A Theory of Corporate Capital Structure and Investment

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This article uses a general equilibrium framework to explore the origins and limitations of financial intermediaries. In the model, investors have a generic lending technology that they can improve at a cost. Those who upgrade become intermediaries to exploit their advantage. However, conflicts with depositors will limit the banks' market presence, and they will only lend to moderately endowed firms while bondholders will finance cash-rich corporations. The article also analyzes the extent to which investors adopt the superior lending technique, the nature of bank competition, and how corporate and bank conditions affect interest rates and investment.

This article investigates why and how financial intermediaries form and why they coexist with bond markets. In the model presented here, investors have a generic lending technology that they can improve at a cost. This improved technology allows investors to reorganize firms more efficiently, so those who specialize will also become intermediaries to spread the superior technology as far as possible. Financial intermediation, however, is limited since corporate loans have some undiversifiable risk that exposes intermediaries to possible failure. Costly bank defaults will raise the intermediary's cost of capital and limit the size of the banking sector.

The dilemma for a company looking for funds is that if it borrows from bondholders it may face high reorganization costs, while if it borrows from a bank it has to pay for the intermediary's higher cost of capital. In this setting, cash-rich companies tap the bond market directly since they are unlikely to default and only want to bypass a costly middleman. Meanwhile, firms with moderate resources borrow from banks since they are more likely to need the intermediary's reorganizational skills. Finally, companies with scarce funds receive no credit.

Recent work on intermediation has developed along two complementary paths: the first strand of literature has argued that banks are good reorganizers. This has been developed by Diamond (1984), Williamson (1986, 1987a, b), Rajan (1992), Chemmanur and Fulghieri (1994), Bolton and Freixas (2000), and supported by Bolton and Sharfstein (1996), who

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show that concentrating debt in the hands of one investor (e.g., a bank) lowers reorganization costs. These predictions have been confirmed empirically by Gilson, Kose, and Lang (1990). The present article shows the obverse result, namely, that good reorganizers will become intermediaries. The second strand of literature was developed by Boyd and Prescott (1986), Diamond (1991), Besanko and Kanatas (1993), Holmstrom and Tirole (1997), Park (2000), and Almazan and Suarez (2003), who argue that intermediaries are good at preventing managers from investing in inferior projects. Although these two functions are not mutually exclusive,[1] the present article emphasizes the first strand of literature, that is, the reorganizational role of investors. This is not because screening is unimportant: it clearly is significant, and proof of this is the existence of venture capitalists and debt covenants. However, there are some reasons why screening is less relevant for debt intermediaries than for equity investors: first, because equity provides a more appropriate incentive to screen projects than does risky debt.[2] In addition creditors are reluctant to become actively involved with borrowers, since such involvement could subordinate their claims [Weintraub and Resnick (1980, ¶ 5.14, note 247)].

This article uses insights from the above-mentioned literature to develop a general equilibrium theory of banking. The advantage of this approach is that it forces us to explain coherently how banks form and compete when their distinguishing trait — superior knowledge — is not randomly allocated by nature but purposefully acquired by rational investors.[3] Thus we gain insight into why the superior lending techniques are adopted and how this adoption affects the stability of bank competition and of further entry. In the model developed here, whoever acquires the superior skills must give up some capital. This sunk cost does not affect competition directly, but it will condition the size of the banking sector. As investors become intermediaries, their leverage qualifies how much they can lend and raises their marginal borrowing costs. These two features — capacity constraints and increasing marginal costs — allow banks to have expost rents even when they engage in Bertrand-like competition.[4] Bank

[1] Indeed, Boyd, Chang, and Smith (1998) develop a model that considers both banking functions.

[2] See Admati and Pfleiderer (1994). Note that security design is a moot issue in Holmstrom and Tirole (1997), Diamond (1991), and Besanko and Kanatas (1993), since there are only two possible payoffs, making risky debt and equity indistinguishable. The question becomes relevant when there are more than two payoffs, as happens in this article.

[3] Boyd and Prescott (1986) also study the endogenous formation of intermediaries, which they define as a group of agents that publicly announces investment, evaluation, and contracting rules to its members, as well as consumption outcomes. The institutional arrangement they obtain resembles a keiretsu because there will be physical investment (done by good investors), third-party project evaluation (by bad investors who become auditors), and outside depositors (those investors left out of the coalition).

[4] See Tirole (1988, p. 211–216) on the resolution of the Bertrand paradox. Winton (1997) takes a careful look at other possible industrial configurations (monopoly, oligopoly, etc.). He finds that it is not always advantageous for banks to increase in size in a finite economy.

rents fall as new intermediaries enter the industry, so there will be an equilibrium where the marginal investor is indifferent between specializing or not, that is, when the present value of bank rents is equal to the cost of specializing. Bond markets will be larger when specialization costs are high or interest rates are low. Banks will be less efficient whenever interest rates rise.

The article is arranged as follows: Section 1 lays down and discusses the underlying assumptions. Section 2 introduces specialized investors and shows that these lenders will become intermediaries, that intermediaries will be limited by conflicts with their depositors, and that firms borrow from banks or bondholders depending on their internal wealth. Section 3 derives the aggregate equilibrium of this economy.

1. Assumptions

Consider an economy with two types of agents, entrepreneurs and investors, who write contracts at time $t = 0$.

Entrepreneurs are risk neutral and can apply their talents to launch a project. This requires a fixed investment that we set to one without loss of generality. The project's expected return is μ. Its actual payoff $s \in [0, \infty]$ is realized at $t = 1$. The stochastic factor s has a density function $f(s)$ satisfying the increasing hazard rate property.[5]

Entrepreneurs are solely differentiated by their internal funds $e < 1$ and are continuously distributed according to a density function $h(e)$. Since entrepreneurs have insufficient funds to complete the project, they must borrow $1 - e$ from outside investors. Entrepreneurs can alternatively invest their funds in a safe asset yielding a gross return r_f.

Investors are competitive and risk neutral, and they can always invest in a safe asset with a gross return r_f. Investors can be subdivided according to the lending technology they adopt. Generic investors (bondholders), whose type is indicated by $\bar{\theta}$, have a generic lending technology and in equilibrium use only their own capital to lend. Specialized investors (banks), whose type is indicated by $\underline{\theta}$, are those investors who improve their lending technology by giving up a fraction λ of their capital. Specialized investors are also called banks because in equilibrium they lend using other people's money in addition to their own.

Although outside investors can intervene in a project to verify its revenues, this will delay any appropriation of funds to time $t = 1 + \Delta(\theta)$, where the delay depends on the investor's type. Gilson, Kose, and Lang (1990) have documented that private and court-administered

[5] The hazard rate is defined as $\rho(s) \equiv f(s)/[1 - F(s)]$. Although the weaker necessary condition is that $s\rho(s)$ increases with s, I will assume $\rho'(s) > 0$ for expositional purposes. This property is widely used in models of incentive contracts. See Fudenberg and Tirole (1991, chap. 7).

interventions are quite prolonged, taking on average 15 and 28 months, respectively, to complete. These delays are costly if investors and entrepreneurs get substandard returns in the interim. In particular, one can show that the present value of the payoff of an intervened project is a fraction of the original income s.[6] Franks and Torous (1994) and Tashjian, Lease, and McConnel (1996) have shown that recovery rates in private reorganizations, in prepackaged bankruptcies, and in formal bankruptcies are 80%, 73%, and 51%. These results imply that intervention is costly and that reorganization costs vary considerably for different procedures. Since there is some flexibility in this formulation, I will define $\theta < 1$ as the fraction of the project's revenues that are lost in a reorganization. In reduced form, the model I have just described is a costly state verification (CSV) framework with proportional costs.[7]

I assume that banks can reorganize projects more efficiently than bondholders, so $\underline{\theta} < \bar{\theta}$. Indeed, Gilson, Kose, and Lang (1990) find that companies who use bank debt are more likely to reorganize privately (i.e., faster and cheaper) than entrepreneurs who use publicly traded debt. To maintain tractability, I rule out bank-bondholder lending coalitions by assuming that such hybrid loans would generate as much verification as if only bondholders had lent. This assumption makes sense if one believes, as Bolton and Sharfstein (1996), that bank verification costs are low because debt is held in a few hands. The world I present is one where entrepreneurs either borrow from one bank (to keep reorganization costs low) or from a possibly large number of bondholders. Park (2000) provides interesting theoretical arguments that explain the simultaneous use of bank and bond finance in highly levered companies. Detragiache, Garella, and Guiso (2000) also explain that firms may avoid single bank arrangements to prevent an intermediary from exploiting its informational monopoly. These arguments notwithstanding, Cantillo and Wright

[6] Formally, if interim returns are $\exp(-k)r_f$ with $k > 0$, then the payoff at $t = 1 + \Delta(\theta)$ is $s[\exp(-k)r_f]^{\Delta(\theta)}$ and its present value from the perspective of $t = 1$ is $\frac{s[\exp(-k)r_f]^{\Delta(\theta)}}{r_f^{\Delta}(\theta)} = s\ \exp[-k\Delta(\theta)] < s$

[7] The CSV setup, introduced by Townsend (1979) and used later by Gale and Hellwig (1985), is based on the simple yet powerful argument that when outside investors have less information than insiders, their intervention is costly and wasteful. This waste is minimized by the debt contract, in which creditors play a passive role as long as a firm meets its obligations. This result fits well with the financing patterns of firms in industrialized countries, as documented by Mayer (1990). One problem with CSV is that debt may no longer be the optimal instrument if one allows stochastic verification. However, Boyd and Smith (1994) find that stochastic verification barely raises welfare, and argue that even minuscule implementation or enforcement costs of stochastic monitoring will make it inferior to deterministic verification. The most serious weakness of CSV is that it may not be renegotiation proof. One can defend CSV in two ways: first, Rubinstein and Wolinsky (1992) show that time-consuming renegotiation widens considerably the set of renegotiation-proof contracts. In our setting, if renegotiation is more prolonged and costly than intervention, the CSV results still obtain. The second argument in favor of the CSV setup is that one can re-create its results using an incomplete contracts framework, as in Hart and Moore (1998). In that article, debt is an optimal renegotiation-proof contract and there is still inefficient liquidation. The CSV framework has some advantages and drawbacks relative to Hart and Moore's setup, but both lead to similar results (use of debt, existence and use of costly intervention), which are taken here as a starting point.

(2000) show empirically that the exclusive choice for bank and bond finance obtains in almost 90% of their firm-year observations.

I define $x = [e\ r_f\ \lambda\ \phi]$ as the vector describing the project's attributes: e is the fraction of the project that is internally financed, r_f is the riskless rate, λ is the cost of specializing, and ϕ is a measure of aggregate corporate wealth. An entrepreneur with attributes x issues securities at time $t = 0$ which promise a contingent payoff $P(s)$. $Y(s)$ is an indicator function dependent of the announced state that equals one if investors intervene the project. Gale and Hellwig (1985) show that the optimal incentive-compatible solution to this problem is the standard debt contract, where the entrepreneur makes a fixed payment $P(s) = b$ in good states $(s \geq b)$, where b is the promised payment to the lender, who does not intervene the project. Bankruptcy is triggered whenever the project's revenues fall below the promised payment, that is, when $s < b$. In default, the entrepreneur is intervened by his creditors and forced to pay as much as he can, so $P(s) = s$ and $Y(s) = 1$ for $s < b$.

2. An Economy with Multiple Investor Types

This section studies the equilibrium when there are two lending technologies. The basic intervention technique is free, but it is not as efficient as the improved technology, which is costly to acquire. One of the key questions that we ask in this section is if a superior lending technology became available, would everyone adopt it? To answer this question, notice that investors can acquire the superior technology in two ways: either directly by becoming a bank, or indirectly by lending to a bank. The section argues that both participation modes are limited by specialization costs and risk. In other words, some investors will specialize and become intermediaries, and yet they will not entirely displace bondholders from the credit markets. Let us review the arguments in more detail.

A generic investor can adopt the superior lending technology directly. To do this he must give up a fraction λ of his capital W_i.[8] Section 2.1 shows that when $\lambda = 0$, all investors specialize and no intermediation occurs, since it is pointless to become a depositor when you yourself can specialize at no cost. Section 2.2 shows that the story changes dramatically if $\lambda > 0$: although λW_i is a sunk cost, it affects specialized investors by reducing their capital and by raising their borrowing costs. Specialization stops when a marginal investor gets as much on a generic technology as he does from using his reduced wealth on a superior technique.

[8] The assumption of proportional specialization costs ensures that heterogeneity among investors plays no role in the model. λ can be viewed in two ways: as a one-time setup expense, or as a recurring cost due to the bank's need to hold liquid assets, as in Repullo (2000).

Generic investors can also share in the superior technology indirectly by lending their funds to specialized investors. This establishes financial intermediation as a way to spread superior lending techniques without forcing everybody to adopt them directly. Intermediation is limited whenever there is an undiversifiable risk component, since this makes banks liable to default. An intermediary will stop borrowing from generic investors when the marginal bankruptcy cost of leverage equals the marginal profit from lending.

If banks are to limit their activities, to whom should they lend? Proposition 2 shows that banks will lend to entrepreneurs with moderate resources, while bond holders will lend to richly endowed entrepreneurs. This section concludes by analyzing how bond markets and banks react to changes in specialization costs, corporate wealth, and interest rates.

2.1 No specialization costs

This subsection assumes that specializing is costless, so $\lambda = 0$. The economic profit functions for investors and entrepreneurs, $u(x;\, b,\, \theta)$ and $v(x;\, b)$, are defined in Equations (1) and (2). They can be decomposed into expected revenue less opportunity cost of capital.

$$u(x;b,\theta)=b[1-F(b)]+\int_0^b[1-\theta]sf(s)ds-[1-e]r_f\equiv R(b,\theta)-[1-e]r_f \quad (1)$$

$$v(x;\, b)=\int_b^\infty[s-b]f(s)ds-er_f\equiv R_v(b)-er_f \quad (2)$$

It can easily be shown that, given our assumptions about $f(s)$, $R(b,\, \theta)$ is quasiconcave and attains a peak at some $\beta(\theta)$ which is independent of entrepreneur attributes. I define a lending equilibrium below, and Proposition 1 shows that a lending equilibrium exists for cash-rich entrepreneurs. Cash-poor entrepreneurs, on the other hand, receive no funds because their high bankruptcy costs prevent potential investors from ever making a profit. The point that separates these two sets is a cash level of $e \equiv c(r_f;\, \theta)$ defined in Proposition 1.

Definition 1. *A lending equilibrium is the lowest promised payment $b(\theta)$ where investors break even and entrepreneurs profit: $b(x;\theta) \equiv min\{b : u(x;\, b,\, \theta) = 0$ and $v(x;\, b) \geq 0\}$*

Proposition 1. *Define the cutoff $e \equiv c(r_f;\, \theta)$ as*

$$c(r_f;\theta)=\begin{cases} 0 & \forall r_f\in[0,R(\beta,\theta)] \\ 1-R(\beta,\theta)r_f^{-1} & \forall r_f\in(R(\beta,\theta),\mu-\int_0^\beta\theta sf(s)ds] \\ \kappa(r_f,\theta)\in(1-R(\beta,\theta)r_f^{-1},1),\dfrac{\partial\kappa}{\partial r_f}>0 & \forall r_f\in(\mu-\displaystyle\int_0^\beta\theta sf(s)ds,\mu) \\ 1 & \forall r_f\in[\mu,\infty). \end{cases}$$

An equilibrium $b(x;\theta) \in ([1-e]r_f, \beta)$ *exists if and only if* $e \in [c(r_f;\theta), 1]$. $\frac{\partial c}{\partial r_f} \geq 0$.

All entrepreneurs borrow when interest rates are low, but some will drop out of the credit market as the riskless rate rises. The reason for this, which was explained in Williamson (1987a), is that an increase in the risk-free rate makes the outside option more attractive for investors, who demand higher payments. This raises expected verification costs and makes the projects — especially those launched by poorly endowed entrepreneurs — less profitable. The market eventually shuts down when the riskless rate equals the project's expected return μ. In contrast, when there are no verification costs, all projects are financed as long as $r_f < \mu$.

2.1.1 Dominance of specialized lender equilibria when $\lambda = 0$. Whenever possible, I use an upper bar and lower bar on the variables to denote that they belong to generic and specialized investors. If $\lambda = 0$, specialized investors will dominate the credit markets, since their profit function is always greater than the payoff for a generic lender, as shown in Equation (3). Furthermore, there will be no intermediation, since there is no point in lending to a specialized investor if you can become one at no cost,

$$u(x;b) = \bar{u}(x;b) + \int_0^b [\bar{\theta} - \underline{\theta}] sf(s)ds \tag{3}$$

Corollary 1 spells out the dominance of specialized investors. Corollary 1(i) shows that when $\lambda = 0$, specialized investors can lend to a greater set of firms than generic investors. It also shows that the specialized investor equilibrium is more efficient than its generic investor counterpart since $\underline{b}(x) < \bar{b}(x)$.

To study how far the specialized technology will spread, we need to define a function $d(x)$ that computes the maximal excess returns that a bank could extract from a project with attributes x. Such a bank would have to charge a rate $b(x, \bar{\theta})$, defined in Proposition 1, to entrepreneurs with access to the bond market. This limit pricing keeps bondholders at bay. The hypothetical bank would also extract all surplus from entrepreneurs with no access to the bond market. The reader should bear in mind that $d(x)$ is a purely theoretical construct because competitive banks will always earn less than $d(x)$. This construct is nevertheless useful to analyze how competitive banks should optimize their resources and compete with each other. The decision function $d(x)$, characterized in Corollary 1(ii), is shown in Figure 1.

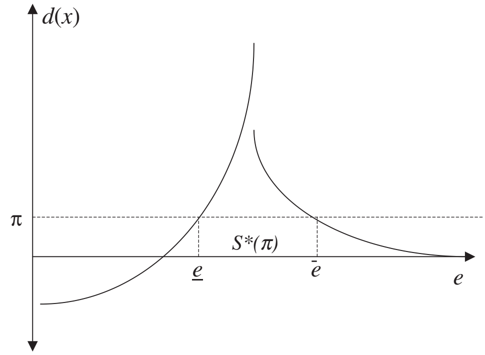

Figure 1
Optimal bank portfolio of loans
The x-axis plots the level of entrepreneurial internal funds, and the y-axis plots the maximum return that a bank lending to such an entrepreneur may get. Note that for entrepreneurs with access to the bond market, this function is decreasing. The level π indicates the banks' extra cost of capital due to intermediation costs. Banks will only lend to entrepreneurs with funds $e \in [\underline{e}, \bar{e}]$.

Corollary 1.

(*i*) *When* $\lambda = 0$ *specialized investors lend to more entrepreneurs and at a lower cost than generic lenders, so* $\underline{c}(r_f) \leq \bar{c}(r_f)$ *and* $\underline{b}(x) < \bar{b}(x)$ *for* $e \in [\bar{c}, 1]$

(*ii*) $d(x)$ *satisfies* $\begin{cases} d(x) < 0 & \text{for } e \in [0, c) \\ d(x) \geq 0,\ d_e(x) > 0 & \text{for } e \in [c, \bar{c}) \\ d(x) \geq 0,\ d_e(x) < 0 & \text{for } e \in [\bar{c}, 1) \\ d(x) = 0 & \text{for } e = 1. \end{cases}$

2.2 Coexistence of generic and specialized lenders

Suppose that it is costly to specialize and that corporate loans have an idiosyncratic and lender-specific shocks.[9] Suppose further that a specialized investor with capital $(1 - \lambda)W_i$ lends to a set S_i of entrepreneurs and charges a different rate $b(x)$ to each $e \in S_i$. The bank's loans and expected revenue are

$$L(S_i) = \int_{S_i} [1 - e] dH(e) de \tag{4}$$

$$\underline{R}(S_i) = \int_{S_i} \underline{R}[b(x)] dH(e) de, \tag{5}$$

[9] An example of a lender-specific factor is an ex ante uncertain verification technology, such that at $t = 1$, investor verification costs are $(\theta + \varepsilon_i)s$, where $E(\varepsilon_i) = E(\varepsilon_i s) = 0$. Another example is the specific way in which macroeconomic shocks affect a given bank.

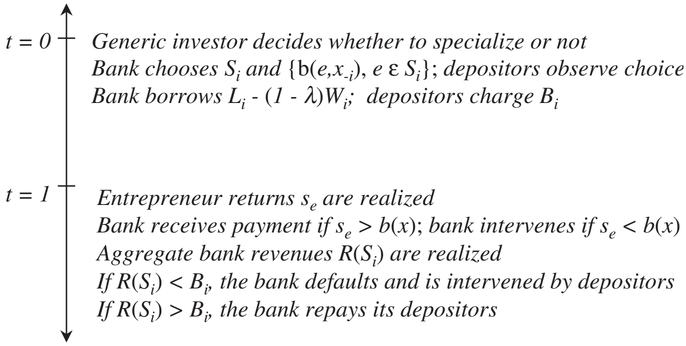

Figure 2
Timeline relating bank and depositors' actions
There are three agents in the economy: entrepreneurs who borrow from banks, who in turn borrow from depositors. At time $t = 0$, all contracts are signed and are publicly observable. At $t = 1$, all outcomes are realized and entrepreneurs or banks may default. If an entrepreneur defaults, its bank will intervene it, while if a bank defaults, then its depositors will interevene it.

where $\underline{R}[b(x)]$ is defined in Equation (1). Under some regularity conditions [see Al-Najjar (1995)], idiosyncratic credit risks are diversified away by a nontrivial set of loans S_i, but the lender-specific factor will remain, and the actual bank revenues will be

$$\tilde{\underline{R}}(S_i) = \underline{R}(S_i)y_i \quad y_i \in [0, v],$$

and the bounded common risk y_i has an expected value of one and a density $g(y_i)$. Furthermore, I assume that this shock is identically distributed for any two banks, so $g(y_i) = g(y_j)$, and that $g(y)$ satisfies the increasing hazard rate property.

I show in Proposition 2(ii) that banks will always lend over and above their remaining capital, so $L(S_i) > [1 - \lambda]W_i$. Note that since y_i is generated by bank-specific factors, $\tilde{\underline{R}}(S_i)$ is private information and thus costly to verify. Thus the optimal contract between the intermediary and generic investors is standard debt, so we can think of these generic investors as bank depositors. The bank borrows $L(S_i) - [1 - \lambda]W_i$ from generic investors, who charge a rate $\bar{r}(S_i)$. The bank owes $[L(S_i) - [1 - \lambda]W_i]$ $\bar{r}(S_i)$, and defaults whenever $y_i < B_i \equiv [L(S_i) - [1 - \lambda]W_i]\bar{r}(S_i)/\underline{R}(S_i)$. Figure 2 restates the sequence of events.

2.2.1 Equilibrium lender behavior. A generic investor has to decide if he should spend λW_i to specialize. If he does, he needs to choose the set S_i of entrepreneurs to whom he will lend, and the rate $b(x)$ that he will charge to each entrepreneur $e \in S_i$. S^* is the union of all the sets where banks lend.

$\{S^*, \{\underline{b})e, x_{-i}) : e \in S^*\}\}$ is a Nash equilibrium if

E1. No bank has an incentive to modify $\underline{b}(x)$ for any $e \in S^*$.
E2. No bank has an incentive to lend to firms $e \notin S^*$.
E3. No bank has an incentive to change its aggregate lending.
E4. The remaining generic investors have no incentive to specialize.

Proposition 2 characterizes this equilibrium:

Proposition 2. *The equilibrium bank behaviour is as follows:*

(*i*) *The equilibrium set and pricing are* $S^*(\pi) = \{e : d(e, x_{-i}) \geq \pi\} = [e, \bar{e}]$ *and* $\underline{b}(x)$ *such that* $\underline{R}[\underline{b}(x)] = [1 - e][r_f + \pi]$. $\underline{e}(x)$ *and* $\bar{e}(x)$ *satisfy* $\underline{e}_\pi \geq 0$, $\underline{e}_{r_f} \geq 0$, $\bar{e}_\pi \leq 0$, $\bar{e}_{r_f} \geq 0$.
(*ii*) *There exists a unique equilibrium premium* $\pi(x) > 0$. $\pi(x)$ *increases in* λ *and* r_f.

The key assumption in Proposition 2(*i*) is the existence of at least two banks in this economy.[10] The proof shows that any lending set and pricing different from that Proposition 2(*i*) produces at least one disgruntled bank that undercuts the others in a close parallel to a Bertrand competition. Intermediaries stop cutting their rates when returns equal their marginal borrowing costs.[11] Proposition 2(*i*) shows that banks lend only to entrepreneurs whose loans have an excess return potential above π, that is, only where banks can cover their marginal intermediation costs. The optimal portfolio $S^*(\pi) = [\underline{e}, \bar{e}]$, plotted in Figure 1, is uniquely identified with the premium π. Banks discard entrepreneurs with scarce funds, who are always unprofitable, and entrepreneurs with abundant resources, since these are contested away by competitive bondholders. Note that when $\lambda > 0$, it is not always the case that banks lend more than bondholders.

Banks compete among themselves until the rate of return for each project $e \in S^*$ has dropped to $r_f + \pi$. Proposition 2(*i*) signs the derivatives of $\underline{e}(x)$ and $\bar{e}(x)$, and shows that aggregate bank loans and expected revenues are

$$L(\pi, r_f) = \int_{\underline{e}}^{\bar{e}} [1 - e]h(e)de \tag{6}$$

$$\underline{R}(\pi, r_f) = \int_{e}^{\bar{e}} \underline{R}(\underline{b}(x))h(e)de = [r_f + \pi]L(\pi, r_f). \tag{7}$$

[10] A similar assumption is made, for example, by Boyd and Prescott (1986, p. 217). Although a monopolist bank will certainly have a different pricing policy, one can show that this will not otherwise change any of the following results.

[11] The reader should be cautioned not to take the Bertrand analogy too literally, since the marginal cost here is an endogenous variable that is determined partly by the banks and partly by general equilibrium considerations.

Proposition 2(*ii*) deals with the equilibrium quantity of loans that is consistent with perfect competition among lenders, profit maximization, and the equilibrium amount of bank loans:

$$\underline{u}(S_i) = [\pi + r_f]L_i \int_{B_i}^{v} [y_i - B_i]g(y_i)dy_i - W_i r_f = 0 \tag{8}$$

$$\begin{aligned}\bar{u}(S_i) = {} & [\pi + r_f]L_i[B_i[1 - G(B_i)] + \int_0^{B_i} [1 - \bar{\theta}]y_i g(y_i)dy_i] \\ & - [L_i - [1 - \lambda]W_i]r_f = 0\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{d\underline{u}(S_i)}{dL_i} &= \frac{\partial \underline{u}(S_i)}{\partial L_i} + \frac{\partial \underline{u}(S_i)}{\partial B_i}\frac{dB_i}{dL_i} \\ &= [\pi + r_f] \int_{B_i}^{v} [y_i - B_i]g(y_i)dy_i - \frac{[1-\lambda]W_i r_f}{L_i[1-\bar{\theta}B_i\rho(B_i)]} = 0\end{aligned} \tag{10}$$

$$\sum_i L_i = L(\pi, r_f) = \int_{\underline{e}}^{\bar{e}} [1 - e]h(e)de. \tag{11}$$

Equations (8) and (9) say that banks and their depositors make no ex ante economic rents. Equation (10) uses Equation (9) to obtain dB_i/dL_i and says that banks set the size of their loan portfolio to maximize profits. Banks and depositors are "small" in that they do not consider the general equilibrium impact on $\underline{b}(x)$ when they modify their leverage. Equation (11) is a market-clearing condition for the equilibrium amount of bank loans. The above equations tell us four things about banks: their leverage, the yield on their borrowings, their return on assets, and their aggregate use of internal capital. Although these variables are determined simultaneously, I will try to explain their genesis as sequentially as possible in what follows.

The optimal bank leverage follows from Equations (9) and (10), which say that an intermediary borrows from generic investors until his marginal bankruptcy cost equals the return on assets $\pi + r_f$. Equation (9) establishes the optimal rate charged by depositors to a bank given its leverage and return on assets. Proposition 2(ii) shows that all banks have the same leverage and pay the same rate $B(\lambda)$ on their borrowings, where $B'(\lambda) > 0$. This makes sense, since any divergence would induce low-cost banks to take away market share from high-cost intermediaries until their marginal costs equalized.

What is the equilibrium return on bank assets? First, note that more specialization lowers the return on intermediary assets, since every time a new investor specializes there is more capital chasing the same loans. In other words, bank leverage declines. This is followed by a round of rate cuts and of loan expansions. In the end, although more loans are made, bank leverage and the return on their assets will decline. To determine how far specialization will go, consider that a generic investor with capital W_i can at most expect to get $W_i r_f$ when he lends in the competitive bond market. Therefore he will stop specializing only when bank revenues equal $W_i r_f$, that is, when Equation (8) holds. In equilibrium, the optimal specialization will determine the return on assets π. The equilibrium premium π, shown in Equation (12), is obtained by combining Equations (8)–(10).

$$\pi \equiv \frac{P(B)r_f}{1-P(B)} > 0,$$
$$P(B) \equiv \bar{\theta}[B\rho(B)\int_B^v [y_i - B]g(y_i)dy_i + \int_0^B y_i g(y_i)dy_i] < 1 \tag{12}$$

$$\pi_{r_f} = \frac{P(B)}{1-P(B)} = \frac{\pi}{r_f} > 0 \quad \pi_\lambda = \frac{P'(B)B'(\lambda)r_f}{[1-P(B)]^2} > 0 \quad \pi_{\lambda r_f} = \frac{P'(B)B'(\lambda)}{[1-P(B)]^2} > 0 \tag{13}$$

One can verify that $P'(B) > 0$, $P(v) = \bar{\theta} < 1$, and $P(0) = 0$. A positive premium implies that intermediaries and bondholders coexist in the credit markets. Equation (13) shows that the premium increases when setup costs or the risk-free rate rise. As setup costs mount, banks need to borrow more to finance the same number of projects. This raises their default rate and premium. As the risk-free rate rises, banks need to promise higher payments to depositors, making bank failures more likely.

In the aggregate, bank loans are extended to all entrepreneurs with potential returns $d(x) \geq \pi$. This will establish the total number of bank loans. The aggregate bank capital is determined by dividing the total bank loans by the bank leverage ratio.

The behavior of this economy is sketched in Figure 3. To understand this diagram, consider a coordinate (r_f, e) that specifies a risk-free rate r_f and an entrepreneur with internal funds e. If the coordinate is in the gray area, the entrepreneur borrows directly from the bond market. If the coordinate is in the white area, the entrepreneur borrows from the bank. If the point is in the black area, the entreprenur is unable to borrow.

Wealthy entrepreneurs (i.e., with funds $e \geq \bar{e}(x)$) tap the bond market directly since they rarely default and need little verification. Generic investors beat intermediaries in this segment, since all that matters is who has the lowest cost of capital. The bond market is enlarged whenever bank setup costs rise or risk-free rates drop. Higher setup costs weaken banks by raising their cost of capital. This induces a contraction in bank loans that is partly offset by an expansion in bonds. Reduced risk-free rates, on the other hand, loosen bondholders' participation constraint.

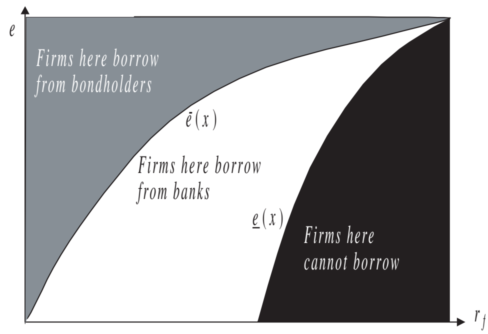

Figure 3
Selection of bank loans and bonds as a function of r_f
The x-axis plots the level of the gross risk-free rate, while the y-axis plots the level of entrepreneurs' internal funds. At very low risk-free rates, all entrepreneurs borrow from bondholders. At moderate risk-free rates, entrepreneurs with scarce internal funds switch to bank funding or are rationed. When the risk-free rate equals the project's expected return, all entrepreneurs become rationed.

This lowers verification and reduces the rents $d(x)$ that banks can extract for entrepreneurs $e \in [\bar{c}, 1]$. Lower rents will force banks to abandon some of their high-grade loans, who will now switch to the bond market.[12]

Entrepreneurs with funds $e \in [\underline{e}(x), \bar{e}(x))$ borrow from banks, since they are more likely to default than their high-grade peers. Entrepreneurs with moderate resources cannot afford to switch away from banks since the gain from eliminating the middleman is outweighed by the loss from more damaging verification at the hand of generic investors. As setup costs λ fall, banks' cost of capital drops, enabling them to steal projects away from bondholders and to fund previously rationed entrepreneurs.

Entrepreneurs with low wealth (i.e., with funds $e < \underline{e}(x)$) receive no credit since neither banks nor bondholders can ever make money from lending to them. The number of excluded entrepreneurs increases as bank setup costs or the risk-free rate rise. Higher setup costs λ raise the banks' cost of capital, spoiling previously borderline projects. An increase in the risk-free rate raises banks' outside opportunities, forcing them to demand a higher payment from entrepreneurs, who will now default more often. More frequent default raises verification costs and spoils borderline projects. An increase in the risk-free rate will also raise banks' premium, compounding the problems for poorly endowed firms.

3. The General Equilibrium of this Economy

This section studies the economy's equilibrium from aggregate investment and savings.

[12] A second-order effect, the decrease in π as r_f drops, is too faint to help banks retain their high-grade customers.

3.1 Aggregate investment

Corporate investment, shown in Equation (14), is the sum of the investments made by entrepreneurs with access to either bank or bondholder finance, that is, with funds $e \in [\underline{e}, 1]$. $\underline{e}(x)$ is characterized in Proposition 2(*i*). To parametrize aggregate corporate wealth, we define the distribution of entrepreneurs as $H(e, \phi)$, where a rise in the first-order stochastic dominance parameter ϕ implies a general increase in corporate capital, so $H_{\phi}(e; \phi) < 0$:

$$I(x) = \int_{\underline{e}}^{1} h(e)de = 1 - H(\underline{e}, \phi) \tag{14}$$

Corollary 2 considers how changes in the risk-free rate, capital losses, and corporate internal funds affect investment.

Corollary 2. *Investment increases with a rise in corporate wealth and with a decrease in the riskless rate or in bank setup costs:* $I_{\phi}(x) > 0$ *and* $I_{x_i}(x) < 0$ *for* $x_i = r_f, \lambda$.

Although corporate wealth is irrelevant in neoclassical investment models, it plays a pivotal role in a CSV framework. For example, if an entrepreneur with funds $e < \underline{e}$ receives an influx $\Delta e > \underline{e} - e$, its investment jumps from zero to one. The aggregate effect very much embodies this intuition.

A rise in the risk-free rate reduces investment because it raises banks' cost of capital and forces intermediaries to cut off cash-poor entrepreneurs from the credit markets. Rationing is compounded because the premium π rises when the riskless rate increases. In a CSV model, as opposed to a frictionless model, an increasing risk-free rate cuts investment even among entrepreneurs with good projects ($\mu > r_f$) because this rise exacerbates agency problems between investors and entrepreneurs. Finally, aggregate investment increases as bank setup costs decrease: smaller λ will lower banks' cost of capital, which allows intermediaries to lend to previously rationed entrepreneurs.

3.2 Aggregate savings

Since this article is premised on the separation between investors and entrepreneurs, it is unreasonable to use a representative agent to study the supply of capital. Instead, I divide agents in two classes: entrepreneurs and investors. For simplicity's sake, I assume that entrepreneurs care only about terminal wealth. Their savings will equal their endowment e, whether they use it to fund their own project (if $e \geq \underline{e}$) or to buy obligations (if $e < \underline{e}$).

Investors care about consumption at time $t = 0$ and $t = 1$. The representative investor is risk neutral with respect to states, but his utility is nonseparable in time: $U = C_0^{1-\alpha} C_1^{\alpha}$. The investor has exogenous endowments E_0 and E_1. Although outside investors cannot own shares in any firm (this is an implication of the CSV model), they will own corporate or bank debt. In addition, those investors who specialize will get bank dividends.[13]

Equation (15) shows C_0 and C_1: consumption at $t = 0$ equals the initial endowment less savings S_a. Expected consumption at $t = 1$ equals the expected return on savings $S_a r_f$ plus the endowment E_1.[14] Equation (16) displays the investor's first-order condition, which determines investor savings S_a.

$$C_0 \equiv E_0 - S_a \quad C_1 \equiv E(\tilde{C}_1) = S_a r_f + E_1 \tag{15}$$

$$[1-\alpha]\frac{C_1}{r_f} = \alpha C_0 \Rightarrow S_a = \alpha E_0 - \frac{[1-\alpha]E_1}{r_f} \tag{16}$$

Aggregate savings S, defined in Equation (17), sums investor and entrepreneur savings.

$$S = S_a + \int_0^1 eh(e)de = \alpha E_0 - \frac{[1-\alpha]E_1}{r_f} + E(e) \tag{17}$$

$$S_{r_f} = \frac{[1-\alpha]E_1}{r_f^2} > 0 \quad S_\phi = -\int_0^1 H_\phi(e,\phi)de > 0 \tag{18}$$

Equation (18) shows that aggregate savings rise with an increase in the risk-free rate. When the risk-free rate is zero, investors try to bring all their consumption to the first period, so $S(0) = -\infty$. As r_f tends to infinity, aggregate savings tend to $\alpha E_0 + E(e)$. Equation (18) also says that an increase in entrepreneur wealth raises aggregate savings. This happens because entrepreneurs have more funds with which to invest. One interesting attribute of this savings equation is its insensitivity to changes in the banks' setup cost λ. This result follows from investors' freedom to specialize: thus if λ increases, there will be less entry to the banking sector, but this would be offset by additional savings in the bond market, so that Equation (16) holds.

Corollary 2 showed that investment decreases as the riskless rate increases: at $r_f = 0$, all entrepreneurs borrow, so $I(0) = 1$, while nobody borrows when $r_f \geq \mu$, so $I(r_f) = 0$. In equilibrium, aggregate investment,

[13] Strictly speaking, one should distinguish between investors who specialize and those who remain generic. Nevertheless, the reader can easily verify that such analysis yields an identical aggregate savings function. If entrepreneurs consume at $t = 0$ and $t = 1$, their savings would be a function of expected dividends, complicating the analysis, but not changing any of our results.

[14] S_a can be used to buy riskless bonds and/or obligations or to create a bank. The expected return of any of these activities (factoring setup costs where necessary) is r_f.

defined in Equation (14), must equal aggregate savings, as shown in Equation (17). Proposition 3 shows how changes in the exogenous variables affect the equilibrium investment and risk-free rate. This proposition is not proved since the results follow directly from Corollary 2 and Equation (18).

Proposition 3. *The equilibrium of the economy behaves as follows:*

(*i*) *An increase in current output E_0 or a decrease in future income E_1 raises investment and reduces the risk-free rate, so* $\frac{dI^*}{dE_0} > 0 \frac{dr_f}{dE_0} < 0$, $\frac{dI^*}{dE_1} < 0 \frac{dr_f}{dE_1} > 0$.

(*ii*) *An increase in bank setup costs reduces investment and real interest rates, so* $\frac{dI^*}{d\lambda} < 0 \frac{dr_f}{d\lambda} < 0$.

(*iii*) *An increase in corporate wealth raises investment. The effect on the interest rate is ambiguous* $\frac{dI^*}{d\phi} > 0 \frac{dr_f}{d\phi} = ?$.

Changes in endowments E_0 and E_1 have a life-cycle impact on savings. For example, a decrease in the current endowment or an increase in the future endowment E_1 prompts investors to dissave in order to compensate for the relative shortfall at $t = 0$. In this situation, the savings curve shifts downward, reducing investment and raising interest rates.

An increase in corporate wealth will lift the investment schedule, as more entrepreneurs gain access to the credit markets. An increase in entrepreneurial funds will also increase entrepreneur's private savings and thus lift the aggregate savings schedule. The simultaneous shifts in both schedules increase investment, but have an uncertain impact on interest rates.

Finally, Proposition 3 shows that higher bank setup costs reduce investment and real interest rates, since a higher λ reduces aggregate investment but leaves the savings function unperturbed.

3.3 Numerical example

This section concludes with a numerical example that brings all the results together. Table 1 shows the parameter values to be used. I assume that bankruptcy costs at the hand of generic and specialized investors destroys 50% and 20% of the project's value, respectively. I assume that specialization costs consume about 5% of the investors' capital. Investors have an

Table 1
Parameter value for model's variables

$\bar{\theta} = 0.5$	$\lambda = 0.05$	$E_0 = 96.8672$
$\underline{\theta} = 0.2$	$\alpha = 0.5$	$E_1 = 97.8359$
$s \sim U[0, 2.5]$	$f(s) = 0.4$	$\forall s \in [0, 2.5]$
$e \sim U[0, 1]$	$h(e) = 1$	$\forall e \in [0, 1]$
$y \sim U[0, 2]$	$g(y) = 0.5$	$\forall y \in [0, 2]$

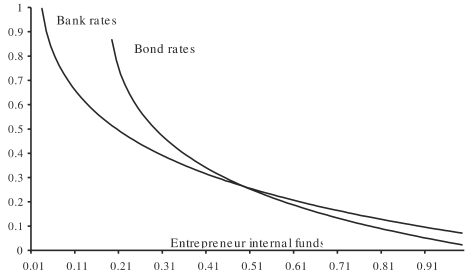

Figure 4
Numerical example of interest rates charged by banks and bondholders to different entrepreneurs
The x-axis plots the entrepreneurial internal funds, and the y-axis plots the interest rate charged to such an entrepreneur by banks and bondholders. The crossing point is at $e = 0.47$, so entrepreneurs with funds less than that but higher than 0.02 will borrow from banks, while entrepreneurs with funds greater than 0.47 will borrow from the bond markets.

initial endowment that grows at 1% per period, and a time preference of 0.5. I assume uniform distributions for the different shocks.

To solve the problem, we begin by using Equations (12), (8), and (9) to obtain banks' risk premium $\pi = 0.0476r_f$, banks' default point $B = 0.1818$, and the ratio of bank capital to total loans $(1-\lambda)\frac{W}{L} = 0.8223$. In the second step, we use the specialized investors' equilibria from Proposition 2(i) to obtain the lowest entrepreneur endowment compatible with an equilibrium ($\underline{e} = 1 - \frac{0.9943}{r_f}$). In the third step, we sum the investment of those entrepreneurs with access to the credit markets, as in Equation (14). We then equate this with aggregate savings from Equation (17) to determine the risk-free rate, which in this example is 2%.

$$I = 1 - e = \frac{0.9943}{r_f} = 0.5\left[96.8672 - \frac{97.8359}{r_f}\right] + 0.5 = S \Rightarrow r_f = 1.02$$

In the last step, we use these results together with Proposition 2(i) and Equation (1) to obtain the interest rates charged by specialized and generic investors to entrepreneurs with wealth e, $\underline{r}(e)$, and $\overline{r}(e)$ and plotted in Figure 4.

$$\underline{r}(e) = \frac{1 - \sqrt{1.224e - 0.224}}{0.6(1-e)}; \quad \overline{r}(e) = \frac{1 - \sqrt{1.0257e - 0.0257}}{0.48(1-e)}$$

Entrepreneurs with wealth $e < 0.02$ are rationed, while entrepreneurs with wealth $e \in [0.02, 0.47]$ borrow from banks. Entrepreneurs with wealth $e > 0.47$ borrow from the bond market. This numerical example shows that when setup costs are positive, it is possible that a majority of entrepreneurs prefer to borrow from the bond market rather than from banks.

4. Conclusion

It is well known that intermediaries play an important role in the economy because of their special attributes. What is not well understood is why, given their advantage at lending, banks are not the only source of corporate funds. A simple answer to this puzzle is that intermediaries are themselves like any firm, since they go bankrupt and they suffer from moral hazard. The present article has explored in detail the consequences of this simple observation and has shown that although banks are more efficient reorganizers than bondholders, they will generally have a higher cost of capital than arm's-length investors. This allows us to understand how companies choose their lenders: cash-rich corporations need not worry about costly interventions, since this is unlikely. The main concern of a well-endowed firm is to borrow from the lender with the lowest cost of capital, that is, directly from bondholders. On the other hand, companies with less cash do need to worry about default and its consequences. These firms prefer to borrow from intermediaries, who are better able to handle financial distress.

This article used an equilibrium approach to characterize bank competition and entry to this industry. The finding was that intermediaries generate ex post rents even when they engage in Bertrand-like competition. These rents, together with the ex ante costs of specializing, determine how many generic investors become banks.

Appendix

Before we begin the proofs, let us redefine lenders' economic profit as

$$u(x;b,\theta) = b[1-F(b)] + [1-\theta]\int_0^b sf(s)ds - [1-e]r_f$$
$$= \int_0^b [1-F(s)]J(x;s,\theta)ds - [1-e]r_f,$$

where $J(x; s, \theta) \equiv 1 - \theta s\rho(s)$ is the so-called virtual surplus that is widely used in mechanism design. The derivative with respect to b yields

$$u_b(x;b,\theta) = [1-F(b)]1 - \theta b\rho(b) = [1-F(b)]J(x;b,\theta).$$

An increasing hazard rate $\rho(s)$ implies that marginal lending costs overtake marginal benefits as b rises. This prevents investors from raising interest rates indefinitely to maximize profits. The economic profit and revenue functions, $u(x; b, \theta)$, $R(b, \theta)$, are quasiconcave and attain a peak at some $\beta(\theta)$ which is independent of x.

Appendix A: Proof of Proposition 1

$S(\theta)$ is the set of attributes where an equilibrium exists for lender θ. Lemma 1 relates the existence of a lending equilibrium and the value of the lender's maximization problem, defined as

$$M(x,\theta) \equiv \max_b \begin{bmatrix} u(x;b,\theta) \\ s.t.\ v(x;b) \geq 0 \end{bmatrix}.$$

In the proof I will drop θ from the notation in the functions' arguments, since it is superfluous.

Lemma 1. *$x \in S$ if and only if $M(x) \geq 0$.*

Proof. (i) If $M(x) < 0$, an investor never lends to a firm with attributes x, since it always loses money in this endeavour. Thus $x \notin S$. (ii) If $M(x) > 0$ there exists a b^* satisfying $b^* \leq \beta$, $u(x;\, b^*) > 0$, and $v(x;\, b^*) \geq 0$. I claim that $u(x;\, b) < 0\ \forall\ b < [1 - e]r_f$. To see this, rewrite $u(x;\, b)$ as $[1 - F(b)][b - (1-e)r_f] + \int_0^b [s - (1-e)r_f]f(s)ds - \int_0^b \theta s f(s)ds$. The first and second terms of this equation are negative for $b < [1 - e]r_f$, and the last term is always negative, so the claim is established. Note that $u_b > 0$ for $b \in ([1 - e]r_f,\, b^* \leq \beta)$. The intermediate value theorem (IVT) holds, so there exists a unique $b \in ([1 - e]r_f,\, b^*)$, where $u(x;\, b) = 0$, $v(x;\, b) > v(x;\, b^*) \geq 0$, and $x \in S$. (iii) If $M(x) = 0$, the rate b^* that solves the maximization also satisfies the definition of an equilibrium and thus $x \in S$. ■

Proof of main theorem. Equations (A.1) to (A.8) characterize $M(x)$:

$$\begin{aligned} M(x) &:= \max_{b \in P(x)} u(x;b) \\ P(x) &= \{b : v(x;b) \equiv R_v(b) - er_f \geq 0\} = \{b : b \leq R_v^{-1}(er_f)\}. \end{aligned} \tag{A.1}$$

The problem can be solved as a constrained maximization due to the quasiconcavity of $\bar{u}(x;b)$:

$$\begin{aligned} L &= u(x;b) + \lambda[R_v^{-1}(er_f) - b] \\ L_b &= [1 - F(b)][1 - \theta b\rho(b)] - \lambda = 0 \\ L_\lambda &= R_v^{-1}(er_f) - b \geq 0 \quad \lambda \geq 0 \\ \lambda L_\lambda &= 0. \end{aligned} \tag{A.2}$$

If $L_\lambda > 0$, the constraint is not binding, and the problem is solved by a β such that $u_b(x;\beta) = 0$, where $u_b(x;\, b) = [1 - F(b)]J(x;\, b)$, and $J(x,\, b)$ is the virtual surplus defined above. The IVT holds since $J(x;\, 0) = 1$, $\lim_{b\to\infty} J(x;\, b) \leq \lim_{b\to\infty}\{1 - \theta b\rho(\epsilon)\} = -\infty$ for $\epsilon > 0$, and $J_b(x;b) < 0$. Hence there exists a unique β where $J(x;\beta) = 0 \Rightarrow u_b(x;\beta) = 0$. If $L_\lambda = 0$, the participation constraint is binding so $v(x;\, b_v) \equiv 0 \Rightarrow b_v \equiv R_v{}^{-1}(er_f)$. Note that $\lim_{b\to\infty} v(x;\, b) = -er_f < 0$ and $v(x;\, 0) = \mu - er_f$, so b_v exists only when the IVT holds, and this happens only if $\mu - er_f > 0$. Equation (A.3) defines the constrained maximization rate $b^*(x)$, while Equation (A.4) determines the relative magnitudes of β and b_v, using the fact that $R'_v(b) < 0$.

$$b^*(x) = \min[b_v = R_v^{-1}(er_f),\, \beta] \tag{A.3}$$

$$\beta < b_v \quad \text{if and only if } R_v(\beta) > R_v(b_v \equiv R_v^{-1}(er_f)) = er_f \tag{A.4}$$

Equation (A.5) displays the derivatives of the dual $M(x) = u(x;\, b^*)$. I claim that $M_e > 0$ and $M_{r_f} < 0$: if $b^* = \beta$, the second argument in Equation (A.5) is zero — this is an application of the envelope theorem — so $\overline{M}_e = r_f > 0$ and $\overline{M}_{r_f} = -(1-e) < 0$. If $b^* = b_v < \beta$, then $v(x;\, b_v) \equiv 0$, and $0 < J(x;\, b_v) < 1$. I use the implicit function theorem to obtain Equations (A.6) and (A.7), which prove the claim. Finally, Equation (A.8) shows the sum of expected revenues [Equations (1) and (2)]. I will now prove Proposition 1 on a case-by-case basis:

$$M_{x_i}(x) = u_{x_i}(x;b^*) + u_b(x;b^*)b^*_{x_i}(x) \tag{A.5}$$

$$M_{x_i}(x) = u_{x_i}(x;b_v) + J(x;b_v)v_{x_i}(x,b_v) \quad \text{when } b^* = b_v \tag{A.6}$$

$$M_e(x) = [1 - J(x;b_v)]r_f > 0 \quad M_{r_f}(x) = -[1 - e(1 - J(x;b_v))] < 0 \tag{A.7}$$

$$R(b) + R_v(b) \equiv \mu - \int_0^b \theta s f(s)ds \tag{A.8}$$

(a) $r_f \in [0, \mu - \int_0^\beta \theta s f(s)ds)$. Define $c(r_f) = \max[0, 1 - R(\beta)r_f^{-1}]$ and evaluate

$$R_v(\beta) - c(r_f)r_f = \min[R_v(\beta), R_v(\beta) + R(\beta) - r_f] = \min[R_v(\beta), \mu - \int_0^\beta \theta s f(s)ds - r_f] > 0.$$

The inequality comes from Equation (A.8) and the assumption that $r_f < \mu - \int_0^\beta \theta s f(s)ds$. Equation (A.4) implies that $b^*(x) = \min[b_v(x), \beta] = \beta$ at $c(r_f)$. At $c(r_f)$, the dual is $M(c, x_{-i}) = \max[R(\beta) - r_f, 0] \geq 0$. Since $M_e(x) > 0$ it is clear that $M(e, x_{-i}) \geq M(c, x_{-i}) \geq 0$ $\forall\, e \in [c, 1]$. Let us consider the other firms: if $r_f \leq R(\beta)$, then $c = 0$ and $M(c, x_{-i}) > 0$, and there are no rationed firms. If $r_f \in (R(\beta), \mu - \int_0^\beta \theta s f(s)ds)$, then $c = 1 - R(\beta)r_f^{-1} > 0$ and $M(c, x_{-i}) = 0$. Further, $M(e, x_{-i}) < M(c, x_{-i}) = 0$ for firms $e \in [0, c)$. These results and Lemma 1 prove the first two lines of Proposition 1.

(b) $r_f \in (\mu - \int_0^\beta \theta s f(s)ds, \mu)$. All firms $e \geq 1 - R(\beta)r_f^{-1}$ have

$$R_v(\beta) - er_f \leq R_v(\beta) + R(\beta) - r_f = \mu - \int_0^\beta \theta s f(s)ds - r_f < 0.$$

The inequality comes from Equation (A.8) and the assumption that $r_f > \mu - \int_0^\beta \theta s f(s)ds$. Equation (A.4) implies that $b^*(x) = \min[b_v(x), \beta] = b_v(x)$ for $e \geq 1 - R(\beta)r_f^{-1}$.

$$M(1 - R(\beta)r_f^{-1}, x_{-i}) = R(b_v) - (1-e)r_f = R(b_v) - R(\beta) < 0 \tag{A.9}$$

$$M(1, x_{-i}) = R(b_v) = b_v[1 - F(b_v)] + \int_0^b s[1-\theta]f(s)ds > 0 \tag{A.10}$$

Equation (A.9) shows that $M(1 - R(\beta)r_f^{-1}, x_{-i}) < 0$ since $R(\beta) > R(b)\ \forall b \neq \beta$. Equation (A.10) shows that $M(1, x_{-i}) > 0$. Since $M_e > 0$, one can invoke the IVT to assert the existence of a unique $\kappa(r_f) \in (1 - R(\beta)r_f^{-1}, 1)$, where $M(\kappa, x_{-i}) \equiv 0$. Further, $M(x) \geq 0$ $\forall e \geq \kappa(r_f)$ and $M(x) < 0\ \forall e < \kappa(r_f)$. This and Lemma 1 prove the third line of Proposition 1. One uses the implicit function theorem for $M(\kappa, x_{-i}) \equiv 0$ and Equation (A.7) to show that $\kappa'(r_f) = -M_{r_f}/M_e > 0$.

(d) $r_f \in (\mu, \infty)$. Combine Equations (1) and (2) to obtain

$$u(x; b) + v(x; b) = \mu - r_f - \int_0^b \bar{\theta} s f(s)ds < 0 \quad \text{if } r_f > \mu.$$

Hence it is never possible to achieve any $u(x;\, b) > 0$ and $v(x;\, b) \geq 0$. This implies that $M(x) < 0$ for all e, and there is no equilibrium for any firm. ∎

Appendix B: Proof of Corollary 1

Proof of Corollaries 1(i) and 1(ii). Equation (B.1) shows that banks profit by simply matching the rate $\bar{b}(x)$ charged by competitive bondholders to firms $e \in [\bar{c}, 1]$:

$$\underline{u}(x; \bar{b}) = \bar{u}(x; \bar{b}) + \int_0^{\bar{b}} [\bar{\theta} - \underline{\theta}] s f(s)ds = \int_0^{\bar{b}} [\bar{\theta} - \underline{\theta}] s f(s)ds > 0. \tag{B.1}$$

I claim that $\bar{b}(x) \leq \underline{\beta}$. Before, I will show that $\bar{\beta} < \underline{\beta}$, defined by $\bar{u}_b(x; \bar{\beta}) \equiv 0$ and $\underline{u}_b(x; \underline{\beta}) \equiv 0$. To establish the relative magnitudes of $\overline{\beta}$ and $\underline{\beta}$, one evaluates specialized investors' marginal payoff $\underline{u}_b(x;\, b)$ at $\bar{\beta}$. Note that $J(x; \bar{\beta}, \underline{\theta}) = J(x; \bar{\beta}, \bar{\theta}) + [\bar{\theta} - \underline{\theta}]\bar{\beta}\rho(\bar{\beta}) = [\bar{\theta} - \underline{\theta}]\bar{\beta}\rho(\bar{\beta}) > 0$. Thus $\underline{u}_b(x; \bar{\beta}) \equiv [1 - F(\bar{\beta})]J(x; \bar{\beta}, \underline{\theta}) > 0$ and a specialized investor must charge $\underline{\beta} > \bar{\beta}$ to set his marginal payoff to zero. The maximization rate for specialized lenders is $\underline{b}^*(x) = \min[b_v(x) = R_v^{-1}(er_f), \underline{\beta}] \geq \bar{b}^*(x) = \min[b_v = R_v^{-1}(er_f), \bar{\beta}]$. Finally, note that $\bar{b}(x) \leq \bar{b}^*(x) \leq \underline{b}^*(x) \leq \underline{\beta}$, which proves the claim.

Equation (B.1) shows that $\underline{u}(x;\, \bar{b}) > 0$. The preceding argument shows that $\underline{u}_b(x; b) > 0$ for $b < \bar{b}(x) \leq \underline{\beta}$. One can also show that $\underline{u}(x;b) < 0$ for $b < [1 - e]r_f$, following the arguments in Lemma 1. The IVT holds, and there exists a unique bank lending rate $\underline{b}(x) \in ((1-e)r_f, \bar{b}(x))$, where $\underline{u}(x;\, \underline{b}) = 0$, $v(x;\, \underline{b}) > 0$, and which drives bondholders out of the market. Further, since $\underline{M}(x) \geq \underline{u}(x; \bar{b}) > 0 \ \forall \ x \in S(\bar{\theta})$, there exists a neighborhood, including $e < \bar{c}(r_f)$, where $\underline{M}(x) > 0$. This follows from the continuity and monotonicity of $\underline{M}(x)$. These results and Lemma 1 imply that the cutoff satisfies $\underline{c}(r_f) \leq \bar{c}(r_f)$. This proves Corollaries 1(*i*) and 1(*ii*). ■

Proof of Corollary 1(iii). The maximal rate that a specialized lender can charge and still deter bondholders is

$$\underline{b}^d(x) = \begin{cases} \underline{b}^*(x) = \min[b_v(x) = R_v^{-1}(er_f), \underline{\beta}] & \text{for } e \in [0, \bar{c}] \\ \bar{b}(x) & \text{for } e \in [\bar{c}, 1], \end{cases} \tag{B.2}$$

where $\underline{b}^*(x)$ solves the specialized investors' maximization, and $\bar{b}(x)$ is defined by $\bar{u}(x;\, \bar{b}) \equiv 0$. The maximal excess return for specialized lenders is

$$d(x) = \frac{\underline{u}(x; \underline{b}^d(x))}{1-e} = \begin{cases} \dfrac{\underline{u}(x; \underline{b}^*)}{1-e} & \text{for } e \in [0, \bar{c}] \\ \displaystyle\int_0^{\bar{b}} \frac{[\bar{\theta} - \underline{\theta}] s f(s) ds}{1-e} & \text{for } e \in [\bar{c}, 1]. \end{cases} \tag{B.3}$$

There is no equilibrium for $e \in [0, \underline{c})$, so Lemma 1 implies that $\underline{M}(x) = \underline{u}(x;\, \underline{b}^*) < 0$ and $d(x) < 0$.

For $e \in [\underline{c}, \bar{c})$, an equilibrium exists, so $\underline{M}(x) = \underline{u}(x; \underline{b}^*) \geq 0$ and thus $d(x) \geq 0$. Proposition 1 also shows that $M_e(x; \theta) = \underline{u}_e(x; \underline{b}^*) > 0$, so $d_e(x) = [(1 - e)\underline{u}_e(x; \underline{b}^*) + \underline{u}(x; \underline{b}^*)]/(1 - e)^2 > 0$.

For $e \in [\bar{c}, 1)$, Equation (B.1) implies that $\underline{u}(x; \bar{b}) > 0$, and thus $d(x) \geq 0$. Equation (B.4) displays $d_e(x)$. This equation uses $\bar{u}(x; \bar{b}) \equiv 0$ to find b_e and to replace $(1 - e)r_f$. The last line of this equation uses integration by parts on the first term in parentheses, and exploits the fact that $sf(s) \equiv s\rho(s)[1 - F(s)]$ for the second term in parentheses:

$$d_e(x) = [\bar{\theta} - \underline{\theta}] \frac{\bar{b} f(\bar{b})(1-e)\frac{d\bar{b}}{de} + \int_0^{\bar{b}} sf(s)ds}{[1-e]^2} \quad \frac{d\bar{b}}{de} = -\frac{u_e}{u_b} = \frac{-r_f}{[1-F(\bar{b})][1-\bar{\theta}\bar{b}\rho(\bar{b})]}$$

$$d_e(x) = -[\bar{\theta} - \underline{\theta}] \frac{\bar{b}\rho(\bar{b})(1-e)r_f - \int_0^{\bar{b}}[1-\bar{\theta}\bar{b}\rho(\bar{b})]sf(s)ds}{[1-e]^2[1-\bar{\theta}\bar{b}\rho(\bar{b})]}$$

$$d_e(x) = -[\bar{\theta} - \underline{\theta}] \frac{\bar{b}\rho(\bar{b})\{\bar{b}[1-F(\bar{b})] + \int_0^{\bar{b}}(1-\bar{\theta})sf(s)ds\} - \int_0^{\bar{b}}[1-\bar{\theta}\bar{b}\rho(\bar{b})]sf(s)ds}{[1-e]^2[1-\bar{\theta}\bar{b}\rho(\bar{b})]}$$

$$d_e(x) = -[\bar{\theta} - \underline{\theta}] \frac{\bar{b}\rho(\bar{b})\{\bar{b}[1-F(\bar{b})] + \int_0^{\bar{b}} sf(s)ds\} - \int_0^{\bar{b}} sf(s)ds}{[1-e]^2[1-\bar{\theta}\bar{b}\rho(\bar{b})]}$$

$$d_e(x) = -[\bar{\theta} - \underline{\theta}] \frac{\bar{b}\rho(\bar{b}) \int_0^{\bar{b}}[1-F(s)]ds - \int_0^{\bar{b}} sf(s)ds}{[1-e]^2[1-\bar{\theta}\bar{b}\rho(\bar{b})]} = -[\bar{\theta} - \underline{\theta}] \frac{\int_0^{\bar{b}}[\bar{b}\rho(\bar{b}) - s\rho(s)][1-F(s)]ds}{[1-e]^2[1-\bar{\theta}\bar{b}\rho(\bar{b})]} < 0.$$

At $e = 1$ $d(x) = 0$. To see this, note that $\bar{u}(x; \bar{b}) \equiv 0$ for $e = 1$ implies that $\bar{b}(x) = 0$. We use l'Hôpital's rule: $\lim_{e\to 1}\{d(x)\} = -\lim_{e\to 1}\{[\bar{\theta} - \underline{\theta}]\bar{b}(x) f(\bar{b}(x))\} = 0$. ■

Appendix C: Proof of Proposition 2

Proof of Proposition 2(i). Lemma 2 defines a decomposable set and shows that such a set admits profitable deviations. Proposition 2(*i*) is an application of Lemma 2.

Definition. An aggregate lending set $\{S, \{b(e, x_{-i})\ e \in S\}\}$ is decomposable if $\exists\, c$, $S^H \neq \emptyset$, $S^L \neq \emptyset$ such that $\frac{R(b^d(x))}{1-e} \geq c \,\forall\, e \in S \cap S^H$, $\frac{R(b^d(x))}{1-e} \geq c \,\forall\, e \in S^H \backslash S$, and $\frac{R(b(x))}{1-e} < c \,\forall e \in S^L$, where $b^d(x)$ is defined in Equation (B.2).

Lemma 2. *A decomposable set* $\{S,\{b(e,x_{-\mathrm{i}})\ e \in S\}\}$ *admits profitable deviations.*

Proof. I assume that there are at least two banks in this economy to allow for competitive behavior. By construction, there is at least one "unlucky" bank who lends to some firms from the "bad" set and who does not exhaust all the good firms in the economy, so and $S_i \cap S^L \neq \emptyset$ and $(S\backslash S_i) \cap S^H \neq \emptyset$. I will show that this "unlucky" bank always deviates. A deviation strategy S'_i, $\{b'(e,x_{-i})e \in S'_i\}\}$ that improves on S_i, $\{b(e,x_{-i})e \in S_i\}\}$ is

$$\begin{array}{lll} \text{(i) } S'_i\backslash(S'_i \cap S_i) \subset S^H & \text{(ii) } S_i\backslash(S'_i \cap S_i) \subset S^L & \text{(iii) } L(S'_i) = L(S_i) \\ \quad I_1 = S'_i \cap S_i & \quad I_2 = S'_i\backslash('S_i \cap S_i) \subset S^H & I_3 = S_i\backslash(S'_i \cap S_i) \subset S^L \end{array}$$

$$\text{For } e \in I_1 \text{ charge } b'(x) = b(x) \text{ so that } \underline{R}(b'(x)) = \underline{R}(b(x))$$

$$\text{For } e \in I_2 \text{ charge } b'(x) \text{ so that } \underline{R}(b'(x)) = [1-e]c$$

$$\text{For } e \in I_3\ b(x) \text{ can only yield } \underline{R}(b(x)) < [1-e]c.$$

The deviating bank drops some loans from the set S^L and picks up some firms in the set S^H, either by undercutting his competitors or by taking good firms to whom nobody lent before. The deviating set has the same amount of aggregate loans as the old portfolio.

Step 1. I will show that the expected bank revenue from S'_i is greater than expected revenue from S_i, so $R'_\mathrm{i} \equiv \underline{R}(S'_i) > \underline{R}(S_i) \equiv R_i$. The assumption that $L(S'_i) = L(S_i)$ implies that whatever loans are dropped (i.e., set I_3) must be made up by the new loans (i.e., set I_2), so $\int_{I_2} N[1-e]dH(e) = \int_{I_3} N[1-e]dH(\mathrm{e})$. Equations (C.1) and (C.2) display $\underline{R}(S'_i)$ and $\underline{R}(S_i)$. Equation (C.3) shows that $\underline{R}(S'_\mathrm{i}) > \underline{R}(S_i)$, given the assumptions on pricing and on the fixed amount of loans.

$$\underline{R}(S'_i) = N\left[\int_{I_1} \underline{R}(b(x))dH(e) + \int_{I_2} \underline{R}(b'(x))dH(e)\right] \equiv R'_i \tag{C.1}$$

$$\underline{R}(S_i) = N\left[\int_{I_1} \underline{R}(b(x))dH(e) + \int_{I_3} \underline{R}(b(x))dH(e)\right] \equiv R_i \tag{C.2}$$

$$\begin{aligned} R'_i - R_i &= N\left[\int_{I_2} \underline{R}(b'(x))dH(e) - \int_{I_3} \underline{R}(b(x))dH(e)\right] \\ R'_i - R_i &> cN\left[\int_{I_2} (1-e)dH(e) - \int_{I_3} (1-e)dH(e)\right] = 0 \end{aligned} \tag{C.3}$$

Step 2. I will show that the bank's payoff is higher with S'_i than with S_i. First, I claim that depositors charge a lower rate to banks with more profitable portfolios, so $\partial B_i/\partial R_i < 0$. These are the regularity conditions that ensure the existence of a lending equilibrium between the bank and its depositors: (a) $g(y_i)$ has an increasing hazard rate; (b) there exists an unconstrained maximization rate $B_u(L_i)$; (c) at the constrained maximizing rate $\mathrm{B}^*(L_i) \leq B_u(L_i)$, $\bar{u}(x;B^*(L_i)) \geq 0$. One can use the arguments in Lemma 1 to prove the existence of a unique equilibrium $B_i \in ([W_i - L_i]r_f, B^*(L))$, where $\bar{u}(x;B_i) \equiv 0$, as defined in Equation (C.4). Equation (C.5) shows that $\partial B_i/\partial R_i$ is negative since $\bar{\theta}B_i\rho(B_i) < 1$ at $B_i < B_u(L_i)$.

$$\bar{u}(x;B_i) = R_iB_i[1-G(B_i)] + \int_0^{B_i} (1-\bar{\theta})R_iy_ig(y_i)dy_i - [L_i - W_i]r_f \equiv 0 \tag{C.4}$$

$$\frac{\partial B_i}{\partial R_i} = -\frac{\bar{u}_{R_i}(x;B_i)}{\bar{u}_{B_i}(x;B_i)} = -\frac{B_i[1-G(B_i)] + \int_0^{B_i}(1-\bar{\theta})y_ig(y_i)dy_i}{R_i[1-G(B_i)][1-\bar{\theta}B_i\rho(B_i)]} < 0 \tag{C.5}$$

$$\underline{u}(S_i') = R_i' \int_{B_i'}^{v} [y_i - B_i'] g(y_i) dy_i - (1-\lambda) W_i r_f \text{ assuming } \lambda W_i \text{ is sunk} \tag{C.6}$$

$$\underline{u}(S_i) = R_i \int_{B_i}^{v} [y_i - B_i] g(y_i) dy_i - (1-\lambda) W_i r_f \text{ assuming } \lambda W_i \text{ is sunk} \tag{C.7}$$

$$\underline{u}(S_i') - \underline{u}(S_i) = R_i' \int_{B_i'}^{B_i} [y_i - B_i'] g(y) dy + \int_{B_i}^{v} ([R_i' - R_i][y_i - B_i] + R_i'[B_i - B_i']) g(y_i) dy_i > 0 \tag{C.8}$$

Equations (C.6) and (C.7) display the bank's expected payoffs for sets S'_i and S_i. Equation (C.8) establishes that a decomposable set S admits profitable deviations since $\underline{u}(S'_i) > \underline{u}(S_i)$. This follows from Equations (C.3) and (C.5), which say that $R'_i > R_i$ and $B_i > B'_i$. ■

I prove Proposition 2(i) by showing that sets different from $S^*(\pi)$ are decomposable.

(I) A set S such that $S_c \cap S^*(\pi) \neq \emptyset$ and $S \cap S_c^*(\pi) \neq \emptyset$ for some π admits profitable deviations. The subscript c stands for the complement of a set, that is, $[0, 1]/S$. First, define $S^L = \{e \in S : R(b(e, x_{-i})) < \pi\}$, $S^H = S^*(\pi)\backslash S^L$, and $c = \pi + r_f$. Note that $S^H \neq \emptyset$ since $S^*(\pi)\backslash S \neq \emptyset$, and $S^L \neq \emptyset$ since $S \cap S_c^*(\pi) \neq \emptyset$. It is clear that this is a decomposable set. A deviating bank lends to new firms and drops some old unprofitable firms.

(II) A set $\{S^*(\pi) = [\underline{e}, \bar{e}]; b(e, x_{-i}) e \in S^*(\pi)\}$ with subsets S^L, S^H, where $\underline{R}(b(x))/[1 - e] < c < R_u(b(x'))/[1 - e']$, admits profitable deviations. It is clear that this is a decomposable set. A deviating bank cuts the rates to firms $e \in S^H$ and drops some firms in $e \in S^L$. Points (I) and (II) prove how the optimal set and pricing policy looks.

Derivatives. Equations (C.9) and (C.10) define the cutoff rates, where $d(x)$ comes from Equation (B.3), and the indicator functions $\underline{I}$, $\bar{I}$ equal one when there exists an interior solution. I assume that $\underline{e} = 0$ when $\underline{I} = 0$ and $e = 0$ when $\bar{I} = 0$. Equation (C.11) shows the derivatives of $\underline{e}(x)$ by implicitly differentiating Equation (C.9), and by using Equation (A.7) to obtain the derivatives of $\underline{M}(\underline{e}; x)$.

$$\underline{e}(x) \equiv \left\{ [d(\underline{e}, x_{-i}) - \pi]\underline{e} \equiv \left[\frac{\underline{M}(\underline{e}, x_{-i})}{1 - \underline{e}} - \pi \right] \underline{e} \equiv 0, \quad e < \bar{c}(r_f) \right\} \tag{C.9}$$

$$\bar{e}(x) \equiv \{ [d(\bar{e}, x_{-i}) - \pi]\bar{e} \equiv 0, \quad e > \bar{c}(r_f) \} \tag{C.10}$$

$$\underline{e}_\pi(x) = \frac{1 - \underline{e}}{\underline{\theta b}^* \rho(\underline{b}^*) r_f + \pi} \underline{I} \geq 0 \quad \underline{e}_{r_f}(x) = \frac{1 - \underline{e\theta b}^* \rho(\underline{b}^*)}{\underline{\theta b}^* \rho(\underline{b}^*) r_f + \pi} \underline{I} \geq 0 \tag{C.11}$$

$$\bar{e}_\pi(x) = \frac{1}{d_e(\bar{e}, x_{-i})} \bar{I} \leq 0 \quad \bar{e}_{r_f}(x) = -\frac{d_{r_f}(\bar{e}, x_{-i})}{d_e(\bar{e}, x_{-i})} \bar{I} = -\frac{[\bar{\theta} - \underline{\theta}] \bar{b} \rho(\bar{b})}{d_e(\bar{e}, x_{-i})} \bar{I} \geq 0 \tag{C.12}$$

Equation (C.12) shows the derivatives of $\bar{e}(x)$ by implicitly differentiating (C.10). The derivatives $d_e(\bar{e}, x_{-i}) < 0$ and $d_{r_f}(\bar{e}, x_{-i}) > 0$ come from Equation (B.4). This concludes the proof of proposition 2(i). ■

Proof of Proposition 2(ii). Equation (C.13) obtains the equilibrium bankruptcy point $B(\lambda)$, which increases in λ, by combining Equations (8) and (10). Equation (C.14) obtains the equilibrium threshold $\pi(\lambda)$ by combining $\lambda = \bar{\theta} B \rho(B)$ and Equations (8) and (9). Equation (C.15) obtains the inverse of the bank's leverage $l(\lambda)$ by combining Equation (9) and the expression for π in Equation (C.14): it implies that in equilibrium, aggregate loans exceed bank capital, so that specialized lenders act as financial intermediaries. Equation (C.16) shows that bank leverage rises with higher capital losses. Note that leverage and the bankruptcy point are identical for any two banks, and that at $\lambda = 0$, $B(0) = 0$ and intermediation collapses so $l(0)=1$. Finally, Equation (C.17) shows the aggregate bank capital in this economy, where $L(\pi, r_f)$ is defined in Equation (6).

$$0=\lambda-\bar{\theta}B_i\rho(B_i)\Rightarrow B_i=B_j=B(\lambda)\quad\text{and}\quad B'(\lambda)=\frac{1}{\bar{\theta}[\rho(B)+B\rho'(B)]}>0$$

$$\frac{r_f}{\pi+r_f}=B[1-G(B)]+\int_0^B[1-\bar{\theta}]y_ig(y_i)dy_i+[1-\bar{\theta}B\rho(B)]\int_B^v[y_i-B]g(y_i)dy_i \tag{C.13}$$

$$\frac{r_f}{\pi+r_f}=1-\bar{\theta}\left[B\rho(B)\int_B^v[y_i-B]g(y_i)dy_i+\int_0^B y_ig(y_i)dy_i\right]$$

$$\pi=\frac{P(B)r_f}{1-P(B)},\ P(B)=\bar{\theta}\left[B\rho(B)\int_{B_i}^v[y_i-B]g(y_i)dy_i+\int_0^B y_ig(y_i)dy_i\right]<1 \tag{C.14}$$

$$l(\lambda)=\frac{[1-\lambda]W_i}{L_i}=1-\frac{B[1-G(B)]+\int_0^B[1-\bar{\theta}]y_ig(y_i)dy_i}{1-P(B)}<1 \tag{C.15}$$

$$\frac{dl}{d\lambda}=-\left[\frac{[1-G(B)][1-\bar{\theta}B\rho(B)]}{[1-P(B)]}+\frac{\int_0^B[1-G(y_i)][1-\bar{\theta}y_i\rho(y_i)]dy_i}{[1-P(B)]^2}P'(B)\right]B'(\lambda)<0 \tag{C.16}$$

$$W=\frac{\int_B^v[y_i-B]g(y_i)dy_i}{1-P(B)}L(\pi,r_f) \tag{C.17}$$

I assume that $\pi=\frac{P(B)r_f}{1-P(B)}\leq d(x)$ for some firm x. If this condition is violated, banks could not find projects with the appropriate rate of return, and there would be no intermediaries in the economy. These equations finish the proof of Proposition 2(ii). ■

Appendix D: Proof of Corollary 2

$$\frac{d\underline{e}}{dr_f}=\underline{e}_\pi(x)\pi_{r_f}+\underline{e}_{r_f}(x)>0 \tag{D.1}$$

$$\frac{d\bar{e}}{dr_f}=\bar{e}_\pi(x)\pi_{r_f}+\bar{e}_{r_f}(x)=\frac{1}{d_e(\bar{e},x_{-i})}\left[\frac{P(B)}{1-P(B)}-\frac{[\bar{\theta}-\underline{\theta}]\bar{b}\rho(\bar{b})}{[1-\bar{\theta}\bar{b}\rho(\bar{b})]}\right]$$

$$=\frac{1}{d_e(\bar{e},x_{-i})r_f}\left[\pi-\frac{[\bar{\theta}-\underline{\theta}]\bar{b}\rho(\bar{b})r_f}{[1-\bar{\theta}\bar{b}\rho(\bar{b})]}\right]=\frac{1}{d_e(\bar{e},x_{-i})r_f}\left(\frac{\int_0^{\bar{b}}[\bar{\theta}-\underline{\theta}]sf(s)ds}{1-\bar{e}}-\frac{[\bar{\theta}-\underline{\theta}]\bar{b}\rho(\bar{b})r_f}{[1-\bar{\theta}\bar{b}\rho(\bar{b})]}\right)$$

$$=\frac{(1-\bar{e})d_e(\bar{e},x_{-i})}{d_e(\bar{e},x_{-i})r_f}=\frac{1-\bar{e}}{r_f}>0 \tag{D.2}$$

$$I_\lambda(x)=-h(e)\underline{e}_\pi(x)\pi_\lambda(x)>0\quad I_{r_f}(x)=-h(e)\frac{d\underline{e}}{dr_f}<0\quad I_\phi(x)=-H_\phi(e,\phi)>0 \tag{D.3}$$

Equation (D.1) uses Equation (13) and Proposition 2(i) to show that the lower cutoff increases with r_f. Equation (D.2) shows that $d\bar{e}/dr_f>0$. The first line uses Equations (13), (C.12), and (12). The second line uses $\pi=d(\bar{e},x_{-i})$ and Equation (B.4). $I_\lambda(x)>0$ follows from Proposition 2(i) and Equation (13). $I_{r_f}(x)<0$ follows from Equation (D.1), and $I_\phi(x)>0$ follows from $H_\phi(e,\phi)<0$ for a generalized cash increase. Equation (D.3) establishes Corollary 2. ■

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