

Structural Models of Corporate Bond Pricing: An Empirical Analysis

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This article empirically tests five structural models of corporate bond pricing: those of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). We implement the models using a sample of 182 bond prices from firms with simple capital structures during the period 1986–1997. The conventional wisdom is that structural models do not generate spreads as high as those seen in the bond market, and true to expectations, we find that the predicted spreads in our implementation of the Merton model are too low. However, most of the other structural models predict spreads that are too high on average. Nevertheless, accuracy is a problem, as the newer models tend to severely overstate the credit risk of firms with high leverage or volatility and yet suffer from a spread underprediction problem with safer bonds. The Leland and Toft model is an exception in that it overpredicts spreads on most bonds, particularly those with high coupons. More accurate structural models must avoid features that increase the credit risk on the riskier bonds while scarcely affecting the spreads of the safest bonds.

The seminal work of Black and Scholes (1973) and Merton (1974) in the area of corporate bond pricing has spawned an enormous theoretical literature on risky debt pricing. One motivating factor for this burgeoning literature is the perception that the Merton model cannot generate sufficiently high-yield spreads to match those observed in the market. Thus the recent theoretical literature includes a variety of extensions and

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improvements, such as allowing for coupons, default before maturity, and stochastic interest rates.^{1,2}

However, the empirical testing of these models is quite limited. Indeed, only a few articles implement a structural model to evaluate its ability to predict prices or spreads. Partly, this reflects the fact that reliable bond price data have only recently become available to academics.³

In this article we compare the Merton model and four newer models to determine the extent to which innovations in structural bond pricing models have improved the pricing of risky bonds. We implement these five structural models using a sample of 182 noncallable bonds with simple capital structures at year-end during the period 1986–1997. The models we consider are a coupon version of the Merton model and the models of Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). For the sake of brevity, henceforth we refer to these models as the M, G, LS, LT, and CDG models, respectively.

These structural models differ in a number of important features, including the specification of the default boundary, recovery rates, coupons, and interest rates. The original Merton model assumes bondholders receive the entire value of the firm in distress and that interest rates are constant; it also can only deal with zero-coupon bonds. Our version of the M model treats a coupon bond as if it were a portfolio of zero-coupon bonds, each of which can be priced using the zero-coupon version of the model.⁴ Also, spot rates are used to discount bond cash flows. The G model differs from the M model in that it treats the coupon on the bond as a compound option. On each coupon date, if the shareholders decide to pay the coupon by selling new equity, the firm stays alive; otherwise default occurs and bondholders receive the entire firm. In the LT model, the firm continuously issues a constant amount of debt with a fixed maturity that pays continuous coupons and, as in the G model, equity holders have the option to issue new equity to service the debt or default.

¹ See, for example, Black and Cox (1976), Titman and Torous (1989), Kim, Ramaswamy, and Sundaresan (1993), Nielsen, Sao-Requejo, and Santa-Clara (1993), Leland (1994, 1998), Briys and de Varenne (1997), Goldstein, Ju and Leland (2001), and Ho and Singer (1982). Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) incorporate strategic defaults into traditional structural models. See also Anderson, Sundaresan, and Tychon (1996), Huang (1997), Acharya *et al.* (1999), Fan and Sundaresan (2000), and Acharya and Carpenter (2002). Duffie and Lando (2001) take into account incomplete accounting information. Garbade (1999) examines managerial discretion. Zhou (2001) and Huang and Huang (2002) incorporate jumps. See Dai and Singleton (2003) for a more complete reference.

² A different approach, which is not the focus of this article, is the reduced-form approach of Jarrow and Turnbull (1995) and Duffie and Singleton (1999). See also Das and Tufano (1996), Duffie, Schroder, and Skiadas (1996), Jarrow, Lando, and Turnbull (1997), Madan and Unal (1998), and Jarrow (2001).

³ See Warga (1991) and Warga and Welch (1993) on the quality of bond price data.

⁴ We implement this “portfolio of zeroes” used by Longstaff and Schwartz (1995) in the M model to make it comparable to the LS and CDG models. An alternative approach that incorporates the conditional probability of default can be found in Duffie and Singleton (1999) and Chen and Huang (2000).

In the event of default, equity holders get nothing and bondholders receive a fraction of the firm asset value (in effect, assuming a liquidation cost in distress). The LS model allows stochastic interest rates that are described by the Vasicek (1977) model. Default occurs when the firm's asset value declines to a prespecified level. In the event of default, bondholders are assumed to recover a constant fraction of the principal and coupon. The CDG model extends the LS model to incorporate a stationary leverage ratio, allowing the firm to deviate from its target leverage ratio only over the short run.

To date, the most extensive implementation of a structural model is found in Jones, Mason, and Rosenfeld (1984; henceforth JMR), who apply the Merton model to a sample of firms with simple capital structures and secondary market bond prices during the 1977–1981 period. JMR find that the predicted prices from the model are too high, by 4.5% on average. The errors are largest for speculative-grade firms, but they conclude that the M model works better for low-grade bonds because it has greater incremental explanatory power for riskier bonds. They also find that pricing errors are significantly related to maturity, equity variance, leverage, and the time period. Ogden (1987) conducts a similar study using prices from new offerings and finds that the M model underpredicts spreads by 104 basis points on average. Both studies conclude that the M model suffers from a nonstochastic interest rate. One reason they may have emphasized this aspect is that they examine bonds from a time period when Treasury rates were particularly volatile.⁵ The high volatility of interest rates also added to the difficulties of pricing the call options that were typically embedded in corporate bonds then.

Lyden and Saraniti (2000) are the first to implement and compare two structural models (the M and LS models) using individual bond prices.⁶ Using prices for the noncallable bonds of 56 firms that were reported in Bridge Information Systems, they find that both the M and LS models underestimate yield spreads. The errors are systematically related to coupon and maturity. Allowing interest rates to vary stochastically has little impact. They conclude that the LS model's prediction errors are related to the estimates of asset volatility.

In addition to implementing the structural models, researchers have examined general patterns predicted by structural models, such as the shape of the credit term structure or the correlation between interest

⁵ Because of double-digit inflation, the Federal Reserve switched to money supply targets during the 1979–1982 period. During this regime, the Federal Funds rate swung wildly from as low as 8.5% to a record 20%, whereas in a typical year the range is considerably less than 3%. See Peek and Wilcox (1987) for an analysis of Volcker's policies.

⁶ The other implementations of structural bond pricing models are found in Wei and Guo (1997) and Anderson and Sundaresan (2000), who implement the M and/or LS models using aggregate data, and Ericsson and Reneby (2001) who implement a perpetual bond model considered in Black and Cox (1976).

rates and spreads (some of these predictions are shared by reduced-form models). These include Sarig and Warga (1989), Helwege and Turner (1999), and He, Hu, and Lang (2000) on the shape of the credit yield curve; Delianedis and Geske (1998) on bond rating changes; Collin-Dufresne, Goldstein, and Martin (2001) and Elton *et al.* (2001) on changes in bond spreads; Duffee (1998), Brown (2001), and Neal, Rolph and Morris (2001) on the relationship between bond spreads and Treasury yields; and Huang and Huang (2002) on real default probabilities implied by structural models. In most cases, these empirical studies do not find support for the models and several conclude that the models severely underpredict spreads.

Contrary to the previous empirical literature, we do not characterize structural models as unable to predict sufficiently high spreads. Using estimates from the implementations we consider most realistic, we agree that the five structural bond pricing models do not accurately price corporate bonds. However, the difficulties are not limited to the underprediction of spreads. The M and G models generate spreads that are too small on average, as previous studies have indicated, but the LS, LT, and CDG models generate spreads that are too high on average. Moreover, most of these models suffer from the problem that predicted spreads are often either ludicrously small or incredibly large, while the average spread prediction error is not particularly informative.

For most of the models, predicted spreads are lowest when the bonds belong to firms with low volatility and low leverage. Holding these factors constant, we find no additional role for maturity. This is in sharp contrast to the previous literature, which claims that structural models are unable to generate sufficiently high spreads on short-maturity bonds. Leland and Toft's model is unusual in that it overestimates bond spreads in most cases, largely because of its simplifying assumptions about coupons. As a result, the model actually tends to overestimate credit risk on shorter-maturity bonds.

While the LS, CDG, and LT models are clearly able to avoid the problem of spreads that are too low on average, they all share the problem of inaccuracy, as each has a dramatic dispersion of predicted spreads. This problem is actually exacerbated if one incorporates stochastic interest rates and costs of financial distress through a face value recovery rule. Possibly a more accurate term structure model than the Vasicek model would help. A third source of dispersion arises in the treatment of coupons. We conclude that the major challenge facing structural bond pricing modelers is to raise the average predicted spread relative to the Merton model, without overstating the risks associated with volatility, leverage, or coupon.

The organization of the article is as follows: Section 1 describes our sample of bonds. Section 2 discusses how we implement the five models

and estimate the model parameters. Section 3 details the predictive ability of the models. We also examine the nature of the model prediction errors for evidence of systematic biases through *t*-tests and regression analysis. Section 4 summarizes the results of the investigation. The formulas for bond prices used in our implementation are given in the appendix.

1. Data

Our goal is to implement the models on a sample of firms with simple capital structures that have bonds with reliable prices and straightforward cash flows. Limiting the sample to firms with simple capital structures is desirable because firms with complicated capital structures raise doubts as to whether pricing errors are related to deficiencies of the models or to the fact that the model does not attempt to price the debt of firms with very complicated liabilities.

We use bond prices in the Fixed Income Database on the last trading day of each December between 1986 and 1997. The Fixed Income Database has bond prices at month-end from January 1973 to March 1998 for all corporate bonds. We choose to analyze Decembers in the 1986–1997 period because a year-end observation on price can be matched up with year-end financial data. The first December that we use, 1986, is chosen because noncallable bonds are rare before that (we only have one bond from December 1986 for this reason).

We restrict our sample to bonds issued by nonfinancial firms, so that the leverage ratios are comparable across firms.⁷ In addition, we exclude gas and electric utilities, as the return on equity, revenues, and thus the risk of the default, are strongly influenced by regulators. The bonds under consideration have standard cash flows—fixed-rate coupons and principal at maturity. We exclude credit-sensitive notes, step-up notes, floating rate debt, foreign currency denominated debt (as well as foreign issuers), and convertible bonds. We also exclude bonds with call options, put options or sinking fund provisions. Following Warga (1991), we eliminate bonds with maturities of less than one year, as they are highly unlikely to trade. There are nearly 8700 noncallable and nonputable bonds that meet these requirements.

To keep the capital structure simple, we choose firms with only one or two public bonds and we exclude bonds that are subordinated. This leaves us with a sample of 628 bonds. Of these, a sizable fraction only have

⁷ In contrast, Lyden and Saraniti (2000) implement their models on bonds issued both by financial and nonfinancial firms. Financial firms, such as banks, routinely have leverage ratios above 90%, whereas only the least creditworthy nonfinancial firms use as much debt.

matrix prices, which are less reliable than trader quotes, and we delete these bonds.

Moreover, we must have firms with publicly traded stock in order to estimate asset volatility. We match by Committee on Uniform Securities Identification Procedures (CUSIP) to the Center for Research in Security Prices (CRSP) database, and require that a firm have five years of stock price data prior to the bond price observation. In addition to eliminating private firms, this rule eliminates bonds issued at the subsidiary level.

Lastly, we check the capital structure in more detail by investigating the firms' liabilities in Moody's CD-ROM product for the years 1996–1999, which has capital structure information for periods ending in December 1994–1997. These data on company histories and financials are an electronic version of the data in the Moody's Industrials Manuals. For the years 1986–1993, we rely on the hard copy of the Moody's manuals. We eliminate all firms that had more than five types of debt (counting each bond as a type of debt) and any firm with different priorities in the debt. A typical firm in the final sample has a public bond or two, bank debt, and commercial paper. We allow firms to have some unusual types of debt (e.g., foreign currency denominated debt) as long as that debt is not an observation in our pricing exercises and the unusual debt totals less than 3% of assets and less than 20% of long-term debt. Our final sample includes 182 bonds.

Table 1 presents summary statistics on the bonds and issuers in the sample. Panel A indicates that the firms in our sample are mostly investment grade, although about a dozen firms have spreads that are representative of the low end of the credit quality spectrum. The firms are also fairly large, with the average market value near \$6.5 billion, and usually have low leverage. Bond maturities range from just over a year to nearly 30 years, but most are in the range of 5 to 10 years. Only one of the bonds is a pure discount bond. Most of the firms are in the manufacturing business (panel B).

A large fraction of the bonds were issued in the early 1990s, reflecting our requirement that the bonds be noncallable (panel C).⁸ Our observations come from a variety of interest rate environments (panel C). Average interest rates [average constant maturity Treasury (CMT) rates across maturities] range from as high as 8.91% to as low as 4.49%; the slope of the Treasury curve is also quite varied over the period, being extremely steep in 1992 but barely positive in 1988 and 1997. In December 1988 the yield curve is nearly inverted (and actually is inverted two months later in February 1989), in advance of

⁸ See Crabbe (1991) and Crabbe and Helwege (1994) on the use of call options over time.

Table 1
Summary statistics on the bonds and issuers in the sample

	Panel A		Minimum	Maximum
	Mean	Std. Dev.		
Years to maturity	8.968	6.929	1.088	29.69
Coupon (%)	7.916	1.506	0	12.25
Bond rating	7.456	3.143	2	24
Yield spread over CMT (bp)	93.53	84.75	22.13	555.6
Asset market value (\$ billions)	6.578	7.472	0.1728	47.973
Market leverage ratio	0.303	0.152	0.086	0.864
Asset volatility (over 150 days)	0.236	0.088	0.085	0.592
Corr between firm value and 3m T-bills	-0.02	0.044	-0.13	0.102
Payout (%)	4.833	2.349	0	12.15
Industry	Panel B			
	No. of bonds	Percent of sample		
Manufacturing	124	68.13		
Mines	16	8.79		
Retail	12	6.59		
Service	15	8.24		
Transportation and utility	10	5.50		
Wholesale	5	2.75		
Observation year	Panel C		Debt issuance deviation	No. of bonds
	Average CMT rate (%)	Yield curve slope		
1986	6.39	0.84	7.68	1
1987	7.75	1.13	-1.43	4
1988	8.91	0.02	-5.97	8
1989	7.81	0.06	-7.72	9
1990	7.48	0.77	-4.73	5
1991	5.55	2.06	-1.33	12
1992	5.06	2.10	5.58	18
1993	4.49	1.56	5.42	15
1994	7.21	0.22	-11.00	19
1995	5.51	0.39	0.08	27
1996	5.78	0.52	-4.59	26
1997	5.65	0.09	5.89	38

The table reports summary statistics on the bonds and issuers in our sample. Bond rating is the average of Moody's and Standard & Poor's ratings, where AAA is 1, AA+ is 2, etc. Bond yield spreads are measured against the constant-maturity Treasury yield. Asset volatility is annualized and is based on equity volatility over the previous 150 trading days. Asset value is the market value of the firm's equity plus total liabilities. Leverage is total liabilities over asset value. Correlation is calculated using equity returns and three month Treasury-bill returns over the previous five years. Payout is a weighted average of the dividend yield (adjusted for share repurchases) and the bond yield, where the weights are the leverage ratio and one minus the leverage ratio. The yield curve slope is the 10-year Treasury rate minus the 2-year Treasury rate. The debt issuance deviation is the residual from a regression of aggregate debt issuance (reported by the Federal Reserve Board) against a time trend over the period January 1986–March 1998.

the 1990–1991 recession, whereas the flat yield curve of 1997 occurs more than two years before the next inverted yield curve (and recession). Not surprisingly, debt issuance is unusually high in December 1997 and quite low in December 1988.⁹

⁹ The debt issuance figures reported in panel C are the residuals from a regression of aggregate nonfinancial debt issuance (reported by the Federal Reserve) against a time trend, using monthly debt issuance data from January 1986 to March 1998.

Because our sample may be considered unusual in that each firm has a simple capital structure, one may wonder if these bonds are representative of the overall bond market. A comparison of our bonds with the 7531 noncallable and nonputable bonds from nonfinancial, nonregulated industries suggests that they are. The two samples have very similar time-series patterns and similar fractions of below-investment-grade bonds (more than 90% of all noncallable bonds are investment grade). Our sample has a slight tendency toward safer companies, and thus toward yields that are a tad lower (7.4% compared to 7.6% for the larger sample). The average bond maturity in our sample is two years shorter than the average overall, reflecting the fact that our longest bond has a maturity that is just under 30 years, and our sample has no century bonds.

Data on bond features are taken from the Fixed Income Database; data on balance sheets and other accounting data are from Compustat; share prices, dividend yields, and the number of shares outstanding are from CRSP. Interest rate data are from the CMT series reported in the Federal Reserve Board's H15 release.

2. Implementations

In this section we first discuss very briefly the implementation of the five models. We then describe how we estimate parameters for these models.

Each of the models has an analytical or quasi-analytical formula for coupon bond prices. Except for the G model, all the pricing formulas are straightforward to implement and are given in Appendix A for the reader's convenience. The Geske formula involves multivariate normal integrals and is not straightforward to implement accurately, especially for long-maturity bonds. Following Huang (1997), we choose to obtain the bond prices for the G model using the binomial method (finite difference methods can also be used). Once we have a set of bond prices predicted by the models, we calculate the corresponding predicted yields as bond equivalent yields assuming semiannual compounding.

Each of the structural models has a set of parameters that must be estimated. Parameters related to firm value and capital structure include the initial levels of debt and assets, the payout parameter, asset return volatility, the speed of a mean-reverting leverage process, and those parameters that characterize the target leverage ratio. In addition, implementation of the models requires estimates of parameters that define the default-free term structure, as well as parameters related to bond characteristics. Below we discuss how to estimate the three types of parameters that are summarized in Table 2.

Table 2
Estimation of model parameters

Parameter	Description	Estimated as	Data source
Bond features			
c	Coupon	Given	FID
T	Maturity	Given	FID
F	Face value	Total liabilities	Compustat
w	Recovery rate	Given	Moody's
Firm characteristics			
V	Firm value	Total liabilities plus market value of equity	Compustat and CRSP
μ_v	Asset returns	Average monthly change in V	Compustat and CRSP
σ_v	Asset volatility	Historical equity volatility adjusted for leverage	Compustat and CRSP
δ	Payout ratio	Weighted average of c and the share repurchase-adjusted dividend yield	Compustat, CRSP and FID
κ_ℓ	Speed of adjustment to target leverage	Coefficient from a regression of changes in log leverage against lagged leverage and r	CRSP, CMT and Compustat
ϕ	Sensitivity of target leverage to interest rates	Coefficient from a regression of changes in log leverage against lagged leverage and r	CRSP, CMT and Compustat
τ	Tax rate	Assumed at 0.35	—
Interest rates			
r	Risk-free rate	NS or Vasicek models	CMT
ρ	Correlation between V and r	Correlation between equity returns and r	CRSP and CMT
σ_r	Interest rate volatility	Vasicek model	CMT

FID is the Fixed Income Database. CRSP is the Center for Research in Security Prices database. CMT is the constant maturity Treasury rate series available on the Federal Reserve Board's website. NS refers to the Nelson and Siegel (1987) model. Total liabilities are used for the default boundaries in most instances, but sensitivity to the assumption is also shown by using the KMV measure of liabilities. Likewise, we also show the sensitivity of the estimates to using historical equity volatility by estimating the models with implied volatility and future volatility.

2.1 Firm value and capital structure parameters

Rather than using the face value of the bond to calculate the default boundary in these models, we use the book value of total liabilities reported in the firm's balance sheet. In most structural models, equity holders earn the residual value of the firm once debt is paid off. Often the payoff amount is assumed to be the face value of the bond being priced. However, equity residual values only begin to accrue once the par value of the bond is paid if there is no other debt in the firm, and the firms in our sample have other debt. Thus all the debt must be paid off before equity has any value. To calculate a leverage ratio, we also need the asset value. This can be estimated as the sum of the

market value of equity and the market value of total debt, the latter being proxied by its book value.¹⁰ The leverage is then measured as total liabilities over the sum of total liabilities and market value of equity. To show the sensitivity of structural models to the leverage ratio, we also implement the LS model using a measure of leverage suggested by KMV Corporation [see Crosbie and Bohn (2002)]¹¹ and we examine implied leverage ratios from the LS model as well.

Parameters κ_ℓ (the mean-reverting parameter), ϕ (characterizing the sensitivity of the leverage ratio to interest rates), and ν (related to the target leverage) can be estimated by regressing leverage on lagged leverage and interest rates. See Appendix B.2 for more details.

Parameter δ measures a firm's yearly payout ratio. This parameter is not part of the original M, G, or LS models, but can easily be added. In the CDG model, the effect of the payout ratio is exactly canceled by the inclusion of a target debt ratio, and the results are the same whether we use the firm's past payout level or assume no payout (see below for more details). The variable δ is meant to capture the payouts that the firm makes in the form of dividends, share repurchase, and bond coupons to equity holders and bondholders. To obtain a good approximation of the value of δ , one must know the firm's leverage, its dividend yield, its share repurchases, and the coupon on its debt. We calculate δ as a weighted average of the bond's coupon and the firm's equity payout ratio, where the weights are leverage and one minus leverage. The equity payout ratio is the dividend yield for firms with no share repurchases in the year of the bond observation, but otherwise the dividend yield is grossed up by the ratio of share repurchases to dividends. If there are no dividends and only share repurchases, a share repurchase yield is calculated for the equity portion of the payout ratio. Share repurchases and yearly dividend levels are reported on Compustat.

A key input parameter in a structural model is σ_v , the asset return volatility, which is unobservable. Asset volatility can be measured by using historical equity volatility and leverage or we can measure asset volatility from bond prices observed at a different point in time. The latter, bond-implied volatility, is analogous to the Black and Scholes implied equity volatility used in option markets. Detailed discussions of these volatility measures are provided in Appendix B.1. We implemented the five structural models using seven estimates of volatility: bond-implied volatility from the previous month's bond price, which is always in November in this sample, and six estimates of volatility based on equity returns. The latter include measures of volatility using 30, 60, 90,

¹⁰ Given that most of the bond prices are close to par, this approximation is expected to be reasonable.

¹¹ KMV is a subsidiary of Moody's.

and 150 trading days before the bond price observation and one based on equity prices over 150 trading days after the bond price is observed, on the assumption that some of the future path of equity price volatility can be anticipated. The sixth equity-based estimate uses the GARCH(1,1) [Bollerslev (1986)] model and 150 days of prior equity returns. Except for those based on bond-implied volatility, the differences in average errors among the various measures are small and largely reflect outliers that occur more often with the shorter windows. We discuss the results of bond implied volatility in more detail in the appendix, but for the sake of brevity in the remainder of the article we report only results for historical volatility based on 150 trading days.

2.2 Interest rate parameters

Interest rate parameters on a given day are estimated using the CMT yield data on that day or over the previous month. (The CMT series, published by the Federal Reserve, is largely a historical series of on-the-run Treasury yields, and is described in more detail on the Federal Reserve Board's web page.) We estimate interest rate parameters either by fitting the Nelson and Siegel (1987; hereafter NS) yield curve model or the Vasicek (1977) model to the CMT yields (see Appendix B.3 for more details).¹²

In the one-factor models of G and LT, the risk-free rate is estimated using the rate from the NS model for a Treasury bond with the same maturity as the corporate bond being priced. For the M model, each coupon is priced with the spot rate whose maturity matches the date of the coupon. In the base case, the NS model is used to generate spot rates. For the sake of comparison, the Vasicek model is also used to calculate spot rates in the M model. In both LS and CDG, interest rate dynamics are described by the Vasicek model; thus we implement these two models using Vasicek estimates to ensure their internal consistency.

The correlation coefficient ρ between asset returns and interest rates in both LS and CDG is approximated by the correlation between equity returns and changes in interest rates. We use 3-month Treasury-bill rates and stock price data over a window of five years in the correlation estimations.

¹² It should be noted that the use of a yield curve model introduces errors in the risk-free rate used as the benchmark for calculating the spread. We ignore the term structure errors when calculating spread prediction errors, as both the predicted yield and the predicted risk-free rate have the same error and it is eliminated with subtraction. While there is no easy way to correct for this additional source of error, it does raise the question of which risk-free rate should be used by a practitioner who prefers to have estimates of yields.

2.3 Parameters related to bond features

Another key input parameter in a structural model is the recovery rate w . Research on recovery rates by Keenan, Shtogrin, and Sobehart (1999) indicates that the average bond recovery rate is 51.31% of face value [see also Altman and Kishore (1996)]. While this figure is conceptually straightforward, its components are not: Some of the loss comes from a decline in firm value and some from the deadweight costs of financial distress. In models with endogenous recovery rates, the two components can be specified separately. Leland and Toft assume that liquidation values are about half of firm value (implying a deadweight cost of 50%), but this figure is still a source of debate in the corporate finance literature [see Andrade and Kaplan (1998)]. Alderson and Betker (1996) analyze estimates provided in bankruptcy cases and conclude that liquidation values are no less than 63% of going concern values. Andrade and Kaplan (1998), however, suggest that the costs of financial distress are lower, likely in the range of 15–20%.

In the original M and G models, bondholders receive 100% of the firm's value in the event of default. In contrast, LT explicitly assumes that bondholders receive only a fraction of the assets in default. In the LS and CDG models, recovery rates as a fraction of face value are, by definition, identical to recovery rates as a fraction of asset value. To follow market convention and to make the results comparable across models, we implement the 51.31% of face value specification for all of the models. However, we also implement the G model with recovery equal to 100% of firm value to show the effects of the face value assumption.

Among the five models examined in the article, the LT model is the only one that considers taxes by incorporating the tax deductibility of interest payments in the model. Following them, we choose a value of the tax rate $\tau = 0.35$.

3. Empirical Results

In Section 3.1 we discuss the ability of the models to fit market prices. We present the percentage pricing errors, the percentage errors in yields, and the percentage errors in yield spreads. We consider the error in spreads to be the most informative measure of model performance because the corporate bond yield is expected to be greater than the Treasury yield in every model.¹³ In addition, we pay particular attention to the standard

¹³ Suppose a corporate bond trades at a yield to maturity (YTM) of 6.5% and a comparable maturity Treasury has a YTM of 6%. A good test of the model is whether it can explain the 50 basis point spread. If the model predicts a YTM of 6.1%, then the spread prediction error is –80%, but the error as a fraction of yield seems small (about –6%). Any model would predict a YTM above 6%, so the percentage error in yields is crediting the model with more predictive power than reasonable. The situation is similar for the pricing errors.

deviation of the spread prediction errors and the average absolute spread prediction errors because all of the models have substantial dispersion in predicted spreads. In Section 3.2 we examine if these models have systematic prediction errors.

3.1 Predicted spreads from the structural models

Table 3 summarizes the prediction errors of the five models. For each of the measures of pricing errors in columns 2 through 7, the numbers in parentheses are standard deviations of the prediction errors. Columns 2, 4, and 6 show measures of model errors, whereas columns 3, 5, and 7 show the absolute values of the errors. We focus our analysis on columns 6 and 7, which report errors in credit spreads.

3.1.1 One-factor models. The first row of this table shows that the M model overprices bonds on average. Our results suggest somewhat less overpricing of bonds than those found by JMR, a difference probably due to our use of a payout ratio and a cost of financial distress (by assuming the recovery rate is 51.31% of face value). When measured by the yield spread error, the average error is negative and indicates that the model has only modest predictive power. All of the spread prediction errors in this table are significantly different from zero.

The estimates for the M model reported in Table 3 are based on risk-free rates from the NS model. Given that some models must use Vasicek estimates, one wonders whether the NS model provides significant benefits. We implemented the M model using rates from the Vasicek model (not shown) and found that the average spread prediction error scarcely differs from that shown in Table 3.

Panels A–C of Figure 1 plot the predicted bond spreads from the M model and the actual bond spreads against maturity for three rating classes. Panel A shows the predictions of the model for bonds rated A or higher; panel B shows BBB-rated bonds; and panel C plots the market and predicted spreads of bonds that are rated below investment grade. All of the rating classes have many examples of extreme overprediction and extreme underprediction of bond spreads, although the cases of underprediction are far more numerous. The tendency toward underprediction appears to be somewhat stronger among the short-maturity bonds, but this pattern only appears in bonds rated BBB or higher.

The “portfolio of zeroes” approach that we use in the M model (and the LS and CDG models) is simple to implement but treats these “zeroes” (coupons) as independent of each other. The G model provides a more rigorous treatment of coupons. Rows 2 and 3 of Table 3 show the predictions from the G model. The results in row 2 assume bondholders receive

Table 3
Performance of the structural models

Bond pricing model	Percentage pricing error, mean (std. dev.)	Absolute percentage pricing error, mean (std. dev.)	Percentage error in yield, mean (std. dev.)	Absolute percentage error in yield, mean (std. dev.)	Percentage error in spread, mean (std. dev.)	Absolute percentage error in spread, mean (std. dev.)
Merton	1.69% (4.94%)	3.67% (3.71%)	-91.30% (17.05%)	91.84% (13.84%)	-50.42% (71.84%)	78.02% (39.96%)
Geske (face recovery)	0.70% (4.89%)	3.22% (3.73%)	-1.71% (15.28%)	8.45% (12.83%)	-29.57% (74.06%)	66.93% (43.12%)
Geske (firm recovery)	2.09% (3.97%)	3.11% (3.23%)	-5.47% (8.46%)	7.58% (6.62%)	-52.92% (48.28%)	65.73% (28.34%)
Leland-Toft	-1.97% (7.54%)	4.06% (6.64%)	15.60% (74.74%)	19.06% (73.92%)	115.69% (490.19%)	146.05% (481.97%)
LS (1-day CMT)	-2.69% (8.19%)	5.63% (6.51%)	6.62% (24.85%)	15.02% (20.86%)	42.93% (171.63%)	124.83% (125.07%)
LS (1-month CMT)	-0.68% (6.94%)	4.56% (5.26%)	1.52% (21.17%)	11.90% (17.55%)	-6.63% (132.53%)	96.83% (90.44%)
CDG (baseline)	-11.21% (13.12%)	12.64% (11.75%)	32.06% (44.41%)	36.74% (40.60%)	269.78% (370.41%)	319.31% (328.42%)
CDG (low κ)	-10.50% (13.03%)	12.09% (11.56%)	30.09% (43.82%)	35.14% (39.86%)	251.12% (362.73%)	304.32% (319.16%)
CDG (low μ)	-3.76% (10.13%)	7.35% (7.90%)	11.00% (31.39%)	20.17% (26.42%)	78.99% (247.95%)	170.16% (196.57%)

The table reports means and standard deviations of the percentage errors in the models' predictions. The percentage errors in prices, yields, and spreads, as well as their absolute values, are calculated as the predicted spread (yield, price) minus the observed spread (yield, price) divided by the observed spread (yield, price). The errors are those generated from implementing the models using 187 bonds with simple capital structures during 1986–1997 with the assumption that recovery rates are 51.31% of face value in default and that asset volatility is measured using 150-day historical volatility.

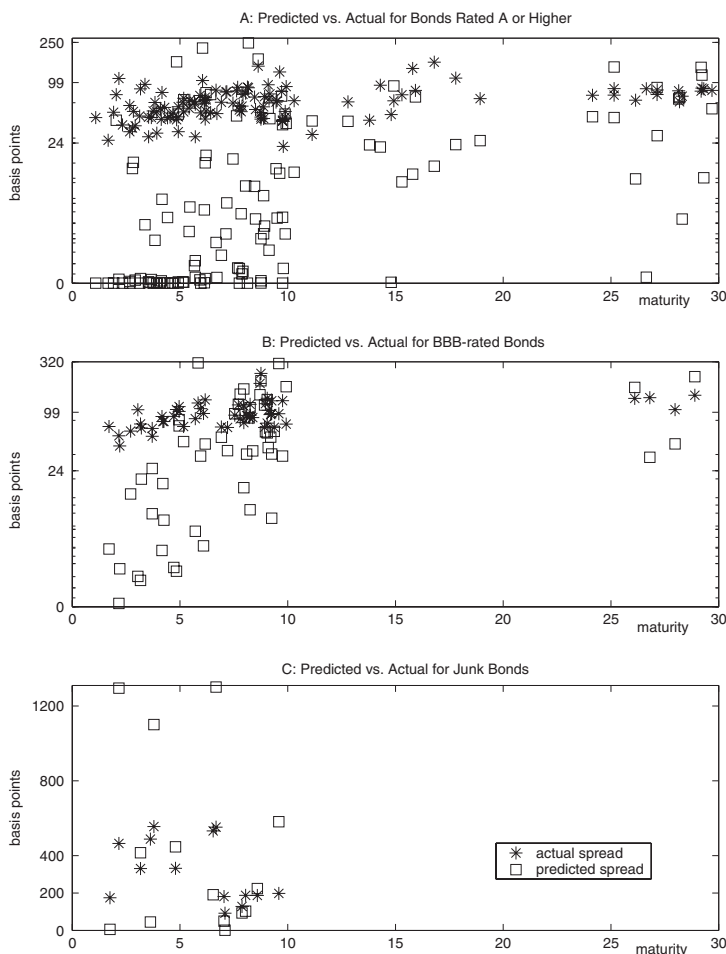


Figure 1

Predicted and actual spreads versus years to maturity: the Merton model. The actual and predicted spreads for the sample of 182 bonds with simple capital structures at year-end during the period 1986–1997. The spreads are plotted against the remaining years to maturity of the bond. The actual spreads are marked with an asterisk and the predicted spreads from the Merton model are denoted with a square. Spreads are calculated in basis points over the Nelson and Siegel (1987) estimates of the Treasury yield curve.

51.31% of the face value of debt, which is an assumption we will use throughout the article. In the next row, we assume that there are no costs of financial distress, so that bondholders receive the entire value of the firm in default.¹⁴

¹⁴ In this model as well as the M and LT models, we actually use the recovery rule $\min(51.31\% \text{ of par, firm value})$, so that the recovery cannot be higher than firm value. Other estimates, not shown, indicate that the firms receiving less than 51.31% of face value are a small fraction of the sample and the average recovery is still close to 51.31%. In addition, we impose the constraint that recovery cannot be more than the amount owed to bondholders.

The average spread prediction error in row 2 of Table 3 is markedly less negative than the average error in the M model under the same recovery assumption. This suggests that the compound option approach to coupons in the G model is an improvement over the “portfolio of zeroes” simplification. This may be due to the equity-holders’ option to pay the coupon, which is incorporated into the G model but ignored in the M model. If shareholders believe it is worthwhile to pay the coupon when the firm is actually insolvent (believing it has a good chance to bounce back), then bondholders face a greater risk of loss (they recover less than they would at the onset of distress). This effect is more important as maturity lengthens, all else being constant (there is more time for the firm to continue a downward spiral). As few junk companies have long-maturity bonds, the effect of the coupon treatment will be smaller for the bonds with high predicted spreads, and this helps the accuracy of the G model (see panels A and B of Figures 1 and 2).

This optionality also automatically takes into account conditional probabilities of default, which are not incorporated in the simple portfolio of zeroes approach. In the latter method, the calculation of the value of each coupon ignores whether previous coupons have been paid or not. This overestimates the probability of default for the firm. Consequently the variance of the estimated spreads may widen because the firms with the highest probability of default are affected by this error the most.

The G model’s option to continue relies on financing with new equity issuance, whereas Ho and Singer (1982) allow coupons to be paid from asset sales. Such financing is even more damaging to bondholders if distress occurs. The impact is expected to be greater for high-grade bonds because they have weaker covenants [Kahan and Tuckman (1993)]. Of the 31 bonds in our sample for which information is available on the SEC’s website, only 2 have covenants preventing this behavior, and only these were rated speculative grade at issuance. While the effect may be small, the variation among the covenants would help reduce the dispersion in spread prediction errors.

In our implementations of the five models, we always assume that bondholders receive a face value recovery rate of 51.31%. We do so because two of the models (LS and CDG) do not allow an alternative rule, making it possible that the other models outperform LS and CDG only because of the added flexibility in specifying recoveries. To show more clearly how recovery rates matter, we implement the G model with and without costs of financial distress. In row 2 we assume bondholders receive 51.31% of face value (which implies a cost of financial distress of about 33%) while in row 3, we assume bondholders receive 100% of the value of the firm (zero cost of distress). Including costs of financial distress in row 2 results in substantially higher average spreads, but with a loss of accuracy (which is apparent from the higher absolute spread errors, as

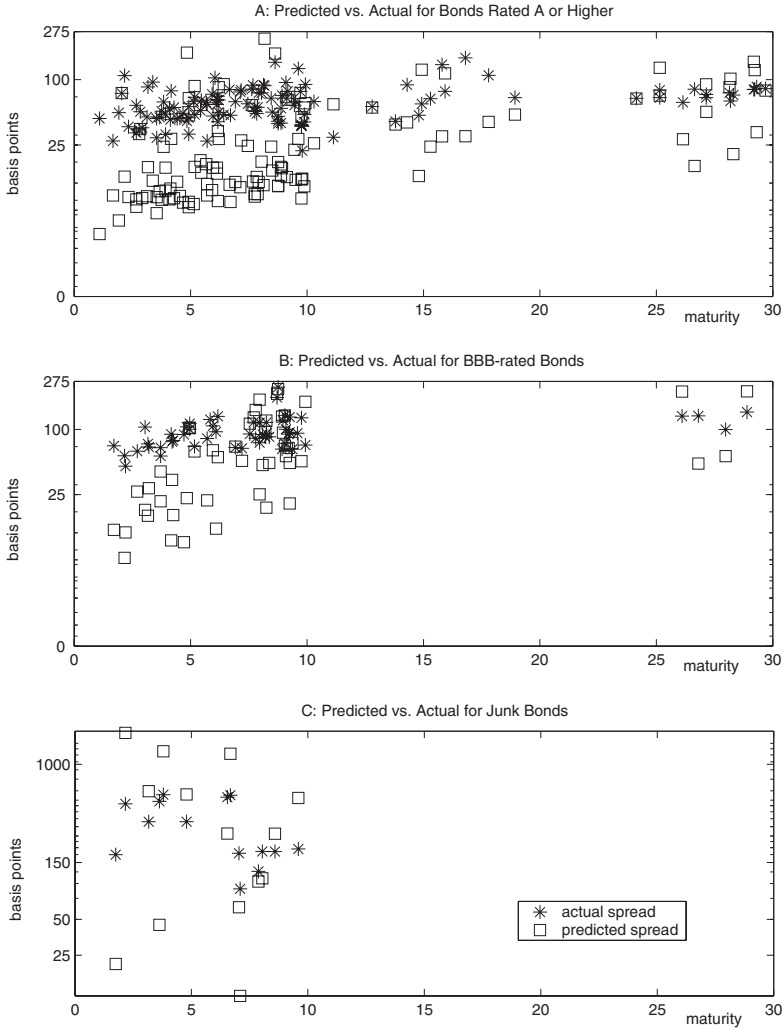


Figure 2
Predicted and actual spreads versus years to maturity: the Geske model. The actual and predicted spreads for the sample of 182 bonds with simple capital structures at year-end during the period 1986–1997. The spreads are plotted against the remaining years to maturity of the bond. The actual spreads are marked with an asterisk and the predicted spreads from the Geske (1977) model are denoted with a square. Spreads are calculated in basis points over the Nelson and Siegel (1987) estimates of the Treasury yield curve.

well as the higher standard deviations of the spread prediction errors). The loss of accuracy occurs because the bonds with a high probability of default (ones whose predicted spreads are already high) become even riskier with the larger losses in default, while the face value recovery rule has little impact on the risk of the bonds with low predicted spreads.

To further illustrate this effect on the dispersion of the predicted spreads, consider the predictions in rows 2 and 3 of Table 3. The average spread predicted by the G model without distress costs is only 53 basis points, or slightly more than half the actual average spread of 93 basis points, whereas the average spread predicted by the model with distress costs is 85 basis points (the medians suggest a similar effect). The actual market spreads range from 22 basis points to a high of 556 basis points. Both implementations result in predictions for dozens of bonds that are less than 10 basis points, although there are somewhat fewer such cases when distress costs are included. The more extreme differences in predictions occur among the riskier bonds. The highest predicted spread for the model with full recovery is 1184 basis points and the second highest predicted spread is 647 basis points (belonging to the bond with the highest actual spread). While these estimates are too high, they are much closer than the estimates from assuming costs of financial distress, which reach 1838 basis points (see Figure 2). Assuming costs of financial distress does help avoid the problem of underprediction of spreads to a large extent, but unfortunately increases the variance of the predicted spreads by extreme overprediction of spreads on the bonds that the model considers to be very risky. A better approach might be to allow costs of financial distress that vary among the bond issuers.

Figure 2 shows the predicted bond spreads from the G model (assuming 51.31% face value recovery). Like the M model, there is evidence of both extreme underprediction of spreads and extreme overprediction, but the problem is less severe here. The G model does better on short-maturity bonds than the M model does. It also has a greater tendency toward overprediction of short-maturity junk bonds, although this group is extremely small.

The results from the G model suggest the compound option approach is a major improvement over the portfolio of zeroes treatment used in the M, LS, and CDG models. The LT model also views coupon bonds as compound options, but has the additional advantage that it has an easy-to-implement closed-form solution.

The LT model, shown in row 4 of Table 3, has a serious tendency to overpredict bond spreads. Every volatility measure that we used in the implementation of the LT model (only one is shown) resulted in a significantly positive average spread prediction error. Moreover, the median of the spread prediction errors summarized in Table 3 is 36% and nearly two-thirds of those spreads are overestimated. However, the standard deviation and the absolute prediction errors indicate a dramatic lack of accuracy.

The LT model predicts a spread for the average bond in our sample that is more than twice the spread actually observed in the market. The highest predicted spread is 5096 basis points, which although way off the mark, at

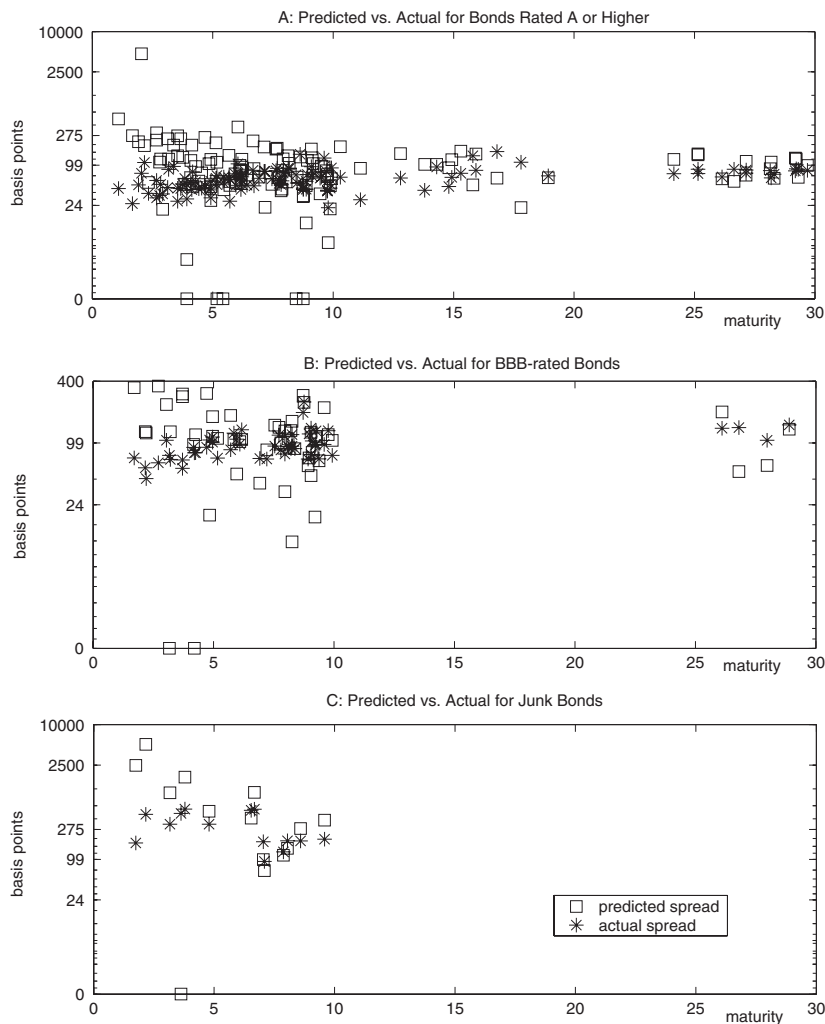


Figure 3
Predicted and actual spreads versus years to maturity: the LT model. The actual and predicted spreads for the sample of 182 bonds with simple capital structures at year-end during the period 1986–1997. The spreads are plotted against the remaining years to maturity of the bond. The actual spreads are marked with an asterisk and the predicted spreads from the Leland and Toft (1996) model are denoted with a square. Spreads are calculated in basis points over the Nelson and Siegel (1987) estimates of the Treasury yield curve.

least belongs to a bond with a market spread of 465 basis points (see Figure 3). Less comforting is the fact that the second highest predicted spread, 4655 basis points, belongs to a bond that only trades at a spread of 75 basis points.

The LT model's assumption of a continuous coupon sharply increases the probability of default on high-coupon bonds. Consider the sole zero-coupon bond in our sample (we set its coupon to one basis point to implement the model). This bond is one of the riskiest in the sample: its market spread is 488 basis points, it is rated B3/CCC+, and both its leverage ratio and asset volatility are above average. Even the M model, which has trouble generating double-digit spreads at all, predicts a spread of 45 basis points for this bond. Yet the LT model generates a predicted yield spread of essentially zero for this bond because the coupon is virtually zero. In contrast, a bond rated AA/Aa2, which has below-average volatility and below-average leverage, carries a market spread of only 62 basis points, but the LT model estimates this bond's spread at 50 basis points. The higher spread owes largely to its 9.25% coupon, which was set at a time when Treasury rates were higher.

Figure 3 shows the predicted spreads of the LT model in comparison to the actual bond spreads. In panel A, the highest predicted spread for a bond rated A or higher is 4655 basis points, which tends to obscure the pattern in other high-grade bonds. Excepting this extreme outlier, the LT model reveals a tendency toward overprediction of spreads that is highest at short maturities. This bias exists for all three rating groups, but is extreme with junk bonds, almost implying a strict downward-sloping credit yield curve. For the investment-grade bonds, the model tends to either overpredict short-maturity bond spreads or to severely underpredict them, while the longer-maturity bonds are priced more accurately.

We have considered the predictive power of three one-factor models, each of which was created without regard for the stochastic nature of risk-free interest rates. Two of these tend to predict spreads that are too low relative to the market, while the third usually overestimates credit risk. Of the three models, only the M model is implemented with any sense of variation in the risk-free rate, yet it is only the LT model with a constant Treasury rate that generates high spreads on average. In the balance of Section 3.1 we consider the impact of explicitly incorporating stochastic interest rates into the structural models.

3.1.2 Two-factor models. The last five rows of Table 3 summarize the results from implementing the LS and CDG two-factor models. Rows 5 and 6 report results for the LS model using two different data sources for the Vasicek model parameters (in row 5, the Vasicek model is estimated using Treasury yields from one day, while in row 6, the model estimates are based on data throughout the month of December). In the implementation of the CDG model, shown in rows 7–9, we use the Vasicek estimates based on one day's data.

The CDG model differs from the LS model in the assumption of a target leverage ratio. We implement the version of the CDG model that assumes that the target depends on interest rates (the two-factor model). To show the sensitivity of the CDG model to parameter choices, we present results from three implementations: row 7 of Table 3 is based on parameters that are estimated from the sample firms' data. These parameters include the firm's mean asset return and the speed of adjustment toward the target leverage ratio. Row 8 assumes a lower speed of adjustment to the mean and row 9 assumes a lower mean asset return.

Both the LS and CDG models have much higher predicted spreads than either the M or G models, and in most implementations (varying in the choice of equity volatility) the average spread error is positive (the average spread error for the LS model shown in row 6 is the lone negative figure). The estimates for the CDG model reveal a strong tendency toward over-estimation of spreads, which is only somewhat mitigated in the last two rows of the table.

The higher average spreads in these two-factor models appear to be a major improvement over the M and G models. However, they come at a substantial expense to accuracy. The absolute spread errors of the LS model in row 5 of Table 3 are nearly double those of the M and G models under the same recovery rate assumption, and they are even higher in the CDG model. The LS and CDG models have substantially fewer bonds that suffer from the kind of serious spread underprediction problem seen in the M model. The median spread error in the LS model using one day to estimate Vasicek is -13% , compared to -62% and -72% for the G and M models, respectively. The median predicted yield spread for the CDG model under the same assumptions is even higher (218 basis points versus 77 basis points for the LS model).

While these two-factor models have truly higher predicted spreads, the range of predicted spreads is extreme: there are 34 bonds for which the LS model's prediction is less than 1 basis point, including 11 bonds where the predicted spread is so close to zero that the prediction error is reported as -100% . The CDG model shown in row 7 can boost the spreads of some of these bonds, as only 22 bonds have a predicted spread of less than 1 basis points and only 10 bonds have a reported error of -100% . In contrast, the lowest predicted spread in the G model is 3 basis points. At the other end of the credit spectrum, the results are equally extreme. The highest predicted spread for CDG is 3179 basis points, quite a bit more than the 2161 basis points predicted for that bond by the LS model, which itself is still much higher than any spread actually found in the sample (see Figures 4 and 5).

We can identify a number of factors that are likely to cause this tremendous dispersion in predicted spreads: the assumption of an

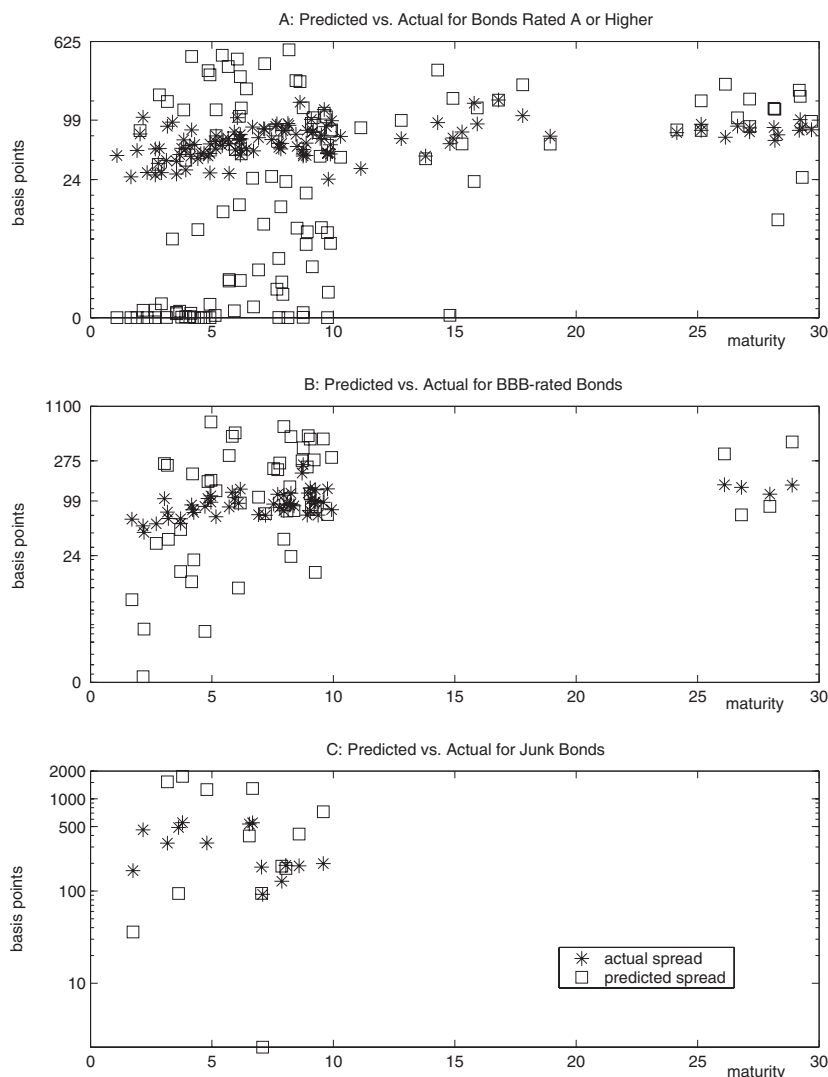


Figure 4

Predicted and actual spreads versus years to maturity: the LS model. The actual and predicted spreads for the sample of 182 bonds with simple capital structures at year-end during the period 1986–1997. The spreads are plotted against the remaining years to maturity of the bond. The actual spreads are marked with an asterisk and the predicted spreads from the Longstaff and Schwartz (1995) model are denoted with a square. Spreads are calculated in basis points over the Vasicek (1977) estimates of the Treasury yield curve.

exogenous default boundary, face value recovery rates, the failure to adjust this portfolio of zeroes approach for conditional probabilities of default, and the use of the Vasicek model. The flat default boundary in LS is likely to make a risky bond even more risky, with little impact on the

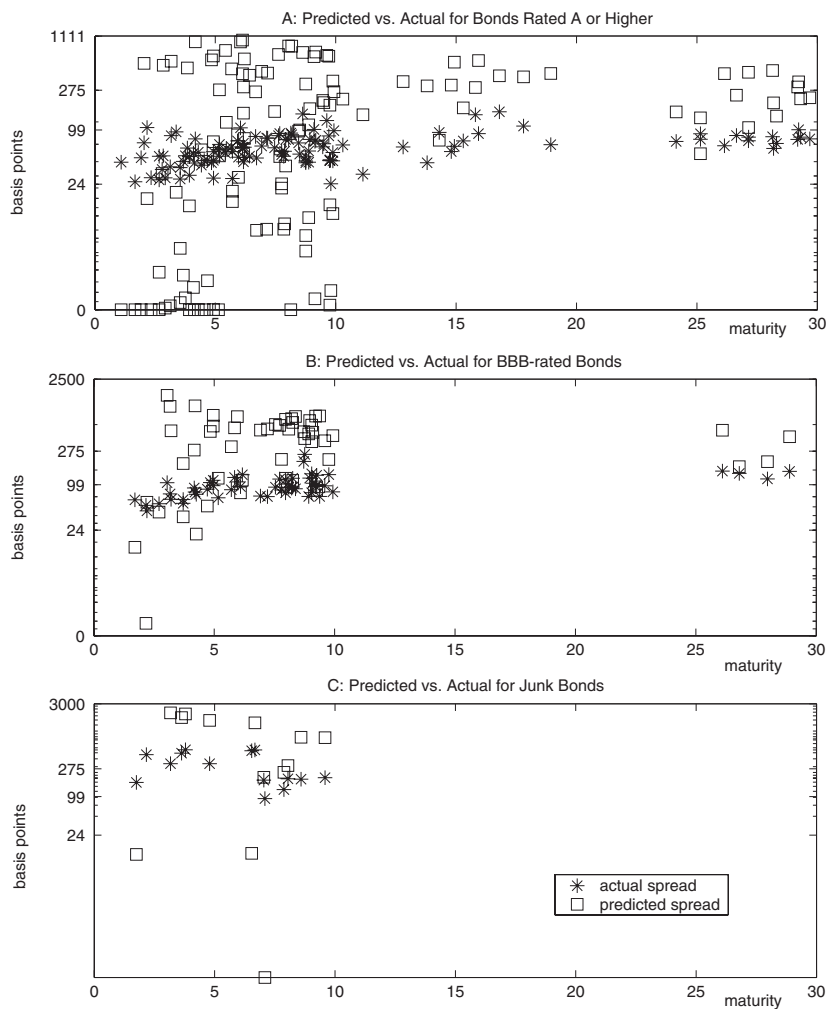


Figure 5
Predicted and actual spreads versus years to maturity: the CDG model. The actual and predicted spreads for the sample of 182 bonds with simple capital structures at year-end during the period 1986–1997. The spreads are plotted against the remaining years to maturity of the bond. The actual spreads are marked with an asterisk and the predicted spreads from the Collin-Dufresne and Goldstein (2001) model are denoted with a square. Spreads are calculated in basis points over the Vasicek (1977) estimates of the Treasury yield curve.

credit risk of a safe bond. We saw in our analysis of the G model, where we were able to hold all other features of the model constant, that the face value recovery rate adds markedly to the dispersion of predicted spreads. Neither the LS and CDG models nor the coupon version of the M model that we implement can reasonably incorporate a recovery rate based on

firm value.¹⁵ A related issue for these two models is how to implement the face value recovery rate rule. Under the simple portfolio of zeroes approach, a default results in bondholders recovering a fraction of the coupon on every future coupon date. This is not consistent with bankruptcy practices. Lastly, as we noted earlier, this portfolio of zeroes approach overestimates the probability of default in assuming independence of the coupons. This may lead to dispersion because the bonds that are already considered risky by the model (i.e., those with a high probability of default on their early coupons) will have an even greater probability of default associated with them, while there are no bonds that would be considered safer as a result.

The LS and CDG models may lose accuracy from the use of the Vasicek model if the interest rate volatility is poorly estimated in this model.¹⁶ In row 6 of Table 3, we show the sensitivity of the LS model to the estimates of the Vasicek model. The implementation in this row uses Treasury data from the entire month in which the bond trades, rather than just a single day. The estimates based on one month of data lead to much lower estimates of interest rate volatility.

The two sets of estimates for the LS model in Table 3 differ both in average predicted spreads and the degree of dispersion in predicted spreads. Consider the estimates for 1997, when the estimate of volatility is 34% using one-day data and is nearly zero using the time-series data. The former group includes six bonds that have predicted spreads of less than a basis point, whereas the less volatile interest rate dynamics result in nine such bonds. Nearly all of the spread estimates for the 1997 sample increase when estimated interest rate volatility rises, but the higher volatility actually exacerbates the overprediction problems on the riskiest bonds with only a small impact on the safest bonds.

The predicted spreads from the LS model from row 5 of Table 3 are shown in Figure 4, plotted against maturity for the three rating groups. Relative to the actual spreads, the LS has a tendency to predict either a very high spread or a very low spread. More often the highest predicted spreads belong to the lowest rated bonds. The dispersion is also more extreme at the shorter maturities, especially in the range of 5–10 years.

The CDG model also exhibits wide dispersion in predicted spreads, but this model differs markedly from the LS model in its predictions for the portion of the sample that have market spreads between 75 and 200 basis points (there are 79 bonds in this group). The CDG model clearly raises

¹⁵ In results not reported, we implemented the LS model with recovery rates that vary by industry [using data from Altman and Kishore (1996)], but like Lyden and Saraniti (2000), we find it does not improve the estimates.

¹⁶ Chan *et al.* (1992) conclude that the Vasicek model is a poor fit for the short-term rate and note that the fit is particularly sensitive to the level of interest rates.

the spread on the typical bond relative to the LS model. The latter model predicts spreads of less than 10 basis points for 10 of these bonds (only 3 in the CDG model) and underpredicts the spread for 35 bonds (only 16 bonds in the CDG model). Of these 79 bonds, the CDG model predicts a spread that is more than double the actual spread for 54 of the bonds, compared to only 27 for the LS model.

Indeed, the results in Table 3 suggest that the CDG model severely overestimates credit risk. Possibly the model might be better implemented with other estimates for the parameters related to the target leverage ratio, such as the speed of adjustment (κ_ℓ) and mean asset returns.

We estimate an average speed of adjustment factor of .10, which is markedly less than the .145 found by Frank and Goyal (2003). Our average may differ because our sample includes more firms from the 1990s. However, raising κ_ℓ would lead to even higher spread errors. In row 8 we show the impact on predicted spreads of lowering each estimated κ_ℓ by 15% (the average κ_ℓ falls from .10 to .085). The estimates shown in row 8 indicate that, for the level of κ_ℓ observed in our sample, the effects of mismeasurement of the speed of adjustment are small. The average spread prediction error falls only by 15–20 percentage points, from a base of 270 percentage points. Lowering κ_ℓ further would make it closer to zero, effectively making the model equivalent to the LS model.

The average asset return in our sample is 1.5% per month, only slightly higher than the median return. Together with our sample's average dividend yield (adjusted for share repurchases), this implies an annual average asset return of 24%, which seems high by historical standards. Our sample starts in 1986 and is dominated by bonds that are priced after 1990, which likely explains the high estimated returns. However, bond market participants may not view the high average asset returns of the 1990s as long-run average rates of return. If so, then a lower mean asset return is more appropriate in the implementation. In the last row of Table 3 we show results from reducing each firm's estimated mean asset return by 45%. This adjustment makes the average asset return in the sample about equal to the return that would lead to equity returns observed over longer periods of time (i.e., consistent with an average equity return of 15% and an equity risk premium of 6%).

By reducing the mean asset return estimate, the problem of bond price underestimation is much smaller. Spreads are now overpredicted by only 79%. The median yield spread is predicted at only 57 basis points and the median spread prediction error is –36%. However, the absolute errors are still quite large, as are the standard deviations. The wide variation in estimated spreads in the CDG model may reflect the added opportunity for parameter estimation error. An alternative to using estimates of mean asset returns and speed of adjustment would be to simply estimate the implied risk-neutral long-term mean of leverage, which not only fits the

data to the spreads, but avoids estimation of the mean asset return in the model (cf. Appendix B.2).

Figure 5 presents the predicted and actual bond spreads for the CDG model from row 7 of Table 3. The highest-rated bonds, shown in panel A of the figure, have a higher fraction of bonds that are considered extremely safe by the model. However, they are not the majority of the bonds in this rating class, and the rest of the bonds are credited with too much risk, especially at the shorter maturities. Bonds rated A or higher also appear to have overpredicted spreads more often among bonds that mature in 10 or fewer years, but once maturity extends beyond 10 years, there is no particular relationship between maturity and spread prediction errors.

3.1.3 Alternative measures of leverage. We have presented pricing errors from the five structural models using parameters that we consider reasonable. One could argue that all of the parameters are subject to measurement error and that the pricing errors are due to the implementation, and such a complaint would naturally focus on asset volatility and leverage. While we have ruled out measurement error in asset volatility as a major issue, we have not yet considered the possibility that our measure of leverage is causing errors. We next summarize the sensitivity of the implementations to the choice of the leverage parameter. For the sake of brevity, we do not consider this parameter for every model.

In estimations (not reported), we also used the bond par value as the strike price in the M model and LS models. Both models had sharply lower estimated spreads as a result, which is unsurprising once one considers the capital structures of the firms in the sample.¹⁷

Another possible way to parameterize leverage is suggested by KMV [see Crosbie and Bohn (2002)], who place a greater weight on short-term obligations probably for the purpose of better predicting the firm's default probability within one year. The logic of this approach is that debts due in the near term are more likely to cause a default. KMV measure the numerator of the leverage ratio as short-term debt plus half the value of long-term debt. Clearly this will also lead to much lower estimates of credit spreads unless the sum of short-term debt and long-term debt is close to total liabilities (as noted earlier, this is not the case in our sample). The most extreme effect of using KMV's leverage measure in our sample is the funeral home firm Hillenbrand, whose liabilities largely consist of prepaid funeral packages that are sold in the form of insurance. As the policies are neither long-term nor short-term debt, the inability to honor

¹⁷ The median ratio of long-term debt to total liabilities is about 30%, and the median face value of the bond is only \$150 million, which is typically a bit more than half of the firm's long-term debt. Thus, par value can easily understate the firm's obligations by 85%.

them would not be considered a source of credit risk by KMV, yet no bondholder would ignore these obligations.

We implemented the LS model using KMV's leverage ratio, and as expected, the lower average leverage leads to much lower predicted spreads. Despite reducing the average predicted spread, the KMV measure could be very useful. By adding a constant to the spreads generated by the KMV measure, predicted bond spreads would fit the data better. For example, the highest predicted spread (without the added constant) falls from 2161 basis points to 870 basis points in the LS model with the KMV measure of debt. The lower variance arises because the reduction in leverage is greatest for the riskiest firms (for bonds rated A or higher, switching to this method reduces leverage by 7%, compared to a 23% decrease for junk-rated firms). Partly this reflects the fact that lower-rated firms do not participate in the commercial paper market and have less short-term debt [see Crabbe and Post (1994)].

Another way to consider the sensitivity of the results to the choice of leverage parameter is to fit the leverage ratio to the bond prices and examine the implied leverage ratio. We generated implied leverage ratios for the LS model, and to speed convergence, we allow two bonds from the same firm on the same day to have different implied leverage ratios. The average implied leverage ratio obtained for the LS model is 35%, a bit higher than the ratio of total liabilities to assets we have been using. Yet the average implied leverage ratio is no more informative than the average spread prediction error. As before, there is substantial variation across credit ratings. Bonds rated A or higher have an implied leverage ratio that is 12% higher than the observed leverage ratio, while BBB bonds and junk bonds have implied leverage ratios that are 5% and 18% below the observed ratios, respectively (all three figures are statistically different from zero). Thus, if there is a better leverage parameter that we could use, it must be a parameter that raises the leverage of the safe bonds while simultaneously lowering the leverage of the riskier bonds.

3.2 Systematic prediction errors

In the previous section we mentioned some examples of bonds with extreme pricing problems. In this section we consider in more detail the question of why the models' predictions are inaccurate. We estimate a multivariate regression analysis of the spread prediction errors to determine which factors cause the weaknesses in the models. Before the regression stage, however, we narrow the list of potential variables by conducting *t*-tests on two subsamples, those with the lowest spread prediction errors and those that have the highest spread prediction errors.

We examine five types of variables that might lead to systematic valuation errors: (1) measures of leverage and capital structure (e.g., bond ratings, book leverage, long-term debt/total liabilities), (2) variables

related to firm value, (3) measures of asset volatility, (4) bond features, and (5) variables that proxy for differences in expected recovery rates [e.g., plant, property and equipment, capital expenditures, and research and development (R&D)]. We examine 26 variables with *t*-tests to determine if any of these areas are problematic in the structural models. Many of these variables are already part of the models.

3.2.1 *t*-Tests to determine systematic errors in spread predictions. Table 4 presents the *t*-tests for the five structural models. The *t*-tests examine differences in characteristics of two groups: those with spread prediction errors that are below the median in the sample and those that have prediction errors that are above the median. The errors analyzed in this table are the percentage differences between the predicted spreads and the observed spreads.

To present the results in a compact format, we only report signed *t*-statistics in Table 4. If the average of a variable is higher in the sample of bonds with the most positive spread prediction errors, then the *t*-statistic will be positive. For example, most models have a significant difference in maturity between the two subsamples, with the lower spreads associated with shorter maturity. Then the *t*-statistic in Table 4 in the maturity row will be a large positive number.

The results of the *t*-tests indicate that all of the models have systematic errors related to leverage, although the problem is less extreme in the LT model. In general, the models do a poor job of pricing safer bonds. The bonds with the smallest predicted spreads have a significantly lower leverage ratio on average (particularly if measured by market leverage). As this factor is included in each of the models, the results suggest that either leverage is measured poorly or that the models do not assign the appropriate risk to each level of leverage. Other leverage-related variables are also significant. For example, the average ratings are sharply different between the two samples for all but the LT model. The bonds with the higher predicted spreads are typically rated BBB+ (nine on this scale), while the bonds with the low predicted spreads have an average rating of about an A- or an A. Further, the spread underprediction problem is typically less severe for the firms where book value of leverage is close to the market value of leverage. It appears that firms with very high market values of equity are often treated by the models as if they are remarkably safe firms, whereas the bond market is not quite so sanguine about their prospects.

The *t*-tests show slight support for CDG's view, in that the difference between the current leverage ratio and the 10-year average ratio is close to statistically significant or actually significant in all but the LT model. When significant, the *t*-tests are always positive, suggesting that bonds with high leverage today receive the highest spreads in most of the models,

Table 4
***t*-Tests of prediction errors from defaultable-bond pricing models**

	Pricing models				
	Extended Merton	Geske	Leland-Toft	Longstaff-Schwartz	Collin-Dufresne and Goldstein
Book leverage	2.95	3.03	1.91	2.88	3.06
Market leverage	6.47	6.35	2.04	6.23	5.76
Abs(market leverage – book leverage)	– 4.28	– 4.06	– 0.33	– 4.12	– 3.39
10-year mean of leverage	5.28	5.12	1.84	4.90	4.48
Current leverage – 10-year mean	1.62	1.69	0.37	1.83	1.81
Long-term debt/total liabilities	2.16	1.03	– 1.31	0.73	1.16
Bond rating	4.85	4.00	– 0.78	2.41	2.84
Difference in ratings	0.22	0.00	– 1.30	0.00	0.22
Total assets (book)	0.18	0.82	1.61	1.32	1.09
Total assets (market)	– 1.73	– 0.82	1.79	– 0.62	– 0.74
Asset growth rate	1.43	1.28	– 0.14	0.36	2.70
Market/book	– 5.90	– 5.29	– 0.30	– 5.47	– 4.52
150-day equity volatility	7.99	7.65	– 0.05	5.13	4.90
150-day asset volatility	6.70	6.23	– 1.68	4.02	3.69
Years to maturity	4.16	2.61	– 0.64	2.46	1.07
Amount outstanding	– 1.47	– 1.05	1.69	– 0.14	0.02
Coupon	2.84	3.30	4.10	3.54	1.93
PPE/assets	3.12	2.40	0.49	1.80	1.37
R&D/assets	– 0.77	0.02	0.13	– 1.08	– 2.41
Cap. exp./assets	2.34	2.06	– 0.69	1.64	1.68
Payout	2.81	3.04	3.86	3.68	0.81
Corr. with 3-month Treasury-bills	1.26	0.78	– 0.03	2.87	3.76
Interest rate volatility	0.10	– 0.03	– 3.51	6.49	4.09
Old bond	0.29	0.29	3.74	2.01	0.86
Two bonds	1.28	1.61	1.61	1.28	0.96
Below par	0.32	0.97	6.17	1.62	1.62

The table reports *t*-statistics for the difference between the average values for the variables reported. The *t*-test compares bonds with spread prediction errors that are above the median for bond pricing models, with bonds whose prediction errors are below the median. Each column reports *t*-statistics for a model. Prediction errors used are percentage errors in predicted spreads using 150-day historical asset volatility. Bond rating is the average bond rating where AAA is 1, AA+ is 2, etc. Amount outstanding is the amount owed on the bond. PPE/assets is plant, property, and equipment over assets. Cap. exp./assets is capital expenditures divided by assets. Payout is a weighted average of the share-repurchased adjusted dividend yield and the bond coupon. Correlation with three-month Treasury-bills is the correlation of returns on equity and interest rates over the previous five years. Interest rate volatility is estimated using the Vasicek model. Old bond is a bond that was issued five or more years ago. Two bonds indicates that a firm has two public bonds in the sample. Below par means the flat price of the bond was below \$100.

even if the market does not perceive this leverage ratio to be normal over the longer term.

Table 4 reveals little evidence that systematic errors occur with firm size. The market values of assets differ significantly in the M model and the LT model, but with opposite signs for the two models.

Although the LT model is again an exception, most models have a strong difference between the market-to-book ratios of the two samples. As the growth rate of the firm, measured by changes in market value, is not a source of mispricing, this suggests that market-to-book is not related to future prospects so much as it is to valuation effects. As many of the bond prices are observed in the booming equity market of the 1990s,

errors associated with high market-to-book may be related to mismeasurement of asset volatility or leverage.

The results on market-to-book are similar to those for the volatility measures (the models that have lower spreads on high market-to-book firms are those with low spreads on low-volatility firms). Most models severely underpredict spreads for firms with low asset volatility, suggesting that all but the LT model are relying rather heavily on volatility as a way to generate higher spreads.

Maturity is a major factor in the M model, but less so in the G and LS models. The maturity *t*-tests are not significant for the LT or CDG models. The M model's underprediction problem is quite severe for low-leverage and low-volatility firms, so unless those firms have many years for leverage and volatility to come into play, there is little chance in that model to generate a high spread. In contrast, the LT model actually has a negative coefficient on maturity, suggesting, if anything, more difficulty in raising the spread on a long-term bond. The literature often proclaims that no structural model can generate sufficiently high spreads on short-maturity bonds, but the LT model does not reveal any such problem. We should note, however, that there is no definitive cutoff for "short-term." While we include in our sample only bonds that have a maturity of more than one year, the problem raised by researchers such as Duffie and Lando (2001) may actually refer to an even shorter horizon.

Coupon is statistically significant in all the models' *t*-tests, but this variable is highly correlated with rating and leverage, and may not be important by itself. Likewise, most models have a difficult time generating high spreads if the payout ratio is low, but the payout ratio reflects the coupon as well.

The *t*-tests for the variables related to recovery rates are contradictory. Sometimes the results indicate that firms that have spreads that are too low have fewer tangible assets, suggesting that they have greater expected losses in default than the model anticipates. But the *t*-tests also indicate that the bonds with the most underpredicted spreads have lower capital expenditures, which suggests few growth opportunities and above-average recovery rates. Moreover, the R&D *t*-test is never significant when these two variables are.

There is no evidence that the change in firm value that occurs with interest rates is a major omission of the M or G models, as neither the correlation nor the interest rate volatility are significantly different across their two groups. The LT model also shows no significance for the correlation, but does have a significant relationship with volatility. As the LT model is so strongly affected by coupon, one must consider whether the higher volatility is just another proxy for coupon. The *t*-test for the three-month correlation is highly significant in the LS and CDG models, suggesting much lower spreads on bonds of firms that have a very

negative correlation with interest rates. A very negative correlation reduces the estimated credit risk of a bond in these models, whereas a very positive correlation leads to a higher estimated spread. Apparently, including the correlation makes the spreads of the bonds with the most negative correlations too low and the spreads of the bonds with positive correlations too high in these two models. Most of the firms have negative correlations, but the positive correlation group is not insubstantial.

Few of the bond-specific variables matter. Old bonds are priced very differently in the LT model only because the average coupon fell so sharply over the sample period. The other models do not have tendencies related to old bonds, even though these bonds are thought to have lower trading volumes and less liquidity.

Note that the current leverage ratio relative to its target has an even higher *t*-statistic in the CDG model, which is surprising given that this model incorporates the target leverage ratio. The asset growth variable, which is the percentage change in the market value of assets over the previous five years, is very significant in the CDG *t*-test and not in other models. Hence it is likely that the target leverage ratio in the CDG model is not all that closely tied to the long-run average leverage ratio, and is more closely tied to asset returns.

3.2.2 Regression analysis. The results of the *t*-tests indicate a number of systematic differences in the prediction errors of the models, often based on variables that are included in the models. One might argue that a combination of factors leads to higher or lower prediction errors and therefore analysis in a multivariate regression setting is more appropriate. Next we estimate regressions using variables that appear to be important from the *t*-tests. We use the actual spread prediction error rather than the absolute value of the error, because we believe the factors that lead to underprediction of spreads are typically not the same as those that lead to overprediction of spreads, which is the assumption when both positive and negative errors are treated equally.¹⁸

Table 5 shows five sets of regressions, two for each of the structural models. Each regression is estimated with 182 bonds whose prices are observed in a variety of different interest rate settings. To control for these different pricing environments, we include four control variables: a measure of the slope of the Treasury yield curve; the level of interest rates (measured by the average of the CMT interest rates at month-end); the volatility of Treasury rates (estimated using the Vasicek model); and a measure of the ease with which corporations can issue bonds. The last is the residual from a regression of aggregate corporate bond issuance against a time trend.

¹⁸ Lyden and Saraniti (2000) estimate regressions using absolute errors.

Table 5
Regression of model prediction errors on firm and bond characteristics

Dependent variables: prediction errors of the bond pricing model

Independent variables	Regression model 1					Regression model 2				
	M	G	LT	LS	CDG	M	G	LT	LS	CDG
Intercept	-3.02 (-9.05)	-2.89 (-7.92)	1.653 (0.43)	-5.44 (-6.80)	-3.97 (-1.55)	-2.03 (-5.47)	-1.78 (-4.53)	3.206 (0.71)	-3.58 (-3.90)	-0.11 (-0.04)
Debt issuance	0.001 (0.09)	-0.002 (-0.30)	-0.017 (-0.20)	-0.031 (-1.70)	-0.030 (-0.52)	-0.006 (-0.83)	-0.011 (-1.36)	-0.032 (-0.35)	-0.045 (-2.44)	-0.052 (-0.88)
Term structure	0.080 (1.48)	0.106 (1.80)	0.482 (0.77)	0.450 (3.46)	0.907 (2.17)	-0.06 (-1.12)	-0.07 (-1.07)	0.164 (0.22)	0.160 (1.05)	0.483 (0.98)
Interest rate	0.081 (1.51)	0.091 (1.56)	-0.49 (-0.79)	0.489 (3.78)	0.754 (1.82)	-0.04 (-0.72)	-0.04 (-0.73)	-0.55 (-0.79)	0.236 (1.67)	0.313 (0.68)
Interest rate volatility	-0.25 (-2.08)	-0.26 (-1.97)	-1.10 (-0.78)	1.421 (4.83)	2.597 (2.75)	-0.25 (-2.14)	-0.26 (-2.16)	-1.23 (-0.86)	1.466 (5.06)	2.681 (2.85)
Years to maturity	0.0104 (2.35)	0.0128 (2.66)	-0.107 (-2.09)	0.0087 (0.83)	0.0052 (0.15)	0.0034 (0.75)	0.0056 (1.18)	-0.100 (-1.82)	-0.003 (-0.32)	-0.008 (-0.22)
Market leverage	2.694 (11.16)	2.910 (11.03)	6.151 (2.20)	5.617 (9.72)	8.816 (4.76)	3.663 (11.54)	4.083 (12.13)	8.110 (2.09)	7.619 (9.72)	13.36 (5.25)
Asset volatility	5.112 (13.19)	4.541 (10.73)	-8.22 (-1.83)	5.260 (5.67)	10.64 (3.58)	6.510 (13.97)	6.189 (12.52)	-4.81 (-0.85)	7.736 (6.72)	17.60 (4.71)
Payout	11.217 (6.68)	10.376 (5.66)	-19.61 (-1.01)	17.032 (4.24)	-11.86 (-0.92)	10.182 (6.07)	9.4402 (5.31)	-21.13 (-1.03)	14.746 (3.56)	-9.392 (-0.70)
Coupon	-0.080 (-3.27)	-0.069 (-2.59)	0.5693 (2.00)	-0.205 (-3.49)	-0.478 (-2.53)	-0.072 (-2.68)	-0.071 (-2.49)	0.3531 (1.07)	-0.179 (-2.69)	-0.538 (-2.49)

Correlation w/3-month Treasury-bills	-1.26 (-1.66)	-1.37 (-1.66)	6.758 (0.77)	-0.84 (-0.46)	4.903 (0.84)	-0.28 (-0.37)	-0.03 (-0.04)	13.24 (1.42)	1.240 (0.66)	9.077 (1.48)
Current leverage — LT leverage						-0.33 (-0.93)	-0.44 (-1.16)	1.416 (0.32)	-0.57 (-0.64)	-3.21 (-1.10)
Bond rating						-0.09 (-5.23)	-0.10 (-5.74)	-0.21 (-0.99)	-0.17 (-3.94)	-0.35 (-2.52)
Size						-1e-05 (-2.20)	-9e-06 (-1.69)	2e-05 (0.26)	-9e-06 (-0.69)	-5e-05 (-1.22)
Two bonds						0.078 (1.16)	0.147 (2.07)	1.314 (1.60)	-0.00 (-0.02)	0.799 (1.49)
PPE/assets						0.057 (0.36)	0.092 (0.54)	0.984 (0.50)	0.091 (0.23)	-2.18 (-1.68)
Old bond						-0.00 (-0.06)	0.123 (0.91)	3.410 (2.17)	-0.02 (-0.09)	0.541 (0.53)
Adj. R^2	0.683	0.644	0.089	0.679	0.299	0.720	0.704	0.105	0.698	0.324

The table reports regression coefficients and their t -values (in parentheses) where the dependent variable is the (signed) spread prediction error (in percentage terms) based on the 150-day historical volatility. Debt issuance is the residual from a regression of aggregate debt issuance against a time trend. Term structure is the difference between 10-year and 2-year Treasury yields. Interest rate is the average of the constant maturity Treasury rates. Interest rate volatility is estimated using the Vasicek model on the day the bond price is observed. Market leverage is total liabilities divided by market value of assets. Asset volatility is calculated using equity returns over the previous 150 trading days. Payout is a weighted average of the share-repurchased adjusted dividend yield and the bond coupon. Correlation with three-month Treasury-bills is the correlation of returns on equity and interest rates over the previous five years. Current leverage minus LT leverage is leverage in the year of the bond price observation less the average leverage over the previous 10 years. Bond rating is the average bond rating where AAA is 1, AA+ is 2, etc. Size is the market value of assets. Two bonds indicates that a firm has two public bonds in the sample. PPE/assets is plant, property, and equipment over assets. Old bond is a bond that was issued five or more years ago.

The columns in Table 5 show regressions of spread prediction errors under two specifications. The first specification includes all of the variables representing parameters in the five models as well as our control variables related to the interest rate environment. The second specification includes all of these variables plus a number of variables that are not included in any of the structural models, but which the *t*-tests indicate might be important. In results not reported, we also estimate regressions adding one variable at a time to determine its incremental impact on the adjusted R^2 of the regression.

The role of the interest rate environment in explaining spread prediction errors is not especially clear from the regressions, as the coefficients seem contradictory across the various models. Of the five models, the LS model is the most sensitive to the interest rate environment. Not only are more of the control variable coefficients significant, but the average Treasury rate and interest rate volatility together (not shown) generate an adjusted R^2 of 0.36 in this model. In contrast, the explanatory power of these variables is well under .05 for the M, G, and LT models and about .15 for the CDG model. The significant negative coefficients on interest rate volatility in the M and G models would suggest that these models would suffer less from underpredicted spreads if they allowed more richness in the Treasury term structure. However, two factors raise doubts about this interpretation. First, the LT model, which assumes a constant interest rate, shows no evidence of underestimating credit spreads when interest rates are volatile (interest rate volatility is not significant in any of the 11 specifications of the LT model that we estimated). Second, the coefficients on interest rate volatility are significant in the LS and CDG models, which should not occur if variables are incorporated correctly in a model (they should not generate systematic errors and only white noise should remain). The positive significant coefficients on interest rate volatility in the LS and CDG models suggest that the two models overstate the role of stochastic interest rates, generating excessively high spreads when the volatility variable is large. Perhaps this problem arises from the lack of precision in Vasicek model estimates.

A second feature of structural models that has often been thought in the literature of causing major pricing errors is maturity, but the regression estimates in Table 5 contradict this view. To the extent that short-term bonds have a spread underprediction problem, as has been emphasized in the literature, the only evidence is in the M and G models. There is no indication that maturity in and of itself affects the LS and CDG models, and maturity actually has a significantly negative coefficient in the LT model in each of the 11 specifications. Moreover, even the M and G models do not exhibit a systematic bias toward underprediction of short-term bonds once the bond rating is introduced into the regression. Bond ratings and maturity are related, as most speculative-grade debt is

issued with a shorter maturity than that typically found on investment-grade bonds. The spread underprediction problem so often associated with short-maturity debt in structural bond pricing models is better described as a problem with very safe bonds, which are hard to price regardless of their maturity.¹⁹

It is typically the case among our five models that a very safe bond is one with low leverage or low asset volatility. The exception is the LT model, where the safety of a bond is more often described by its coupon, but even this model relies heavily on leverage to generate higher spreads. In each of the models we examine, the firm's leverage ratio is a major determinant of the spread prediction error. All five models have significantly positive coefficients on leverage in all the estimated specifications. Moreover, the adjusted R^2 always rises considerably when leverage is included. Leverage is universally important in these structural models in generating higher predicted spread: A firm with low leverage is unlikely to have spreads as high as those observed in the bond market.

For all but the LT model, risk is also sharply delineated by the firm's asset volatility. Like leverage, asset volatility is significantly positive in every specification of these four models. And the explanatory power of the regressions is again clearly higher when volatility is considered. This is especially true of the M and G models, which have few other sources of risk available to generate high spreads.

The LT model instead relies on coupon and leverage to define risk. In the second specification shown in Table 5, the coefficient on coupon is not significant, although it is still positive. The reason is that the indicator variable for old bonds, which we included to capture liquidity, is highly correlated with coupons. In our sample, Treasury rates fall sharply from the beginning of the sample to the end, so that old bonds have high coupons.

The final variable to consider that is already part of a structural model is the correlation of firm value with interest rates. This variable is not important in any of the models. The fact that even the LS and CDG models do not have significant coefficients suggests that the correlation has a weak impact on the model. We recalculated the estimation errors in the LS model under the assumption that the correlation between firm value and interest rates was zero to check on the impact of the correlation estimates. These results (not shown) indicate that the correlation plays a small role in the model. For example, the average spread prediction error when eliminating the correlation differs by only two percentage points from that shown in Table 3. This may owe to the relatively low correlations among the firms in our sample, which are all well below .15 in magnitude.

¹⁹ We also estimated regressions with an indicator variable for bonds with less than five years to maturity. This variable was not significant for any of the models.

The second set of regression specifications reported in Table 5 includes the other variables that were significant in the *t*-tests. Likely most of these were significant because they were related to volatility or coupon, as few are consistently significant once other factors are considered. Although we see from the two implementations of the G model that the recovery rate assumption has a large impact on the prediction errors, this factor is not well captured by plant, property, and equipment, or our other measures of recovery rates.

The only reliably significant variable added to the second set of regressions is bond rating. Most of the models have a spread underprediction problem with bonds that the credit rating agencies perceive as risky and this risk is not captured by volatility or leverage. The rating agencies acknowledge the impact of leverage, and to a lesser extent, volatility, but low ratings are also associated with financially aggressive management (e.g., who might undertake a share repurchase) and firms with volatile leverage (e.g., cyclical companies). Ratings can also be reduced for firms with a declining franchise value (such as IBM or Kellogg's, who lost their AAA ratings with decreases in market share), firms that are poorly managed or those that have high technology risk.

Our results indicate that many of the models have very low predicted credit spreads for bonds belonging to safe firms. Indeed, the negative coefficients on the intercepts in the regressions suggest the M, G, and LS models may simply underestimate spreads on all bonds. If corporate bonds have a large and constant liquidity premium, part of the spread underprediction problem may owe to liquidity factors that are not part of any of these models.²⁰ To consider this possible explanation, we add 25 basis points to each predicted spread as a liquidity premium. We find this only helps the M and G models. For the other models, adding a constant for liquidity largely exacerbates the problem of overpredicted spreads, and in the LT model actually causes a higher standard deviation of spread prediction errors. Of these three, only the LS model enjoys a decrease in the average absolute spread with the liquidity premium, but the decrease is only 6 percentage points.

4. Conclusion

This article directly tests five corporate bond pricing models using a sample of noncallable bonds belonging to firms with simple capital structures between 1986 and 1997. In particular, we implement the models of Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001). We examine each

²⁰ See, for example, Huang and Huang (2002) and Janosi, Jarrow, and Yildirim (2002) on recent attempts to estimate how much of the corporate bond yield spread is due to credit risk.

of these models under similar assumptions to compare their abilities to predict corporate bond spreads. Then we consider whether the predicted spreads are prone to systematic errors.

We find that all the models have substantial spread prediction errors, but their errors differ sharply in both sign and magnitude. In particular, the average error is a rather poor summary of a model's predictive power, as the dispersion of predicted spreads is quite large (as seen in the high standard deviations and large average absolute prediction errors). All five models tend to generate extremely low spreads on the bonds that the models consider safe (usually low leverage and low asset volatility) and to generate very high spreads on the bonds considered to be very risky.

Although both the M and G models share a tendency toward underestimation of corporate bond spreads on average, the fact that the problem is less severe for the G model suggests that the endogenous default boundary of the G model is a major improvement. The option to make coupon payments in distress helps to improve the dispersion of predicted spreads relative to other models. Moreover, we can see by altering the recovery rate assumption in the G model that the dispersion of predicted spreads is exacerbated by the use of a fixed face value recovery rate.

In contrast, the LT model overpredicts spreads on average. The problem is widespread and is not sensitive to the parameter estimates. Rather, it owes largely to the assumption of a continuous coupon. Indeed, this model actually overpredicts spreads on shorter-maturity bonds, a very surprising result given the extant literature. We should note, however, that our regressions do not show a significant relationship between maturity and prediction errors in any of the other models, holding other factors constant. The previous literature may have overemphasized maturity in and of itself as a cause of mispricing.

The two-factor models of LS and CDG incorporate stochastic interest rates and a correlation between firm value and interest rates. We find that the correlation is not very important empirically. Stochastic interest rates do raise the average predicted spreads, but the results are rather sensitive to the interest rate volatility estimates from the Vasicek model. While other features of the LS model also lead to high spreads on average, a sizable fraction of the bonds in the sample still have the problem of extremely low predicted spreads.

The CDG model might alleviate the problem of excessive dispersion in predicted spreads if the most underprediction occurs among firms with leverage ratios below their targets and the overpredicted spreads belong to bonds with unusually high leverage ratios. The CDG model helps somewhat in this regard, but it tends toward overestimation of credit risk overall. The model requires the estimation of several parameters that are not in the other models, and thus may suffer more from measurement errors.

Despite the fact that most models generate their highest spreads on junk bonds, our regression results always point to a significantly lower predicted spread for lower-rated bonds, *ceteris paribus*. While the rating agencies heavily weight leverage, they also consider the potential for default from other sources and these may improve the models.

In conclusion, it is clear that these five structural bond pricing models have difficulty in accurately predicting credit spreads. However, the difficulties are not limited to the underprediction of spreads, as the previous literature implies. Underprediction of spreads often stems from the fact that the main channels for default are through high current leverage ratios, high asset volatility, or high payout ratios. Factors that help to raise these bonds' predicted spreads, such as the volatility of interest rates, raise estimated spreads on most bonds, but often by a small amount on the safe bonds and by even more on the riskiest bonds. Interest rate volatility may require a better model than the Vasicek model if excess dispersion of spreads is to be avoided. The modeling of the coupon is another important issue, as we found little evidence favoring the simple portfolio of zeroes approach used in the M, LS, and CDG models or the continuous coupon approach of the LT model. And the recovery rate assumption can affect the variance of the spread prediction errors greatly. We conclude that the focus of future research efforts should be on raising spreads on the safer bonds without raising them too much for the riskiest bonds.

Appendix A: Formulas for Prices of Defaultable Bonds

For completeness we include in this appendix all of the formulas for bond prices used in our model implementations except the formula for the Geske model.

Let V_t , K_t , and r_t be the time t values of the firm's assets, total liabilities, and the risk-free interest rate, respectively. Assume that

$$dV_t = (r_t - \delta)V_t dt + \sigma_v V_t dZ_{1t}^{\mathbb{Q}} \quad (1)$$

$$dr_t = (\alpha - \beta r_t) dt + \sigma_r dZ_{2t}^{\mathbb{Q}} \quad (2)$$

$$d \ln K_t = \kappa_\ell [\ln(V_t/K_t) - \nu - \phi(r - \theta)] dt, \quad (3)$$

where σ_v , δ , α , β , σ_r , κ_ℓ , ν , and ϕ are constants, $\theta = \alpha/\beta$, and $Z_1^{\mathbb{Q}}$ and $Z_2^{\mathbb{Q}}$, two one-dimensional standard Brownian motion processes under the risk-neutral measure $\mathbb{Q}$, are assumed to have a constant correlation coefficient of ρ .

All five of the structural models examined in this study assume Equation (1), and except CDG, all assume κ_ℓ is zero. The G and LT models assume a constant interest rate. The LS and CDG models assume the dynamics of the (default-free) interest rate can be described by the Vasicek model given in Equation (2). The M model allows for any term structure model.

A.1 The extended Merton model

Consider a defaultable bond with maturity T and unit face value that pays semiannual coupons at an annual rate of c . For simplicity, assume $2T$ is an integer. Let T_n , $n = 1, \dots, 2T$, be the n th coupon date.

In the extended Merton model, $K_t = K \forall t \in [0, T]$ and default is triggered if the asset value is below K on coupon dates. The price of a coupon bond is equal to the portfolio of zeroes

and can be written as follows:

$$P^M(0, T) = \sum_{i=1}^{2T-1} D(0, T_i) E^{\mathbb{Q}}[(c/2)I_{\{V_{T_i} \geq K\}} + \min(wc/2, V_{T_i})I_{\{V_{T_i} < K\}}] \\ + D(0, T) E^{\mathbb{Q}}[(1+c/2)I_{\{V_T \geq K\}} + \min(w(1+c/2), V_T)I_{\{V_T < K\}}], \quad (4)$$

where $D(0, T_i)$ denotes the time 0 value of a default-free zero-coupon bond maturing at T_i , $I_{\{\cdot\}}$ is the indicator function, $E^{\mathbb{Q}}[\cdot]$ is the expectation at time 0 under the $\mathbb{Q}$ measure, and w is the recovery rate.

It is known that

$$E^{\mathbb{Q}}[I_{\{V_i \geq K\}}] = N(d_2(K, t)) \quad (5)$$

$$E^{\mathbb{Q}}[I_{\{V_i < K\}} \min(\psi, V_i)] = V_0 D(0, t)^{-1} e^{-\delta t} N(-d_1(\psi, t)) + \psi [N(d_2(\psi, t)) - N(d_2(K, t))], \quad (6)$$

where $\psi \in [0, K]$, $N(\cdot)$ represents the cumulative standard normal function and

$$d_1(x, t) = \frac{\ln(\frac{V_0}{x D(0, t)}) + (-\delta + \sigma_v^2/2)t}{\sigma_v \sqrt{t}}; \quad d_2(x, t) = d_1(x, t) - \sigma_v \sqrt{t}. \quad (7)$$

Given a term structure $D(0, \cdot)$, Equation (4) along with Equations (5) and (6) can then be used to calculate the price of a defaultable coupon bond under Merton's assumptions.

Notice that any term structure model is allowed in this extended Merton model. In our implementation, $D(0, \cdot)$ is obtained using the Nelson and Siegel (1987) model (in the base case) and also the Vasicek (1977) model (see Section B.3).

A.2 The Leland and Toft model

In LT, coupons are paid continuously and the total coupon is c per year. All the formulas given in this subsection are from Leland and Toft (1996) (although the notation here is somewhat different). The value of a defaultable bond is given by

$$P^{LT}(0, T) = \frac{c}{r} + \left(1 - \frac{c}{r}\right) \left(\frac{1 - e^{-rT}}{rT} - I(T)\right) + \left(wV_B - \frac{c}{r}\right) J(T), \quad (8)$$

where

$$I(T) = (G(T) - e^{-rT} F(T)) / (rT) \\ J(T) = \frac{1}{z \sigma_v \sqrt{T}} [-e^{(z-a)b} N(q_-(T)) q_-(T) + e^{-(z+a)b} N(q_+(T)) q_+(T)] \\ G(T) = e^{(z-a)b} N(q_-(T)) + e^{-(z+a)b} N(q_+(T)) \\ F(T) = G(T) |_{z=a}$$

with

$$a = \frac{r - \delta}{\sigma_v^2} - \frac{1}{2}; \quad b = \ln(V_0/V_B); \quad z = \left(a^2 + \frac{2r}{\sigma_v^2}\right)^{1/2}; \quad q_{\mp}(t) = \frac{-b \mp z \sigma_v^2 t}{\sigma_v \sqrt{t}}.$$

The default boundary V_B is given in Equations (11) and (13) of LT.

A.3 The CDG and LS models

In CDG, Equations (1)–(3) are assumed to hold. The value of a defaultable bond that pays semiannual coupons is given by

$$P^{CDG}(0, T) = \left(\frac{c}{2}\right) \sum_{i=1}^{2T-1} D(0, T_i) [1 - w_{\ell} Q^{\mathbb{R}_i}(0, T_i)] + \left(1 + \frac{c}{2}\right) D(0, T) [1 - w_{\ell} Q^{\mathbb{R}_T}(0, T)], \quad (9)$$

where $D(0, T_i)$ denotes the time 0 value of a T_i -maturity default-free zero-coupon bond given by the Vasicek (1977) model, $Q^{\mathbb{F}_T}(0, T_i)$ represents the time 0 default probability over $(0, T]$ under the T_i -forward measure [e.g., Geman, El Karoui, and Rochet (1995) and Jamshidian (1989)], and w_ℓ , the loss rate, equals $1 - w$. In CDG, the loss rate on coupons is 100%. Here we use the same loss rate on both coupons and principal. (This reduces the model's overprediction error in spreads.)

One can see from Equation (9) that the key step in implementing the CDG model is to determine $Q(0, \cdot)$. It can be calculated as follows:

$$Q^{\mathbb{F}_T}(t_0, T) = \sum_{i=1}^n q(t_{i-\frac{1}{2}}, t_0), \quad t_0 = 0, \quad t_i = iT/n, \quad (10)$$

where for $i = 1, 2, \dots, n$,

$$q(t_{i-\frac{1}{2}}, t_0) = \frac{N(a(t_i; t_0)) - \sum_{j=1}^{i-1} q(t_{j-\frac{1}{2}}, t_0) N(b(t_i; t_{j-\frac{1}{2}}))}{N(b(t_i; t_{i-\frac{1}{2}}))} \quad (11)$$

$$a(t_i; t_0) = -\frac{M(t_i, T | X_0, r_0)}{\sqrt{S(t_i | X_0, r_0)}} \quad (12)$$

$$b(t_i; t_j) = -\frac{M(t_i, T | X_{t_j})}{\sqrt{S(t_i | X_{t_j})}} \quad (13)$$

and where the sum on the right-hand side of Equation (11) is defined to be zero when $i = 1$, $X \equiv V/K$, and

$$\begin{aligned} M(t, T | X_0, r_0) &\equiv E_0^{\mathbb{F}_T}[\ln X_t]; \\ S(t | X_0, r_0) &\equiv \text{var}_0^{\mathbb{F}_T}[\ln X_t]; \\ M(t, T | X_u) &\equiv M(t, T | X_0, r_0) - M(u, T | X_0, r_0) \frac{\text{cov}_0^{\mathbb{F}_T}[\ln X_t, \ln X_u]}{S(u | X_0, r_0)}, \quad u \in (t_0, t) \\ S(t | X_u) &= S(t | X_0, r_0) - \frac{(\text{cov}_0^{\mathbb{F}_T}[\ln X_t, \ln X_u])^2}{S(u | X_0, r_0)}, \quad u \in (t_0, t). \end{aligned}$$

This approach is in the spirit of LS and used by CDG to implement the one-factor version of their model.

Under the T -forward measure,

$$\begin{aligned} e^{\kappa_\ell t} \ln X_t &= \ln X_0 + \bar{\nu}(e^{\kappa_\ell t} - 1) + \int_0^t [(1 + \phi \kappa_\ell) r_u - \rho \sigma_r \sigma_r B(t, T)] e^{\kappa_\ell u} du \\ &\quad + \int_0^t \sigma_v e^{\kappa_\ell u} dZ_{1u}^{\mathbb{F}_T} \end{aligned} \quad (14)$$

$$\begin{aligned} r_t &= r_0 e^{-\beta t} + \left(\frac{\alpha}{\beta} - \frac{\sigma_r^2}{\beta^2} \right) (1 - e^{-\beta t}) + \frac{\sigma_r^2}{2\beta^2} e^{-\beta T} (e^{\beta t} - e^{-\beta t}) + \sigma_r e^{-\beta t} \int_0^t e^{\beta u} dZ_{2u}^{\mathbb{F}_T} \\ &= E_0^{\mathbb{F}_T}[r_t] + \sigma_r e^{-\beta t} \int_0^t e^{\beta u} dZ_{2u}^{\mathbb{F}_T}, \end{aligned} \quad (15)$$

where

$$\bar{\nu} \equiv (\nu - \phi\theta) - (\delta + \sigma_v^2/2)/\kappa_\ell \quad (16)$$

$$B(t, T) = \frac{1}{\beta} (1 - e^{-\beta(T-t)}). \quad (17)$$

Equations (14) and (15) can then be used to calculate $M(t, T | X_0, r_0)$ and $\text{cov}_0^{\mathbb{F}T}[\ln X_t, \ln X_u]$ and thus the expectations in Equations (11)–(13).

It follows from Equation (14) that

$$\begin{aligned} e^{\kappa_\ell t} E_0^{\mathbb{F}T}[\ln X_t] &= \ln X_0 + \bar{\nu}(e^{\kappa_\ell t} - 1) + \int_0^t (1 + \phi\kappa_\ell) e^{\kappa_\ell u} E_0^{\mathbb{F}T}[r_u] du \\ &\quad - \frac{\rho\sigma_v\sigma_r}{\beta} \left[\frac{e^{\kappa_\ell t} - 1}{\kappa_\ell} - e^{\beta T} \frac{e^{(\kappa_\ell + \beta)t} - 1}{\kappa_\ell + \beta} \right]. \end{aligned} \quad (18)$$

Equation (15) can then be used to evaluate explicitly the integral on the right-hand side of Equation (18) and hence to arrive at $M(t, T | X_0, r_0)$. Similarly

$$\begin{aligned} \text{cov}_0^{\mathbb{F}T}[\ln X_t, \ln X_u] e^{\kappa_\ell(t+u)} &= \sigma_v^2 E_0^{\mathbb{F}T} \left[\int_0^t e^{\kappa_\ell v} dZ_{1v}^{\mathbb{F}T} \int_0^u e^{\kappa_\ell v} dZ_{1v}^{\mathbb{F}T} \right] \quad (\equiv I_1) \\ &\quad + \sigma_v(1 + \phi\kappa_\ell) E_0^{\mathbb{F}T} \left[\int_0^t e^{\kappa_\ell v} dZ_{1v}^{\mathbb{F}T} \int_0^u e^{\kappa_\ell v} r_v dv \right] \quad (\equiv I_2) \quad (19) \\ &\quad + \sigma_v(1 + \phi\kappa_\ell) E_0^{\mathbb{F}T} \left[\int_0^u e^{\kappa_\ell v} dZ_{1v}^{\mathbb{F}T} \int_0^t e^{\kappa_\ell v} r_v dv \right] \quad (\equiv I_3) \\ &\quad + (1 + \phi\kappa_\ell)^2 \text{cov}_0^{\mathbb{F}T} \left[\int_0^t e^{\kappa_\ell v} r_v dv, \int_0^u e^{\kappa_\ell v} r_v dv \right] \quad (\equiv I_4), \end{aligned}$$

where $\forall t \geq u$,

$$\begin{aligned} I_1 &= \frac{\sigma_v^2}{2\kappa_\ell} (e^{2\kappa_\ell u} - 1) \\ I_2 &= (1 + \phi\kappa_\ell) \frac{\rho\sigma_v\sigma_r}{\kappa_\ell + \beta} \left[\frac{e^{2\kappa_\ell u} - 1}{2\kappa_\ell} - \frac{e^{(\kappa_\ell - \beta)u} - 1}{\kappa_\ell - \beta} \right] \\ I_3 &= (1 + \phi\kappa_\ell) \frac{\rho\sigma_v\sigma_r}{\kappa_\ell + \beta} \left[\frac{1 - e^{(\kappa_\ell - \beta)t}}{\kappa_\ell - \beta} + \frac{e^{2\kappa_\ell u} - 1}{2\kappa_\ell} + e^{(\kappa_\ell + \beta)u} \frac{e^{(\kappa_\ell - \beta)t} - e^{(\kappa_\ell - \beta)u}}{\kappa_\ell - \beta} \right] \\ I_4 &= (1 + \phi\kappa_\ell)^2 \frac{\sigma_r^2}{2\beta} \left[-\frac{(e^{(\kappa_\ell - \beta)t} - 1)(e^{(\kappa_\ell - \beta)u} - 1)}{(\kappa_\ell - \beta)^2} + (e^{(\kappa_\ell + \beta)u} - 1) \frac{e^{(\kappa_\ell - \beta)t} - e^{(\kappa_\ell - \beta)u}}{\kappa_\ell^2 - \beta^2} \right. \\ &\quad \left. - \frac{\beta}{\kappa_\ell^2 - \beta^2} \frac{e^{2\kappa_\ell u} - 1}{\kappa_\ell} + \frac{1}{\kappa_\ell^2 - \beta^2} (1 - 2e^{(\kappa_\ell - \beta)u} + e^{2\kappa_\ell u}) \right]. \end{aligned}$$

The LS model can be nested within CDG. The price of a defaultable bond in the LS model can be obtained by setting κ_ℓ to zero. The resultant formulas are omitted here for brevity.

Appendix B: Parameter Estimation

In this appendix we discuss estimation of asset return volatility, parameters in the mean-reverting leverage process, and interest rate process parameters.

B.1 Asset return volatility

One estimate of asset (return) volatility σ_v can be obtained from historical equity (return) volatility σ_e using the relationship $\sigma_e = \sigma_v \frac{V_t}{S_t} \frac{\partial S_t}{\partial V_t}$, where S_t denotes the market value of equity at time t . Specifically, given $\partial S_t / \partial V_t$ and σ_e , the above formula can be used to solve for σ_v . One simple scheme is to use $\partial S_t / \partial V_t = N(d_1(K_t, t))$ —a result of the Merton (1974) model—where d_1 is defined in Equation (7). One estimate of σ_e is the sample volatility of daily equity returns over a particular horizon. [The RiskMetrics™ (1994) estimate of σ_e was used in an earlier analysis and did not produce sharply different results.] The GARCH(1,1) specification is also used to estimate σ_e .

Bond-implied volatility is analogous to the Black and Scholes option-implied volatility used in option markets. Namely, given a structural model, one chooses the asset volatility such that the model price for the bond fits today's bond market price. This bond-price implied volatility is then used as an estimate for a future date's asset volatility. In our analysis, bond-implied volatility is estimated using bond prices in each November during our sample period. Except for four newly issued bonds, all the bonds have trader quotes in November. All but four of the quotes indicate that prices varied between November and December in our sample period.

We implemented the models using bond-implied volatility and the pricing errors are smaller than those associated with historical equity volatilities that we use in most of the article. However, despite the greater accuracy, bond-implied volatility may not be so much a measure of volatility as a catch-all for errors in the structural model. If prices do not change much from one month to the next, then the estimate of the bond-implied volatility will include the effects of all the factors that are not in the structural model. Thus we expect this method to generate more accurate predicted prices in the next month, absent major news. One way to consider the usefulness of the bond-implied volatility approach is to compare spread prediction errors from this method to the errors in spreads from a random walk model (where we simply assume the December spread will be November's spread). For the M model, the average spread prediction error from the random walk method is 1.9%, which is an improvement over the 13% average error obtained by using the bond-implied volatility approach.

Asset volatility can also be estimated based on the equity value. Under the KMV approach [Crosbie and Bohn (2002)], one solves both the asset value and the asset volatility simultaneously using both the equity value and equity volatility as the target. (We implemented this method using a subsample in an earlier version of the article and found the KMV estimates are close to those based on historical equity volatility.) A related approach, which we do not implement, is to fit the market price of equity only [Black (1985)].

B.2 Parameters in a mean-reverting leverage process

Here we discuss how to estimate parameters κ_ℓ , ϕ , and ν defined in Equation (3) in the CDG model. One can see from Equation (14) that κ_ℓ and ϕ can be estimated by regression. However, this would not produce a direct estimate of $\bar{\nu}$ defined in Equation (16) since the regression is under the physical measure, whereas Equation (14) is defined under the T -forward measure.

Under the physical measure denoted by $\mathbb{P}$,

$$\begin{aligned} dV_t &= (\mu_v - \delta) V_t dt + \sigma_v V_t dZ_{1t} \\ d \ln X_t &= [\mu_v + \kappa_\ell \bar{\nu} + \kappa_\ell (\phi r_t - \ln X_t)] dt + \sigma_v dZ_{1t}, \end{aligned}$$

where μ_v is a constant, the constant $\bar{\nu}$ is as defined earlier, and Z_1 is a one-dimensional standard Brownian motion under $\mathbb{P}$. Let $\alpha_\ell \equiv \mu_v + \kappa_\ell \bar{\nu}$. A regression of the change in the log leverage ratio against log leverage lagged one period and the interest rate will then generate

parameter estimates: $\hat{\alpha}_\ell$, $\hat{\kappa}_\ell$ and $\hat{\phi}_\ell$. If μ_v can be estimated, then $\bar{\nu}$ can be estimated as follows:

$$\hat{\bar{\nu}} = (\hat{\alpha}_\ell - \hat{\mu}_v) / \hat{\kappa}_\ell.$$

In our implementation, monthly market leverage ratios and three-month CMT data over the prior 10 years are used in the regressions and μ_v is estimated by the mean return of the asset value over the prior 10 years.

B.3 Interest rate parameters

Let $y(t, T; \Theta_r)$ denote the spot rate at time t with term equal to $T - t$ predicted by a particular model characterized by its parameter set Θ_r . To fit the model to CMT rates on day t , one chooses parameters in Θ_r to minimize the sum of errors squared, where the error is measured as the deviation between the model yield and the market yield. Given the estimated spot curve $y(t, \cdot; \Theta_r)$ at t , the model price of a default-free coupon bond can be easily computed.

In the Nelson and Siegel (1987) model,

$$y(t, T; \Theta_r) = \beta_0 + \delta_1(\beta_1 + \beta_2) \frac{(1 - e^{-(T-t)/\delta_1})}{T - t} - \beta_2 e^{-(T-t)/\delta_1}, \quad (20)$$

where $\Theta_r = (\beta_0, \beta_1, \beta_2, \delta_1)$, and β_0 and δ_1 need to be positive.

In the Vasicek (1977) model,

$$y(t, T; \Theta_r) = -\ln D(t, T)/(T - t) = -A(t, T) + r_t B(t, T), \quad (21)$$

where $\Theta_r = (\alpha, \beta, \sigma_r, r_t)$, $B(t, T)$ is defined in Equation (17), and

$$A(t, T) = \left(\frac{\sigma_r^2}{2\beta^2} - \frac{\alpha}{\beta} \right) (T - t) + \left(\frac{\sigma_r^2}{\beta^2} - \frac{\alpha}{\beta} \right) \frac{e^{-\beta(T-t)} - 1}{\beta} - \frac{\sigma_r^2}{2\beta^2} \frac{e^{-2\beta(T-t)} - 1}{2\beta}. \quad (22)$$

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