Equilibrium Investment Strategies and Output Price Behavior: A Real-Options Approach

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The effects of competitive interactions on investment decisions and on the dynamics of the price of a nonstorable commodity are studied in a model of incremental investment with time to build and operating flexibility. I find that an increase in uncertainty may encourage firms to increase their capacity. Furthermore, I show that it may be optimal to invest in additional capacity during periods in which part of the operational capacity is not being utilized. The impact of competition on the properties of the endogenous output price is dramatic. For example, I find that price volatility may be increasing in the number of competitors in the industry.

The essence of the real-option approach to analyzing investment under uncertainty is that many investment and operating decisions can be structured in terms of the options they create. For example, the opportunity to invest in a project or to expand production facilities is like a call option on the investment opportunity. The underlying asset is the present value of the net operating cash flows from the investment and the exercise price is the investment outlay. The option is exercised by making the investment expenditure. Once the real options embedded in investment opportunities are identified the valuation models of financial options can be used for real investment analysis. One key difference is that, unlike financial options, real options may not be proprietary, so that its exercise may involve strategic preemptive motives. Most of the articles on real options applications that are concerned with strategic exercise policies focus on the effect of competition on the value of the option to invest and ignore the operating decisions that may arise once the investments are completed. However, operating decisions like the ability to vary output in response to changes in demand, costs, and other variables are valuable real options.

This article studies the effect of competitive interactions among firms in a given industry on investment decisions and on the price of a nonstorable

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commodity. I analyze strategic behavior in a real-options framework that includes realistic features of investment projects, such as time to build, operating flexibility, and capacity choice.

Consider investment in electricity generation. The construction of new power plants can take 6 to 10 years to complete. The capacity of a power generating facility is determined by, among other considerations, future demand. An electric utility can expand its generating capacity in response to an increase in expected future demand. Producers can vary the amount of energy generated in response to changes in demand or costs. Other operating options that are common in the electricity supply industry include the choice of the fuel to burn. Finally, an electricity producer has to formulate its investment strategies considering the strategies of other electric utilities operating in its area of service.

Time to build is an important characteristic of many investment projects. Besides electricity generation, there are many examples of investment with significant construction lags. Grenadier (1995a) reports lags of about two to three years for office building construction. Majd and Pindyck (1987) note that the construction of an underground mine, or the development of a large petrochemical plant can take at least five or six years to complete. Pindyck (1991) observes investment lags of 5 to 10 years in pharmaceuticals and aerospace. Real option models with time to build include Majd and Pindyck (1987), Grenadier (1995a), and Bar-Ilan and Strange (1996). These models study the investment decisions of an individual firm and their focus is on a single project of a given fixed size, like the decision to build a new factory or a new office building.

Capacity choice and operational flexibility are important features of many real investments. For example, the owner of real estate can select the scale or density at which to develop his property. This choice is often subject to legal limitations, such as zoning regulations. Williams (1991) features a model of real estate development as an option that analyzes both the optimal timing and the scale of the development. Many land development projects are examples of "lumpy" investments, where the size of the project is chosen when the investment occurs. Investment can also proceed incrementally, where the initial size of the project can be expanded later. For example, a firm must decide how large a factory to build and when to expand it. Pindyck (1988) develops a model of irreversible and incremental investment in which the firm has the option to not utilize the incremental unit when demand is low. He and Pindyck (1992) extend Pindyck's model to allow for technology choice. An example of technology choice arises when a firm can use one or two variable inputs in production. An electric utility, for instance, can choose between a coal-fired plant, an oil-fired plant, or a plant that can burn either fuel. The problem is to decide which type and how much capital to install. These models of capacity choice assume that investment is instantaneous and the firm is a monopoly.

In many real investment contexts, the payoffs from a firm's investment are affected by the investment strategies of its competitors. Models of irreversible investment under uncertainty and competitive equilibrium include Dixit (1991, 1993), Leahy (1993), Grenadier (1995b), and Williams (1993). All these articles assume that the investment is instantaneous. Grenadier (2000) derives a competitive equilibrium with time to build, and Grenadier (1996) studies an oligopolistic industry equilibrium with construction lags. These last two articles by Grenadier assume that the firms' capacity is always fully used.

The closest work on equilibrium investment strategies using option pricing methods to this article is in Baldursson (1998) and Grenadier (2002). Baldursson studies an oligopoly were firms facing a stochastic linear demand curve use capacity as a strategic variable. The equilibrium framework of Baldursson is very similar to my model. However, Baldursson assumes that investment is instantaneous and that installed capacity is always fully utilized. The examples analyzed in Baldursson (1998) indicate that qualitatively the price process will be the same in oligopoly and perfect competition. Grenadier (2002) develops an approach to solving for the investment equilibrium that is applicable to the more general specification of demand and considers time to build. However, like in Baldursson (1998), Grenadier (2002) does not assume flexibility in the use of the installed capacity, and the resulting output price behavior is the same for different numbers of firms in the industry.

My model extends the classic real-options model of irreversible investment and capacity choice by introducing time to build and competition [see, e.g., Pindyck (1988) and He and Pindyck (1992)]. A firm must decide how much capacity to build initially and when to expand it later. The firm has the option to not use any incremental unit of capacity if demand falls. My analysis makes several interesting contributions. First, I show that with time to build, more uncertainty may encourage the firm to hold more capacity. Furthermore, the firms' optimal capacity may be larger under uncertainty than under certainty. These results contrast with models of incremental investment that assume no construction lags and find a negative effect of uncertainty on capacity choice [see, e.g., Pindyck (1988) and Dixit and Pindyck (1994, chap. 11)].

The optimal capacity choice under uncertainty involves a trade-off. On the one hand, the irreversibility of investment induces firms to hold less capacity as a buffer against unanticipated drops in demand. On the other hand, the potential profits lost by not having enough capacity to satisfy an unanticipated increase in demand creates an incentive to hold more capacity. However, when new capacity can be used immediately, the income from any additional capacity depends on current demand, which is independent of uncertainty. Therefore, without construction lags, more uncertainty reduces the level of capacity that firms are willing to hold.

With time to build there is a lag between the time at which new capacity is installed and the time at which it can be used in production. More uncertainty increases the likelihood of extreme future demand states. Since firms have the option to not utilize any additional capacity when demand falls, the profit from incremental capacity is always positive. This means that the opportunity cost of insufficient capacity rises with uncertainty. Thus the net effect of uncertainty on the level of optimal capacity is ambiguous. An increase in uncertainty may encourage firms to hold more capacity.

I also find that the level of optimal capacity may increase with a longer investment lag. This result has implications for the dynamics of capacity utilization. Specifically, the frequency of times in which all the completed capacity is used may be decreasing in the length of the investment lag. This result is consistent with the chronic excess capacity that has been observed in commercial real estate and electricity generation, where construction lags are long. I discuss the dynamics of capacity in Section 3.

The second main result refers to the dynamics of capacity choice and capacity utilization. I show that firms may find it optimal to invest in additional capacity even during periods in which part of their current operational capacity is not being used. Thus I provide a new insight for an observed phenomenon in electricity generation, in which the decision to start the construction of a new power plant is often made even in the presence of excess capacity. The intuition is that firms expand their capacity in anticipation of a stronger future demand even if current demand is not enough to operate at full capacity. I elaborate further on this result in Section 3.

The third important finding in this article is that more competition does not change the above results on capacity choice and capacity expansion. This latter result contrasts with the analysis in Pindyck (1993) which shows that competition has a depressing effect on irreversible investment even when the opportunity cost of insufficient capacity is increasing in uncertainty. I elaborate further on this point in Section 5.

Finally, although the dynamics of capacity are qualitatively the same under oligopoly and perfect competition, the properties of the equilibrium output price can be remarkably different. Specifically, I show that during the periods in which the industry's operational capacity is fully used, the output price can increase to a very high level relative to the price when completed capacity is not fully used. The relative size of these spikes in the price series is increasing in the number of firms in the market. Furthermore, the dynamics of capacity utilization induces heteroscedasticity in the output price process, and the price volatility may be increasing in the number of competitors in the industry.

The presence of spikes and heteroscedasticity are empirically stylized facts about commodity price series [see Deaton and Laroque (1992)]. Extensive research literature explains these properties through the dynamics of storage. For a comprehensive treatment, see Williams and Wright (1991). In my model these properties follow from the interaction of time to build, capacity choice, flexibility and competition. Thus my analysis complements the storage-based models. Furthermore, my model provides a useful framework for studying the prices of commodities for which storage is relatively expensive, such as electricity for example. I discuss the output price dynamics in Section 4.

This article is organized as follows. Section 1 presents my model of capacity choice with time to build and operating flexibility. The properties of the optimal capacity choice are examined in Section 2, and the dynamics of capacity are analyzed in Section 3. Section 4 examines the effect of competition on investment and output prices, and Section 5 compares my findings to recent research on competitive investment under uncertainty. Section 6 concludes.

1. A Model of Capacity Choice with Time to Build and Operating Flexibility

My model extends the classic model of irreversible investment and capacity choice of Pindyck (1988) by introducing time to build and strategic behavior. In this section I examine the capacity choice problem of a monopolist. I initially abstract from competition to focus on how the interaction of time to build and operating flexibility affect the firm's investment problem. The extension to oligopoly and perfect competition is presented in Section 4.

Consider a firm that produces a nonstorable commodity whose price is a function of the level of output and a stochastic demand shock. Specifically I assume the following simple form for the inverse demand curve:

$$P = Y - \gamma q, \tag{1}$$

where P is the output price, Y is an exogenous shock to demand, q is the level of output, and γ is a nonnegative constant describing the slope of the linear demand function.[1,2] This assumed functional form implies that changes in the variable Y will be reflected in parallel shifts to the demand curve. Thus Y can be thought of as the relative strength of the demand side of the market. Conditions affecting the strength of demand include the

[1] The analysis presented in this section can be extended to any demand function linear in Y or a power function of Y. The qualitative results would be the same. I use Equation (1) for analytical convenience.

[2] He and Pindyck (1992) and Grenadier (1995a) also incorporate linear demand functions into a real option framework. A linear demand is also used in several recent studies of the electricity industry. See, for example, Green and Newbery (1992) and Wolfram (1999).

level of industrial production, household income, etc. The demand shock evolves as a geometric Brownian motion,

$$dY(t) = \mu Y(t)dt + \sigma Y(t)dZ(t), \tag{2}$$

where μ is the instantaneous proportional change in Y per unit time, σ is the instantaneous standard deviation per unit time, and Z is a standard Wiener process. Both μ and σ are constant. Thus the intercept grows at a constant rate which is perturbed in continuous time by a normally distributed random variable. Despite this assumed symmetric perturbation in the exogenous source of uncertainty, I show below that the equilibrium output price process will exhibit realistic features such as mean reversion, spikes, and heteroscedasticity.

The firm operates a simple production technology. Each unit of operational capacity can produce one unit of output per unit time at a cost of

$$C(q) = c_1 q + 0.5 c_2 q^2. \tag{3}$$

This cost function implies a linear marginal production cost in which the nonnegative constants c_1 and c_2 are, respectively, the intercept and the slope.[3] At any time t the firm chooses output to maximize its current profit. It follows from the assumed production technology that the output chosen cannot exceed the current level of operational capacity, which is denoted by $O(t)$. Thus,

$$\pi(O(t), Y(t)) = \max_{0 \leq q \leq O(t)}[(Y(t) - \gamma q)q - c_1 q - 0.5 c_2 q^2] \tag{4}$$

is the instantaneous profit that the firm earns when the operating capacity level is $O(t)$ and the demand parameter is $Y(t)$.

The firm can add units of capacity at a sunk cost of k per unit. However, there is a lag between the time at which the firm decides to invest in a new unit and the time at which this unit can be used in production. I denote this investment lag or time to build by h, where $h \geq 0$. Thus a unit of capacity purchased at time t will be ready to use in production at time $t + h$.

With time to build, at any point in time t there may be both operational units, whose total number is $O(t)$, and units that are currently under construction. Let $N(t)$ denote the number of units that are under construction. These are the units that began construction during the interval $(t - h,\ t]$ and, consequently, will be completed during the interval $(t,\ t + h]$. Let $K(t)$ denote "committed capacity", where $K(t) = O(t) + N(t)$.

[3] A quadratic cost function and its associated linear marginal cost curve have been widely used to estimate the actual marginal cost of a power plant in empirical studies of competitive electricity markets. See, for example, Green and Newbery (1992). An increasing marginal cost curve is consistent with the fact that power companies use cheaper forms of energy first, using most costly plants (gas combustion turbines) only when demand is high.

Investment is irreversible; technically this means that the process for the committed capacity, $\{K(t)\}$, is nondecreasing.[4]

The firm's problem is to choose the path of capacity expansion that maximizes the present value of its future cash flows. We assume that the firm is risk neutral. This means that future cash flows are discounted at the risk-free rate of interest, r, which is assumed constant.[5] To ensure that the value of the firm is finite we must have $r > \mu$.

As in Pindyck (1988) and He and Pyndyck (1992), I approach the solution to this problem by examining the firm's incremental investment decision.[6] The opportunity to invest in an additional unit of capacity is analogous to a perpetual American call option. The underlying asset is the value of an extra unit of capacity and the exercise price is the cost of investing in this unit, k.

Therefore the solution to the firm's capacity choice problem involves two steps. First, the value of an extra unit of capacity must be determined. Second, the value of the option to invest in this unit must be determined together with the decision rule for exercising this option. This decision rule is the solution to the optimal capacity problem.

1.1 The value of a marginal unit of capacity

The value of a marginal unit of committed capacity is the present value of the expected flow of profits from this unit. At any time t this value depends on committed capacity, $K(t)$, and the demand shock, $Y(t)$, but it does not depend the amount of capacity that is currently under construction, $N(t)$, and the times at which these units will be completed and become operational. The reason is that, because of the construction lag, the decision to invest in an extra unit of capacity at time t does not influence the profit flow over the interval $(t,\ t+h)$. Since all the units under construction at time t will be completed by time $t+h$, the profit flow during the period $[t, t+h)$ depends only on the entry times over the period $[t-h, t)$, while the

[4] The assumption of total irreversibility is done for tractability. The model can be extended to allow the firm to sell part of its capacity. However, none of the basic results of the model are altered under partial irreversibility. Furthermore, assuming that the installed capacity depreciates over time at a given rate does not affect the model results qualitatively.

[5] Risk neutrality is now a standard assumption in most of the literature on real options. This assumption can easily be relaxed by adjusting the drift rate μ to account for a risk premium, as in Cox and Ross (1976). Alternatively we can assume that there exists an asset whose return is perfectly correlated with Y. This asset can be traded continuously in infinitesimal increments and with zero transaction costs. Thus markets are dynamically complete in the sense that contingent claims written on Y can be priced by taking the expectation of the discounted cash flows under the risk-neutral probability measure.

[6] The value of the firm can be derived as the solution to an optimal instantaneous control problem. He and Pindyck (1992) show that the solution to this type of capacity choice problem can be obtained by examining the firm's incremental investment decision. My solution extends their approach by including an investment lag. The details are available upon request.

profit flow after time $t+h$ depends only on the current level of committed capacity.[7] Therefore, for any t, we have $O(t+h)=K(t)$.

For given a current committed capacity K, and demand Y, $\Delta G(K, Y, h)$ denotes the value of a marginal unit of capacity. It follows from Equations (1), (3), and (4) that the profit from an additional unit of committed capacity at any $t \geq h$ is

$$\Delta\pi(K, Y(t)) = \max[Y(t) - (2\gamma + c_2)K - c_1, 0]. \tag{5}$$

Thus, after it is completed, each incremental unit of capacity will be used only when the additional profit it generates is positive. This implies that the firm can costlessly shut down the additional unit of capacity during times of weak demand and costlessly bring it back on line during times of high demand. Stopping and restarting costs can be introduced into our framework, but this would complicate the presentation without altering any of the fundamental insights. Furthermore, by assuming costless suspension, we obtain an analytical expression for the value of the incremental unit of committed capacity.[8]

From Equation (5), the profits from the marginal unit at any time $t \geq h$ are identical to the payoff to a call option at maturity, for which the underlying asset follows the process in Equation (2) and the exercise price is $(2\gamma + c_2)K + c_1$. Therefore the marginal unit of capacity can be valued by using the Black–Scholes formula to value each of these options, and then summing these values by integrating over time. However, it is easier to value an incremental unit of capacity as a simple contingent claim that depends on Y.

To find $\Delta G(K, Y, h)$ we first determine the value of the marginal unit after it is completed and can be used in production. The value of an incremental unit of operational capacity when the current level of completed capacity is O is denoted by $\Delta H(O, Y)$. The procedure for obtaining $\Delta H(O, Y)$ is standard. The details can be found in chapter 6 of Dixit and Pindyck (1994). The solution gives the following expression for the value of an additional unit of operational capacity,

$$\Delta H(O,Y) = \begin{cases} A(O)\,Y^{\alpha} + \dfrac{Y}{r-\mu} - \dfrac{(2\gamma+c_2)O+c_1}{r} & \text{for } Y \geq (2\gamma+c_2)O+c_1 \\ B(O)\,Y^{\beta} & \text{for } Y \leq (2\gamma+c_2)O+c_1, \end{cases} \tag{6}$$

[7] See Grenadier (2000) for a formal proof that in a model of incremental investment with time to build, knowledge of only the current levels of the demand shock and of committed capacity are sufficient for solving the investment problem.

[8] This assumption is not far from reality. In the electricity industry, gas-fired peaking plants are flexible enough to be switched on and off at short notice in response to market conditions.

where α and β are, respectively, the negative and positive roots of the characteristic equation

$$\frac{\sigma^2}{2}\xi(\xi-1)+\mu\xi-r=0 \tag{7}$$

and the constants A and B are given by

$$\begin{aligned} A(O) &= \frac{[(2\gamma+c_2)O+c_1]^{1-\alpha}}{\beta-\alpha}\left(\frac{\beta}{r}-\frac{\beta-1}{r-\mu}\right) \\ B(O) &= \frac{[(2\gamma+c_2)O+c_1]^{1-\beta}}{\beta-\alpha}\left(\frac{\alpha}{r}-\frac{\alpha-1}{r-\mu}\right). \end{aligned} \tag{8}$$

The expression for $\Delta H(O, Y)$ is interpreted as follows. If the marginal unit is always used its value is $Y/(r-\mu)-[(2\gamma+c_2)O-c_1]/r$. Thus, when $Y>(2\gamma+c_2)O+c_1$, $A(O)Y^\alpha$ is the value of the option to not use the unit should Y decrease. Similarly, when $Y<(2\gamma+c_2)O+c_1$, the unit is not used and $B(O)Y^\beta$ is the value of the option to use it should Y increase.

Once we have the value of an incremental unit of operational capacity, the value of a unit of capacity under construction $\Delta G(K, Y, \theta)$, where $0<\theta\leq h$ is the remaining time until the unit becomes operational, is obtained as follows

$$\begin{aligned} \Delta G(K, Y, \theta) &= E\left[\int_\theta^\infty e^{-rt}\Delta\pi(K, Y(t))dt \mid Y(0)=Y\right] \\ &= e^{-r\theta}E\left[\int_0^\infty e^{-rt}\Delta\pi(K, Y(t+\theta))dt \mid Y(0)=Y\right] \\ &= e^{-r\theta}E[\Delta H(K, Y(\theta)) \mid Y(0)=Y]. \end{aligned} \tag{9}$$

In general, at any time t

$$\Delta G(K(t), Y(t), \theta)=e^{-r\theta}E_t[\Delta H(K(t), Y(t+\theta))], \tag{10}$$

where $E_t[\cdot]=E[\cdot\mid Y(t)]$.

Appendix A derives the solution to this conditional expectation,

$$\begin{aligned} \Delta G(K, Y, \theta) = {} & (1-\Phi(v-\alpha\sigma\sqrt{\theta}))A(K)Y^\alpha+(1-\Phi(v-\sigma\sqrt{\theta}))\frac{Ye^{-(r-\mu)\theta}}{r-\mu} \\ & -(1-\Phi(v))\frac{[(2\gamma+c_2)K+c_1]e^{-r\theta}}{r}+\Phi(v-\beta\sigma\sqrt{\theta})B(K)Y^\beta, \end{aligned} \tag{11}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and v is defined as

$$v = v(K, Y, \theta) = \frac{\log[(2\gamma + c_2)K + c_1] - \log Y - (\mu - \sigma^2/2)\theta}{\sigma\sqrt{\theta}}. \tag{12}$$

1.2 The decision to invest in the marginal unit

Having valued the marginal unit of committed capacity, we can now value the option to invest in this unit. Let $\Delta F(K, Y)$ denote the value of this option when the current committed capacity level is K. The exercise price of this option is equal to the cost of construction. Therefore, for any level of committed capacity K, $\Delta F(K, Y)$ is a perpetual American call option whose value depends on Y. Hence there will be a threshold value at which it will be optimal to exercise this option. Specifically, for any K there exists a threshold, $Y(K)$, such that the option to build an additional unit of capacity will be exercised the first time that Y equals or exceeds $Y(K)$.

Using standard methods, it is easy to show that $\Delta F(K, Y)$ satisfies the following partial differential equation [see Pindyck (1988)]:

$$\frac{\sigma^2}{2} Y^2 \Delta F_{YY} + \mu Y \Delta F_y - r\Delta F = 0. \tag{13}$$

The solution is subject to the following boundary conditions:

$$\begin{aligned} &\Delta F(K, 0) = 0 \\ &\Delta F(K, Y(K)) = (G(K, Y(K), h) - k \\ &\Delta F_Y(K, Y(K)) = \Delta G_Y(K, Y(K), h). \end{aligned} \tag{14}$$

The first boundary condition arises because $Y = 0$ is an absorbing barrier for the process described in Equation (2), and therefore the option to invest has no value at that point. This implies the following functional form for the option to invest in a marginal unit of capacity,

$$\Delta F(K, Y) = D(K) Y^\beta \quad \text{for } Y < Y(K), \tag{15}$$

where β is the positive root of Equation (7) and $D(K)$ is a "constant" to be determined. The last two boundary conditions form the system of equations that must be solved to get the values of $Y(K)$ and $D(K)$. They are the value-matching and the smooth-pasting condition, respectively, and they imply that $Y(K)$ is the value of Y that maximizes the value of the option to invest.

Using the functional forms for $\Delta F(K, Y(K))$ and $\Delta G(K, Y(K), h)$ this system becomes

$$
\begin{aligned}
D(K)\,Y(K)^{\beta} = {} & (1-\Phi(v_K-\alpha\sigma\sqrt{h}))A(K)\,Y(K)^{\alpha} \\
& + (1-\Phi(v_K-\sigma\sqrt{h}))\frac{Y(K)e^{-(r-\mu)h}}{r-\mu} \\
& - (1-\Phi(v_K))\frac{[(2\gamma+c_2)K+c_1]e^{-r\theta}}{r} \\
& + \Phi(v_K-\beta\sigma\sqrt{h})B(K)\,Y(K)^{\beta}-k
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\beta D(K)\,Y(K)^{\beta-1} = {} & (1-\Phi(v_K-\alpha\sigma\sqrt{h}))\alpha A(K)\,Y(K)^{\alpha-1} \\
& + A(K)\,Y(K)^{\alpha}\frac{\phi(v_K-\alpha\sigma\sqrt{h})}{Y(K)\sigma\sqrt{h}} \\
& + (1-\Phi(v_K-\sigma\sqrt{h}))\frac{e^{-(r-\mu)h}}{r-\mu} + \frac{\phi(v_K-\sigma\sqrt{h})}{\sigma\sqrt{h}}\frac{e^{-(r-\mu)h}}{r-\mu} \\
& - \frac{\phi(v_K)[(2\gamma+c_2)K+c_1]e^{-rh}}{rY(K)\sigma\sqrt{h}} \\
& + \Phi(v_K-\beta\sigma\sqrt{h})\beta B(K)\,Y(K)^{\beta-1} \\
& - \frac{\phi(v_K-\beta\sigma\sqrt{h})}{Y(K)\sigma\sqrt{h}}B(K)\,Y(K)^{\beta},
\end{aligned} \tag{17}
$$

where $\nu_{\mathrm{K}} = \nu(K, Y(K), \theta)$, and $\phi(\cdot)$ is the standard normal density function. Eliminating $D(K)$ we are left with an equation for the investment threshold. A reduced expression for this equation is given by

$$\beta[\Delta G(K, Y(K), h) - k] - Y(K)\Delta G_Y(K, Y(K), h) = 0. \tag{18}$$

This last equation cannot be solved analytically to get an expression for $Y(K)$, however, it is easily solved numerically. In the next section I illustrate the solution to the investment problem.

1.3 The firm's optimal capacity

The function $Y(K)$ is the firm's optimal investment rule, if Y and K are such that $Y > Y(K)$, the firm should add capacity, increasing K until $Y = Y(K)$. Equivalently, given the current level of the demand shock Y, we can determine the firm's optimal capacity by rewriting Equation (18) in terms of $K(Y)$,

$$\beta[\Delta G(K(Y), Y, h) - k] - Y\Delta G_Y(K(Y), Y, h) = 0. \tag{19}$$

In the next section I analyze the properties of the solution to Equation (19).

2. Characteristics of the Optimal Capacity Choice

In the previous section I derived the solution to the firm's optimal capacity choice. Assuming that the firm has no initial capacity, the solution to Equation (19) gives the optimal level of capacity to build for a current level of demand Y. Because there is no closed-form expression for $K(Y)$, it is necessary to solve Equation (19) numerically in order to understand the properties of the firm's capacity choice. I examine the effects of demand volatility, σ, and of the length of the construction lag, h, on $K(Y)$. The numerical results presented in this and the next two sections were obtained through numerical simulations under a wide range of parameter values. To illustrate the analysis I look at the specific example of an electricity producer.

The capital cost for new power plants depends on the type of technology used in generating electricity. A recent analysis by the International Energy Agency (IEA) finds that the capital cost for a new nuclear plant is about \$2000 per kilowatt, a new coal-fired plant costs about \$1200 per kilowatt, while a combined-cycle gas plant costs about \$500 per kilowatt (see IEA (2001)]. For the base case I set the capital cost at $k = \$1200$ per kilowatt. The marginal cost of supplying an additional unit of electricity depends on the production technology and the type of fuel used in generation. Power companies use cheaper forms of energy first, using their most costly plants (gas combustion turbines) only when demand is high. The observed shape of the marginal cost curve, also known as the supply stack, is relatively flat for a low level of output and steep when output is high. In the model, output is constrained by the level of operating capacity. This makes the marginal cost curve vertical when output is equal to operating capacity. Thus the model accounts for the rapid increases in marginal costs that have been observed when production levels approach capacity. I will set the intercept and the slope of the marginal cost curve at $c_1 = 200$ and $c_2 = 0.5$. Regarding the slope of the demand curve, the usual practice in studies of competitive electricity markets is to set this parameter such that it is consistent with the observed price elasticity for electricity demand at the typical values of price and quantity [see, e.g., Borenstein and Bushnell (1999) and Wolfram (1999)]. For the central case I will use $\gamma = 50$. As for the average annual growth rate of the demand shift variable, I set $\mu = 0.03$.[9] Finally, I set the risk-free rate at $r = 0.06$. The fundamental characteristics of the results presented below are not affected by varying these parameters within realistic ranges.

To analyze the effect of uncertainty on capacity choice when it takes time to build, I first examine the solution to the benchmark case in which

[9] The IEA estimates that the world demand for electricity will grow at an annual average rate of 2.8% between 1997 and 2020. See, IEA (2000).

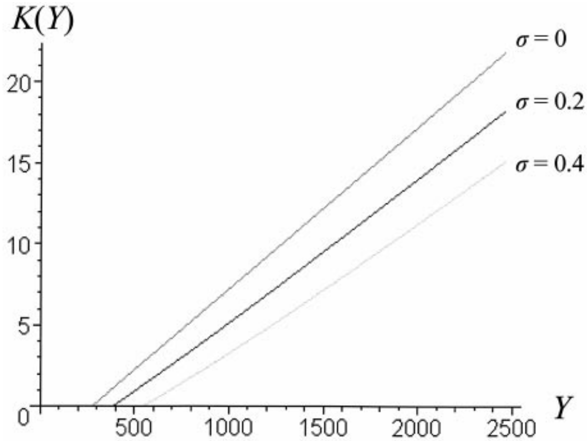

Figure 1
Optimal capacity as a function of *Y* with instantaneous investment
This figure shows the capacity choice $K(Y)$ as a function of the demand parameter Y for different levels of demand volatility σ. The assumed parameter values are $\mu = 0.03$, $\gamma = 50$, $c_1 = 200$, $c_2 = 0.5$, $r = 0.06$, $k = 1200$, $h = 0$.

there is no investment lag. Figure 1 shows $K(Y)$ when investment is instantaneous ($h = 0$) for different levels of demand volatility. Since the output price is strictly increasing in Y, so is $K(Y)$. For any level of demand, optimal capacity is smaller when future demand is more uncertain. As shown in Figure 1, the curve $K(Y)$ for $\sigma = 0.4$ is below that for $\sigma = 0.2$, which is below $K(Y)$ for $\sigma = 0$. The firm's capacity choice is decreasing in demand volatility when new capacity can be used immediately. I explain the intuition behind this result below.

The effect of uncertainty on the level of optimal capacity is remarkably different when it takes time to build. Figure 2 shows $K(Y)$ when the investment lag is three years ($h = 3$) for different levels of demand volatility. Uncertainty has a negative effect on the level of optimal capacity for low demand values. However, as the level of demand increases, more uncertainty may encourage the firm to build more capacity. As shown in Figure 2, as Y increases, the curve $K(Y)$ for $\sigma = 0.4$ will be eventually above $K(Y)$ for $\sigma = 0.2$. Furthermore, the capacity curves for $\sigma = 0.2$ and 0.4 will eventually be above the curve for $\sigma = 0$. This implies, for example, that for Y large enough, a firm facing an annual level of demand volatility of 0.4 will invest more than an otherwise identical firm facing no demand uncertainty. Thus the firm may invest more under uncertainty than under certainty. It is important to note that these results do not rely on the assumption of a positive trend in the demand process. Similar results can be obtained for a trendless demand process ($\mu = 0$).[10]

[10] Examples available from the author.

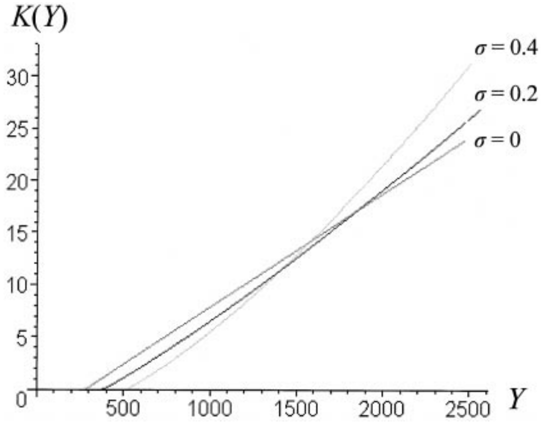

Figure 2
Optimal capacity as a function of Y with time to build
This figure shows the capacity choice $K(Y)$ as a function of the demand parameter Y for different levels of demand volatility σ. The assumed parameter values are $\mu = 0.03$, $\gamma = 50$, $c_1 = 200$, $c_2 = 0.5$, $r = 0.06$, $k = 1200$, $h = 3$.

Uncertainty affects the firm's optimal capacity choice in two opposing ways. On the one hand, the irreversibility of investment creates a cost of excess capacity. Specifically, if future demand decreases, the firm will find that it is holding more capacity than it would have chosen had the decrease in demand been anticipated. More uncertainty increases the likelihood of lower future demand, and thus the cost of excess capacity. On the other hand, the value of an additional unit of capacity includes the option to not utilize the unit when demand decreases. This option offsets the effect of uncertainty on the cost of excess capacity as follows. For any given level of demand volatility, a firm that can shut down part of its operational capacity when demand is low will hold more capacity than an otherwise identical firm that has to use all its completed capacity. However, to determine the net effect of an increase in demand volatility on the firm's capacity choice, we have to compare the effects of uncertainty on both the cost of excess capacity and the opportunity cost of insufficient capacity.

The firm's output in every period is constrained by the number of operational units. The potential profits lost from not having enough capacity create an opportunity cost that is decreasing in the level of capacity. When new capacity can be used immediately, the opportunity cost of insufficient capacity depends on current demand, which is independent of uncertainty. Therefore, since the cost of excess capacity is increasing in demand uncertainty, with no investment lags, more uncertainty reduces the level of optimal capacity.

With time to build, there is a lag between the time at which a new unit of capacity is purchased and the time at which it can be used in production.

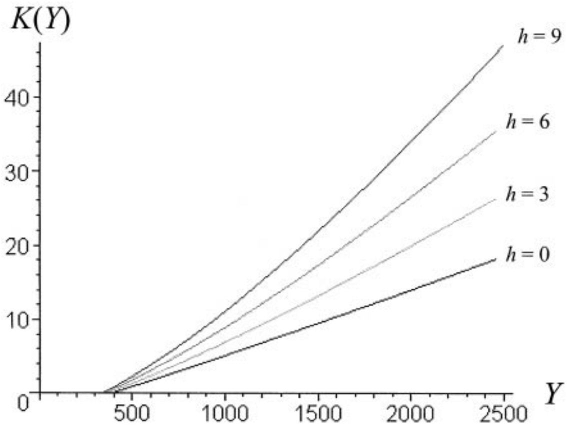

Figure 3
Optimal capacity as a function of Y for different lags
This figure shows the capacity choice $K(Y)$ as a function of the demand parameter Y for different lengths of the investment lag h. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.2$, $\gamma = 50$, $c_1 = 200$, $c_2 = 0.5$, $r = 0.06$, $k = 1200$.

More uncertainty increases the likelihood of extreme levels of future demand. But the firm need not use the additional unit of capacity when demand decreases. Therefore the opportunity cost of insufficient capacity rises with uncertainty. Thus the net effect of uncertainty on the level of optimal capacity is ambiguous. More uncertainty raises the level of optimal capacity if the increased opportunity cost of insufficient capacity outweighs the increased cost of excess capacity.

The ability to vary output by not using the marginal unit of capacity at times when demand is low is essential to generating the above result. Without this option the firm has to operate the unit at a loss during periods of extremely low demand that may result from a high level of uncertainty. In Appendix B I analyze the capacity choice of an otherwise identical firm that does not have this operating option. This implies that completed capacity is always fully used. For this case I derive the firm's optimal capacity choice in closed form and show that the level of committed capacity is decreasing in demand volatility.

Figure 3 shows the effects of changes in the length of the investment lag on the optimal level of capacity, $K(Y)$, for the base case parameters, with $\sigma = 0.2$. For any level of demand, optimal capacity is increasing in the length of the investment lag.

In general, the net effect of a longer construction lag on capacity is ambiguous. On one hand, as the construction time increases, the present value of the flow of profits from an additional unit of capacity decreases. On the other hand, the likelihood of extreme levels of demand at the time of completion is increasing in the construction lag. This implies that both the cost of excess capacity and the opportunity cost of insufficient capacity increase with the length of the investment lag.

3. The Dynamics of Capacity Expansion and Capacity Utilization

In this section I analyze the dynamics of committed capacity, operational capacity, and output. When the current level of demand is Y, the solution to Equation (19) gives the optimal level of committed capacity, $K(Y)$, for a firm that has no capacity. Alternatively, since investment is irreversible, at any time t, $K(Y(t))$ is the level of "desired capacity." Thus the irreversibility constraint implies that $K(t) \geq K(Y(t))$ for all t, and $K(t) = K(Y(t))$ if $dK(t) > 0$, where $dK(t) \geq 0$ denotes the infinitesimal increment to capacity at time t.[11] Therefore, for any time t, the level of committed capacity $K(t)$ may depend on the whole past history of the demand process $\{Y(s), 0 \leq s \leq t\}$. In addition, since output price is strictly increasing in Y, so is the desired committed capacity $K(Y)$. Finally, the process for the committed capacity $\{K(t), t \geq 0\}$ is nondecreasing because we assume that the installed capacity does not depreciate.

With these properties we can characterize the dynamics of committed capacity. The firm begins with no capacity at $t = 0$, it observes $Y(0)$, and begins the construction of $K(Y(0))$ units of capacity. Then, if Y increases from its initial value, the firm will expand its initial level of committed capacity. After demand has reached a temporary maximum, the level of committed capacity remains fixed until demand reaches its historic high again. Thus capacity expansion occurs when demand is increasing and only when it is increasing above previous highs. Once we have obtained the path of committed capacity, the path of operational capacity follows the identity $O(t) = K(t - h)$. Given $O(t)$ the output choice is

$$Q(t) = \min\left[\frac{1}{2\gamma + c_2}(Y(t) - c_1), O(t)\right].$$

This follows from Equation (4).

I illustrate the dynamics of capacity expansion and capacity utilization for the base case parameters introduced in the previous section. The upper graph in Figure 4 shows a sample path for the demand level, $Y(t)$. The lower graph in Figure 4 shows the paths of committed capacity, $K(t)$, completed capacity, $O(t)$, and output, $Q(t)$, that result from the demand path shown in the upper graph. The times at which the paths $O(t)$ and $Q(t)$ coincide are the times when operational capacity is fully utilized.

[11] As mentioned in note 6, the optimal path of capacity expansion, $\{K(t), t \geq 0\}$, can be obtained as the solution to an optimal instantaneous control problem. This form of optimal control problem implies that the stochastic process for $K(t)$ is continuous.

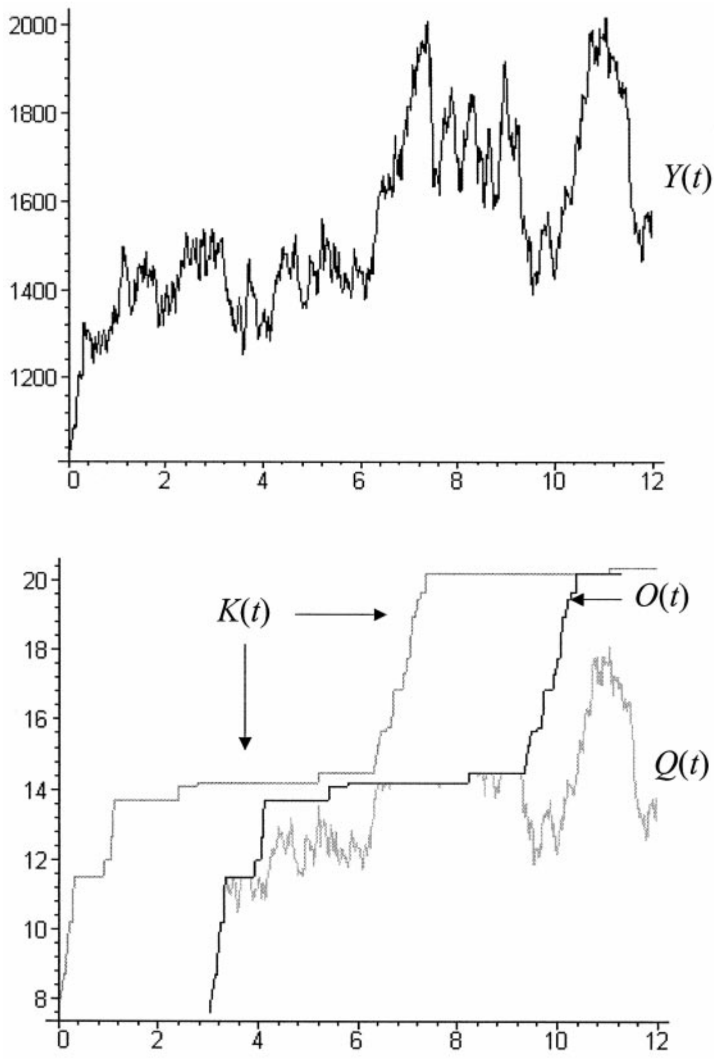

Figure 4
The dynamics of committed capacity, operational capacity, and output
The upper graph shows a sample path of demand and the lower graph shows the corresponding paths of committed capacity, operational capacity, and output. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.15$, $\gamma = 50$, $r = 0.06$, $k = 1200$, $h = 3$, $c_1 = 200$, $c_2 = 0.5$, $Y(0) = 1000$.

Figure 5 shows the paths of capacity expansion, $\Delta K(t)$, and the fraction of completed capacity that is used in production, $Q(t)/O(t)$, that result from the demand path realization of Figure 4. The completed capacity is fully used when $Q(t)/O(t) = 1$. The flat parts on the path $Q(t)/O(t)$ correspond to the periods of time in which the operational capacity is fully

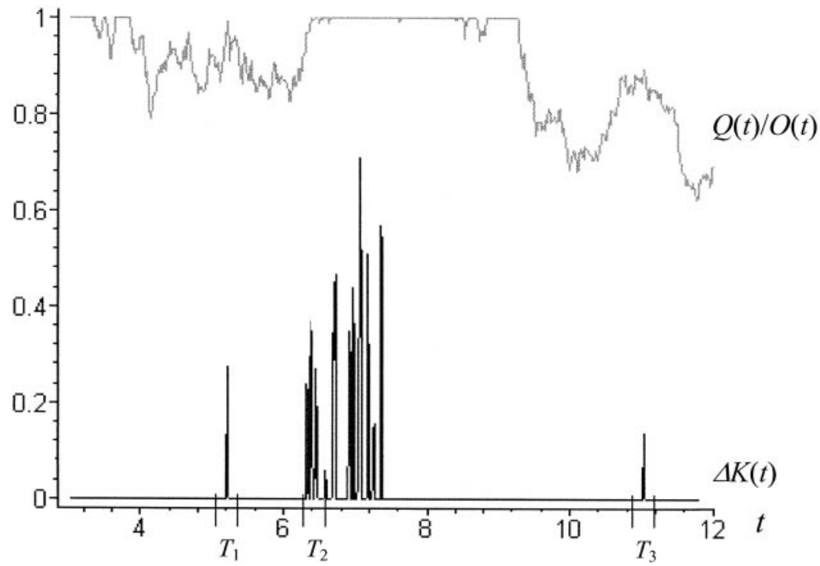

Figure 5
The dynamics of capacity expansion and capacity utilization
This figure shows the paths of capacity expansion and capacity utilization that result from the demand path of Figure 4. $\Delta K(t)$ is the change in committed capacity during the interval $[t, t+\Delta t]$. The operational capacity and output at time t are $O(t)$ and $Q(t)$, respectively, and $Q(t)/O(t)$ is the fraction of completed capacity that is used in production at time t. The flat parts on the path, $Q(t)/O(t)$, correspond to the periods in which all the operational capacity is fully utilized. When completed capacity is fully used $Q(t)/O(t)=1$. During the time intervals T_1, T_2, and T_3, corresponding to [5.21, 5.23], [6.33, 6.41], and [11.04, 11.06], respectively, the firm starts the construction of new capacity when the current operational capacity is not fully utilized. The assumed parameter values are $\mu=0.03$, $\sigma=0.15$, $\gamma=50$, $r=0.06$, $k=1200$, $h=3$, $c_1=200$, $c_2=0.5$, $Y(0)=1000$.

used. Notice that there are periods of capacity expansion in which part of the operational capacity is not used in production. This behavior occurs whenever $Y(t) > Y(K(t))$, and $Y(t) < (2\gamma+c_2)O(t)+c_1$. In Figure 5 this occurs during the intervals T_1, T_2, and T_3. At the beginning of the interval T_2, the firm expands its capacity when 3% of its completed capacity is idle. During T_3 the firm adds more capacity when 11% of its operational capacity is not used. It is worth noting that this result does not rely on the assumed positive trend in the demand process. A similar investment behavior can be obtained for a trendless demand process ($\mu=0$).[12]

The result that the firm may find it optimal to invest in additional capacity when its completed capacity is not fully used contrasts with the dynamics of capacity utilization found in Pindyck (1988). The reason is that Pindyck assumes that new capacity can be used immediately. Hence it is not optimal to add more capacity only to keep it idle for some time. Thus all capacity will be utilized during expansions.

[12] Details available from the author.

With time to build, the decision to invest in additional capacity depends on the expected demand after the construction is completed. Hence, even if current demand is not enough to operate at full capacity, an increase above historic levels induces the firm to add more capacity in anticipation of a stronger future demand.

Therefore, by allowing for flexibility in the use of the operational capacity and a construction lag, the simple model of capacity choice studied in this article offers a rational explanation for the observed behavior of several markets in which new units are built when there is current excess capacity. The electricity supply industry is one example of such markets. Bar-Ilan and Strange (1996) notice that new power plants continue being built even in the presence of excess capacity. However, the model studied in Bar-Ilan and Strange (1996) suggests only a part of the explanation for this phenomenon because their analysis focuses on the effect of construction lags on the timing of an investment in a single project of a given fixed size.

Grenadier (1995a) presents a model that attempts to explain the recurrence of cycles of overbuilding in real estate markets, in which large quantities of newly completed buildings enter the market during periods of already high vacancy rates. This overbuilding phenomenon is different from the behavior discussed in the last paragraph. More precisely, in the context of the model, the overbuilding phenomenon would correspond to new capacity being completed at a time when demand is insufficient to use it. The analysis in Grenadier (1995a) focuses on the probability of overbuilding, which he defines as the probability (as of the beginning of the construction) that a new building will be completed at a time when demand is insufficient to rent a specified fraction of its units. Furthermore, Grenadier (1995a) studies the decision to invest in a single building of fixed capacity and does not address the dynamics of capacity utilization.

The length of the construction lag plays a crucial role in the dynamics of capacity utilization. The top graph in Figure 6 compares the path of capacity expansion when investment is instantaneous with the path of capacity expansion for a three-year construction lag that results from the realization of demand of Figure 4. Under the assumed parameters, the firm holds less capacity when new units can be used immediately. As a result, there will be periods of low demand when capacity is fully utilized. A large drop in demand is required for capacity utilization to fall below 100%. Thus, as can be seen in the middle graph in Figure 6, when investment is instantaneous, capacity will be fully used most of the time. In contrast, for a lag of three years there are fewer periods in which the completed capacity is fully used, as can be seen in the bottom graph of Figure 6. This behavior is consistent with the chronic excess capacity that has been observed in commercial real estate and electricity generation, where construction lags are long.

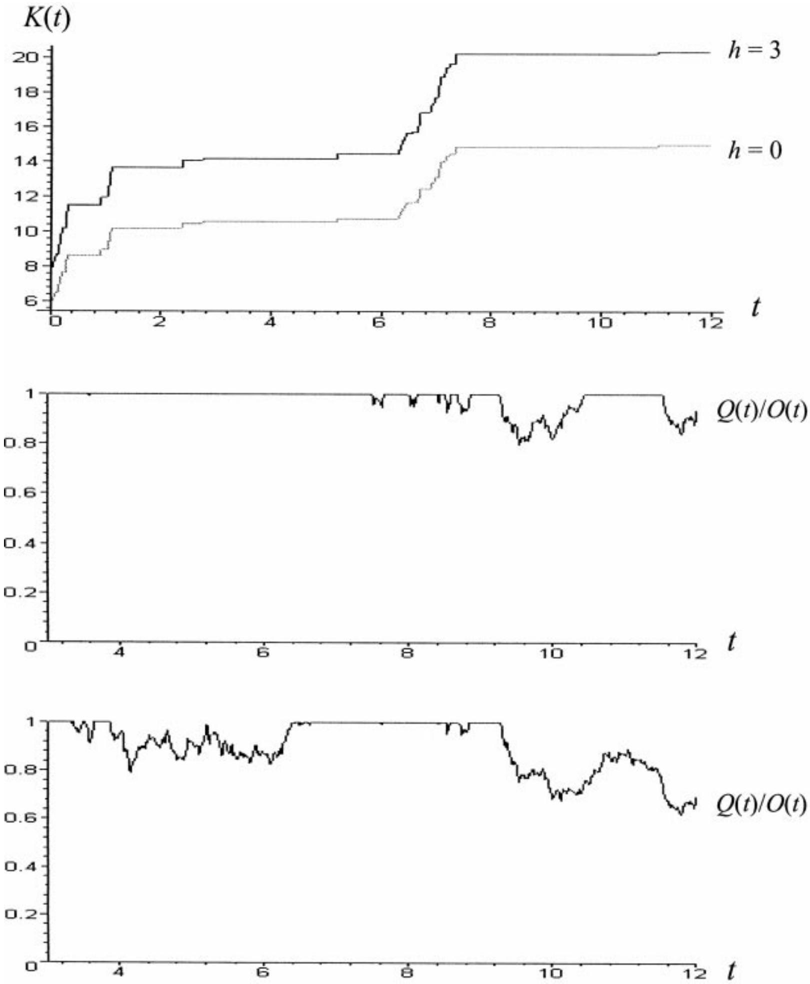

Figure 6
The dynamics of capacity and capacity utilization with and without a construction lag
The graphs in this figure compare the dynamics of capacity expansion and capacity utilization with and without construction lags. The top graph shows the paths of committed capacity, $K(t)$, for $h = 0$ and for $h = 3$. The middle and bottom graphs show the fraction of completed capacity that is used in production, $Q(t)/O(t)$, for $h = 0$ and $h = 3$, respectively. The flat parts on the path, $Q(t)/O(t)$, correspond to the periods in which all the operational capacity is fully utilized. When completed capacity is fully used $Q(t)/O(t) = 1$. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.15$, $\gamma = 50$, $r = 0.06$, $k = 1200$, $h = 3$, $c_1 = 200$, $c_2 = 0.5$, $Y(0) = 1000$.

4. Capacity Choice with Time to Build and Operating Flexibility in Oligopoly

In this section I extend the model of Section 1 to the case of an oligopolistic industry. The main difference from the monopoly case is that, in an oligopoly, the state space for each firm includes not only its capacity level but also the capacity levels of the other firms in the market. Committed capacity is the strategic variable and each firm must condition its capacity choice on the strategies of its competitors.

Consider an industry composed of n firms. For each firm i, the value of an incremental unit of committed capacity is denoted by $\Delta G_i(K_i, K_{-i}, Y, h)$, where K_i is the committed capacity of firm i and $K_{-i} = (K_1, \ldots, K_{i-1}, K_{i+1}, \ldots K_n)$ denotes the strategies of firm i's competitors. The option to invest in an incremental unit of committed capacity is denoted by $\Delta F_i(K_i, K_{-i}, Y)$. Given the current demand, Y, and level of committed capacity of its competitors, K_{-i}, firm i's reaction function, $K_i(Y, K_{-i})$, is its optimal choice of committed capacity. This function is the solution to the system of equations

$$\begin{aligned} \Delta F_i(K_i(Y, K_{-i}), K_{-i}, Y) &= \Delta G_i(K_i(Y, K_{-i}), K_{-i}, Y, h) - k \\ \frac{\partial \Delta F_i(K_i(Y, K_{-i}), K_{-i}, Y)}{\partial Y} &= \frac{\partial \Delta G_i(K_i(Y, K_{-i}), K_{-i}, Y, h)}{\partial Y}. \end{aligned} \tag{20}$$

An n-tuple of strategies $(K_1^*, \ldots, K_n^*)$ is a Nash industry equilibrium if

$$K_i^* = K_i(Y, K_{-i}^*), i = 1, \ldots, n.$$

To determine the instantaneous profit flow of each firm, note that at any time t the firms play a static Cournot game. Each firm chooses its output to maximize its current profit. This choice depends on current demand and it is constrained by the firm's current operational capacity. In addition, each firm must condition its output choice on the output choices of the other firms, which are also constrained by their operational capacity levels. The operational capacity of firm i is denoted by O_i, and $O_{-i} = (O_1, \ldots, O_{i-1}, O_{i+1}, \ldots, O_n)$ summarizes the operational capacity of firm i's competitors. To simplify the analysis I assume that the industry is composed of n identical firms. That is, all firms have the same cost function and investment cost. I also assume that all the firms start with the same initial capacity and thus they all have the same size at any time. Thus our analysis focuses on a symmetric Nash equilibrium.

Let $K(t)$ and $O(t)$ denote the total industry committed and operational capacity, respectively. Since all firms are identical, it follows that $K_i(t) = K(t)/n$ and $O_i(t) = O(t)/n$. Hence, $\Delta G_i(K_i, K_{-i}, Y) = G_i(K, Y)$ and $\Delta F_i(K_i, K_{-i}, Y) = F_i(K, Y)$.

From the assumed demand and cost functions of Equations (1) and (3) we have that at any time t, firm i's profit given the industry operating capacity $O(t)$ is

$$\pi_i(O(t), Y(t)) = \max_{0 \leq q_i \leq O(t)/n}[(Y(t) - \gamma \sum_{j=1}^{n} q_j)q_i - c_1 q_i - 0.5 c_2 q_i^2]. \tag{21}$$

Then it follows from the assumption of symmetric equilibrium that when the industry-committed capacity is K, the additional profit from an incremental unit of capacity at any future time $t \geq h$ is

$$\Delta\pi_i(K, Y(t)) = \max\left[Y(t) - \frac{(n+1)\gamma + c_2}{n}K - c_1, 0\right], \quad i = 1, \ldots, n. \tag{22}$$

This is similar to the marginal profit of Equation (5), the only difference is that the coefficient multiplying the capacity is $[(n+1)\gamma + c_2]/n$ instead of $2\gamma + c_2$. Therefore, to get the solution to the capacity choice problem in oligopoly, we make this substitution in Equations (8), (11), (12), (16), and (17). The capacity choice problem under perfect competition is obtained by taking the limit as n increases to infinity. Thus the properties of the optimal capacity choice that I previously presented in a monopoly context in Section 2 are the same under oligopoly and perfect competition.

I will now analyze the effect of increasing competition on the dynamics of capacity expansion and capacity utilization, and on the behavior of the output price. Although an analytic solution in closed form for the industry-committed capacity is impossible, I can prove several interesting results on the relation between the number of firms in the industry and the dynamics of capacity expansion and capacity utilization. In the analysis that follows K^n, O^n, and Q^n denote the committed capacity, operational capacity, and output of an n-firm industry, respectively. The first result I prove is that the industry capacity is increasing in the number of firms in the market. Comparing Equations (5) and (22), it is easily verified by substitution that for any number of firms n,

$$K^n = \frac{n(2\gamma + c_2)}{(n+1)\gamma + c_2}K^1, \tag{23}$$

and consequently,

$$O^n = \frac{n(2\gamma + c_2)}{(n+1)\gamma + c_2}O^1, \tag{24}$$

thus the capacity of an n-firm industry is equal to the monopoly capacity times an expression that is greater than one, increasing in n, and converges to $2 + c_2/\gamma$ as n increases to infinity.

However, the investment threshold for new capacity, $Y(K^n)$, is independent of the number of firms in the market. To prove this result it suffices to show that $Y(K^n) = Y(K^1)$ for all n. Solving Equations (23) and (24) for K^1 an O^1 and substituting into Equations (8), (11), (12), (16), and (17) proves this result. It is worth comparing this result on the timing of capacity expansion with that of Grenadier (2002). He finds that the investment threshold for additional capacity is decreasing in the number of firms in the industry. The difference arises because Grenadier analyzes the effect of competition on the timing of new investment for a given level of current committed capacity that is independent of the number of firms in the market. We can obtain Grenadier's result by solving Equation (18) for a given level of committed capacity, K, following the substitutions below Equation (22); the resulting investment threshold $Y(K)$ is decreasing in n. Thus the result that increasing competition leads firms to exercise their expansion options sooner in Grenadier (2002) is equivalent to my result that the industry committed capacity is increasing in the number of firms in the market.

Finally, the length of periods at which capacity is fully utilized, and the percentage of capacity utilization, does not vary with the number of firms in the industry. To obtain this result, first notice that given the industry operating capacity, O^n, the output for an n-firm industry is

$$Q^n = \min\left[\frac{n}{(n+1)\gamma + c_2}(Y - c_1), O^n\right]. \tag{25}$$

This follows from Equation (21) and the assumption of symmetry. Then Equations (24) and (25) imply that $Q^n/O^n = Q^1/O^1$ for any n.

Figure 7 plots the paths of capacity expansion for different numbers of firms in the industry that result from the realization of demand in Figure 4. As shown above, committed capacity is increasing in the number of firms in the market, and the times at which new capacity is added are the same regardless of the number of firms. In sum, the properties of the optimal capacity choice and the dynamics of capacity expansion and

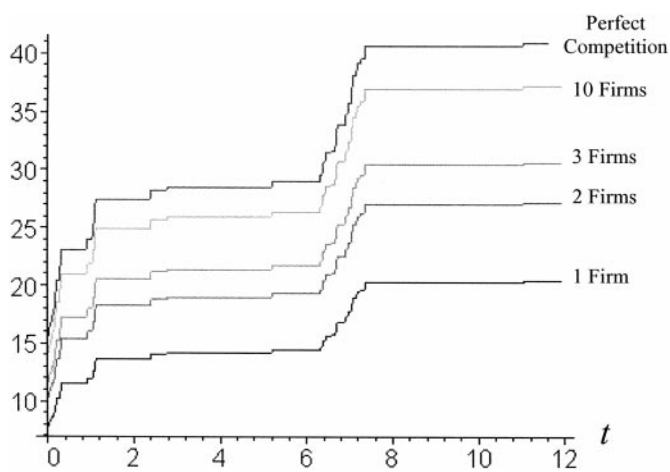

Figure 7
The dynamics of capacity expansion for different numbers of firms in the industry
This figure shows the paths of capacity expansion for a monopoly, a duopoly, a 3-firm oligopoly, a 10-firm oligopoly, and perfect competition that result from the same realization of demand. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.15$, $\gamma = 50$, $r = 0.06$, $k = 1200$, $h = 3$, $c_1 = 200$, $c_2 = 0.5$, $Y(0) = 1000$.

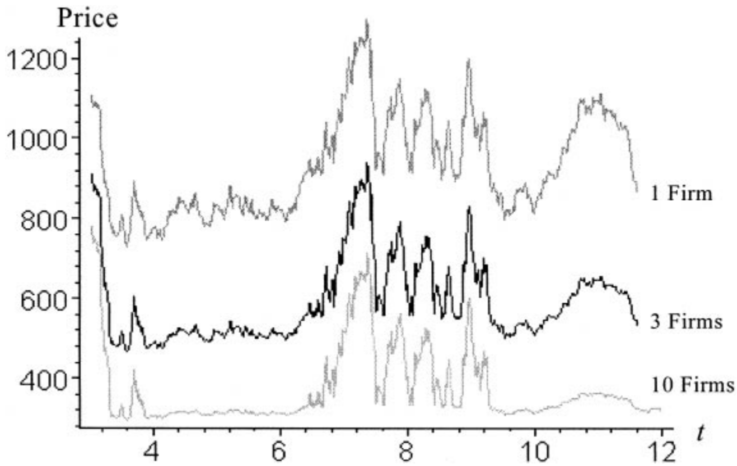

Figure 8
The path of output price for different numbers of firms in the industry
This figure shows the output price paths for a monopoly a 3-firm oligopoly and a 10-firm oligopoly that result from the same realization of demand. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.15$, $\gamma = 50$, $r = 0.06$, $k = 1200$, $h = 3$, $c_1 = 200$, $c_2 = 0.5$, $Y(0) = 1000$.

capacity utilization are qualitatively the same under oligopoly and perfect competition.

In contrast, the properties of the endogenous price process change significantly as the number of firms in the industry increases. Figure 8 shows the price paths for a monopoly, a 3-firm oligopoly, and a 10-firm oligopoly that result from the realization of the demand process of Figure 4. A standard consequence of having a larger number of firms in the market is a reduction in the output price. As can be seen from Figure 8, the price path for 1 firm is completely above the path for 3 firms, which is above the path for 10 firms.

The most remarkable property of the resulting price paths is that during the periods in which the industry's operational capacity is fully utilized the output price can increase to a very high level relative to the price during periods when capacity is not completely used. The relative size of these spikes in the price series gets larger with more firms in the industry. Furthermore, the resulting price series is heteroscedastic with higher (lower) volatility when prices are high (low). This heteroscedasticity is a direct consequence of the dynamics of capacity utilization. More precisely, there are two possibilities or regimes of price volatility, depending on whether or not the current operational capacity is fully used. The difference between the price volatility in these two regimes is increasing in the number of firms in the industry.

Figure 9 displays the paths of percent change in price, $dP(t)/P(t)$, corresponding to the price paths of Figure 8. Each graph in Figure 9

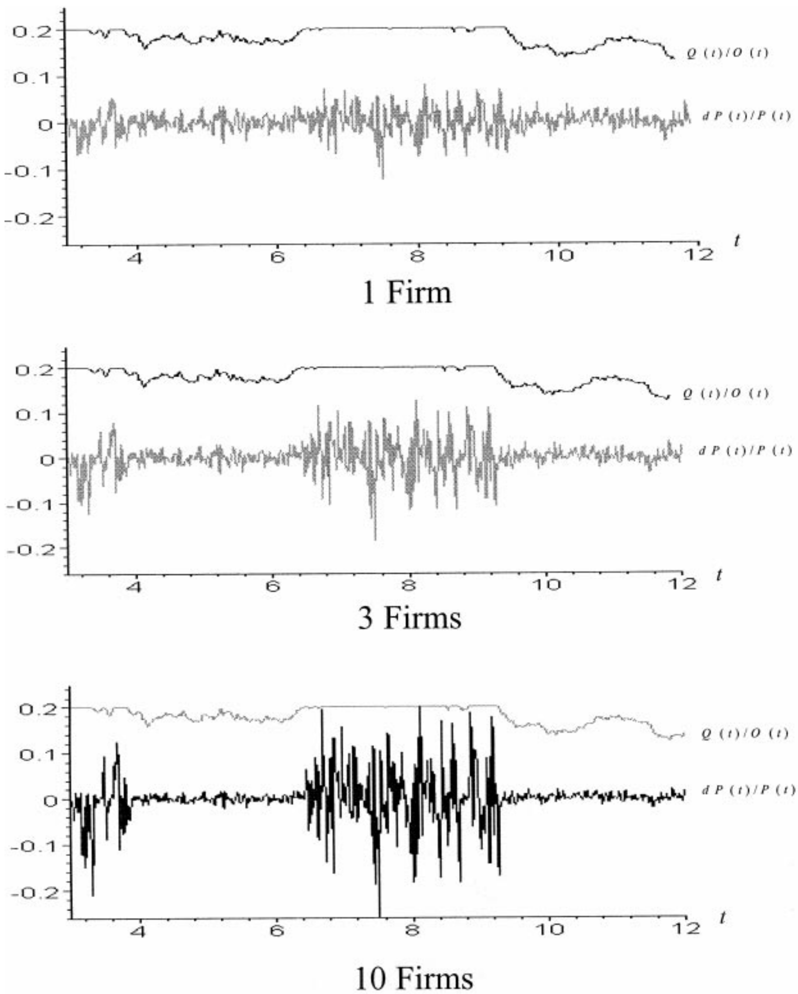

Figure 9
Capacity utilization and the relative changes in output prices
Each graph in this figure shows the path of capacity utilization, $Q(t)/O(t)$, and the change in the relative output price, $dP(t)/P(t)$, that result from the same realization of demand. The paths on the top graph correspond to a monopoly, those on the middle graph are for a 3-firm oligopoly, and the paths on the bottom are for a 10-firm oligopoly. The units on the vertical axis correspond to $dP(t)/P(t)$. The flat parts on the path, $Q(t)/O(t)$, correspond to the periods in which all the operational capacity is fully utilized. The assumed parameter values are $\mu = 0.03$, $\sigma = 0.15$, $\gamma = 50$, $r = 0.06$, $k = 1200$, $h = 3$, $c_1 = 200$, $c_2 = 0.5$, $Y(0) = 1000$.

also includes the path of capacity utilization, $Q(t)/O(t)$. The flat parts in the paths of capacity utilization correspond to the periods in which the industry's capacity is fully used. When capacity is not fully used, the relative variations in price decrease as the number of firms increases. In contrast, when the industry is operating at full capacity, the relative variations in price are larger as the number of firms increases.

To prove the above results with respect to the output price behavior, let P_n be the output price for an n-firm industry. It follows from Equations (1) and (25) that

$$P_n = \max\left[\frac{(\gamma + c_2)Y + n\gamma c_1}{(n+1)\gamma + c_2},\, Y - \gamma O^n\right]. \tag{26}$$

When the operational capacity is enough to satisfy current demand, the effect of a change in the demand shock on the output price is $\partial P_n/\partial Y = (\gamma + c_2)/((n+1)\gamma + c_2)$, which is less than one, decreasing in n, and converges to zero as n approaches infinity. In contrast the effect of a change in the demand shock on the output price when capacity is fully used is $\partial P_n/\partial Y = 1$ for all n.

Thus, as the number of firms in the market increases, the effect of a shock in demand on the output price is dissipated among a larger number of firms when the industry capacity is not fully used. However, when demand is very high and capacity is insufficient, the output price response to a change in the demand parameter is not affected by the degree of competition in the industry. Therefore the difference between the variations in the output price when capacity is fully used and when it is partially used is increasing in the number of firms in the industry.

As illustrated in Figure 8, the endogenous output price paths are characterized by the presence of spikes that occur during the periods in which the industry's operational capacity is fully utilized. The relative size of these spikes in the price series gets larger with more firms in the industry. An expression relating the size of the price spike to the number of firms in the market can be obtained by looking at the difference between the actual output price and the price that would result if there were always sufficient capacity to satisfy the demand. For n firms this difference is

$$P_n - \frac{(\gamma + c_2)Y + n\gamma c_1}{(n+1)\gamma + c_2}, \tag{27}$$

where P_n is given by Equation (26). As n increases to infinity, the difference in Equation (27) converges to $\max[0, Y - \gamma O^c - c_1]$, where O^c is the operating capacity of the perfectly competitive industry. Thus when the industry capacity is fully used, the size of the spike for perfect competition is $Y - \gamma O^c - c_1$. It follows from Equations (24), (26), and (27) that the size of the price spike for n firms is $(n\gamma/((n+1)\gamma + c_2))(Y - \gamma O^c - c_1)$, which is increasing in n.

The existence of rare but violent explosions in prices is a stylized fact of commodity prices [see Williams and Wright (1991) and Deaton and

Laroque (1992)]. Commodity price series are characterized by the presence of occasional spikes. In our model these spikes occur when the completed capacity is fully used. Thus the frequency of these spikes depends on the dynamics of capacity utilization, which are affected by the length of the investment lag. As was discussed in Section 3, the frequency of the periods in which capacity is fully used may be decreasing in the length of the construction lag. Hence the sudden increases in output price may be rare for longer lags.

The properties of the commodity price series have been extensively analyzed within the context of models of competitive storage [e.g., Williams and Wright (1991) and Deaton and Laroque (1992)]. In essence, these models attribute the observed asymmetry in price changes - increases in price, while rarer, are larger than the typical price decreases — to the fact that storage itself is asymmetric. Low prices are associated with a large stockpile able to buffer price changes, while very high prices are associated with the stockpile being empty and unable to buffer price changes.

My model does not consider storage because we assume that consumption equals industry output in every period. Therefore the analysis would apply to commodities for which storage is relatively expensive. One such commodity is electricity. Thus my model complements the models of competitive storage cited above.

It is worth noting that sudden price increases occur in competitive electricity markets. In particular, during the week of June 22–26, 1998, the wholesale electricity markets in the midwestern United States experienced a dramatic price increase. A study by the Federal Energy Regulatory Commission [FERC (1998)] highlighted insufficient generating and transmission capacity among the main causes of this price spike. In addition, the same study concludes that although a number of companies have announced plans for some new generation plants in the Midwest, the operational conditions that led to the June 1998 price spike are likely to recur. The reason cited is that the minimum lead time for installing new capacity is at least one year (for natural gas-fired combustion turbines). For many larger plants, the lead time will be much longer. Thus the model implications appear to conform to actual behavior in wholesale electricity markets.

The result that output price volatility is increasing in the number of firms in the industry is consistent with the cross-country evidence on electricity prices of Wolak (2000). He studies the behavior of spot electricity prices in England and Wales, Norway, the state of Victoria in Australia, and New Zealand. Wolak finds that electricity supply industries with a larger component of private participation in the generation market tend to have more volatile prices. It is particularly interesting how the recent behavior of the Victoria power market conforms to my prediction on the relation between output price behavior and the number of firms in the industry. Restructuring and privatization of the electricity supply industry in the state of Victoria in 1994 took place at the power station level. Each power station was formed into a separate entity to be sold. Wolak reports that entry by new generators and changes in firm ownership as more new generating companies have been privatized has led to an increase in volatility of Victoria Power Exchange prices. Wolak suggests that part of this price volatility may be explained by attempts to exercise market power. My model shows that the observed behavior of electricity prices in competitive markets can be explained by the interaction of time to build, operating flexibility, and capacity choice.

5. Relation to Previous Research on Competitive Investment Decisions

In this section I compare my findings with other recent articles that study irreversible investment and industry equilibrium. Baldursson (1998) studies an oligopoly where firms face a linear demand curve similar to the one in my model. Firms use capacity as their strategic variable. Baldursson assumes that investment is instantaneous and that the installed capacity is always fully utilized so production equals capacity at each instant. It follows from these assumptions that more uncertainty induces the firms to hold less capacity and delays the decision to expand their installed capacity. Furthermore, the examples presented in Baldursson (1998) indicate that the qualitative nature of the price process will be the same in oligopoly and in competitive equilibrium. In contrast, by allowing for a construction lag and flexibility in the use of the installed capacity, my model of industry equilibrium generates more interesting implications about the industry effect on investment and output price.

The option that allows the firms to let part of their completed capacity go unused when demand is low is crucial in obtaining the industry equilibrium results. Without any operational flexibility, the net effect of uncertainty on industry investment is always negative, even with construction lags. The model studied in Grenadier (2000) illustrates this point. Grenadier derives a dynamic competitive equilibrium for a market subject to time to build. Grenadier assumes that all the industry capacity is always fully utilized and derives the firms' investment rule in closed form. The resulting investment threshold is increasing in demand volatility, and therefore more uncertainty decreases the level of committed capacity. Furthermore, the output price properties derived in Grenadier (2000) for a competitive equilibrium with time to build are identical for oligopoly, as demonstrated in Grenadier (2002).

In the latter article, Grenadier develops an approach to solving for the equilibrium investment strategies in oligopoly that is applicable to more general specifications for the inverse demand function and the stochastic process for the demand shock. The focus of Grenadier (2002) is on the effect of competition on the option value of waiting to invest. Grenadier shows that increased competition leads firms to exercise their options to invest in additional capacity sooner. As discussed above, this result is equivalent to my result that the industry committed capacity is increasing in the number of firms in the market.

Finally, I mention above that increased competition does not affect the comparative statics of the optimal capacity choice that were presented in Section 2 in a monopoly context. In particular, I find that more uncertainty may encourage firms to hold more capacity. I explained that the ability to vary output by not using the marginal unit of capacity at times when demand is low is essential to generating this result. Our findings, that the encouraging effects of uncertainty on investment are robust to increased competition, contrast to those of Pindyck (1993) on competitive investment under uncertainty. Pindyck shows that competition has a depressing effect on investment, even when the marginal product of capital is increasing in uncertainty. In the example presented by Pindyck, the marginal product of capital is convex in price, which is affected by a stochastic demand shift factor. By Jensen's inequality, increased uncertainty about future demand increases the expected marginal product of capital and hence increases the incentive to invest. However, when demand increases, existing firms will expand or new firms will enter until the market clears. The increased supply limits any price increase under good demand outcomes. Furthermore, since investment is irreversible, it is impossible to prevent the price from falling under bad demand outcomes by reducing capital. Therefore the equilibrium response of firms offsets the increases in the value of a unit of capital that results from the convexity of the marginal profit function. Pindyck presents a two-period example in which investment is left unchanged by a mean-preserving spread in the demand shift variable. When the number of periods is more than two, the initial investment is lower when future demand is more uncertain.

Contrary to the findings of Pindyck (1993), competition does not affect the net effect of uncertainty on irreversible investment in my model. The marginal product of capital is convex in both models, the difference in results arises because the production technologies are different. Pindyck assumes a Cobb–Douglas production function.[13] This technology allows the substitutability of capital with other factors or inputs, such as labor for example. Thus, at any time, the installed capacity is always fully used, and the firm changes its output level by varying other production factors to maximize its instantaneous profit. This ensures that the marginal product

[13] Pindyck's purpose is to show that the encouraging effect of uncertainty on investment found by Abel (1983) and Caballero (1991) is not robust to increased competition. These two article assume a Cobb–Douglas production function.

of capital is convex in the stochastic demand factor. In contrast, capital is the only factor in production in my model, and the marginal product is convex because the firm can vary output by not using the marginal unit of capital when demand falls. More uncertainty increases the likelihood of extreme demand outcomes at the time of completion. New entry offsets any increase in the marginal profits under good demand outcomes, but the option to not utilize the marginal unit when completed prevents the marginal profits from falling below zero under bad demand outcomes. Thus it follows from the interaction of irreversibility and flexibility in capacity use that increased competition does not change our results on the effect of uncertainty on investment.

6. Conclusion

This article examines the effect of competitive interactions among firms in a given industry on investment decisions and on the price of a nonstorable commodity. Unlike the vast majority of the articles on the strategic exercise of real options that focus on the effect of competition on the value of the option to invest, I also analyze the effect of operating flexibility once the investment is completed. Specifically, this article presents a model of investment under uncertainty that includes time to build, capacity choice, and flexibility in the use of the installed capacity, and considers the effect of competition. This model provides new insights into the effects of uncertainty on capacity choice and on the dynamics of capacity expansion and capacity utilization that are not available in previous models that do not incorporate one or more of these four realistic features.

The main results are the following. First, more uncertainty may encourage firms to hold more capacity. Similarly the optimal capacity choice may be increasing in the length of the construction lag. This implies that a longer lag may reduce the frequency of times in which the operational capacity is fully used. Second, firms may find it optimal to invest in additional capacity, even during times in which their current capacity is not fully used. These optimal responses by firms are consistent with the actual behavior of capacity utilization and capacity expansion in real estate and electricity generation. Finally, there are two regimes for the changes in the output price depending on whether or not the operational capacity is fully used and the difference between the price volatility in these two regimes increases as the industry becomes more competitive.

Appendix A

This appendix derives the value of a unit of capacity under construction, $\Delta G(K, Y, \theta)$. I follow the procedure outlined in the appendix of Bar-Ilan and Strange (1996).

The expectation in Equation (10) is

$$E[\Delta H(K(t), Y(t+\theta))] = \int_w^\infty \left(A(K(t))(Y(t+\theta))^\alpha + \frac{Y(t+\theta)}{r-\mu} - \frac{w}{r}\right) f(Y(t+\theta))dY(t+\theta)$$
$$+ \int_0^w \left(B(K(t))(Y(t+\theta))^\beta\right) f(Y(t+\theta))dY(t+\theta), \tag{28}$$

where $w = (2\gamma + c_2)K(t) + c_1$ and $f(Y(t+\theta))$ denotes the probability density function (pdf) of the demand at time $t+\theta$ when the construction is completed, given the current demand $Y(t)$.

Since Y follows the geometric Brownian motion process specified in Equation (2), the distribution of future demand $Y(t+\theta)$, given the current demand $Y(t)$, is lognormal

$$\log Y(t+\theta) \approx N\left(\log Y(t) + \left(\mu - \frac{\sigma^2}{2}\right)\theta, \sigma^2\theta\right). \tag{29}$$

The solution for the expected value of Equation (28) uses the following two properties of the lognormal distribution. [See note 11 in Bar-Ilan and Strange (1996) for references].

(i) If $z = \log x$ is distributed $N(g, s^2)$, then the uth moment of x around zero is

$$E(x^u) = e^{ug + u^2 s^2/2}. \tag{30}$$

(ii) When z is truncated below $\log x_0$, then the uth moment of x around zero is

$$E(x^u) = \frac{1 - \Phi(v - us)}{1 - \Phi(v)} e^{ug + u^2 s^2/2}, \tag{31}$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution and v is defined as

$$v = \frac{1}{s}(\log x_0 - g). \tag{32}$$

The derivation of the value of the incremental unit of capacity when the time remaining to its completion is uses the above properties of the lognormal distribution with

$$g = \log Y(t) + \left(\mu - \frac{\sigma^2}{2}\right)\theta, \quad s = \sigma\sqrt{\theta}, \quad \text{and} \quad x_0 = w.$$

Equation (31) implies

$$\int_w^\infty (Y(t+\theta))^u f(Y(t+\theta))dY(t+\theta) = (1 - \Phi(v - u\sigma\sqrt{\theta}))E[(Y(t+\theta))^u] \tag{33}$$

$$\int_0^w (Y(t+\theta))^u f(Y(t+\theta))dY(t+\theta) = \Phi(v - u\sigma\sqrt{\theta})E[(Y(t+\theta))^u]. \tag{34}$$

Therefore Equations (6), (28), (33), and (34) imply

$$\Delta G(K(t), Y(t), \theta) = e^{-r\theta}\Big\{A(K)(1 - \Phi(v - \alpha\sigma\sqrt{\theta}))E[Y^\alpha(t+\theta)]$$
$$+ (1 - \Phi(v - \sigma\sqrt{\theta}))\frac{E[Y(t+\theta)]}{r-\mu}$$
$$- (1 - \Phi(v))\frac{w}{r} + B(K)\Phi(v - \beta\sigma\sqrt{\theta})E[Y^\beta(t+\theta)]\Big\}, \tag{35}$$

Equation (30) implies

$$E[Y^\alpha(t+\theta)] = Y^\alpha(t)e^{[\alpha(\mu-\sigma^2/2)+\alpha^2\sigma^2/2]\theta}$$
$$E[Y^\beta(t+\theta)] = Y^\beta(t)e^{[\beta(\mu-\sigma^2/2)+\beta^2\sigma^2/2]\theta}$$
$$E[Y(t+\theta)] = Y(t)e^{\mu\theta}. \tag{36}$$

Since α and β are the roots of the characteristic [Equation (7)], we have

$$\begin{aligned} E[Y^{\alpha}(t+\theta)] &= Y^{\alpha}(t)e^{r\theta} \\ E[Y^{\beta}(t+\theta)] &= Y^{\beta}(t)e^{r\theta} \\ E[Y(t+\theta)] &= Y(t)e^{\mu\theta}. \end{aligned} \tag{37}$$

Combining Equations (35) and (37), we get the expression for the value of the incremental unit under construction,

$$\begin{aligned} \Delta G(K, Y, \theta) = {} & (1 - \Phi(v - \alpha\sigma\sqrt{\theta}))A(K)Y^{\alpha} + (1 - \Phi(v - \sigma\sqrt{\theta}))\frac{Ye^{-(r-\mu)\theta}}{r-\mu} \\ & - (1 - \Phi(v))\frac{[(2\gamma + c_2)K + c_1]e^{-r\theta}}{r} + \Phi(v - \beta\sigma\sqrt{\theta})B(K)Y^{\beta}, \end{aligned} \tag{38}$$

where

$$v = v(K, Y, \theta) = \frac{\log[(2\gamma + c_2)K + c_1] - \log Y - (\mu - \sigma^2/2)\theta}{\sigma\sqrt{\theta}}. \tag{39}$$

Appendix B

The investment rule for additional capacity and the optimal capacity choice are the solutions to Equations (18) and (19), respectively. These equations do not admit a closed-form solution for all parameter values. However, there are two benchmark cases for which we can characterize analytically the solution to each of these equations and their properties.

The first case of interest is when demand is certain, $\sigma = 0$, and $\mu \geq 0$. The benefit of delaying is growing if $\mu > 0$, while the cost of investing is constant, therefore $\Delta F(K, Y) > 0$ for all K. The value of the option to invest if exercised at T is

$$\Delta F(K, Y) = \left[\left(\frac{Ye^{\mu(T+h)}}{r-\mu} - \frac{(2\gamma + c_2)K + c_1}{r}\right)e^{-rh} - k\right]e^{-rT}. \tag{40}$$

The optimal exercise time T^* solves

$$\frac{\partial \Delta F(K, Y)}{\partial T} = -Ye^{-(r-\mu)(T^*+h)} + [(2\gamma + c_2)K + c_1]e^{-r(T^*+h)} + rhe^{-rT^*} = 0. \tag{41}$$

To find the value of Y for which it is optimal to invest immediately, we set $T^* = 0$; this yields

$$Y(K) = [(2\gamma + c_2)K + c_1 + rke^{rh}]e^{-\mu h}. \tag{42}$$

This implies the following path of committed capacity,

$$K(t) = \max\left\{\frac{Y(t)e^{\mu h} - c_1 - rke^{rh}}{2\gamma + c_2}, 0\right\}. \tag{43}$$

Since $Y(t) = Y(t-h)\, e^{\mu h}$ and $O(t) = K(t-h)$, the path of operational capacity is

$$O(t) = \max\left\{\frac{Y(t) - c_1 - rke^{rh}}{2\gamma + c_2}, 0\right\}. \tag{44}$$

Solving for Y in Equation (44), we have $Y(t) > (2\gamma + c_2)O(t) + c_1$. In particular, $Y(K) > (2\gamma + c_2)O(t) + c_1$ for all K and O. Thus expansion always occurs when the completed capacity is fully used. With no demand uncertainty, future demand can be perfectly forecasted so that each marginal unit of capacity is always used when completed.

The second special case is when the firm does not have the option to reduce output if the level of demand decreases. Since installed capacity is always fully utilized, the value of an incremental unit of capacity when $0 < \theta \leq h$ is the remaining construction time is

$$\Delta G(K, Y, \theta) = \frac{Ye^{-(r-\mu)\theta}}{r-\mu} - \frac{[(2\gamma + c_2)K + c_1]e^{-r\theta}}{r}. \tag{45}$$

The value of the investment threshold is

$$Y(K) = \frac{\beta}{\beta - 1}\frac{r-\mu}{r}\left[(2\gamma + c_2)K + c_1 + rke^{rh}\right]e^{-\mu h}. \tag{46}$$

In this case, a lag between the addition of an incremental unit of capacity and its completion does not change the effect of uncertainty on investment when the firm has to use all its operating capacity. More demand uncertainty raises $\beta/(\beta - 1)$, and thus always raises $Y(K)$. Notice also that $Y(K)$ is equal to the certainty trigger times a factor $\beta(r-\mu)/(\beta-1)r$, which is greater than one for all parameters values. Therefore, when the firm does not have the option to reduce output, the investment triggers is always greater than the certainty trigger.

By solving for K in Equation (46) we get the firm optimal capacity given the current value of the demand shock,

$$K(Y) = \max\left\{\frac{r}{2\gamma + c_2}\left[\frac{Ye^{\mu h}}{r-\mu}\left(1 - \frac{1}{\beta}\right) - ke^{rh} - \frac{c_1}{r}\right], 0\right\}. \tag{47}$$

More demand uncertainty reduces β. This implies that the firm's optimal capacity is decreasing in demand uncertainty. Given a demand path $Y(t)$, Equation (47) gives the firm's desired committed capacity, $K(Y(t))$. Because investment is irreversible, the path of committed capacity, $K(t)$, is such that $K(t) \geq K(Y(t))$ for all t. Furthermore, since we do not allow for depreciation of the installed capacity, investment occurs only when demand reaches a new high. This implies that the firm's path of optimal committed capacity is

$$K(t) = \max\left\{\frac{r}{2\gamma + c_2}\left[\frac{e^{\mu h}}{r-\mu}\left(1 - \frac{1}{\beta}\right)\sup\{Y(s), 0 \leq s \leq t\} - ke^{rh} - \frac{c_1}{r}\right], K(0)\right\}. \tag{48}$$

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