

Fund Families as Delegated Monitors of Money Managers

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Because a money manager learns more about her skill from her management experience than outsiders can learn from her realized returns, she expects inefficiency in future contracts that condition exclusively on realized returns. A fund family that learns what the manager learns can reduce this inefficiency cost if the family is large enough. The family's incentive is to retain any given manager regardless of her skill but, when the family has enough managers, it adds value by boosting the credibility of its retentions through the firing of others. As the number of managers grows, the efficiency loss goes to zero.

1. Introduction

Open-end mutual funds intermediate much of the investment in financial securities. A number of papers have examined the potential moral hazard and information asymmetry problems that can arise between the investors and the portfolio managers of these funds (just *managers*, from here on).¹ However, a manager does not usually work directly for investors, but rather for a mutual-fund family like Fidelity. The existence of this

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¹ Important papers include Bhattacharya and Pfleiderer (1985), Starks (1987), Kihlstrom (1988), Huberman and Kandel (1993), Heinkel and Stoughton (1994), Admati and Pfleiderer (1997), Huddart (1999), and Das and Sundaram (2002).

additional layer of agency is a significant complication whose economic role, other than possibly economies of scale and scope, has to date been somewhat of a puzzle.

We provide a resolution that develops the intuition that investors learn less about manager quality than managers can learn themselves. In such a setting, we argue, the fund family can act as a validation mechanism for the manager by conveying to investors the manager's own information about her skill, which she could not otherwise convey. But just as all managers have an incentive to say that they are skilled, fund families have an incentive to say that all the managers they hire are skilled. So why would investors believe fund families any more than managers? We show how mutual-fund families can credibly convey manager quality with a commitment to fire a return-dependent fraction of managers. Investors know that when it comes time to bite the bullet, the fund family will fire the worst managers first. The remaining managers are thus of better expected quality, and an investor can use this information to make a better-informed decision about whether to invest in a fund or not. The family does not completely solve the problem since the fraction to be fired is determined *ex ante* and can differ from the *ex-post* fraction that is unskilled. But as the number of funds monitored by the family grows, the per-manager likelihood of a difference goes to zero and, in the limit, managers are fired if and only if they are unskilled, as a first-best scenario would dictate.

Our analysis centers around a two-period model of the money-management problem with risk-neutral investors, managers, and monitors. We assume that the economic surplus created by money management is all captured by managers and fund families while investors only earn normal returns. While in principle this surplus could be shared with investors, our assumption follows from the work of Berk and Green (2002). In their model, the scarcity of investment skill causes capital to flow to investment opportunities until they are completely dissipated, which allows the manager to capture all the surplus. We also assume that everybody has the same information about manager quality at the outset but that managers, through their trading experience, learn their skills faster and more precisely than investors do. The latter part of this assumption is key. It captures the fact that managers know about many dimensions of their trading strategies that investors can only imperfectly infer. For example, managers know why they take their positions, they know the positions they choose not to take, and they know the precise timing and size of each of their trades. In the same spirit, at the end of the model's first period, investors update based on the fund's first-period return, while the manager learns whether she is skilled or not.

In the absence of monitoring, an adverse selection problem emerges since a manager who generates a high return by luck, rather than skill, has

no incentive to reveal this to investors. Instead, she pools with the skilled managers, who have no credible way of signaling their skills, and she is consequently rehired by investors.² Similarly, a manager who experiences a low return because of bad luck, not bad skill, cannot credibly signal this information to investors and is stuck in the pool of low-return managers who are not rehired. In both cases, the under-informed investors make hiring decisions that fail to generate all the available surplus.

In this context, enabling investors to learn a manager's skill at the same pace as the manager herself increases the economic surplus that the manager's activities are expected to generate *ex ante*. In the first-best scenario, the investors have the same information as the manager, and so can avoid hiring an unskilled manager or letting a skilled manager go. Because managers retain any surplus, they stand to gain from investors learning their skills at a faster pace, and so would like to commit *ex ante* to revealing their information to investors as they learn it. Their problem is to do so credibly, without revealing important private information about their trading strategies.

Our paper's main contribution is to show that managers can solve this credibility problem by joining fund families, who act as delegated monitors of money managers. Families learn what their managers learn, namely their skills. In other words, we assume that the manager's information can be communicated to or observed by the family they join. Clearly, the bulk of this information cannot be shared with individual investors, as it would lead to front-running. Instead, it is much safer and economical for the manager to disclose this sensitive information to closely-watched board members affiliated with a fund family. The credibility of families is not immediate, however, because their incentives regarding negative information resemble those of their managers; they too gain from investors believing the best of the managers. However, the credibility problem is alleviated when the fund family has multiple fund managers under its umbrella. For example, suppose that two managers perform badly in a given period. When these managers operate outside of a fund family, they can no longer manage money: investors infer from their returns that further investment is not worthwhile, even if it is possible that bad luck was responsible for the bad performance. When these managers operate within a fund family, the family can choose to fire one of these two managers. This firing is relatively cheap since investors would not have invested with the fired manager anyhow. At the same time, the firing provides useful information to investors about the retained manager: when retaining a manager, the family prefers keeping one who is skilled (and was unlucky) than one who is unskilled, as it can

² Limited liability enables us to rule out negative compensation, which would permit some signaling to take place.

expect this manager to generate more money in the future. So, although investors do not observe the skills of a retained manager, they know that she is likely more skilled than the fired manager. In other words, when managers can be fired by a fund family, investors can update about manager skills not only through their past performance but also through the retention decisions of the family they belong to. This can make the hiring of a skilled but unlucky manager possible, reducing the amount of economic surplus lost to adverse selection.

The paper goes on to show that the monitoring implicitly performed by a fund family through its firing and retention decisions improves as more funds join the family. This is because both type-1 (keeping an unskilled manager) and type-2 (firing a skilled manager) errors can be smaller when more managers are compared to each other. In the limit, as the number of funds grows, a commitment by the family to fire the expected fraction of unskilled managers, conditional on the first-period return, results in exactly the unskilled managers being fired. As a result, the full first-best surplus is recaptured, though managers must share this gain with the family.

Except for this limiting result, the monitoring mechanism that we characterize is imperfect. This is not surprising given Diamond's (1984) insight that the delegation of monitoring to a self-interested third-party brings agency problems of its own. Diamond (1984) and Krasa and Villamil (1992) show that the pooling of monitored agents can improve the efficiency of delegated monitoring. Our paper points out that applying a similar mechanism to the money-management industry yields an interesting result: money managers pooled into fund families can be monitored more credibly and efficiently, and thus can manage money more profitably.

This mechanism is reminiscent of the one at work in the models of internal capital markets developed by Gertner, Scharfstein, and Stein (1994) and by Stein (1997). In these models, a corporation's headquarters gets to pick the relative winners and losers in the competition by the firm's divisions for fixed resources. This better ex-post allocation of resources makes ex ante financing cheaper and more efficient. The fund family performs a similar function in our model, not so much by allocating resources, but by picking which managers get to remain under its umbrella. As a monitor of money managers, our fund family is similar to the monitor of Alchian and Demsetz (1972) who participates in the profits of the team (the set of funds), can observe what each team member (manager) observes, and can alter the membership of the team (i.e., fire managers). However, the presence of the monitor in our model is the result of a completely different force. In Alchian and Demsetz (1972), because the output of each team member cannot be individually observed, organizing into a team creates a free-rider problem that monitoring is designed to reduce. Effort is costless in our model so there is no free-rider

problem; instead, the creation of a team of funds is what makes monitoring possible.

The process by which managers get to stay employed is similar to the way workers get promoted in the literature on tournaments.³ For example, whereas a fixed pre-specified fraction of workers get promoted in Malcomson's (1984) model of a firm, a fixed pre-specified fraction of money managers remain employed in our model of a fund family. The important difference between the two models lies in their purpose: tournaments within firms are designed to motivate workers to exert effort while the implicit ability tournaments that take place within fund families make allocations of capital to their funds by investors more efficient. Indeed, the firing of some fraction of a fund family's managers can boost the expected ability of those remaining, as perceived by investors who ultimately choose whether or not to have their money managed.

Several authors have analyzed delegated portfolio-management problems in which investors seek to learn about a manager's skills over time. In Huberman and Kandel (1993) and Huddart (1999), risk-averse managers know their skill *ex ante*, and choose their portfolios knowing that investors will infer skill from their portfolios' returns. Both studies find pooling and separating equilibria, where the separating equilibria involve skilled managers investing in the one risky asset more aggressively, so that, win or lose, they are identifiable *ex post* as the skilled ones. Heinkel and Stoughton (1994) and Farnsworth (2001) also address the situation where a manager knows her ability and investors try to infer it, but add moral hazard as a problem that the compensation contract must resolve. The use of a benchmark return proves to be an integral part of the resulting contract. Similarly, Holmström (1999) analyzes a multi-period contracting problem between a principal and an agent, at the beginning of which neither knows the agent's ability. The paper concentrates on the agent's effort patterns throughout her relationship with the agent. In all these models, the manager is able to affect what investors learn about her through the (portfolio and/or effort) choices she makes in the early periods. Although these effects could be incorporated into our model, we rule them out in order to isolate the informational role played by fund families in the money-management industry. The learning aspect of our model is similar to that of Lynch and Musto (2003), which is not about monitoring but about fund flows. Their model assumes that both managers and investors learn only from past returns, while our model allows managers to learn more than investors.

Although Sharpe (1981) and Barry and Starks (1984) do not discuss the potential monitoring benefits of pooling managers under one monitoring

³ See Lazear and Rosen (1981), Green and Stokey (1981), Nalebuff and Stiglitz (1983), Malcomson (1984, 1986), Rosen (1986), and Bhattacharya and Guasch (1988). We would like to thank an anonymous referee for pointing out this analogy.

entity, they do discuss the advantages for investors of contracting with multiple managers simultaneously. In particular, Sharpe (1981) argues that investing with more than one money manager achieves skills diversification, that is, it makes the ex-post value of fund managing more predictable, which is a benefit to risk-averse investors. Barry and Starks (1984) show that risk sharing can improve when an investor contracts with more than one risk-averse money manager at a time. A recent paper by Pichler (2002) shows how these two benefits can complement each other within a team of money managers.

To our knowledge, the economics of the fund family as an organization have not been treated theoretically. Empirical work is also relatively sparse. Khorana and Servaes (1999) look at fund family decisions to open new funds. Massa (2003) shows that the organization of funds into families creates positive externalities across these funds which in turn affect their performance and fees. Finally, Nanda, Wang, and Zheng (2004) study families' strategies to attract flows to their funds.⁴ As far as we know, the monitoring role of fund families remains untested, but our model yields several testable implications that should provide guidance for future empirical work. Furthermore, our theory's predictions distinguish it from the stories of economies of scale and scope that the aforementioned papers are ultimately based on.

The rest of the paper is organized as follows. In Section 2, we define and motivate the two-period model. In Section 3, we show the efficiency loss incurred in the absence of monitoring. In Section 4, we allow the manager to work in a fund family and discuss the agency problem that prevents effective monitoring by a one-fund family. We then show when and how a two-fund family can recover value and generalize the result to N funds. Section 5 contains a discussion of the model and its predictions. Finally, Section 6 summarizes and concludes. All proofs are in Appendix A.

2. The Model

There are two periods ($t = 1, 2$). At the start of each period, a different group of A small risk-neutral investors, each with a dollar, make their investment decision: they can invest in the market directly, or they can pool their money into a mutual fund managed by a risk-neutral manager. In both periods, direct investments in financial markets yield an unconditional return which is either $x \in (0, 1]$ or $-x$ with equal probabilities. So, uninformed investments have a zero expected return.

⁴ Some empirical work addresses a question related to the organization of funds into families, namely the role of fund managers' employers and boards. For example, Khorana (1996) documents the relation between return and continued employment, Chevalier and Ellison (1999) show demotion and separation following poor return, and promotion following high return, and Tufano and Sevick (1997) study the effect of board composition on the fees charged by mutual funds.

If instead investors choose to invest in the mutual fund for a period, the fund manager generates signals which are potentially informative about the returns of traded financial assets in that period.⁵ Skilled managers generate excess expected returns. More precisely, a manager with skill $\alpha \in (0, 1)$ improves the probability that returns are positive from $\frac{1}{2}$ to $\frac{1+\alpha}{2}$, and so generates an average return of $\frac{1+\alpha}{2}x + \frac{1-\alpha}{2}(-x) = x\alpha > 0$. Unskilled managers, on the other hand, cannot improve on the unconditional return distribution; their expected returns are zero. Managers are skilled with probability ϕ , and the random variable

$$\tilde{a} = \begin{cases} \alpha, & \text{prob. } \phi \\ 0, & \text{prob. } 1 - \phi \end{cases} \quad (1)$$

denotes a manager's skill, which is initially unknown.⁶ In each period t , therefore, the return that a manager generates on behalf of the fund's investors is given by

$$\tilde{r}_t = \begin{cases} +x, & \text{prob. } \frac{1+\tilde{a}}{2} \\ -x, & \text{prob. } \frac{1-\tilde{a}}{2} \end{cases}. \quad (2)$$

The cost of operating the fund is $k > 0$, which is absorbed equally by each of the investors. This cost represents expenses incurred by the fund for transacting, hiring personnel, buying computer equipment, and so on. We assume that k is large relative to each investor's investible funds, so that it is prohibitively expensive for an individual investor to form his own fund. The net profit that a manager with A dollars under management generates in period t is $A\tilde{r}_t - k$. Thus a manager with a probability p of being skilled is expected to generate a net profit of $Ax\alpha p - k$. We also refer to this quantity as the expected net value or expected surplus of the fund.

At the beginning of the first period, neither the manager nor the investors know whether the manager is skilled. They all use the same prior probability of ϕ that the manager is skilled, and all calculate the expected net value she creates to be $Ax\alpha\phi - k$. At the beginning of the second period, investors update their beliefs about the manager using the public information available at that time, namely the information contained in first-period returns. At the same time, the manager updates her beliefs with what she has privately learned about herself in the first period. This information allows her to update more accurately than investors, as she knows not only her return but also the portfolio that generated it, why she bought that portfolio, how the realized returns of different assets compare to her ex ante expectations, and so on. For

⁵ We assume there is no private effort cost to abstract from moral hazard considerations.

⁶ Avery and Chevalier (1999) also study a model in which the manager initially does not know her own ability. Their analysis, which does not consider the agency aspect of the manager's position, concentrates on the herding behavior of the manager over time.

simplicity, we assume that the manager's private information allows her to learn her skill perfectly after the first period.

Each period starts with the manager announcing a compensation contract that she is willing to work for, and the period's investors accepting or rejecting the offer. We make three assumptions about contracting. First, compensation is allowed to depend only on the realized return of the fund. Given that returns can take only two possible values in each period, this contract is fully characterized by the wage that will be paid by the investors to the manager in each of the two states. Second, there is no negative compensation: instead of paying, the manager would just file bankruptcy. Third, we adopt Berk and Green's (2002) result that the marginal dollar invested in a mutual fund generates no excess return in equilibrium. In our setting, this means that the contract leaves investors with zero expected surplus.

Because investors and the manager agree at the time of first-period contracting that the probability of a high first-period return is $\frac{1+\alpha\phi}{2}$, they all calculate the same expected wage for a given contract, and therefore any contract with expectation $Ax\alpha\phi - k$ under these probabilities serves equally well. Without negative compensation, the manager cannot be hired in the first place if this quantity is negative, so we assume that the parameters satisfy the restriction

$$\phi \geq \frac{k}{Ax\alpha}. \quad (3)$$

Because the manager has private information after the first period about the probability of a high second-period return (it is $\frac{1+\alpha}{2}$ if the manager learned she is skilled and $\frac{1}{2}$ otherwise), there is a signaling problem with second-period contracts. Among the contracts under which investors expect zero surplus, skilled managers prefer the contract with the lowest payment for low returns. The equilibrium contract pays the minimum for low returns, which under our assumptions is zero. Thus, the second-period contract the manager offers takes the form⁷

$$\tilde{w}_2 = \begin{cases} \omega, & \text{if } \tilde{r}_2 = +x \\ 0, & \text{if } \tilde{r}_2 = -x. \end{cases} \quad (4)$$

Since ω is the contract's only moving part, we sometimes refer to the second-period contract simply as ω . Under the assumption that investors get zero expected surplus, the ω offered by the manager is the one at which investors calculate an expected net return of zero and the manager's expected compensation exactly equals the expected surplus.

⁷ A formal proof that this contract is signaling-proof is provided in Appendix B.

Finally, the manager is assumed to receive nothing if she is not managing money, i.e., her reservation wage is assumed to be zero. As a result, even an unskilled manager prefers a chance at receiving this fee to not managing money, and so a manager who discovers she is unskilled has no incentive to reveal this information to investors.

3. Equilibrium in the Absence of Monitoring

3.1 Strategies and equilibrium

The chronology of this economy in each period is illustrated in Figure 1. Each period t begins with the manager offering investors to manage their money for one period in return for a compensation contract $\tilde{w}_t$. The investors are free to decide whether or not to hire the money manager. If hired, the manager collects information and invests the investors' money. The first period ends with investors and manager receiving their first-period payoffs and the manager observing her ability. The second period repeats the process, except that second-period investors can use first-period returns to update their beliefs about the manager's ability. The economy ends after the second-period payoffs.

The manager makes one decision per period, which is the compensation contract she offers, and the investors' one decision in each period is to accept or reject. Since the manager and investors are symmetrically informed at the outset, first-period contracting is straightforward: as mentioned above, the equilibrium contract has an expected value of $Ax\alpha\phi - k$, regardless of its form, and the investors accept it. From here on, we focus exclusively on the second period, and accordingly drop the time subscript so as to minimize notation. The second-period contract has an important dependence on investors' beliefs about the manager.

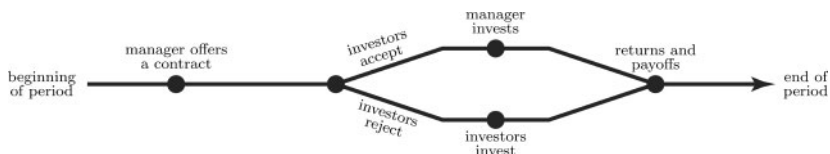


Figure 1

Sequence of events in each period

In each period, the manager offers a contract to the investors, who can accept or reject it. If the investors accept, the manager invests their money; otherwise, they make their own investments. Payoffs are realized at the end of the period.

Lemma 1. Suppose that, at the beginning of the second period, investors assign a probability p that the manager is skilled. The manager is then hired by the investors if and only if

$$p \geq \frac{k}{Ax\alpha}. \quad (5)$$

The manager's compensation in that period is then

$$\omega = \omega(p) \equiv \frac{2(Ax\alpha p - k)}{1 + \alpha p}. \quad (6)$$

The second-period investors expect the manager to generate a surplus of $Ax\alpha p - k$ on average, so condition (5) simply states that the manager will be hired when this expectation is non-negative. This happens when the investors' beliefs about the manager's skill, p , are large enough. The same condition can also be written as $x\alpha p \geq \frac{k}{A}$, leading to an alternative interpretation: investors pool their money into the managed fund when the fund's expected return exceeds the fund's per-dollar fees. When that is the case, the manager extracts all surplus by charging $\omega(p)$ for a positive return. Note that $\omega(p)$ is increasing in p , so that both the probability of, and compensation for, a high return increase as the manager's apparent skill grows.

What is the manager's apparent skill? Investors start with prior belief ϕ and then update with the manager's realized return via Bayes' rule.

Lemma 2. After first-period fund returns of $+x$, investors assign a probability of

$$\phi_{+x} \equiv \frac{(1 + \alpha)\phi}{1 + \alpha\phi} \quad (7)$$

to the manager being skilled. After first-period fund returns of $-x$, investors assign a probability of

$$\phi_{-x} \equiv \frac{(1 - \alpha)\phi}{1 - \alpha\phi} \quad (8)$$

to the manager being skilled.

The connection between skill and returns delivers $\phi_{+x} > \phi > \phi_{-x}$. Since ϕ is already assumed high enough for the manager to be hired, this immediately implies that the manager is retained after good performance and that her expected compensation conditional on good performance (but *not* on whether she is skilled) is her expected surplus $Ax\alpha\phi_{+x} - k$. On the other hand, ϕ_{-x} could be either less or greater than $\frac{k}{Ax\alpha}$. From Lemma 1, we know that the manager is retained only if it is greater, with an expected compensation conditional on poor performance of $Ax\alpha\phi_{-x} - k$. The next section calculates the second-period compensation the manager

expects at the outset, before anything is learned about her ability, by combining these quantities.

3.2 Manager surplus and first-best

The manager's expected second-period compensation at the outset, which we denote π_0 , is the probability of good performance times the expected compensation conditional on good performance, plus the probability of bad performance times the expected compensation, if any, conditional on bad performance. Alternatively, because the manager extracts all rents, her expected second-period compensation is simply the expected surplus that her activities generate.

Before we perform this calculation, it is useful to describe the first-best scenario for this model. In that scenario, investors learn about managerial skills at the same pace as the manager. This means that second-period investors know $\tilde{a}$, and so no longer require first-period returns when they make their contracting decisions. Clearly, with this information, investors would hire the manager only when she is skilled. The ex ante probability of this happening is ϕ , and the expected net surplus generated by a skilled manager is $Ax\alpha - k$. Thus, in this first-best scenario, the expected second-period compensation of the manager is $\phi(Ax\alpha - k)$.

As the following proposition shows, when the manager learns her ability faster than investors do, the second-period compensation that she can expect at the outset is smaller than $\phi(Ax\alpha - k)$. Retaining an unskilled manager and letting a skilled manager go both destroy value, and value destruction reduces the surplus than can be captured by the manager ex ante.

Proposition 1. *If $\phi_{-x} < \frac{k}{Ax\alpha}$, then*

$$\pi_0 = \phi(Ax\alpha - k) - \frac{1-\alpha}{2}\phi(Ax\alpha - k) - \frac{1}{2}(1-\phi)k. \quad (9)$$

If $\phi_{-x} \geq \frac{k}{Ax\alpha}$, then

$$\pi_0 = \phi(Ax\alpha - k) - (1-\phi)k. \quad (10)$$

To better understand the two sources of value destruction [and their associated negative terms in Equations (9) and (10)], we turn to Table 1, which decomposes the manager's expected compensation in the second period. Because first-period returns ($\tilde{r}_1$) and skill ($\tilde{a}$) can each take two possible values, there are four possible first-period outcome combinations. For each of these outcomes, the table shows the hiring decisions made by the second-period investors, depending on whether they learn at the same pace as the manager (first-best) or not (asymmetric learning).

Table 1
Manager's expected compensation in the second period

		Asymmetric learning							
		First-best		$\phi_{-x} < \frac{k}{Ax\alpha}$			$\phi_{-x} \geq \frac{k}{Ax\alpha}$		
Outcome ($\tilde{r}_1, \tilde{a}$)	Probability of outcome	Hired?	Ex ante surplus	Hired?	Ex ante surplus	Lost surplus	Hired?	Ex ante surplus	Lost surplus
$(+x, \alpha)$	$\frac{1+\alpha}{2}\phi$	yes	$Ax\alpha - k$	yes	$Ax\alpha - k$	0	yes	$Ax\alpha - k$	0
$(+x, 0)$	$\frac{1}{2}(1 - \phi)$	no	0	yes	$-k$	k	yes	$-k$	k
$(-x, \alpha)$	$\frac{1-\alpha}{2}\phi$	yes	$Ax\alpha - k$	no	0	$Ax\alpha - k$	yes	$Ax\alpha - k$	0
$(-x, 0)$	$\frac{1}{2}(1 - \phi)$	no	0	no	0	0	yes	$-k$	k

For each of the four possible first-period outcomes, this table shows the hiring decision made by the second-period investors as well as the associated expected surplus for that decision when investors learn the manager's skill at the same pace as she does (first-best) and slower than she does (asymmetric learning). The lost surplus from asymmetric learning is the difference. There are two cases for asymmetric learning: $\phi_{-x} < \frac{k}{Ax\alpha}$, in which case the manager is hired only after good performance, and $\phi_{-x} \geq \frac{k}{Ax\alpha}$, in which case the manager is always rehired.

When $\phi_{-x} < \frac{k}{Ax\alpha}$ and investors do not learn $\tilde{a}$, they hire the manager only if $\tilde{r}_1 = +x$ since the manager is not worth hiring after a bad performance. When they do learn $\tilde{a}$ the same investors ignore $\tilde{r}_1$, and instead hire the manager only if $\tilde{a} = \alpha$. Since the average net surpluses that skilled and unskilled managers generate are $Ax\alpha - k > 0$ and $-k$ respectively, it is clear that inefficiencies occur when an unskilled manager is hired ($\tilde{r}_1 = +x, \tilde{a} = 0$) or a skilled manager is not retained ($\tilde{r}_1 = -x, \tilde{a} = \alpha$). The table also reports the surplus lost to these contracting inefficiencies. When multiplied by the probability of their corresponding first-period outcome, these inefficiencies yield the two negative terms in Equation (9).

A similar argument can be made to obtain the negative term in Equation (10). When $\phi_{-x} \geq \frac{k}{Ax\alpha}$, the manager is hired by the second-period investors regardless of her first-period performance. Inefficiencies then occur when an unskilled manager is hired, that is, in the ($\tilde{r}_1 = +x, \tilde{a} = 0$) and ($\tilde{r}_1 = -x, \tilde{a} = 0$) outcomes. These two outcomes have a combined probability of occurrence of $1 - \phi$, and the surplus lost to the hiring of an unskilled manager is k , as the last column of Table 1 shows. Multiplying these two terms together yields the negative term in Equation (10).

The analysis in this section shows how the manager loses from inefficient contracting. In the remainder of the paper, we show how delegated monitoring can eliminate at least some of this inefficiency and so provide a valuable service to the manager.

4. Delegated Monitoring

A manager's alternative to managing without a monitor is to join a fund family. A fund family can in principle bring several benefits, such as economizing on marketing and investor relations; the one we focus on here is the

fund family's potential to monitor. We expand the model to multiple managers, and we allow them to join a stylized fund family: in return for receiving a fraction of the manager's wage, the fund family intermediates her initial contract with the public, learns her skill along with her, and then either retains or fires her. If the family retains exactly the skilled managers, this arrangement recovers exactly the first-best surplus (gross of the family's share). But it is unlikely that the family can be induced to fire exactly the unskilled managers. What is more interesting and meaningful is whether the family adds value in the context of realistic incentives for the family to retain or fire. In this section, we introduce the concept of a fund family, discuss the incentive problem that it faces, and then resolve the problem. The key to the resolution is to allow a family to monitor multiple funds.

4.1 Fund family defined

The manager can join a fund family at the outset. To join, the manager negotiates a fraction μ of her second-period wages, if any, that will go to the family. The fixed fraction means that the family has limited liability since the manager has limited liability. We also assume that the family is risk-neutral just like the manager. The family offers contracts to investors. As before we assume that investors pay all expected surplus, so the first-period expected wage is again $A\alpha\phi - k$. The family has access to the same information as the manager and so learns her skill over the first period, and then retains or fires her. There is no moral hazard with the monitor; once engaged, the monitor learns what the managers learn. To eliminate moral hazard, it is enough that the monitor's action of monitoring is observable. For simplicity, we further assume that monitoring is costless and the family's opportunity cost of monitoring is zero. Second-period investors also pay all expected surplus so, if investors calculate a probability p that the manager is skilled, then her second-period wage is $\omega(p)$, as calculated in Lemma 1.

The fraction μ plays a dual role in the model. First, because the family potentially benefits from a manager only when she manages money, it prefers the investors' opinion of a manager to be high enough that they choose to invest their money with her. Second, conditional on the manager working (i.e., controlling for the public belief about the manager), the family prefers the manager's actual skill to be high, as this increases the likelihood of high returns and their associated positive revenue. We do not model the negotiation over μ ; we only assume that if monitoring creates expected value, then the negotiated μ splits this expected value between the family and the manager. The split is likely to depend on the relative scarcity of monitoring and managing, i.e., the relative bargaining power of the parties.⁸ Finally, note that μ cannot depend on the manager's skill, which is unknown at the time negotiation takes place.

⁸ If monitoring is costly to the family because of foregone revenue or monitoring-related expenditures, then μ must recoup these costs in expectation for monitoring to be economical for the family. At the same time, $(1 - \mu)$ must be large enough to allow the manager to do better than she can do without monitoring.

We assume that there is no value added by replacing a fired manager with a new one. Since the family would presumably earn revenue from such a replacement, the implicit assumption is that the replacement cost equals this subsequent revenue.

4.2 The agency problem

The best monitoring can do is eliminate all contracting inefficiency by retaining a manager if and only if she is skilled. With such monitoring in place, second-period investors would on average pay the first-best amount $\phi(Ax\alpha - k)$ for the manager's services. In particular, this monitoring eliminates all inefficiency losses represented by the negative terms in Equations (9) and (10). So first-best monitoring adds a positive amount of expected value which the family and manager can share through the μ they negotiate.

If a family's incentives were to retain only skilled managers then the analysis of delegated monitoring would end here. However, because firing the manager brings zero net revenue, the family has no natural incentives to fire. For example, a family of one fund would never fire a manager with a good first-period performance even if it discovers that this manager is unskilled. The family gets zero from firing her, but gets whatever investors are willing to pay for a good-performing manager from retaining her. Thus a manager gets no efficiency gains from being a family's only manager as the family's incentives are then the same as the manager's.

One way to drive a wedge between the incentives of the managers and those of the families is to allow a family to monitor multiple managers. The rest of this section shows how this can work, first in the simplest two-manager case, and then in the general N -manager case.

4.3 The two-fund family

Instead of one manager in the family, now there are two managers, each with A dollars. The two managers have independently distributed skills and returns, and we allow them and the family to contract at the outset on a firing policy. The firing policies we allow are commitments by the family to fire a stated number of good performers and a stated number of bad performers (both numbers could be zero), conditional on the performance realization. With two managers, there are three possible performance realizations: both good, both bad, and one good and one bad. The firing policy, therefore, decides whether 0, 1, or 2 managers are fired if both have good first-period performance, how many are fired if both have bad performance, and whether the good performer, the bad performer, or both are fired in the latter scenario. Commitment is assumed to be costless: it is costless for the family to

contract on the firing policy, and the family honors the contract at no additional cost.⁹

Firing policies add value to the extent they contain information about the family's private observation of skill. Conditional on the first-period performance realizations, the public knows exactly how many good and how many bad performers must be fired by the family. Because the family receives a fixed fraction of the manager compensation in the second period and that compensation is contingent on a good return, the family always prefers to fire an unskilled manager rather than a skilled manager, given that a manager must be fired. So a family's firing policy can add value when the family has a choice whom to fire because the public learns about a surviving manager's skill from the fact that she was not fired.

Solving this two-manager problem is simplified by several observations. First, the family cannot add value by committing to firing both managers in a given outcome: without replacement, the family would never get rid of its only two sources of revenue; it can only do better by firing either zero or one manager. Second, the family does not commit to firing the poor performer (or to firing the good performer) when one performance is good and the other bad, because doing so does not tell the public anything more than it already knows from first-period returns. So the firing policy pays off only if both performances are identical (both good or both bad), and the decisions to make at the outset are whether to fire zero or one in each of these two circumstances.

If two managers generate the same return $r \in \{-x, +x\}$ in the first period, then for each manager, the public calculates the same probability ϕ_r that she is skilled. Table 2 shows the three possible combinations of managers' skills — both skilled, both unskilled, or one of each — together with the public's assessment of their probabilities of occurring. Assuming the family must fire one of its managers at the beginning of the second period, the family has no choice but to fire a skilled (an unskilled)

Table 2
Firing strategy of the fund family

Managers' skills	Probability	Fire	Retain
(Skilled, skilled)	ϕ_r^2	Skilled	Skilled
(Skilled, unskilled)	$2\phi_r(1 - \phi_r)$	Unskilled	Skilled
(Unskilled, unskilled)	$(1 - \phi_r)^2$	Unskilled	Unskilled

After both managers generate returns of $r \in \{-x, +x\}$ in the first period, the fund family learns whether or not they are skilled. The second-period investors can calculate the probability of each potential combination using the information from first-period returns. If the family must fire one manager, then it prefers to retain a skilled manager if possible.

⁹ We do not allow all possible firing policies. In particular, we do not allow policies where the family decides after the first period how many good performers or bad performers to fire. Such policies push the model into a complex signaling environment.

manager when both turn out to be skilled (unskilled). The monitoring gain comes from the other skill combination: the family is better off retaining the skilled manager even if the two managers look identical to investors. Knowing this, investors can update the probability that the *retained* manager is skilled: it is simply the sum of the first two probabilities in the table, namely

$$\phi_r^2 + 2\phi_r(1 - \phi_r) = \phi_r(2 - \phi_r).$$

Clearly, this probability is greater than ϕ_r , the investors' beliefs *before* they learn about the firing of the other manager. Whether this boost in investor beliefs is worth the firing of one potential source of revenue depends on the relative values of ϕ_r and $\frac{k}{Ax\alpha}$, as the next proposition summarizes.

Proposition 2. *If both managers have the same first-period return $r \in \{-x, +x\}$, then the second-period outcomes with and without delegated monitoring depend on ϕ_r and $\frac{k}{Ax\alpha}$ as follows.*

- (a) $\phi_r(2 - \phi_r) < \frac{k}{Ax\alpha}$: Neither manager works the second period, with or without delegated monitoring. The value created is zero.
- (b) $\phi_r < \frac{k}{Ax\alpha} \leq \phi_r(2 - \phi_r)$: Neither works the second period without delegated monitoring, but in a fund family one is fired and the other works. The value created is $\frac{1}{2}[Ax\alpha\phi_r(2 - \phi_r) - k]$ per manager.
- (c) $\phi_r^2 < \frac{k}{Ax\alpha} \leq \phi_r$: Both work without delegated monitoring, but with a delegated monitor one is fired and the other works. The value created is $\frac{1}{2}(k - Ax\alpha\phi_r^2)$ per manager.
- (d) $\frac{k}{Ax\alpha} \leq \phi_r^2$: Both work with or without a delegated monitor. The value created is zero.

If the managers are not monitored and therefore hired or not hired simply on the strength of their returns, there are just two regions in which $\frac{k}{Ax\alpha}$ can fall relative to ϕ_r . As shown in Lemma 1, either $\frac{k}{Ax\alpha} > \phi_r$ so they are both dismissed by investors, or $\frac{k}{Ax\alpha} \leq \phi_r$ so they are both retained. Proposition 2 shows that monitoring inserts a region between the both-hired and both-dismissed regions in which monitoring adds value and only one manager is retained.

The family's monitoring proves to be valuable when $\frac{k}{Ax\alpha}$ and ϕ_r are relatively close [cases (b) and (c)], that is, when investors are most uncertain about the value of money management. When $\phi_r < \frac{k}{Ax\alpha} \leq \phi_r(2 - \phi_r)$ in case (b), public beliefs about each manager are not large enough for investors to have their money managed in the second period. However, the boost from firing one manager is large enough for contracting with the other manager to occur. When $\phi_r^2 < \frac{k}{Ax\alpha} \leq \phi_r$ in case (c), it is again the case that one manager is fired and the other works, but the reason is

different. In this region, both managers would be rehired were they not monitored. Monitoring creates value because it reduces the expected loss from hiring unskilled managers more than it increases the expected loss from firing skilled managers. Still, the family's monitoring cannot add any value when $\frac{k}{A\alpha\alpha}$ is either very large or very small (cases (a) and (d) respectively). In the first case, the fund fees per unit of return are such that the fund is prohibitively expensive for investors to consider, even with the increase in investor beliefs that firing a manager creates. In the latter case, the fees are so small that investors are willing to pay a lot to have their money professionally managed, and so sacrificing a source of revenue is too costly to get the boost in investor beliefs.

All these results are illustrated in Figure 2. In particular, the figure shows how value creation centers around $\frac{k}{A\alpha\alpha} = \phi_r$ and gets (linearly) smaller as $\frac{k}{A\alpha\alpha}$ gets away from ϕ_r . A useful perspective on the result is that monitoring adds value if $\frac{k}{A\alpha\alpha}$ is in the range $[\phi_r^2, \phi_r(2 - \phi_r)]$. The width of this range is $2\phi_r(1 - \phi_r)$, which is proportional to the variance of the manager's skill conditional on her return.

4.4 The N -fund family

We now generalize the fund family to N managers. In this general case, there are $N + 1$ possible first-period outcomes, one for each possible number $M_{+x} \in \{0, 1, \dots, N\}$ of good performers (the number of bad performers is $M_{-x} = N - M_{+x}$). The firing policy dictates, for each of these outcomes, how many good performers to fire and how many poor performers to fire. As before, it is common knowledge that the family prefers retaining skilled managers to unskilled managers when that is possible. We characterize the optimal firing policy for a given $M_r, r \in \{-x, +x\}$ and show that the efficiency outcome approaches first-best as N grows.

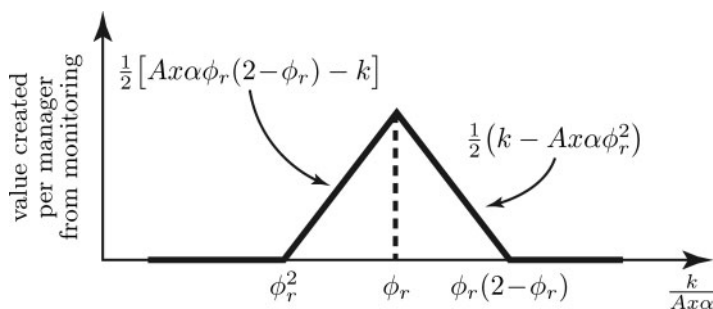


Figure 2

Gains from monitoring two managers

When two managers generate the same return $r \in \{-x, +x\}$ in the first period, firing one of these two managers creates some value as long as $\phi_r^2 < \frac{k}{A\alpha\alpha} \leq \phi_r(2 - \phi_r)$.

When choosing its firing policy, the fund family seeks to recover as much of the contracting inefficiency of Section 3 as possible. The following proposition characterizes the optimal number $F_r(M_r)$ of M_r managers with the same first-period return r who will be fired. The proposition uses $B(I, J, p)$ to denote the binomial cumulative distribution function (cdf) for J draws, I successes, and probability p of success.¹⁰

Proposition 3. *If there is an integer F , $0 < F < M_r$, that satisfies*

$$B(F - 1, M_r, 1 - \phi_r) < \frac{k}{Ax\alpha} < B(F, M_r, 1 - \phi_r), \quad (11)$$

then it is optimal for the family to commit to fire $F_r(M_r) = F$ of the M_r managers who make return $r \in \{-x, +x\}$ in the first period. If $B(0, M_r, 1 - \phi_r) > \frac{k}{Ax\alpha}$, then the optimal number to fire is zero; if $B(M_r - 1, M_r, 1 - \phi_r) < \frac{k}{Ax\alpha}$, then no managers work the second period regardless of monitoring.

The tradeoff used to derive this result is intuitive: as the family commits to fire a larger number of managers, the publicly-perceived skill of a remaining manager is higher (implying a larger per-manager surplus), but there are fewer managers working. The optimal choice of F , the number to fire, minimizes the total expected value loss relative to the first-best. Expected value loss occurs because skilled managers are fired and unskilled managers are retained. With each additional manager fired, the expected value loss per manager from retaining the manager when unskilled goes down while the expected value loss per manager from firing the manager when skilled goes up. When F is less than the optimum, the former outweighs the latter for an additional firing, while the converse is true for all F at least as large as the optimum. So the optimal F is the smallest number of firings for which an additional firing causes the following: the expected value loss per manager from retaining an unskilled manager goes down by less than the amount by which the expected value loss per manager from firing a skilled manager goes up. The last part of Proposition 3 simply points out that, just as in the two-manager case, there are parameters for which firing no one is optimal and for which firing everyone is optimal, and these are cases for which monitoring does not add any value. As the following result demonstrates, these parameter ranges shrink as M_r increases.

Lemma 3. *For any $\phi_r > 0$ the optimal number F of managers to fire satisfies Equation (11) for M_r large enough.*

¹⁰ That is, $B(I, J, p) \equiv \sum_{i=0}^I \binom{J}{i} p^i (1-p)^{J-i}$.

Organizing funds under the umbrella of a fund family adds value when monitoring affects the rehiring decisions of investors. Lemma 3 shows that this is the case for a fund family with a large enough cross-section of similar fund managers. Indeed, when many managers perform similarly, the family always finds it optimal to fire some managers and retain the others, thereby avoiding the no-monitoring outcome. In the previous section, we showed that monitoring by a two-fund family adds value if $\frac{k}{Ax\alpha}$ lands in a range $2\phi_r(1 - \phi_r)$ wide. Lemma 3 shows that M -fund families add value if $\frac{k}{Ax\alpha}$ is between $B(0, M_r, 1 - \phi_r)$ and $B(M_r - 1, M_r, 1 - \phi_r)$, that is, if it is between $\phi_r^{M_r}$ and $1 - (1 - \phi_r)^{M_r}$, which expands to the entire unit interval as M_r grows.

The important quantity in all the above results is the ratio $\frac{k}{Ax\alpha}$ which, when rewritten as $\frac{k/A}{x\alpha}$, is seen to be the ratio of the per-dollar administrative expense to the per-dollar value added by skill. So as the cost of running a fund increases relative to the value added by skill, whether from k increasing or A or x decreasing, managers prefer policies with more ex-post firing. The effect of α by itself depends on whether performance is good or bad. If α is higher then there is less firing after good performance for two reasons: the loss from firing a skilled manager is higher and the likelihood that the performance came from a skilled manager is higher. The effect on firing after bad performance is ambiguous because, while the loss from firing a skilled manager is higher, the likelihood that the performance came from a skilled manager is now lower.

The firing policy described in Proposition 3 minimizes contracting inefficiencies for every possible first-period outcome. Inefficiencies remain because the actual number of unskilled managers differs from the expected number. However, as the number N of funds in the family grows, the actual fraction of managers who are unskilled converges to the expected fraction, both for the good performance managers and for the bad performance managers. Thus we expect contracting inefficiencies to be reduced as the size of the fund family increases. In fact, as we show next, the remaining inefficiency goes to zero as the number of managers in the fund family grows. We start with the following observation.

Lemma 4. *As M_r grows, $\frac{F_r(M_r)}{M_r}$ converges to $1 - \phi_r$.*

As the number of managers with a given return increases, the fraction of managers fired converges to the fraction of these managers expected ex ante to be unskilled. This observation leads to our final result.

Proposition 4. *As the number of funds in the family grows, contracting efficiency converges to first-best.*

In the limit, the entire efficiency loss is recaptured through family monitoring. More precisely, in the limit, only the skilled managers are retained by the family for the second period and, because investors understand this, they are willing to pay each retained manager the first-best expected surplus generated by a single manager, $Ax\alpha - k$. We reach this conclusion even though we consider only those firing policies that commit to the relation between two future observable and verifiable events: the number of managers with a given performance and the number of those fired.

5. Implications of the Model

Our theoretical model provides an economic rationale for mutual-fund families. In fact, its central prediction is that these families will exist. The theory also yields some more precise and rejectable predictions that can be taken to the data. These predictions help us explore the results of the model more deeply and, at the same time, distinguish it from other theories.

The theory predicts that retention and firing grow more efficient as the family grows larger. This should be apparent in a number of ways. The growing efficiency of retention means that the expected skill of a retained manager grows with the number of managers in the family. Therefore, her future performance should be better. Fund flows are not dynamic in our model but if funds' net returns equilibrate through new investment, as in Berk and Green (2002), then for a given return history, the retained manager should attract more new investment if her family is larger. In the limit, all information is in being retained, rather than in performing well. Thus the following implications emerge.

Implication 1. *The expected skill of a retained manager grows with the number of managers in the family.*

Implication 2. *The fund return that a retained manager is expected to generate gross of fees grows with the number of managers in the family.*

Implication 3. *Flows respond relatively less to fund performance and more to retention decisions, as the associated fund family grows.*

A manager benefits more from monitoring if less is known about her skill. Monitoring adds more value early in a manager's career since more is known about her skill if she has a longer track record. In fact, in the context of our model, an infinitely long track record fully reveals a manager's ability. We should therefore expect managers to work in fund families earlier, rather than later, in their careers.

Implication 4. *Managers are more likely to work in fund families early in their careers.*

Families use information about manager quality over and above that in the fund's return history. Consequently, the firing behavior of families is expected to differ from that of investors with respect to stand-alone funds. Proposition 3 provides even more specific predictions about these differences.

Implication 5. *Families sometimes fire managers after a return history that would not cause a stand-alone fund to fold. Conversely, families sometimes retain managers after a return history that would cause a stand-alone fund to fold.*

Admittedly, there is very little evidence, if any, showing that contracts between families and their funds explicitly require families to close a return-dependent fraction of their funds, as in our model. Even so, this lack of evidence does not necessarily mean that families are not performing the type of monitoring function we describe. Even if commitments to fire are not made explicitly, it is possible that reputational considerations motivate families to fire a return-dependent fraction of managers. In a multi-period setting, investors may avoid investing with families who have failed in their operating histories to fire the requisite return-dependent fractions of their managers.

Implication 6. *Surviving families fire more than non-surviving families.*

6. Conclusion

Decades of empirical studies have established that it is very hard to estimate portfolio-manager skill from realized portfolio returns. The portfolio manager herself has more than her portfolio's returns to learn from: she knows why she chose the initial portfolio, how and why she traded it, how the realizations of individual securities' returns compare with what she expected, and so on. She learns more from a period of experience than outsiders learn from a period of portfolio returns, so it stands to reason that she enters the next period knowing more about her skill than outsiders know. A manager with a good return could privately know she is *not* worth hiring, and a manager with a poor return could privately know she *is* worth hiring. The problem is how to transmit this information credibly to investors when the low-ability manager has no incentive to reveal her low ability to investors. We show how monitoring by a fund family can credibly reveal information about manager quality to investors and we demonstrate how multiple-fund monitoring by a single

family is crucial to its effectiveness. In particular, a one-manager fund family has little credibility since the family's incentives are exactly those of the manager. We show how increasing the number of funds monitored by the family can avoid this incentive problem. We make this point in the context of money management but the principle applies more generally. The key elements of the economic situation are that the agent learns about skill over time, that someone else could also learn at sufficiently low cost, and that a number of other agents are in a similar situation. This could describe the situation of, for example, assistant professors.

We analyze the situation where a manager's performance is publicly observable from the moment she starts managing. In practice, there is some discretion because the family could have the manager handle house money, or pretend money, for a while. The manager would intuitively have to cross a threshold of promise, as calculated from this practice experience, to start handling public money. Outsiders could infer this threshold and therefore know, as the model assumes, the manager's promise at that point. So the model still applies in the bigger picture where managers practice first, but it would be interesting to see what the optimal threshold is in this framework.

We do not claim to have identified the only reason why fund managers may want to gather under the umbrella of a fund family. There are undoubtedly reasons other than the value that more credible monitoring creates (e.g., marketing reasons and economies of scale and scope). However, our framework could also be useful to study other aspects of the money-management industry. For instance, it may be interesting to consider when a fund manager would be better off leaving a fund family to manage independently (e.g., start a hedge fund). Our analysis suggests that managers who believe their skill has been established beyond a doubt would see little benefit from monitoring, as would managers who perceive little difference between what they will privately learn about their skill and what their returns will convey.

Finally, the result that monitoring captures more value as the number of funds grows may relate to the recent development of "funds of funds."¹¹ Many funds have recently launched with the sole intention of investing in existing hedge funds. Given the close relation that investors can enjoy with the hedge funds they patronize, this trend may reflect a demand from hedge-fund managers for credible monitoring. How managers sort into those who manage independently, those who manage partly for funds of funds, and those who manage within mutual-fund families is a promising area for future research.

¹¹ We are grateful to Zhiwu Chen for pointing out this possibility to us.

Appendix A

Proof of Lemma 1. Let us denote the investors' information set at the beginning of the second period by $\mathcal{I}$. Since investors assign a probability p that the manager is skilled, they expect her to generate an average net surplus of

$$\begin{aligned} & A(E[\tilde{r} \mid \mathcal{I}, \tilde{a} = \alpha] \Pr\{\tilde{a} = \alpha \mid \mathcal{I}\} + E[\tilde{r} \mid \mathcal{I}, \tilde{a} = 0] \Pr\{\tilde{a} = 0 \mid \mathcal{I}\}) - k \\ & = A[x\alpha p + 0(1 - p)] - k = Ax\alpha p - k \end{aligned}$$

with their money if they hire her. The manager will be hired by the investors if and only if this last quantity is positive; this is equivalent to (5). In that case, since the investors assign a probability of

$$\begin{aligned} & \Pr\{\tilde{r} = +x \mid \mathcal{I}, \tilde{a} = \alpha\} \Pr\{\tilde{a} = \alpha \mid \mathcal{I}\} + \Pr\{\tilde{r} = +x \mid \mathcal{I}, \tilde{a} = 0\} \Pr\{\tilde{a} = 0 \mid \mathcal{I}\} \\ & = \frac{1 + \alpha}{2} \phi + \frac{1}{2} (1 - \phi) = \frac{1 + \alpha p}{2} \end{aligned}$$

to the manager generating positive returns in the period, the manager extracts all of the surplus by announcing a compensation contract of

$$\omega = \omega(p) \equiv \frac{2(Ax\alpha p - k)}{1 + \alpha p}.$$

This completes the proof. ■

Proof of Lemma 2. This result follows from a simple application of Bayes' rule:

$$\begin{aligned} \phi_{+x} & \equiv \Pr\{\tilde{a} = \alpha \mid \tilde{r}_1 = +x\} \\ & = \frac{\Pr\{\tilde{r}_1 = +x \mid \tilde{a} = \alpha\} \Pr\{\tilde{a} = \alpha\}}{\sum_{a=0,\alpha} \Pr\{\tilde{r}_1 = +x \mid \tilde{a} = a\} \Pr\{\tilde{a} = a\}} \\ & = \frac{\frac{1+\alpha}{2} \phi}{\frac{1+\alpha}{2} \phi + \frac{1}{2} (1 - \phi)} = \frac{(1 + \alpha)\phi}{1 + \alpha\phi}. \end{aligned}$$

Similarly,

$$\begin{aligned} \phi_{-x} & \equiv \Pr\{\tilde{a} = \alpha \mid \tilde{r}_1 = -x\} \\ & = \frac{\Pr\{\tilde{r}_1 = -x \mid \tilde{a} = \alpha\} \Pr\{\tilde{a} = \alpha\}}{\sum_{a=0,\alpha} \Pr\{\tilde{r}_1 = -x \mid \tilde{a} = a\} \Pr\{\tilde{a} = a\}} \\ & = \frac{\frac{1-\alpha}{2} \phi}{\frac{1-\alpha}{2} \phi + \frac{1}{2} (1 - \phi)} = \frac{(1 - \alpha)\phi}{1 - \alpha\phi}. \quad \blacksquare \end{aligned}$$

Proof of Proposition 1. If $\phi_{-x} < \frac{k}{Ax\alpha}$, then the manager works only after a good first-period return so π_0 is the probability of a good first-period return times the expected surplus conditional on a good first-period return, i.e., $\frac{1+\alpha\phi}{2} \left[Ax\alpha \left(\frac{(1+\alpha)\phi}{1+\alpha\phi} \right) - k \right]$, which rearranges to (9).

If $\phi_{-x} \geq \frac{k}{Ax\alpha}$, then the manager works the second period no matter what, so π_0 is the same as the first-period expected surplus $Ax\alpha\phi - k$, which rearranges to (10). ■

Proof of Proposition 2. If $\phi_r(2 - \phi_r) < \frac{k}{Ax\alpha}$, then Lemma 1 tells us that the probability that the retained manager is skilled, conditional on her return and the fact that she was retained is too low for her to work the second period, so neither manager works the second period.

If $\phi_r < \frac{k}{Ax\alpha}$, we know from Lemma 1 that the managers cannot work the second period without monitoring. However, if $\phi_r(2 - \phi_r) \geq \frac{k}{Ax\alpha}$, then the retained manager's probability of being skilled is high enough to work. The total surplus for the retained manager is $Ax\alpha\phi_r(2 - \phi_r) - k$, which is shared equally by the two managers at the outset.

If $\phi_r \geq \frac{k}{Ax\alpha}$ then, from Lemma 1, both managers would work the second period after making r in the first period if there were no firing policy and would get the expected surplus $Ax\alpha\phi_r - k$. If the fund family commits to firing one of them then the expected surplus of the retained manager is $Ax\alpha\phi_r(2 - \phi_r) - k$, and each manager's expected surplus at the outset, conditional on both managers making r in the first period, is $\frac{1}{2}[Ax\alpha\phi_r(2 - \phi_r) - k]$. So the expected value added per manager by the firing policy is $\frac{1}{2}[Ax\alpha\phi_r(2 - \phi_r) - k] - (Ax\alpha\phi_r - k) = \frac{1}{2}(k - Ax\alpha\phi_r^2)$. This value is positive if and only if $\phi_r^2 < \frac{k}{Ax\alpha}$.

Finally, if $\phi_r^2 \geq \frac{k}{Ax\alpha}$, then the expected value created by firing one of the two managers, calculated in the previous paragraph, is negative. So neither is fired and both work (since ϕ_r is then greater than $\frac{k}{Ax\alpha}$). ■

Proof of Proposition 3. We prove the result for a given first-period return $r \in \{-x, +x\}$ and a given number M of managers who generate that return. Let U represent the number of the M managers who are unskilled. Suppose that the family commits to firing F of M managers who make return r , and let $T_1(F)$ and $T_2(F)$ be the probabilities that a manager calculates at the outset that she will be fired when skilled and retained when unskilled, respectively. The family maximizes expected value by minimizing total contracting inefficiency, i.e., by choosing F to minimize

$$T_1(F)(Ax\alpha - k) + T_2(F)k.$$

When the family's policy is to fire F managers, some skilled managers end up getting fired when fewer than F of the M managers are unskilled; that is, $F - U$ skilled managers get fired when $U < F$. Thus the expected number of skilled managers get fired is

$$\sum_{U=0}^{F-1} \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} (F - U),$$

and so a manager's ex ante probability of being fired while skilled is

$$T_1(F) = \frac{1}{M} \sum_{U=0}^{F-1} \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} (F - U).$$

This means that the commitment to fire an additional manager increases the ex ante probability of being fired while skilled by

$$T_1(F + 1) - T_1(F) = \frac{1}{M} \sum_{U=0}^F \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} = \frac{1}{M} B(F, M, 1 - \phi_r).$$

Similarly, when the family's policy is to fire F managers, unskilled managers are retained when more than F managers are unskilled; that is $U - F$ unskilled managers are retained when $U > F$. The expected number of unskilled managers who are retained is

$$\sum_{U=F+1}^M \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} (U - F),$$

and so a manager's ex ante probability of being retained while unskilled is

$$T_2(F) = \frac{1}{M} \sum_{U=F+1}^M \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} (U - F).$$

Firing an additional manager increases the ex ante probability of being retained while unskilled by¹²

$$\begin{aligned} T_2(F + 1) - T_2(F) &= -\frac{1}{M} \sum_{U=F+1}^M \binom{M}{U} (1 - \phi_r)^U \phi_r^{M-U} \\ &= -\frac{1}{M} \left[1 - B(F, M, 1 - \phi_r) \right]. \end{aligned}$$

So if the family moves from a policy of firing F to a policy of firing $F + 1$, total efficiency loss increases by

¹² Of course, this quantity is negative, reflecting the fact that firing an additional manager actually decreases the likelihood that an unskilled manager is retained.

$$\begin{aligned} & [T_1(F+1) - T_1(F)](Ax\alpha - k) + [T_2(F+1) - T_2(F)]k \\ &= \frac{1}{M} \left[B(F, M, 1 - \phi_r)Ax\alpha - k \right]. \end{aligned}$$

This quantity is increasing in F and has the same sign as $B(F, M, 1 - \phi_r) - \frac{k}{Ax\alpha}$. So if there is an interior solution for F , it is the first F for which $B(F, M, 1 - \phi_r) > \frac{k}{Ax\alpha}$, that is the F that satisfies

$$B(F-1, M, 1 - \phi_r) < \frac{k}{Ax\alpha} < B(F, M, 1 - \phi_r). \quad (12)$$

For each of the $M - F$ retained managers, outside investors will calculate a probability of being skilled greater than $\frac{k}{Ax\alpha}$, since the probability that *all* retained managers are skilled is $B(F, M, 1 - \phi_r)$ which, from Equation (12), is greater than $\frac{k}{Ax\alpha}$. So F managers are fired and $M - F$ work the second period. If $B(0, M, 1 - \phi_r) > \frac{k}{Ax\alpha}$ then firing zero is optimal. If $B(M-1, M, 1 - \phi_r) < \frac{k}{Ax\alpha}$, then the probability that at least one manager is skilled is below $\frac{k}{Ax\alpha}$, so no manager can work no matter how many are fired. ■

Proof of Lemma 3. The value of $B(0, M, 1 - \phi_r)$ is ϕ_r^M , which asymptotes to 0 as M grows so, for M large enough, it is below $\frac{k}{Ax\alpha}$. Similarly, the value of $B(M-1, M, 1 - \phi_r)$ is $1 - (1 - \phi_r)^M$, which asymptotes to 1 as M grows so, for M large enough, it is above $\frac{k}{Ax\alpha}$. ■

Proof of Lemma 4. Let the random variable $\tilde{U}$ represent the number of unskilled managers when M managers generate the same first-period return $r \in \{-x, +x\}$. This random variable follows a binomial distribution $\text{Bin}(M, 1 - \phi_r)$. Let $\varepsilon_1 = \frac{1}{2} \frac{k}{Ax\alpha}$. From the weak law of large numbers, we know that for any $\delta \in (0, 1)$ there is an m_1 such that $M > m_1$ implies

$$\Pr \left\{ \frac{\tilde{U}}{M} \leq (1 - \phi_r) - \delta \right\} < \varepsilon_1 < \frac{k}{Ax\alpha}. \quad (13)$$

From Lemma 3, we know that for M large enough ($M > m'_1$, say),

$$\frac{k}{Ax\alpha} < B(F, M, 1 - \phi_r) = \Pr \left\{ \frac{\tilde{U}}{M} \leq \frac{F}{M} \right\}. \quad (14)$$

Together, Equations (13) and (14) imply that

$$\frac{F}{M} > (1 - \phi_r) - \delta \quad (15)$$

for $M > \max(m_1, m'_1)$.

Similarly, if we let $\varepsilon_2 = \frac{1}{2} \left(1 - \frac{k}{Ax\alpha}\right)$, we know that for the same δ as above there is an m_2 such that $M > m_2$ implies

$$\Pr\left\{\frac{\tilde{U}}{M} \geq (1 - \phi_r) + \delta\right\} < \varepsilon_2 < 1 - \frac{k}{Ax\alpha}. \quad (16)$$

From Lemma 3, we also have $\Pr\{\tilde{U} \leq F - 1\} = B(F - 1, M, 1 - \phi_r) < \frac{k}{Ax\alpha}$ for M large enough ($M > m'_2$, say), which is equivalent to

$$\Pr\left\{\frac{\tilde{U}}{M} \geq \frac{F}{M}\right\} > 1 - \frac{k}{Ax\alpha}. \quad (17)$$

Together, Equations (16) and (17) imply that

$$\frac{F}{M} < (1 - \phi_r) + \delta \quad (18)$$

for $M > \max(m_2, m'_2)$. So, for $M > \max(m_1, m'_1, m_2, m'_2)$, equations (15) and (18) imply that $(1 - \phi_r) - \delta < \frac{F}{M} < (1 - \phi_r) + \delta$. Since we can make δ arbitrarily small, this means that $\frac{F}{M}$ converges to $1 - \phi_r$, as desired. ■

Proof of Proposition 4. There are two elements to the proof: (i) contracting efficiency for M managers who all make return $r \in \{-x, +x\}$ converges to first-best as M goes to infinity; (ii) the effect of the number N of managers on the distribution of M , combined with (i), implies convergence as N goes to infinity.

- (i) Contracting efficiency for M managers who make r converges to first-best as M goes to infinity if the expected fraction of unskilled managers who are retained and the expected fraction of skilled managers who are fired converge to zero as M goes to infinity. As in the proof of Lemma 4, we let the random variable $\tilde{U}$ represent the number of unskilled managers when M managers generate the same first-period return $r \in \{-x, +x\}$.

Expected fraction of retained unskilled managers goes to zero. Let us denote the fraction of retained unskilled managers by $\tilde{f}_U \equiv \frac{\max(0, \tilde{U} - F)}{M}$. For any $\delta \in (0, 1)$, we have

$$\begin{aligned}
 E[\tilde{f}_U] &= \Pr\{\tilde{f}_U \leq \delta\} E[\tilde{f}_U \mid \tilde{f}_U \leq \delta] + \Pr\{\tilde{f}_U > \delta\} E[\tilde{f}_U \mid \tilde{f}_U > \delta] \\
 &\leq 1 \cdot \delta + \Pr\{\tilde{f}_U > \delta\} \cdot 1 = \delta + \Pr\{\tilde{f}_U > \delta\} \\
 &= \delta + \Pr\left\{\frac{\tilde{U} - F}{M} > \delta\right\} = \delta + \Pr\left\{\frac{\tilde{U}}{M} > \frac{F}{M} + \delta\right\}.
 \end{aligned} \tag{19}$$

From Lemma 4 (and its proof), we know we can choose an m such that $M > m$ implies both that $(1 - \phi_r) - \frac{\delta}{2} < \frac{F}{M} < (1 - \phi_r) + \frac{\delta}{2}$ and also that $\Pr\left\{\frac{\tilde{U}}{M} > (1 - \phi_r) + \frac{\delta}{2}\right\} < \delta$. These inequalities imply

$$\begin{aligned}
 \delta &> \Pr\left\{\frac{\tilde{U}}{M} > (1 - \phi_r) + \frac{\delta}{2}\right\} = \Pr\left\{\frac{\tilde{U}}{M} > \left[(1 - \phi_r) - \frac{\delta}{2}\right] + \delta\right\} \\
 &\geq \Pr\left\{\frac{\tilde{U}}{M} > \frac{F}{M} + \delta\right\},
 \end{aligned}$$

so that $E[\tilde{f}_U]$ in Equation (19) is smaller than 2δ . Since we can make δ arbitrarily small, this implies that the expected fraction of retained unskilled managers goes to zero.

Expected fraction of fired skilled managers goes to zero. Let us denote the fraction of fired skilled managers by $\tilde{f}_S \equiv \frac{\max(0, F - \tilde{U})}{M}$. For any $\delta \in (0, 1)$, we have

$$\begin{aligned}
 E[\tilde{f}_S] &= \Pr\{\tilde{f}_S \leq \delta\} E[\tilde{f}_S \mid \tilde{f}_S \leq \delta] + \Pr\{\tilde{f}_S > \delta\} E[\tilde{f}_S \mid \tilde{f}_S > \delta] \\
 &\leq 1 \cdot \delta + \Pr\{\tilde{f}_S > \delta\} \cdot 1 = \delta + \Pr\{\tilde{f}_S > \delta\} \\
 &= \delta + \Pr\left\{\frac{F - \tilde{U}}{M} > \delta\right\} = \delta + \Pr\left\{\frac{\tilde{U}}{M} < \frac{F}{M} - \delta\right\}.
 \end{aligned} \tag{20}$$

Again, from Lemma 4, we know we can choose an m such that $M > m$ implies both that $(1 - \phi_r) - \frac{\delta}{2} < \frac{F}{M} < (1 - \phi_r) + \frac{\delta}{2}$ and also that $\Pr\left\{\frac{\tilde{U}}{M} < (1 - \phi_r) - \frac{\delta}{2}\right\} < \delta$. These inequalities imply

$$\begin{aligned}
 \delta &> \Pr\left\{\frac{\tilde{U}}{M} < (1 - \phi_r) - \frac{\delta}{2}\right\} = \Pr\left\{\frac{\tilde{U}}{M} < \left[(1 - \phi_r) + \frac{\delta}{2}\right] - \delta\right\} \\
 &\geq \Pr\left\{\frac{\tilde{U}}{M} < \frac{F}{M} - \delta\right\},
 \end{aligned}$$

so that $E[\tilde{f}_S]$ in Equation (20) is smaller than 2δ . Since we can make δ arbitrarily small, this implies that the expected fraction of fired skilled managers goes to zero. This completes the first part of the proof.

- (ii) To finish the proof, observe that the number $\tilde{M}$ of managers who generate a first-period return of r follows a binomial distribution with N draws and probability of success p equal to either $\frac{1-\phi\alpha}{2}$ for $r = -x$ or to $\frac{1+\phi\alpha}{2}$ for $r = +x$. Thus from the weak law of large numbers we know that $\frac{\tilde{M}}{N}$ converges in probability to p . From the first part of the proof, we know that for any $\delta \in (0, 1)$ there is an m such that $\tilde{M} > m$ implies that the expected fraction of the $\tilde{M}$ managers who are skilled and fired or unskilled and retained is less than $\frac{\delta}{2}$. The convergence of $\frac{\tilde{M}}{N}$ to p means that there is an n such that $N > n$ implies that $\Pr\left\{\frac{\tilde{M}}{N} \leq \frac{m}{N}\right\} < \frac{\delta}{2}$ (since $\frac{m}{N}$ goes to zero and p stays the same as N grows). So the expected fraction of managers who are skilled and fired or unskilled and retained is less than $\frac{\delta}{2} + \frac{\delta}{2} = \delta$. Therefore, since we can make δ arbitrarily small, efficiency losses converge to zero as N goes to infinity. This completes the proof. ■

Appendix B: Proof that $\tilde{w}_2$ in Equation (4) is Signaling-Proof in the Second Period

At the beginning of the second period, the manager knows whether or not she is skilled. The investors do not, and so update the probability that the manager is skilled to, say, $\phi' \in (0, 1)$. Any contract offered to the investors by a skilled manager can and has to be imitated by the unskilled manager (i.e., the managers pool in equilibrium): otherwise, the manager is immediately identified as unskilled, and she is not hired by the second-period investors.

In equilibrium, any contract offered to the second-period investors extracts all of the available surplus, denoted S for this proof. So, if the contract offered by the manager pays $\omega_{+x} \geq 0$ for positive returns and $\omega_{-x} \geq 0$ for negative returns, it will be the case that, in this pooling equilibrium,

$$\frac{1 + \alpha\phi'}{2}\omega_{+x} + \frac{1 - \alpha\phi'}{2}\omega_{-x} = S,$$

which implies that ω_{+x} must satisfy

$$\omega_{+x} = \omega_{-x} + \frac{2(S - \omega_{-x})}{1 + \alpha\phi'}. \quad (21)$$

The expected compensation of a skilled manager is then

$$\frac{1 + \alpha}{2} \omega_{+x} + \frac{1 - \alpha}{2} \omega_{-x}$$

which, using (21), is equal to

$$\frac{1 + \alpha}{1 + \alpha\phi'} S - \frac{\alpha(1 - \phi')}{1 + \alpha\phi'} \omega_{-x}.$$

This last expression is clearly decreasing in ω_{-x} , so the skilled manager will want to reduce it as much as possible. Given limited liability, it will be the case that the skilled manager prefers a contract with $\omega_{-x} = 0$ to any other contract allowing investors to break even. Thus the equilibrium second-period contract is of the form specified in Equation (4). ■

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