currencies, by examining the co-variation between risk premia in different national capital and foreign exchange

With thanks to an anonymous referee, David Backus, Kent Daniel, Bernard Dumas, Campbell Harvey (the editor), Bob Hodrick, Lisa Kramer, Lim Kian Guan, Raman Uppal, and seminar participants at the AFA 2005 Annual Meeting, Columbia University, the Kraus Conference, Penn State, Singapore Management University, Stockholm Institute for Financial Research, and University of Pennsylvania for helpful comments. Brennan thanks Lancaster University, and Xia thanks the Singapore Management University for their hospitality and acknowledges financial support from the Weiss Center for International Financial Research and the Rodney L. White Center for Financial Research at the Wharton School and the Wharton-SMU Research Center of the Singapore Management University. Yihong Xia, who was an assistant professor at the Wharton School of University of Pennsylvania, died on August 6, 2005. Address correspondence to the Michael Brennan, Anderson School, UCLA, 110 Westwood Plaza, Los Angeles, CA 90095-1481. E-mail: mbrennan@anderson.ucla.edu.

markets. The basic theoretical framework relies on the principle that the absence of arbitrage implies the existence of a pricing kernel for any *numeraire* that prices payoffs denominated in that *numeraire*, and the volatility of the pricing kernel is the maximum Sharpe ratio for returns denominated in that *numeraire*. Under the assumption that correlations of interest rates, exchange rates, and other rates of return, with pricing kernels, are constant over time, variation in risk premia is associated with variation either in the volatility of the risks themselves or in the overall level of risk premia that is captured by the volatility of the pricing kernel for returns denominated in a given currency. In a multiple currency setting, there is a separate pricing kernel for each currency and, with perfectly integrated capital markets, a simple relation obtains between the pricing kernels for different currencies and the exchange rates between them.

We show that no-arbitrage in integrated international markets implies not only Covered Interest Parity but also a cross-currency Risk Premium Restriction which relates risk premia for returns denominated in different currencies. When the pricing kernel correlations are constant, this Risk Premium Restriction specifies a linear relation between the volatilities of the pricing kernels for returns denominated in any two currencies and the volatility of the exchange rate between the currencies. In this setting, the foreign exchange risk premium can be expressed in two equivalent fashions. First, the arithmetic risk premium can be expressed as the sum of the covariance of the domestic price level innovation with the nominal exchange rate innovation and a term that is proportional to the product of the volatilities of the exchange rate and of the domestic pricing kernel. Second, the logarithmic risk premium can be expressed as a quadratic function of the volatilities of the pricing kernels of the two countries.

The theoretical setup of our article is closely related to recent papers by Ahn (2004), Backus, Foresi, and Telmer (2001), Bansal (1997), and Nielsen and Saà-Requejo (1993) that link interest rates and foreign exchange rates using reduced-form term structure factor models. All of these papers are based on complete affine models of the term structure of interest rates. These models impose certain constraints on the relation between the interest rate and the risk premium which, as shown by Backus, Foresi, and Telmer (2001), severely limit the ability of the models to capture the forward premium anomaly. The analysis in this article on the other hand is based on an *essentially* affine model of the term structure which allows for *independent* variation in risk premia and interest rates.¹ This additional model flexibility allows us to generate reasonable estimates of interest rates and to obtain estimates of bond market risk

¹ The distinction between a completely affine and an essentially affine model is that the compensation for interest rate risk can vary independently of interest rate volatility in an essentially affine model but not in a complete affine model. See Duffee (2002) for more details.

premiums that are significantly related to the excess returns on foreign currency investment. We report the first extensive empirical tests of the relations between, first, foreign exchange *risk premia* and the risk premium in the corresponding two bond markets and second, between foreign exchange *risk* and the risk premium in the corresponding two bond markets.

To generate testable implications about market integration, we use only domestic bond yields to estimate the volatility of the pricing kernel for each currency. As a result, our estimates of the pricing kernel volatility do not rely on a specific hypothesis about market integration. We assume that both the drift and the volatility of each pricing kernel, as well as the corresponding expected rate of inflation, follow simple Ornstein-Uhlenbeck processes: this assumption gives rise to an essentially affine Gaussian model of the term structure of interest rates in which yields on default-free zero-coupon nominal bonds are linear functions of the real interest rate, the expected rate of inflation, and the volatility of the pricing kernel.² This allows us to estimate the unobservable volatility of the pricing kernel for each currency up to a constant of proportionality, from data on domestic zero-coupon government bond yields and inflation rates.

The empirical analysis is conducted using monthly bond yield and inflation data for the U.S. Dollar (USD), Canadian Dollar (CAD), Deutsche Mark (DM), British Pound (BP), and Japanese Yen (JY), and the exchange rates between these currencies, for the period from January 1985 to May 2002. The term structure model provides a reasonable fit to bond yields for all currencies, although the prolonged decline in the real interest rate for the JY during the 1990s does not accord well with the mean-reverting assumption underlying the model, and the estimate of the mean reversion parameter for the real interest rate in Japan is close to zero.

Empirical investigation of the cross-currency Risk Premium Restriction reveals strong comovement of the estimated pricing kernel volatilities for all currencies except the JY. Country pairs with closer geographic proximity tend to have stronger comovement. Moreover, the comovement is stronger in the second half of the sample period, which suggests that capital market integration may have improved—the R^2 for regressions of monthly changes of one pricing kernel volatility on another and on the change of exchange rate volatility, for the period October 1994 to May 2002, range from 18 to 49% except for currency pairs that include the JY.

Motivated by the first specification of the foreign exchange risk premium, the excess return to foreign currency investment was regressed on estimates of the product of the domestic pricing kernel volatility and the exchange rate volatility and on the estimated exchange rate volatility. The coefficient of the product term was found to be significant at the 10% level

² This model was developed in Brennan, Wang, and Xia (2004). See also Brennan and Xia (2003).

or better in seven out of the twenty regressions. In the regressions that involved the volatility of the USD pricing kernel, this product term was significant at the 5% level in three out of four cases, whereas for the CAD pricing kernel volatility, the term was insignificant in every case. The coefficient of the exchange rate volatility was significant in twelve out of the twenty regressions.

To evaluate the predictive power of the second expression for the foreign exchange risk premium, the difference between the log spot exchange rate at the end of the month and the one-month log forward rate at the beginning of the month was regressed on linear and quadratic terms in the estimated volatilities of the pricing kernels for the two currencies in a GARCH(1,1) framework in which a quadratic function of the two estimated kernel volatilities was included in the variance equation as the Risk Premium Restriction specifies. The pricing kernel volatilities were significant at the 6% level or better in eight out of the ten regressions with the R^2 ranging from 6 to 39%. Moreover, the coefficients of the pricing kernel volatilities were jointly significant in the variance equation for nine out of the ten regressions. However, the lagged squared error was significant in four out of the ten variance equations, suggesting that our estimates of the pricing kernel volatilities are not perfect instruments for the exchange rate volatility as the cross-currency Risk Premium Restriction implies.

Taken together, these three sets of results provide both significant evidence of comovement of risk premia for claims denominated in different currencies and support for the hypotheses that the foreign exchange risk premium can be expressed as a function either of the domestic pricing kernel volatility and the volatility of the exchange rate or as a quadratic function of the volatilities of the pricing kernels associated with the two currencies.

Given this success, it is natural to ask whether the variation in the estimated pricing kernel and exchange rate volatilities can account for the forward premium puzzle identified by Fama (1984). First, we show that when we calculate the foreign exchange risk premium using parameter estimates derived from bond yields (as well as the time series of the estimated pricing kernel volatilities), the variation in the foreign exchange risk premium and the correlation of the premium with the expected change in the exchange rate satisfy the Fama (1984) necessary conditions for a risk premium-based explanation of the puzzle. However, when we include the forward premium in the regression of log spot minus forward rates on the pricing kernel volatilities, the coefficient of the forward premium is significant (and negative) in four out of the ten regressions. Two of these regressions involve the JY for which we have reason to believe that our estimates of the pricing kernel volatility may be especially poor. The other two involve the CAD, and for these we

have no explanation. Thus, our empirical analysis supports Fama's diagnosis of the forward premium puzzle in that it finds strong evidence of the time variation in risk premia that is required to explain the puzzle, although the puzzle remains evident for four exchange rates associated with CAD and JY.

The rest of the article is organized as follows. Section 1 derives the theoretical relations between the pricing kernels for different currencies and the exchange rate between them and relates the foreign exchange risk premium to the pricing kernel and exchange rate volatilities. Section 2 discusses the details of data construction and reports descriptive statistics. Section 3 describes the estimation procedure for the term structure model and discusses the time series of state variable estimates for the different currencies. Section 4 compares the empirical relations between pricing kernel and exchange rate volatility estimates with the theoretical predictions and assesses the relations between the volatility estimates and the foreign exchange risk premium and the ability of the volatility estimates to account for the forward premium puzzle. Section 5 summarizes and concludes.

1. Asset Pricing in a Multi-Currency Setting

In the absence of arbitrage, there exists a pricing kernel for any *numeraire* that prices all payoffs in terms of that *numeraire*. Consider a world in which asset prices follow diffusion processes.³ Let m and m^* denote the (real) pricing kernels corresponding to the domestic and foreign consumption bundles as *numeraire*, and write the dynamics of these pricing kernels as:

$$\frac{dm}{m} = -r(X)dt - \eta(X)dz, \quad (1)$$

$$\frac{dm^*}{m^*} = -r^*(X)dt - \eta^*(X)dz^*. \quad (2)$$

with $m_0 = m_0^* = 1$. It is understood that the diffusion and drift coefficients of Equations (1) and (2) may depend on a set of unspecified state variables denoted by X .

The definition of the pricing kernel implies that any real return process $\frac{dV}{V}$ with volatility σ_V has an instantaneous expected return given by:

³ See Huang (1985) for sufficient conditions for this setup.

$$\begin{aligned} E\left(\frac{dV}{V}\right) &= -E\left(\frac{dm}{m}\right) - \text{Cov}\left(\frac{dm}{m}, \frac{dV}{V}\right) \\ &= rdt + \eta\sigma_V\rho_{Vm}dt \end{aligned}$$

where, ρ_{Vm} is the correlation between innovations to the asset return and the pricing kernel. It then follows that $r(r^*)$ is the domestic (foreign) instantaneous real risk-free rate and, because $|\rho_{Vm}| \leq 1$, $\eta(\eta^*)$, the volatility of the pricing kernel for the domestic (foreign) economy, is the maximum Sharpe ratio for returns calculated using the domestic (foreign) consumption basket as *numeraire*.

Let s denote the real exchange rate expressed in units of domestic purchasing power (units of U. S. consumption basket) per unit of foreign purchasing power (units of UK consumption basket), and write the stochastic process for the exchange rate as

$$\frac{ds}{s} = e(X)dt + \sigma_s(X)dz_s, \quad (3)$$

where, again the dependence of the drift and diffusion coefficients on X is to be understood.

The definition of the foreign pricing kernel implies that for any real foreign gross return realized between time t and time $t + \tau$, $x_{t,t+\tau}^*$, the following relation holds

$$m_t^* = E_t(m_{t+\tau}^* x_{t+\tau}^*) \quad (4)$$

Moreover, expressing the return on the foreign asset in terms of domestic purchasing power, the definition of the domestic pricing kernel implies that if foreign real returns can be freely converted into domestic purchasing power:

$$m_t = E_t\left(m_{t+\tau}^* x_{t+\tau}^* \frac{s_{t+\tau}}{s_t}\right). \quad (5)$$

A sufficient condition for (4) and (5) to hold simultaneously is that

$$m^* = ms.$$

This condition is also necessary if markets are complete so that both m and m^* are unique and one of the three variables, m, m^* , and s , is redundant. If markets are incomplete, there is an infinite number of pricing kernels m and m^* satisfying Equations (4) and (5), but the minimum-variance pricing kernel for each *numeraire* derived from the projection of the pricing kernels onto the space of all (domestic and foreign) asset returns in that *numeraire* is unique and satisfies the above

condition.⁴ In what follows, m and m^* are to be understood as minimum variance pricing kernels. The relation $m^* = ms$ thus implies that

$$-r^*dt - \eta^*dz^* = -rdt + edt - \eta\sigma_s\rho_{sm}dt - \eta dz + \sigma_s dz_s. \quad (6)$$

Equality of the two stochastic processes in (6) requires that their drift and volatility coefficients be the same:

$$E\left(\frac{ds}{s}\right) = edt = (r - r^* + \eta\sigma_s\rho_{sm})dt, \quad (7)$$

$$(\eta^*)^2 = \eta^2 + \sigma_s^2 - 2\eta\sigma_s\rho_{sm}. \quad (8)$$

Equation (7) expresses the drift of the real exchange rate as the sum of the real interest differential and a risk premium which is equal to the instantaneous covariance of the real exchange rate with the domestic pricing kernel, whereas Equation (8) relates the squared volatility of the two pricing kernels to the variance of the real exchange rate and the covariance of the exchange rate with the domestic pricing kernel.

Let $s^* \equiv 1/s$ denote the real exchange rate expressed in terms of the number of units of foreign purchasing power per unit of domestic purchasing power (e.g., “U.K. goods per unit of U.S. good”); then $ds^*/s^* = (-e + \sigma_s^2)dt - \sigma_s dz_s$. Symmetry then implies that

$$(\eta)^2 = (\eta^*)^2 + \sigma_s^2 + 2\eta^*\sigma_s\rho_{sm^*}, \quad (9)$$

which, combined with (8), yields the following relation between the volatility of the pricing kernels for two currencies and the volatility of the exchange rate between them:

$$\rho_{sm^*}\eta^* = \rho_{sm}\eta - \sigma_s. \quad (10)$$

Because $\eta(\eta^*)$, the volatility of the minimum variance pricing kernel, is equal to the maximum Sharpe ratio or risk premium per unit of risk for returns denominated in the given consumption *numeraire*, (10) is a condition on the maximum risk premia in the two countries which we refer to as the (cross-currency) *Risk Premium Restriction*. The restriction implies that if there is any uncertainty associated with the exchange rate, that is $\sigma_s \neq 0$, then the exchange rate uncertainty must be priced by the domestic or the foreign pricing kernel or by both: that is, if $\sigma_s \neq 0$ then $\eta^*\rho_{sm^*} \neq 0$ or $\eta\rho_{sm} \neq 0$, or both. This is a consequence of Siegel’s paradox, which

⁴ See Backus, Foresi, and Telmer (2001), Brandt *et al.* (2003), and Saà-Requejo (1993).

says that foreign exchange risk cannot be priced at zero from both domestic and foreign perspective because of Jensen's inequality.

Finally, combining Equations (7) and (8) leads to the following stochastic process for the log exchange rate:⁵

$$d \ln s = \left[r - r^* + \frac{1}{2} \eta^2 - \frac{1}{2} (\eta^*)^2 \right] dt + \sigma_s dz_s. \quad (11)$$

To extend the analysis to nominal exchange rates, let $P_t(P_t^*)$ denote the domestic (foreign) price levels, and assume that the stochastic processes for P_t and P_t^* are of the form:⁶

$$\frac{dP}{P} = \pi dt + \sigma_P dz_P, \quad (12)$$

where π , the expected rate of inflation, will in general depend on the unspecified vector of state variables X .

Let S_t denote the nominal spot exchange rate expressed in terms of domestic currency per unit of foreign currency (e.g., "dollars per pound"). Note that one unit of the foreign consumption basket, which is equivalent to P_t^* units of foreign currency, can be exchanged for s_t units of the domestic consumption basket or equivalently for $s_t P_t$ units of domestic currency. The absence of arbitrage then implies that the real and nominal exchange rates are related to the foreign and domestic price levels by $S_t P_t^* = s_t P_t$. As shown earlier, the real exchange rate s_t is determined by the pricing kernels m and m^* by $m^* = m s$ if the domestic and foreign economies are fully integrated. Therefore, the nominal exchange rate can be directly related to the domestic and foreign pricing kernels by:

$$S_t \frac{m_t}{P_t} = \frac{m_t^*}{P_t^*}.$$

Applying Ito's Lemma to the above equation leads to the following nominal foreign exchange rate process:

$$\frac{dS}{S} = E dt + \sigma_S dz_S.$$

where the drift and innovation are given by:

$$E dt \equiv (R - R^* + \eta \sigma_S \rho_{Sm} + \sigma_{SP}) dt, \quad (13)$$

⁵ Similar equations also appear in Ahn (2004), Backus, Foresi, and Telmer (2001), Bansal (1997), and Brandt and Santa-Clara (2002).

⁶ Similar to Solnik (1974) and Adler and Dumas (1983) but in contrast with Grauer, Litzenberger, and Stehle (1976), we do not assume Purchasing Power Parity in our setting.

$$\sigma_S dz_S = \eta dz_m - \eta^* dz_m^* + \sigma_P dz_P - \sigma_P^* dz_P^*, \quad (14)$$

$\sigma_{xy} \equiv \sigma_x \sigma_y \rho_{xy}$ denotes the covariance between the innovations to variables x and y , and the domestic (foreign) instantaneous nominal risk-free rate, R (R^*), is given by

$$R \equiv r + \pi - \eta \sigma_P \rho_{Pm} - \sigma_P^2.$$

Equation (13), which is the nominal counterpart of the real relation (7), expresses the expected change in the *nominal* exchange rate as the sum of three terms: the nominal interest rate differential, $R - R^*$; the covariance of the nominal exchange rate with the domestic price level, σ_{SP} ; and a time varying exchange rate risk premium, $\eta \sigma_S \rho_{Sm}$, which is equal to the covariance of the *nominal* exchange rate with the *real* domestic pricing kernel. A direct implication of Equation (13) is that the exchange rate risk premium, defined as the risk premium on an unhedged position in an investment that is riskless in terms of foreign currency, $E - (R - R^*)$, is a function of the volatility of the domestic pricing kernel, η , and the volatility of the nominal exchange rate, σ_S :

$$E - (R - R^*) = \sigma_P \rho_{SP} \sigma_S + \rho_{Sm} (\sigma_S \eta). \quad (15)$$

Equation (15) shows that time variation in the exchange rate risk premium can arise from time variation in the covariance of price changes with the nominal exchange rate and from time variation in the covariance of the nominal exchange rate with the pricing kernel. In our empirical work, we shall treat the volatility of the price level, σ_P , and the correlations between the price level and the exchange rate, and between the nominal interest rate and the pricing kernel, ρ_{SP} and ρ_{Sm} , as constant, and seek to explain the variation in the exchange rate risk premium in terms of variation in the volatility of the exchange rate, σ_S , and of the pricing kernel, η .

From Equations (13) and (14), it is easy to derive the nominal counterparts to equations (10) and (11):

$$\sigma_S = \eta \rho_{Sm} - \eta^* \rho_{Sm^*} + (\sigma_P \rho_{SP} - \sigma_P^* \rho_{SP^*}), \quad (16)$$

$$d \ln S = \left[(R - R^*) + \left(\frac{1}{2} \sigma_P^2 - \frac{1}{2} \sigma_P^{*2} \right) + \frac{1}{2} \eta^2 - \frac{1}{2} (\eta^*)^2 + \sigma_P \rho_{Pm} \eta - \sigma_P^* \rho_{P^*m^*} \eta^* \right] dt + \sigma_S dz_S, \quad (17)$$

The advantage of analyzing the stochastic process for the *log* exchange rate as in Equation (17) is that the volatility of the nominal exchange rate cancels out of the drift term so that the expected change in the log

exchange rate depends only on the interest rate differential, which is equal to the forward premium due to Covered Interest Parity and on the volatilities of the domestic and foreign pricing kernels. The corresponding expression for the (log) exchange rate risk premium is then

$$E(d \ln S) - (R - R^*) = \left(\frac{1}{2} \sigma_P^2 - \frac{1}{2} \sigma_{P^*}^2 \right) + \frac{1}{2} \eta^2 - \frac{1}{2} (\eta^*)^2 + \sigma_P \rho_{Pm} \eta - \sigma_{P^*} \rho_{P^*m^*} \eta^*. \quad (18)$$

Following Fama (1984), we decompose the forward premium under this log specification into two terms:

$$\ln F_{t,\tau} - \ln S_t = [\ln F_{t,\tau} - E_t(\ln S_{t+\tau})] + [E_t(\ln S_{t+\tau}) - \ln S_t] \equiv p_t + q_t,$$

where $F_{t,\tau}$ denotes the τ -period forward rate at time t , p_t denotes the negative of the risk premium at time t :

$$p_t = - \left[\left(\frac{1}{2} \sigma_P^2 - \frac{1}{2} \sigma_{P^*}^2 \right) + \frac{1}{2} \eta_t^2 - \frac{1}{2} (\eta_t^*)^2 + \sigma_P \rho_{Pm} \eta_t - \sigma_{P^*} \rho_{P^*m^*} \eta_t^* \right] \tau, \quad (19)$$

and q_t is the expected change in the exchange rate:

$$q_t = [(R_t - R_t^*)\tau - p_t] \tau. \quad (20)$$

The expressions for p_t and q_t follow directly from Covered Interest Parity and equation (17).

The forward premium puzzle is the empirical finding of a negative correlation between the current forward premium, $\ln F_{t,\tau} - \ln S_t$, and the subsequent change in the spot rate, $\ln S_{t+\tau} - \ln S_t$. Fama (1984, p. 327) shows that the forward premium puzzle implies that the (negative of the) foreign exchange risk premium, p_t , must satisfy the following two *Fama conditions*:

$$(F1) \quad \text{Var}(p_t) > \text{Var}(q_t),$$

$$(F2) \quad \text{Cov}(p_t, q_t) < 0.$$

That is, the variance of the risk premium must exceed the variance of the expected rate of change in the exchange rate, and the risk premium (the negative of p_t) must be positively correlated with the rate of change in the exchange rate.

When unexpected inflation is unpriced ($\rho_{Pm} = \rho_{P^*m^*} = 0$), the expressions for p_t and q_t reduces to

$$p_t = c - \frac{1}{2} [\eta_t^2 - (\eta_t^*)^2],$$

$$q_t = [(r + \pi) - (r^* + \pi^*)] + \frac{1}{2} [\eta_t^2 - (\eta_t^*)^2].$$

Then, the two Fama conditions are satisfied if $\frac{1}{2} [\eta_t^2 - (\eta_t^*)^2]$ is more volatile than, and negatively correlated with, the nominal interest rate differential $[(r + \pi) - (r^* + \pi^*)]$. As Backus, Gregory, and Telmer (1993) point out, this condition cannot be satisfied in a *complete* affine term structure model, because such a model requires that η^2 be an affine function of $r(\pi)$ if $r(\pi)$ follows a square root process or that η^2 be constant if both r and π follow Vasicek processes. Thus, the *complete* affine model is not able to deliver the high volatility of risk premia in conjunction with low volatility for interest rates that is required to account for the forward premium puzzle with realistic interest rate behavior. In contrast, this requirement is easily satisfied in an *essential* affine model in which η follows a separate process from r and π . Such a model will be estimated in Section 3 and in Section 4, we shall show that the resulting estimates of the risk premia satisfy these two conditions for most currency pairs.

The empirical tests that we shall report below rely on estimates of the domestic and foreign pricing kernel volatilities or maximum Sharpe ratios, η and η^* . If these were computed using the returns on both domestic and foreign assets converted into a common currency, then the Risk Premium Restriction (16) would hold by construction because, in the absence of arbitrage, it is always possible to find m and m^* such that $m^* = m_s$, and the restriction is a direct consequence of this relation. Therefore, to derive testable implications of market integration, we shall assume that the correlations between domestic (foreign) bond returns and the domestic (foreign) currency pricing kernel are constant. This assumption, which implies that domestic (foreign) bond risk premia are proportional to $\eta(\eta^*)$, enables us to estimate $\eta(\eta^*)$ up to a constant of proportionality using only domestic (foreign) bond yields without making any assumption about market integration, and these independent estimates of η and η^* will satisfy the linear Risk Premium Restriction only if the markets are integrated.

In Section 4, we report estimates of the empirical counterpart of the nominal Risk Premium Restriction (16) and examine the ability of our estimates of the pricing kernel and exchange rate volatilities to account for time variation in the foreign exchange rate risk premium as implied by (15) and (18). The data and the estimation of η and η^* are discussed in the following two sections.

2. Data Construction and Description

The basic data used to estimate the pricing kernel volatilities consist of estimated zero coupon Treasury bond yields on the second day of each month from January 1985 to May 2002 for the United States, United Kingdom, Germany, Canada, and Japan. The sample period and the number of countries are limited by the availability of government bond data for a sufficient period.

Data on bond prices, coupon rates, and coupon, issue, and redemption dates for all available government bonds outstanding on the given date were taken from Datastream. Most bonds in the United States, United Kingdom, Canada, and Japan pay semi-annual coupons: those that did not were eliminated from the sample. In Germany, most bonds pay annual coupons and those that did not were also excluded. Finally, all stripped zero-coupon and floating-rate bonds and bonds that were callable or extended to dates beyond the original redemption dates were excluded.

For each country, a cubic spline was fitted each month to the yields on all of the sample bonds with maturities up to twenty years.⁷ No extrapolation was used in the estimation, so that the longest possible extracted zero-coupon bond yield for a given month is always less than or equal to the longest maturity bond available for that month. For the United States, the United Kingdom, and Canada, zero-coupon bond yields for maturities of six months, 1, 2, 3, 5, 7, and 10 years were obtained for each month, whereas for Germany and Japan, the maximum maturity of the zero-coupon bond yields was only, respectively seven years and eight years for each month. The cubic spline approach has been used previously by McCulloch (1975) to fit the U.S. term structure and by Litzenberger and Rolfo (1984) (LR) to study tax effects on yield curves in different countries. The procedure used here follows LR but ignores capital gains and income tax effects, because the model developed in Sections 2 and 3 assumes no taxes or other frictions.⁸

Where possible, the estimated zero-coupon constant maturity bond yields were compared with data from other sources. For the United States, our estimated yield curve was compared with the Fama–Bliss bond yields from Center for Research in Security Prices (CRSP) which are available only for maturities of one, two, three, four and five years up to December 2001 and also with bond yield data provided directly by Bliss for all maturities up to December 2000. Our estimates have sample means and standard deviations that are very similar to those of these two data sets. The correlations of bond yields between the two data sets are above 0.95 for maturities of one, two, three, four, and five years but are only 0.7 for 10-year yields. For the U.K. yield curve, our estimates were compared with those published by the Bank of England for the same sample period for maturities of 1, 2, 3, 5, 7, 10, and 15 years: the correlations are all above 0.9, but the sample means of our estimates are slightly higher. Because the data from the Bank of England are

⁷ Bliss (1996) tests and compares five different methods for estimating the term structure. He finds that the unsmoothed Fama–Bliss method does the best but that differences between this and the cubic spline approach are small. The cubic spline approach seems to be the approach most widely used in empirical studies of U.S. yields.

⁸ We also estimated the term structure by specifying a capital gain tax rate of 0% and an income tax rate of 33%. The estimated after-tax yield curve was highly correlated with the before-tax curve but with lower sample means. LR found that the minimum absolute standard error of estimate does not vary much with the assumed capital gains tax rate.

available for the whole sample period, we used this data set to estimate the U.K. state variables.

Table 1 reports summary statistics for the estimated zero-coupon bond yields for the different currencies. The yield curves for USD, CAD, JY, and DM are on average upward-sloping. For example, the average zero coupon yield for the USD increases from 6.07% per year for the six-month bond to 7.41% for the 10-year bond. The sample standard deviation decreases with maturity, from 1.64% for the six-month bond to 0.82% for the 10-year bond. On the other hand, the average yield curve for the BP is slightly hump-shaped and almost flat: increasing from 8.02% for the one-year bond to 8.14% for the seven-year bond and then decreasing to 7.88% for the 15-year bond. The sample standard deviation also decreases slightly with maturity. Overall, JY bonds have the lowest average yields at around 3–4%, whereas the BP has the highest average yields at around 8%.

Table 1
Summary statistics of fitted zero-coupon constant maturity bond yields

Bond maturity (year)	0.5	1	2	3	5	7	10	CPI inflation	Excess market return
The United States									
Mean (% per year)	6.07	6.14	6.27	6.41	6.69	6.96	7.41	3.08	9.97
Standard deviation (% per year)	1.64	1.59	1.58	1.61	1.53	1.19	0.82	0.77	15.04
Autocorrelation	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.34	0.00
Canada									
Mean (% per year)	6.99	7.09	7.18	7.45	7.70	7.84	8.01	2.78	5.50
Standard deviation (% per year)	2.53	2.43	2.26	2.14	1.99	1.88	1.82	1.13	14.84
Autocorrelation	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.09	0.09
The United Kingdom (from Bank of England)									
Mean (% per year)	8.02	8.03	8.06	8.11	8.14	8.10	7.88	3.80	6.16
Standard deviation (% per year)	2.65	2.35	2.21	2.13	2.12	2.10	1.97	1.63	16.73
Autocorrelation	0.99	0.98	0.98	0.98	0.98	0.99	0.99	0.24	0.03
Germany									
Mean (% per year)	5.25	5.59	5.69	5.67	5.84	6.17		1.96	7.45
Standard deviation (% per year)	1.90	1.64	1.50	1.45	1.38	1.24		0.91	19.87
Autocorrelation	0.97	0.98	0.98	0.99	0.99	0.98		0.26	0.07
Japan									
Mean (% per year)	2.98	2.92	2.95	3.12	3.52	3.92		0.84	1.67
Standard deviation (% per year)	2.37	2.36	2.32	2.23	2.05	1.89		1.46	20.31
Autocorrelation	0.99	0.99	0.99	0.99	0.99	0.99		0.18	0.10

This table reports summary statistics for fitted zero-coupon constant maturity bond yields. The bond yields are estimated from government coupon bonds using a cubic spline. The raw government coupon bond data are collected from Datastream. The sample is from January 1985 to May 2002.

The bond yields are highly persistent with first-order autocorrelations of 0.98 or above. Yields for different maturities are also highly correlated (not reported here), particularly for nearby maturities. The shortest and the longest maturity bond yields have a correlation of 0.74 for the USD, 0.80 for the BP, 0.87 for the CAD, and 0.63 for the DM. For the JY, the six-month and eight-year yields have a correlation of 0.96, and the correlations between other yields are even higher, suggesting either that a single factor model may capture the dynamics of the JY term structure or more plausibly that the level of rates in Japan has shifted down by so much that slope effects appear small in comparison.

Table 1 also reports summary statistics for monthly inflation rates and for excess equity market index returns for the countries corresponding to each currency. Inflation rates were calculated from Consumer Price Index data obtained from Datastream. The equity market excess returns were calculated from the Datastream total market index returns. For our sample period, the estimate of the equity premium for Japan is only 1.7%, although the estimate doubles if the data from 1980 to 1985 are included. On the other hand, the U.S. equity premium estimate during this period is almost 10%, reflecting the bull market of the 1990s. Average inflation rates range from a low of only 0.84% per year in Japan to a high of 3.8% in the United Kingdom.

Table 2 reports summary statistics for spot and one-month forward exchange rates and the one-month Treasury-Bill rates for each currency, all of which were taken from Datastream.⁹ Cross-rates between currencies other than the USD were calculated by taking ratios of USD exchange rates. Finally, following Schwert (1989), the monthly volatilities of the exchange rates were estimated as the square root of the sum of the squared deviations of the daily foreign exchange rate changes over the month from the sample mean change for the month. These “historical” estimates are available for the whole sample period for each currency pair. As a robustness check, we also collected the one-month implied volatilities calculated from over the counter (OTC) foreign exchange options on the last day of each month and published on the website of the Federal Reserve Bank of New York; the average of the bid and ask implied volatility was used. The implied volatilities are available only for the period October 1994 to May 2002 and for the currency pairs between the USD and the BP, DM,¹⁰ JY and CAD, and the pairs between the DM and the BP or the JY.¹¹

⁹ Since one-month Treasury Bill rates for the JY are available from Datastream only from December 1993, one-month bill rates from the Bank of Japan were used for the earlier period.

¹⁰ The implied volatility of the DM against USD, JY, or BP was replaced by the corresponding implied volatilities for the Euro starting from January 1999.

¹¹ The ex post estimated conditional volatility from a GARCH(1,1) model was also used as an alternative estimate of exchange rate volatility as a further robustness check. Jorion (1995) reports that, despite their limitations, implied volatility estimates outperform those from GARCH models even with “ex post” parameter estimates.

Table 2
Summary statistics of foreign exchange rates and interest rates

Currency	Mean	Std deviation	Autocorrelation
1. Change in the spot rate: $S_{t+1}/S_t - 1$ (% per month)			
Canadian Dollar/German Mark	0.33	3.56	0.010
Canadian Dollar/Japanese Yen	0.48	3.90	0.014
Canadian Dollar/British Pound	0.25	3.17	-0.007
Canadian Dollar/U.S. Dollar	0.09	1.38	-0.061
German Mark/Japanese Yen	0.20	3.37	0.015
German Mark/British Pound	-0.04	2.48	0.109
German Mark/U.S. Dollar	-0.13	3.36	0.014
Japanese Yen/British Pound	-0.14	3.60	0.053
Japanese Yen/U.S. Dollar	-0.26	3.63	0.031
British Pound/U.S. Dollar	-0.07	3.13	-0.070
2. Forward premium: $100 \times (\ln F_t - \ln S_t)$ (% per month)			
British Pound/U.S. Dollar	0.23	0.21	0.744
Canadian Dollar/German Mark	0.16	0.28	0.629
Canadian Dollar/Japanese Yen	0.33	0.26	0.092
Canadian Dollar/British Pound	-0.14	0.21	0.139
Canadian Dollar/U.S. Dollar	0.09	0.21	0.470
German Mark/Japanese Yen	0.17	0.22	0.496
German Mark/British Pound	-0.31	0.26	0.821
German Mark/U.S. Dollar	-0.04	0.29	0.746
Japanese Yen/British Pound	-0.47	0.22	0.322
Japanese Yen/U.S. Dollar	-0.24	0.26	0.494
3. One-month Treasury-bill rates (% per year)			
British Pound	8.34	3.16	0.985
Canadian Dollar	6.78	2.96	0.977
German Mark	5.19	1.91	0.989
Japanese Yen	2.99	2.69	0.995
U.S. Dollar	5.27	1.55	0.886
4. Historical volatility of exchange rates (% per month):			
British Pound/U.S. Dollar	2.63	1.06	0.571
Canadian Dollar/German Mark	3.12	0.94	0.291
Canadian Dollar/Japanese Yen	3.18	1.18	0.382
Canadian Dollar/British Pound	2.86	0.90	0.461
Canadian Dollar/U.S. Dollar	1.27	0.47	0.477
German Mark/Japanese Yen	2.86	1.19	0.625
German Mark/British Pound	2.09	0.83	0.448
German Mark/U.S. Dollar	2.92	0.96	0.379
Japanese Yen/British Pound	3.08	1.15	0.506
Japanese Yen/U.S. Dollar	2.95	1.11	0.366
5. One-month implied volatility of exchange rates (% per month):			
British Pound/German Mark	2.45	0.58	0.735
British Pound/U.S. Dollar	2.40	0.44	0.603
Canadian Dollar/U.S. Dollar	1.68	0.46	0.824
German Mark/Japanese Yen	3.48	0.98	0.834
German Mark/U.S. Dollar	3.07	0.64	0.718
Japanese Yen/U.S. Dollar	3.47	0.91	0.716

Summary statistics are reported for one-month spot exchange rate changes, $S_{t+1}/S_t - 1$, one-month Treasury-bill rates, R_t , and one-month forward premia, $\ln F_t - \ln S_t$. Forward and spot exchange rates are calculated from beginning of month U.S. Dollar rates taken from Datastream. The sample, which starts in January 1985, ends in May 2002 for British Pound, Canadian Dollar, and Japanese Yen and ends in December 1998 for the DM due to the introduction of the Euro. The DM Treasury-bill rates are from Bloomberg and the JY treasury bill rates are imputed from other treasury yields. All other data except for implied volatilities are as of the beginning of the month and are from Datastream. Implied volatilities are measured at the end of the month from October 1994 to May 2002 and are from the Federal Reserve Bank of New York.

Consistent with previous studies, changes in spot rates for all four country pairs are highly volatile and have very low autocorrelation. The lowest monthly standard deviations are for CAD–USD (1.38%) and for DM–BP (2.5%). The standard deviations for all other rates are in excess of 3% per month, and the highest is 3.90% for CAD–JY. The autocorrelation of the changes in spot rates is negative in three out of ten cases, but in all cases the absolute value is less than 0.1. In contrast, forward premia exhibit low sample standard deviations and much higher autocorrelations: for example, the forward premium for the DM against the BP has a monthly standard deviation of only 0.26% but an autocorrelation of 0.82.

The table also reports summary statistics on the historical and implied volatilities for exchange rates. The mean implied one-month volatilities range from 1.68% per month for the CAD–USD rate to 3.48% for the DM–JY rate, whereas the corresponding numbers for the historical volatilities are 1.27% for the CAD–USD and 2.86% for the DM–JY rates. Although the historical volatilities have larger sample standard deviations, their autocorrelations, from 0.36 to 0.57, are generally much smaller than those of the corresponding implied volatilities, which range from 0.60 to 0.82; this may reflect greater measurement errors in the historical estimates.

3. Estimating Pricing Kernel Volatilities

3.1 A simple essentially affine model

To estimate the pricing kernel volatility, η , and related parameters for each country from panel data on bond yields, it is necessary to place some further structure on the dynamics of the pricing kernel and the inflation rate. We follow Brennan, Wang, and Xia (2004) (BWX) in assuming that the real interest rate, r , and the maximum Sharpe ratio, η , follow Ornstein-Uhlenbeck processes, so that the stochastic process for the pricing kernel may be written as:

$$\frac{dm}{m} = -r dt - \eta dz_m \quad (21)$$

$$dr = \kappa_r(\bar{r} - r)dt + \sigma_r dz_r \quad (22)$$

$$d\eta = \kappa_\eta(\bar{\eta} - \eta)dt + \sigma_\eta dz_\eta \quad (23)$$

The expected rate of inflation, π , is also assumed to follow an Ornstein-Uhlenbeck process:

$$\frac{d\pi}{\pi} = \kappa_{\pi}(\bar{\pi} - \pi)dt + \sigma_{\pi}dz_{\pi}. \quad (24)$$

Although the Gaussian process assumptions for r , π , and η are simple and are made mainly to facilitate the estimation, the fact that r and η follow two separate processes distinguishes the current model from the complete affine term structure models that have been considered previously in the international finance literature.¹² As discussed extensively in Backus, Foresi, and Jelmer (2001), the conditional means and variances of the log pricing kernels in complete affine models are linear in the *same* set of state variables, which introduces a constraint on the behavior of the short rate and the variability of the risk premium in the economy. In contrast, our model allows the short rate, r , and the risk premium of the economy as captured by η , to vary separately. This additional flexibility can potentially yield better estimates of the interest rate and the risk premium.

3.2 Estimation procedure

Under the above assumptions, the nominal yield on a zero-coupon (default-free) bond of maturity τ is a linear function of the state variables, r , π , and η :

$$-\frac{\ln N}{\tau} = -\frac{\hat{A}(\tau)}{\tau} + \frac{B(\tau)}{\tau}r + \frac{C(\tau)}{\tau}\pi + \frac{\hat{D}(\tau)}{\tau}\eta. \quad (25)$$

where the coefficients, $\hat{A}(\tau)$, $B(\tau)$, $C(\tau)$, and $\hat{D}(\tau)$ are functions of the parameters of the joint stochastic process for the pricing kernel (21, 22, and 23), realized inflation (12), and the expected rate of inflation (24), as given in BWX.

In principle, it is possible to estimate the parameters of the system (21–24) by Maximum Likelihood using equation (25) and yields on three bonds of different maturities. However, the choice of bonds to use in the estimation is arbitrary, and there is no guarantee that the estimates will be consistent with the yields of other bonds. Therefore, to minimize the consequences of possible model mis-specification and measurement errors in the fitted bond yield data, we allow for errors in the pricing of individual bonds and use a Kalman filter/MLE method to estimate the time series of the unobservable (latent) state variables r , π , and η , and their dynamics, from data on bond yields.¹³

¹² See, for example, Backus, Foresi, and Telmer (2001) and Saà-Requejo (1993).

¹³ The Kalman filter/MLE is one of the two most widely used estimation approaches in the empirical term structure literature. See Duffee and Stanton (2002) for a detailed examination of term structure estimation methods.

In summary, there are three transition equations for the unobserved state variables, r , η , and π , that are the discrete time versions of Equations (22), (23), and (24). There are $n + 1$ observation equations based on the yields at time t , $y_{\tau_j,t}$, on n bonds with maturities τ_j , $j = 1, \dots, n$. The first n observation equations are derived from Equation (25) by the addition of measurement errors, ϵ_{τ_j} :

$$\begin{aligned} y_{\tau_j,t} &\equiv -\frac{\ln N(t, t + \tau_j)}{\tau_j} \\ &= -\frac{\hat{A}(t, \tau_j)}{\tau_j} + \frac{B(\tau_j)}{\tau_j} r_t + \frac{C(\tau_j)}{\tau_j} \pi_t + \frac{\hat{D}(\tau)}{\tau} \eta_t + \epsilon_{\tau_j}(t). \end{aligned} \quad (26)$$

The last observation equation is based on the realized inflation rate at time t :

$$\frac{\Delta P}{P} = \pi \Delta t + \epsilon_P(t).$$

This final observation equation is used to identify r and π which enter the bond yield Equation (26) symmetrically.

The measurement errors, $\epsilon_{\tau_j}(t)$, are assumed to be serially and cross-sectionally uncorrelated, and to be uncorrelated with the innovations in the transition equations. To reduce the number of parameters to be estimated, the variance of the yield measurement errors was assumed to be of the form: $\sigma^2(\epsilon_{\tau_j}) = \sigma_b^2$, where σ_b is a single parameter to be estimated. This is equivalent to the assumption that the measurement error variance of the log price of the bonds is proportional to the bond maturity.¹⁴ In addition, it was assumed that the errors in the observation equations are uncorrelated with the innovations of the state variables, that is, $\rho_{ir} = 0$, $\rho_{i\pi} = 0$, and $\rho_{i\eta} = 0$ ($i = \epsilon_1, \dots, \epsilon_n$, and ϵ_P).

The long run means of the state variables were set exogenously to facilitate identification and estimation. More specifically, $\bar{\pi}$ for each currency was set equal to the sample mean of the corresponding CPI inflation rate, which is 3% for the USD, 3.8% for the BP, 2.8% for the CAD, 2% for the DM, and 1% for the JY; $\bar{r}$ was set equal to the difference between the sample means of the one-month Treasury-bill rate and the CPI inflation rate, which is 2.6% for the United States, 4.6% for the United Kingdom, 4% for Canada, 3% for Germany, and 2.5% for Japan;¹⁵ and $\bar{\eta}$ was set equal to 1.2 times the sample mean of the country's equity market Sharpe ratio, which is 0.62 for the United States, 0.58 for the United Kingdom,

¹⁴ Brennan, Wang, and Xia (2004) estimate the system by assuming that $\sigma^2(\epsilon_{\tau_j}) = \sigma_b^2/\tau_j$ so that the measurement error variance of the log price is independent of maturity.

¹⁵ Strictly speaking, $r = R - \pi + \eta\sigma_P\rho_{Pm} + \sigma_P^2$ in our model. Because we impose $\rho_{Pm} = 0$ in the estimation and σ_P^2 is negligible, we use $\bar{r} \approx \bar{R} - \bar{\pi}$ to derive the long run mean of the real interest rate.

0.44 for Canada, 0.46 for Germany, and 0.21 for Japan.¹⁶ $\bar{\eta}$ was set 20% higher than the realized equity market Sharpe ratio to allow for the fact that the equity market is only one component of the investment opportunity set. Note that for any asset i , only the product $\rho_{im}\eta$ is identified in the estimation—therefore errors in the predetermined values of $\bar{\eta}$ will be offset by errors in the estimated correlations. On the other hand, when we relate η to foreign exchange risk premia in Section 5, we must acknowledge that our estimates of η for any currency are empirically identified only up to a (positive) multiplicative constant, so that no restrictions can be placed on the magnitudes of the coefficients of η . Finally, $\sigma(\epsilon_P)$ was set to the sample standard deviation of realized CPI inflation rates, and ρ_{mP} was set to zero to reduce the number of parameters to be estimated.

3.3 Estimation results

Table 3 reports estimates of the standard error of the bond yield measurement error, σ_b , and of the parameters of the stochastic processes for the real interest rate, the expected inflation rate, and the real pricing kernel volatility for each of the five currencies. In all five currencies, σ_b is highly significant, falling in the range of 18–48 basis points, which is comparable to values found in previous studies of the U.S. term structure.

The volatility of the real interest rate innovations, σ_r , is in the range of 63–92 basis points per year for the CAD, the DM, the JY, and the BP; for the USD, on the other hand, it is 277 basis points. The high volatility of r for the USD is offset by strong mean reversion: the estimate of the USD κ_r (0.290) is more than twice as high as that of the next highest currency (the BP) and implies a half life for innovations of about 2.4 years, as compared with almost five years in the BP, almost six years in the CAD, and more than 10 years in the DM where the mean reversion parameter is not significant.¹⁷ As we shall see below, there is evidence of mean reversion in the estimates of r for all currencies except the JY, whose estimated value of r declines fairly steadily from 1990 so that there is little evidence of mean reversion in this sample period. It is then not surprising to find the estimate of JY κ_r close to zero and insignificant; the Ornstein-Uhlenbeck assumption clearly fails to capture the behavior of the real interest rate in Japan during this sample period. The estimated correlation

¹⁶ Note that the sample means of the ex post equity market Sharpe ratios and the Treasury bill rates reported here differ from those reported in Tables 1 and 2, because these means are calculated using data starting from January 1980 instead of January 1985. The longer sample period was chosen to improve the efficiency of the estimates for $\bar{r}$, $\bar{\pi}$, and $\bar{\eta}$. The estimates for $\bar{r}$, $\bar{\pi}$, and $\bar{\eta}$ are similar in the long and the shorter sample for the CAD, BP, and DM. For the JY and the USD, the estimates of $\bar{r}$ and $\bar{\pi}$ are also similar but the estimates of $\bar{\eta}$ are significantly different in the two samples. The estimates for $\bar{\eta}$ based on the sample from January 1985 are 0.80 for the USD and 0.10 for the JY.

¹⁷ The estimates of the USD κ_r (σ_r) of 0.290 (2.77%) compare with values of 0.069 (0.966%) reported by Brennan and Xia (2006) for the (similar) sample period 1983–2000 but with a different set of zero-coupon yield estimates, and by Brennan, Wang, and Xia (2004) of 0.074 (1.11%) for the period 1952–2000.

Table 3
Parameter estimates of the simple term structure model

The United States							
Estimate	σ_b 0.48%	σ_r 2.77%	σ_π 0.81%	σ_η 0.193	κ_r 0.290	κ_π 0.002	κ_η 0.292
<i>t</i> -ratio	(52.82)	(10.92)	(5.15)	(1.88)	(1.97)	(0.70)	(3.84)
Estimate	ρ_{rr} 0.027	$\rho_{r\eta}$ -0.413	ρ_{rm} -0.801	$\rho_{\pi\eta}$ -0.199	$\rho_{\pi m}$ -0.276	$\rho_{\eta m}$ 0.919	
<i>t</i> -ratio	(0.11)	(0.54)	(6.22)	(0.69)	(1.66)	(2.57)	
Pre-set value	$\bar{r}$ 2.62%	$\bar{\pi}$ 3.00%	$\bar{\eta}$ 0.62	σ_P 0.77%	ρ_{Pm} 0.00	ML 9334.6	
Canada							
Estimate	σ_b 0.38%	σ_r 0.78%	σ_π 0.74%	σ_η 0.196	κ_r 0.119	κ_π 0.000	κ_η 0.073
<i>t</i> -ratio	(50.65)	(6.77)	(4.64)	(1.48)	(3.35)	(0.68)	(2.44)
Estimate	ρ_{rr} -0.080	$\rho_{r\eta}$ -0.181	ρ_{rm} -0.865	$\rho_{\pi\eta}$ -0.254	$\rho_{\pi m}$ 0.122	$\rho_{\eta m}$ 0.915	
<i>t</i> -ratio	(0.37)	(0.70)	(2.51)	(0.91)	(1.23)	(2.09)	
Pre-set value	$\bar{r}$ 4.00%	$\bar{\pi}$ 2.78%	$\bar{\eta}$ 0.44	σ_P 1.33%	ρ_{Pm} 0.00	ML 8561.5	
The United Kingdom							
Estimate	σ_b 0.44%	σ_r 0.63%	σ_π 0.92%	σ_η 0.207	κ_r 0.143	κ_π 0.000	κ_η 0.104
<i>t</i> -ratio	(49.74)	(10.35)	(12.94)	(2.45)	(4.68)	(0.76)	(1.89)
Estimate	ρ_{rr} -0.104	$\rho_{r\eta}$ -0.234	ρ_{rm} -0.714	$\rho_{\pi\eta}$ -0.191	$\rho_{\pi m}$ 0.178	$\rho_{\eta m}$ 0.833	
<i>t</i> -ratio	(0.75)	(1.14)	(3.49)	(0.78)	(1.93)	(4.38)	
Pre-set value	$\bar{r}$ 4.56%	$\bar{\pi}$ 3.78%	$\bar{\eta}$ 0.58	σ_P 1.63%	ρ_{Pm} 0.00	ML 8522.2	
Germany							
Estimate	σ_b 0.34%	σ_r 0.83%	σ_π 1.15%	σ_η 0.281	κ_r 0.067	κ_π 0.799	κ_η 0.002
<i>t</i> -ratio	(44.14)	(5.93)	(4.91)	(0.75)	(1.17)	(4.44)	(0.68)
Estimate	ρ_{rr} -0.101	$\rho_{r\eta}$ -0.217	ρ_{rm} -0.984	$\rho_{\pi\eta}$ -0.142	$\rho_{\pi m}$ 0.257	$\rho_{\eta m}$ 0.939	
<i>t</i> -ratio	(0.33)	(0.65)	(3.41)	(0.54)	(0.51)	(2.12)	
Pre-set value	$\bar{r}$ 3.02%	$\bar{\pi}$ 2.45%	$\bar{\eta}$ 0.46	σ_P 0.98%	ρ_{Pm} 0.00	ML 7405.9	
Japan							
Estimate	σ_b 0.18%	σ_r 0.92%	σ_π 0.40%	σ_η 0.065	κ_r 0.001	κ_π 0.119	κ_η 0.048
<i>t</i> -ratio	(44.78)	(4.52)	(1.42)	(1.76)	(0.75)	(1.86)	(1.35)
Estimate	ρ_{rr} -0.154	$\rho_{r\eta}$ -0.259	ρ_{rm} -0.979	$\rho_{\pi\eta}$ -0.237	$\rho_{\pi m}$ 0.343	$\rho_{\eta m}$ -0.146	
<i>t</i> -ratio	(0.27)	(1.05)	(9.16)	(0.42)	(0.39)	(0.23)	
Pre-set value	$\bar{r}$ 2.54%	$\bar{\pi}$ 1.14%	$\bar{\eta}$ 0.21	σ_P 1.62%	ρ_{Pm} 0.00	ML 7969.4	

This table reports estimates of the parameters of the stochastic process of the investment opportunity set, Equations (22–24), obtained from a Kalman filter applied to inflation rates and bond yields. The state variables are r , the real interest rate, π , the expected rate of inflation, and η , the volatility of the pricing kernel or the Sharpe ratio of the economy. In the table, m denotes the pricing kernel, and P is the price level. Asymptotic *t*-ratios are in parentheses.

between innovations in the real interest rate and the pricing kernel, ρ_{rm} , is negative and highly significant for all five currencies, suggesting that there is a significant real interest rate risk premium, and long-term bonds command a positive premium in all five currencies.

The estimated volatilities of innovations in expected inflation rates are highly significant and are in the range of 74 to 115 basis points except for the JY where the point estimate of 40 basis points is not significant. Consistent with previous findings that expected inflation follows almost a random walk, the estimates of κ_{π} are very small and insignificant for the Anglo-Saxon currencies, the CAD, the USD, and the BP. In contrast, the estimate for the DM is much larger and significant, perhaps reflecting the strong anti-inflationary bias of the Bundesbank. We place little weight on the JY estimate because it is clear that expected inflation, like the real interest rate, does not conform to the O-U assumption underlying the estimation during the sample period. Finally, none of the estimates of $\rho_{\pi m}$ is significant so that there is no evidence of a risk premium associated with expected inflation.

The point estimate of the volatility of the innovation in η , σ_{η} , is in the range of 0.19 to 0.28 for all currencies except the JY where it is only 0.06—we conjecture that this reflects the generally poor fit of the model to the JY yields during this sample period. The point estimate is highly significant only for the BP; the lack of significance and the point estimate of 0.193 for the USD are a little surprising, because the stochastic nature of risk premia in the USD has been widely documented, and Brennan, Wang, and Xia (2004) and Brennan and Xia (2003) both report highly significant estimates of σ_{η} of 0.424 and 0.301 for the periods 1952–2000 and 1983–2000. Just as for the real interest rate, the estimate of the mean-reversion parameter, κ_{η} , is highest for the USD and lowest for the DM and the JY for which the model does not fit so well. The half life of innovations in η is almost 9.5 years for the CAD, around 6.7 years for the BP, and about 2.4 years for the USD. Finally, $\rho_{\eta m}$ is positive and significant for all currencies except the JY for which, as mentioned earlier, the model does not fit well. It is interesting to note that, with the exception of the JY, the signs of ρ_{rm} and $\rho_{\eta m}$ are opposite and are consistent across currencies, so that the risk premia for these fundamental investment opportunity set risks are priced in a consistent fashion across currencies.

In summary, the estimation results display an encouraging consistency across currencies except for the JY where the post-bubble economy has not conformed well to the model assumptions about the real interest rate or inflation.

Table 4 reports summary statistics on the time series of the estimated state variables, r , π , and η , that are obtained from the Kalman filter for each of the currencies. Note first that the sample mean of a state variable

Table 4
Summary statistics for state variable estimates

Currency	Mean	Standard deviation	Auto correlation
1. Instantaneous real risk-free rate: r			
U.S. Dollar	3.21%	1.70%	0.964
Canadian Dollar	3.15%	2.72%	0.992
British Pound	4.34%	2.40%	0.989
German Mark	2.91%	1.54%	0.987
Japanese Yen	1.12%	2.40%	0.986
2. Expected Inflation: π			
U.S. Dollar	2.79%	0.71%	0.970
Canadian Dollar	3.69%	1.94%	0.986
British Pound	3.52%	2.10%	0.984
German Mark	2.34%	0.96%	0.974
Japanese Yen	1.70%	0.46%	0.974
3. Maximum Sharpe ratio: η			
U.S. Dollar	0.145	0.450	0.989
Canadian Dollar	0.754	0.940	0.995
British Pound	0.686	0.711	0.992
German Mark	0.468	0.803	0.994
Japanese Yen	0.306	0.186	0.995

This table reports summary statistics for the state variables r , π , and η , estimated from government bonds in the five countries. A cubic spline was used to estimate zero-coupon yields for maturities of 0.5, 1, 2, 3, 5, 7, and 10 years from bond prices taken from Datastream. The state variables were estimated from these yields using the Kalman filter algorithm. The sample is from January 1985 to May 2002. All data are as of the beginning of the month and are in annual terms.

estimate reported in Table 4 may be quite different from the pre-set long run mean reported in Table 3, because the Kalman filter trades off the model fit to the time series of the state variables against the cross-sectional fit to bond yields of different maturities. Particularly noticeable are the sample means for $\hat{\eta}$ of 0.145 for the USD and 0.754 for the CAD as compared with our pre-set estimates of $\bar{\eta}$ of 0.62 and 0.44, respectively. The standard deviation of the estimated real interest rates, ranging from 1.5% per year for the DM to 2.7% for the CAD, is slightly smaller than those of their respective one-month Treasury-bill rates except for the USD. The standard deviation for the η series ranges from 0.19 for the JY to 0.94 for the CAD, indicating a large variation in the volatility of the pricing kernel. All state variable estimates exhibit strong persistence with the autocorrelation in every case being above 0.96. This is inconsistent with the few large κ estimates reported in Table 3 and again is because of the competing demands of fitting the time series of state variable estimates and the cross-section of yields.

The state variable estimates are plotted in Figures 1 to 3. The time series of real interest rate estimates shown in Figure 1 display considerable volatility for all five currencies, ranging in all cases from a high of between 6 and 8% to a low of between 0 and minus 2%. All the series

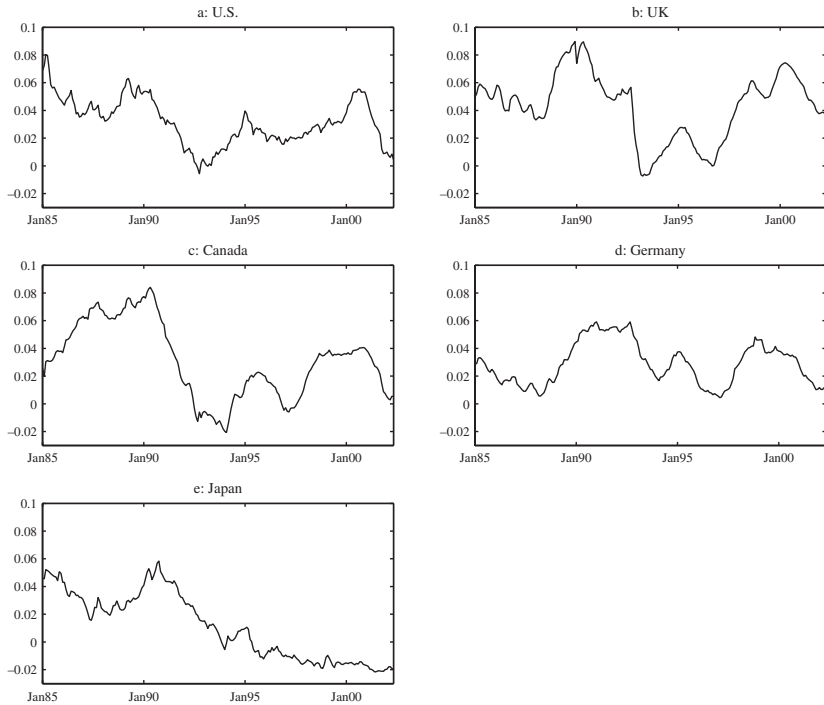


Figure 1

Time series of real interest rate estimates

The figure plots the estimated real interest rate for the United States (a), the United Kingdom (b), Canada (c), Germany (d), and Japan (e) from January 1985 to May 2002. The series are filtered out from the bond yield and Inflation data.

except that for the JY display strong mean reversion. Although there are common elements in the series, currency-specific factors are clearly relevant also. Thus, although the patterns for the USD and the CAD rates are broadly similar after 1990, the USD rate drops steeply from about 8% in the early part of the sample period, whereas the CAD rate is rising sharply from an initial value of around 2%. The BP rate generally tracks the USD rate after about 1985 but with a lag: the rates in both currencies rise strongly in the late 1990s only to fall after 2000 with the decline in the stock markets. The DM rate displays a broadly similar pattern but with a period of elevated rates following the German re-unification in 1990 and with an earlier decline toward the end of the 1990s reflecting the sluggishness of the German economy during this period. The JY interest rate shows the most anomalous pattern, declining almost monotonically from around 5% in 1990 to minus 2% by the end of the sample period.

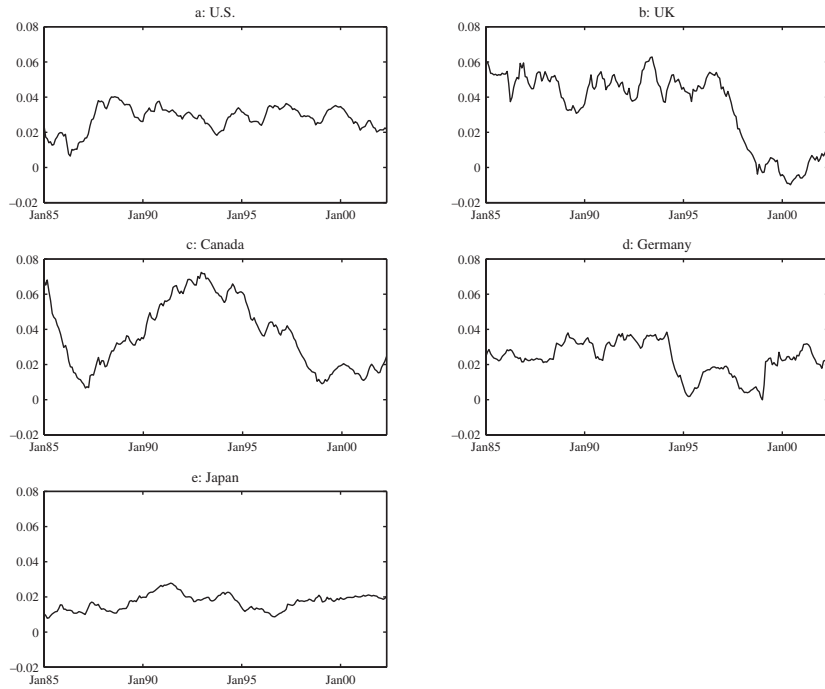


Figure 2

Time series of expected inflation estimates

The figure plots the estimated expected Inflation for the United States (a), the United Kingdom (b), Canada (c), Germany (d), and Japan (e) from January 1985 to May 2002. The series are filtered out from the bond yield and Inflation data.

Figure 2 plots the time series of expected inflation estimates for the period of January 1985 to May 2002. The expected inflation estimates for the USD, the JY, and the DM exhibit much lower volatility than the real interest rates: they vary only from around 0.5 to around 4%. On the other hand, the expected rate of inflation for the BP moved around in a tight range between 3 to 6% until 1997 when it fell rapidly to below zero and then moved up to less than 2%; expected inflation in the CAD has long swings in the much larger range of 0.5 to over 7%.

Figure 3 plots the time series of estimated pricing kernel volatilities for the five currencies. The pricing kernel volatilities for the USD, the BP, and the DM all reach their lows at the peak of the stock market boom in the 1999–2000 period with the USD decline starting around 1993, the BP decline around 1995, and the DM decline around 1996. The three pricing kernel volatilities then all rise rapidly in the post stock market boom period toward the end of the sample. The CAD and the JY pricing kernel

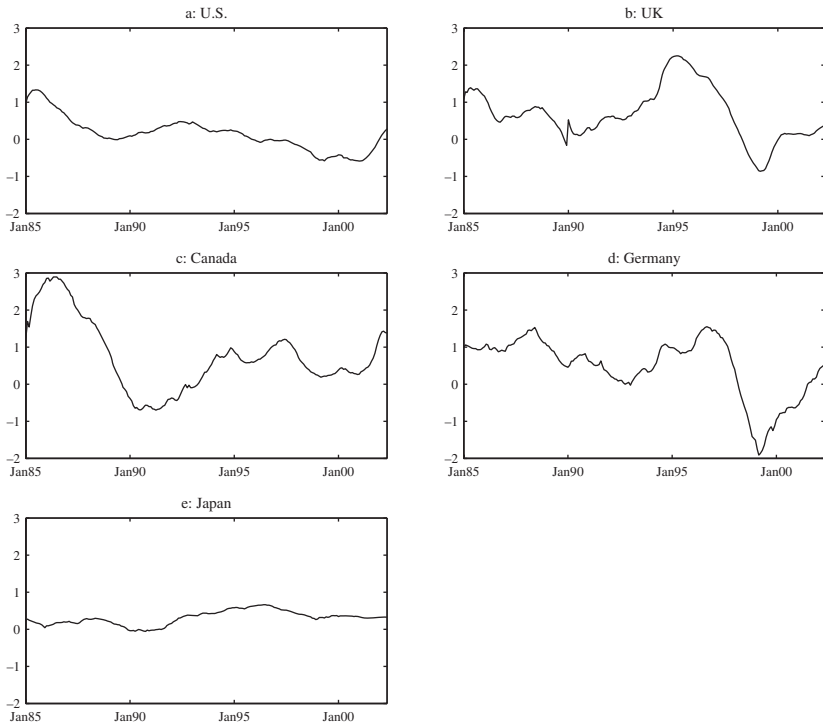


Figure 3

Time series of maximum Sharpe ratio estimates

The figure plots the estimated Sharpe ratio for the United States (a), the United Kingdom (b), Canada (c), Germany (d), and Japan (e) from January 1985 to May 2002. The series are filtered out from the bond yield and inflation data.

volatilities also decline in the second half of the 1990s but not as dramatically as those of the other three currencies. For both the CAD and the JY, the lows (in both cases below zero) are reached in 1990. In the first half of the 1990s, pricing kernel volatilities for all currencies are generally high and increasing.¹⁸ Finally, in contrast to the unrelated movements in the CAD and the USD real interest rates up to 1990, the pricing kernel volatilities for the corresponding currencies display significant comovement, first rising sharply, then declining from 1986 to 1990, and finally rising at the end of the sample period.¹⁹

¹⁸ This is consistent with Bekaert and Hodrick (1992), who report large Hansen–Jagannathan bounds for the pricing kernel volatility inferred from foreign exchange and equity returns.

¹⁹ Ilmanen (1995) reports very high correlations between the estimated excess returns on bonds in different currencies that are obtained by projecting the excess returns on the bonds on a common set of instruments.

To assess how well the simple Gaussian term structure model fits the domestic interest rates, we calculate the theoretical instantaneous nominal interest rate, $R \equiv r + \pi - \eta\sigma_P\rho_{Pm} - \sigma_P^2$, from the estimated state variables as well as the parameter estimates reported in Table 3. The estimated nominal interest rates R , which are nonnegative for all five currencies throughout the sample period, are plotted along with the thirty-day Treasury-bill rates for each currency in Figure 4. Because the thirty-day Treasury-bill rate is not used in the estimation (the shortest maturity yield used is a six-month yield), this provides an out-of-sample assessment of the goodness-of-fit of the model. Although there are a few large mispricings, the figures show that the model implied nominal interest rate tracks the corresponding Treasury rates closely during most of the sample period in all five currencies, suggesting that the simple Gaussian term structure model provides a reasonably good fit for the data.²⁰

4. Empirical Analysis

4.1 Comovement in risk premia

The Risk Premium Restriction (16), which holds in an integrated capital market, is an exact linear relation between the volatility of domestic and foreign real pricing kernels and the volatility of the nominal exchange rate. Although it is unrealistic to expect the restriction to hold exactly when pricing kernel volatilities are estimated from separate sets of national bond yields, it is of interest to see how closely the pricing kernel volatilities that we have estimated for the different currencies are related to each other and to the volatility of the exchange rate between the currencies. Thus consider the following empirical specification corresponding to Equation (16):

$$\eta_{j,t} = c_0 + c_1\eta_{i,t} + c_2\sigma_{S,t} + \epsilon_t, \quad (27)$$

where, $\eta_{j,t}$ ($\eta_{i,t}$) is the estimated pricing kernel volatility for currency j (i), and $\sigma_{S,t}$ is the volatility of the exchange rate between currencies i and j . The theoretical values of the coefficients in (27) are

$$c_0 = \frac{\sigma_P\rho_{SP} - \sigma_{P^*}\rho_{SP^*}}{\rho_{Sm^*}}, \quad c_1 = \frac{\rho_{Sm}}{\rho_{Sm^*}}, \quad c_2 = -\frac{1}{\rho_{Sm^*}}.$$

Although the Kalman filter estimates of the parameters of the Ornstein-Uhlenbeck processes for η imply that these variables have stationary distributions, the estimated time series η 's have first order autocorrelations that are close to unity, and unit root test fails to reject the null of a unit root for all five series. A Johansen cointegration test,

²⁰ See Tang and Xia (2004) for a detailed examination of the goodness-of-fit of different affine term structure models in these five countries.

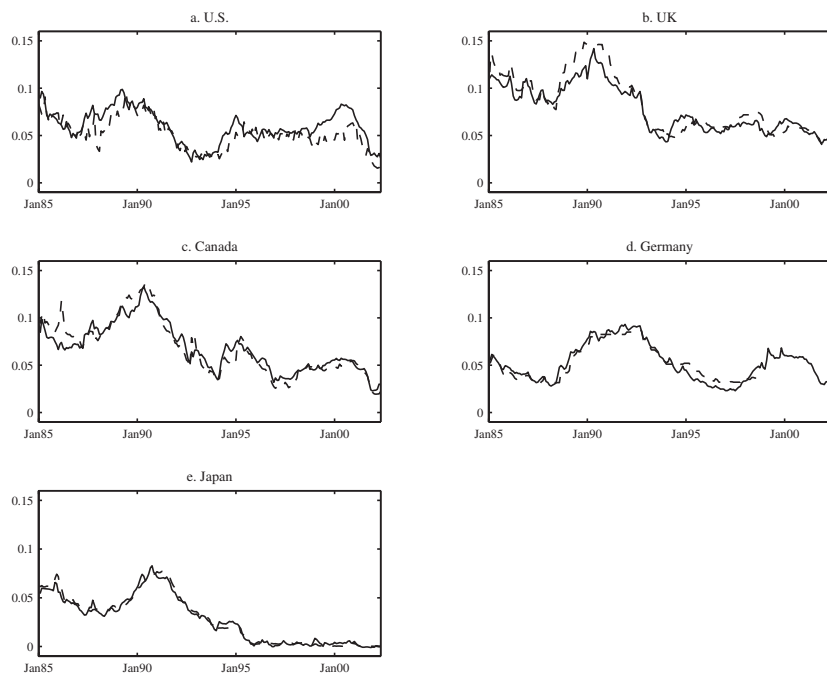


Figure 4

Time Series of Nominal Interest Rate Estimates and the Thirty-day Treasury-bill rates

The figure plots the thirty-day Treasury-bill rates together with the estimated instantaneous nominal interest rate for the United States, Canada, the United Kingdom, Germany, and Japan from January 1985 to May 2002. The nominal interest rate series are calculated from the estimated state variables r , π , and η , whereas the thirty-day Treasury bill rates are from Datastream. Solid line represents estimated interest rate and dashed line represents the Treasury-bill rates.

however, fails to reject the null of no-cointegration between the η s for all currency pairs except for the pairs of USD–DM and USD–JY. In this case, Hamilton (1994) recommends two approaches to avoid the possibility of spurious results from a simple regression of (27).

The first approach estimates the regression using first differences in place of levels:²¹

$$\Delta\eta_{i,t} = c_0 + c_1\Delta\eta_{i,t} + c_2\Delta\sigma_{S,t} + \varepsilon_t. \quad (28)$$

Table 5 reports the results from the above regression for the whole period and for two subperiods.

²¹ The regression in levels yields broadly similar coefficient estimates for c_1 but much higher R^2 than the first difference regression.

Table 5
Regression of pricing kernel volatility and exchange rate volatility innovations

No.	<i>j</i>	<i>i</i>	Sample period: January 1985–May 2002				Sample period: January 1985–September 1994				Sample period: October 1994–May 2002			
			<i>c</i> ₀	<i>c</i> ₁	<i>c</i> ₂	<i>R</i> ² (%)	<i>c</i> ₀	<i>c</i> ₁	<i>c</i> ₂	<i>R</i> ² (%)	<i>c</i> ₀	<i>c</i> ₁	<i>c</i> ₂	<i>R</i> ² (%)
1	CAD	DM	0.001 (0.14)	0.269 (4.02)	0.270 (0.79)	6.72	−0.004 (0.25)	0.231 (1.35)	0.618 (1.15)	0.89	0.007 (0.86)	0.296 (5.09)	−0.250 (0.81)	28.07
2	CAD	JY	0.000 (0.03)	1.030 (2.22)	0.341 (1.57)	4.58	−0.007 (0.43)	1.158 (1.82)	0.798 (2.38)	5.68	0.007 (0.73)	0.833 (1.58)	−0.168 (0.83)	1.85
3	CAD	BP	0.002 (0.18)	0.268 (1.84)	−0.097 (0.23)	6.69	−0.005 (0.38)	0.241 (1.32)	−0.199 (0.34)	3.90	0.013 (1.22)	0.395 (2.83)	0.253 (0.57)	18.23
4	CAD	USD	0.004 (0.48)	1.046 (3.58)	1.807 (1.49)	19.25	0.002 (0.13)	0.905 (1.75)	3.664 (2.20)	12.97	0.005 (0.74)	1.179 (5.36)	−1.134 (1.73)	49.32
5	DM	CAD	−0.002 (0.25)	0.276 (2.63)	0.096 (0.23)	6.59	0.001 (0.12)	0.091 (1.36)	−0.171 (0.34)	0.41	−0.011 (0.74)	1.001 (3.60)	0.690 (1.06)	28.37
6	DM	JY	−0.003 (0.26)	0.920 (2.00)	0.225 (0.44)	3.10	−0.001 (0.08)	0.523 (1.40)	0.365 (0.72)	1.71	0.000 (0.02)	2.095 (1.60)	0.078 (0.10)	5.19
7	DM	BP	−0.001 (0.14)	0.385 (1.97)	−0.189 (0.53)	14.54	0.000 (0.06)	0.152 (1.13)	−0.142 (0.32)	4.11	0.014 (1.10)	1.059 (5.00)	−0.702 (1.12)	41.31
8	DM	USD	0.000 (0.00)	0.652 (2.60)	−0.386 (1.04)	5.95	0.000 (0.03)	−0.151 (0.76)	−0.126 (0.28)	−0.97	−0.006 (0.42)	1.623 (3.95)	−0.692 (1.25)	26.02
9	JY	CAD	0.000 (0.19)	0.050 (2.21)	0.019 (0.24)	4.28	0.003 (0.95)	0.053 (1.69)	−0.004 (0.03)	4.51	−0.003 (1.51)	0.047 (2.23)	0.037 (0.51)	1.83
10	JY	DM	0.000 (0.27)	0.043 (2.47)	−0.047 (0.49)	3.09	0.003 (0.87)	0.060 (1.58)	−0.078 (0.45)	1.50	−0.002 (1.48)	0.035 (1.85)	−0.013 (0.13)	5.20
11	JY	BP	0.001 (0.32)	0.053 (1.71)	−0.023 (0.31)	5.14	0.002 (0.79)	0.035 (1.04)	−0.010 (0.09)	0.89	−0.001 (0.68)	0.085 (4.09)	−0.020 (0.20)	14.54
12	JY	USD	0.001 (0.27)	0.040 (0.96)	0.048 (0.62)	−0.26	0.003 (0.86)	0.017 (0.25)	0.077 (0.60)	−1.49	−0.003 (1.40)	0.088 (2.05)	0.013 (0.16)	2.83

13	BP	CAD	-0.004 (0.43)	0.283 (4.30)	0.318 (1.00)	6.81	0.009 (0.85)	0.231 (3.11)	0.560 (1.43)	4.27	-0.022 (1.89)	0.508 (3.81)	-0.392 (0.83)	18.34
14	BP	DM	-0.002 (0.34)	0.398 (4.95)	0.025 (0.06)	14.50	0.008 (0.82)	0.374 (1.98)	-0.099 (0.18)	4.07	-0.016 (1.87)	0.402 (6.03)	0.463 (1.10)	41.35
15	BP	JY	-0.004 (0.47)	1.150 (2.77)	0.427 (0.99)	5.49	0.006 (0.59)	0.742 (1.61)	0.776 (1.02)	1.79	-0.014 (1.20)	1.936 (2.69)	0.098 (0.34)	14.54
16	BP	USD	-0.001 (0.07)	0.752 (4.36)	0.361 (0.98)	8.24	0.012 (1.16)	0.641 (2.64)	0.516 (1.12)	3.90	-0.019 (1.72)	0.979 (4.79)	-0.138 (0.36)	24.23
17	USD	CAD	-0.003 (0.93)	0.177 (3.53)	0.040 (0.11)	18.04	-0.006 (1.17)	0.108 (1.73)	0.131 (0.24)	8.94	-0.002 (0.46)	0.426 (6.75)	0.583 (1.38)	49.09
18	USD	DM	-0.003 (0.77)	0.105 (3.01)	0.378 (1.90)	7.28	-0.006 (1.22)	-0.043 (0.80)	0.355 (1.26)	0.80	0.001 (0.27)	0.170 (4.53)	0.382 (1.81)	26.75
19	USD	JY	-0.003 (0.80)	0.139 (0.96)	0.117 (1.15)	-0.17	-0.006 (1.22)	0.043 (0.25)	0.012 (0.10)	-1.69	0.002 (0.29)	0.556 (1.64)	0.200 (1.33)	3.55
20	USD	BP	-0.003 (0.81)	0.116 (2.18)	0.167 (0.95)	8.31	-0.007 (1.36)	0.075 (1.37)	0.214 (1.03)	4.11	0.005 (0.77)	0.265 (3.54)	0.106 (0.36)	24.25

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; JY, Japanese Yen; USD, U.S. Dollar.
The table reports parameter estimates from regressions for

$$\Delta\eta_{j,t} = c_0 + c_1\Delta\eta_{i,t} + c_2\Delta\sigma_{S,t} + \epsilon$$

where, $\Delta\eta_{i,t} \equiv \eta_{i,t} - \eta_{i,t-1}$ is the first difference of the currency i pricing kernel volatility, and $\Delta\sigma_{S,t} \equiv \sigma_{S,t} - \sigma_{S,t-1}$ is the first difference of the foreign exchange volatility between currency i and j . Newey-West t -statistics are in parentheses.

Two features stand out in the regression results. First, the regression R^2 is much higher in the second subperiod than in the first; the average adjusted R^2 rises from 3.0 to 21.4%. The much stronger results in the second subperiod are consistent with a significant improvement in market integration over time. Second, the results are much worse when the JY is one of the currencies. For the whole period, as well as for the two subperiods, the regressions involving the JY always have the weakest regression result: for the second sub-period, the average R^2 for the regressions that involve the JY η is 6.2% as compared with 31.3% for the regressions that do not involve JY. While this is consistent with the finding of Brandt, Cochrane, and Santa-Clara (2003) that their international risk-sharing index is lowest for Japan, it also seems likely that the poor fit of the term structure model to JY yields, and corresponding errors in the JY estimates of η , are responsible. For the whole period and the two subperiod regressions the highest R^2 is for the CAD–USD pair, and the second highest is for the BP–DM pair. This is consistent with the earlier observation that geographical proximity leads to more integrated markets.

In sixteen (eighteen) out of the twenty regressions for the whole sample period, $\hat{c}_1$ is significantly different from zero at 5% (10%) significance level, implying that $\hat{\rho}_{Sm}/\hat{\rho}_{Sm^*} \neq 0$ so that the spot rate is priced by either the domestic or the foreign pricing kernels: the only two regressions yielding insignificant $\hat{c}_1$ are those between the USD and the JY. All estimates of c_1 are positive, implying that ρ_{Sm} and ρ_{Sm^*} have the same sign. However, $\hat{c}_2$, the estimated coefficient of $\Delta\sigma_{S,t}$, is insignificant in all of the regressions. This may reflect the poor quality of our estimates of exchange rate volatility.²²

The second estimation approach recommended by Hamilton (1994) is to include lagged values of both the dependent and the independent variables in the regression:

$$\eta_{j,t} = c_0 + c_1\eta_{i,t} + c_2\sigma_{S,t} + c_3\eta_{j,t-1} + c_4\eta_{i,t-1} + \epsilon_t, \quad (29)$$

This regression was estimated using data for the whole period as well as for the two subperiods. The results, which are omitted for conciseness, can be summarized as follows. The regression $\bar{R}^2$ is very high for all regressions because both $\eta_{i,t}$ and $\eta_{j,t}$ are highly autocorrelated. The point estimates for c_1 are very similar to those reported in Table 5 and the t -statistics are slightly higher. Again, all regressions except for those between the USD and the JY produce significant estimates of c_1 in both the whole and the second subperiods, whereas $\hat{c}_1$ is mostly insignificant in

²² In results not reported here, the same regressions were estimated using implied volatilities from currency option prices and estimated conditional volatilities from the GARCH(1,1) model of Section 5.3 in place of the historical estimates of the exchange rate volatility. Implied volatilities are available only for BP–DM, DM–JY and the currencies against USD for the period of October 1994 to May 2002. Results similar to those summarized in Table 5 were obtained for both of these exchange rate volatility proxies.

the first subperiod. In this regression, however, the point estimates of c_2 are quite different from those reported in Table 5, and now four (six) out of the twenty regressions yield significant (at 5%) estimates of c_2 in the whole (second) period regression.

In summary, with the exception of the JY, we have found strong relation between the volatilities of the pricing kernels estimated for the different currencies and a measure of exchange rate volatility as specified by the Risk Premium Restriction. In addition, the relation is stronger in the second subperiod, reflecting the possibility that capital market integration has improved in the later period.

4.2 The Foreign exchange risk premium

In this subsection, we examine the ability of the empirical counterparts of expressions (15) and (18) to capture variation in the foreign exchange risk premium. Although Equations (15) and (18) are theoretically equivalent, their empirical analogues may differ. In Equation (15), the predicted risk premium depends on the exchange rate volatility and the volatility of only the domestic pricing kernel, whereas in Equation (18) the prediction depends on the volatilities of both pricing kernels but not on the exchange rate volatility. As we have seen in the previous subsection, the Risk Premium Restriction (16) which equates the two expressions does not hold exactly when the empirical estimates of exchange rate and pricing kernel volatilities are used.

Substituting the realized appreciation, $(S_{t+1} - S_t)/S_t$, for the expected appreciation, E_t , and using Covered Interest Parity, $(F_{t,\tau} - S_t)/S_t = (R - R^*)\tau$, to replace the interest differential, $R - R^*$, Equation (15) yields the following regression specification:

$$\frac{S_{i,j,t+\tau} - F_{i,j,t,\tau}}{S_{i,j,t}} = c_0 + c_1(\eta_{i,t}\sigma_{S,t}) + c_2\sigma_{S,t} + \epsilon_t, \quad (30)$$

where $\tau = 1/12$ for monthly data, $S_{i,j,t}$ and $F_{i,j,t,\tau}$ denote the spot and τ -period (one-month) forward exchange rates between currencies i and j at the beginning of month t measured in units of currency i per unit of currency j , $\eta_{i,t}$ is the volatility of the pricing kernel for currency i , and $\sigma_{S,t}$ is the realized volatility of the exchange rate during month t . Equation (15) implies that $c_1 = \rho_{Sm}\tau$ and $c_2 = \sigma_P\rho_{SP}\tau$.

For all country pairs, the unit-root null hypothesis is strongly rejected for the product term $\eta\sigma_S$, for σ_S , and for $(S_{i,j,t+\tau} - F_{i,j,t,\tau})/S_{i,j,t}$. Therefore, regression (30) was estimated using OLS: the results are reported in Table 6. Fifteen out of the twenty regressions are significant at the 5% level or better, providing strong evidence that time variation in foreign exchange risk premia is associated with time variation in our estimates of foreign exchange risk and pricing kernel volatility. The weakest results are for the

Table 6
Regression of excess returns to foreign currency investment on domestic pricing kernel and exchange rate volatilities

No.	<i>i</i>	<i>j</i>	<i>c</i> ₀	<i>c</i> ₁	<i>c</i> ₂	<i>R</i> ² (%)	<i>F</i> -statistic <i>p</i> -value
1	CAD	DM	−0.001 (0.17)	0.077 (0.87)	0.105 (0.42)	−0.48	0.550
2	CAD	JY	−0.026 (2.85)	0.122 (1.17)	0.766 (2.49)	6.25	0.000
3	CAD	BP	0.019 (1.63)	0.144 (1.60)	−0.649 (1.48)	3.01	0.016
4	CAD	USD	−0.002 (0.76)	−0.041 (0.52)	0.223 (0.92)	−0.40	0.557
5	DM	CAD	−0.003 (0.36)	0.221 (1.30)	−0.137 (0.47)	−0.11	0.406
6	DM	JY	−0.019 (2.29)	−0.057 (0.25)	0.761 (2.68)	5.72	0.003
7	DM	BP	0.016 (3.86)	0.466 (2.48)	−1.034 (4.04)	11.66	0.000
8	DM	USD	0.003 (0.36)	0.044 (0.25)	−0.224 (0.72)	−0.82	0.725
9	JY	CAD	0.024 (2.95)	0.733 (1.39)	−0.979 (3.45)	4.97	0.002
10	JY	DM	0.018 (2.49)	0.215 (0.45)	−0.722 (2.47)	3.96	0.013
11	JY	BP	0.028 (3.22)	0.587 (1.15)	−0.987 (2.72)	6.06	0.001
12	JY	USD	0.019 (2.63)	0.965 (1.73)	−0.945 (3.17)	4.74	0.003
13	BP	CAD	−0.023 (1.78)	−0.221 (1.83)	0.842 (1.79)	4.18	0.005
14	BP	DM	−0.020 (3.66)	−0.206 (1.32)	1.096 (3.09)	9.92	0.000
15	BP	JY	−0.030 (3.07)	−0.053 (0.39)	0.942 (2.71)	7.48	0.000
16	BP	USD	−0.013 (1.25)	−0.284 (1.84)	0.568 (1.27)	2.24	0.036
17	USD	CAD	0.003 (0.76)	−0.069 (0.41)	−0.176 (0.73)	−0.54	0.645
18	USD	DM	0.003 (0.39)	0.523 (2.84)	−0.134 (0.47)	3.20	0.025
19	USD	JY	−0.027 (3.10)	0.701 (3.85)	0.899 (2.74)	10.93	0.000
20	USD	BP	0.025 (2.61)	0.757 (4.29)	−0.974 (2.41)	8.34	0.000

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; JY, Japanese Yen; USD, U.S. Dollar. The table reports parameter estimates for

$$\frac{S_{i,j,t+\tau} - F_{i,j,t,\tau}}{S_{i,j,t}} = c_0 + c_1 \hat{\eta}_{i,t} \sigma_S + c_2 \sigma_S + \epsilon.$$

where, $S_{i,j,t}$ and $F_{i,j,t,\tau}$ denote the spot and τ -period (one-month) forward exchange rates between currencies i and j at time t measured in currency units of i per currency unit of j , and $\eta_{i,t}$ is the volatility of the real pricing kernel for currency i at time t . Newey-West adjusted t -statistics are in parentheses. Coefficient estimates significant at the 10% level are reported in bold face. The p -value of the regression's F -statistic to test the joint significance of the regressors is reported in the last column. A value less than 0.05 means that the regressors are jointly significant at 5% level.

CAD which is associated with four out of the five insignificant regressions; the other insignificant regression is for the USD–DM risk premium which is not predicted by the DM η regression but is predicted by the USD η regression. For the JY regressions σ_S is always significant, but the term involving the JY η is significant (at the 10% level) only in the regression for the USD.

The estimated coefficient of σ_S is significant at the 5% (10%) level in ten (eleven) out of the twenty regressions; excluding the regressions involving CAD, the coefficient is significant in eight (nine) out of twelve regressions. This is consistent with Brandt and Santa-Clara (2002) who found that the implied volatility from currency option prices predicts the change in the exchange rate but contrasts with the results in Baillie and Bollerslev (1989, 1990), Bekaert and Hodrick (1993), and Domowitz and Hakkio (1985), who found that the conditional exchange rate volatility estimated from a GARCH model has no or only weak association with the drift of the exchange rate. The coefficient of the product term $\eta\sigma_S$ is significant at 5% (10%) in only four (seven) regressions; the proportion rises to four (six) out of twelve regressions if those involving the CAD are excluded.

Despite the fact that the exchange rate volatility enters both regressors, these regressions provide general support for the hypothesis that exchange risk premia are related to time variation in both the exchange risk and the volatility of the pricing kernel. They also support the view that time variation in foreign exchange returns is due to time variation in general risk premia, because the pricing kernel volatility estimate is derived from the estimated time-varying risk premia in other (bond) markets.

The second specification for the (log) risk premium, given by Equation (18), involves only the volatilities of the two pricing kernels (η^2 and η). Because both η^2 and η are highly persistent and the Johansen test rejects the null of no cointegration between $\ln S - \ln F$ and $\eta_i, \eta_j, \eta_i^2, \eta_j^2$ for all currency pairs, leads and lags of the first difference of the independent variables are added to yield the DLS regression specification due to Stock and Watson (1993):²³

$$\begin{aligned} \ln S_{i,j,t+\tau} - \ln F_{i,j,t,\tau} = & c_0 + c_1 \eta_{i,t}^2 + c_2 \eta_{j,t}^2 + c_3 \eta_{i,t} + c_4 \eta_{j,t} + \sum_{p=-2}^2 c_{6,p} \Delta \eta_{i,t-p}^2 \\ & + \sum_{p=-2}^2 c_{7,p} \Delta \eta_{j,t-p}^2 + \sum_{p=-2}^2 c_{8,p} \Delta \eta_{i,t-p} + \sum_{p=-2}^2 c_{9,p} \Delta \eta_{j,t-p} + \epsilon_t, \end{aligned} \quad (31)$$

where, Δ is the first difference operator, and i denotes the domestic currency, whereas j denotes the foreign currency. If the true pricing kernel volatilities were observable, then $c_1 = -c_2 = 1/24$ under the null. Because

²³ OLS regressions yield qualitatively similar results.

the pricing kernel volatilities were empirically estimated only up to a scale factor, c_1 and c_2 are not constrained except for their sign: $c_1 > 0$ and $c_2 < 0$. If unexpected inflation is priced so that $\rho_{Pm} \neq 0$ and $\rho_{P^*m^*} \neq 0$, then the model also predicts that $c_3 \neq 0$ and $c_4 \neq 0$.

The Risk Premium Restriction for nominal exchange rates (16) implies that the variance of the error term in Equation (31) is a quadratic function of η_i and η_j under the no-arbitrage hypothesis in integrated markets: this suggests the following empirical specification for the variance term of the above regression:

$$\sigma_{S,t}^2 = d_0 + d_1\epsilon_{t-1}^2 + d_2\sigma_{S,t-1}^2 + d_3\eta_{i,t}^2 + d_4\eta_{j,t}^2 + d_5\eta_{i,t} + d_6\eta_{j,t} + d_7\eta_{i,t}\eta_{j,t}, \quad (32)$$

If Equation (16) held exactly for the empirical estimates of the pricing kernel and exchange rate volatilities, then we should expect that $d_1 = d_2 = 0$.

Equations (31) and (32) constitute an extended GARCH(1,1) model for the exchange rate risk premium, which was estimated by maximum likelihood.²⁴ The results are reported in Table 7. Because the regression estimates for the two definitions of the exchange rate, S and $1/S$, are identical except for the signs of the coefficients, we report the results for only ten regressions, one for each currency pair. Panel A reports the results for the mean Equation (31). The Wald test for the mean equation tests the joint significance of the η and η^2 terms for the risk premium: the test statistic is significant at the 5% (10%) level for seven (eight) out of the ten regressions. The regressions for which the test statistic is not significant at the 10% level are for the CAD–DM (as in Table 5) and the BP–USD pairs. With the exception of the BP–USD regression, the R^2 range from 13 to 26%. The coefficients of many of the individual η and η^2 terms are insignificant. This may reflect errors in the measurement of η and multicollinearity between η and η^2 . However, eight out of the ten c_1 estimates satisfy the restriction that $c_1 > 0$, and seven out of the ten estimates of c_2 satisfy the restriction that $c_2 < 0$. Overall, these results provide strong evidence that the foreign exchange risk premium depends on the Sharpe ratios of the relevant currencies as we would expect in integrated capital markets.

Turning to the variance equation whose estimates are reported in Panel B, the Wald test for the joint significance of the η and η^2 terms rejects the null of no significance for eight out of the ten regressions, the exceptions being the CAD–DM and the DM–USD exchange rates. This provides strong evidence that, despite the general lack of importance of the historical estimate of the exchange rate volatility found in the regressions reported in Table 5, a quadratic function of η_i and η_j carries information about

²⁴ Using Eviews 5.0 software. The Bollerslev-Wooldridge heteroskedasticity-consistent covariance matrix was used in the estimation.

Table 7

Regression of log excess returns to foreign currency investment on estimated pricing kernel volatilities

Panel A: Mean equation										Wald test	
No.	<i>i</i>	<i>j</i>	<i>c</i> ₀	<i>c</i> ₁	<i>c</i> ₂	<i>c</i> ₃	<i>c</i> ₄	<i>R</i> ² (%)	<i>p</i> -value		
1	CAD	DM	0.011 (1.56)	0.003 (1.18)	0.000 (0.03)	-0.001 (0.20)	-0.017 (0.94)	22.77	0.134		
2	CAD	JY	-0.016 (2.00)	0.013 (2.67)	-0.154 (2.64)	-0.027 (2.32)	0.120 (2.36)	15.49	0.001		
3	CAD	BP	0.001 (0.41)	0.000 (0.14)	-0.005 (1.98)	0.004 (0.68)	0.009 (1.95)	12.61	0.058		
4	CAD	USD	0.001 (1.15)	-0.003 (3.22)	0.008 (1.66)	0.002 (1.16)	-0.001 (0.33)	26.32	0.000		
5	DM	JY	0.020 (2.59)	0.037 (4.16)	-0.006 (0.11)	-0.054 (3.87)	-0.018 (0.49)	39.38	0.000		
6	DM	BP	-0.005 (0.74)	0.003 (0.45)	-0.002 (0.36)	0.010 (0.82)	-0.002 (0.13)	13.47	0.010		
7	DM	USD	-0.004 (0.46)	0.011 (1.38)	0.019 (0.71)	-0.003 (0.21)	-0.021 (0.79)	21.99	0.005		
8	JY	BP	0.004 (0.79)	0.210 (2.95)	-0.012 (2.26)	-0.073 (1.90)	0.007 (0.83)	10.05	0.017		
9	JY	USD	-0.003 (0.78)	-0.024 (0.42)	-0.020 (1.63)	0.034 (1.05)	-0.003 (0.44)	13.63	0.035		
10	BP	USD	0.002 (0.54)	0.001 (0.34)	-0.017 (1.52)	-0.003 (0.44)	0.001 (0.07)	6.50	0.424		
Panel B: Variance equation										Wald test	
No.	<i>i</i>	<i>j</i>	<i>d</i> ₀	<i>d</i> ₁	<i>d</i> ₂	<i>d</i> ₃	<i>d</i> ₄	<i>d</i> ₅	<i>d</i> ₆	<i>d</i> ₇	<i>p</i> -value
1	CAD	DM	0.674 (0.75)	14.624 (0.10)	494.403 (0.77)	-0.048 (0.50)	-0.182 (0.31)	-0.628 (0.79)	-0.220 (0.48)	0.842 (0.79)	0.980
2	CAD	JY	-0.183 (510.52)	-79.202 (2.61)	1009.736 (31.21)	0.091 (167.34)	-2.766 (14.45)	-0.303 (1007.44)	1.781 (15.06)	0.755 (604.05)	0.000
3	CAD	BP	0.055 (3.42)	-65.411 (1.80)	1009.752 (23.01)	0.017 (37.32)	-0.029 (1.98)	-0.018 (0.69)	0.051 (2.83)	-0.030 (0.97)	0.000
4	CAD	USD	0.256 (5.58)	-126.279 (4.08)	-677.215 (5.55)	-0.032 (1.18)	-0.027 (0.22)	0.081 (2.83)	-0.205 (3.20)	0.053 (0.51)	0.000
5	DM	JY	0.074 (1.02)	-24.931 (0.91)	977.948 (17.05)	0.015 (0.45)	0.359 (0.85)	-0.048 (0.58)	-0.201 (2.35)	-0.016 (0.05)	0.037
6	DM	BP	0.065 (3.40)	-21.607 (0.82)	923.366 (30.92)	-0.144 (3.74)	-0.150 (4.22)	-0.049 (1.49)	0.131 (2.05)	0.251 (4.31)	0.000
7	DM	USD	0.479 (2.29)	340.446 (2.49)	172.992 (0.88)	0.068 (0.35)	-0.349 (0.62)	-0.135 (0.43)	0.860 (1.02)	-0.459 (0.54)	0.187
8	JY	BP	1.683 (3.71)	20.308 (0.91)	-1008.957 (26.99)	21.786 (3.46)	0.919 (1.52)	-2.866 (0.90)	-0.128 (0.13)	-8.055 (3.37)	0.000
9	JY	USD	-0.038 (0.89)	-35.407 (0.87)	1004.433 (19.03)	-0.478 (1.68)	-0.036 (0.39)	0.428 (1.85)	0.186 (1.07)	-0.600 (1.11)	0.016
10	BP	USD	1.279 (4.26)	306.562 (2.41)	-116.300 (0.67)	0.018 (0.13)	-0.256 (0.54)	-0.433 (1.38)	1.641 (2.80)	-1.205 (1.86)	0.001

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; JY, Japanese Yen; USD, U. S. Dollar. The table reports the DLS parameter estimates for the GARCH(1,1) regressions given in Equations (31) and (32). Panel A reports the coefficient estimates for the mean equation except those in front of the leads and lags of the first difference terms, which are omitted for brevity. Panel B reports the coefficient estimates multiplied by 1000 for ease of presentation. Bollerslev-Wooldridge heteroscedasticity-consistent *t*-statistics are in parentheses. The χ^2 *p*-value of the Wald test for the null $c_1 = c_2 = c_3 = c_4 = 0$ is reported in the last column of Panel A, and the *p*-value of the Wald test for the null $d_3 = d_4 = d_5 = d_6 = d_7 = 0$ is reported in the last column of Panel B. Estimates of coefficients of the terms involving η_s that are significant (at 10% level) are reported in bold face.

exchange rate volatility as the theoretical Equation (16) predicts. On the other hand, the existence of GARCH effects, as shown by the statistical significance of d_1 and d_2 in several of the regressions, implies that our estimates of η_i and η_j are not sufficient statistics for σ_S as the theoretical relation implies.

Finally, we estimate the system by regressing $\ln S_{i,j,t+\tau} - \ln F_{i,j,t,\tau}$ on the theoretical value of the risk premium, $-p_t$, derived in Section 2. The estimated coefficient for the risk premium term is significant at 10% (5%) in eight (six) out of the ten regressions, which is consistent with the results from the Wald test reported in Table 7. While the point estimates of the risk premium coefficient are all significantly different from the model-implied value of unity, eight out of the ten are positive, which is consistent with the theoretical sign restriction, and we note that the estimated risk premium depends on η and η^* which were estimated only up to a positive multiplicative constant.

In summary, the results discussed in this subsection are consistent with the qualitative implications of the theoretical Equations (18) and (16) that the volatilities of the foreign and domestic pricing kernels carry information about both the foreign exchange risk premium and the volatility of the exchange rate. The rejection of the quantitative restrictions of the model may reflect the remaining frictions between national capital markets, which prevents the theoretical no-arbitrage condition from holding perfectly. It is also likely to be due to errors in the η series caused by estimation error and model mis-specification.

4.3 The forward premium puzzle

Table 8 reports regressions of change in the log spot rate on the log forward premium for the period January 1985 to May 2002. The slope coefficients are negative in six (seven) out of the ten OLS (GARCH) regressions, of which four (five) are significantly different from the theoretical value of unity. This is consistent with the evidence of Fama (1984) and others on the forward premium puzzle.

In this subsection, we consider first whether the risk premia generated by our estimates of the pricing kernel parameters satisfy conditions, ($F1$) and ($F2$), which Fama (1984) has shown are necessary for a risk premium-based explanation of the puzzle. Then, we examine whether the puzzle remains after taking account of the time variation in exchange rate risk premia associated with the pricing kernel volatilities.

The Fama conditions. In order to test whether the Fama conditions are satisfied by our estimates of the pricing kernel parameters, the time series of p_t was calculated for each currency pair using equation (19) from the monthly estimates of η_t along with the estimates for inflation volatility, σ_P , and the correlation with the pricing kernel, ρ_{Pm} , both of which are reported in Table 3.

Table 8
Regression of log spot rate change on log forward premium

No.	<i>i</i>	<i>j</i>	Results from OLS			Results from GARCH(1,1)		
			c_0	c_1	$R^2(\%)$	c_0	c_1	$R^2(\%)$
1	CAD	DM	0.004 (1.14)	0.523 (0.43)	-0.44	0.004 (1.40)	0.120 (0.13)	1.03
2	CAD	JY	0.005 (1.36)	-0.396 (0.42)	-0.41	0.013 (2.18)	-1.929 (1.12)	1.67
3	CAD	BP	-0.001 (0.56)	-2.438 (1.71)	2.15	-0.004 (1.56)	-4.228 (2.65)	0.96
4	CAD	USD	0.001 (0.93)	-0.073 (0.17)	-0.47	0.001 (1.14)	-0.584 (1.34)	2.17
5	DM	JY	0.001 (0.21)	0.230 (0.19)	-0.58	0.000 (0.10)	-1.141 (1.36)	3.01
6	DM	BP	-0.001 (0.40)	0.060 (0.06)	-0.60	0.000 (0.03)	0.332 (0.41)	9.15
7	DM	USD	-0.003 (1.19)	1.179 (0.98)	0.39	-0.001 (0.61)	0.402 (0.57)	6.39
8	JY	BP	-0.007 (1.24)	-0.993 (0.98)	-0.13	-0.023 (3.36)	-4.387 (3.16)	3.22
9	JY	USD	-0.007 (2.25)	-1.461 (1.63)	0.61	-0.008 (2.58)	-2.670 (2.57)	2.30
10	BP	USD	0.002 (0.72)	-1.224 (0.91)	0.23	0.001 (0.48)	-0.463 (0.40)	-0.51

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; JY, Japanese Yen; USD, U. S. Dollar. The table reports parameter estimates from regression estimates of

$$\ln S_{i,j,t+\tau} - \ln S_{i,j,t} = c_0 + c_1 (\ln F_{i,j,t,\tau} - \ln S_{i,j,t}),$$

where, $S_{i,j,t}$ and $F_{i,j,t,\tau}$ denote the spot and τ -period (one-month) forward exchange rates between currencies i and j at time t measured in units of currency i per unit of currency j . In the GARCH(1,1) regression, we use the DLS regression and include the η terms in the variance equation to enable an easier comparison with results in Table 10. Values in the parentheses are the Newey-West-adjusted t -statistics for the OLS regression results and the Bollerslev-Wooldridge heteroscedasticity-consistent t -statistics for the GARCH(1,1) regression results.

The time series of q_t was obtained by subtracting the estimate of p_t from the theoretical interest rate differential $R_t - R_t^*$. Table 9 reports the sample standard deviations of p_t and q_t and the correlations between these variables.

The correlations are all negative (and close to minus one) so that condition $F2$ is satisfied for all currency pairs. Moreover, the standard deviation of p_t exceeds that of q_t as required by condition $F1$ for all currency pairs except the CAD–DM and BP–USD pairs. With these two exceptions, our estimates of the risk premia that have been constructed from bond yields and inflation data show sufficient variability to potentially account for the forward premium puzzle.²⁵

²⁵ Attempts to explain the puzzle with general equilibrium models which assume time-varying uncertainty in fundamentals have met with little success. For example, Bekaert (1996, p. 460) concludes after one such exercise that “Not a single experiment in all my simulations yields negative correlations (between the forward premium and subsequent changes in the spot exchange rate).” Engel (1996) provides an extensive survey of the empirical evidence.

Table 9
Variability of the foreign exchange risk premium and the expected appreciation rate

No.	Currency pair	Standard deviation		Correlation
		p_t	q_t	$\text{Corr}(p_t, q_t)$
1	CAD–DM	2.2309	2.2373	–0.99995
2	CAD–JY	2.2678	2.2621	–0.99999
3	CAD–BP	2.6042	2.5976	–0.99999
4	CAD–US	1.9970	1.9981	–0.99995
5	DM–JP	0.7294	0.7251	–0.99963
6	DM–BP	1.3301	1.3289	–0.99985
7	DM–US	0.8304	0.8194	–0.99961
8	JP–BP	1.2214	1.2205	–0.99996
9	JP–US	0.4412	0.4354	–0.99847
10	UK–US	1.3617	1.3569	–0.99989

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; UK, United Kingdom; US, United States.

This table reports standard deviations of the foreign exchange risk premium, p_t , and the expected appreciation rate, q_t , and the correlations between p_t and q_t . p_t is calculated from Equation (19), using the estimates for inflation volatility, σ_P , and correlation with the pricing kernel, ρ_{Pm} , reported in Table 3, along with the monthly estimates of η_t . q_t is equal to the difference between the interest rate differential and p_t .

Regression analysis. We now consider explicitly whether the forward premium puzzle remains after accounting for the time variation in risk premia that is captured by the estimated pricing kernel volatilities. For this purpose, we estimate the coefficient of the forward premium in the GARCH system examined in Section 5.2 by relaxing the constraint and moving the forward premium to the right hand side of the mean equation:

$$\begin{aligned} \ln S_{i,j,t+\tau} - \ln S_{i,j,t} = & c_0 + c_1 \eta_{i,t}^2 + c_2 \eta_{j,t}^2 + c_3 \eta_{i,t} + c_4 \eta_{j,t} + c_5 (\ln F_{i,j,t,\tau} - \ln S_{i,j,t}) \\ & + \sum_{p=-2}^2 c_{6,p} \Delta \eta_{i,t-p}^2 + \sum_{p=-2}^2 c_{7,p} \Delta \eta_{j,t-p}^2 + \sum_{p=-2}^2 c_{8,p} \Delta \eta_{i,t-p} \\ & + \sum_{p=-2}^2 c_{9,p} \Delta \eta_{j,t-p} + \sum_{p=-2}^2 c_{10,p} \Delta (\ln F_{i,j,t-p,\Delta} - \ln S_{i,j,t-p}) + \epsilon_t, \end{aligned}$$

$$\sigma_{S,t}^2 = d_0 + d_1 \epsilon_{t-1}^2 + d_2 \sigma_{S,t-1}^2 + d_3 \eta_{i,t}^2 + d_4 \eta_{j,t}^2 + d_5 \eta_{i,t} + d_6 \eta_{j,t} + d_7 \eta_{i,t} \eta_{j,t}, \quad (34)$$

Panel A of Table 10 reports the mean equation estimates. The coefficient of the forward premium, c_5 , is estimated with considerable error. While its theoretical value is unity, six out of the ten estimates are negative, and two of them are in excess of 2. Four out of the ten estimates are significantly different from unity; these are for the USD–CAD, the BP–CAD, the BP–JY, and the DM–JY exchange rates. We have already observed that the

Table 10
Regression of log spot rate change on log forward premium and pricing kernel volatilities

Panel A: Mean equation										Wald test	
No.	<i>i</i>	<i>j</i>	c_0	c_1	c_2	c_3	c_4	c_5	$R^2(\%)$	<i>p</i> -value	
1	CAD	DM	0.012 (1.57)	0.001 (0.43)	− 0.007 (0.70)	0.003 (0.41)	− 0.013 (0.78)	1.828 (0.80)	29.52	0.030	
2	CAD	JY	−0.015 (1.22)	0.012 (2.27)	−0.187 (2.71)	−0.026 (2.04)	0.138 (2.34)	0.599 (0.33)	17.53	0.009	
3	CAD	BP	−0.007 (1.91)	0.000 (0.14)	−0.002 (0.48)	0.005 (0.68)	0.004 (0.72)	−4.043 (2.09)	14.17	0.350	
4	CAD	USD	0.006 (5.15)	−0.001 (1.46)	0.004 (0.98)	−0.006 (2.87)	0.008 (3.72)	−2.836 (4.69)	26.05	0.000	
5	DM	JY	0.034 (3.39)	0.037 (3.89)	0.037 (0.63)	−0.072 (4.42)	−0.010 (0.29)	−4.652 (2.13)	44.83	0.000	
6	DM	BP	0.000 (0.00)	0.008 (0.91)	0.001 (0.11)	0.010 (0.52)	−0.011 (0.65)	2.571 (1.58)	18.42	0.016	
7	DM	USD	0.004 (0.63)	0.015 (2.03)	0.022 (0.87)	−0.014 (1.02)	−0.029 (1.09)	−0.685 (0.36)	23.05	0.002	
8	JY	BP	−0.047 (3.12)	0.185 (2.41)	−0.008 (1.21)	−0.050 (1.29)	0.006 (0.60)	−8.656 (3.54)	11.15	0.013	
9	JY	USD	−0.008 (1.50)	0.005 (0.08)	−0.019 (1.18)	0.007 (0.22)	0.010 (0.79)	−2.216 (0.96)	15.64	0.497	
10	BP	USD	−0.009 (1.33)	−0.003 (1.33)	−0.002 (0.22)	0.009 (1.78)	−0.025 (2.01)	3.447 (1.50)	7.29	0.084	
Panel B: Variance equation										Wald test	
No.	<i>i</i>	<i>j</i>	d_0	d_1	d_2	d_3	d_4	d_5	d_6	d_7	<i>p</i> -value
1	CAD	DM	1.793 (2.68)	143.850 (1.92)	−444.376 (1.09)	−0.139 (1.04)	−0.819 (2.32)	−1.328 (1.94)	−0.688 (1.61)	2.230 (2.13)	0.240
2	CAD	JY	−0.005 (0.04)	−56.415 (1.51)	947.400 (11.81)	0.017 (0.39)	−1.408 (1.98)	−0.052 (0.36)	0.862 (1.90)	0.298 (1.08)	0.000
3	CAD	BP	0.052 (1.93)	5.748 (0.23)	931.917 (27.46)	0.024 (1.68)	−0.021 (1.44)	−0.019 (0.55)	0.051 (1.52)	−0.046 (1.43)	0.006
4	CAD	USD	0.257 (48.45)	−172.637 (4.32)	−424.534 (2.81)	−0.052 (3.03)	−0.160 (1.98)	0.070 (3.20)	−0.192 (4.44)	0.145 (2.56)	0.000
5	DM	JY	0.174 (0.63)	−67.154 (1.00)	544.298 (1.03)	−0.104 (1.42)	1.475 (0.86)	0.242 (1.21)	−0.227 (0.42)	−0.487 (0.82)	0.072
6	DM	BP	0.158 (1.10)	14.074 (0.25)	858.793 (15.79)	−0.036 (0.44)	−0.110 (5.11)	−0.218 (1.44)	0.079 (0.62)	0.210 (1.90)	0.000
7	DM	USD	0.538 (5.80)	809.808 (4.26)	31.084 (1.18)	−0.289 (3.78)	−0.001 (0.00)	0.098 (0.61)	−0.028 (0.06)	−0.136 (0.32)	0.000
8	JY	BP	0.558 (1.01)	−1.086 (0.02)	−150.220 (0.14)	18.094 (1.04)	0.868 (1.07)	−0.337 (0.19)	−0.003 (0.00)	−7.801 (1.06)	0.937
9	JY	USD	1.156 (0.76)	−27.963 (0.55)	−148.876 (0.11)	−5.810 (0.85)	1.026 (0.64)	3.249 (0.86)	−2.062 (0.58)	4.849 (0.52)	0.952
10	BP	USD	0.732 (4.02)	440.798 (3.27)	87.172 (0.62)	0.009 (0.12)	−0.150 (0.62)	−0.267 (1.50)	0.972 (2.89)	−0.739 (1.90)	0.000

BP, British Pound; CAD, Canadian Dollar; DM, Deutsche Mark; UK, United Kingdom; US, United States.

The table reports the DLS parameter estimates for the following GARCH(1,1) regressions given in Equations (33) and (34). Panel A reports the coefficient estimates for the mean equation except those in front of the leads and lags of the first difference terms, which are omitted for brevity. Panel B reports the coefficient estimates multiplied by 1000 for ease of presentation. Bollerslev-Wooldridge heteroscedasticity-consistent *t*-statistics are in parentheses. The $\chi^2 p$ -value of the Wald test for the null $c_1 = c_2 = c_3 = c_4 = 0$ is reported in the last column of Panel A, and the *p*-value of the Wald test for the null $d_3 = d_4 = d_5 = d_6 = d_7 = 0$ is reported in the last column of Panel B. Significant (at 10% level) coefficient estimates in front of the terms involving γ s are reported in bold face.

pricing kernel volatility is likely to be especially badly estimated for the JY. However, there is no obvious explanation for the CAD results. A Wald test for the significance of the η s in the presence of the forward premium variable rejects the null in eight out of ten regressions. Thus, it seems that the pricing kernel volatilities pick up a component of the foreign exchange risk premium that is orthogonal to the forward premium variable, and we conjecture that the remaining explanatory power of the forward premium is due to errors in our estimated pricing kernel volatilities as well as the failure of the maintained assumption that the correlations of returns and exchange rates with the pricing kernels are constant. The evidence of association between the pricing kernel volatilities and the volatility of the exchange rate reported in Panel B is slightly weaker than that reported in Table 7.

Nevertheless, these results taken together support the view that foreign exchange risk is priced, and that both the risk premia and the risk vary with the overall level of risk premia for payoffs in a given currency, as measured by the volatility of the pricing kernel. Although the estimated pricing kernel volatilities do not 'solve' the forward premium puzzle, these terms are clearly and significantly associated with the dynamics of the foreign exchange rate and its risk premium. Given the restrictive form of the term structure model which was used to estimate the pricing kernel volatilities, and the strong maintained assumptions that inflation volatility as well as all correlations with the pricing kernel are constant over time, the evidence is supportive of a rational risk-based explanation of the forward premium puzzle.

5. Conclusion

In this article, we have shown that, in the absence of arbitrage in integrated capital markets, there exists a simple *Risk Premium Restriction* between the volatilities of the pricing kernels (or maximum Sharpe ratios) for returns denominated in different currencies and the volatility of the exchange rate between them. We have also shown that the foreign exchange risk premium can be expressed in two alternative but theoretically equivalent forms. First, the arithmetic risk premium can be expressed as the sum of two terms: one that is proportional to the volatility of the exchange rate and the other that is proportional to the product of the volatility of the domestic pricing kernel and the volatility of the exchange rate. Second, when the premium is written in logarithmic form, it can be expressed as a quadratic function of the volatilities of the pricing kernels of the two currencies without involving the exchange rate volatility.

The volatilities of the pricing kernels (Sharpe ratios) associated with five major currencies were estimated from (monthly) panel data on zero-coupon bond yields and inflation using the parsimonious three-factor essentially affine Gaussian term structure model proposed by Brennan,

Wang, and Xia (2004) which allows for independent time-variation in the volatility of the pricing kernel and the real interest rate. This term structure model implies that bond yields are linear functions of the instantaneous real interest rate, the expected rate of inflation and the pricing kernel volatility, so that the Kalman filter approach yielded estimates of these three state variables for each currency/country.

The empirical relations between the estimated pricing kernel volatilities for the different currencies were investigated by regressing the first difference of the volatility of the pricing kernel for one currency on the first difference of the volatility of the pricing kernel for another currency and the first difference of the estimated volatility of the exchange rate between the two currencies. With the exception of the JY, strong relations were found between the pricing kernel volatilities estimated for different currencies: the relation becomes stronger in the second half of the sample period, and there is weak evidence that it is improved by the inclusion of the historical estimate of the volatility of the exchange rate between the currencies.

Regressions of the realized excess return on foreign currency investment on the estimated exchange rate volatility and the product of the exchange rate volatility and the estimated volatility of the domestic pricing kernel were significant in fifteen out of the twenty cases. The term constructed from the product of the domestic pricing kernel volatility and the exchange rate volatility was significant at the 10% level in seven out of twenty regressions and in none of the regressions that involve the CAD pricing kernel volatility. The exchange rate volatility itself is significant at the same level in twelve out of twenty regressions. Thus, there is strong evidence of predictability in the exchange risk premium, but these regressions in which the dependent variable is the excess return to foreign currency investment frequently fail to identify significant separate contributions to the risk premium from time-varying exchange rate volatility and from time-varying pricing kernel volatility.

GARCH model regressions, in which the dependent variable is the log excess return to foreign currency investment, revealed strong evidence that, as implied by the assumption of capital market integration, the foreign exchange risk premium is predicted by a quadratic function of the η s of the two currencies: eight out of ten regressions were significant at the 5% level. Furthermore, the variance equation of the GARCH model shows that these same η s also provide information about the volatility of the exchange rate as the (cross-currency) Risk Premium Restriction implies.

It was shown that, for the model parameters estimated from the bond yield and inflation data, the time series of theoretical foreign exchange risk premia has sufficient volatility to potentially provide an explanation for the "forward premium puzzle." However, when the (log) returns to

foreign currency investment are regressed on the forward premium and linear and squared terms in η_i and η_j , four out of the ten coefficients of the forward premium are negative and significantly different from their theoretical value of unity: these significant rejections of the model are associated with the JY and CAD. In the case of the JY, the pricing kernel volatilities are especially likely to be subject to error, because the mean reverting model of interest rates that underlies our estimation does not fit the JY data well during this sample period.

Nevertheless, these results support the notion that risk premia in foreign exchange markets vary with the general level of risk premia in the corresponding bond markets as is to be expected in integrated capital markets. Given the remaining frictions between national capital markets and the restrictive nature of the essentially affine term structure model that was used to estimate the pricing kernel volatilities, the results are clearly supportive of a rational explanation of the forward premium puzzle. However, further work is required to identify the volatility of the pricing kernels more precisely and to allow for time variation in the correlations of state variable innovations with the pricing kernels which were assumed to be constant throughout the article.

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