Whence GARCH? A Preference-Based Explanation for Conditional Volatility

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We develop a preference-based equilibrium asset pricing model that explains low-frequency conditional volatility. Similar to Barberis, Huang, and Santos (2001), agents in our model care about wealth changes, experience loss aversion, and keep a mental scorecard that affects their level of risk aversion. A new feature of our model is that when perturbed by unexpected returns, investors become temporarily more sensitive to news. Gradually investors become accustomed to the new level of wealth, restoring prior levels of risk aversion and news sensitivity. The state-dependent sensitivity to news creates the type of volatility clustering found in low-frequency stock returns. We find empirical support for our model's predictions that relate the scorecard to conditional volatility and skewness.

Over a decade ago, stock prices were commonly characterized as a "random walk with a drift." This description seemed not only to describe stock price movements, but also to be backed by solid theory. Competition among investors should make prices efficient, eliminating any predictability. The last decade, however, has produced some new empirical facts; for example, stock returns seem to be more predictable than a random walk, to be higher than consumption models can explain, and to vary more than cash flow models can justify.[1] Perhaps the most robust new fact is that stock return volatility is serially correlated. Stylized facts about volatility clustering include the following: (1) both "good" news (positive shocks) and "bad" news lead to higher levels of volatility [Engle (1982) and Bollerslev (1986)], (2) bad news tends to increase future volatility more than good news [Nelson (1991)], (3) the effect of news on volatility has a transitory (rapid decay) and more permanent (slow decay) component [Engle and Lee (1999)], and (4) volatility appears to have an effect on the risk premium [Merton (1980) and French, Schwert, and Stambaugh (1987)].

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[1] See Cochrane (1999) for a review of the "new facts in finance."

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Nearly all empirical work in finance published this decade acknowledges and accounts for volatility clustering in returns. Volatility autocorrelation is typically modeled using autoregressive conditional heteroscedasticity (ARCH) [Engle (1982)], generalized ARCH (GARCH) [Bollerslev (1986)], stochastic volatility [Anderson (1994)], Markov switching [Hamilton (1988, 1989)], nonparametrics [Pagan and Ullah (1988)], or some extension of these statistical models.[2] The motivation for using conditional volatility models is their utility — they fit the data. Whereas our statistical knowledge of the degree and nature of volatility clustering is impressive, our theoretical knowledge of *why* volatility clusters seems paltry. The default explanation for time-dependent volatility is that news about economic fundamentals itself is clustered, as if an *exogenous* news supplier works overtime in some periods and takes vacations in others [see, e.g., Anderson and Bollerslev (1997) and Campbell and Hentschel (1992)]. This default explanation is problematic, given the regular nature of macroeconomic news releases and Schwert's (1989) and Pagan and Schwert's (1990) finding that stock return volatility is only weakly related with economic fundamentals.

Several recent articles develop *endogenous* explanations for volatility clustering using heterogeneous traders, where the trading process, not the fundamental news, induces volatility autocorrelation. Brock and LeBaron (1996) show that if diverse traders adapt their trading strategies on a slower time scale than trading takes place, volatility will be autocorrelated. Kelly and Steigerwald (1999), Cao, Coval, and Hirshleifer (2002), and Gaunersdorfer and Hommes (2000) show how the entrance of sidelined investors can induce time-dependent volatility. Peng and Xiong (2001) show how changes in the rate that financial analysts digest news can result in GARCH-like returns. In these models, the *generation* of fundamental news follows an i.i.d. random process, but the *arrival* of news in stock prices through trading is clustered. For three reasons, these explanations are incomplete. First, frictions and feedbacks tend to be symmetric, thus these models are not able to explain the good/bad news asymmetry. Second, market frictions are short-lived, thus they are candidates for explaining the short-lived clustering found in high-frequency data. They are not useful, however, in explaining the more permanent news effect on volatility found by Ding and Granger (1996), and Bollerslev and Mikkelsen (1996). Third, Engle and Lee (1999) and Kim, Morley, and Nelson (2003) show that only the long-run component of volatility clustering affects the risk premium. Our preference explanation complements the frictions and feedback explanations since it models the asymmetric volatility clustering at low frequencies. Furthermore, our

[2] Pagan and Schwert (1990) compare, contrast, and test the GARCH, Markov switching, and nonparametric representations of stock return volatility.

model has the advantage of simultaneously explaining both the existence of volatility clustering and why it is priced.

The leverage models of Black (1972, 1976) and Christie (1982) predict higher volatility after bad news, since firms operate with higher leverage. In contrast to the stylized facts, they also predict lower volatility after good news. The state-uncertainty models of David (1997) and Veronesi (1999) are also capable of explaining shifts in conditional heteroskedasticity through bayesian learning or gradual resolution of uncertainty. However, they too incorrectly predict lower volatility after good news.

We propose a preference-based asset pricing model capable of explaining many of the new empirical facts in finance, including many of the stylized findings about conditional volatility. Since preference models have been used to explain long-run stock predictability and *excess volatility*, it seems natural that such models should be extended to explain *volatility clustering* and its pricing as well. Our model can be viewed as an extension of two extant models. On the one hand, we extend the preference model of Barberis, Huang, and Santos (2001; hereafter BHS) by allowing investors who are away from their customary level of wealth to not only become more or less risk averse, but also to be more sensitive to news. On the other hand, we extend the volatility feedback model of Campbell and Hentschel (1992) by endogenizing the cause of volatility autocorrelation.

Following BHS, our model allows investors to derive utility from fluctuations in wealth as well as from consumption. The level of financial utility derived from portfolio fluctuations depends on a slow-moving measure of prior investment performance (a mental scorecard). When the portfolio is below the benchmark, investors become more risk averse. In addition, the utility increases from gains are smaller than the utility decreases from losses (loss aversion). Preferences over financial utility have an advantage relative to preferences over consumption utility due to the historically low correlation between consumption growth and stock returns. A large stock move has little effect on consumption, on risk aversion tied to consumption, and consequently, on subsequent stock volatility. In contrast, with preferences over financial wealth, a stock price move can induce a sufficiently large change in risk aversion and sensitivity to impact subsequent stock volatility.

We modify BHS in two distinct ways and the resulting new model explains features of conditional volatility. First, whereas BHS investors use the mental scorecard to measure how far ahead or behind their portfolio is, our investors use it to measure how far away from the expected, customary, or habit level the portfolio is. Second, we allow investors to become more attentive and sensitive to news when their portfolio has been perturbed. Gradually investors become accustomed or adapted to the new level of wealth, restoring the normal sensitivity to news. Our time-varying sensitivity to news generates volatility clustering even when news itself follows an i.i.d. random process.

Our asset pricing model explains the asymmetry in volatility autocorrelation. Campbell and Hentschel's (1992) volatility feedback model *exogenously* imposes the clustered arrival of news and then endogenously explains the asymmetry. A large piece of bad news has a direct and indirect effect. Directly, the bad news lowers stock prices. Indirectly, the news signals volatile times, causing discount rates to increase and prices to fall even further. For good news, the indirect (volatility feedback) effect offsets, rather than compounds, the direct effect. Thus volatility feedback implies an asymmetry in the correlation between stock price changes and future volatility. Our model maintains the feedback effect but generates clustering *endogenously*. Rather than presupposing that news is clustered, our agents cause returns to be clustered as they become temporarily news sensitive or attentive when perturbed from their customary level of wealth.

Like BHS, we start by writing down a utility function. We then numerically solve the resulting asset pricing model and show that solutions to and simulations of our model are internally consistent with the new empirical facts. However, as Shefrin and Statman (1994) imply, preference models are capable of explaining nearly anything, and to be useful must be structured so that they can be falsified. We structure our model so that it makes specific refutable predictions that go beyond the new empirical facts that motivated our model in the first place. Like Campbell and Hentschel (1992), we finish by taking our refutable predictions to the data. We find support for our model's predictions regarding conditional volatility and skewness, and limited support for its predictions about excess returns.

Our article is organized as follows. In Section 1 we modify the standard consumption-based model to include wealth-varying degrees of risk aversion and sensitivity to news. In Section 2 we develop our model's implications for equilibrium asset prices. In Section 3 we numerically solve the model, illustrate the solutions, and show, using simulated data, how our model captures many of the new empirical facts, including GARCH, in monthly returns. In Section 4, we develop our model's new refutable predictions with respect to volatility, excess returns, and skewness, and test the predictions on U.S. stock data. Section 5 concludes.

1. The Model

1.1 A Lucas-type economy

Following Lucas (1978), identical, infinitely lived agents with a total mass of one maximize the following time-additive utility function by allocating wealth between consumption, C_t, a risky asset, S_t, and a risk-free asset, B_t, in each period t. The amount consumed and invested is subject to a series of feasibility constraints $C_t + S_t + B_t \leq Y_t + S_{t-1} R_t + B_{t-1} R_{f,t}$, where Y_t is income from an exogenous nonfinancial source (e.g., labor) and $S_{t-1}R_t$ ($B_{t-1}R_{f,t}$) is the value of the risky (risk-free) asset carried over from $t-1$ to t. We denote the gross return on the risk-free and the risky asset from t to $t+1$ as $R_{f,t+1} = 1 + r_{f,t+1}$ and $R_{t+1} = 1 + r_{t+1}$, respectively, and note that $R_{f,t+1}$ is known to the investor at time t. Following BHS, our agents maximize utility not only from consumption, $U(C_t)$, but also from unexpected fluctuations in the value of their financial wealth, $F(W_{t+1})$,

$$\max\ E\left[\sum_{t=0}^{\infty} \delta^t U(C_t) + b_0 \bar{C}_t^{-\gamma} \delta^{t+1} F(W_{t+1})\right], \tag{1}$$

where δ^t is the subjective discount rate that captures investors' exogenous preference for immediacy, $\bar{C}_t$ is aggregate per capita consumption, and $b_0 \bar{C}_t^{-\gamma}$ scales financial utility to consumption utility even if aggregate wealth increases over time. Higher levels of $b_0 > 0$ make financial utility more important relative to consumption utility, whereas $b_0 = 0$ makes financial utility irrelevant, reducing Equation (1) to the standard consumption utility model.

For simplicity, we assume the risk-free asset is in zero net supply and the supply of the risky asset is normalized to one. We choose the standard two-income economy since the historical correlation between aggregate consumption and dividend growth rates is weak. Dividends, D_t, and Y_t are exogenously determined and nonstorable; consequently, in aggregate, $\bar{C}_t = Y_t + D_t$. Aggregate consumption and dividends follow lognormal processes given by

$$\Delta c_t = g + v_t$$

$$\Delta d_t = g + \varepsilon_t,$$

where $c_t = \ln(\bar{C}_t)$, $v_t \sim$ i.i.d. $N(0, \sigma_v^2)$, $d_t = \ln(D_t)$, $\varepsilon_t \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$, and corr $(v_t, \varepsilon_{t+j}) = \rho$ for $j = 0$ and corr $(v_t, \varepsilon_{t+j}) = 0$ for $j \neq 0$. Following BHS, we assume equal expected growth rates, g, for financial and non financial income. Importantly, our two sources of news, v and ε, are not serially correlated. That is, new information or news is not clustered in time.

We define financial gains and losses relative to an expected level as follows:

$$W_{t+1} = S_t X_{t+1}, \tag{2}$$

where $X_{t+1} = R_{t+1} - E_t(R_{t+1})$, or the return shock. Investors experience financial utility, $F(W_{t+1})$, when the return on the risky asset is greater than expected and disutility when it is less than expected. Thus $F(W_{t+1})$ takes its sign from X_{t+1}.

1.2 Utility functions

With regard to consumption, we make the standard power utility assumption, $U(C) = C^{1-\gamma}/(1-\gamma)$. Power utility leads to a constant relative risk aversion parameter, γ, which measures the local curvature of the utility function and also defines the rate of intertemporal substitution. By itself, utility from the traditional consumption model is unable to justify historical risk premiums without extreme levels of risk aversion — levels that are inconsistent with observed low risk-free rates. Furthermore, the standard model does not predict any of the new empirical facts in finance, such as time-varying expected returns, high ex post volatilities of the market return, and, relevant to our article, the clustering of volatility.[3] By adding a second source of utility, BHS overcomes the consumption model's deficiency with time- or wealth-varying degrees of risk aversion.[4]

With regard to wealth fluctuations, we make the following financial utility assumptions:

$$F(W_{t+1}) = \lambda(z_t, X_{t+1})W_{t+1}, \tag{3}$$

$$\begin{aligned} \lambda(z_t, X_{t+1}) &= k(a_0 - a_1 z_t) \quad &&\text{for } X_{t+1} < 0, \\ &= a_0 - a_1 z_t &&\text{for } X_{t+1} \geq 0, \end{aligned} \tag{4}$$

where $k > 0$, $a_0 > 0$, and $a_1 > 0$, and where $z_t \in (-\infty, \infty)$ is a variable that performs two roles.[5] First, z_t acts as an approximate mental scorecard of prior investment performance, although movements in z_t are not necessarily one-for-one with return shocks. Second, the absolute value of z_t measures the degree of attentiveness investors pay to news affecting their portfolios. We call z_t the scorecard and motivate and formalize its two roles below. $\lambda(z_t, X_{t+1})$ controls how much psychological reward or financial utility our investors receive from a gain and how much disutility from a loss; consequently $\lambda(z_t, X_{t+1})$ measures our investors' level of risk aversion.

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[3] For early work on the equity premium puzzle, see Mehra and Prescott (1985), Weil (1989), and Hansen and Jagannathan (1991). For a review of the overall shortcomings of the standard consumption-based asset pricing model, see Cochrane (1997, 2001).

[4] Another approach, introduced by Campbell and Cochrane (1999), is to allow investors to derive utility from consumption relative to a slowly evolving habit level of consumption. Other solutions to the equity premium puzzle include Epstein and Zin (1989), Abel (1990), Constantinides and Duffie (1996) and Bansal and Yaron (2003).

[5] We also impose the restriction $\lambda(z_t, X_{t+1}) = (a_0 - a_1)$ for all $z_t > 1$ and $\lambda(z_t, X_{t+1}) = k(a_0 - a_1)$ for all $z_t < 1$ to maintain investor risk aversion for all levels of the scorecard. Although in theory $z_t \in (-\infty, \infty)$, when we solve the model we choose parameter values to scale z_t between -1 and 1. Our restriction proves to be innocuous since more than 99% of the distribution of the scorecard in our solution is contained in set $[-1, 1]$.

Equations (1) through (4) introduce our preference or behavioral assumptions via the financial utility function. Alternatively, one could directly make consumption utility vary, for example, by making the relative risk aversion parameter, γ, move through time. This alternative is problematic due to the low correlation between consumption growth and stock returns. A large stock movement may have little effect on consumption and consequently may be unable to induce the changes in risk aversion and sensitivity necessary to cause subsequent stock volatility changes. In contrast, when investors experience risk and loss aversion over financial wealth and when these aversions depend on past portfolio performance, a large movement in stock prices induces sufficient variation in aversion and sensitivity to cause subsequent stock volatility.

Two characteristics of our wealth-varying risk aversion are important: loss aversion and scorecard dependence. First, k measures the degree of loss aversion. For any level of the scorecard, the "hurt" felt from a loss is greater than the "help" felt from an equivalently sized gain. Second, a_1 measures how the past performance affects the magnitude of the utility derived from gains and losses. After a sequence of poor returns, the mental scorecard, z_t, is negative and large, $\lambda(z_t, X_{t+1})$ is positive and large, and gains and losses are felt more intensely. Thus investors are more risk averse after past losses. In contrast, after past gains, investors feel gains and losses less intensely and are therefore less risk averse. a_0 is the investors' baseline level of financial utility derived from gains. The baseline level is intensified by k for a loss and by $a_1 z_t$ after past losses, and is mitigated by $a_1 z_t$ after past gains. Loss aversion is consistent with Kahneman and Tversky's (1979) prospect theory. Scorecard dependence is consistent with Thaler and Johnson's (1990) house money effect. Loss aversion and scorecard dependence are illustrated in Figure 1.

Figure 1 shows the financial utility derived from gains and losses for three different levels of the scorecard. The kinks in the three $\lambda(z_t, X_{t+1})$ functions shown in Figure 1 are driven by loss aversion. With $k = 1.5$, a loss's impact on utility is 50% greater that a gain's impact, regardless of past performance. The relative slopes of the three $\lambda(z_t, X_{t+1})$ functions are driven by scorecard dependence. After past losses, say $z_t = -0.5$ illustrated by the dotted line, shocks have a much bigger impact on utility than after past gains, say $z_t = 0.5$ illustrated by the dashed line.

We define the law of motion for z_t, the investors' scorecard of prior performance, as

$$z_{t+1} = \phi z_t + h(z_t) X_{t+1} \tag{5}$$

$$h(z_t) = a_2 |z_t| + a_3, \tag{6}$$

where $0 < \phi \leq 1$ is the decay or memory parameter, $h(z_t)$ measures the scorecard's sensitivity to wealth shocks, $a_2 > 0$ is the scorecard's impact

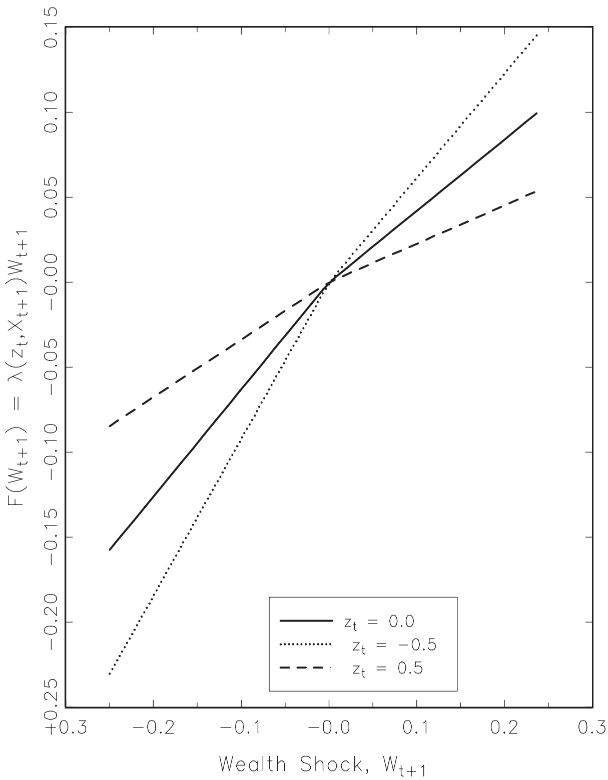

Figure 1
Financial utility function, $F(W_{t+1})$, with $a_1 = 0.4$ and $k = 1.5$.

on sensitivity to shocks, and $a_3 > 0$ is the state-independent or base level of the scorecard's sensitivity to shocks.

Three characteristics of the scorecard are important; the first two follow BHS. First, investors harken back only so far when remembering past portfolio shocks. Investors gradually become accustomed or adapted to new levels of wealth; thus z_t decays toward a steady state of zero. For example, levels of ϕ close to one indicate that investors have long memories (or persistent habits), and, once perturbed, investors remain so for a long time. Second, changes in the scorecard are driven by return shocks. Positive surprises increase and negative surprises decrease the scorecard.

The third scorecard characteristic is unique to our model. In our model, the amount that a positive shock increases and a negative shock decreases the scorecard varies with the level of the scorecard. Investors' responses to news vary over time or, more specifically, vary over levels of past performance. After a large shock or several shocks of the same sign, investors become perturbed or aroused. When perturbed, the investors are more attentive and sensitive to subsequent news.

1.3 Psychological support

Standard neoclassical economists typically assume that the utility derived from consumption depends only on the absolute level and characteristics of the goods consumed. However, many researchers improve on the success of the standard models by deviating from this norm and allowing the utility derived from consumption to depend on the context surrounding the consumption. For example, early work by Veblen (1899) and Duesenberry (1949) argues that utility from consumption depends on an *external* reference point based on the consumption of others. An external and relative reference point or scorecard is the key behavioral assumption of the recent "habit" [Campbell and Cochrane (1999); hereafter CC] and "keeping up with the Jones" [Abel (1990)] models in finance. Closer to our assumption, Helson's (1964) adaptation-level theory shows how individuals evaluate experiences relative to an *internal* reference point or historical scorecard. For example, Pollak (1970, 1976) and Becker and Murphy (1986) show how the utility derived from consumption today depends on past levels of consumption. Kahneman and Tversky (1979) find that people use mental scorecards or "anchors" when making estimates of value and that deviations from these anchors are conservative. An internal scorecard of past portfolio performance is the key behavioral assumption of BHS. Thus our assumption that investors keep a mental scorecard based on past experience is not novel; in contrast, our assumption about how the scorecard moves is.

A unique feature of our model is that investors' attention and sensitivity to financial news is heightened when their scorecard is perturbed.[6] Since this behavior assumption is new, we support the assumption with literature from two branches of psychology: attention and sensitization/habituation. Kahneman (1973) shows that organisms have limited cognitive resources. Given limitations on attention, he finds that an "organism selectively attends to some stimuli, or aspects of stimulation, in preference to others" (p. 3). Kahneman finds that the presentation of a sudden, unexpected, and intense stimulus triggers an orienting response. The orienting response includes extra effort to analyze the stimulus at the expense of other ongoing activities, and particular to our assumption, "an orientation toward probable sources of future significant information ... and a transient increase of arousal ... that allocates [cognitive] capacity ... in advance of relevant stimulation" (p. 48). Orientation and

[6] In Veronesi's (1999) and David's (1997) models, investors' response to news is also contextual, depending on the degree of perceived uncertainty about the economic state and the degree of profitability, respectively.

arousal define a state of preparation for and heightened sensitivity to future stimuli that parallels our assumption regarding a conditional sensitivity to financial news.[7]

Related to Kahneman's (1973) theory on attention is Hilgard and Marquis' (1940) theory of sensitization. Thompson, Donegan, and Lavond (1988, p. 251) define sensitization as, "a process by which presentation of a stimulus can increase responsiveness to subsequent stimulus presentation ... Its effects are often not specific to a particular stimulus dimension or response system and are often thought to reflect a general state change, such as an increase in arousal." Kupfermann (1974, p. 17) associates the aroused state with "an orienting response and increased responsiveness that continues even when the arousing stimulus is no longer present." Thompson and Spencer (1966) document evidence of sensitization and contrast it with habituation. Whereas the initial stimulus temporarily increases sensitivity, repeated stimulation results in a diminishing response or accustomization.

Our use of adaptation, attention, and sensitization theories can be illustrated with a weather analogy. Adaptation explains why a professor from the Midwest (say, the University of Michigan) presenting a paper in the Sunbelt (say, Arizona State University) in January wears short sleeves to lunch while the hosts wear jackets or sweaters. The response to temperature depends on the deviation from a reference temperature or temperature scorecard based on recent experience. Adaptation level and habituation theory also explain why the same professor, while on a nine-month visit to Arizona State University, would wear a jacket to lunch in January. The professor would become accustomed or adapted to warmer temperatures, and, on a relative basis, January would seem cool. Furthermore, attention and sensitization theories explain why, on the initial visit to Arizona, weather becomes salient for the Michigan professor who may watch the weather report on the news every evening, add and subtract layers of clothes during the day, and make frequent adjustments to the thermostat at night. Admittedly none of the psychology books or articles we cite deal directly with investors' sensitivity to their portfolios. Nevertheless, we note that our preference assumption, which states that the attention and sensitivity to news temporarily increases after a shock, is not without precedent.

2. Equilibrium

Since z_t is the single state variable, we construct a one-factor Markov equilibrium. Our investors choose consumption, C, and investment, S and

[7] Our model has parallels to Barber and Odean's (2002) attention model. Barber and Odean show that investors, confronted with a large cross section of stocks, ration their attention (and consequently purchases) to those stocks in the news and those with recent large price swings. In contrast, our investors ration attention across time, giving more attention to their portfolio after large price swings.

B, allocations to maximize expected lifetime utility as in Equation (1), restated below:

$$\max E\left[\sum_{t=0}^{\infty}\delta^t U(C_t) + b_0 \bar{C}_t^{-\gamma}\delta^{t+1}F(W_{t+1})\right],$$

where

$$\begin{aligned} F(W_{t+1}) &= \lambda(z_t, X_{t+1})W_{t+1}, \\ \lambda(z_t, X_{t+1}) &= k(a_0 - a_1 z_t) \quad \text{for } X_{t+1} \le 0, \\ &= a_0 - a_1 z_t \qquad \text{for } X_{t+1} \ge 0. \end{aligned}$$

The transition of the "scorecard" state variable, z, follows $z_{t+1} = \phi z_t + (a_2|z_t| + a_3)X_{t+1}$. The first-order conditions of Equation (1) for an optimal consumption and investment choice produce the following Euler equation for the risky asset:

$$1 = \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} + F'(W_{t+1})\right]. \tag{7}$$

Equation (7) has the standard interpretation: the marginal utility cost of consuming one less unit and investing that unit at time t should equal the expected marginal utility benefit from the proceeds of selling that investment at time $t+1$. However, in our model the proceeds generate both consumption and financial utility. Substituting Equation (2) into Equation (3) and taking the derivative with respect to S_t yields the marginal utility of financial wealth, $F'(W_{t+1}) = \lambda(z_t, X_{t+1})\ [R_{t+1} - E_t(R_{t+1})]$, which can be substituted into Equation (7) to yield

$$\begin{aligned} 1 &= \delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{t+1} + \lambda(z_t, X_{t+1})[R_{t+1} - E_t(R_{t+1})]\right] \\ &= \frac{\delta}{1 + \delta E_t[\lambda(z_t, X_{t+1})]E_t(R_{t+1})} E_t\left[\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \lambda(z_t, X_{t+1})\right\} R_{t+1}\right] \\ &= E_t[m_{t+1}R_{t+1}], \end{aligned} \tag{8}$$

where the pricing kernel for the stock is $m_{t+1} = \kappa_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \lambda(z_t, X_{t+1})\right]$ and $\kappa_t = \frac{\delta}{1+\delta E_t[\lambda(z_t, X_{t+1})]E_t(R_{t+1})}$.[8]

Because financial utility is only a function of the risky asset performance, the Euler equation for the risk-free asset reduces to Mehra and Prescott's (1985) risk-free rate,

$$R_{f,t+1} = \frac{1}{\delta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}, \tag{9}$$

[8] Beginning with Equation (8), we fold the parameter b_0 into the function $\lambda(z_t, X_{t+1})$.

which is constant. Furthermore, since our model generates the risk premium via financial utility and loss aversion rather than high levels of the consumption risk aversion coefficient, our model can have both a low constant risk-free rate and a high equity premium, a combination that is puzzling for the standard model.

Multiplying Equation (8) by P_t and dividing by D_t yields the following price/dividend equation:

$$\frac{P_t}{D_t}(z_t) = E_t\left[m_{t+1}\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}\right]. \tag{10}$$

Before generating a numerical solution to the model, we discuss some of the model's qualitative intuition. For example, assume that an investor receives a *negative* dividend shock, which not only leads to a negative return shock, X_{t+1}, but also impacts the return through its effect on risk aversion and sensitivity. The bad news drives up risk aversion through $\lambda(z_t, X_{t+1})$, resulting in an additional price decline. Also, assuming that the investor's scorecard was at its average level of zero, the negative dividend shock increases the investor's sensitivity to subsequent shocks and increases expected volatility. The higher expected volatility feeds back, causing a further negative impact on prices. That is, bad news about dividends poses a triple threat to prices: the bad news itself, the resulting increase in risk aversion, and the resulting increase in sensitivity to news. The second and third threats allow returns to be more volatile than the shocks to fundamentals alone would allow. Furthermore, the impact of bad news lingers because the investors' heightened sensitivity decays slowly over time; consequently reactions to future dividend shocks will be greater than if the investor was not perturbed.

If instead the investor experiences a *positive* shock, the contemporaneous shock itself is amplified by the accompanying reduction in risk aversion — a double threat. However, as in Campbell and Hentschel (1992), this increase in price is tempered by the increase in expected volatility. Even though the sensitivity of the scorecard is symmetric to both positive and negative shocks, the behaviorally induced increase in prices after positive shocks will be smaller than the behaviorally induced price decline after a negative shock. Consequently, although our model has symmetric sensitivity, it will produce asymmetric responses to news, a result consistent with empirical evidence [Nelson (1991)].

3. Solving the Model

The parameter values we assume when solving the model numerically are given in Table 1. The first six rows of Table 1 report parameter values that are associated with the traditional consumption model; the next six rows report parameters that are unique to our model. We set the values of

Table 1
Model parameter values

Parameter	Variable	Value
Traditional:		
Consumption and dividend growth rate	g	1.89%[a]
Subjective discount factor	δ	0.98[a]
Standard deviation of consumption innovations (ν)	σ_ν	3.5%[a]
Standard deviation of dividend innovations (ϵ)	σ_ε	11.2%[a]
Correlation coefficient between ν_t and ϵ_t	ρ	0.2
Consumption utility power	γ	2.0
Model specific:		
Memory of scorecard	ϕ	0.92
State-dependent degree of risk aversion	a_1	0.4
State-induced scorecard sensitivity to shocks	a_2	5.0
Base-level scorecard sensitivity to shocks	a_3	2.3
Relative importance of financial wealth	b_0	1.0
Loss aversion	k	1.5

[a]Corresponds to annualized numbers. Traditional parameters are from Campbell and Cochrane (2000) or Barberis, Huang, and Santos (2001). Model-specific parameters are chosen to calibrate the model to fit the post war U.S. stock market data.

observable parameters, g, δ, σ_ν, σ_ε, and ρ, equal to their historical U.S. values taken from either CC or BHS. We also set γ equal to two, following CC. The loss-aversion parameter, k, is set equal to 1.5, which is lower than values used in BHS.

In our base-case solution, we choose the model-specific parameters ϕ, a_1, a_2, a_3, and b_0 to meet two objectives: (1) to match certain moments of the postwar data (such as the equity premium and clustered volatility) and (2) to achieve stable solutions to the model.[9] The second objective is important because of the explosive nature of our model. After an exogenous shock, our investors become sensitive to news, exacerbating the response to the next exogenous shock.

3.1 Numerical solution

Solving our model requires a solution to Equation (10) as well as the scorecard's law of motion [Equation (5)]. Description of the equilibrium requires that we simultaneously solve both equations due to their self-referential nature. Since both price-dividend ratios and the scorecard are endogenous, we use an iterative method similar to that of BHS to obtain solutions for the model. Specifically we guess a solution for the pricing equation, Equation (10), and use this solution to construct a transition relationship for z_t, Equation (5), which satisfies the intermediate pricing solution. Given the new transition relationship for z_t, we update the

[9] We add a restriction to the model which reduces the number of parameters we need to choose. Specifically, we impose the restriction that $\lambda(z_t, X_{t+1}) = 0$ for $z_t = 1$, meaning that the investor becomes risk neutral as the scorecard increases. Given this condition and a choice of a_1, the implication is that $a_0 = a_1$. For this reason we drop any discussion of the parameter choice of a_0.

pricing solution and iterate until both the pricing solution and the transition relationship converge.

We can restate Equation (10), conditional on levels of z_t, as

$$\frac{P_t}{D_t}(z_t) = E_t\left\{\left(\kappa_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} + \lambda(z_t, X_{t+1})\right]\right)\frac{D_{t+1}}{D_t}\left(1+\frac{P_{t+1}}{D_{t+1}}(z_{t+1})\right)\right\}$$
$$= \kappa_t E_t\left\{\left[\left(G^{-\gamma} e^{-\gamma\rho\frac{\sigma_v}{\sigma_\epsilon}\varepsilon_{t+1}+\frac{\gamma^2}{2}(1-\rho^2)\sigma_v^2}\right) + \lambda(z_t, X_{t+1})\right]\right.$$
$$\left.\times\, Ge^{\varepsilon_{t+1}}\left(1+\frac{P_{t+1}}{D_{t+1}}(z_{t+1})\right)\right\}, \tag{11}$$

where $G = \ln(g)$ and solve Equation (11) numerically using parameter values from Table 1. Our price-dividend function and scorecard law of motion can then be used to solve for various conditional return moments, including volatility.

Risk premium. The conditional expected stock return is

$$E_t(r_{t+1}) = E_t\left(\frac{\frac{P_{t+1}}{D_{t+1}}(z_{t+1})+1}{\frac{P_t}{D_t}(z_t)} \times \frac{D_{t+1}}{D_t}\right). \tag{12}$$

Because the expected return on the stock enters into Equation (10), we must solve for Equation (12) at the same time we solve Equations (5) and (10). Using numerical integration, we check for convergence in the expected return relationship in Equation (12) as we do for the price-dividend ratio and the transition relationship for z_t. Figure 2 illustrates the negative relationship between conditional expected returns and z_t. When our investors' scorecards are low ($z_t < 0$), the investors "feel" changes in financial wealth more intensely, exhibit high levels of risk aversion, and consequently, require a high expected return on stocks. Figure 2 shows that the model allows the annual expected return to vary from a high of about 16% to a low of about 5.5%.

From Equation (9) the risk-free rate is

$$R_{f,t+1} = \left[E_t\left\{G^{-\gamma} e^{-\gamma\rho\frac{\sigma_v}{\sigma_\epsilon}\varepsilon_{t+1}+\frac{\gamma^2}{2}(1-\rho^2)\sigma_v^2}\right\}\right]^{-1}. \tag{13}$$

Since Equation (13) shows that $R_{f,t+1}$ is independent of the scorecard, z_t, the risk-free rate in Figure 2 is constant. As our investors realize a sequence of positive shocks, their aversion to risk declines. Figure 2 shows that at very high levels of z_t, our investors approach risk neutrality and the risk premium approaches zero. In general, however, our investors' concern for wealth and fear of losses is capable of generating a risk premium without increasing the coefficient of risk aversion, γ, to puzzling levels.

Standard deviation. The expected standard deviation of returns, $E_t(\sigma_{t+1})$, conditional on the scorecard, z_t, can be calculated using

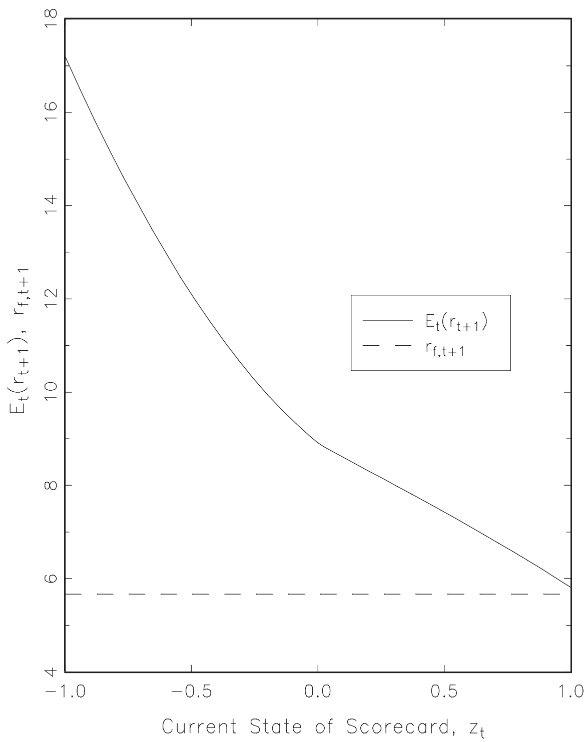

Figure 2
Annualized conditional expected stock return, $E_t(r_{t+1})$, and the risk-free rate, $r_{f,t+1}$, as a function of the scorecard, z_t.

numerical integration techniques given solutions to Equations (5), (10), and (12). Figure 3 graphs the relationship between the standard deviation of returns, $E_t(\sigma_{t+1})$, and z_t. Figure 3 shows that as our investors become perturbed in either direction, their sensitivity to shocks and consequently the expected standard deviation of stock returns increase. However, negative shocks lead to greater volatility than do positive shocks. This asymmetry results from the fact that shocks (positive or negative) increase sensitivity and has an effect of reducing prices, which exacerbates price declines after negative shocks but tempers price increases after positive shocks. Figure 3 shows how the annualized expected standard deviation of stock returns can vary from 12.5% to 23%. We also include Figure 4, which plots the relationship between the current period's return shock (X_t) and next period's expected return standard deviation $E_t(\sigma_{t+1})$.[10] This

[10] We construct the "news impact curve" by simulating a stream of 100,000 dividend shocks and obtain the resulting return shocks (X_t), scorecard levels (z_t), and expected volatility ($E_t\ (\sigma_{t+1}|z_t)$) each period. We then construct a kernel-weighted nonparametric estimate of $E_t\ (\sigma_{t+1}|X_t)$ using the simulated data and plot the results in Figure 4.

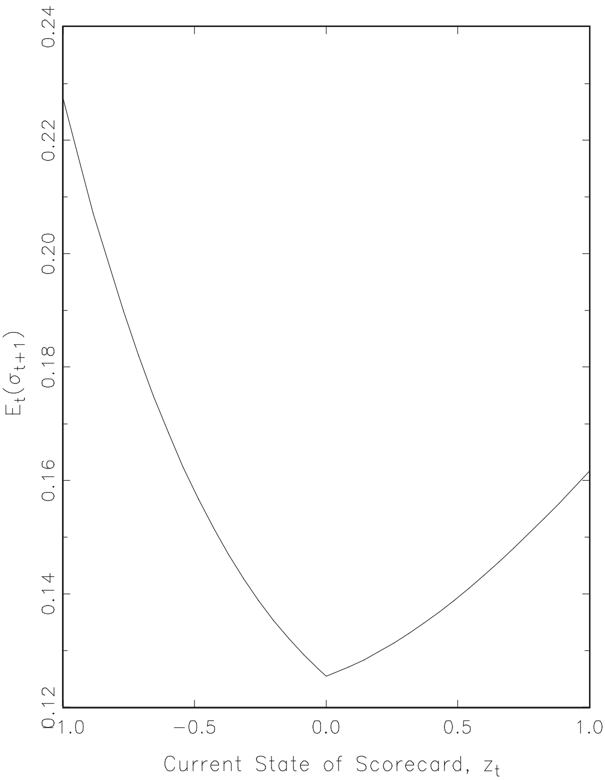

Figure 3
Annualized conditional expected standard deviation of stock returns, $E_t(\sigma_{t+1})$, as a function of the scorecard, z_t.

graph is provided as a comparison to the "news impact curve" of Pagan and Schwert (1990). Figure 4 illustrates that in our model (1) both negative and positive news generally lead to increased volatility and (2) negative news generates greater volatility than positive news. The fact that, in our model, good news on average leads to increases in volatility is unique among current theoretical explanations and is consistent with empirical studies, such as Pagan and Schwert (1990).

Skewness. Skewness, conditional on the scorecard, follows the relationship illustrated in Figure 5, where $E_t(\mu_{3,t+1}) = E_t\left[\frac{(r-\bar{r})^3}{\sigma^3}\right]$.[11] Intuitively, the predominantly negative slope of the conditional skewness curve in Figure 5 is related to the law of motion for the scorecard. When the

[11] We also find, but do not present, negative skewness in the unconditional return distributions.

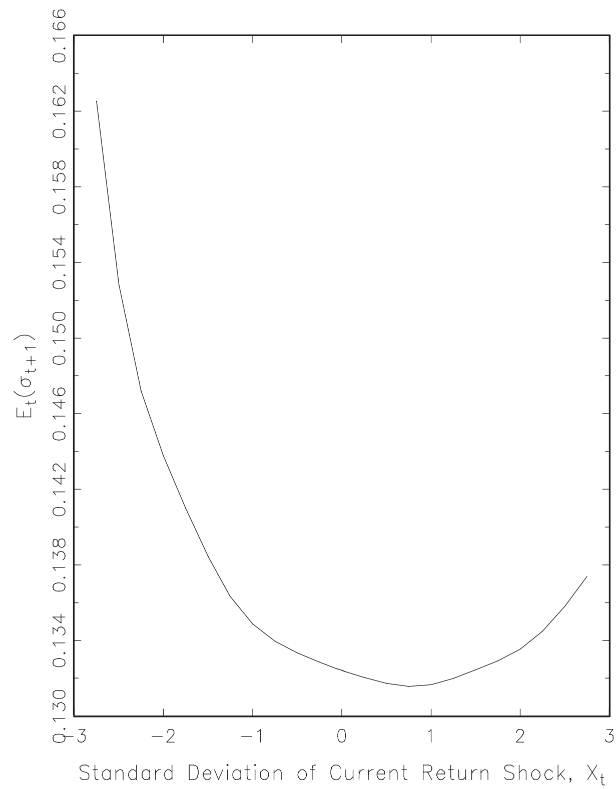

Figure 4
News impact curve, annualized conditional expected standard deviation of stock returns, $E_t(\sigma_{t+1})$, as a function of stock return shocks.

scorecard is negative, a reversion of the scorecard toward zero is expected as the investor becomes accustomed to the new level of wealth. This movement toward zero will have a positive impact on prices and returns independent of the news regarding dividends, thus inducing a positive skew, or more accurately, a less negative skew. On the other hand, when the scorecard is positive the expected movement in the scorecard is a decline toward zero, causing a decline in prices and increasing the negative skewness.

3.2 Simulations

We now conduct a simulation exercise to further investigate the model's properties. We begin by simulating a sample path of 10,000 monthly dividend shocks, ε_{t+1}, based on the historical standard deviation reported in Table 1. We then use this series of exogenous shocks to develop a time series of price dividends and returns using the solutions to Equations (11) and (5). For each simulation we find the average level of key variables,

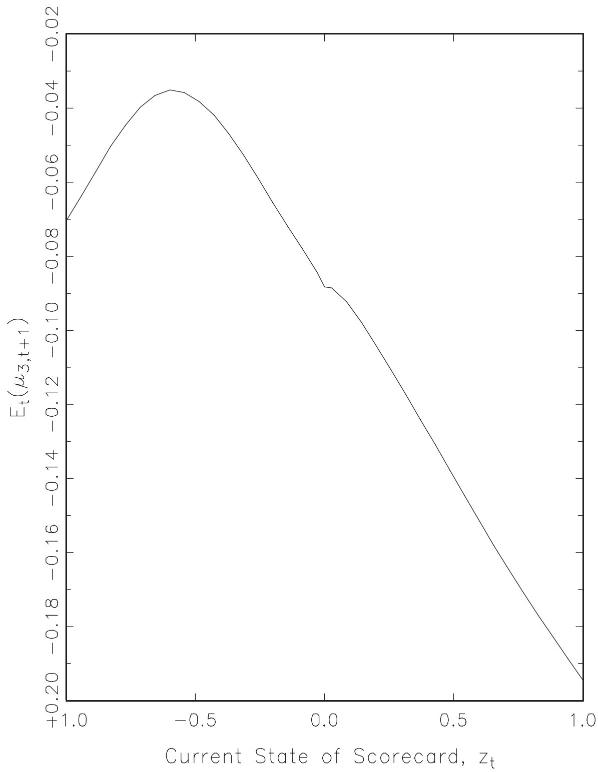

Figure 5
Annualized conditional expected skewness coefficient of stock returns, $E_t(\mu_{3,t+1})$, as a function of the scorecard, z_t.

such as the risk-premium and price-dividend ratios, as well as the average parameter estimates from an EGARCH(1,1) model on the simulated stock returns using a typical EGARCH formulation,

$$r_t = c + e_t,$$

$$\mathrm{Var}(e_t) = h_t,$$

$$\ln(h_t) = c_0 + c_1 \ln(h_{t-1}) + c_2 \left| \frac{e_{t-1}}{h_{t-1}} \right| + c_3 \frac{e_{t-1}}{h_{t-1}}.$$

For comparison purposes, the first row of Table 2 reports estimates from the historical monthly value-weighted Center for Research in Security Prices (CRSP) returns from 1952 to 2000.

The choice of monthly returns warrants discussion due to an apparent conflict. On one hand, GARCH is typically considered a high-frequency phenomenon, often measured in days; on the other hand, changes in

Table 2
Simulation results and sensitivity analysis

$$r_t = c + e_t, \mathrm{var}(e_t) = h_t$$

$$\ln(h_t) = c_0 + c_1 \ln(h_{t-1}) + c_2 \left| \frac{e_{t-1}}{h_{t-1}} \right| + c_3 \frac{e_{t-1}}{h_{t-1}}$$

	c_1	p-value	c_2	p-value	c_3	p-value	σ	r	$r{-}r_f$	P/D
CRSP monthly data (1952–2000)	0.939	0.000	0.142	0.000	−0.062	0.013	18.35	12.52	6.86	24
Simulations:										
Consumption utility	−0.219	0.410	−0.008	0.365	0.001	0.534	11.27	5.48	−0.18	30.23
Habit persistence	1.000	0.000	0.069	0.000	−0.119	0.000	15.20	7.58	6.64	18.30
Preference	0.873	0.000	0.060	0.000	−0.054	0.000	13.80	9.96	4.30	13.26
Sensitivity analysis										
$k = 1.25$	0.058	0.574	0.001	0.585	0.001	0.680	11.85	7.08	1.42	21.42
$\phi = 0.90$	0.195	0.483	0.020	0.274	0.001	0.344	12.37	8.47	2.81	15.41
$a_1 = 0.38$	0.222	0.235	0.023	0.232	−0.014	0.309	12.75	8.34	2.68	16.11
$a_2 = 4.75$	0.332	0.247	0.017	0.241	0.015	0.266	12.71	9.25	3.59	13.75
$a_3 = 2.2$	0.098	0.388	0.028	0.307	0.002	0.458	12.85	9.12	3.46	13.90
$b_0 = 0.98$	0.063	0.251	0.025	0.294	−0.001	0.419	12.80	8.80	3.20	15.68

"Consumption utility" refers to the case where $b_0 = 0$, similar to Mehra and Prescott (1985). "Habit persistence" refers to the Campbell and Cochrane (1999) model where the last four elements of that row are taken directly from their article. "Preference" refers to our model where parameters are defined in Table 1. Results from simulating sample paths for given parameter values and estimating an EGARCH(1,1) model on the CRSP value-weighted index returns for the first row and various simulated returns for subsequent rows. All numbers reported below the "Simulations" heading are average values from 10 simulations for each set of parameter values where in each simulation we construct a sample of 10,000 observations. The columns labeled "p-value" report the results of a t-test for the c_1, c_2, and c_3 parameters being equal to zero and are based on standard errors as calculated in Bollerslev and Wooldridge (1992). The standard deviations and return results are in percent, and the last four columns have been annualized.

behavior or attitudes are considered slow moving, often measured in quarters or years. We note four findings related to this apparent conflict in our use of monthly horizons. First, volatility clustering is evident at monthly horizons. The first row of Table 2 shows that both the ARCH parameter, c_2, and the GARCH parameter, c_1, are statistically significant (p-values less than .001). Second, we note the considerable evidence of low-frequency or long-horizon serial correlation in volatility. Ding and Granger (1996, p. 199) document two volatility components and note that, "Some volatility components may have a very big short-run effect, but die out very quickly. Some of them may have a relatively smaller short-run effect but they last for a long time period." Bollerslev and Mikkelsen (1996, p. 173) find "that the empirical autocorrelation of absolute returns exhibit fairly rapid decay at short lags, while the rate of decay for longer lags is much slower." Engle and Lee (1999) confirm the finding of a long-run component of stock return volatility and show that only the long-run component affects the size of the risk premium.[12] We model

[12] Hamilton and Susmell (1994) find related evidence for long-run persistence in volatility regimes using a Markov-switching model. Kim, Morley, and Nelson (2003) show that only the persistent changes in volatility effect the equity risk premium.

this long-run, priced volatility component. Third, whereas most models of time-varying risk premiums are tied to relatively stable consumption growth shocks, our varying risk aversion is tied to relatively volatile stock price changes. For example, investors' attention to their portfolios and perhaps their degree of risk aversion reasonably could have increased noticeably between October 1 and November 1, 1987, after large stocks fell 20% and small stocks fell 30%. More recently, in February 2000, small stocks gained 24% and the following month larger stocks increased 10%. Such large monthly jumps may induce higher sensitivity in investors. Fourth, many investors fund their portfolios and receive performance reports on a monthly basis. In short, the priced component of clustered volatility is evident over monthly horizons and investors conceivably make changes to their scorecards at monthly horizons.

After reporting historical data, Table 2 reports average parameter values from 10 simulations each of three models: the traditional model, the habit model, and our new preference model. In the simulations referred to as "consumption utility" we set $b_0 = 0$, which collapses our model to the traditional consumption-only model tested by Mehra and Prescott (1985). In the simulations referred to as "habit persistence" we simulated CC's (1999) model.

The first six columns of Table 2 show the degree and significance of a given model's ability to capture clustered volatility and the last four columns show the model's ability to generate some of the new facts in finance. The second row of Table 2 shows that the "consumption utility" model fails to capture any of the features of historical return volatility reported in the first row. This is not the case for the CC "habit persistence" model. The average memory component of volatility, c_1, is 1.0, even greater than the memory found in the historical data. The habit explanation of volatility autocorrelation is not unexpected, but should be interpreted with caution. Specifically, memory in volatility in the CC model is driven by an assumed, but counterfactual, high correlation between consumption growth and stock returns. While the "habit persistence" model generates average values of the c_2 and c_3 coefficients that show a strong relationship between return shocks and subsequent volatility, the relationship implied by the average values also vary from that observed in the data. In particular, since c_3 is larger in absolute magnitude than c_2, positive news leads to declines in return volatility.[13]

Simulations of our "preference" model result in an average GARCH parameter, c_1, of 0.873 (p-value less than .001), providing a strong memory component in volatility, although somewhat less in magnitude than the coefficient estimated from historical monthly returns ($c_1 = 0.939$). Our simulations also lead to an average ARCH parameter, c_2, of 0.060.

[13] This same criticism applies to the BHS model, as can be seen in their Figure 6.

Relative to the historical data, our model's level of c_2 is low, but it has the predicted sign and is highly significant (average p-value less than .001). The negative sign on c_3, and the fact that $|c_3| < |c_2|$, shows that our model also generates the asymmetric relationship between good and bad news found in the historical data.

The simulations show that our preference model has a well-behaved financial utility function, especially given that we explicitly chose parameters to match conditional second moments in return data. We find, but do not report, the average level of $\lambda(z_t, X_{t+1})$ to be 0.430 with a standard deviation of 0.014. So while our model is able to capture many of the conditional features of equity returns, the model does not require exorbitantly high variation in financial utility to accomplish this design.

The simulations show that our model captures some of the new empirical findings in finance. Relative to the traditional consumption-only model, our model has higher average levels of return volatility (annualized return standard deviation of 13.80% rather than 11.27%) and a reasonable equity premium (annualized premium of 4.30% rather than -0.18%). Because of the fear of losses, stocks are less attractive to investors in our model than in the traditional model (consumption utility). Our model yields an annualized average price dividend ratio of 13.26 rather than 30.23 and, not reported, an average Sharpe ratio of 0.174 rather than 0.133. The attractiveness of stocks in our model varies over time, whereas the only source of variation in the traditional model is from the exogenous dividend stream; consequently the standard deviation of the price-dividend ratio, not reported, is 0.824 in our model and zero, by definition, in the consumption-only model. The standard deviation of the scorecard in our base case is 0.328. Although the scorecard is zero on average, scores of 1.0 or -1.0 (three standard deviations away from zero) are possible. In our simulations we observe rare occurrences of scorecards above 1.0 and below -1.0.[14]

The last six rows of Table 2 illustrate the sensitivity of our preference model's simulation results to changes in the six model-specific parameter values. In each row we change one of the original parameters in Table 1, then run 10 simulations and report the average values of the results. In each case we perturb a parameter in a direction that decreases the importance of behavioral responses. That is, we explore the model as we move parameters in the direction of the traditional, consumption-only model. The sensitivity results in Table 2 show that as our investors become less behaviorally motivated, the degree of volatility clustering and the ability of the model to explain the new empirical facts is diminished, in some cases quite dramatically. In particular, our model's results appear quite sensitve to choice of k, ϕ, and a_1. However, we find that when we decrease

[14] This empirical distribution of the scorecard informs the x-axis scale of Figures 2–5.

parameters b_0, a_2, and a_3, the general volatility features of the model remain.

4. Testing the Scorecard's Predictive Ability

Prior to this section we reverse-engineered a model to generate return moments, such as clustered volatility, found in historical returns, and then solved and simulated the model. We have shown that our model is consistent with the well-documented conditional second moment features of returns that motivated the model. We now turn our focus to testing the theoretical model using new predictions about conditional moments. First, we test our scorecard's ability to predict conditional volatility. Second, we compare our scorecard with other preference scorecards and test its ability to predict conditional excess returns and skewness. When solving and simulating the model, we intentionally searched for parameter values that generated returns with moments similar to those found in the historical data. Those parameters are now fixed. Thus when testing our model's new predictions about conditional volatility, excess return, and skewness, we cannot search over model parameters or "mine" the data to provide better forecasts.

4.1 The scorecard and conditional volatility

An obvious prediction of our theoretical model is that return volatility is clustered. However, the clustering of returns motivated our model in the first place, consequently a test for clustering itself cannot be used as a test of our model. To this end, new and refutable predictions are needed. Our model makes such predictions. Our theoretical model links past volatility to future volatility in a very specific way — through the investors' scorecard, as illustrated in Figure 3.

First, our theoretical model predicts that the further from zero a weighted sum of past shocks, z_t, becomes, the greater the next period's volatility. Prior statistical descriptions of clustered volatility generally relate large shocks to future large shocks. In contrast, our model specifically predicts a cumulative effect; for example, two successive large shocks of the same sign will have a much bigger impact on future volatility than two successive shocks with opposite signs. Second, our theoretical model predicts that negative levels of z_t will be associated with news having a greater impact on future volatility than positive levels of z_t. Note that our second prediction is not just a restatement of the GARCH asymmetry that helped motivate our model. GARCH asymmetry suggests that a single piece of good news will be followed by high levels of volatility, but a single piece of bad news will be followed by even higher levels of volatility. In contrast, our model conditions volatility asymmetrically on the scorecard. For example, when z_t is close to zero, bad news will have little effect on future volatility, whereas after a string of good news, another piece of good news will be followed by very high levels of volatility.

We test our two volatility predictions using two empirical models. Our first empirical model, following Pagan and Schwert (1990), regresses an estimate of conditional return volatility, $\hat{e}_t^2$, on an estimate of the lagged scorecard, $\hat{z}_{t-1}$, allowing for the predicted nonlinearity

$$\hat{e}_t^2 = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N \hat{z}_{t-1} + \eta_t,$$

where N is a binary variable equal to one when $\hat{z}_{t-1}$ is negative and zero otherwise, and where η_t is the regression residual. Our model predicts $\beta_1 > 0$ and $-(\beta_2 + \beta_1) > \beta_1$. We test these predictions against null hypothesis 1, H_1: $\beta_1 = 0$ and $-(\beta_2 + \beta_1) - \beta_1 = 0$. In our second empirical model, we explore whether our measure of the scorecard has predictive power beyond Nelson's (1991) EGARCH model. The EGARCH (1,1) model allows returns to be asymmetrically correlated with future volatility. Specifically, empirical model 2 regresses an estimate of conditional return volatility, $\hat{e}_t^2$, on an estimate of the lagged scorecard, $\hat{z}_{t-1}$, and on predictions of conditional volatility, $\hat{\sigma}_t^2$, from an EGARCH (1,1) model:

$$\hat{e}_t^2 = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N \hat{z}_{t-1} + \beta_4 \hat{\sigma}_t^2 + \eta_t,$$

where β_3 is reserved for Table 4.

We construct $\hat{e}_t^2$ using the squared residuals from a simple regression of monthly value-weighted index returns on a constant, $r_t = c + e_t$. We construct $\hat{z}_t$ from $z_t = \phi z_{t-1} + (a_2\ |z_{t-1}| + a_3) X_t$. We start our scorecard at zero in December 1925 and use the parameter estimates in Table 1 ($\phi = 0.92$, $a_2 = 5.0$, and $a_3 = 2.3$) to recalculate the scorecard each subsequent month, and let X_t be the deviation of the portfolio return from its unconditional average.[15] Following Nelson (1991), our forecast of $\hat{\sigma}_t^2$ is from the EGARCH (1,1) model, $\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \alpha_2 \left| \frac{e_{t-1}}{\sigma_{t-1}} \right| + \alpha_3 \frac{e_{t-1}}{\sigma_{t-1}}$.

The first two rows of Table 3 report the results of estimating empirical models 1 and 2, respectively, using postwar (1952–2000) data.[16] We test our model's predictions on monthly U.S. value-weighted CRSP returns. After reporting the base case, we also report results from alternative time periods and the results of sensitivity analysis with respect to $\hat{e}_t^2$ and $\hat{z}_{t-1}$. All our tests use Newey and West's (1987) *t*-statistics, a correction for generalized serial correlation.

In the 1952–2000 regression, β_1 is positive, β_2 is negative, and $|\beta_2|$ is greater than $2 \times |\beta_1|$, as predicted. However, only β_2 is significantly

[15] Starting the scorecard in December 1925 at 1 or -1 has no material effect on the results reported in Table 3.

[16] Our January 1952 break point corresponds with a new Federal Reserve/Treasury accord and is the postwar separation date used by Campbell (1991) and Campbell and Hentschel (1992).

Table 3
Tests of the scorecard's ability to predict conditional volatility, $\hat{e}^2$

Model 1: $\hat{e}_t^2 = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \eta_t$

Model 2: $\hat{e}_t^2 = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \beta_4 \hat{\sigma}_t^2 + \eta_t$

Scenario	Model	β_1	β_2	β_4	$\bar{R}^2$	F-test
1952–2000	Model 1	0.0045	−0.0252	—	0.095	31.65
		(0.241)	(0.000)			(0.000)
	Model 2	0.0060	−0.0253	0.2932	0.098	8.45
		(0.139)	(0.000)	(0.008)		(0.000)
1927–2000	Model 1	0.0119	−0.0467	—	0.104	114.60
		(0.014)	(0.000)			(0.000)
	Model 2	0.0056	−0.0305	0.5537	0.119	4.43
		(0.095)	(0.005)	(0.000)		(0.012)
1803–1925	Model 1	0.0095	−0.0410	—	0.076	61.52
		(0.000)	(0.000)			(0.000)
	Model 2	0.0059	−0.0277	0.6306	0.116	7.99
		(0.004)	(0.000)	(0.000)		(0.000)
$a_2 = 4.5$	Model 1	0.0021	−0.0248	—	0.109	36.87
1952–2000		(0.841)	(0.069)			(0.000)
	Model 2	0.0028	−0.0243	0.1984	0.109	13.96
		(0.786)	(0.069)	(0.135)		(0.000)
$a_3 = 2.0$	Model 1	0.0051	−0.0283	—	0.098	32.79
1952–2000		(0.244)	(0.000)			(0.000)
	Model 2	0.0068	−0.0285	0.2940	0.101	9.76
		(0.139)	(0.000)	(0.008)		(0.000)
$\phi = 0.85$	Model 1	0.0034	−0.0323	—	0.099	33.18
1952–2000		(0.450)	(0.002)			(0.000)
	Model 2	0.0045	−0.0324	0.1321	0.098	5.99
		(0.312)	(0.002)	(0.298)		(0.002)
Enhanced $\hat{e}^2$	Model 1	0.0043	−0.0238	—	0.089	29.64
1952–2000		(0.233)	(0.000)			(0.000)
	Model 2	0.0061	−0.0240	0.3587	0.095	6.86
		(0.127)	(0.001)	(0.004)		(0.001)

Our estimate $\hat{e}^2$ is the squared residuals from a simple regression, $r_t = c + e_t$, where r_t is the CRSP value-weighted stock index return in month t. Our estimate of $\hat{z}_t$ is from $z_t = \phi z_{t-1} + (a_2|z_{t-1}| + a_3)X_{t-1}$. For the samples starting in 1952 and 1926, we start the scorecard at zero in December 1925 and use the values of ϕ, a_2, and a_3 given in Table 1. For the sample starting in 1803, we start the scorecard at zero in December 1801 and leave out August–November 1914 because the NYSE was closed due to the war. X_t is the deviation of the return from its unconditional average, and N is a binary variable equal to one when $\hat{z}_{t-1}$ is negative and zero otherwise. Our estimate of $\hat{\sigma}_t^2$ is from Nelson's (1991) EGARCH(1,1) model of returns. Pre-1926 data are from Schwert (1990). F-tests are for null hypothesis 1: $\beta_1 = 0$ and $-(\beta_1 + \beta_2) - \beta_1 = 0$ and p-values are in parentheses and are based on Newey and West (1987) standard errors.

different from zero (p-value less than .001). The adjusted $\bar{R}^2$ is 9.5%; for comparison purposes, the $\bar{R}^2$ for an EGARCH(1,1) model on the same data is just over 2%. Null hypothesis 1, H_1: $\beta_1 = 0$ and $-(\beta_2 + \beta_1) - \beta_1 = 0$, is rejected with a p-value of less than .001 using a chi-squared Wald test. Figure 6 illustrates the combined effect of $\hat{z}_{t-1}$ on $\hat{e}_t^2$ over ranges of the scorecard found in historical return data and closely resembles the effect, illustrated in Figure 3, found in our theoretical model. Historically, conditional volatility is lowest when the scorecard is near zero and increases as investors are perturbed. Furthermore, the historical relationship in Figure 6 shows that negative perturbations have a greater influence than positive perturbations.

When volatility forecasts from the EGARCH model are included (empirical model 2), the $\bar{R}^2$ increases slightly to 9.8%, H_1 is still significantly rejected, and β_4 is significant. Thus both the EGARCH forecast of volatility and the scorecard are significant, indicating that our preference explanation and other explanations (clustered news, for example) may both be causing volatility autocorrelation. The scorecard's value in predicting volatility is not diminished when competing with an EGARCH(1,1) model. In addition to the Table 3 regressions, we also estimated an EGARCH(1,1) model augmented by the scorecard, $\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + \alpha_2 \left| \frac{e_{t-1}}{\sigma_{t-1}} \right| + \alpha_3 \frac{e_{t-1}}{\sigma_{t-1}} + \alpha_4 \hat{z}_{t-1} + \alpha_5 N\hat{z}_{t-1}$. In this unreported regression, α_4 and α_5 are both statistically significant, and the $\bar{R}^2$ of the regression increases from 2.3% to 13.9% when the two scorecard terms are included.

We check the robustness of the 1952–2000 results by looking at two alternative time periods; the first alternative time period is 1927–2000. Whereas the CRSP data start in January 1926, our regression starts in January 1927 to make the first value of $\hat{z}_t$ meaningful. As with the post war data, the scorecard can be used to predict conditional volatility; H_1 is rejected, the $\bar{R}^2$ is more than 10%, and both β_1 and β_2 are significant. Our second alternative time period is based on the 1802–1925 pre-CRSP data of Schwert (1990); we start the scorecard at zero in December 1801 and begin our tests in January 1803 to allow the scorecard to move away from zero. For Schwert's pre-CRSP data, both beta coefficients are statistically significant with and without the EGARCH estimates of conditional volatility. Thus the significance of the scorecard's ability to predict volatility clustering actually increases as we use out-of-sample data from before 1952.

In the first three time periods of Table 3, our proxy for the scorecard is built up using $\phi = 0.92$, $a_2 = 5.0$, and $a_3 = 2.3$. These behavioral parameter estimates were found when calibrating the theoretical model, but are not directly observable. We check for robustness by examining alternative measures of ϕ, a_2, and a_3. In each scenario we decrease one behavioral parameter, making the scorecard and financial utility less important. The changes to the behavioral parameters only slightly diminish the scorecard's ability to predict conditional volatility.

We also use Pagan and Schwert's (1990) more robust method for constructing a measure of conditional volatility, $\hat{e}_t^2$. The Pagan and Schwert method first regresses returns on 12 monthly dummy variables, then regresses these first-pass residuals, u_t on lagged values, $u_t = \sum_{i=1}^{10} \pi_i u_{t-i} + e_t$ then squares the second-pass residuals. The two enhancements control for seasonality and simple autoregression in the residuals. Test results using the enhanced measure of conditional volatility are reported in the last two rows of Table 3. In both empirical models, H_1 is rejected, and both coefficients have the predicted sign.

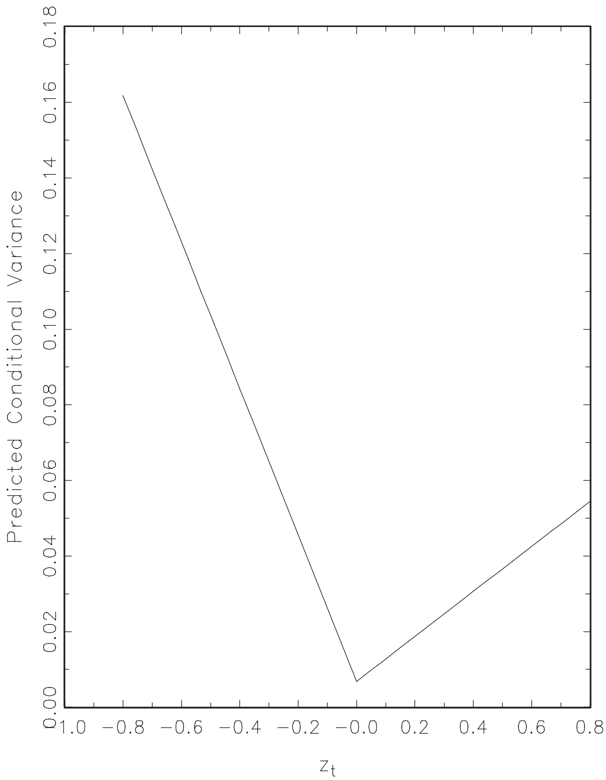

Figure 6
Empirical model of the predicted relationship between the scorecard, z_t, and annualized conditional variance.

4.2 Competing scorecards, conditional excess returns, and skewness

Lettau and Ludvigson (2001) posit that investors, in an effort to smooth their consumption path, invest less and consume more (out of labor income and existing investments) when expected returns are high. Consequently the log consumption-aggregate wealth ratio should be able to forecast stock returns. One can think of deviations from the consumption-aggregate wealth ratio trend as a competing scorecard. Lettau and Ludvigson (2001) develop a proxy for the log consumption-aggregate wealth ratio, $\widehat{cay}$, that is significantly correlated with future short- and intermediate-horizon excess stock returns. They find that the proxy has more predictive ability than extant covariates, such as the dividend yield. Furthermore, in a follow-on study, Lettau and Ludvigson (2002) show that $\widehat{cay}$ not only predicts excess returns, but predicts the volatility of excess returns, and hence Sharp ratios.

CC, too, have a competing scorecard. In their model, investors' degree of risk aversion is influenced by a scorecard they call the surplus

Table 4
Competing scorecards' (z, *cay*, and *scr*) ability to predict conditional volatility, excess return, and skewness

Model 3: *Dependent variable* $= \beta_0 + \beta_1\hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \beta_3\hat{z}^2_{t-1} + \eta_t$

Model 4: *Dependent variable* $= \beta_0 + \beta_1\hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \beta_3\hat{z}^2_{t-1} + \beta_4 c\hat{a}y_{t-1} + \beta_5 s\hat{c}r_{t-1} + \eta_t$

Dependent variable	Model	β_1	β_2	β_3	β_4	β_5	$\bar{R}^2$	F-test
Panel A: Monthly data (1959–2000)								
$\hat{e}^2_t$	Model 3	0.0060	−0.0271	—	—	—	0.096	27.837
		(0.243)	(0.001)					(0.000)
	Model 4	0.0059	−0.0272	—	−0.0169	—	0.099	11.69
		(0.235)	(0.000)		(0.062)			(0.000)
$r_t - r_{f,t}$	Model 3	−0.0859	0.0929	—	—	—	0.000	1.051
		(0.156)	(0.322)					(0.350)
	Model 4	−0.0970	0.0942	—	0.5343	—	0.029	1.545
		(0.094)	(0.285)		(0.000)			(0.213)
$\hat{\mu}_3$	Model 3	−1.6910	—	−1.8070	—	—	0.038	11.043
		(0.000)		(0.066)				(0.000)
	Model 4	−1.6633	—	−1.7497	0.0887	—	0.035	9.022
		(0.000)		(0.071)	(0.956)			(0.000)
Panel B: Quarterly data (1952–2000)								
$\hat{e}^2_t$	Model 3	0.0263	−0.0579	—	—	—	0.079	9.208
		(0.045)	(0.000)					(0.000)
	Model 4	0.0263	−0.0558	—	−0.1252	0.0301	0.138	25.923
		(0.045)	(0.000)		(0.000)	(0.217)		(0.000)
$r_t - r_{f,t}$	Model 3	−0.0573	−0.0717	—	—	—	0.002	1.206
		(0.665)	(0.692)					(0.301)
	Model 4	−0.0608	−0.0993	—	1.7549	−0.4532	0.089	1.479
		(0.623)	(0.569)		(0.007)	(0.112)		(0.230)
$\hat{\mu}_3$	Model 3	−1.7985	—	−1.6031	—	—	0.042	5.262
		(0.013)		(0.266)				(0.005)
	Model 4	−1.8710	—	−1.7187	−3.4957	−1.2493	0.038	11.899
		(0.012)	—	(0.239)	(0.259)	(0.525)		(0.000)

See notes to Table 3. $c\hat{a}y_t$ is Lettau and Ludvigson's (2002) proxy for the log consumption:aggregate wealth ratio. $s\hat{c}r_t$ is the Campbell and Cochrane (1999) surplus consumption ratio. $\hat{e}^2_t$ is an estimate of return volatility, $r_t - r_{f,t}$ is our estimate of excess return, and $\hat{\mu}_{3,t}$ is an estimate of return skewness. F-tests in the first two rows of each panel are for null hypothesis 1: $\beta_1 = 0$ and $-(\beta_1 + \beta_2) - \beta_1 = 0$. F-tests in the third and fourth rows of each panel are for the null hypothesis 2: $\beta_1 = \beta_2 = 0$. F-tests in the fifth and sixth rows of each panel are for the null hypothesis 3: $\beta_1 = \beta_3 = 0$ and p-values are in parentheses and are based on Newey and West (1987) standard errors.

consumption ratio, $scr_t = \ln[(C_t - X_t)/X_t]$, where their X_t is a slow-moving, time-varying, habit level of consumption. For example, in a boom, consumption is well above the habit level, investors are less risk averse, and future returns are expected to be low. CC do not formally test their scorecard, but do provide a graph (their Figure 10) that visually relates their scorecard's prediction to historical levels of the price-dividend ratio.

We now compare and contrast the ability of $\hat{z}$, $\widehat{cay}$, and $\widehat{scr}$ to predict not only conditional volatility, but also conditional excess returns and conditional skewness. Our data on $\widehat{cay}$ come from Lettau and Ludvigson, and are available monthly starting in January 1959 and quarterly starting in the first quarter of 1952. Data on consumption (nondurable plus service) are available quarterly. We obtain consumption data from Lettau and Ludvigson and population data from the Census Bureau to create per capita consumption. We create a quarterly measure of $\widehat{scr_t}$ using CC's equations (3) and (10) plus the parameter values in their Table 1, and assume that the ratio begins at the steady state in December 1951.

Specifically, using monthly and quarterly data we test empirical models 3 and 4:

$$\textit{dependent variable} = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \beta_3 \hat{z}_{t-1}^2 + \eta_t,$$

$$\textit{dependent variable} = \beta_0 + \beta_1 \hat{z}_{t-1} + \beta_2 N\hat{z}_{t-1} + \beta_3 \hat{z}_{t-1}^2 + \beta_4 \widehat{cay_{t-1}} + \beta_5 \widehat{scr_{t-1}} + \eta_t,$$

where the dependent variable is an historical estimate of volatility, excess return, or skewness.

The results of our competing scorecard tests are reported in Table 4 with monthly results in panel A and quarterly results in panel B. Comparing the first two regressions of panel A shows that adding $\widehat{cay}_{t-1}$ as an independent variable has little effect on our scorecard's relationship with conditional volatility. β_1 and β_2 change only slightly from model 3 to 4 and collectively remain significant (H_1 is rejected). β_4 is significantly negative (p-value .062), confirming Lettau and Ludvigson's (2001) finding that when consumption is temporarily below its trend relationship with financial assets and labor income, stocks tend to be volatile. Our scorecard actually performs better in the quarterly regressions (panel B) than in the monthly regressions. Specifically, β_1 becomes significantly positive (p-value .045) and β_2 remains significantly negative, regardless of whether the two competing scorecards are included. Furthermore, $|\beta_2| > 2\beta_1$ so H_1 is rejected in favor of our model's two predictions. Thus, in quarterly returns, the scorecard predicts volatility and its asymmetry. In contrast to $\hat{z}$ and $\widehat{cay}$, CC's scorecard, $\widehat{scr}$, is not correlated with conditional volatility confirming that memory in stock return volatility is unlikely to be generated by consumption growth shocks.

Our model makes two new predictions about expected excess returns conditional on the scorecard. First, z_t is negatively correlated with future conditional excess returns. When z_t is negative (positive), a large (small) reward is needed to induce investors to bear the risk of stocks; consequently expected returns should be high (low). Second, this negative correlation is stronger when z_t is negative than when it is positive. When the scorecard is negative, investors demand a high premium for two reinforcing reasons: the investors themselves are unusually risk averse and stocks (due to investor sensitivity to news) are extra volatile. When the scorecard is positive, the latter force cancels some of the former's effect on the conditional excess return. Our theoretical model's two new predictions are that both β_1 and β_2 will be negative in empirical models 3 and 4 when the dependent variable is the excess return. We compare these two predictions against null hypothesis two: H_2: $\beta_1 = 0$, $\beta_2 = 0$.

Panel A shows that for the monthly excess return model 3, β_1 has the predicted sign, however, β_1 and β_2 are insignificant and H_2 cannot be rejected (p-value 0.350). In contrast, after including the competing scorecards in model 4, the predictive negative relationship between the scorecard and excess returns becomes significant ($\beta_1 = -0.097$, p-value .094). For the quarterly data in panel B, both coefficients have the predicted sign, but neither is significant.[17] The same is true of β_5; the higher the surplus consumption ratio, the lower the subsequent quarter's excess return; however, the p-value is .112. In contrast, $\widehat{cay}$ is positively and significantly related to next month's and next quarter's excess return.

Given the triple effect of bad news but the double effect of good news, our theoretical model makes two new predictions regarding conditional skewness. The first prediction is a negative relationship between the scorecard and conditional skewness. From Equation (5), when z_t is large and negative, investors expect that in the next period the scorecard will take a large step back toward zero since ϕ is less than one. This expected large step toward less risk aversion and less sensitivity to news conditionally dampens the effect of bad news, making large negative returns less likely. On the other hand, when the scorecard is large and positive, the ϕ-induced large step back toward zero exacerbates bad news, causing conditionally negative skewness. The second prediction is that the scorecard/skewness relationship will be concave because the size of the expected ϕ-induced step toward a scorecard of zero is larger when the scorecard is further away from zero.

Our estimate of each month's skewness coefficient, $\hat{\mu}_{3,t}$, is created from within-month daily returns in the spirit of French, Schwert, and Stambaugh (1987); that is, $\hat{\mu}_{3,t} = \left[\frac{1}{N}\sum_{n=1}^{N}(r_{n,t} - \bar{r}_t)^3\right]/\sigma^3$, where $r_{n,t}$ is the market return on day n of month t, $\bar{r}_t$ is the average daily return in month t, N is the number of trading days in month t, and $\sigma_t^2 = \sum_{n=1}^{N} r_{n,t}^2 + 2\sum_{n=1}^{N} r_{n,t}r_{n+1,t}$. Regarding skewness, our model's two new predictions are that both β_1 and β_3 are negative in empirical models 3 and 4 when the dependent variable is $\hat{\mu}_{3,t}$. We compare these two predictions against null alternative three: H_3: $\beta_1 = 0$, $\beta_3 = 0$.

With regard to skewness within a month, our scorecard's predictive ability is significant with or without the consumption-aggregate wealth ratio's trend deviations. In both empirical models 3 and 4, β_1 and β_3 are

[17] We also explored whether our measure of the scorecard has predictive power beyond that of financial and macroeconomic indicators previously found to predict excess returns. The additional dependent variables were the dividend-yield ratio, the default spread (difference between BAA and AAA corporate bond yields), the relative short-term interest rate (3-month Treasury bill yield less its 12-month moving average), and the term spread (difference between the long-term Treasury bond yield and the 3-month Treasury bill yield). Including these varibles in the excess return regression increased the significance of β_1. Relative to this common set of so-called predictors, the scorecard's performance is noteworthy. For example, β_1's p-value was typically lower than the p-values of the additional independent variables.

negative and significant, and hypothesis three, H_3: $\beta_1 = 0$ and $\beta_3 = 0$ is strongly rejected (p-value less than .001). In the quarterly regressions, both coefficients retain the predicted sign, but β_3 loses its significance. In contrast to our scorecard, neither $\widehat{cay}$ nor $\widehat{scr}$ can be used to predict quarterly skewness; β_4 and β_5 are not significant. The general conclusions from Table 4 are that $\hat{z}$ seems to be better at predicting conditional volatility and skewness, whereas Lettau and Ludvigson's $\widehat{cay}$ measure is better at predicting excess returns. CC's $\widehat{scr}$ is not significantly related to quarterly conditional volatility, excess returns, or skewness.[18]

We have empirically tested three sets of predictions relating our scorecard to clustered volatility, excess returns, and skewness. However, our theoretical model potentially captures other stylized facts about stock returns. We cite one example. Figures 4 and 5 together suggest that our theoretical model is capable of explaining the "Sharpe ratio volatility puzzle" documented by Lettau and Ludvigson (2002). Many of the newer asset pricing models, such as CC and BHS, unlike the traditional CAPM, are capable of generating a time-varying equity risk premium. However, these newer theoretical models require that the equity risk premium vary positively with stock market volatility. Evidence of the comovement of time-varying excess returns and volatility is mixed. For example, Lettau and Ludvigson (2002) and Li (2001) document a negative relationship, whereas Campbell and Hentschel (1992) and Kim, Morley, and Nelson (2003) report a positive relationship. Our model predicts a positive relationship half the time (when scorecards are negative) and allows for a negative relationship the other half of the time.

5. Conclusion

In contrast to the historical description of stock prices as a "random walk with a drift," Cochrane (1999) summarizes many of the "new facts in finance." Stocks exhibit time-varying risk premia that are on average higher than models allow, with excess ex post volatility and serial correlation in volatility. The serial correlation in volatility has a high-frequency component and, most relevant to our model, a low-frequency component. We develop a preference-based equilibrium model that provides an explanation for the empirical observances of conditional volatility as well as many of the other new facts. Agents in our theoretical model derive utility

[18] Tallarini and Zhang (2000) find that the surplus consumption ratio "matches reasonably well the mean stock returns but it fails to match the higher moments such as variance, skewness, and kurtosis." Li (2001) finds that the surplus consumption ratio can predict a small portion of the changes in excess return; however, this ability is found after searching over three memory parameters, four horizons, linear and nonlinear models, and monthly, quarterly, and annual data aggregation levels. Menzly, Santos, and Veronesi (2001) find that the surplus consumption ratio does well in cross-section tests.

from both consumption and wealth changes, similar to BHS. Utility from wealth changes depends on a mental scorecard of past investment performance. After past unexpected gains (losses) the scorecard is positive (negative), investors are less (more) risk averse, and the risk premium required to entice them to hold stocks is small (large). Thus the scorecard can explain time- or wealth-varying excess returns.

Unique to our model, as investors' expected wealth is perturbed in either direction (positive or negative scorecard), they become more sensitive to news, and stock prices react more to news than when the investors are not sensitive (or returns are as expected). Because investors' sensitivity to news is persistent, return shocks lead to greater return volatility, and this heightened volatility decays slowly over time.

We numerically solve our model and show that it is consistent with time-varying excess returns, high risk premia, skewness, and clustered volatility. We generate artificial data from our model and find that these data display many of the characteristics found in the new empirical literature. We also structure our model so that it not only generates known stock returns characteristics, but also generates new and specific refutable predictions.

We test several of these new predictions on monthly U.S. stock data. Specifically, we feed our model actual return data and create a time series of the scorecard, then test whether the scorecard can predict volatility, excess returns, and skewness. First, we find our scorecard is significantly correlated with future volatility in ways predicted by our model. As the scorecard deviates from zero, the next period's volatility increases and negative deviations (past losses) predict higher volatility than positive deviations. Second, high levels of our scorecard predict low excess returns (past gains induce less risk aversion) as predicted; however, negative scorecards do not always predict high excess returns. Third, our scorecard is significantly and negatively correlated with return skewness as predicted by our theoretical model.

In essence, by making a new sensitivity assumption, we allow a preference model to explain conditional volatility in low-frequency returns. Yet much work remains to be done. Laboratory experimentation with financial rewards would help to substantiate our sensitivity assumption. Our model could be further tested against its predictions regarding conditional Sharp's ratios. While we use monthly data, one could start with high-frequency data in connection with an estimation procedure such as fractionally integrated GARCH to parse out the low-frequency volatility factor, which would also allow for alternative forms of our behavioral assumptions (e.g., hypergeometric decay rather than geometric decay in the scorecard). Nevertheless, building on the success of preference models in explaining long-run stock predictability and excess volatility, we find that such models can be extended to explain volatility clustering as well.

Appendix A

We provide a description of the variables used in our empirical analysis as well as our sources for each data series below. In general, we use the variable definitions and sources common in the literature. Unlike many recent empirical articles, we are not just interested in testing our model on postwar data; consequently we occasionally deviate from normal sources in order to test our model over longer horizons.

Monthly returns (1802–2000), r_t
We use the return (with dividends) on the CRSP value-weighted portfolio for our measure of monthly return over the period 1926–2000. In addition, we use Schwert's (1990) index return data for our measure of return over the period 1802–1925.

Daily returns (1885–2000), r_t
We use the return (with dividends) on the S&P 500 index portfolio for our measure of daily return over the period July 1962–2000. Our source of these data is CRSP. In addition, we use Schwert's (1990) index return data over the period 1885–1962. Schwert's data contain daily stock returns for the Dow Jones composite portfolio from February 16, 1885, through January 3, 1928, and for the S&P's composite portfolio from January 4, 1928, through July 2, 1962.

Risk-free rate (1926–2000), $r_{f,t}$
We use the return on the 30-day Treasury bill as our measure of the risk-free rate. Our source is the Ibbotson SBBI database.

Excess return (1802–2000), $r_t - r_{f,t}$
We use the difference between monthly or quarterly return and the risk-free rate as our measure of excess returns. Due to the lack of interest rate data over the period 1802–1925, we use actual, and not excess return, over this period.

Consumption wealth ratio (1959–2000), $\widehat{cay}_t$
We use Lettau and Ludvigson's (2001) monthly and quarterly log consumption-aggregate wealth ratio, which is a linear function of aggregate consumption, labor income, and asset wealth. See Lettau and Ludvigson (2001) for a more detailed description of the data.

Consumption (1959–2000)
We use a quarterly consumption measure that includes nondurables and services. Our source is Lettau and Ludvigson (2001), who obtain the data from the U.S. Department of Commerce's Bureau of Economic Analysis.

Population (1959–2000)
We use a measure defined as the total population, all ages including armed forces overseas. This series is used to create per capita consumption and in the creation of our surplus consumption state variable (see CC). Our source is the U.S. Department of the Commerce's Census Bureau.

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