

Multiple Unit Auctions and Short Squeezes

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This article develops a theory of multiunit auctions where short squeezes can occur in the secondary market. Both uniform and discriminatory auctions are studied and bidders can submit multiple bids. We show that bidders with short and long pre-auction positions have different valuations in an otherwise common value setting. Discriminatory auctions lead to more short squeezing and higher revenue than uniform auctions, *ceteris paribus*. Asymptotically, as the auction size approaches infinity, the two formats lead to equivalent outcomes. Shorts employ more aggressive equilibrium bidding strategies. Most longs strategically choose to be passive. Free riding on a squeeze by small, long players has no impact on these results, but affects revenue in discriminatory auctions.

The problem of how to organize the sale of many identical units is often solved in practice by holding an auction where bidders can submit multiple bids for multiple units. Such auctions are important, not least because the auctioned assets often play prominent roles in the wider economy. Examples include auctions of treasury securities, electricity, gold, and money. The scale of these auctions and the frequency with which they are held add to their significance. An important feature of these auctions in practice is that bidders often have established forward positions in the underlying asset before the auction is actually held. Players that are short must cover their positions either by buying in the auction or in the postauction market. The risk of leaving it to the postauction market lies in the chance that a few bidders buy so much in the auction that they obtain market power in the secondary market. This power can be used to

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ask exorbitant prices when short players come to buy, that is, to implement a short squeeze. This is known as the loser's nightmare [Simon (1994)]. In this article, we contribute to the theory of multiunit auctions by showing how a potential short squeeze affects bidders' valuations and strategies and the auction outcome. We model the two most commonly used multiunit auction formats, namely uniform price and discriminatory price auctions.

To get a perspective on the importance of the problem we are studying, over the period 1998–2002 the U.S. Treasury held more than 800 auctions with a total nominal value of \$12.7 trillion. Moreover, primary dealers (who must bid in the auctions) often enter these auctions with substantial short positions as a result of preauction demand for the to-be-issued security by pension funds and other institutions [*Joint Report* (1992), Bikhchandani and Huang (1993), Simon (1994), Nyborg and Sundaresan (1996)]. Empirical evidence by Sundaresan (1994) suggests that short squeezing is a regular feature of this market. Direct evidence where a primary dealer bought most of the auction and subsequently squeezed the shorts is provided by the notorious Salomon squeeze:¹

... the two-year notes became so scarce that the dealers who owned the notes charged exorbitant fees and financing costs when lending them to short-sellers. From small bond arbitrage operations in Chicago to the New York powerhouses, bond traders across America were badly burned. "The arbs were hurt the worst; several of the smaller shops went out of business." ... The pain was so severe and the cries of foul play so loud that the two-year note squeeze became the talk of the bond market for weeks. (*Wall Street Journal*, August 19, 1991)

While this auction was discriminatory, since October 1998, all U.S. Treasury auctions have been uniform.

Another important example of multiple unit auctions is repo auctions, which are used, for instance, by the European Central Bank (ECB) to channel euro-denominated liquidity into the banking sector. ECB repo auctions are held every week and the typical size is around 90 billion euros. Since July 2000, they have been discriminatory.² In these auctions, financial institutions submit bids for how much they would like to borrow on a collateralized basis from the central bank at a given interest rate. Some banks participating in repo auctions have a liquidity shortfall, perhaps as a result of their normal day-to-day activities. If these banks

¹ See the *Joint Report on the Government Securities Market* (1992). Jegadeesh (1993) and Jordan and Jordan (1996) provide further discussion.

² Until June 2000, the ECB used to conduct repo auctions as fixed-rate tenders. The interaction between short squeezing in the interbank market and bidder behavior in fixed-rate tenders is studied by Nyborg and Strebulaev (2001). The ECB experience with discriminatory auctions has been studied by Bindseil, Nyborg, and Strebulaev (2002).

do not manage to cover their liquidity shortfall in the auction they must borrow in the interbank market or at the marginal lending facility (discount window). That a squeeze on liquidity can occur is suggested by the spikes observed in interbank rates around the end of the reserve maintenance period [see Hartmann, Manna, and Manzanares (2001) for European evidence and Hamilton (1996) for U.S. evidence].

The possibility of a short squeeze in the postauction market has importance for bidders in the auction as well as for the seller. From a short bidder's perspective, an important question is how to bid to avoid being squeezed. Long players are interested in how to bid in order to implement a squeeze or potentially free ride on somebody else who will implement a squeeze. Auction revenue may be larger when the chance of a squeeze is larger, since this is likely to involve more aggressive bidding. Thus a seller may view an auction procedure that promotes squeezing as desirable. Under a squeeze, however, prices (or interest rates) are being distorted away from their competitive levels and volatility increases. This may be undesirable for many sellers such as sovereign treasuries and central banks. For example, the U.S. Treasury has expressed a low award concentration as an auction objective and, to that end, does not allow individual dealers to buy more than 35% in the auction.³ Whatever the seller's objective may be, it is important to establish the extent to which short squeezing, price distortion, and the seller's revenue depend upon the auction format, the size of the auction, and preauction allocations.

To address these issues we develop a model where a multiple unit auction of a homogeneous asset is followed by postauction trading. At the time of the auction, bidders may already have long or short positions in the to-be-auctioned asset. Because our aim is to focus on the interaction between strategic behavior in the postauction market and the auction itself, preauction allocations are exogenously given. Short squeezing may happen in the postauction market if some bidders are so large that they have market power. In both uniform and discriminatory auctions, bidders compete by simultaneously submitting collections of bids. Since the model is cast in the context of borrowing, individual bids consist of a quantity that the bidder wishes to borrow and an interest rate. But our analysis and results apply equally and in full to treasury auctions and more generally to security or commodity auctions where players buy the underlying asset outright and bids are price-quantity pairs. The bids with

³ The 35% rule takes into account bidders' when-issued positions. As illustrated by the Salomon scandal, the U.S. Treasury will punish dealers that attempt to get around this rule. The *Joint Report* (p. C-7) informs us that Salomon was prohibited from bidding in U.S. Treasury auctions on behalf of its customers for an indeterminate time. In addition, Salomon was fined nearly \$300 million [Sundaresan (1997, p. 72)]. In treasury auctions in other countries, for example, Sweden, there are no such limits and indeed it happens from time to time that a single bidder buys the entire auction [Nyborg, Rydqvist, and Sundaresan (2002)].

the largest interest rates are hit first, until supply is exhausted. In uniform auctions, all bidders pay the same “market-clearing” rate; in discriminatory auctions, bidders pay what they bid. Our main analysis covers the case where there is only one short player, but we also discuss the case of multiple shorts.

Our first finding deals with the valuation of the auctioned assets. In principle, our model is cast in a common value setting, since in the absence of a squeeze, the competitive rate (or price) will prevail in the secondary market. However, we show that the possibility of a squeeze introduces a fundamental valuation asymmetry between short and long bidders as well as between different types of long players. Short bidders tend to have downward-sloping valuation schedules, large long players tend to have upward-sloping valuation schedules, and small long players tend to have flat valuation schedules. The varying marginal valuations of bidders has important consequences for equilibrium bidding strategies and outcomes.

This also has more general ramifications; it shows that one has to be careful in categorizing auctions to be of the common- or private-value varieties. Typically the auction literature has taken this categorization to be exogenously given. Our analysis shows that players’ valuations can depend on their endowments and on the market structure. Unlike the exogenous private and independent values model studied by Vickrey (1961) and others, *our setting has endogenous private values and these are not independent.*

Our main result is that there is a trade-off between discriminatory and uniform auctions. Expected revenue is higher under discriminatory auctions,⁴ but the equilibrium probability of a short squeeze is lower under uniform auctions. Thus discriminatory auctions involve a higher incidence of transactions at noncompetitive rates in both the auction itself and the secondary market. Postauction volatility is therefore predicted to be higher under discriminatory auctions, which is consistent with the empirical findings of Nyborg and Sundaresan (1996) in a study of the U.S. Treasury experiment with uniform versus discriminatory auctions in the 1990s.

These results arise solely from the threat of a short squeeze. We do not consider the relative merits of uniform versus discriminatory auctions on other dimensions such as the winner’s curse [see, e.g., Milgrom and Weber (1982)] or the extent to which they may lead to monopsonistic market power among bidders [Wilson (1979), Kyle (1989), Back and Zender (1993)]. Our article draws on the literature on short squeezing, particularly on Dunn and Spatt (1984) and Cooper and Donaldson (1998). However, our main emphasis is on multiunit auctions, and we are not aware of any other model that examines the impact of a potential short squeeze on

⁴ See Section 3.1 for a discussion of how this relates to the empirical evidence on auction revenue.

equilibrium in *auctions where bidders can submit multiple bids for multiple units*. The work most closely related to ours is Chatterjea and Jarrow (1998), who model a preauction forward market, a single bid auction, and a postauction market in which a short squeeze may occur.⁵ However, our article differs substantially from theirs on several important dimensions. For example, (i) we model true multiple unit auctions, (ii) there can be any number of strategic players, and (iii) all players can participate in the auction regardless of their preauction position in the underlying asset. While Chatterjea and Jarrow study the case that a dealer attempts to squeeze a short player who cannot participate in the auction, *we study how short and long players compete in the auction in the face of a potential short squeeze*, which is the most relevant scenario in many contexts. For example, in U.S. Treasury auctions, many primary dealers are often short in the when-issued market and are also the main participants in the auction (*Joint Report*, 1992).

A significant issue raised in the short-squeezing literature is the extent to which “small” long players are able to free ride on a squeeze by a “large” long player. Kyle (1984) posits that small longs would be able to sell all their units well above the competitive price before the short squeezer would be able to sell any units. This is formalized by Cooper and Donaldson (1998) within an explicit market structure. The ability of smaller players to free ride on a squeeze is also well recognized by traders we have interviewed. This is related to the well-known point that when there are positive externalities, smaller players can do better (on a per unit basis) than larger players, because the latter will often internalize the externality [Olsen and Zeckhauser (1966), Bergstrom, Blume, and Varian (1986)]. A famous example from the finance literature is the ability of small shareholders to free ride on the monitoring efforts of a large shareholder [Shleifer and Vishny (1986)]. However, using a similar framework to Cooper and Donaldson, Dunn and Spatt (1984) provide a short-squeezing model without free riding. The contrast arises because of differences in the market microstructures. This raises an important question. How is bidding in the auction affected by the scope for free riding? To study this we employ a generalized short squeezing model that nests the models of Dunn and Spatt, and Cooper and Donaldson as special cases. Surprisingly we find that the main qualitative features of the auction equilibrium are not sensitive to the scope for free riding.

We explicitly characterize auction equilibria when there is one short. There are pure strategy equilibria for uniform auctions, but only mixed

⁵ Other models of market cornering and short squeezing include Kyle (1984), Jarrow (1992, 1994) and Kumar and Seppi (1992). These articles do not model auctions. Models of treasury auctions with either a when-issued or a resale market include Bikhchandani and Huang (1989) and Viswanathan and Wang (2000). These articles do not consider short squeezes. For a recent survey on auction theory, see Klemperer (1999).

strategy equilibria for discriminatory auctions. In equilibrium under either format, the threat of a squeeze induces shorts to submit collections of bids with a higher expected mean rate (or price) and more dispersion than longs. This is a result of the differential valuations of shorts and longs. In fact, because they place such high value on the first few units, shorts bid so aggressively for them that most longs optimally choose not to participate in the auction. This nonparticipation result is quite surprising, particularly when the scope for free riding is large, since a small long would not lose his ability to free ride in the secondary market if he were to buy a small amount in the auction.

Under discriminatory auctions, the equilibrium probability of a short squeeze decreases with the auction size, increases with the market power of the largest long players, and decreases with the scope for free riding, *ceteris paribus*. As a result, price distortions, postauction volatility, and revenue per unit sold tend to be smaller when the auction size is large or market power is small. Finally, discriminatory and uniform auctions are asymptotically equivalent in that their outcomes converge as the auction size grows.

The rest of the article is organized as follows. Section 1 describes the model. Section 2 contains the private valuations result and the equilibrium analysis. Section 3 draws out empirical predictions. Section 4 discusses extensions, and Section 5 concludes. The appendix contains all proofs.

1. The Model

We construct a three-date model where at date 1 there is a multiple unit auction, at date 2 there is trading in a secondary market, and at date 3 trades are settled and payoffs are collected. There are N players with initial positions in an underlying asset of $Y_0 = \{y_{n,0}\}_{n=1}^N \in \mathcal{Z}^N$, where $\mathcal{Z}$ denotes the set of integers. Initial allocations of individual players can be negative as well as positive. However, the total initial supply, $Q_0 = \sum_{n=1}^N y_{n,0}$, is nonnegative.⁶ Initial allocations are common knowledge. We refer to players with negative (positive) positions as *short* (*long*).

We will think of the underlying asset as being money, and players with negative initial positions must refinance the loans that these positions

⁶ The assumption that $Q_0 \geq 0$ is realistic, but not essential. $Q_0 < 0$ would mean that some units of the physical underlying asset or long forward contracts on it would be held by players outside the model (see Section 4). The assumption that initial allocations are integer quantities corresponds to the fact that, in multiple unit auctions in practice, there is usually a quantity multiple. For example, in U.S. Treasury auctions, individual bids must be for quantities in increments of \$1000 nominal. In ECB repo auctions, the quantity multiple is 0.1 million euros. In addition to being realistic, this discreteness assumption means that the type of underpricing equilibria in the uniform auction studied by Wilson (1979) and Back and Zender (1993) does not exist in our model [see Kremer and Nyborg (2004)]. Underpricing could be recovered by introducing a tick size for prices [see Back and Zender (1993) for an example]. However, Kremer and Nyborg (2004) show that if the tick size is "small," then equilibrium underpricing is at most one tick. Our analysis could be viewed as representing the limiting case as the tick size on prices goes to zero.

represent by obtaining the required funds either by borrowing in the auction at date 1 or in the secondary market at date 2. All loans made at these two dates mature at date 3. The auction is held by an agency that does not participate actively in the game. This agency can be thought of as the central bank (or treasury) and the N primary players can be thought of as commercial (or investment) banks. One could equally think of the underlying asset as being a commodity or security and the auction as being a *reverse repo auction* of the asset. In this case, players would be bidding to borrow the commodity until date 3.⁷

Denote the award to the n th player in the auction by $y_{n,1}$ and the quantity-weighted average interest rate that he must pay on this amount by $a_{n,1}$. Let $y_{n,2} = y_{n,0} + y_{n,1}$ denote a player's holdings at the beginning of date 2. Players with $y_{n,2} < 0$ must borrow at date 2 to cover their short positions. This opens up the possibility of short squeezing.⁸ Denote the quantity-weighted average interest rate at which the n th player lends or borrows at date 2 by $a_{n,2}$. Thus the total interest earnings or payments to player n at date 3 are

$$\pi_n = a_{n,2}y_{n,0} + (a_{n,2} - a_{n,1})y_{n,1}, \quad (1)$$

where all variables except $y_{n,0}$ will be determined endogenously. The objective of a player is to maximize π_n .⁹

Date 1: uniform and discriminatory auctions

The size of the auction is $Q \in \mathcal{Z}_+$. A bidder can make any number of bids such that the total quantity he demands is less than or equal to Q . An individual bid is an ordered pair $(r, q) \in [R_l, \infty) \times \Omega$, specifying an interest rate and a quantity, respectively, where $\Omega = \{1, \dots, Q\}$ and R_l represents the central bank's reservation rate.

Denote the set of bids submitted by player n by

$$\mathbf{b}_n = \{(r_{n,i}, q_{n,i})\}_{i=1}^{m(n)}, \quad (2)$$

where $m(n)$ is the total number of bids submitted by the player. These bids can be ordered into a demand function $x_n(r) = \sum_{i=1}^{m(n)} q_{n,i} \mathbf{1}_{[r_{n,i} \geq r]}$, which is a left continuous decreasing step function. The aggregate demand schedule is $X(r) = \sum_{n=1}^N x_n(r)$. The stop-out rate, r_s , is the highest rate at which

⁷ If the security or commodity were purchased outright instead of through a repo, algebraically and economically, the model and the results would be unaffected (see notes 8 and 9). In the context of when-issued markets and treasury auctions, date 2 would represent the issue date.

⁸ In other markets, squeezing is possible if substitutes cannot be used to settle positions. For example, in U.S. Treasury when-issued markets, only securities with particular CUSIPs can serve this function.

⁹ In the case where the underlying asset is a security or commodity and is purchased outright instead of borrowed through a reverse repo, the objective function of Equation (1) can also be interpreted in terms of prices. In this case, $a_{n,i}$ would be a quantity-weighted average price.

supply is exhausted (or R_l if no such rate exists). Specifically, since $X(r)$ is a left continuous step function,

$$r_s = \begin{cases} \max\{r \mid X(r) \geq Q\} & \text{if } \{r \mid X(r) \geq Q\} \neq \emptyset \\ R_l & \text{otherwise.} \end{cases} \quad (3)$$

The auctioned supply is allocated to the highest bids. Bids above the stop-out rate are awarded in full, while bids at the stop-out rate are rationed (pro rata). To formalize this, let $dx_n(r_s)$ be the marginal demand of player n at the rate r_s and let $x_n(r_s^+)$ denote his demand at prices above r_s .¹⁰ Then the n th player receives an auction award of

$$y_{n,1} = x_n(r_s^+) + \frac{dx_n(r_s)}{\sum_{i=1}^N dx_i(r_s)} \left[Q - \sum_{i=1}^N x_i(r_s^+) \right]. \quad (4)$$

The difference between uniform and discriminatory auctions lies in the rate that winning bidders must pay. In a uniform auction, all winning bidders pay the stop-out rate. Therefore, the interest costs on the winning bids of the n th bidder will be

$$\text{interest in uniform auction} = a_{n,1} y_{n,1} = r_s y_{n,1}. \quad (5)$$

In a discriminatory auction, winning bidders pay what they bid. Thus the n th bidder's interest costs are the sum of the interest on its winning bids, that is, interest in discriminatory auction = $a_{n,1} y_{n,1}$

$$= \sum_{i=1}^{m(n)} q_{n,i} r_{n,i} 1_{[r_{n,i} > r_s]} + [y_{n,1} - x_n(r_s^+)] r_s. \quad (6)$$

Hence for discriminatory auctions, $a_{n,1}$ can be larger than the stop-out rate.

Date 2: the secondary market and short squeezing

Although our primary focus is on the auction itself, what makes our analysis new and interesting is the possibility of short squeezing in the secondary market. We model short squeezes in a reduced form that nests the models of Dunn and Spatt (1984) and Cooper and Donaldson (1998) as special cases. In these articles, there is a short squeeze if and only if some player has market (monopoly) power, which using our notation is defined as follows:

Definition 1. Let $\mathcal{N}$ denote the set of all N players. For $t \in \{0, 2\}$, the “market power” of player n is

$$z_{n,t} \equiv \max \left[0, - \sum_{i \in \mathcal{N} \setminus n} y_{i,t} \right]. \quad (7)$$

¹⁰ Formally, (i) $x_n(r_s^+) = \sum_{i=1}^{m(n)} q_{n,i} 1_{[r_{n,i} > r_s]}$, and (ii) $dx_n(r_s) = \sum_{i=1}^{m(n)} q_{n,i} 1_{[r_{n,i} = r_s]}$, so $dx_n(r_s) = 0$ if bidder n places no bids at r_s .

This says that at date 2, the market power of the n th player is the units of the underlying asset held by that bank that the shorts need to cover their positions and cannot obtain from other players. Shorts and “small” longs have no market power. If no player has market power, all units trade at the competitive rate of $R_0 \geq R_l$. However, if a player has market power over z units, he can lend z units at $R_h > R_0$ to the shorts.¹¹ After shorts have refinanced, longs lend any remaining units of the underlying asset, perhaps to retail clients, at the competitive rate.

The question that remains is: In the case of a short squeeze, what is the transaction rate for those units that shorts need to cover and over which no player has monopoly power? There is no consensus answer in the literature. In Dunn and Spatt (1984), the equilibrium transaction rate on these units is R_0 . In contrast, in Cooper and Donaldson (1998), when there is a unique largest long player, all other longs are able to lend all their units at R_h , as suggested by Kyle (1984). In other words, in equilibrium, small players free ride on the squeeze of a large long player, and shorts get squeezed on all units they need to cover. The different conclusions reached by these articles is due to differences in trading mechanisms. The general point is that the extent to which “small” longs are able to free ride on a squeeze depends on how the market is organized.¹²

To study the impact of different levels of free riding, we introduce $\delta \in [0, 1]$ as a *measure of the scope for free riding*, where $\delta = 0$ denotes no free riding (Dunn and Spatt) and $\delta = 1$ denotes full free riding (Cooper and Donaldson). There are three types of longs at date 2: (i) *small longs* with no market power; (ii) *intermediate longs* with positive market power but not the largest; and (iii) the X *largest longs* with the largest positive market power. We denote these by L_0 , L_1 , and L_2 , respectively. If no player has market power, $X=0$ and L_1 and L_2 are empty. Given

¹¹ Following much of the literature [e.g., Kyle (1984), Dunn and Spatt (1984), Vila (1989), Cooper and Donaldson (1998)], we take the squeeze rate, R_h , as given. In the context of money markets, R_h would be the central bank’s marginal lending facility, or perhaps a lower rate determined by bargaining (see note 12). In most countries or currency areas, the competitive rate is typically straddled by the lending and deposit facilities of the central bank. In other contexts, R_h would reflect the price of the “fancy” good [see, e.g., Salant (1984)]. In treasury auctions, R_h would be determined by the penalty from failing to deliver on a trade from the when-issued market.

¹² An alternative market structure, suggested by a referee, is that a strategic long bargains with a strategic short [see also Vila (1989), where a strategic long makes take it or leave it offers and therefore has all bargaining power]. These two players would share the bargaining surplus and free riding would be eliminated (unless bargaining is inefficient). The squeeze rate would now depend on the players’ bargaining powers. However, as in Vila (1989), it would not depend on the players’ positions under Rubinstein (1982) or (asymmetric) Nash bargaining, when disagreement means that the short must use the marginal lending facility or go to the “fancy” good market (proof available from the authors upon request). Under this bargaining approach, the squeeze rate may therefore be viewed as exogenous for the purpose of our analysis.

the history of the game up to date 2, payoffs to long players are as follows:¹³

$$\pi_n = \begin{cases} [R_0 + \delta 1_{[X \geq 1]}(R_h - R_0)]y_{n,2} - a_{n,1}y_{n,1} & \text{if } n \in L_0 \\ R_h z_{n,2} + [R_0 + \delta(R_h - R_0)][y_{n,2} - z_{n,2}] - a_{n,1}y_{n,1} & \text{if } n \in L_1 \\ R_h z_{n,2} + R_0(y_{n,2} - z_{n,2}) - a_{n,1}y_{n,1} & \text{if } n \in L_2. \end{cases} \quad (8)$$

Notice that when δ is relatively large, small and intermediate longs do better on a per unit basis than the largest longs and, in some cases, may even do better in absolute terms.

The aggregate payoff to the shorts is determined by Equation (8) and the fact that the total gross payoff to all players (i.e., net of interest due from units obtained in the auction) is

$$(Y_L - Y_S)R_0, \quad (9)$$

where $Y_L = \sum_{n \in \mathcal{N}_2^+} y_{n,2}$, $Y_S = \sum_{n \in \mathcal{N}_2^-} |y_{n,2}|$, and where $\mathcal{N}_t^+ = \{i | y_{i,t} \geq 0\}$ denotes the set of long players and $\mathcal{N}_t^- = \{i | y_{i,t} < 0\}$ denotes the set of short players at date t . Define $Y_0 = \sum_{n \in L_0} y_{n,2}$, $Y_1 = \sum_{n \in L_1} (y_{n,2} - z_{n,2})$, and $Z_L = \sum_{n \in \mathcal{N}_2^+} z_{n,2}$. By subtracting from Equation (9) the gross payoffs to the various long players using Equation (8) and adding the shorts' interest costs from units obtained in the auction, we find that the aggregate payoff to the shorts is¹⁴

$$\begin{aligned} \sum_{n \in \mathcal{N}_2^-} \pi_n = & -R_h Z_L - R_0(Y_S - Z_L) - \delta 1_{[X \geq 1]}(R_h - R_0)(Y_0 + Y_1) \\ & - \sum_{n \in \mathcal{N}_2^-} a_{n,1}y_{n,1}. \end{aligned} \quad (10)$$

The first term in Equation (10) represents the squeeze itself on Z_L units. The second term is what the shorts would pay to cover the remaining units that they are short, if there were no free riding. The third term represents the free riding. This says that the number of units that no player has market

¹³ In their analysis, Cooper and Donaldson consider only the case where there is at most one player with market power. A proof that the general case (when many longs have market power) yields payoffs as stated in Equation (8) with $\delta = 1$ is available from the authors upon request. Note that we are focusing on their "endgame/delivery process," since, for our purposes, the dynamic aspects of their model are of secondary importance. Dunn and Spatt's model is only developed for one short. However, the general point is that there may be a trading mechanism that delivers the competitive price on those units over which players do not have monopoly power. In a more standard, nonsqueeze setting, Allen and Hellwig (1986) have shown in a model where capacity-constrained sellers choose prices as strategies that the equilibrium price converges in distribution to the competitive price as the number of sellers increases.

¹⁴ In deriving Equation (10), we have used the following implications of the definition of market power: (i) for any player with market power at date 2, $y_{n,2} - z_{n,2} = Y_L - Y_S$; and (ii) $Z_L = Y_L - Y_0 + |L_2|(Y_S - Y_L) + |L_1|(Y_S - Y_L)$, where $|L_1|$ is the number of intermediate longs. Note also that (ii) implies that for $X \geq 1$, $Y_1 + Y_0 + Y_S - Z_L - (X - 1)(Y_L - Y_S)$. This shows that the aggregate payoff to the shorts at date 2 is increasing in X when $\delta > 0$, ceteris paribus. This is a feature of Cooper and Donaldson's model. Details are available from the authors upon request.

power over and on which the shorts pay the squeeze premium, $R_h - R_0$, is equal to $Y_0 + Y_1$ (the aggregate “no market power position”).

Equation (7) also defines market power at date 0. This is what date 2 market power would be if no units were auctioned at date 1. To retain his entire date 0 market power, a long player will need to buy all units in the auction. To guarantee that no player will have market power at date 2, it is sufficient for the shorts to buy a total of

$$Z \equiv \max\{z_{n,0} \mid n \in \mathcal{N}\}$$

units in the auction. This parameter will play a central role in the auction analysis.

2. Auction Equilibrium

As a benchmark, observe that if all players have long positions initially, the unique equilibrium outcome under either auction format is that all units are bought at R_0 . For example, all players submit bids for Q units at a rate of R_0 . In this section we study the more interesting case that there are both short and long players. We assume there is only one short at date 0, but many longs. We address the question as to whether the short or the auctioneer benefit from competition among many longs. The answer is not obvious: longs may bid aggressively in order to implement a squeeze or passively in order to free ride on somebody else’s squeezing efforts. Multiple shorts are discussed in Section 4.

Without loss of generality, let $y_{2,0} \geq y_{3,0} \geq \dots \geq y_{N,0} \geq 0 > y_{1,0}$. Hereafter we refer to player 1 as the “short (player)” and the others as the “longs.” The number of longs with positive market power at date 0 is denoted by K . So only players $2, \dots, K+1$ have market power initially. We will be interested in longs with the largest market power: let there be M of them, $M \leq K$, with numbers $n=2, \dots, M+1$. Each of the M largest longs has initial market power of $z_{n,0} = z_{2,0} = Z$.

We assume that the auction size is *large*, in the sense that $Q > Z$. Thus the short can *cover* (buy Z units or more) in the auction if he bids sufficiently aggressively. However, it is not a foregone conclusion whether in equilibrium he will buy enough to avoid being squeezed. In contrast, if the auction size is *small* ($Q < Z$), there will always be a squeeze at date 2 under any auction format, since the auction does not offer the chance for the short to cover. In this case, at least two players value each auctioned unit at R_h and competition drives the auction rate up to R_h .¹⁵ Large

¹⁵ When $Q = Z < |y_{1,0}|$, the probability of a short squeeze depends on δ . If $\delta = 0$, the short and all longs with market power value all units at R_h . In equilibrium, they bid this and there is a squeeze for sure because of rationing. If $\delta = 1$, the probability is zero because if the short buys all Z units, he avoids being squeezed on $|y_{1,0}|$. Thus, he is willing to bid above R_h . When $Q = Z = |y_{1,0}|$, there is a squeeze for sure because of rationing.

auctions are not only more interesting and challenging to study, but we also believe they are the most relevant scenario in practice.

2.1 Private valuations

The setting in our article is a seemingly common value since, in the absence of a short squeeze, the prevailing rate is R_0 for all units. In addition, in the event of a squeeze, the extra interest paid by a short player goes directly to a long. However, we show here that there is a simple but fundamental asymmetry between short and long players which leads them to value the auctioned units differently.

We ask the following question: How much is player n willing to pay to obtain an additional unit in the auction? We keep the total number of units in the auction fixed at Q , so if player n gets an additional unit, some other player gets one unit less. The player's willingness to pay for an additional unit is the difference between his gross payoff with the unit and without; that is, $V_n(q) = a_{n,2}^*(q)(y_{n,0} + q) - a_{n,2}^*(q-1)(y_{n,0} + q - 1)$, where $a_{n,2}^*(q)$ is the date 2 equilibrium quantity weighted average rate at which the n th player lends or borrows his $y_{n,0} + q$ units. We refer to $V_n(q)$ as player n 's *valuation schedule*.

To illustrate the difference in valuations between shorts and longs, we consider initially the case that there are only two bidders, one short (player 1) and one long (player 2). In this case, $z_{2,2} = \max[0, -y_{1,2}]$ and the market power of the long at date 0 is $Z = -y_{1,0}$. Hence, for a given outcome in the auction, $\{y_{1,1}, y_{2,1}, a_{1,1}, a_{2,1}\}$, Equation (10) tells us that trading in the secondary market will yield the following date 3 payoff for the short:

$$\pi_1 = \begin{cases} (R_h - a_{1,1})y_{1,1} - ZR_h & \text{if } y_{1,1} < Z \\ (R_0 - a_{1,1})y_{1,1} - ZR_0 & \text{if } y_{1,1} \geq Z. \end{cases} \quad (11)$$

This shows that the short values the first Z units he wins in the auction at R_h and the last $Q - Z$ units at R_0 . Intuitively the short needs to win only Z units to avoid being squeezed, and any additional units that he wins can be lent at R_0 .

Using Equation (8) and $y_{1,1} = Q - y_{2,1}$, the long's payoff can be written as

$$\pi_2 = \begin{cases} (R_0 - a_{2,1})y_{2,1} + y_{2,0}R_0 & \text{if } y_{2,1} \leq Q - Z \\ (R_h - a_{2,1})y_{2,1} + y_{2,0}R_0 - (Q - Z)R_h & \text{if } y_{2,1} > Q - Z. \end{cases} \quad (12)$$

This shows that the long values the first $Q - Z$ units he wins in the auction at R_0 and the last $Q - Z$ units at R_h . The intuition is that the long needs to win at least $Q - Z + 1$ units to implement a squeeze. If he wins less, he can lend all his units at R_0 . In contrast, every unit above $Q - Z$ that he wins can be lent at R_h in the postauction market.

This illustrates that the possibility of short squeezing in the secondary market can give rise to differential valuations in the primary market in a seemingly common value setting. *In particular, the short's valuation schedule is decreasing, while the long's is increasing.* The idea that a potential short squeeze can give rise to private valuations in multiunit auctions has been suggested by Sundaresan (1994) [see also Back and Zender (1993)]. The varying marginal valuations, both across and within bidders' valuation schedules, are the reason why uniform and discriminatory auctions will lead to different outcomes.

For $N > 2$, the value of an additional unit to a player depends on how the remaining units are distributed. For example, the value to the short from capturing a q th unit in the auction is (i) R_h if he will be squeezed regardless of whether he wins $q - 1$ or q units; (ii) R_0 if he will not be squeezed; or (iii) $R \geq R_h$ if by winning this extra unit the short goes from being squeezed to not being squeezed. To see this, suppose $N = 3$, initial allocations are $\{-8, 12, 5\}$, $Q = 5$, and $\delta = 1$. If the short gets two units and the largest long gets three units, the short will be squeezed on six units at date 2. If the short now captures a third unit, he goes from being squeezed to not being squeezed and can borrow five units at date 2 at R_0 . So here the short values this third unit at $R = 6R_h - 5R_0 > R_h$. But in general it is impossible to unequivocally specify the short's valuation schedule. However, because the same fundamental forces are at work as when $N = 2$, the short generally values the first few units higher than the last unit, which he always values at R_0 , since $Q > Z$.

For longs, we need to distinguish between "small" and "large" players; that is, those with and without market power at date 0, respectively. Only large longs can implement a squeeze at date 2, since if a small long buys the entire auction, $z_{n,2} = 0$ for all n . The value to a large long from winning a q th unit is (i) R_h if he will be implementing a squeeze, (ii) $R_0 + \delta(R_h - R_0)$ if he will free ride on a squeeze; (iii) R_0 if none of the longs will be able to implement a squeeze (e.g., because the short wins the other $Q - q$ units); or (iv) $R \leq R_0$ if by winning this extra unit the player will stop somebody else implementing a squeeze. We cannot unequivocally specify a large long's valuation schedule. However, in general, a large long values the first few units lower than the last unit, which he always values at R_h , since if he buys all units he will implement a short squeeze for sure.

The value to a small long from capturing a q th unit can also be $R_0 + \delta(R_h - R_0)$ (if he free rides on a squeeze), R_0 (if there is no squeeze), or $R \leq R_0$ (if he stops a squeeze). Unlike a large long, the small long generally values the first few units higher than the last unit, which he always values at R_0 . This opposite pattern is a result of the small long's inability to implement a short squeeze.

The above discussion illustrates the conditional nature of valuation schedules. It also illustrates that the short has a strong incentive to bid

aggressively for a few units only, in order to try to avoid being squeezed. Small longs may also benefit from bidding aggressively for a few units provided that they can free ride on somebody else's squeezing efforts.

The most long player n would be willing to pay for Q units in the auction is (per unit) $\bar{R}_{Qz_{n,0}} \equiv [(Q - z_{n,0})R_0 + z_{n,0}R_h]/Q$. This is the quantity weighted average interest rate the long would earn from winning all Q auctioned units. $\bar{R}_{Qz_{n,0}}$ is increasing in $z_{n,0}$ and therefore has a maximum at

$$\bar{R}_{QZ} \equiv \frac{(Q - Z)R_0 + ZR_h}{Q} < R_h. \quad (13)$$

This average, or break-even, value for each of the M largest longs turns out to be an important parameter in the analysis of equilibrium in discriminatory auctions.

2.2 Uniform auctions

Recall that a pure strategy for player n in the auction specifies a set of bids, as represented by Equation (2) with $\sum_{i=1}^{m(n)} q_{n,i} \leq Q$, as a function of the player's initial allocation. The equilibrium concept is Nash equilibrium in either pure or mixed strategies.

Theorem 1. *In uniform auctions the following is an equilibrium: Player 1 (the short) submits $\mathbf{b}_1^u = \{(R_h, Z), (R_0, Q - Z)\}$ and all other players submit $\mathbf{b}_n^u = \{(R_0, Q)\}$. In this equilibrium (i) the stop-out rate is R_0 and the revenue to the seller is QR_0 ; (ii) for every n , the payoff is $\pi_n = y_{n,0}R_0$; and (iii) there is no short squeeze in the secondary market.*

Intuitively the short is exploiting the differential valuations by bidding for the Z units he needs to cover at a very high price (say R_h). Faced with this, the longs can do no better than being *passive* (placing no bids above R_0) because of their lower valuations. There is no cost to the short from bidding so aggressively since he only pays the stop-out rate for all units he wins. The combination of differential valuations and uniform price thus eliminates short squeezing. That the short pays the stop-out rate rather than what he actually bids allows him to costlessly employ a strategy which, in a sense, preempts a short squeeze. As a consequence, all units in the auction are sold at the competitive rate, and there are no interest rate distortions or excess volatility in the secondary market. This is true regardless of the scope for free riding. The theorem also establishes that there are no benefits to new entrants from bidding aggressively in the auction, since it covers the case that some bidders have zero initial position and since the short's strategy is unaffected by the number of bidders.

Theorem 2 (uniqueness). *In uniform auctions, the outcome in Theorem 1 is the unique pure-strategy equilibrium outcome.*

When $N=2$, it is straightforward that there are no mixed-strategy equilibria that admit squeezing. However, with multiple players it is possible to construct mixed-strategy equilibria with squeezing by letting a large player bid for Q units at $\bar{R}_{QZ}$ with a very small probability and having a third player with no market power bid for $Q-1$ units using a uniform distribution with all mass right below $\bar{R}_{QZ}$. These equilibria would vanish if there were a positive probability that the large player trembled and submitted his bid slightly below $\bar{R}_{QZ}$ or the short bid for one unit just above R_0 , since then the small long would do better by doing nothing. Thus the mixed-strategy squeezing equilibria are not robust.¹⁶

2.3 Discriminatory auctions

Compared with uniform auctions, the analysis of discriminatory auctions is more complicated because there is no equilibrium in pure strategies. This is more than a technical result. It also implies that in any equilibrium, the probability, Θ , of a squeeze is positive.

Lemma 1. *There is no equilibrium in pure strategies in discriminatory auctions.*

This is a consequence of the different marginal valuations outlined above and the fact that bidders in discriminatory auctions pay what they bid. To see this, suppose for example that the short submits bids for Z units at R_h and $Q-Z$ units at R_0 , as he does in the uniform auction. The best response for each long would be to be passive (bid at R_0). But then the short could do better by demanding Z units at a rate marginally above R_0 , to avoid being squeezed. But then some long with market power could improve his payoff by being *active* (bid above R_0 with positive probability), etc.

Theorem 3. *Suppose the auction is discriminatory. For each of the M long players with the largest market power at date 0, there is an equilibrium in which only that player and the short are active, with all other players being passive. In particular, for each $n=2, \dots, M+1$, the following is equilibrium:*

(i) *The short submits $\mathbf{b}_1^* = \{(\tilde{S}, Z), (R_0, q_{1,2}^*)\}$, where $q_{1,2}^* \in \{0, \dots, Q-Z\}$ and $\tilde{S}$ is a random variable with support $[R_0, \bar{R}_{QZ}]$ and cumulative distribution function*

$$F(S) = \frac{Q-Z}{Z} \frac{S-R_0}{R_h-S}. \quad (14)$$

¹⁶ They are not “truly perfect” [Kohlberg (1981)], since they do not survive rate perturbations. Truly perfect equilibrium differs from Selten’s (1975) perfect equilibrium in that the equilibrium point must survive perturbations, or trembles, from every direction. Fudenberg and Tirole (1992) discuss this further.

(ii) Player n submits $\mathbf{b}_n^* = \{(\tilde{L}, Q)\}$, where $\tilde{L}$ is a random variable with support $[R_0, \bar{R}_{QZ}]$ and cumulative distribution function

$$G(L; \delta) = \frac{Z(R_h - \bar{R}_{QZ}) + \delta(R_h - R_0)(|y_{1,0}| - Z)}{Z(R_h - L) + \delta(R_h - R_0)(|y_{1,0}| - Z)}. \quad (15)$$

(iii) Every other player i submits $\mathbf{b}_i^* = \{(R_0, Q)\}$.

Expected equilibrium payoffs are (a) the short: $E[\pi_1] = y_{1,0}R_0 - (\bar{R}_{QZ} - R_0)Z$; (b) active long: $E[\pi_n] = y_{2,0}R_0$; (c) passive long: for $i \notin \{1, n\}$, $E[\pi_i] = y_{i,0}[R_0 + \Theta\delta(R_h - R_0)]$; (d) auctioneer: $E[\pi_A] = QR_0 + Z(\bar{R}_{QZ} - R_0) - \sum_{i \geq 3} y_{i,0}(R_h - R_0)\delta\Theta$.

This theorem describes M sets of equilibria, where only the short and one of the largest longs are active. Since the M largest longs are indistinguishable from each other and since the exact amount the short bids for at R_0 does not affect payoffs, the equilibria described in Theorem 2 are observationally equivalent. The two active banks play mixed strategies and a short squeeze occurs with positive probability.¹⁷

In equilibrium, the short splits his bids into a high bid for the Z units he needs to avoid being squeezed and a bid for some additional, less valuable units at the competitive rate, R_0 . This parallels the result for uniform auctions. But now the randomness of the short's strategy affords the longs with the opportunity to implement a short squeeze. So the active long submits a single bid for the entire auction at a rate that is above R_0 with positive probability. This reflects that the long needs to buy all units to maximize the value of a squeeze, since his valuation schedule is increasing. Figure 1 illustrates that the rate distribution used by the short for Z units, $F(S)$, first order stochastically dominates that used by the long for Q units, $G(L; \delta)$. Intuitively this happens because the short values the first Z units higher on average than the long values all Q units. Furthermore, in the event of no squeeze, the long ends up buying $Q - Z$ units, which he values at only R_0 . As a response to this, the long chooses $G(L; \delta)$ to have a mass point at R_0 , as seen in Figure 1. In other words, the long bids above R_0 only part of the time.

A surprising feature of equilibrium is that most long players strategically choose not to participate actively in the auction. When the scope for free riding is large, bidders prefer not being the largest long at date 2 because they do better by free riding. But why do these bidders not attempt to augment their positions by bidding above R_0 for a small number of units? When the scope for free riding is small, why do they

¹⁷ If the squeeze rate, R_h , were to depend on players' positions, then if the long bids for Q units at $\tilde{L} \geq R_0$ and the short bids for Z units, the players' equilibrium positions in the event of a squeeze will be common knowledge. Thus the equilibrium squeeze rate can be computed by bidders in the auction. Hence it is possible to construct equilibria as in Theorem 3 that have similar qualitative features (an example is available from the authors upon request). Endogenous R_h would have no impact on uniform auctions, since the short would still bid for Z units at a "very high" rate.

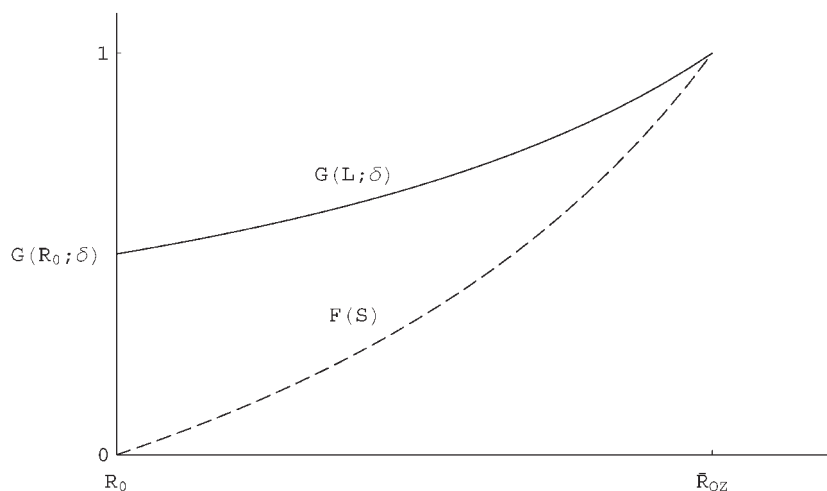


Figure 1

Distribution functions for the short, $F(S)$, and active long, $G(L; \delta)$.

not compete in the auction to be the player with the largest market power at date 2? In either case, most longs decide to be completely passive because the risk that the short will manage to cover in the auction is too large.¹⁸ The short is simply too aggressive to make it worthwhile for most longs to participate actively.

The short's aggressiveness is reflected in the equilibrium expected payoff of the active long, which is just $y_{2,0}R_0$. In other words, the long does not earn any rents from his ability to squeeze—his quasi-rents when a squeeze occurs are offset by the loss from overpaying in the event of no squeeze. The short's equilibrium expected payoff equals the value of his initial holding less the amount $(\bar{R}_{QZ} - R_0)Z$. So his expected interest costs on the Z units he needs to cover are less than his willingness to pay. These results can be understood by an analogy to private-value, single-unit auctions. In such auctions, the bidder with the highest valuation wins and pays the valuation of the second highest player (in expectation). In our case, the short has the higher valuation and his payoff is the same as if he got Z units at the long's valuation, $\bar{R}_{QZ}$, for sure. Indeed, this is the most the short would need to pay to guarantee no squeeze. This also provides intuition for why the short's equilibrium payoff is unaffected by the scope for free riding.

The private-value, single-unit auction analogy also sheds light on why most longs are passive. The active long values the auctioned units higher

¹⁸ Jehiel and Moldovanu (1996) show, in a single unit, private values setting that negative (or positive) externalities can lead to "nonstrategic participation." Our multiunit auction result is driven by the fear that free riding will not be possible and, to a smaller extent, the desire to free ride.

than the other longs, by virtue of being largest. So if competition from the short drives the rents of the active long to zero, then bidders with lower valuations also cannot expect positive rents from buying in the auction. By being passive they earn positive abnormal returns when there is some scope for free riding. An implication of the passivity of most longs is that Theorem 3 is robust to the possibility of market entry by new players, since it covers the case that some players have $y_{i,0} = 0$ and since players' strategies are not functions of N .

The equilibrium in Theorem 3 can be described as a "single-bid equilibrium" in the sense that no bidder submits more than one bid where the rate is above R_0 with positive probability. Single-bid equilibria are attractive because they are relatively simple. The equilibrium identified in the uniform auction is also a single-bid equilibrium. It is also outcome unique. Here we establish a similar, but somewhat weaker result for the discriminatory auction.

Theorem 4 (uniqueness). *In discriminatory auctions, the equilibria described in Theorem 3 are the only single-bid equilibria with only two active bidders.*¹⁹

This essentially means that there is a unique two-bidder, single-bid equilibrium in the discriminatory auction, since the equilibria in Theorem 3 are observationally equivalent.²⁰ Theorem 4 also establishes that there is no single-bid equilibrium where one of the smaller longs with market power is active and all other longs are passive. The intuition has its roots in how the short tailors his strategy according to the market power of the active long. The proof of the theorem shows that in a hypothetical equilibrium where the active long has $z_{n,0} < Z$, the short is not as aggressive as when $z_{n,0} = Z$. The implication is that one of the largest longs could step in and earn positive rents. This can be understood with reference to the analogy to private-value, single-unit auctions above. In Theorem 3, the mixed strategies of the two active players have support $[R_0, \bar{R}_{QZ}]$, which is intuitive since $\bar{R}_{QZ}$ is the active long's average valuation of Q

¹⁹ To be precise, the equilibria described in the theorem are unique up to trivial variations, such as where some passive bidders do not submit bids at all, demand less than Q at R_0 , or submit bids below R_0 , etc.

²⁰ While we cannot say for sure that there are no other Nash equilibria than those identified in Theorem 3, it is straightforward to use the theorem to construct correlated equilibria where several longs are active. Suppose, for example, that $M \geq 2$ and let the correlation device $\bar{\sigma}$ be a random variable that is uniformly distributed on $\{2, 3, \dots, M\}$. Then it is equilibrium for player n to be active as in Theorem 3 if and passive otherwise. One could also let these M players observe an additional signal, $\bar{l}$, drawn from Equation (15). It is equilibrium for player n to bid $\bar{l}$ if $\bar{\sigma} = n$ and $\bar{l} - \epsilon$ (for $\epsilon \in (0, \bar{l} - R_0)$) otherwise. This works because the player chosen to be the most aggressive is indifferent between bidding $\bar{l}$ or being passive. Furthermore, this coordination among the longs does not affect the short's best reply since his payoff is independent of the identity of the most aggressive long.

units. Theorem 4 also establishes that there is no two-bidder, single-bid equilibrium with different support.

3. Implications and Predictions

In this section we draw out empirical implications and predictions of our model, using Theorems 1 and 3. Our findings are divided into two parts: auction performance and bidder behavior. It should be emphasized that the predictions of our model are, by design, pure implications of the loser's nightmare.

3.1 Auction performance

Our main results are that discriminatory auctions lead to a higher probability of a short squeeze than uniform auctions and consequently a higher expected revenue, *ceteris paribus*. These results arise from the different valuations of shorts and longs and from the different pricing rules of the two auction formats. Relative to discriminatory pricing, uniform pricing reduces the costs to shorts from submitting a few very high bids to get the units they need to avoid being squeezed. Such balloon bids also discourage longs from bidding aggressively. Uniform auctions therefore have the twin effects of reducing the probability of a short squeeze and bringing revenue down.

The central role of the short in our analysis differentiates it from Chatterjea and Jarrow (1998), where a potential squeezer is not permitted to bid in the auction. This also means that the trade-off between discriminatory and uniform auctions on revenue versus short squeezing is not a feature of their model. Like us, Back and Zender (1993) also predict a lower revenue in uniform auctions, but this arises from imperfect competition rather than a reduction in the probability of a short squeeze.

That auction revenue is predicted to be higher in discriminatory auctions is interesting, not least because at first glance it appears to be counterfactual. Empirical studies of U.S. Treasury auctions show that markups (defined as the difference between auction and when-issued yields) are larger in discriminatory auctions than in uniform auctions [Nyborg and Sundaresan (1996), Malvey and Archibald (1998)]. This suggests a smaller revenue under discriminatory auctions. However, as pointed out by Nyborg and Sundaresan (1996), the problem with this inference is that when there is the potential for a short squeeze or when a squeeze is on, the when-issued yield will contain a "squeeze premium." In this case, the markup is not an accurate reflection of auction revenue. Hence revenue may be larger under discriminatory auctions even though the markup is larger, which is what would be happening under our model. The larger revenue is a pure reflection of the increased likelihood of a short squeeze.

Short squeezing leads to higher volatility in the secondary market, since some units change hands above R_0 . Hence our model predicts that volatility tends to be higher after discriminatory auctions. This is consistent with the empirical evidence [Nyborg and Sundaresan (1996)]. In the context of treasury auctions, we would expect a short squeeze to manifest itself through the specialness of the on-the-run security. We would therefore expect a positive relation between specialness and volatility after discriminatory auctions.

Next, we look closer at the equilibrium probability, Θ , of a short squeeze in discriminatory auctions. For $N = 2$ or $\delta = 0$, Theorem 3 implies that $\Theta = Z/(2Q)$, which leads to the intuitively appealing results that Θ is increasing in the market power of the long and decreasing in auction size. However, when $N > 2$ and $\delta > 0$, we have

$$\begin{aligned}\Theta &= \int_{[R_0, \bar{R}_{QZ}]} F(L) dG(L; \delta) \\ &= \frac{(Q - Z) \left(-Z(|y_{1,0}| - Z)\delta + (Q\psi - Z^2) \ln \left[\frac{Q\psi - Z^2}{\psi(Q - Z)} \right] \right)}{Q(|y_{1,0}| - Z)^2 \delta^2},\end{aligned}\quad (16)$$

where $\psi = Z(1 - \delta) + |y_{1,0}|\delta$.²¹ So in the general case, the equilibrium probability of a short squeeze depends on the exogenous parameters in quite nontrivial ways. The change in Θ with respect to Q can be decomposed into two effects:

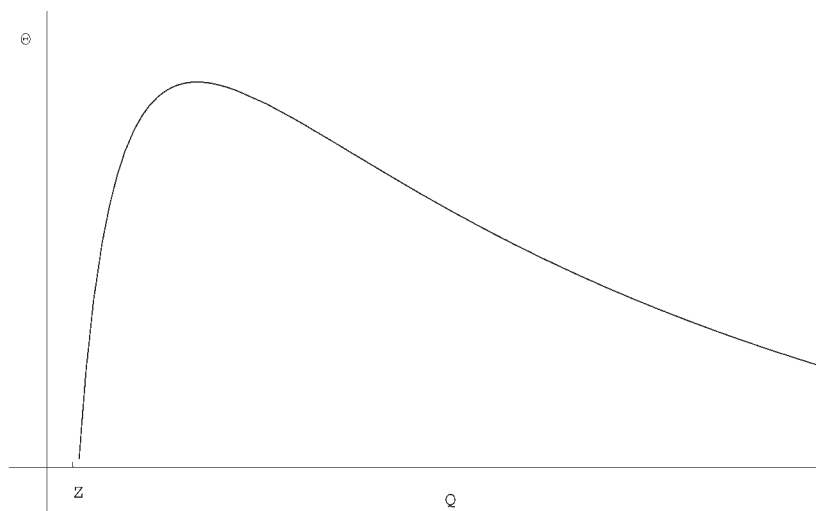
$$\frac{\partial \Theta}{\partial Q} = \int_{R_0}^{\bar{R}_{QZ}} \frac{\partial F(L) G'(L; \delta)}{\partial Q} dL + G'(\bar{R}_{QZ}; \delta) \frac{\partial \bar{R}_{QZ}}{\partial Q}.\quad (17)$$

The first term is a “direct” effect; it captures the effect an increase in Q has on $F(\cdot)$ and $G(\cdot; \delta)$, keeping the upper bound of these distributions, $\bar{R}_{QZ}$, constant. The second term is an “indirect” effect; it captures the effect of the decrease in $\bar{R}_{QZ}$ that results from an increase in Q . It can be shown that the “direct” effect is always positive and the “indirect” effect is always negative. A similar decomposition can be done for Z , but in this case the “direct” effect is negative and the “indirect” effect is positive.

Proposition 1. Suppose $N > 2$ and $\delta > 0$. (i) There is $\bar{Q}$ such that for all $Q > \bar{Q}$, the discriminatory auction equilibrium probability of a short squeeze, Θ , is strictly decreasing in the auction size, Q . $\bar{Q}$ approaches Z as δ goes to zero. (ii) There is $\bar{Z}$ such that for all $Z < \bar{Z}$, Θ is strictly increasing in Z . $\bar{Z}$ approaches Q as δ goes to zero. (iii) Θ is strictly decreasing in the absolute value of the short’s initial holding, $|y_{1,0}|$.

The first statement of the proposition shows that the indirect effect dominates for sufficiently large Q . This can be understood by recalling

²¹ For $N > 2$ and $\delta > 0$, $\lim_{\delta \rightarrow 0} \Theta = Z/(2Q)$.

**Figure 2**

Probability of short squeeze as a function of auction size, for $N > 2$ and $\delta > 0$.

that $\bar{R}_{QZ}$ is also the long's break-even value; it is the maximum that he is willing to pay for Q units. The valuation of the short, however, for the first Z units is unaffected by Q . So as $\bar{R}_{QZ}$ falls, the valuation gap between the active long and the short becomes increasingly large, hence the long's willingness to be aggressive falls relative to that of the short. The upshot is that the probability of a short squeeze falls when the auction becomes larger.

The second statement of the proposition has a similar intuition, but in this case $\bar{R}_{QZ}$ is increasing in Z . Hence the equilibrium probability of a short squeeze tends to increase as the market power of the largest long increases.

The third statement of the proposition can be understood by noting that for $|y_{1,0}|$ there is no "indirect" effect, since $\bar{R}_{QZ}$ does not depend on $|y_{1,0}|$. The proposition shows that, *ceteris paribus*, as the short's position grows, the "direct" effect is negative. This is because more of the benefits to the longs from short squeezing will go to free riders, thus leading the active long to bid less aggressively. It is important to keep in mind that this is a comparative statics result. In practice, it is likely that Z and $|y_{1,0}|$ are positively correlated, but here Z is being kept constant.

Figure 2 depicts the probability of a short squeeze as a function of auction size.²² The figure shows the typical situation when $N > 2$ and $\delta > 0$

²² Although Z , Q , and $|y_{1,0}|$ are integers, Figures 2, 3, and 4 are drawn as smooth curves for expositional purposes.

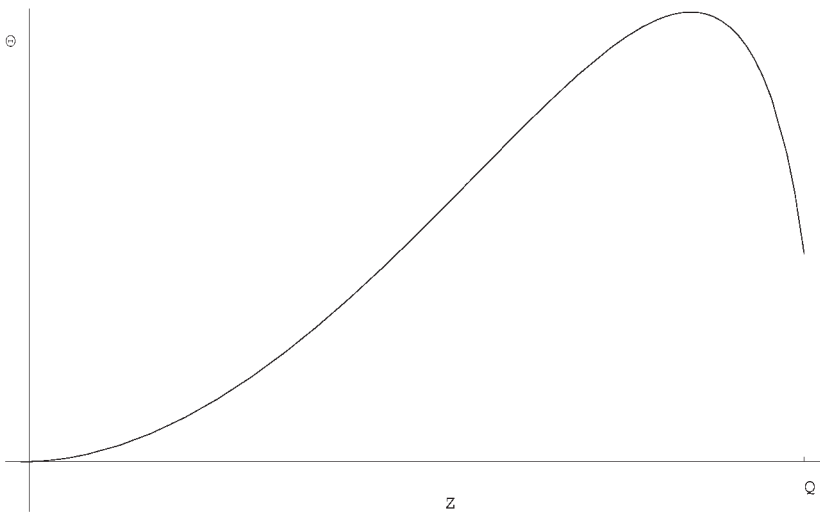


Figure 3
Probability of short squeeze as a function of the market power of the largest long at date 0, for $N > 2$ and $\delta > 0$.

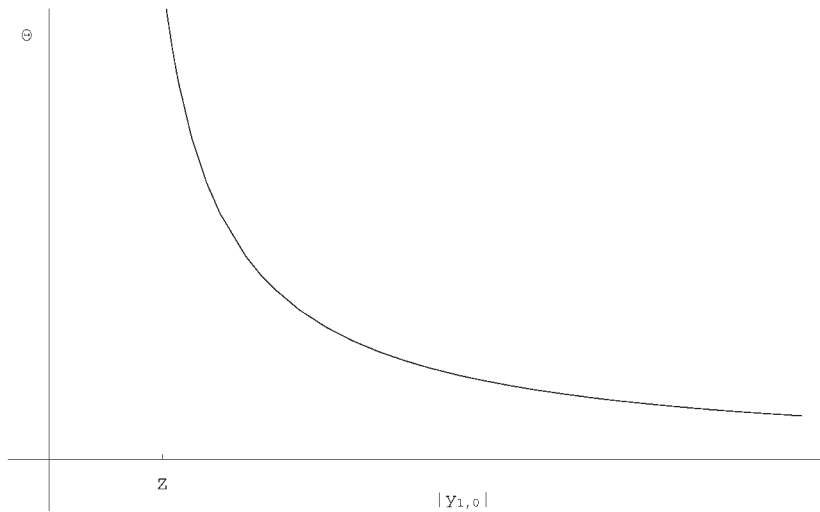


Figure 4
Probability of short squeeze as a function of the absolute value of the short's position at date 0, for $N > 2$ and $\delta > 0$.

that the probability of a short squeeze is initially increasing and then decreasing. Figure 3 depicts a similar, but opposite pattern for Θ as a function of the largest long's market power. Figure 4 shows the effect on Θ of changing the short's initial holding.

Although the probability of a short squeeze is nonmonotonic in auction size, the short's equilibrium expected payoff is monotonically increasing in Q , since $\bar{R}_{QZ}$ is decreasing in Q (see Theorem 3). In the region where Θ is increasing in Q there is downward pressure on the short's expected equilibrium payoff as Q rises. But this is more than offset by the lower cost of funds in the event of no squeeze.

Proposition 2. *In discriminatory auctions, expected revenue per unit is strictly decreasing in auction size.*

This is quite intuitive since the valuation gap between the short and active long is increasing in auction size. In particular, as Q increases, the long's valuation $\bar{R}_{QZ}$ falls. The short therefore needs to pay less to avoid being squeezed and this translates into lower revenue.

Proposition 3. *As either $\frac{Q}{Z} \rightarrow \infty$ or, for $\delta > 0$, $\frac{|v_{1,0}|}{Z} \rightarrow \infty$, equilibrium payoffs and the probability of a short squeeze approaches those in the uniform auction.*

The asymptotic equivalence is intuitive since when Q becomes very large with respect to Z , the cost of implementing a squeeze becomes very large relative to the benefit. It is just not worth it. What happens is that the break-even value for the long approaches the competitive rate and the active long and the short's bidding strategies “diverge”; that is, the short places almost all the mass of his bid for Z units in a neighborhood of $\bar{R}_{QZ}$, while the long's mass point at R_0 approaches one. Hence the probability of a short squeeze approaches zero, as in the uniform auction. This is illustrated in Figures 2 and 4.

3.2 Bidder behavior

We focus on measures of individual bidder behavior that have been used in the empirical literature; for example, the number of bids, total quantity demanded, and quantity weighted mean price and variance [Gordy (1999) and Nyborg, Rydqvist, and Sundaresan (2002)]. Our findings delineate the differences in behavior between short and long bidders and are testable by someone possessing the appropriate dataset.

When players use mixed strategies, as in our model in discriminatory auctions, what is most easily observed by an econometrician are the realizations of these mixed strategies. Our model delivers sharp predictions regarding these realizations, which can be tested by examining the bids submitted by short and long bidders in a given auction.

Proposition 4. *Given any realization of the equilibrium strategies of the players, the number of bids and the quantity weighted variance of*

the bids submitted by the short are greater than or equal to those of any long, irrespective of the auction mechanism. In uniform auctions, the mean rate of the bids submitted by the short is larger than that of any long.

The first statement is a simple consequence of the observations in Theorems 1 and 3 that the short submits at least one bid and the longs submit at most one bid each. The second statement follows directly from the result that in uniform auctions, the short submits a bid at R_h , or higher, and (possibly) one at R_0 , while the longs submit their bids at R_0 .

In discriminatory auctions, it would be meaningless to compare the short and active long's mean bids in a single auction since their bids are stochastic. However, since most longs are passive, we would expect to see the short place bids at higher mean rates than most longs. The next proposition compares mean bids across auctions for the active long and short by integrating over $G(L; \delta)$ and $F(S)$, respectively.

Proposition 5. *In discriminatory auctions, quantity weighted mean bids satisfy: (i) The expected value is higher for the short than the active long (with equality if and only if $N=2$ or $\delta=0$); (ii) for both players, the expected value is strictly decreasing in auction size, Q ; and (iii) for $N=2$, the variance is larger for the long than for the short.*

The difference in the expected mean bids of the short and active long is a consequence of differences in valuations. When there are no free riders, the short's and active long's average valuations are the same. In the presence of free riders the short's average valuation is higher, since he then values the first few units above R_h . This leads the short to have a higher equilibrium mean bid than the active long. Part (ii) is a consequence of the price-quantity trade-off faced by the active long: As the auction size grows, he has to buy more units to squeeze on the same number of units. Consequently the long's willingness to bid aggressively drops. In equilibrium, the short adjusts his bids downward in response to the long's less aggressive bidding. Finally, when $N=2$, the short's variance is smaller in part because the short always places a proportion of his bids at R_0 . In addition, the fact that the long has a mass point at R_0 augments the long's variance.

We close this section by examining the equilibrium impact of the scope for free riding, on both bidder behavior and auction performance. Theorems 1 and 3 show that the impact of δ is quantitative but not qualitative. The only effect is on the active long's choice of distribution in discriminatory auctions. Here we trace the implications of this.

Proposition 6. *In discriminatory auctions, as the scope for free riding, δ , increases, (i) $G(L; \delta)$ strictly increases; (ii) Θ strictly decreases; (iii) expected revenue strictly decreases.*

Part (i) shows that as the scope for free riding increases, the active long becomes less aggressive, in the sense of first-order stochastic dominance. Put another way, the short becomes more aggressive in relative terms, which is intuitive since an increase in δ translates into an increase in the cost to the short in the event of a squeeze. The reason that it is the long's, and not the short's, strategy that is affected by δ is that the two active players tailor their strategies according to the valuations of the other—and δ affects the short's valuation, but not the active long's. Parts (ii) and (iii) are simple consequences of part (i).

4. Extensions

4.1 Preauction market

Given our results that preauction allocations affect auction performance and payoffs, it is natural to ask how these positions are formed and what is the influence of the auction format. To make this extension realistic, one would have to recognize that in many contexts the set of players in the forward market may be different than in the auction itself. For example, in repo auctions, banks are subject to client withdrawals and deposits. In treasury auctions, players that are excluded from the auction may trade in the when-issued market, perhaps because of hedging motives [Nyborg and Sundaresan (1996)]. Different bidders may end up with unequal positions, perhaps because they have relations with different clients.

Intuition based on our model would suggest that when-issued prices will tend to be higher under discriminatory auctions than under uniform auctions, reflecting the larger likelihood of a squeeze. As a result, the open interest in the preauction market may be lower under discriminatory auctions. This could explain the finding that when-issued volume is considerably larger under uniform auctions [Nyborg and Sundaresan (1996)]. A lower open interest may also serve to reduce the incidence of short squeezes and make the performance of discriminatory auctions appear more similar to uniform auctions. The cost would be reduced hedging under the discriminatory format.

4.2 Multiple shorts

Preauction trading may result in multiple bidders with short positions. To study this we need to specify how the aggregate payoff to the shorts in Equation (10) is shared across the shorts. As a special case, suppose there is a unique largest long ($X=1$) and the scope for free riding is at its maximum ($\delta=1$). Then the gross payoff to short player n is $y_{n,2}R_h$ if

there is a squeeze and $y_{n,2}R_0$ if not. Since the shorts need to buy Z units in the auction to avoid being squeezed, uniform auction equilibria are characterized by some shorts bidding R_h for Z units in aggregate, with the remaining bids placed at R_0 . As in Theorem 1, there are no squeezes. For discriminatory auctions, the differences in players' valuation schedules still mean that there are no pure-strategy equilibria, implying a positive equilibrium probability of a squeeze.

4.3 Negative initial aggregate position

Due to preauction demand by hedgers who do not participate in the auction, bidders in the auction may have a negative initial aggregate position ($Q_0 < 0$). This would necessitate a change in the definition of market power, since in the absence of an auction a long player would have monopoly power over all his units (assuming hedgers would not sell back into the market). That is, player n 's date 0 market power would be $z_{n,0} = \max[0, y_{n,0}]$. Furthermore, shorts would have to buy a total of $|Q_0|$ units in the auction to avoid being squeezed. Hence $|Q_0|$ would essentially take on the role of Z . The combination of $Q_0 < 0$ and multiple shorts raises the possibility that players that are short at date 0 will be able to implement a squeeze at date 2 if they buy the entire auction. Hence shorts may have U-shaped valuation schedules. However, our general results on revenue and short squeezing under uniform versus discriminatory auctions remain true.

4.4 Private information

Suppose that players do not know each others' positions but that $Q_0 > 0$ is common knowledge. By the definition of market power, $z_{n,0} = \max[0, y_{n,0} - Q_0]$. Hence each long knows his own market power, but shorts cannot recover them (except when $N=2$). *This illustrates that longs have a fundamental informational advantage over shorts.*

To study the auction, suppose for simplicity that positions will be fully revealed at the start of date 2 and that there is no free riding. In uniform auctions, the following is an equilibrium: (i) one of the shorts bids R_h for $Q - 1$ units and R_0 for 1 unit (and all other shorts are passive); (ii) long player n bids (a) R_0 for Q units if $z_{n,0} < Q$ or (b) R_h for Q units if $z_{n,0} \geq Q$. Thus if $Q > Z$, there is no short squeeze and all units trade at R_0 . If $Q \leq Z$, there is a squeeze for sure. Thus the equilibrium outcome is the same as in the full information case. Discriminatory auctions are more complicated. Among other things, equilibrium will depend on the shorts' beliefs about Y_0 . We conjecture that the informational advantage held by the longs will lead to positive equilibrium rents for active longs, unlike in the full information case where they earn no rents. Private information on positions could then help to provide a motivation for the building up of large long stakes prior to the auction.

5. Conclusion

In this article, we have studied the impact of a potential short squeeze in the secondary market on equilibrium bidding strategies in a multiunit auction. What is important for our analysis is not *the actual occurrence* of a short squeeze, but the *possibility* of one. We have established closed-form solutions for equilibrium strategies and outcomes and found that: (1) Players who have the opportunity to squeeze, who have the potential to free ride on a squeeze, who are not affected by a squeeze, or who risk being squeezed, have different marginal valuations of the auctioned asset in an otherwise standard common value environment. (2) There is a trade-off between the two auction types that are most frequently used in practice. Discriminatory auctions lead to higher revenue than uniform auctions, but this revenue advantage comes at the cost of a higher incidence of short squeezes and consequently a more volatile secondary market. There may be no straight answer to the problem of choosing the auction format; it will depend on the aims and preferences of the auctioneer. Sellers who prefer maximizing revenue would do better with discriminatory auctions; sellers who prefer minimizing market distortions would do better with uniform auctions. (3) In discriminatory auctions, price distortion and the probability of a squeeze depend on the size of the auction, market power of the largest longs, and the scope for free riding. (4) In both types of auctions, shorts bid more aggressively than longs. (5) The scope for free riding does not affect the qualitative nature of equilibria in the two auction formats.²³

We have motivated our model with reference to the fact that many large and important multiunit auctions in financial and commodity markets are embedded in larger structures, where bidders often enter the auction with short positions and must cover in the auction or, failing that, in the after-market where they can be squeezed. Our model is also applicable to secondary market situations where a remedy against a short squeeze is sought.²⁴ A recent squeeze took place in the gold markets at the end of February 2001:

[Gold lease] rates have been squeezed considerably by the requirements of borrowers versus a lack of lenders. Yesterday afternoon, there was

²³ Insofar as the possibility of short squeezing can be a factor that limits arbitrage activity when markets are overvalued [see, e.g., Duffie, Garleanu, and Pedersen (2002) and Jones and Lamont (2002)], our article implies that efficiency can be improved when additional securities are injected into the market, particularly through uniform auctions.

²⁴ Well-known short squeezes in commodity markets include the Hunt brothers' silver squeeze in the late 1970s and the Sumitomo copper squeeze in the 1990s. Other examples in various markets include 30-year U.S. Treasury bonds (*The Economist*, February 12, 2000), the German government bond futures market (*eFinancial News*, March 26, 2001) ["Deutsche Bank makes Euro 50m from Bobl squeeze."], ABN AMRO (on the Lander 5 bond) (*Euroweek*, May 15, 1998), Japanese government bond futures market (*Barron's*, October 28, 1996) ["A giant short squeeze involving Japanese government bonds futures last

substantial lending from one of the central banks... (Dow Jones Newswires, February 2001)

The jump in the one-month gold lease rate from its normal level of less than 1% to above 4.5% in a matter of days is testimony to this particular short squeeze. That the 12-month rate was at most around 2% during this time is also consistent with a short squeeze. One may attempt to alleviate a short squeeze by releasing additional stock to the market, as indeed seems to have happened in our gold squeeze example. However, this is likely to be a blunt instrument unless one can find a way of actually selling to the shorts rather than the longs. In practice, trying to locate and verify who is short may be difficult, costly, and time consuming. Our analysis suggests that the uniform auction may be an effective mechanism that achieves the objective of selling to the shorts, and the auction can be organized quickly and relatively cheaply. This parallels Vickrey's (1961) finding for single-unit auctions that second-price auctions are efficient, even when bidders' valuations are asymmetrically distributed.

Appendix

Proof of Theorem 1. Under the proposed strategies, $r_s = R_0$ and $y_{1,1} \geq Z$, implying that $\forall i$ $a_{1,i} = R_0$ [by Equation (5)] and there is no squeeze. Thus, auction revenue is $R_0 Q$ and $\forall i$ $\pi_i = y_{i,0} R_0$ [by Equation (8)]. This establishes (i), (ii), and (iii). To see that the proposed strategies constitute equilibrium, suppose that all players except long player i use them. Thus if player i does nothing, $r_s = R_0$ and $X(R_0) \geq Q$. Therefore if he does not implement a squeeze, player i cannot do better than submitting b_i^u . To implement a squeeze, player i must win at least $Q' \geq Q - z_{i,0} + 1$ units in the auction. Given b_i^u , he must pay at least R_h for each of these units. Since his valuation of them is at most $\bar{R}_{QZ} < R_h$ each, his payoff will be less than $y_{i,0} R_0$. Thus, since b_i^u is clearly a best response to $\{b_i^u\}_{i=2}^N$, these strategies form an equilibrium.

Proof of Theorem 2. Under pure strategies, the auction outcome is known with certainty. So it suffices to show that in equilibrium there is no squeeze for sure. Suppose by contradiction that there is a squeeze. Then $r_s \leq \bar{R}_{QZ}$, since otherwise no long would find it worthwhile to implement a squeeze. The short's payoff is $-r_s y_{1,1} + z R_h + y_{1,0} R_0$, where $z = \max\{z_{n,2}\}$. But he can improve on this by adding the bid $(r_s + \epsilon, z)$ for arbitrarily small $\epsilon > 0$, since his payoff is then arbitrarily close to $-r_s(y_{1,1} + z) + y_{1,0} R_0$ and since $r_s \leq \bar{R}_{QZ} < R_h$. This is a contradiction.

Proof of Lemma 1. Suppose there is a pure-strategy equilibrium where $z_{i,2} = 0 \forall i$. It would be inconsistent with this for any long to pay more than R_0 for any unit in the auction, and

month... apparently prompted actions by Japanese regulators. The squeeze... was the largest in the history of global financial futures markets..."], NYMEX gas futures (*Futures*, February 1996), Apex Oil (*Forbes*, May 24, 1982). Backwardation in commodity futures is also often attributed to short squeezes or potential short squeezes. In the equity markets, small cap/small float stocks seem to be particularly vulnerable to short squeezes. Examples include MicroStrategy (*Washington Business Journal*, June 16, 2000), Solv-Ex (*Business Week*, August 5, 1996), Presstek (*Forbes*, June 17, 1996).

therefore, for the short to pay more than $R_0 + \epsilon$ for any unit, where $\epsilon \geq 0$ is arbitrarily small. Hence the highest bid is at most $R_0 + \epsilon$ and is made by the short. But then any long with $z_{i,0} > 0$, say, player 2, could do better by submitting a bid for Q units at $R_0 + \epsilon + \epsilon_1$, where $\epsilon_1 > 0$ is arbitrarily small. This is a contradiction. Suppose next that there is an equilibrium where there is a squeeze for sure and let $z_{i,2} = \max\{z_{n,2}\}$. This means that player i buys more than $Q - z_{i,0}$ units in the auction. Define r_z to be the rate at which he wins his last (fraction of a) unit. Equilibrium requires that $r_z \leq \bar{R}_{QZ} < R_h$. Hence the short can do better by demanding an extra unit at $r_z + \epsilon$, where $\epsilon > 0$ is arbitrarily small. This is a contradiction.

Proof of Theorem 3. We will prove the theorem for $q_{1,2}^* = Q - Z$. The case that $q_{1,2}^* < Q - Z$ follows immediately. Denote $\mathbf{b}^* = \{\mathbf{b}_i^*\}_{i=1}^N$. We wish to show that $\forall i$ $\mathbf{b}_i^*$ is a best reply to $\mathbf{b}_{-i}^*$. Under $\mathbf{b}^*$, $\forall i$ $X_{-i}(R_0) \geq Q$. Therefore no bidder can do better than under $\mathbf{b}^*$ by bidding below R_0 . Furthermore, since the short values the first Z units at least at R_h and the last $Q - Z$ units at R_0 , he cannot do better than letting one of his bids be $(R_0, Q - Z)$, as specified by $\mathbf{b}_1^*$.

Next we address the strategies of the two active bidders. We will first show that $F(S)$ and $G(L; \delta)$ must be given by Equations (14) and (15), respectively. Observe that $F(S)$ has no mass points. Therefore, under $\mathbf{b}^*$, either the active long will win Q units and squeeze on Z units, or he will win fewer than Q units and squeeze on no units. In the former case, the payoff to the active long is $\pi_n = (Q - Z)R_0 + ZR_h - QL \geq 0$ if and only if $L \leq \bar{R}_{QZ}$ (by definition of $\bar{R}_{QZ}$). Hence the long cannot do better than having $G(\bar{R}_{QZ}; \delta) = 1$. Furthermore, since $G(L; \delta)$ does not have a mass point at $\bar{R}_{QZ}$, the short cannot do better than having $F(\bar{R}_{QZ}) = 1$. Hence, neither bidder can improve his payoff by expanding the supports of $F(S)$ or $G(L; \delta)$.

Since $F(S)$ and $G(L; \delta)$ are continuous distributions, the short's expected payoff is

$$E[\pi_1] = \int_{[R_0, \bar{R}_{QZ}]} \{G(S; \delta)(-ZS - (|y_{1,0}| - Z)R_0) + (1 - G(S; \delta))(-ZR_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0)))\} dF(S), \quad (18)$$

since, under $\mathbf{b}^*$, $X = 1$ in the event of a squeeze [see Equation (10)]. The active long's expected payoff is

$$E[\pi_n] = (y_{2,0} + Q - Z)R_0 + \int_{[R_0, \bar{R}_{QZ}]} \{F(L)(ZR_h - QL) + (1 - F(L))(Z - Q)L\} dG(L; \delta). \quad (19)$$

For $\mathbf{b}^*$ to be equilibrium, it is necessary that the integrands of these two expressions are independent of S and L , respectively. In particular, the active long's strategy must satisfy

$$G(S; \delta)(-ZS - (|y_{1,0}| - Z)R_0) + (1 - G(S; \delta))(-ZR_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0))) = C_1, \quad (20)$$

where C_1 is a constant. Since $G(\bar{R}_{QZ}; \delta) = 1$, it follows that $C_1 = -Z\bar{R}_{QZ}$.

From this, a bit of algebra shows that $G(L; \delta)$ as given by Equation (15) is the unique solution to Equation (20). Similarly the short's strategy must satisfy

$$F(L)(ZR_h - QL) + (1 - F(L))(Z - Q)L = C_2, \quad (21)$$

where C_2 is a constant. Since $F(\bar{R}_{QZ}) = 1$, it follows that $C_2 = (Z - Q)R_0$, from where it is found that $F(L)$, as given by Equation (14), is the unique solution to Equation (21). This establishes that $\mathbf{b}_1^*$ and $\mathbf{b}_n^*$ are best replies to $\mathbf{b}_{-1}^*$ and $\mathbf{b}_{-n}^*$, respectively, provided that the bidders cannot improve their payoffs by splitting their bids further.

We first examine whether the long might improve his payoff by splitting his bid. Under $\mathbf{b}^*$, the expected payoffs of the active long is $E[\pi_n] = y_{2,0}R_0$. Suppose the long deviates by

submitting Q bids, $\{(L_k, 1)\}_{k=1}^Q$, where $L_k \geq L_{k+1}$ and $L_k \in [R_0, \bar{R}_{QZ}]$. Then²⁵

$$E[\pi_n] = R_0 y_{2,0} + \frac{1}{Z} \left((Q - Z) \sum_{k=Q-Z+1}^Q L_k - Z \sum_{k=1}^{Q-Z} L_k \right). \quad (22)$$

The second and the third parts of this expression have the same number of terms $((Q - Z)Z)$. Since $L_k \geq L_{k+1}$, the sum of the last two terms is at most zero. Hence, no matter how the long splits his bid for Q units, he cannot improve on his expected payoff of $R_0 y_{2,0}$.

Suppose now that the short deviates from b_1^* by submitting Z bids (assuming $Z \geq 2$), $\{(S_k, 1)\}_{k=1}^Z$, where $S_k \geq S_{k+1}$ and $S_k \in [R_0, \bar{R}_{QZ}]$. Then²⁶

$$E[\pi_1] = \sum_{k=1}^Z G(S_k; \delta) (R_h - S_k) - Z R_h - (1 - G(S_Z; \delta)) (|y_{1,0}| - Z) \delta (R_h - R_0) - (|y_{1,0}| - Z) R_0. \quad (23)$$

This is strictly increasing in S_Z . Hence the short will set $S_Z = S_{Z-1}$. Proceeding in the same vein, we get $S_k = S_{k-1}$, $k = 2 \dots Z - 1$. Thus the short maximizes his payoff by bidding S_1 for all Z units and therefore cannot improve his payoff by deviating from b_1^* .

We now turn to the strategies of the players with $z_{i,0} = 0$, that is, players $i = K + 2, \dots, N$. Assume that $\delta = 1$, since this will maximize the potential benefits to such a player from bidding above R_0 . If player i bids for 1 unit at $l > R_0$, his expected payoff is

$$E[\pi_i] = (R_0 - l) \int_{[R_0, l]} (1 - F(L)) dG(L; \delta) + (R_h - l) \int_{[R_0, l]} F(L) dG(L; \delta).$$

Note that $E[\pi_i | l = R_0] = 0$ and $\frac{\partial E[\pi_i]}{\partial l} = C[Q(l - R_0) - Z(R_0 - R_h)]$, where $C = \frac{(R_0 - R_h)^2 (Q - Z)}{QZ(R_h - L)^3}$ and so $\frac{\partial E[\pi_i]}{\partial l} < 0$ for $l < \bar{R}_{QZ}$. Hence $E[\pi_i] < 0$ for all $l > R_0$. This expands trivially if player i bids for more than 1 unit up to $Z - 1$ units. There is no benefit from bidding for Z or more units, since if player i were awarded this, there would be no squeezing. This establishes that $\forall i \in \{K + 2, \dots, N\}$, b_i^* is a best reply to b_{-i}^* .

Next, suppose that $i = 2, \dots, K + 1$ and $i \neq n$. Suppose first that $Z \geq 2$. Then if a player bids for one unit at a rate $l \in (R_0, \bar{R}_{QZ}]$, the change in his expected payoff relative to what he gets by being passive (excess payoff) is

$$\Delta \pi_i = (R_0 - l) \int_{[R_0, l]} (1 - F(L)) dG(L; \delta) + [\delta(R_h - R_0) + (R_0 - l)] \int_{[R_0, l]} F(L) dG(L; \delta).$$

Thus $\frac{\partial \Delta \pi_i}{\partial l} = \frac{\partial G(l; \delta)}{\partial l} [R_0 - l + \delta(R_h - R_0)F(l) - G(l; \delta)] = C[R_0(Z - Q)\delta + R_h(Z\delta - 1) - |y_{1,0}|\delta] + L(Z + (Q + |y_{1,0}|)\delta - 2Z\delta)$, where $C > 0$ (see note 28). Since $\frac{\partial^2 \Delta \pi_i}{\partial l^2} > 0$ and $\frac{\partial \Delta \pi_i}{\partial l} |_{l=R_{QZ}} = C \times \frac{(R_h - R_0)(Q - Z)(Z(2\delta - 1) - |y_{1,0}|\delta)}{Q} < 0$, it follows that $\frac{\partial \Delta \pi_i}{\partial l} < 0$. Therefore, since $\Delta \pi_i |_{l=R_0} = 0$, the player is worse off by bidding for one unit at $l > R_0$. This expands trivially to the case of bidding for up to $Z - 1$ units. Suppose now that $Z \geq 1$ and consider bids by player i for $Q' \geq Z$ units. Note first that it is not profitable to bid for $Q' \leq Q - z_{i,0}$ units, since

²⁵ We start the calculation of Equation (22) by noting that $E[\pi_n] = R_0 y_{2,0} + (Q - Z)R_0 + F(L_Q)(ZR_h - \sum_{k=1}^Q L_k) + (F(L_{Q-1}) - F(L_Q))((Z - 1)R_h - \sum_{k=1}^{Q-1} L_k) + \dots + (F(L_{Q-Z+1}) - F(L_{Q-Z}))((Z - Z + 1)R_h - \sum_{k=1}^{Q-Z+1} L_k) + \dots + (1 - F(L_1))(-\sum_{k=1}^{Q-Z} L_k) = R_0 y_{2,0} + \sum_{k=Q-Z+1}^Q F(L_k)(R_h - L_k) - \sum_{k=1}^{Q-Z} L_k$.

²⁶ We start the calculation of Equation (23) by noting that $E[\pi_1] = G(S_Z; \delta)((\sum_{k=1}^Z S_k - (|y_{1,0}| - Z)R_0) + (G(S_{Z-1}; \delta) - G(S_Z; \delta))((\sum_{k=1}^{Z-1} S_k) - R_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0))) + \dots + (G(S_1; \delta) - G(S_2; \delta))(-S_1 - (Z - 1)R_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0))) + (1 - G(S_1; \delta))(-ZR_h - (|y_{1,0}| - Z)(R_0 + \delta(R_h - R_0)))$.

there will be no squeezing at all when $l > L$. Suppose therefore that player i bids for $Q' \in [Q - (z_{i,0} - 1), Q]$ units. Then his expected excess payoff is at most²⁷

$$\begin{aligned} \Delta \pi_i &= [(Q' - Q + z_{i,0})R_h + (Q - z_{i,0})R_0 + y_{i,0}R_0 - Q'l - y_{i,0}R_0 \\ &\quad - y_{i,0}\delta(R_h - R_0)]\Pr\{l > L > S\} + [(Q' - Q + z_{i,0})R_h \\ &\quad + (Q - z_{i,0})R_0 - Q'l + y_{i,0}R_0 - y_{i,0}R_0]\Pr\{l > S > L\}. \\ &\quad + [(Q - Z)(R_0 - l) + y_{i,0}R_0 - y_{i,0}R_0]\Pr\{S > l > L\}. \\ &= [(Q' - Q + z_{i,0})R_h + (Q - z_{i,0})R_0 - Q'l]F(l)G(l; \delta) \\ &\quad + (Q - Z)(R_0 - l)G(l; \delta)(1 - F(l)) - y_{i,0}\delta(R_h - R_0) \int_{[R_0, l]} F(L) dG(L; \delta). \end{aligned} \quad (24)$$

The last term is always nonpositive, and the sum of the first two terms is less than $\gamma(z_{i,0} - Z)$, where $\gamma > 0$ (since $Q' \leq Q$).²⁸ Therefore player i does not improve his payoff, since $Z \geq z_{i,0}$. Along the same lines as above, player i cannot earn a positive excess payoff by splitting his bids. This establishes that $\forall i \in \{2, \dots, K\} \setminus \{n\}$, $\mathbf{b}_i^*$ is a best reply to $\mathbf{b}_{-i}^*$.

The expected equilibrium payoffs to the short and active long are calculated from Equations (18) and (19), using the known values of C_1 and C_2 . Player i , $i \notin \{1, n\}$, gets $y_{i,0}(R_0 + \delta(R_h - R_0))$ in the event of a squeeze (probability Θ) and $y_{i,0}R_0$ in the event of no squeeze. Hence $E[\pi_i]$ is as stated in the theorem. Note that aggregate payoff must sum to $(Q + \sum_{i=1}^N y_{i,0})R_0$. The seller's expected payoff is calculated by subtracting the payoffs of all other players and using the definition of Z .

Proof of Theorem 4.

Case 1: The short and one of the largest longs are the two active players. We want to show that they must use the strategies described in Theorem 3. Since the value of a squeeze to the long is maximized when he wins Q units, he must submit $\mathbf{b}_L = \{(\bar{L}, Q)\}$, where the distribution of $\bar{L}$, $G(L)$, has positive mass above R_0 . Therefore, by the proof of Theorem 3, the short cannot do better than submitting $\mathbf{b}_S = \{(R_0, Q), (\bar{S}, Z)\}$, where the distribution of $\bar{S}$, $F(S)$, has positive mass above R_0 .²⁹ Hence, by the proof of Theorem 3, it only remains to show that the support of F and G is $[R_0, \bar{R}_{QZ}]$ and that F and G do not have coinciding mass points. Clearly the lower bound of the support is R_0 and the upper bound is at most $\bar{R}_{QZ}$. Consider an upper bound $\bar{R} < \bar{R}_{QZ}$. Solving for $F(S)$ and $G(L; \delta)$ as before (Theorem 3), we find that both F and G have mass points at R_0 . But then, for some $\epsilon > 0$, the long could increase his payoff by placing his mass point at $R_0 + \epsilon$. Similarly if F and G have coinciding mass points above R_0 , at least one of the players could improve his payoff by moving the mass point up slightly.

Case 2: The short and one of the nonlargest longs (player i) are the two active players. First note that if $z_{i,0} = 0$, a squeeze happens with zero probability. By the proof of Lemma 1, this is inconsistent with equilibrium. Suppose next that $z_{i,0} > 0$. Then as in Case 1, the short and long must use strategies $\mathbf{b}_S$ and $\mathbf{b}_L$, where $F(S)$ and $G(L; \delta)$ are given by Equations (14) and (15), respectively, but where Z should be replaced by $z_{i,0}$ and the upper bound of the support is $\bar{R}_{Qz_i}$. Given these strategies, if player 2 (who is supposed to be passive) deviates by

²⁷ It is less in the event that $l > L > S$ and $X = 2$.

²⁸ $\gamma = \frac{(l - R_0)(R_h - R_0)^2(Q - Z)(Q(|y_{1,0}| \delta - Z(\delta - 1)) - Z^2)}{Q(R_0 - l)Z(LZ + R_0(|y_{1,0}| - Z)\delta + R_h(Z(\delta - 1) - |y_{1,0}| \delta))} > 0$. $C = \frac{(R_h - R_0)^2(Q(|y_{1,0}| \delta - Z(\delta - 1)) - Z^2)}{Q(R_h - L)(LZ + R_0(|y_{1,0}| - Z)\delta + R_h(Z(\delta - 1) - |y_{1,0}| \delta))} > 0$.

²⁹ The relevant part of the proof is where it is shown that the short will not split [see Equation (23)]. The argument is applicable here as well since it does not rely on the functional form of G .

submitting (l, Q) for some $l > R_0$, his expected excess payoff is

$$\begin{aligned} \Delta\pi_2 = & [ZR_h + (Q - Z)R_0 - Q]F(l)G(l; \delta) - y_{2,0}\delta(R_h - R_0) \int_{[R_0, l]} F(L)dG(L; \delta) \\ & + [(Z - z_{i,0})R_h + (Q - Z)R_0 - (Q - z_{i,0})l]G(l; \delta)(1 - F(l)) \\ > & [ZR_h + (Q - Z)R_0 - Q]F(l)G(l; \delta) + (Q - Z)(R_0 - l)G(l; \delta)(1 - F(l)) \\ & - (|y_{1,0}| - z_{i,0})(R_h - R_0)F(l)(G(l; \delta) - G(R_0; \delta)), \end{aligned} \quad (25)$$

since (a) $(Q - Z)(R_0 - l) < (Z - z_{i,0})R_h + (Q - Z)R_0 - (Q - z_{i,0})l$; (b) $y_{2,0} \leq |y_{1,0}| - z_{i,0}$; (c) $\int_{[R_0, l]} F(L)dG(L; \delta) < F(l)[G(l; \delta) - G(R_0; \delta)]$ (the latter is true since $F(L)$ is monotonically increasing); and (d) $\delta \leq 1$. Thus, by direct substitution of $F(L)$ and $G(L; \delta)$ in the right-hand side of Equation (25) we get

$$\begin{aligned} & \frac{(l - R_0)(R_h - R_0)(Q(z_{i,0}(\delta - 1) - |y_{1,0}| \delta) + z_{i,0}^2)}{Q(R_h - l)(Lz_{i,0} + R_0(|y_{1,0}| - z_{i,0})\delta + R_h(z_{i,0}(\delta - 1) - |y_{1,0}| \delta))} \\ & \times \left[\frac{(l - R_0)(Q - z_{i,0})(|y_{1,0}| - z_{i,0})}{|y_{1,0}|} + \frac{(l - R_h)Q(Z - z_{i,0})}{z_{i,0}} \right]. \end{aligned}$$

The first factor is negative $\forall l$. By continuity there is sufficiently small $\epsilon > 0$ such that the second factor is negative when $l = R_0 + \epsilon$ (the first fraction equals 0 when $l = R_0$ and the second fraction is negative). Hence player 2 increases his payoff from the proposed deviation. This establishes that the active long cannot have $z_{i,0} < Z$.

Case 3: Two longs (players i and j) are the two active players. By supposition, the short is passive. This will be shown to lead to a contradiction. Note that $z_{i,0} = z_{j,0} = 0$ would be inconsistent with equilibrium, since then neither i nor j would be able to implement a squeeze and therefore would do better by being passive. So let $z_{i,0} > 0$. Call it “bidding to squeeze” when player n bids above R_0 for at least $Q - z_{n,0} + 1$ units. Let R_n be the lower bound of the support of the distribution of rates when player n bids to squeeze. Clearly $R_n \leq R_{QZ} < R_h$ for $n = i, j$. If both players bid to squeeze for sure, then the short can improve his payoff with the bid $(\max[R_i, R_j] + \epsilon, 1)$ for sufficiently small $\epsilon > 0$. If only player n bids to squeeze for sure, then the short can improve his payoff with the bid $(R_n + \epsilon, 1)$ for sufficiently small $\epsilon > 0$. If neither player bids to squeeze for sure, then we argue that whenever player n bids above R_0 , but not to squeeze, we must have $\delta > 0$, since he could only benefit from such a bid if he could free ride on a squeeze. We refer to this as “bidding to free ride.” Let $\hat{r} > R_0$ be in the support of the distribution of rates when player n bids to free ride. Note that conditional on free riding, player n values each unit at R_n . Since player n must at least break even (in expectation) when he bids to free ride at $\hat{r}$, the short would do better than breaking even by bidding for Z units at $\hat{r}$, since when $\delta > 0$, he values the first Z units above R_n . Finally, suppose that neither player bids to free ride or bids to squeeze for sure, then player i can do better with deviation where he demands Q for sure at $R_0 + \epsilon$, for sufficiently small $\epsilon > 0$. Thus the two active bidders cannot both be longs.

Proof of Proposition 1. Part (i): Using Equation (16), we obtain

$$\frac{\partial \Theta}{\partial Q} = \frac{-(|y_{1,0}| - Z)Z(Q + Z)\delta + (Q^2\psi - Z^3)\ln\left[\frac{Q\psi - Z^2}{\psi(Q - Z)}\right]}{Q^2(|y_{1,0}| - Z)^2\delta^2}. \quad (26)$$

Since $\ln(1 + x) \leq x - \frac{x^2}{2} + \frac{x^3}{3}$, we may get an upper bound on Equation (26) by substituting the above approximation for the “ln” term, where $x = \frac{Z(\psi - Z)}{\psi(Q - Z)}$:

$$\frac{\partial \Theta}{\partial Q} \leq \frac{Z^2(|y_{1,0}| - Z)^2\delta^2}{6(Q - Z)^3\psi^3Q^2(|y_{1,0}| - Z)^2\delta^2} \times [a_1Q^3 + o(Q^2)], \quad (27)$$

where $a_1 = -(1 - \delta)Z(Z + 2|y_{1,0}|\delta) - Z^2\delta^2 < 0$. Therefore $\exists \bar{Q}$ such that $\forall Q > \bar{Q}$, Equation (27) is negative, which is sufficient to establish part (i). The limiting result follows since $\lim_{\delta \rightarrow 0^+} \Theta = Z/2Q$. The proof of part (ii) goes along the same lines. Part (iii): $\frac{\partial \Theta}{\partial |y_{1,0}|} = \int_{R_0}^{\bar{R}_{QZ}} \frac{\partial F(L)G'(L, \delta)}{\partial |y_{1,0}|} dL < 0$, since the integrand is less than zero for any $L \in (R_0, \bar{R}_{QZ}]$.

Proof of Proposition 2. Using $Z = |y_{1,0}| - \sum_{i \geq 3} y_{i,0}$ and Equation (16) in the formula for $E[\pi_A]$ in Theorem 3, we get

$$\frac{\partial \frac{E[\pi_A]}{Q}}{\partial Q} = \frac{(R_h - R_0)Z(2Z^2 - Q(\psi + Z))}{Q^3(|y_{1,0}| - Z)\delta} \times \ln \left[\frac{Q\psi - Z^2}{\psi(Q - Z)} \right].$$

Since $Q > Z$ and $\psi \geq Z$, the factors are of different signs. The proposition follows.

Proof of Proposition 3. As $\frac{Q}{Z} \rightarrow \infty$, $\bar{R}_{QZ} \rightarrow R_0$ and therefore both $F(S)$ and $G(S; \delta)$ go to R_0 . Therefore $E[\pi_A] \rightarrow Q R_0$ and $\forall n$ $E[\pi_n] \rightarrow y_{n,0} R_0$. Now, as $\frac{Q}{Z} \rightarrow \infty$, $G(R_0; \delta) \rightarrow 1$. Therefore, since $F(R_0) = 0$, $\Theta \rightarrow 0$. For $\delta > 0$ the case that $\frac{|y_{1,0}|}{Z} \rightarrow \infty$ follows along the same lines.

Proof of Proposition 4. The proposition follows from inspection of the strategies in Theorems 1 and 3.

Proof of Proposition 5. (i) Let $\mathbf{m}_S$ and $\mathbf{m}_L$ denote the quantity-weighted mean bids of the short and long, respectively. For the short, we let $q_{1,2}^* = Q - Z$, since this minimizes $\mathbf{m}_S$. We have

$$E[\mathbf{m}_S] = \frac{1}{Q} \int_{[R_0, \bar{R}_{QZ}]} (SZ + q_{1,2}^* R_0) dF(S) = R_0 + (R_h - R_0) \left(Z - (Q - Z) \ln \frac{Q}{Q - Z} \right) / Q,$$

$E[\mathbf{m}_L] = G(R_0; \delta) R_0 + \int_{[R_0, \bar{R}_{QZ}]} L dG(L; \delta) = (Z((Q - Z)R_0 + R_h Z) - (R_h - R_0)(Q\psi - Z^2)) \ln \frac{Q\psi}{Q\psi - Z^2} / (QZ)$, and where $\psi = Z(1 - \delta) + |y_{1,0}|\delta$. Direct calculation shows that

$$\lambda \equiv E[\mathbf{m}_S] - E[\mathbf{m}_L] = \frac{R_0 - R_h}{Q - Z} \left[(Q - Z)Z \ln \left(\frac{Q}{Q - Z} \right) - (Q\psi - Z^2) \ln \left(\frac{Q\psi}{Q\psi - Z^2} \right) \right].$$

We want to show that $\lambda \geq 0$. Now, for $N = 2$, $|y_{1,0}| = Z$ and so $\psi = Z$. For $\delta = 0$, $\psi = Z$ as well. When $\psi = Z$, we see that $\lambda = 0$. For $N > 2$ and $\delta > 0$, $\frac{\partial \lambda}{\partial |y_{1,0}|} = \frac{\delta}{\psi} \frac{R_h - R_0}{Q - Z} (Z^2 - Q\psi(1 + \ln(\frac{Q\psi}{Q\psi - Z^2})))$. Since $\ln(\frac{Q\psi}{Q\psi - Z^2}) > 0$ and $\psi > Z$, $\frac{\partial \lambda}{\partial |y_{1,0}|} < 0$. Since $|y_{1,0}| > Z$ and $\lambda|_{|y_{1,0}|=Z} = 0$, the statement follows. (ii) If $q_{1,2}^*$ is fixed, we have $\frac{\partial E[\mathbf{m}_S]}{\partial Q} = \frac{-R_h Z + (R_h - R_0) \ln(1 - \frac{Z}{Q})}{Q^2} < 0$. If $q_{1,2}^* = Q - Z$, we have $\frac{\partial E[\mathbf{m}_S]}{\partial Q} = \frac{(R_0 - R_h)Z \ln \frac{Q}{Q - Z}}{Q^2} < 0$. For the active long, we have $\frac{\partial E[\mathbf{m}_L]}{\partial Q} = \frac{(R_0 - R_h)Z \ln \frac{Q\psi}{Q\psi - Z^2}}{Q^2} < 0$. (iii) If $N = 2$, $q_{1,2}^* = Q - Z$, since otherwise the long can buy units below R_0 , which is not consistent with equilibrium. Therefore

$$\begin{aligned} \sigma^2[\mathbf{m}_S] - \sigma^2[\mathbf{m}_L] &= \frac{1}{Q} \left(\int_{[R_0, \bar{R}_{QZ}]} \left(S + \frac{Q - Z}{Q} R_0 - E[\mathbf{m}_S] \right)^2 dF(S) \right. \\ &\quad \left. - \int_{[R_0, \bar{R}_{QZ}]} (L - E[\mathbf{m}_L])^2 dG(L; \delta) - \frac{Q - Z}{Q} (R_0 - E[\mathbf{m}_L])^2 \right) \\ &= \frac{1}{Q^2} (R_0 - R_h)^2 (Q - Z) \varphi, \end{aligned}$$

where $\varphi \equiv Z(Z - 2Q) + 2Q(Q - Z) \ln \frac{Q}{Q - Z}$. To establish the proposition, it suffices to show that $\varphi < 0$. Now, $\frac{\partial \varphi}{\partial Z} = 2(Z - Q) \ln \frac{Q}{Q - Z}$ and $\frac{\partial^2 \varphi}{\partial Z^2} = \frac{2Z}{Z - Q} < 0$. Therefore, since $\frac{\partial \varphi}{\partial Z}|_{Z=0} = 0$, $\frac{\partial \varphi}{\partial Z} < 0$ for $Z > 0$. Hence, since $\varphi|_{Z=0} = 0$, we have $\varphi < 0$ for $Z > 0$.

Proof of Proposition 6. Part (i): $\frac{\partial G(L; \delta)}{\partial \delta} = \frac{(\bar{R}_{QZ} - L)(R_h - R_0)(|y_{1,0}| - Z)Z}{(LZ + R_0(|y_{1,0}| - Z)\delta - R_h(Z(1 - \delta) + |y_{1,0}|\delta))^2} > 0$. Part (ii):

Proceeding as in the proof of Proposition 1(i), an upper bound on $\frac{\partial \Theta}{\partial \delta}$ using $\ln(1+x) < x$ is: $\frac{\partial \Theta}{\partial \delta} \leq -\frac{Z^2}{Q\delta\psi} < 0$. Part (iii): $\frac{\partial E[\pi_A]}{\partial \delta} = -\sum_{i \geq 3} y_{i,0}(R_h - R_0)(\Theta + \frac{\partial \Theta}{\partial \delta}\delta) = -\sum_{i \geq 3} y_{i,0}(R_h - R_0) \frac{(Q-Z)Z}{Q(|y_{1,0}| - Z)^2 \delta^2 \psi} \eta$, where

$$\eta = Z(|y_{1,0}| - Z) - (Q - Z)\psi \ln \left[\frac{Q\psi - Z^2}{(Q - Z)\psi} \right].$$

Direct calculation shows that $\eta|_{|y_{1,0}|=Z} = 0$, $\frac{\partial \eta}{\partial |y_{1,0}|} \Big|_{|y_{1,0}|=Z} = 0$ and $\frac{\partial^2 \eta}{\partial |y_{1,0}|^2} > 0$. Therefore $\eta \geq 0$ and the statement follows.

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