Household Portfolio Diversification: A Case for Rank-Dependent Preferences

Valery Polkovnichenko
University of Minnesota and The Federal Reserve Bank of Minneapolis

The proliferation of novel preference theories in financial economics is hampered by a lack of non-experimental evidence and by the theories' additional complexity which has not been shown to be critical in applications. In this article I present arguments in support of preferences with *rank dependency*. Using the Survey of Consumer Finances data, I document two widespread patterns inconsistent with expected utility: (*i*) many households *simultaneously* invest in well-deversified funds and in poorly-diversified portfolios of stocks; and (*ii*) some households with substantial savings do not invest anything in equities. I show that portfolio choice models with rank-dependent preferences, plausibly parameterized and under fully rational assumptions, are *quantitatively* consistent with the observed diversification. These results call for further efforts to integrate the models of rank-dependent preferences in portfolio theory and asset pricing.

The decision theoretic literature has long acknowledged the shortcomings of the expected utility theory of Von Neumann and Morgenstern (1944). Several generalizations of the theory are inspired by the experimental evidence and aimed at improving its descriptive accuracy. However, there are few applications of new models of preferences in finance, and expected utility continues to dominate the field.[1] Two principal hurdles slow the proliferation of novel preferences in financial economics: first, the lack of empirical (non-experimental or "field") data confirming that the behavior motivating non-expected utilities is widespread outside the laboratory settings; second, the additional and often significant complexity of the alternative preferences which has not been shown to be critical in applications where the models based on expected utility fall short of reasonable predictions. This article presents empirical and theoretical

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[1] A notable exception is the recursive utility of Epstein and Zin (1989) which is commonly used to distinguish between risk aversion and intertemporal substitution

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arguments to help overcome these two hurdles for a class of *rank-dependent* preferences.

The economic essence of rank-dependent preferences is their ability to accommodate probability-dependent risk attitude; that is, the preferences are consistent with both risk-averse and risk-seeking behavior depending on the probability distribution of outcomes. Such behavior, often observed in experimental settings, is at odds with the expected utility model.[2] In this article, I argue that examples of such behavior are also common in the data on household portfolio diversification from the Survey of Consumer Finances (henceforth SCF). I perform portfolio choice simulations to show that rank-dependent preferences help explain the empirical evidence better than expected utility does. Thus, rank dependency appears to be not just a minor aberation but rather a quantitatively important feature of preferences.

In the empirical section of the article, I present data from the SCF showing two wide-spread and persistent patterns in household portfolio diversification: (*i*) some households invest significant fractions of wealth *simultaneously* in well-diversified mutual funds and in undiversified portfolios of individual stocks; (*ii*) some households with substantial savings do not have any investments in equities, either directly or through mutual funds. I also demonstrate that households which own undiversified portfolios of individual stocks are well aware of the higher risk associated with such investments, and they consider this risk when deciding on portfolio allocation.

In the theoretical part of the article, using a sample of past returns, I construct simulations of optimal portfolios for several rank-dependent models of preferences. These are the *anticipated utility* model of Quiggin (1982), the *dual theory of choice* model due to Yaari (1987), and the *cumulative prospect theory* model of Tversky and Kahneman (1992). I find that, in a range of plausible parameterizations, the models predict the observed diversification. Depending on the particular choice of preference parameters, the models accommodate a variety of patterns of portfolio choice, including full diversification which is predicted by the expected utility, simultaneous holdings of undiversified and diversified assets, and non-participation in equity markets. Moreover, for the range of parameter values obtained from the experimental studies unrelated to

[2] One of the earliest references to such behavior is Marschak's (1938, p. 320) description of agents' preferences: "they dislike ... situations in which amounts of meat can be anything within a wide range; [...] they like 'long odds,' i.e., high positive skewness of yields." From the preferences axiomatization prospective, probability varying risk attitude constitutes a violation of the independence axiom in expected utility theory. A classical text-book example of such violation is Allais' Paradox [Allais (1979)]. Experimental literature documenting probability-dependent risk attitude is extensive, see for example, Starmer and Sugden (1989), Battalio, Kagel and Jiranyakul (1990), Tversky and Kahneman (1992), Kachelmeier and Shehata (1996), and references therein. Shoemaker (1982), Camerer (1995), and Starmer (2000) provide excellent surveys of this literature

portfolio choice, the rank-dependent models predict simulated portfolios *quantitatively* similar to those found in the data. In the present context, under-diversification represents and attempt to "get ahead" by hoping to capture large but unlikely extreme gains, gains which are only possible in a relatively undiversified portfolio. The non-participation in equity markets emerges because of *first-order* risk aversion in rank-dependent preferences, which implies that sufficiently risk-averse individuals, regardless of their wealth, may choose to avoid risky investments altogether.

The empirical results presented here contribute to a growing body of research documenting that individual portfolio choice is not efficient in the usual mean–variance sense. Beginning with Blume and Friend (1975), several empirical studies have documented concentrated holdings of stocks in individual portfolios, particularly in retirement plans.[3] Outside retirement plans, Kelly (1995) documents the small number of different companies held directly by households in the 1983 SCF data. Goetzmann and Kumar (2001) find substantially concentrated portfolios in the individual brokerage accounts. The current empirical literature on diversification has two main limitations. First, the samples used often are not representative of the entire population. Second, diversification is considered only for a part of the household portfolio. The present article examines diversification in the context of the overall financial portfolio, using representative cross-sections spanning almost two decades. This helps in evaluating the economic significance of the observed regularities in portfolio choice and establishing how widespread and persistent they are.

The literature exploring portfolio choice with non-expected utility preferences is almost as old as the expected utility theory itself. Apparently, the first attempt to formally address the issue of varying risk attitude was due to Friedman and Savage (1948). Following their pioneering work, portfolio theories were developed in which the investors not only want protection from risk but also want to have a "shot at riches." Among the first theories of this type were Roy's (1952) *safety-first* and Markowitz's (1952) *customary wealth* portfolio theories.[4] Prospect theory has been applied in portfolio choice and asset pricing by Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), and Gomes (2003). For tractability reasons, rank dependency and diversification are not emphasized in this literature. Epstein and Zin (1991) used rank-dependent utility in a representative agent setting to address the equity premium puzzle and focused on the role of first-order risk aversion. The quantitative experiment in this article was inspired by the results in Shefrin and

[3] Poor diversification in retirement plans offering employer stock was documented in several papers, for example by Benartzi (2001), Benartzi and Thaler (2001), Liang and Weisbenner (2002), Mitchell and Utkus (2002).

[4] Several authors subsequently explored portfolio selection using these theories, for example, Telser (1955), Pyle and Turnovsky (1970), Arzac (1974), Bawa (1978), Arzac and Bawa (1977).

Statman (2000) who show the optimality of undiversified portfolios in the context of Roy's safety-first theory modified to include rank dependency. The present article is different from Shefrin and Statman (2000) in three main respects. First, the focus here is on the quantitative aspect of diversification. Second, portfolio choice is obtained by simulations and not via assumptions about the distribution of returns or market completeness. Finally, I use different preferences, primarily because of the availability of parameter estimates for quantitative experiments.

The explanation of portfolio diversification based on preferences is certainly not the only one possible. Frequently mentioned alternative explanations include: familiarity bias, ignorance, overconfidence, and informational frictions.[5] While some of these hypotheses are empirically supported, all of them are simply qualitative in nature and explain the bias toward undiversified portfolios without quantifying it. The advantages of the preference-based explanation over the existing alternatives are that it provides sharp and testable quantitative predictions for portfolio allocation, including comparative statics, and that it does so without abandoning rationality. Perhaps one of the greatest advantages of the preference-based explanation is that the discipline of formal modeling allows for integration of portfolio choice into common applications, including models of consumption and savings or the equilibrium analysis of asset prices.

The rest of the paper is organized as follows: Section 1 describes individual diversification using the SCF data. This section also provides direct evidence on the relevance of preference-based explanation for diversification. Section 2 provides an overview of the rank-dependent choice models used in simulations and explains why expected utility, even with preferences for skewness, cannot offer a consistent explanation for the observed diversification. Section 3 describes simulations of portfolio choice for different rank-dependent preferences and presents comparative statics results. Section 4 concludes.

1. Diversification of Individual Portfolios

In this section, I analyze financial portfolios of U.S. households using the Survey of Consumer Finances for the years 1983, 1989, 1992, 1995, 1998, and 2001. These data sets contain detailed information about wealth

[5] Benartzi (2001) presents the results supporting either familiarity bias or ignorance of investors in retirement plans. Benartzi and Thaler (2001), Choi *et al.* (2001a,b), and Liang and Weisbenner (2002) support the notion of implicit investment advice in the investment options offered in retirement plans. Huberman (2001) and Grinblatt and Keloharju (2001) provide support for familiarity bias. Consistent with familiarity, Health and Tversky (1991) find preferences for bets on distributions where subjects have their own judgment. Odean (1999), Barber and Odean (2001), and Goetzmann and Kumar (2001) present empirical evidence in favor of overconfidence. Merton (1987) presents a model with information-collection costs

composition for representative samples of American households, along with population weights calibrated for computations of aggregates.[6] My primary interest is the extent of diversification of individual households, that is, whether investors own any equity at all, and if they do, how diversified their equity portfolios are. I distinguish between *direct* and *indirect* stock investors. The former invest some amount in the stocks of individual publicly traded companies; the latter hold equities only through mutual funds, pension plans, retirement accounts, and other well-diversified vehicles.[7] Indirect stockholders are essentially well diversified because they only own shares of mutual funds. So the focus of this discussion about equity portfolio diversification is on direct investors. However, in constructing the direct investors' financial portfolios, I account for all of their liquid financial assets, including diversified equity holdings, if any. The presentation of the data is organized in order of increasing detail about portfolios.

1.1 The cross-section of equity ownership

The portfolio composition and diversification of households may look qualitatively different depending on their wealth. To control for the influence of wealth, I divide the samples into five cohorts sorted by liquid financial assets (FA). Here, liquid financial assets include: checking, savings, and money market accounts; CDs; publicly traded equities; mutual funds; bonds (government and corporate); annuities and trusts; and pension assets in accounts which permit either withdrawal and/or use of the assets as collateral. I adjust the assets' value for inflation using the CPI with 1989 as the base year. There are four cohorts (2 through 5) referred to in the tables:[8]

2. $0 < FA \leq \$10{,}000$
3. $\$10{,}000 < FA \leq \$100{,}000$
4. $\$100{,}000 < FA \leq \$1$ million
5. $FA > \$1$ million

Table 1 shows for each cohort the financial assets, equity value, and number of households as percentages of the respective aggregates in each year. The missing entries in rows corresponding to 1983 are attributable

[6] The SCF data contains statistical replications of each observation. For example, in the years used for this paper, there were five replications per actual interview. Replications are designed to minimize the effects of nonresponse in surveys. More information is provided on the SCF site http://www.federalreserve.gov/boarddocs/surveys. Except when noted otherwise, I use all replicated observations and weights to compute reported statistics. None of the reported results are affected by whether I use all replications or one average observation per five replications.

[7] If pension plan savings include employer stock, the individual is still classified as a direct investor.

[8] Cohort 1 with no financial assets, $FA = 0$, is omitted from all tables. The cell "All" for the column counting total households in Table 1 differs from 100% by the percentage of households in this cohort.

Table 1
Distributions of financial assets and equity

SCF year and cohort	*Financial assets (FA)*				*Equity (% of aggregate)*			*Households (% of population)*		
	Median Values ($)		% of aggregate							
	Non-stockholders	Stockholders	All	Stockholders	All	Indirect	Direct	All	Stockholders	With direct stocks
1983										
2	1158	4602	3.2	–	–	–	0.7	54.4	–	5.0
3	22,105	32,622	21.6	–	–	–	7.8	27.8	–	9.8
4	251,368	312,105	35.2	–	–	–	32.5	6.0	–	4.0
5	1,069,578	1,949,210	40.0	–	–	–	59.1	0.5	–	0.5
All	2894	31,631	100.0	–	–	–	100.0	88.6	–	19.3
1989										
2	1050	4080	1.9	0.5	0.6	0.4	0.2	44.7	6.2	2.7
3	23,000	30,000	18.8	10.5	11.8	7.7	4.1	31.0	16.3	7.4
4	177,000	204,100	47.4	40.1	47.1	21.5	25.6	10.3	8.3	6.2
5	1,281,970	1,580,870	31.9	30.4	40.5	16.7	23.8	0.8	0.8	0.7
All	2800	35,100	100.0	81.8	100.0	46.3	53.7	86.8	31.6	16.9
1992										
2	854	4188	2.2	0.8	0.9	0.7	0.3	48.3	9.1	73.0
3	25,127	35,000	20.8	13.8	15.2	11.4	3.8	29.4	18.3	7.8
4	256,239	284,615	47.5	39.1	45.7	26.5	19.2	10.2	8.1	5.6
5	1,423,675	1,505,726	29.6	28.6	38.5	12.4	26.2	0.7	0.7	0.6
All	1880	34,230	100.0	82.5	100.0	50.9	49.4	88.6	36.2	17.0

1995										
2	940	4388	1.9	0.8	0.9	0.7	0.1	46.5	10.9	2.6
3	22,410	33,252	17.7	12.3	13.6	10.8	2.8	31.1	19.8	77.2
4	274,608	306,974	44.1	38.4	44.1	31.6	12.5	10.5	8.9	4.8
5	1,836,018	1,725,517	36.4	34.2	41.6	17.9	23.7	0.7	0.7	0.6
All	1959	30,721	100.0	85.6	100.0	60.8	39.2	89.0	40.4	15.3
1998										
2	776	4212	1.1	0.5	0.4	0.3	0.1	41.9	10.1	1.9
3	25,868	34,294	14.2	10.7	10.3	8.5	1.9	33.5	24.0	8.1
4	243,754	288,543	41.3	38.2	38.7	27.4	11.4	14.6	13.3	8.1
5	1,752,793	1,865,099	43.2	43.1	50.5	21.7	28.8	1.4	1.4	1.2
All	1656	41,463	100.0	92.4	100.0	57.8	41.9	91.4	48.9	19.2
2001										
2	1200	6000	0.85	0.38	0.3	0.3	0.1	41.4	11.1	2.4
3	36,500	53,200	10.1	7.9	7.6	6.4	1.2	31.8	23.1	7.5
4	271,000	327,800	43.4	40.6	43.7	32.2	11.3	17.5	16.0	9.9
5	4,056,200	2,787,000	45.6	45.3	48.6	22.7	25.9	1.7	1.7	1.4
All	1490	69,650	100.0	94.2	100.0	61.8	38.4	92.4	52.0	21.2

Source: Survey of Consumer Finances. Financial assets and cohorts subdivision are defined in the paper. A household is classified as an owner of equity if it owns equity directly or indirectly in mutual funds, pension plans or IRA's.

to the fact that one cannot identify all owners of equity for that year. In 1983, the survey did not ask about the allocation of mutual fund investments across stocks, bonds, and other securities. As a result, it is impossible to tell whether a household had any equity unless it was directly held.[9]

The first two columns of Table 1 show the median dollar value of financial assets for households with and without equities. Households with equities are substantially wealthier, and the disparity in liquid wealth across households with and without equities is larger for the lower wealth cohorts. The second two columns of Table 1 show the distribution of liquid assets across population and across equity-owning households. The liquid assets are concentrated and the aggregate share of stockholder's liquid wealth increased steadily from 1989 (81.8%) to 2001 (94.2%).

The next three columns of Table 1 show the distributions of total, direct, and indirect holdings of equities. The wealthiest cohort holds more equity directly than indirectly; all other cohorts invest more indirectly (except the fourth in 1989). Over the period for which the SCF allows us to distinguish between direct and indirect stockholders, the total value of equity shifted substantially toward indirect equity holdings. This could be because of a decrease in the relative cost of investing in equities indirectly and because of the proliferation of defined contribution retirement plans.

The last three columns of Table 1 show the percentage of households in each cohort, and stockholders and direct stockholders separately. Ownership of equities consistently increased between 1989 and 2001; the increase was concentrated in the lowest wealth cohorts. Note that the percentage of households with direct equity investments did not change significantly since 1983.

Non-participation in the equity market observed in Table 1 is inconsistent with friction-less portfolio choice based on expected utility. Various participation costs may prevent poor households from investing in equity, but there is also a stable group of relatively rich households who do not own any equities, either directly or through mutual funds.[10] Note that the median wealth of *non-stockholders* in each cohort is higher than the median wealth of *stockholders* in the next poorer cohort. The fraction of non-stockholders in cohorts 3–5 (ignoring cohort 2) varies between 10 and 16% of the population over the years. Thus, although participation costs may very well explain why poor households (cohort 2) do not invest in equities, they are inconsistent with non-participation of the richer

[9] Beginning in 1989, the survey began to ask how the mutual funds and pension assets are invested: mainly in equities, bonds, or some combination. In computing the values of equity holdings for households with funds having equities and bonds, I assumed 50% of such funds to be in equities.

[10] The cost of participation may be due to direct transaction cost or indirect cost of learning about equities and opportunities to invest in them. For example, Gomes and Michaelides (2005) assume such participation cost in their model.

households in cohorts 3–5. In the theoretical sections of the paper, I will explore whether rank-dependent preferences help account for this finding. To summarize, Table 1 indicates that the number of direct equity owners is significant and, through 2001, this group of households had substantial wealth invested in equities. I now proceed to a more detailed analysis of the portfolios of the direct investors.

1.2 Household equity portfolios

In addition to the values of direct and indirect stockholdings, the SCF includes questions about the number of directly held stocks and the value of the stock of current and former employers. These questions allow us to determine, to some extent, how well-diversified household portfolios are.

Table 2 provides median statistics for equity portfolios. If the household has no equity, either direct or indirect, it is not included in the subsample used to construct Table 2. Using the remaining observations, the medians are computed *conditional on having nonzero* amount of a particular type of equity, either direct or indirect. The first two columns show the median fraction of directly held stock (relative to the *total equity* held by a household) and the median number of different companies held directly. In the late 1980s and early 1990s and in all cohorts, directly held stocks accounted for well over two-thirds of the total equity portfolios of direct household investors. In 1995, a shift toward indirect stock holdings began. However, the fraction of direct equity holdings was still significant throughout 2001.

For cohorts 2–4, even in 2001, the number of different companies held was alarmingly small. Most direct household investors held the stocks of only one or two different companies. The median for all cohorts was two in 1983 and did not change until 2001 when it rose to three. Note also that for the households in the wealthier cohorts 3–5, directly held stocks are only part of their equity portfolio (column 1). The rest of the portfolio is invested in diversified funds. This is inconsistent with the hypothesis that investors are unaware of diversified opportunities. Most direct investors do hold diversified funds, but they combine them with substantial investments in undiversified portfolios of individual stocks.

The number of different stocks held alone does not accurately describe the extent of diversification. One has to look further at how overall household financial wealth is invested. The last three columns of Table 2 present this information and show the median fractions of total household financial wealth invested in equities—all equities, held indirectly and directly. The median fraction of wealth invested in equity is relatively uniform across the cohorts. Across cohorts, median direct stockholdings range from 15 to 33% of financial assets. While diversification improves with wealth, not all wealthy households, even in 2001, were well diversified. For example, the second wealthiest cohort 4 had a

Table 2
Equities in household portfolios from the Survey of Consumer Finances

	Equity portfolios of direct investors		*Equity relative to total financial assets*		
SCF year and cohort	Direct fraction (relative to all equity owned)	Number of Stocks	All equity	Indirect	Direct
1983					
2	–	1	–	–	0.29
3	–	1	–	–	0.17
4	–	4	–	–	0.23
5	–	15	–	–	0.47
All	–	1	–	–	0.21
1989					
2	1.00	1	0.33	0.29	0.26
3	1.00	1	0.32	0.29	0.14
4	0.88	4	0.35	0.19	0.16
5	0.75	6	0.37	0.19	0.19
All	1.00	2	0.33	0.26	0.17
1992					
2	1.00	1	0.40	0.37	0.33
3	0.68	1	0.38	0.35	0.14
4	0.64	4	0.40	0.28	0.16
5	0.83	10	0.41	0.13	0.25
All	0.78	2	0.39	0.34	0.17
1995					
2	1.00	1	0.46	0.44	0.33
3	0.65	2	0.48	0.44	0.16
4	0.52	3	0.50	0.38	0.17
5	0.77	15	0.54	0.18	0.26
All	0.68	2	0.49	0.38	0.20
1998					
2	1.00	1	0.45	0.43	0.33
3	0.50	2	0.54	0.47	0.19
4	0.39	4	0.60	0.44	0.18
5	0.51	15	0.70	0.39	0.33
All	0.49	2	0.53	0.46	0.21
2001					
2	1.00	1	0.46	0.43	0.34
3	0.41	2	0.54	0.49	0.17
4	0.29	4	0.66	0.49	0.15
5	0.50	14	0.70	0.33	0.26
All	0.40	3	0.57	0.48	0.18

Table reports median values. Financial assets and cohorts subdivision are defined in the paper. The medians are computed conditional on owning nonzero amount of a particular type of equity, direct or indirect.

median of four different companies' stocks, and the median fraction of total wealth held directly in stocks varied between 15 and 23% from 1983 to 2001.

Figure 1 shows the cumulative distributions of direct investors and the cumulative value of directly held stocks as a function of the number of stocks in portfolios. The distributions remained essentially unchanged over the 18 years covered by the surveys. Over the time period covered, about 80% of direct investors held stocks of five or fewer different companies and about 90% held fewer than ten companies' stocks. The

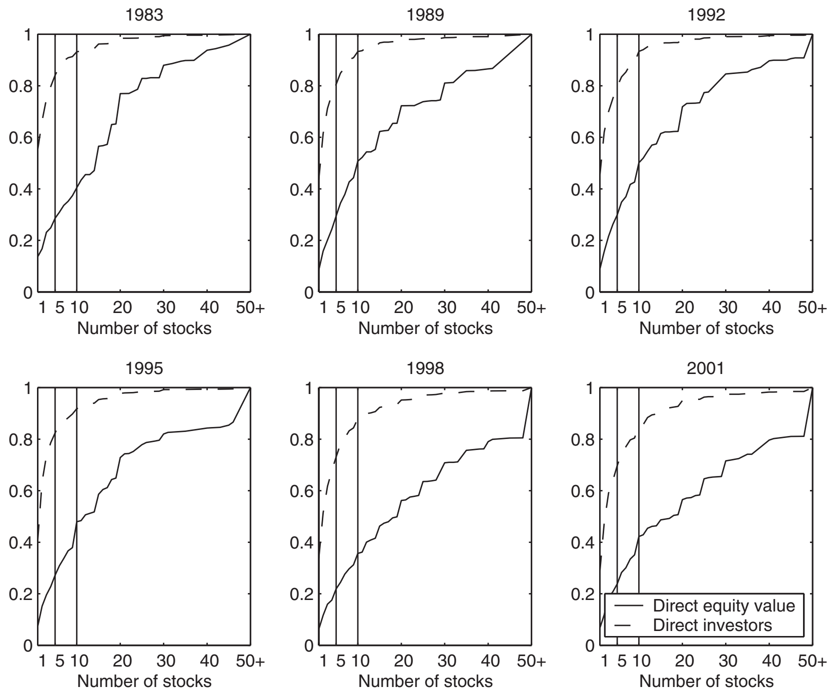

Figure 1
Cumulative distributions of direct investors and direct equity value as functions of the number of stocks in direct portfolios from the Survey of Consumer Finances.

distributions start at around 40% or more of investors holding the equity of only one company. Because of the skewness of the wealth distribution, the aggregate wealth invested in undiversified portfolios is not very large. However, the number of undiversified households is quite significant, even in the last survey in 2001. Furthermore, undiversified portfolios are significant at the *individual* level, which is the appropriate scale for the purpose at hand. It is also important to understand that rank-dependent preferences explored later in this paper may have new implications for the aggregate analysis (e.g., asset pricing) even though the lack of diversification *per se* may not. To illustrate how common undiversified portfolios are, consider the survey in 1998. According to Figure 1, about 75% of direct investors held equity in five or fewer companies. This corresponds to about 14.2% of total population or 30% of all equity owners. Because in 1998 the survey was designed to aggregate to approximately 100 million households, there were about 14 million such households.

In part, poor diversification is attributable to investments in employer stock which could arise from stock-based compensation and/or from matching in pension plans.[11] On average, 7% of households (or about 30% of direct stockholders) owned employer stock. Employer stock is quite a significant fraction of equity portfolios (up to 100%), especially in the less-wealthy cohorts. Relative to financial assets, the fraction of employer stocks for all cohorts declined from 18 to 9% between 1983 and 2001. While at least a half of the investors with only one stock appear to have employer stock, the diversification picture is not driven by employer stock. The direct stockholders without employer stock are still numerous. This is also consistent with the data for 1985 studied by Mitchell and Utkus (2001) who report that 401(k) plans with employer stocks were a small fraction of the total number of plans, assets, and participants. In the SCF data, poor diversification persists throughout the 1980s and early 1990s when 401(k) plans and stock-based compensation were not as common as in the mid-late 1990s (Figure 1).

From the SCF data, I conclude that most direct stockholders invest substantial fractions of their wealth in undiversified portfolios of individual stocks and often combine such investments with well-diversified mutual funds. In the next section, I present additional empirical evidence to support the preference-based approach to portfolio diversification.

1.3 Do investors understand the risk of directly held stocks?

There are several biases (e.g., familiarity, overconfidence) which were empirically shown to play a role in undiversified portfolio choice. Toward this end, the SCF data may be used to provide empirical evidence consistent with the preference-based explanation for diversification. According to the biases, investors are not aware of the risks of under-diversification because they otherwise would hold well-diversified portfolios. Here I present evidence to the contrary and link diversification with risk attitude which is consistent with preference-based explanation.

I use regressions to determine if direct investors are aware of the higher risk associated with undiversified portfolios of individual stocks. I consider two measures of diversification as dependent variables: (*i*) the fraction of total financial wealth (FA) invested in directly held stocks (DIR) and denoted as $\left(\frac{DIR}{FA}\right)$; (*ii*) the fraction of *risky* assets (RIS) invested in directly held stocks denoted as $\left(\frac{DIR}{RIS}\right)$. The first measure detects if investors realize the overall riskiness of individual stocks and the second if they realize their riskiness *relative* to the other risky investments. Financial wealth is defined as before, but risky financial wealth (RIS) excludes some safe assets, such as checking, savings, and money market accounts, CDs, defined benefit pensions, and various government and municipal bonds

[11] The SCF questions do not distinguish between these sources.

and mutual funds and pension plans that hold only such bonds. To eliminate investors with "unintended" lack of diversification, for example, due to employer stock match in pension plans, I restrict the sample to households who report trading individual stocks at least three times in the preceding calendar year. Such households willingly own stocks outside their pension plans and therefore are not forced to hold undiversified portfolios. Furthermore, to eliminate the effect of multiple imputations on the significance of the results, I average each five imputations per respondent to obtain a single observation and use such observations without the SCF weights. These procedures restrict the sample to 600–700 direct investors per survey.[12]

In each regression, there are four independent variables. The first two variables are the log of total financial assets [log(*FA*)] and the education of the most educated member in the household (*EDU*).[13] Both education and wealth are likely to be related to the psychological biases because wealthy and more educated individuals may be less ignorant and less prone to biases. Also, if there is a systematic effect of education on portfolio choice, this would indicate that investors are aware of the differences between directly held stocks and other portfolio components. To link portfolio allocation to risk preferences and to further determine whether investors realize higher risk from directly held stocks, I use two additional variables. The first is the number of dependents in the household (*DEP*) and the second is the self-reported risk attitude (*RA*) ("wiliness to hold financial risks"). The surveys ask "How much financial risk are you willing to take?" The four potential responses coded 1 through 4 are as follows:

1. Take substantial risks, expect to earn high returns
2. Take above average risks, expect to earn above average returns
3. Take average risks, expect to earn average returns
4. Not willing to take any financial risks

The number of dependents and the risk attitude are proxies for the willingness to take financial risk. The regressions are estimated as follows:

$$\frac{DIR}{FA} = a_0 + a_1 \log(FA) + a_2 EDU + a_3 DEP + a_4 RA$$
$$\frac{DIR}{RIS} = b_0 + b_1 \log(FA) + b_2 EDU + b_3 DEP + b_4 RA$$

[12] The first survey in 1983 was excluded because it does not contain information on the risk profile of mutual funds necessary to determine the value of risky part of the portfolio (*RIS*).

[13] Education is coded as zero for incomplete high school, 1 for at least high school, 2 at least for college, and 3 for graduate or professional degree.

If the investors are biased or ignorant and do not realize that individual stocks are risky (overall or relative to other risky assets), then the last two dependent variables should not be significant. Negative coefficients on *DEP* and *RA* would indicate that households are aware of the risk of individual stocks and consider this risk in portfolio allocation. Additionally, the significance of the coefficient on education also would indicate that the investors are aware of the differences between directly held stocks and other investments.

Table 3 shows the results of the two regressions. The significance of the coefficients is marked as *, **, and *** to indicate significance levels of at least 10, 5, and 1% respectively. The direct portfolios are relatively larger for wealthier households. More educated households have lower fractions of direct equity, and this effect is always statistically and economically significant. This is consistent with more educated investors being *aware* of the risks of directly held stocks. Both the number of dependents and the risk attitude reduce the exposure to directly held stocks. The number of dependents is not always significant, but whenever it is significant it has the expected negative sign and is economically important. Particularly significant (statistically and economically) and consistent over time is the effect of self-reported risk attitude. The marginal effect of changing risk attitude by one category is in the range of 2.5–7.1% for the total fraction of direct equity and 1.5–8.2% for the fraction of direct equity in risky investments. The averages of these coefficients across surveys are 4.5 and 4.3%, which is significant relative to the median allocations to direct

Table 3
Cross-sectional variation in direct equity relative to financial assets as a function of risk attitude

SCF year/Direct equity realtive to	log(*FA*)	Education	Number of dependents	Risk attitude	Adj. R^2 (%)
1989					
Total FA	0.007	−0.033**	0.019	−0.071***	4.6
Risky FA	0.0001	−0.058**	0.027	−0.082***	4.9
1992					
Total FA	0.024***	−0.029**	−0.026**	−0.025*	4.3
Risky FA	0.018**	−0.056***	−0.027**	−0.037**	4.0
1995					
Total FA	0.011*	−0.036**	−0.008	−0.025*	1.6
Risky FA	0.004	−0.064***	−0.009	−0.015	2.2
1998					
Total FA	0.027***	−0.043***	−0.023**	−0.050***	7.6
Risky FA	0.013**	−0.052***	−0.027**	−0.054***	4.6
2001					
Total FA	0.041***	−0.030**	−0.005	−0.038**	9.7
Risky FA	0.027***	−0.048***	−0.016	−0.027*	3.9

The dependent variables are fractions of direct equity relative to total and risky financial assets. Financial assets and the independent variables are defined in the text. The sample is restricted to direct equity holders who traded direct equity at least three times in the year preceeding the survey. The regressions are unweighted and each five SCF implications are collapsed into one observation by averaging across implications. Asterisks*, **, and *** indicate the significance level at least at 10, 5, and 1% respectively.

equity from Table 3. Similarly large marginal effects are observed for the education variable.

Thus, it appears that direct investors are well aware of the additional risk associated with directly held equity. The investors do not simply overlook this risk, but rather balance their direct equity portfolios according to their personal risk tolerance. Although investors seem to understand the risk of individual stocks, a caveat still applies here that under-diversification may partly be a result of over-confidence in stock-picking abilities which would bias the investor's estimate of expected returns. However, stocks are not the only assets investors *knowingly* use to create undiversified portfolios. There are financial securities *specifically structured* to have large risk. Such securities are not mean–variance efficient, yet their creation was prompted by investors' demand. For example, Swedish lottery bonds pay lottery tickets in lieu of interest coupons (Green and Rydquist, 1999); British premium bonds have regular prize drawings among their holders. Equity Participation Notes or Equity Linked Notes (EPNs or ELNs) are another kind of intrinsically undiversified securities, combining low-interest bonds with the high upside potential of options. Similarly, Liquid Yield Options Notes (LYONs) created by Merrill Lynch (see McConnell and Schwartz (1992)) combine a safe bond-like asset with the highly volatile payoff of a call option. In the remaining sections of the paper, I show that holding such undiversified securities and undiversified portfolios of individual stocks may be explained by the preferences with rank dependency.

2. Rank-dependent Theories of Choice, Expected Utility, and Preferences for skewness

The traditional expected utility model is inconsistent with experimental evidence in several ways and there is now a large literature on modifications of preferences needed to explain the empirical observations. In the context of diversification, the phenomenon of varying risk attitude, wherein people exhibit risk aversion in some choices and risk taking in others, is particularly relevant. A well-known type of *rank-dependent* preferences is designed to capture such varying risk attitude. In this section, I review main features of rank-dependent preferences and compare them to the expected utility. It should be noted that this review is very brief, only enough to clarify the simulations and results in the following sections. It is not intended as a survey of the vast experimental and theoretical literature on this type of preferences, and I refer interested readers to the sources cited for more details.[14]

[14] Some good starting points are the surveys by Shoemaker (1982), Camerer (1995) and Starmer (2000).

2.1 Rank-dependent utility (RDU) and the dual theory of choice

The experimental violations of the Independence Axiom of the expected utility theory prompted the creation of several rank-dependent preferences designed to capture varying risk attitude.[15,16] One of the widely known and general models with rank dependency is Rank-Dependent Utility (RDU) proposed by Quiggin (1982). In his model, utility is weighted by the decision-weighting function that depends on the probabilities of outcomes and their relative ranking.[17] Yaari (1987) independently proposed the "dual theory of choice", a special case of Quiggin's RDU with linear utility of outcomes and rank-dependent decision weights. Rank dependency is also present in the cumulative prospect theory discussed later.

Let $i = 1, \ldots, N$ index possible outcomes (wealth or consumption) ordered from lowest to highest. Every outcome is assigned a decision weight w_i. The utility functional is given by:

$$V_{RDU} = \sum_{i=1}^{N} u(c_i)w_i, \tag{1}$$

where $u(\cdot)$ is the outcome utility defined on consumption c_i in state i. In Yaari's model $u(c) = c$, but in a general formulation, $u(\cdot)$ is simply an increasing and continuous function. The decision weights w_i are constructed using a strictly increasing and differentiable function $g(\cdot):[0,1] \rightarrow [0,1]$, s.t. $g(0) = 0$ and $g(1) = 1$, defined on the cumulative probability of outcomes P_i as follows:

$$\begin{aligned} w_1 &= g(P_1) \\ w_i &= g(P_i) - g(P_{i-1}) \text{ for } i = 2, \ldots, N-1 \\ w_N &= 1 - g(P_{N-1}). \end{aligned} \tag{2}$$

Thus, the decision weight in each outcome depends both on the ranking of the outcome and on the entire probability distribution of possible outcomes. A special case when $g(P) = P$ coincides with expected utility. The form of $g(\cdot)$determines how the decision maker transforms objective probabilities into subjective decision weights and capture probability-

[15] If lotteries q and r are probability vectors over a common domain of outcomes and $q \succeq r$, then for any other such lottery s and any parameter $p \in [0,1]$, independence axiom requires for compound lotteries that $p \cdot q + (1-p) \cdot s \succeq p \cdot r + (1-p) \cdot s$. This axiom is systematically violated in experimental studies (e.g., Allais' Paradox).

[16] Several other models without rank dependency also conform with varying risk attitude, for example, the weighted utility of Chew and MacCrimmon (1979), the generalized expected utility [Machina (1982)], and the theory of disappointment aversion by Gul (1991). However, rank dependency has received relatively more attention from the recent decision theoretic literature and, most importantly for the purpose at hand, there was accumulated experimental data which allows to calibrate parameters of such preferences.

[17] Axiomatizations of this utility were provided, among others, by Yaari (1987) and Abdellaoui (2002).

dependent risk attitude.[18] Quiggin (1993) shows that when the weighting function is such that $g(P) \geq P$, the agent is risk averse. If the inequality is reversed, the agent is risk seeking. In particular, concave g corresponds to risk aversion and convex g to risk seeking. A mixture of convex and concave segments (inverse S-shape) results in risk aversion over relatively bad outcomes and risk seeking over good outcomes. Such inverse S-shaped weighting functions have received empirical support in the experimental decision literature.

An important and distinct property of rank-dependent utilities is that of "first order" risk aversion which may help explain rich investor's non-participation in the equity market. All expected utility functions are *locally risk neutral* near a riskless outcome. Thus, some amount of risky assets with a positive risk premium is *always* desirable, even for the most risk averse expected utility agents.[19] This is different for RDU because, in general, the weighting function is nonlinear in probabilities. To be more specific, consider an example in which the investor has a certain amount, x, and invests a small amount, k, in the asset with random return given by $1 + \alpha + \tilde{\epsilon}$ where α is a constant equal to the expected return of the asset and the random variable $\tilde{\epsilon}$ has two equally likely outcomes, $\pm\epsilon$. The expected utility of the investor's payoff, $x_k = x + k\alpha + k\tilde{\epsilon}$, may be approximated as follows:

$$\begin{aligned} EU(x_k) &\approx U(x+k\alpha) + \frac{1}{2}U''(x+k\alpha)(k\epsilon)^2 = U(x+k\alpha) + O\Big((k\epsilon)^2\Big) \\ &\approx U(x) + U'(x)k\alpha + O\big(k^2;\epsilon,\alpha\big) \end{aligned}$$

Note that because the last term in the expected utility is quadratic in k, as long as $\alpha > 0$, there will be some (possibly very small) $k > 0$ such that the utility of acquiring k units of risky asset exceeds the utility of "staying put" with the certain amount x. Thus, regardless of risk aversion, the expected utility investor will purchase some amount of the risky asset with a positive risk premium. Now consider the same situation with RDU investor. RDU functional can be written using (1) as

$$\begin{aligned} V_{RDU}(x_k) &\approx u(x+k\alpha) - (2g(0.5)-1)u'(x+k\alpha)k\epsilon \\ &= u(x) + u'(x)k\{\alpha - 2(g(0.5)-0.5)\epsilon\} + O\big(k^2\big). \end{aligned}$$

[18] Related to this formulation of subjective weights is Gul's (1991) disappointment aversion theory. His model is very similar to some special cases of RDU and CPT when considered over binary gambles. Gul's model is simpler since it is designed to minimize deviations from EU theory and yet admit the behavior observed in Allais' paradox. In contrast, rank-dependent preference are not focused on just explaining Allais type of behavior and aim to accommodate more complex patterns in choice under risk.

[19] See Yaari (1987), Segal and Spivak (1990), and Quiggin (1993) who discuss the implications of first-and second-order risk aversion for portfolio choice and insurance.

If $\alpha < 2(g(0.5) - 0.5)\epsilon$, then the RDU investor optimally will hold none of the risky asset. Note that this inequality depends on the risk aversion of the investor via $g(\cdot)$. The higher is $g(0.5) - 0.5$, the less likely is the investor to purchase the risky asset. The reason for the different behavior predicted by the expected utility is that expected utility depends on the "riskiness" parameter ϵ only in the second- and higher order terms. In contrast, the RDU function depends on ϵ in the first- and higher order terms, hence the terms "second" and "first" order risk aversion.

2.2 Cumulative prospect theory (CPT)

One of the principal differences between other rank-dependent generalizations of expected utility and prospect theory is in the treatment of outcomes.[20] Rather than considering an absolute level of wealth, prospect theory borrows from the earlier idea of *customary wealth* suggested by Markowitz (1952). In line with experimental evidence, prospect theory assumes that agents value their wealth not in absolute terms, but *relative* to the *status quo*. The exact specification adopted in Tversky and Kahneman (1992) is a piecewise power function:

$$v(x) = \begin{cases} x^{\alpha}, & \text{if } x \geq 0 \\ -\lambda(-x)^{\beta}, & \text{if } x < 0, \end{cases}$$

where x is gain or loss relative to *status quo*. The parameter $\lambda > 1$ captures the effect known as *loss aversion*, which is another salient feature of individual choice. Gains and losses have asymmetric effects around the *status quo*: holding their absolute value constant, losses hurt more than gains help. Using experimental data, Kahneman and Tversky have estimated the parameters for this value function to be $\alpha = \beta = 0.88$ and $\lambda = 2.25$.

The value function for a complex gamble with multiple outcomes is a weighted average of the value function $v\,(x)$ over all possible gains (+) and losses (–):

$$V_{CPT} = \sum_{i \in \text{gains}} \pi_i^+ v(x_i) + \sum_{i \in \text{losses}} \pi_i^- v(x_i)$$

In their most recent version of CPT (1992), following the idea of RDU, the weights were defined as a function of the cumulative distribution over ranked outcomes.

Tversky and Kahneman (1992) defined weights as $\pi_i^{\pm} = w^{\pm}(P_i) - w^{\pm}(P_{i^*})$, where P_i is the probability of all outcomes at least as good (bad) as i and P_{i^*} is the probability of all outcomes i^* which are strictly better (worse) than i. They suggested the following functional form for $w^{\pm}(P)$:

[20] Axiomatic preference foundation for the CPT is provided recently by Wakker and Zank (2002).

$$w^{\pm}(P) = \frac{P^{\gamma}}{(P^{\gamma} + (1-P)^{\gamma})^{1/\gamma}} \tag{3}$$

and estimated γ to be 0.61 for gains and 0.69 for losses. For better analytical tractability in applications, the weights are often replaced by the probability weights, which correspond to $\gamma = 1$ in Equation (3). However, the weighting function plays an important role in prospect theory and, as in RDU, captures the variable attitude toward risk. As I show later, omitting this feature of preferences results in very different portfolio choices.

To illustrate the concept of varying risk attitude, Figure 2 shows certainty equivalents (CE) for two simple gambles with \$1 gain and \$1 loss as a function of probability p for four types of value functions: risk-neutral; risk-averse expected utility; the CPT with parameters mentioned above; and a special case of the CPT with $\gamma = 1$.[21] For gain lotteries, the RDU and the CPT functionals coincide if we define $u(c) = c^{\alpha}$ for RDU.

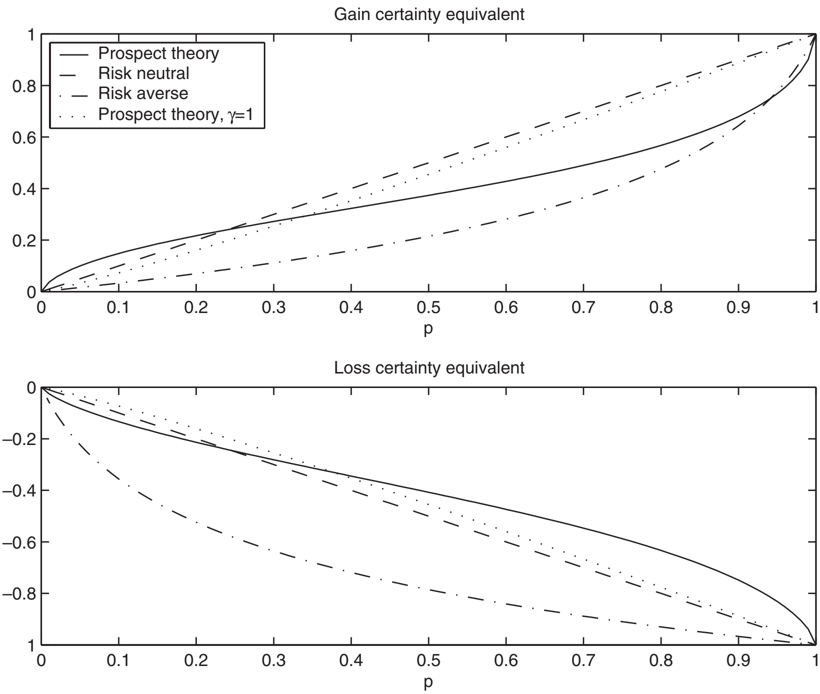

Figure 2
Certainty equivalents of simple lotteries with \$1 gain and \$1 loss with probability *p* for prospect theory, its special case with $\gamma = 1$ (linear probability weighting), risk neutral, and risk averse expected utility.

[21] I used the exponential expected utility $-e^{-\alpha x}$ with $\alpha = 3$ to illustrate the risk-averse CE.

Therefore, the top panel of Figure 2 also shows varying risk attitude for the RDU preferences. The risk-neutral CE is equal to the expected value of the lotteries which are the diagonals of the two boxes. The risk-averse CE is below the diagonals for gains and losses and for all probabilities. The risk-seeking CE (not shown) would always be above the risk-neutral diagonal. Note that a special case with linear decision weights ($\gamma = 1$) does not have probability-varying risk attitude because only the shape of $v(x)$ matters for risk attitude when the decision weights are linear.

As the probability p changes, the risk attitude of the investor with a rank-dependent value function may vary between risk aversion and risk taking. For CPT, in the domain of gains, the individual is risk taking for low probabilities and risk averse for high probabilities. In the domain of losses, it is the reverse: risk aversion for low probabilities and risk taking for high probabilities. Similar variation in risk attitude is found in other experimental studies, including experiments with high stakes such as in Kachelmeier and Shehata (1992) who describe experiments conducted in China with prizes as much as three times the monthly incomes of their subjects. Given such evidence, there is reason to believe that the behavior found in the experiments goes beyond playing lotteries and gambling in real life. A related example is pointed out by Lopes (1987), that farmers often plant safer crops to guarantee a certain minimum income and simultaneously plant crops with more uncertainty about their price but with the potential for high payoff.

Note that, as in the RDU case, the concept of first-order risk aversion applies to the CPT preferences as well but, for brevity sake, it will not be shown here. However, in the comparative statics section, I will demonstrate that there is a range of parameters for which the CPT implies optimal portfolios with no exposure to risky assets.

2.3 Diversification and preferences for skewness

It is possible to model the desire for a skewed distribution of wealth or consumption in the context of an expected utility with "preferences for skewness". It is important to understand that preferences for skewness are not identical with risk seeking and defined as a utility with non-negative third derivative $u'''(\cdot) \geq 0$. As noted by Kraus and Litzenberger (1976) and many other authors [e.g., Arditti (1967) and Tsiang (1972)], the utility function with non-negative third derivative has non-decreasing absolute risk aversion which implies that demand for risky assets is non-decreasing in wealth.[22] Kraus and Litzenberger (1976) used an unconditional CAPM, and Harvey and Siddique (2000) used a conditional CAPM to show that explicitly incorporating preferences for skewness by using cubic

[22] This feature was also linked to the prudence of preferences (Kimball, 1990) which gives rise to the precautionary savings motive.

approximations of expected utility greatly improves the fit of asset pricing models.[23] In view of the success, of the mean–variance-skewness utility in asset pricing, a natural question is: Can we use the approximate mean-variance-skewness utility as a replacement for the true underlying utility function in portfolio-choice problems? Also, since the cubic approximation is convex on the entire segment to the right of the approximation point, can one construct other utilities with only a *local* convexity to generate undiversified portfolios? For many reasons, the answer to both questions is no.

The main stumbling point of the expected utility model is the linearity in probabilities which is present in *all* expected utility functions. Because of that, any attempt to model risk seeking with expected utility requires (at least) local convexity in the utility function. Quiggin (1993, pp. 45–47) discusses the utility functions which combine convex and concave segments, such as the Friedman and Savage utility. The problem with such utilities is that in the convex region, the variance-increasing gambles improve utility, and the investors plunge their wealth entirely into the "most undiversified" portfolio available. When the most undiversified asset is a lottery ticket, as Quiggin (1993) shows, the investors gamble until their wealth is outside the convex region. Once the wealth is outside the convex region, the portfolio is well diversified and the investor does not want to gamble anymore. Clearly, the observed diversification does not switch from one extreme to the other—some risk taking is observed across all wealth groups, and there is typically no complete plunging of wealth into undiversified portfolios. Rank-dependent preferences avoid this chain of consequences by making the value function non-linear in probabilities and separating risk attitude from the marginal utility of wealth.

The cubic or mean-variance-skewness utility has the same problems as the locally convex utilities just discussed. In particular, if the approximate mean-variance-skewness utility is confronted with assets of high enough skewness, it predicts a 100% allocation to the most undiversified portfolio available. On the basis of simulated returns from the quantitative experiments in the next section and a cubic approximation to the CRRA utility function, the approximation predicts 100% investment in individual stocks, *regardless* of the value of the risk-aversion coefficient. This result is a consequence of sufficiently high skewness of individual stock returns in simulations combined with convexity in the mean-variance-skewness utility.[24]

[23] Cubic approximation for expected utility involves first three moments of the returns distribution: mean, variance, and skewness. It results in quadratic stochastic discount factor (SDF). In contrast, the standard linear SDF may be derived from the second-order approximation to the expected utility function (mean–variance utility).

[24] Particularly, for the simulated stock returns distribution and the third-order approximation to the CRRA utility function with any risk aversion, it is the case that $\frac{u'''(\bar{R})}{6}E\left(\tilde{R}-\bar{R}\right)^3 > \frac{u''(\bar{R})}{2}E\left(\tilde{R}-\bar{R}\right)^2$, where $\tilde{R}$ is the random stock return and $\bar{R}$ the expected stock return. At the same time, the skewness of the diversified investment (fund) is quite small and unattractive, so the fund is abandoned completely and the optimal allocation is 100% to stock, regardless of risk aversion.

3. Portfolio Choice with Rank-Dependent Preferences

In this section, I quantitatively evaluate the diversification of investors with rank-dependent preferences. We know that rank-dependent preferences have embedded risk taking. But is there "enough" risk taking so that, when captured by a reasonably calibrated choice problem, one can predict the portfolio diversification observed in the data? To impose discipline on the experiment, the parameters for the weighting functions vary over a range around the estimates obtained in the experimental decision studies, unrelated to portfolio choice.

3.1 Simulation of Returns Distributions

I consider simulations for the simplest set of assets that could potentially result in an undiversified portfolio: an equity fund, a single stock, and a T-bill. The choice of a stock or fund is not essential here. Neither is the fact that there is only one stock and only one fund. The central point is that portfolio choice is made by the investor with access to both diversified and undiversified investment opportunities. Whether the undiversified asset is a portfolio of a few stocks or only one is not important. Shefrin and Statman (2000) show theoretically that, when markets are complete, the optimal portfolios of investors with rank-dependent preferences are not diversified. Thus, enlarging the universe of assets is not going to change the conclusions—investors still would construct an "optimal asset" with idiosyncratic risk using available securities. To abstract from the complex process of selecting stocks and funds, I consider only one stock and one fund, both simulated as random selections. To create a returns distribution for the stock and fund, I use data from the Center for Research in Securities Prices (CRSP) 1926–1996 monthly returns file. I use both nominal and real returns to check robustness. Qualitatively, the results are not affected by this choice and, for brevity sake, I report only the results with nominal returns.

The stock return (r_s) is simulated as follows: First, I select a random month t in the interval from 01/1926 to 12/1996. Then, I select a security from among those traded during the 11 months following month t.[25] The security may be selected *equally likely* from those available (EL stock) or, as in Brennan and Torous (1999), with the probability proportional to the market value, a *value weighted* selection (VW stock). The latter is intended to model the choice of a company which is more known and visible. I then record the cumulative annual return on the security selected for months t through $t + 11$.

The fund return (r_i) is simulated as follows: For the same month t, 100 random stocks are chosen equally likely without replacement from among

[25] This is introducing a very small survivorship bias. The requirement for the security to be traded during the next year is not very restrictive, and the selection from the remaining companies is unbiased. When the investment horizon is longer, greater attention is paid to selection bias.

those traded during the 11 months following month t. For the months t through $t + 11$, the returns of the stocks are *value weighted* with the same weights constructed from the relative capitalizations in January of the year of month t. The 12-month cumulative value weighted return is then recorded as funds' return. Note that the fund always has value-weighted returns and the same fund return distribution is used with two different methods of simulating individual stock returns. The annual risk-free rate (r_f) is constructed from the yields on 30-day T-bills for month t and the 11 months following. Thus, r_f simulates the annual yield for a money market fund.

I simulate 200,000 random triples of returns $r = (r_s, r_i, r_f)$. The average (*nominal*) returns are [12.7% (16.6%), 12.9%, 4.0%], where the number in brackets corresponds to the EL stock. The standard deviations of returns are [36.0% (63.7%), 24.0%, 3.4%]. The Sharpe ratios for the two types of stock selections and for the fund are [0.24 (0.19), 0.37], and the correlations of the stocks with the fund are [0.60 (0.46)]. The stocks have much lower Sharpe ratios, their returns are much more volatile, and they have more extreme realizations in the tails of the distributions. Figure 3 shows the histograms for the simulated distributions of the returns on risky

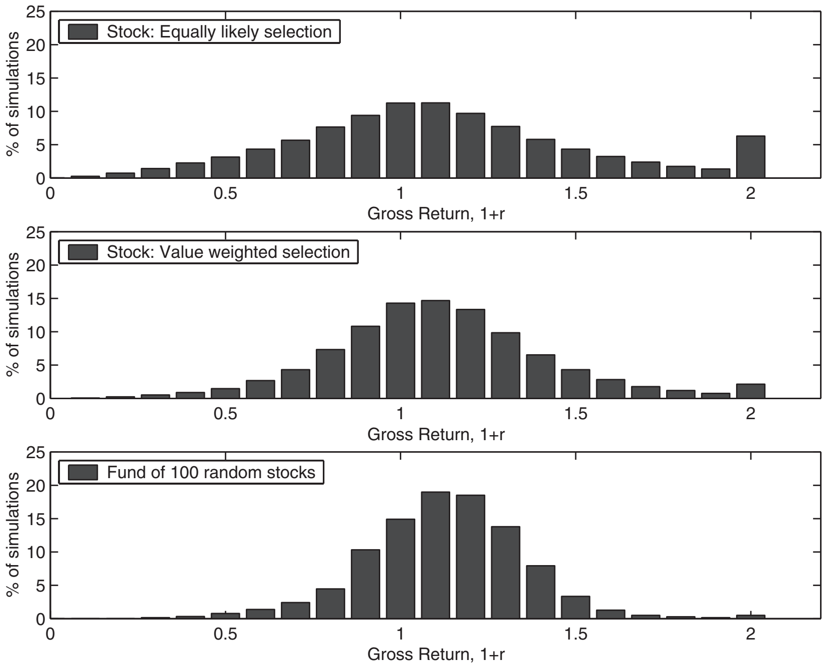

Figure 3
Histograms of the log returns for the simulated distributions of risky assets
Top panel—equally likely stock selection, middle panel—value-weighted stock selection, and lower panel—fund of 100 randomly selected companies (value-weighted returns).

assets. Note that VW stock and EL stock are markedly different. Since small capitalization companies are represented more in the EL stock, it has a higher mean return and higher standard deviation.

Simulated returns then are used to construct distributions of total returns for various portfolio weights $w_p = (w_s, w_i, w_f)$ subject to the constraint $w_s + w_i + w_f = 1$ in $[0, 1] \times [0, 1] \times [0, 1]$. I use a grid with 2% increments over the admissible portfolio weights. Since the value functions in simulations are homogeneous in wealth, I evaluate them on portfolio returns $r_p = r \cdot w_p'$ for CPT and on gross returns $1 + r_p$ for RDU. For each set of weights w_p, I construct a simulated portfolio return distribution by weighting simulated return triples. A cumulative distribution is computed on the grid with 2% increments and 200 points, starting with –100% (total wealth loss).

To check the robustness of the results for the CPT investors, I also repeat the simulations using a five-year horizon. The assumptions underlying the simulations make the stock less attractive and the fund more attractive. As before, I simulate returns using the 1926–1996 CRSP monthly file. But for a five-year horizon, the returns on individual stocks often become unavailable. When the return becomes unavailable, with probability 0.5, I replace it with another return, selected randomly, and with probability 0.5, I set it to –1 (total loss). This "bankruptcy" procedure is conservative in that it makes stock less attractive. About half of the delistings in the data are related to mergers and issue exchanges, and the other half are delisted for other reasons.[26] When a company is withdrawn from public trading, the return is not always –1, as assumed; shareholders still may be able to recover some value. On the other hand, the fund return is simulated so that the fund is more attractive because of survivorship bias: stocks in the fund that become unavailable always were replaced with new random selections.

3.2 Portfolio choice of the RDU investors

To conform with experimental studies of RDU, I use a power (CRRA) specification for the utility of outcomes:

$$u(w) = \frac{w^{1-\alpha}}{1-\alpha},\ \alpha > 0,$$

and the weighting function given in Equation (3). Experimental studies by Camerer and Ho (1994), Tversky and Kahneman (1992) and Wu and Gonzales (1996) have estimated the parameters of rank-dependent value

[26] Specifically, I counted as "bankruptcy" the delisting codes 400 (liquidation), 500 (stopped trading, unknown reason), 550 through 588 (dropped from trading for reason other than to migrate to the other exchange), and 700 (delisted by SEC). The number of such delistings in the 1997 CRSP file is about 6000, about half of all delistings, hence the use of probability 0.5 in the random replacement rule.

functions. They used simple binary gambles to solicit certainty equivalents. Since RDU and CPT formulations coincide for the two-outcome gain lotteries, one can use a range of values around the estimates from all of these studies to investigate the potential of the RDU to account for undiversified portfolios. The range of point estimates of the weighting function parameter γ is [0.56, 0.74] and the power parameter $1 - \alpha$ is in the range [0.32, 0.88] or $\alpha \in [0.12, 0.68]$. Such low-power parameters correspond to nearly risk-neutral behavior. Partly this may be due to the fact that gains-only gambles in the experiments involve relatively small stakes. For numerical simulations, to evaluate comparative statics and robustness, I allow somewhat wider ranges: $\gamma \in [0.2, 1]$ and $\alpha \in [0, 4]$.

Figure 4 shows the *optimal* portfolio fractions allocated to stock and fund for various parameters of the value function. The top panels show the optimal allocations for the portfolios with the VW stock and the bottom panels with the EL stock. When $\gamma = 1$, the simulations correspond to the expected utility case (with preferences for skewness $u'''(\cdot) > 0$). The expected utility portfolios consist only of the fund and the risk-free asset. Only in the EL stock case when risk aversion is well below 1, close to the

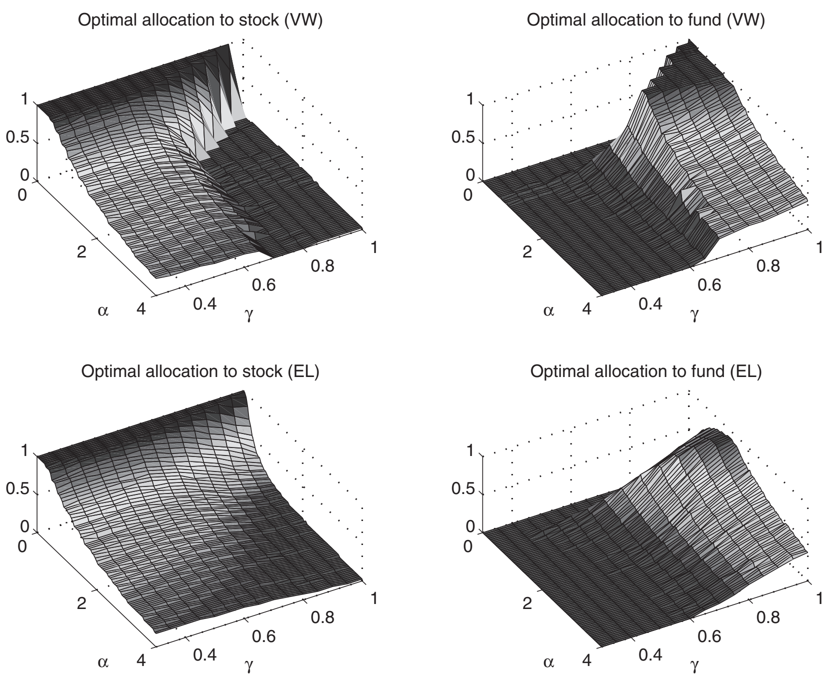

Figure 4
Optimal portfolio allocations for various parameters of the rank-dependent expected utility
Portfolios consist of a single stock (either value weighted (VW) or equally likely (EL) selection), an index fund of 100 randomly selected companies (value-weighted returns) and a T-bill.

risk neutral case, is there some significant investment in the individual stock.[27] This only happens for the EL stock because its expected return is higher than that of the fund. For the VW stock, the expected utility portfolio is well diversifed regardless of risk aversion.

As γ declines, the decision-weighting function acquires a more pronounced inverted S-shape. This introduces risk seeking for low-probability favorable events but maintains risk-averse behavior for low-probability unfavorable events. One can see that, in the range of γ from the experimental studies (0.5–0.8), the optimal portfolios have a significant fraction invested in the stock. The fraction allocated to stock and total equity declines naturally with the utility curvature parameter α which controls the marginal utility of wealth. The decision-weighting function parameter γ controls the composition of the equity portfolio by changing the degree of risk seeking. For higher values of γ, the equity portion of the optimal portfolio is tilted to the fund; for the lower values of γ, it is tilted to the stock. When $\gamma = 0.6$, approximately in the middle of the experimental estimates, the allocation to the VW stock ranges from 65 to 17% as α goes from 1 to 4. At the same time fund allocation is about one-third of that and varies from 17 to 5%, and the allocation to the risk-free asset varies from about 20 to 80%. When $\alpha < 0.5$, approaching constant marginal utility of wealth, the optimal portfolios quickly become the least diversified possible.

Since the EL stock has a "thicker" right tail, the risk taking is more pronounced for the EL stock than for the VW stock. For any given value of α, as γ declines from 1, the allocation to the EL stock picks up earlier, at the higher values of γ, than for the VW selection. For any given set of parameters, the equity portion of the optimal portfolio with the EL stock is also more tilted toward the stock than for the VW stock. Again, when $\gamma = 0.6$, in the EL stock selection, the allocation to stock varies from 60 to 15%, and the allocation to fund varies from 10 to 3% as curvature α goes from 1 to 4. Thus, for $\gamma = 0.6$, the fund is only 14–17% of the total equity allocation of the portfolio with the EL stock, while with the VW stock the fund's fraction of total equity is about 20–24%.

Note that because in the simulated distribution the risk free return was volatile, it was not possible to generate exactly zero equity investments for the range of parameters considered here. If the volatility of the risk-free return were zero (e.g., when the asset's maturity matches investor's horizon), non-participation in equity would occur for reasonable values of the power parameter. In the simulations with CPT, non-participation in the

[27] Even if (nearly) risk-neutral expected utility preferences are accepted as a defense, there is a problem with such explanation. Risk neutrality *jointly* implies high total allocation to equities and within equities high allocation to the individual asset with the highest expected return (which could be a stock). However, the observed portfolios are less extreme than risk neutrality would suggest. Total exposure to stocks is not necessarily very aggressive but *within* equity portion the portfolios are tilted toward undiversified assets. While this pattern naturally emerges with rank-dependent preferences, the expected utility with near risk neutrality can not replicate it.

equity market is more pronounced because of the added feature of loss aversion. Non-participation with CPT occurs in a wider range of parameters because the risk-free asset never entails losses. Overall, the RDU model suggests that substantial allocation to undiversified individual assets is consistent with the experimentally estimated decision-weighting functions. While the total exposure to equities is controlled primarily by the power parameter α, when γ is in the range of experimental point estimates, the equity part of the optimal portfolio is tilted toward individual stock.

3.3 Portfolio choice of the CPT investors

There are two important assumptions that have to be made to implement the simulations for the CPT investor: one about the horizon over which to compute gains and losses, and another about the accounting method for gains and losses. Benartzi and Thaler (1995) argue that the relevant horizon is somewhere between 8 and 13 months. They find that the CPT investors who evaluate their portfolios approximately once a year would be indifferent between holding an equity index and a risk-free bond. Annual evaluation seems most natural for individual investors, and the results of Benartzi and Thaler (1995) support this view. I use an annual horizon first; then, to check for robustness, I also compute the portfolios selected by an investor with a five-year horizon.

Another important component of the CPT is the accounting method for gains and losses. There are two possible accounting methods: the single-account method considers gain or loss of total wealth; the multiple-account method separately tracks each asset in the port-folio. Here I use the single-account method. The most important motivation for this is that to calibrate the model I would like to use the preference parameters obtained in experiments. Multiple-account preferences require the specification of additional structure with no experimental data to guide calibration.[28]

First I consider the simulations with parameter estimates obtained by Tversky and Kahneman (1992). In addition, I compute the values of CRRA expected utility with the risk-aversion coefficient of 2. Figure 5 (top left panel) shows the CPT value function for portfolios with the VW stock, fund and T-bill. The maximum value is achieved at approximately 22% invested in the stock, 12% in the fund, and the rest in T-bills. The equity component of the optimal portfolio is invested 64.7% in the stock and 35.3% in the fund. The top right panel of Figure 5 shows the values of expected utility for the same portfolios. Expected utility dictates optimal investment in the fund (about 80%) and the risk-free asset.

[28] Shefrin and Statman (2000) show theoretically that within their framework, the single-account and the multiple-account methods produce portfolios similarly undiversified. Although the preferences in their paper are not identical to the CPT, their result supports the belief that using the multiple-account method would not change any of the conclusions. The key feature of the CPT preferences which generates under-diversification is rank dependency, not the accounting method for gains and losses.

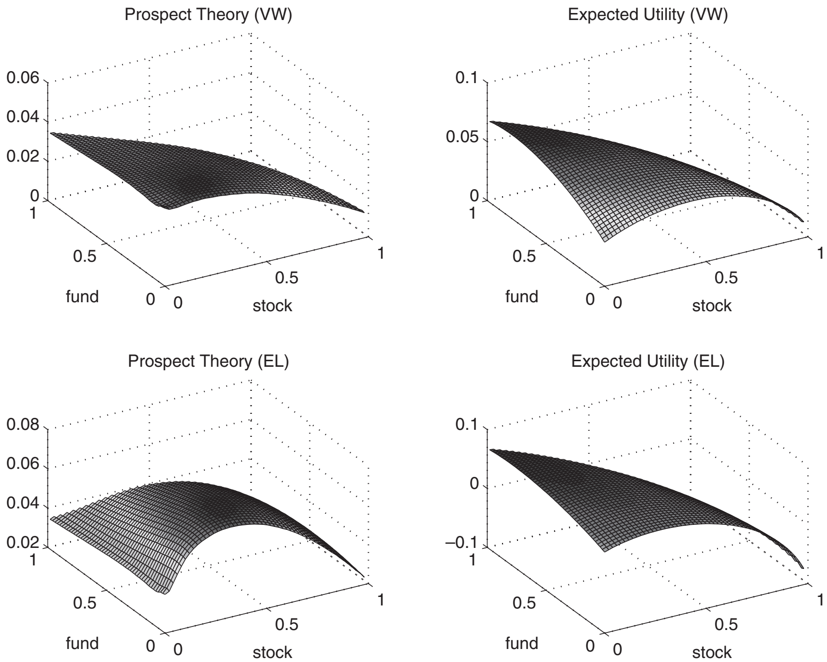

Figure 5
Prospect value function and expected utility
Portfolios consist of a single stock (either value weighted (VW) or equally likely (EL) selection), an index fund of 100 randomly selected companies (value-weighted returns) and a T-bill. Prospect value function parameters are from Tversky and Kahneman (1992). Expected utility value functions assume CRRA utility with the risk-aversion coefficient of 2.

The lower left panel of Figure 5 shows the results of the same exercise, but with the EL stock. Now the CPT investor holds even more stock than for the VW stock. The optimal portfolio is located at approximately 38% in the stock, 18% in the fund, and the rest in T-bills. The equity portfolio is invested 67.9% in the stock and 32.1% in the fund. The lower right panel of Figure 5 shows the values of expected utility for the same portfolios and, again, it is optimal to invest primarily in the fund and the risk-free asset.[29]

Figure 5 illustrates the key driving force behind portfolio selection of the CPT investors. Recall that the CPT investors are attracted to gains with small probabilities. It is especially evident in EL stock, because there the simulation is more affected by high outlier returns on small capitalization firms. The optimal portfolio is invested partially in the

[29] Some small amount of EW stock is desirable under EU due to diversification across stock and fund, but the fraction of stock is very small, under 4%.

stock because the stock also has a possibility for substantial loss. Recall that the CPT investors are risk averse over losses of small probabilities. Thus, for a CPT investor, the portfolio choice is a tradeoff between high outlier returns from undiversified part of the portfolio and the potential losses which have to be insured by holding diversified assets.

3.3.1 Comparative statics of the CPT-optimal portfolios. I now perform comparative static analysis of the optimal portfolios for the CPT investors using the range of estimates from Camerer and Ho (1994), Kahneman and Tversky (1992), and Wu and Gonzales (1996). Because the outcome value function in the CPT has to be finite at zero, I consider only the power function x^{α} where $\alpha \in [0,1]$. Since the experimental studies typically use only gain lotteries for estimation, the parameters for losses and gains have to be assumed equal. I impose this restriction in the comparative statics simulations and discuss it later in more detail. I also expand somewhat the domain of parameters to provide a sense of stability of the results. I consider the ranges of $\gamma \in [0.25, 1]$ and $\alpha \in [0.2, 1]$. There is less information on the loss-aversion parameter λ since loss gambles are relatively harder to stage in experiments. Given that λ is less critical for risk seeking, I do not vary λ in the reported simulations and I set it to 2.25, as before.

Figure 6 shows the optimal allocations to stock and fund for various α and γ. The top two panels show the optimal allocations to stock and fund for the portfolios with the VW stock. The bottom two panels show the same allocations for the portfolios with the EL stock. As the curvature parameter α declines, the portfolios become more conservative. With respect to γ, the allocations are not monotonic. For γ close to 1, the equity part of the portfolios is mostly invested in the fund, but it may contain some small amount of stock. That occurs when loss aversion is the sole driving force behind portfolio selection, since the value function is weighted almost linearly, as for expected utility. For tractability in applications, it is often assumed that $\gamma = 1$, in which case there is no probability varying risk attitude in preferences. The simulations show that portfolio choice is very sensitive to this simplification and that optimal portfolios are well diversified while this is not the case in a general specification.

As γ and/or α decline, both risky assets disappear completely from the optimal portfolios. There is relatively more non-participation in equity for the VW stock than for the EL stock: the EL stock is more attractive because of the heavier right tale of its return distribution. Thus, for some plausible parameter values, the CPT is consistent with the observations of non-participation in equity markets regardless of an investor's wealth. Note also that the estimates of Camerer and Ho (1994) ($\alpha = 0.32$ and $\gamma = 0.56$) and Wu and Gonzalez (1996) ($\alpha = 0.52$ and $\gamma = 0.74$) correspond to non-participation in the equity market for either VW or EL stock. Nelson and

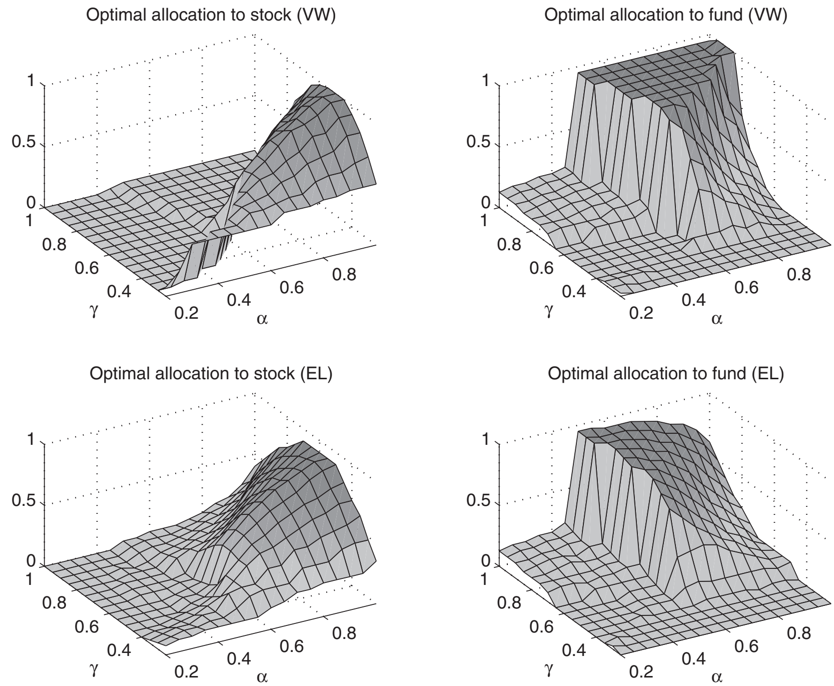

Figure 6
Optimal portfolio allocations for various parameters of the prospect theory value function
Portfolios consist of a single stock (either value weighted (VW) or equally likely (EL) selection), an index fund of 100 randomly selected companies (value-weighted returns), and a T-bill.

Stowe (2002) also note that these parameters "imply virtually no risk seeking over unlikely gains and no risk aversion over unlikely losses [...] so that individuals will purchase neither lottery tickets nor insurance." For sufficiently low γ, risk seeking is strong enough, and the equity portfolio becomes tilted toward the stock. In general, undiversified portfolios containing a significant fraction invested in the stock are optimal for α near one.

It should be noted that the comparative statics shown on Figure 6 do not subsume the results from the CPT simulations using the parameters of Tversky and Kahneman (1992). In the comparative statics, there are constraints on parameters which introduce potential problems, so a word of caution is in order. Unfortunately, there is no alternative experimental data to guide the comparative statics for the fully specified five-parameter version of the CPT. The experimental estimates, other than in Tversky and Kahneman (1992), do not correspond to the fully specified CPT and are based only on the "gains" side of the theory which requires only two parameters. Because of these limitations, all parameters for losses in the comparative statics simulations are forced to be the same as for gains. Thus, when risk seeking for gains of small probabilities is increasing (γ_g), the risk aversion over losses of small probability (γ_l) is increasing at the same time and at the same rate. While this appears to be a substantial constraint for the CPT, searching over a five-dimensional grid without further guidance from experimental work is hardly a productive exercise. For this reason, more weight should be given to the comparative statics with RDU which does not require asymmetry between losses and gains and to the results in the previous section with a fully-specified CPT. Moreover, Nelson and Stowe (2002) argue that the functional form of the decision weights in the CPT may be questioned and conclude that alternative functional forms have to be explored. Thus, as it stands, the CPT is far from being refined by the experimental choice literature, and more tests and estimations with experimental and non-experimental data will be needed.

3.3.2 On the effects of longer horizon and background risk. The horizon over which losses and gains are evaluated may affect the CPT-optimal portfolios. Previously shown simulations assume that portfolio performance is evaluated on an annual basis (myopically). Benartzi and Thaler (1995) show that myopic CPT investors put less in equities than expected utility maximizing investors because of the short-run risk of loss. They also show that myopic CPT investors reverse their decisions: as the evaluation period increases, the investors become "more aware" of the equity premium and their optimal portfolios move toward all equity. There may be some basis for believing that diversification improves as the evaluation horizon lengthens because individual stocks have an increasing probability of complete loss over long horizons.

Using the simulations for a five-year horizon and Kahneman and Tversky parameters, I find that, consistent with results of Benartzi and Thaler (1995), the optimal portfolio of the CPT investor becomes 100% equity. However, the portfolio is still undiversified. The optimal portfolios are about 50% in the fund and 50% in the stock, both for the VW and for the EL stock simulations. Thus, increasing the evaluation horizon for the CPT investor does not improve diversification. The fraction of wealth invested in the stock is higher than with a one-year horizon because with a five-year horizon the insurance portion of the portfolio appears to be better for the CPT investors. With a longer horizon, the CPT investors recognize potential long-run gains from the equity premium and completely abandon the risk-free asset. As a result, the expected return on the diversified (insurance) part of the portfolio goes up and the CPT investor can afford more of the undiversified asset.

A final remark related to horizon is in order. The portfolio strategies considered in the simulations are simple "buy and hold" rules. Although this may be a sensible assumption for a short horizon, actual portfolio dynamics with rank-dependent preferences are likely be more complicated for longer horizons. A formal investigation of this topic is beyond the scope of the present paper. In a dynamic environment with rank-dependent preferences, the extent of diversification is likely to vary over time, but as long as the preferences have some risk seeking, undiversified assets will be attractive.

Another interesting issue is the effects of background risk, for example, from labor income, on portfolio diversification. While the static model is not able to address fully the range of questions associated with background risk, some preliminary experiments can be done here. I have repeated the simulations of comparative statics with RDU and CPT models augmented to include financial and non-financial wealth (human capital). Human capital was assumed to be non-tradable with idiosyncratic standard deviation of 10% per year.[30] The model was simulated for several cases where the weight of the non-tradable component in total wealth ranged from 0.1 to 0.9. This variation captures the effect of age on portfolio with higher weight corresponding to younger investors who have most of their wealth in the form of human capital. The main findings from the augmented model can be summarized as follows. As non-financial wealth becomes more significant, financial portfolios shift toward equities. For investors with human capital weight above 0.4–0.5, the allocations of financial wealth typically range from 100% to fund to 100% in stock with virtually no investments in the risk-free asset. However, as before, portfolios remain undiversified for a wide range of utility parameters. The tilt toward equity is explained by the fact that idiosyncratic risk of human capital is uncorrelated with equity returns. Additionally, for higher fractions of human wealth, non-participation in equities observed earlier in the CPT case disappears. This suggests that the explanation of non-participation in equity market based on first-order risk aversion may be more relevant for the older and/or wealthier investors for whom financial portfolios represent the bulk of their wealth. This effect goes in the "right direction" because for wealthier investors the explanation of nonparticipation based on fixed costs is not plausible. Some interesting questions related to background risk require a dynamic model and remain for future research, for example, the variation of diversification in the presence of liquidity constraints and the effects of correlation of transitory income shocks and stock returns.

[30] This figure is consistent with empirical estimates from PSID for volatility of permanent income shocks reported in Cocco, Gomes, and Maenhout (2004). The expected return on human wealth was assumed to be zero, that is, the stock of human capital may simply go up or down by 10%. Assuming positive human capital return/growth (as would be the case for young investors) makes financial portfolios even more aggressive over all range of parameters.

4. Conclusion

While the decision-theoretic literature acknowledges the importance of rank dependency in preferences, this is driven mainly by the experimental evidence. Financial economics has not yet embraced this new type of preferences, partly because their relevance in applications has not been shown. This paper identifies individual portfolio choice as an area where the complexity of rank-dependent preferences is justified by a significant improvement upon the predictions of the expected utility. This calls for further integration of rank-dependent preferences in portfolio theory and asset pricing. Shefrin and Statman (2000) propose a portfolio choice model which is one step in this direction, but more research is needed to better understand such models. The implications for portfolio choice in incomplete markets and the effects of investor heterogeneity would be particularly interesting. In asset pricing, Epstein and Zin (1991) have used rank-dependent preferences to address the equity premium puzzle in a representative agent setting. More research in this direction would help understand the aggregation of such preferences in heterogeneous agents economies. On the experimental side, more information is needed for a realistic calibration of preferences with probability-dependent risk attitude. Aside from estimating parameterizations of the existing models, more work is needed to explore alternative functional forms.

The preference-based explanation is certainly not the only possible reason for the lack of diversification. However, varying risk attitude is one of the most persistent features of human decision-making and cannot easily be dismissed as a motive for holding undiversified portfolios. Individual stocks, while probably the most common, is only one among many ways to reduce diversification. As mentioned earlier, there are several types of lottery bonds and synthetic securities combining bonds and options that are not mean–variance efficient. Such examples, along with household portfolio data, suggest that the desire for a "long shot" at getting rich affects every day investors just as much as it affects casino gamblers, lottery players, and the subjects in experimental decision studies. This should not be surprising; after all, once gamblers are done gambling, lottery players have torn unlucky tickets, and the experimental study subjects answered their questionnaires, they all go home and decide how to invest their savings.

References

Abdellaoui, M., 2002, "A Genuine Rank-Dependent Generalization of the Von Neumann-Morgenstern Expected Utility Theorem," *Econometrica*, 70, 717–736.

Allais, M., 1979, "The Foundations of a Positive Theory of Choice Involving Risk and a Criticism of the Postulates and Axioms of the American School," in M. Allais, and O. Hagen (eds), *Expected Utility Hypothesis and the Allais' Paradox; Contemporary Discussions of Decisions under Uncertainty with Allais' Rejoinder*, pp. 25–145, Reidel, Dordrecht. Netherlands.

Arditti, F. D., 1967, "Risk and the Required Return on Equity," *Journal of Finance*, 22, 19–36.

Arzac, E. R., 1974, "Utility Analysis of Chance-Constrained Portfolio Selection," *Journal of Financial and Quantitative Analysis*, 8, 993–1007.

Arzac, E. R., and V. S. Bawa, 1977, "Portfolio Choice and Equilibrium in Capital Markets with Safety-First Investors," *Journal of Financial Economics*, 4, 277–288.

Barber, B., and T. Odean, 2001, "Boys Will Be Boys: Gender, Overconfidence, and Common Stock Investment," *Quarterly Journal of Economics*, 116, 261–292.

Barberis, N., M. Huang, and T. Santos, 2001, "Prospect Theory and Asset Prices," *The Quarterly Journal of Economics*, 116, 1–54.

Battalio, R. C., J. H. Kagel, and K. Jiranyakul, 1990, "Testing Between Alternative Models of Choice under Uncertainty: Some Initial Results," *Journal of Risk and Uncertainty*, 3, 25–50.

Bawa, V. S., 1978, "Safety-First, Stochastic Dominance, and Optimal Portfolio Choice", *Journal of Financial and Quantitative Analysis*, 13, 255–271.

Benartzi, S., 2001, "Excessive Exptrapolation and the Allocation of 401(K) Accounts to Company Stock," forthcoming in *Journal of Finance*, 56, 1747–1764.

Benartzi, S., and R. H. Thaler, 1995, "Myopic Loss Aversion and the Equity Premium Puzzle," *The Quarterly Journal of Economics*, 110, 73–92.

Benartzi, S., and R. H. Thaler, 2001, "Naive Diversification Strategies in Retirement Saving Plans," *American Economic Review*, 91, 79–98.

Blume, M. E., and I. Friend, 1975, "The Asset Structure of Individual Portfolios and some Implications for Utility Functions," *Journal of Finance*, 30, 585–603.

Brennan, M. J., and W. N. Torous, 1999, "Individual Decision-Making and Investor Welfare," *Economic Notes*, 28.

Camerer, C. F., 1995, "Individual Decision Making," in J. H. Kagel, and A. E. Roth (eds), *Handbook of Experimental Economics*, pp. 587–703, Princeton University Press, Princeton, NJ.

Camerer, C. F., and T. Ho, 1994, "Violations of the Betweenness Axiom and Nonlinearity in Probability," *Journal of Risk and Uncertainty*, 8, 167–196.

Chew, S., and K. MacCrimmon, 1979, "Alpha-Nu Choice Theory: A Generalization of Expected Utility Theory," *Working Paper 669, University of British Columbia.*

Choi, J. J., D. Laibson, B. C. Madrian, and A. Metrick, 2001a, "Defined Contribution Pensions: Plan Rules, Participants Choices, and the Path of Least Resistance," *NBER Working Paper Series 8655*.

Choi, J. J., D. Laibson, B. C. Madrian, and A. Metrick, 2001b, "For Better or For Worse: Default Effects and 401(K) Savings Behavior," *NBER Working Paper Series 8651*.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout, 2004, "Consumption and portfolio choice over the life-cycle," forthcoming in *The Review of Financial Studies*.

Epstein, L. G., and S. E. Zin, 1989, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Epstein, L. G., and S. E. Zin, 1991, "'First-Order' Risk Aversion and the Equity Premium Puzzle," *Journal of Monetary Economics*, 26, 387–407.

Friedman, M., and L. J. Savage, 1948, "The Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, 56, 279–304.

Goetzmann, W. N., and A. Kumar, 2001, "Equity Portfolio Diversification," *NBER Working Paper Series No. 8686*.

Gomes, F., and A. Michaelides, 2005, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," forthcoming in *Journal of Finance*, 60, 869–904.

Gomes, F. J., 2003, "Loss Aversion and the Demand for Risky Assets," *forthcoming in Journal of Business*.

Green, R., and K. Rydqvist, 1999, "Ex-Day Behavior with Dividend Preference and Limitations to Short-Term Arbitrage: The Case of Swedish Lottery Bonds," *Journal of Financial Economics*, 53, 145–187.

Grinblatt, M., and M. Keloharju, 2001, "How Distance, Language, and Culture Influence Stockholdings and Trades," *Journal of Finance*, 56, 1053–1073.

Gul, F., 1991, "A Theory of Disappointment Aversion," *Econometrica*, 59, 667–686.

Harvey, C. R., and A. Siddique, 2000, "Conditional Skewness in Asset Pricing Tests," *Journal of Finance*, 55 (3), 1263–95.

Heath, C., and A. Tversky, 1991, "Preference and Belief: Ambiguity and Competence in Choice under Uncertainty," *Journal of Risk and Uncertainty*, 4, 5–28.

Huberman, G., 2001, "Familiarity Breeds Investment," *Review of Financial Studies*, 14, 659–680.

Kachelmeier, S. J., and M. Shehata, 1992, "Examining Risk Preferences under High Monetary Incentives: Experimental Evidence from the People's Republic of China," *American Economic Review*, 82, 1120–41.

Kelly, M., 1995, "All their Eggs in One Basket: Portfolio Diversification of U.S. Households," *Journal of Economic Behavior and Organization*, 27, 87–96.

Kimball, M., 1990, "Precautionary Savings in the Small and in the Large," *Econometrica*, 58, 53–73.

Kraus, A., and R. Litzenberger, 1976, "Skewness Preferences and the Valuation of Risk Assets," *Journal of Finance*, 31, 1085–1100.

Liang, N., and Weisbenner, S., 2002, "Investor Behavior and the Purchase of Company Stock in 401(k) Plans - The Importance of Plan Design", working paper series 2002-36, Board of Governors of the Federal Reserve System (U.S.).

Lopes, L. L., 1987, "Between Hope and Fear: The Psychology of Risk," in L. Berkowitz *et al.* (eds), *Advances in Experimental Social Psychology*, pp. 255–295, Academic Press, San Diego, CA, USA.

Machina, M. J., 1982, "Expected Utility Without the Independence Axiom," *Econometrica*, 50, 277–323.

Markowitz, H., 1952, "The utility of wealth," *Journal of Political Economy*, 60, 151–158.

Marschak, J., 1938, "Money and the Theory of Assets," *Econometrica*, 6, 311–325.

McConnell, J., and E. Schwartz, 1992, "The Origin of LYONs: A Case Study in Financial Innovation," *Journal of Applied Corporate Finance*, 4, 40–47.

Merton, R. C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42, 483–510.

Mitchell, O. S., and S. P. Utkus, 2002, "Company Stock and Retirement Plan Diversification," *Pension Research Council Working Paper Series (PRC WP 2002-4)*.

Nelson, W., and J. Stowe, 2002, "A Further Examination of Cumulative Prospect Theory Parameterizations," *The Journal of Risk and Uncertainty*, 24 (1), 31–46.

Neumann, J. V., and O. Morgenstern, 1944, *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, NJ.

Odean, T., 1999, "Do Investors Trade Too Much," *American Economic Review*, 89, 1279–1298.

Pyle, D. H., and S. J. Turnovsky, 1970, "Safety-First and Expected Utility Maximization in Mean-Standard Deviation Portfolio Analysis," *The Review of Economics and Statistics*, 52, 75–81.

Quiggin, J., 1982, "A Theory of Anticipated Utility," *Journal of Economic and Behavioral Organization*, 3, 323–343.

Quiggin, J., 1993, *Generalized Expected Utility Theory: The Rank-Dependent Model*, Kluwer: Norwell, MA, USA.

Roy, A. D., 1952, "Safety-First and the Holding of Assets," *Econometrica*, 20, 431–449.

Segal, U., and A. Spivak, 1990, "First Order versus Second Order Risk Aversion," *Journal of Economic Theory*, 51, 111–25.

Shefrin, H., and M. Statman, 2000, "Behavioral Portfolio Theory," *Journal of Financial and Quantitative Analysis*, 35, 127–51.

Shoemaker, P., 1982, "The Expected Utility Model: Its Variants, Purposes, Evidence and Limitations," *Journal of Economic Literature*, 20, 529–563.

Starmer, C., 2000, "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature*, 38, 332–382.

Starmer, C., and R. Sugden, 1989, "Violations of the Independence Axiom in Common Ratio Problems: An Experimental Tests of some Competing Hypotheses," *Annals of Operations Research*, 19, 79–102.

Telser, Lester, 1995, "Safety First and Hedging", *Review of Economic Studies*, 23, 1–16.

Tsiang, S., 1972, "The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference, and the Demand for Money," *American Economic Review*, 62 (3), 354–71.

Tversky, A., and D. Kahneman, 1992, "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323.

Wakker, P. P., and H. Zank, 2002, "A Simple Preference Foundation of Cumulative Prospect Theory with Power Utility," *European Economic Review*, 46, 1253–1271.

Wu, G., and R. Gonzalez, 1996, "Curvature of the Probability Weighting Function," *Management Science*, 42, 1676–1690.

Yaari, M. E., 1987, "The Dual Theory of Choice under Risk," *Econometrica*, 55, 95–115.