# Financial Innovation and Information: The Role of Derivatives When a Market for Information Exists

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We study the effects of financial innovation in a model of endogenous information acquisition. We determine the conditions under which the introduction of a derivative written on an existing stock increases or decreases the incentive to purchase information. We show that financial innovation produces some effects which hold across informational structures and others which differ. The former coincide with the few empirical results that are robust in the literature (effects on prices, risk premia, and volatility), while the latter coincide with the ones that differ experiment by experiment (effects on volume, correlation between volume and volatility, and market informational efficiency).

Traditional models in finance have a hard time explaining the empirical findings on financial innovation. The effects of the introduction of the same type of contract (such as a futures or an option), written on the same financial asset (e.g., stocks or bonds), can be very different, depending on the financial market into which the new asset has been introduced (e.g., over-the-counter or regulated stock exchange) and on the timing of its introduction. While the empirical studies in general find a decrease in risk premia and volatility and an increase in asset prices, the results in terms of trading volume, correlation between volume and volatility, and degree of market efficiency are mixed and often contradictory [Edwards (1988a, b), Harris (1989), Damodaran (1990), Damodaram and Lim (1991)]. A reduction in the volatility of the underlying asset is accompanied in some cases by an increase in trading, but by a decrease in trading in other cases. Furthermore, the introduction of a derivative affects the forecastability of asset returns, and thus also changes the degree of market efficiency.

We try to explain these results by showing how the effects of the introduction of a derivative depend both on the market informational structure

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and on the existence of a market for information. We consider two scenarios that proxy for alternative informational structures. In each scenario there are two types of agents, one of whom is initially uninformed but is allowed to purchase information. The two scenarios differ in terms of the nature of the other class of agents in the economy. In the first scenario, this class is also uninformed, whereas in the second, this class starts off better informed. Thus, in the first case, there is initially low informational asymmetry and some agents may want to purchase additional information to gain an informational advantage. In the second case, information is initially concentrated in the hands of a few (high informational asymmetry) and the less informed agents may be willing to buy information simply to reduce their informational disadvantage. Thus the demand for information will depend on the informational structure of the market. At the same time, the amount of information purchased determines, in equilibrium, the informational structure of the market itself and asset equilibrium conditions. This implies that a change in the amount of information affects prices differently, depending on whether information is purchased to reduce or increase an information asymmetry.

We show that financial innovation, by changing the incentive to purchase information and inducing a reallocation of the risky asset among classes of differentially informed agents, produces some effects which are the same in both scenarios as well as some that differ. The effects that do not change with the information structure coincide with the few empirical results that are robust in the literature (effects on prices, risk premia, and volatility), while the ones that change coincide with the ones that differ experiment by experiment (effects on volume, correlation between volume and volatility). We consider, for the first time to our knowledge, endogenous information acquisition in a continuous-time setting where there exist both a stock and a derivative asset and we describe the relationship between financial innovation and market informational structure. We generate patterns in empirically observable variables (prices, volatilities, trading volume, correlation between volume and volatility, asset returns autocorrelation) that are consistent with stylized facts.

There is a large literature that analyzes the effects of introducing a derivative in incomplete markets, depending on the particular type of market incompleteness. In the case of allocational incompleteness, the introduction of the derivative asset is shown to improve the allocational spanning of the economy [Ross (1976), Detemple and Selden (1986), Geanakoplos (1990)]. In the case of informational incompleteness, the derivative has been analyzed as an additional signal [Grossman (1977, 1988), Back (1993), Huang and Wang (1997)] or as a focal point capable of coordinating people's expectations [Guesnerie and Rochet (1993), Brennan and Cao (1996)]. In this article, we also consider informational incompleteness, but we endeavor to endogenize the decision to purchase information. One limitation of many existing models, even ones which explicitly describe the learning process of the investors, is that they generally consider agents' information sets as exogenously given. Alternatively, the learning process is modeled as passive filtering based on the direct observation of market signals, such as prices and dividends.

The articles most closely related to this research are those by Grossman (1977) and Cao (1999). Both authors look at the interaction between the introduction of a derivative and the incentive to purchase information, but they reach contradicting results. In a standard two-period model, Grossman shows that the introduction of a future always reduces the incentive to purchase information. The intuition is that, once the new asset is traded, agents are less willing to incur a cost in order to observe an outside signal, as they can observe it for free by directly looking at the price of the futures contract. Cao, on the other hand, finds that the introduction of an option-type contract will always raise the incentive to purchase information. As agents purchase more information, the riskiness of the assets falls, their prices rise, and therefore the wealth and utility of the agents increases.

In both Grossman's and Cao's models, the effects due to the change in the level of informational asymmetry are not analyzed and no spillover of information to the less informed agents is considered. Also, information is purchased only for the purpose of gaining an informational advantage, while the case where information is purchased to reduce the informational disadvantage vis-à-vis more informed agents is not considered. Furthermore, given that these models are static in nature, they do not account for the intertemporal hedging of learning risk.[1] That is, the dynamic effects generated by a change in agents' perception of risk and their desire to ensure against it. These effects strongly affect agents' portfolio allocations and the determination of asset pricing in equilibrium [Brennan (1998), Brennan and Xia (1997), Lewellen and Shanken (1998)].

Our analysis here involves a fully dynamic setup where information is purchased at each point in time and the individual pays a fee per unit of time during which the signal is acquired.[2] The decision to purchase information can be thought of as, say, setting up a research department or subscribing to a newsletter. Information is short-lived, and continuously updated. Unlike a standard two-period model, the value of the assets is never fully revealed. The purchase of information reduces the amount of resources available to the investor, directly affecting his portfolio/consumption decision. While this

[1] While Grossman's model is explicitly static, Cao's is based on a series of rounds of trade. The private signal does not change over time, while public signals are received at each round of trade. This implies that the intrinsic value of the private signal necessarily decreases over time, regardless of agents' actions. The value of information is, therefore, not determined at equilibrium on the basis of the contingent evolution of the economy.

[2] This makes our model different from Cao's (1999). In his model the signal is acquired only once, before the first round of trading starts. This implies that its value is not linked to the evolution of the economy. Therefore dynamic portfolio rebalancing decisions will not be affected by such information. Furthermore, in the presence of exogenously imposed public informational arrival, its value must decrease over time.

adds to the complexity of the problem, it also enriches the model, as at each point of time the portfolio rebalancing decision is taken on the basis of a new flow of information. Indeed, the price of the assets and their intrinsic values change over time depending on the evolution of the economy and on the newly acquired information. The amount of information existing in the market and the way it is disseminated among agents matters and has to be properly incorporated, in the same way as agents' attitudes toward risk and the level of fundamental riskiness are accounted for in the standard models. To our knowledge, only Detemple and Kihlstrom (1987), in a single-agent framework, explicitly deal with the problem of optimal intertemporal purchase of information, but they do not explore its equilibrium implications.

We are able to show conditions under which the introduction of a derivative increases the incentive to purchase information and conditions under which it reduces it. Grossman's and Cao's results become specific subcases of our setup. We argue that the change in the incentive to purchase information also modifies the effects of the introduction of a derivative on the degree of informational efficiency of the market. The effects depend on whether there is a lot of information concentrated among few people or there is less information, but it is better disseminated across market participants.

The structure of the article is dictated by the two crucial "ingredients" that affect our results: the market informational structure and the purchase of information. First, we demonstrate how conditioning on different informational structures helps to explain the effects due to financial innovation. Then we show the differential effects induced by allowing agents to optimally choose the information they want to purchase. The model is solved through simulations.

# 1. The Model

## 1.1 The economy

***Assumption 1.*** *We assume an exchange economy, as in Wang* (1993), *where an exogenously given production process yields a real stream of dividends D. The stocks* ($S$) *represent a claim to this technology that agents can freely trade. Dividends are a function of the underlying productivity process* ($\pi$) *as well as of some specific shock. The former represents the systematic component, while the latter is the idiosyncratic part. The dividend process can be represented as*

$$dD = (\beta\pi - kD)\,dt + b_D\,d\mathbf{z}, \tag{1}$$

*where the state variable $\pi$ follows an Ornstein Uhlenbeck process,*

$$d\pi = a_\pi(\bar{\pi} - \pi)\,dt + b_\pi\,d\mathbf{z}, \tag{2}$$

*where* $\mathbf{b}_D = (\sigma_D, 0, 0, 0, 0, 0, 0)$, $\mathbf{b}_\pi = (0, \sigma_\pi, 0, 0, 0, 0, 0)$, *and* $\mathbf{z}$ *is a seven-element vector process defined below. Agents can borrow and lend at an exogenously determined interest rate* $r$.

***Assumption 2.*** *We first consider an economy where the only available asset is the stock (with a price* $P_S$*). We then treat the case where a derivative (with price* $P_F$*) is introduced. Both assets are infinitely divisible. The stock is in positive net supply, with a stationary long-run supply level normalized to* 1, *while the derivative is in zero net supply. For both assets, there are short-run deviations, due to noise trading. We assume two sources of noise trade (*$\theta_1$ *and* $\theta_2$*) represented by the Ornstein Uhlenbeck processes,*

$$d\boldsymbol{\theta} = -\mathbf{a}_\theta \boldsymbol{\theta}\, dt + \mathbf{b}_\theta\, d\mathbf{z}, \tag{3}$$

*where* $\mathbf{a}_\theta = (\mathbf{a}_1, \mathbf{a}_2)'$, $\mathbf{b}_\theta = (\mathbf{b}_{\theta_1}, \mathbf{b}_{\theta_2})'$,[3] *and* $\boldsymbol{\theta} = (\theta_1, \theta_2)'$. *We can identify them as discretionary and nondiscretionary noise trading, or simply as noise trading coming from different classes of investors. This assumption keeps the market from being fully revealing, even after the derivative has been introduced, without the need to introduce an additional source of uncertainty linked to the new asset. To keep the notation more compact, we define the vector of unknown state variables as* $\boldsymbol{\nu} = (\pi, \boldsymbol{\theta})$. *The Brownian process* $\mathbf{z}$ *is a seven-element vector,* $\mathbf{z} = (z_D, z_\pi, z_{\theta_1}, z_{\theta_2}, z_{\varepsilon_\pi}, z_{\varepsilon_{\theta_1}}, z_{\varepsilon_{\theta_2}})$, *where* $\varepsilon$ *refers to the errors specific to each signal and is defined below. We assume, without loss of generality, that the stochastic components of* $d\mathbf{z}$ *are not correlated with each other.*

***Assumption 3.*** *Agents differ in terms of their information sets. The generic agent* $j$ *is endowed with an exponential utility function,* $U^j(c^j(s), s) = -e^{-\zeta^j s - \gamma^j c^j(s)}$, *where* $\gamma^j$ *and* $\zeta^j$ *are, respectively, the degree of risk aversion and rate of time preference and* $c(t)$ *is consumption at time* $t$.

***Assumption 4.*** *We consider three classes of agents: "fully informed agents," "information purchasing agents," and "fully uninformed agents," identified, respectively, with the superscripts* $f$, $i$, *and* $u$. *We use the superscript* $j$ *whenever we refer to a generic agent, with no need to specify the type. The fully informed agents have full knowledge (knowing* $\boldsymbol{\nu}$*). All other agents do not have full knowledge and use the available signals to make inferences. While the fully uninformed agents can only observe market signals (dividends and asset prices), the information purchasing agents can also purchase "outside*

[3] We have that $\mathbf{a}_1 = (a_{\theta_1}, 0)$ and $\mathbf{a}_2 = (0, a_{\theta_2})$ and the vectors of the diffusion coefficients are $\mathbf{b}_{\theta_1} = (0, 0, \sigma_{\theta_1}, 0, 0, 0, 0)$ and $\mathbf{b}_{\theta_2} = (0, 0, 0, \sigma_{\theta_2}, 0, 0, 0)$. It is worth noting that the two sources of noise trading are not "asset specific." This implies that the introduction of the derivative does not change the total amount of noise trading in the economy, but only modifies the way it is allocated among the different assets. We thank M. Brennan and J. Ingersoll for suggesting this way of modeling noise trading.

*signals" on both the main underlying process* ($\boldsymbol{\pi}$) *and the stochastic asset supply* ($\boldsymbol{\theta}$). *The vector of signal* ($\boldsymbol{\varphi} = (\varphi_\pi, \varphi_{\theta_1}, \varphi_{\theta_2})$) *follows*

$$d\boldsymbol{\varphi} = \boldsymbol{\xi} d\boldsymbol{\nu} + d\boldsymbol{\varepsilon}, \tag{4}$$

*where* $\boldsymbol{\xi} = (\boldsymbol{\xi}_\pi, \boldsymbol{\xi}_{\theta_1}, \boldsymbol{\xi}_{\theta_2})$. *That is, the outside signals are unbiased predictors of the underlying unknown sources of uncertainty* ($\boldsymbol{\nu}$), *subject to a set of normally distributed random sources of noise* $\varepsilon$ *which follow*

$$d\boldsymbol{\varepsilon} = \frac{\mathbf{b}_\varepsilon}{\sqrt{\Upsilon}} d\mathbf{z}, \tag{5}$$

*where* $\mathbf{b}_\varepsilon = (\mathbf{b}_{\varepsilon_\pi}, \ \mathbf{b}_{\varepsilon_{\theta_1}}, \ \mathbf{b}_{\varepsilon_{\theta_2}})$. *The quantity of information purchased* ($\Upsilon$) *reduces the noise of the signal. The decision to purchase information can be interpreted as setting up a research department or subscribing to a newsletter where, in each period, the purchaser pays a fee to observe a signal.*

This analysis would also carry through with just one signal on $\pi$. Indeed, this approach encompasses more limited information acquisition settings which can be modeled as limiting cases as $\mathbf{b}_{\varepsilon_{\theta_1}}$ and $\mathbf{b}_{\varepsilon_{\theta_2}}$ go to infinity. Nevertheless, we think that the present, more encompassing, specification better represents the fact that research departments in many financial institutions also try to forecast liquidity shocks and that new companies have been set up to provide information on them (newsletters and flows reports).[4] Moreover, information on liquidity trading can be more relevant than information on fundamentals [Ito, Lyons, and Melvin (1998), Cao and Lyons (1999)].

The type of agents and their degrees of informativeness define the informational structure of the market. We use them to characterize two alternative scenarios. In the first scenario, there are two classes of agents (information purchasing agents and fully uninformed agents). Both are initially uninformed and then one of them is allowed to purchase a set of signals. In the second scenario, a strong asymmetry of information exists from the start. There are two classes of agents: one class is fully informed, while the second can only filter information from prices and dividends. The uninformed also have the possibility of purchasing a set of signals to reduce their informational disadvantage. In both scenarios we consider a continuum of agents and $\omega$ represents the mass of information purchasing ones.

The structure of the economy is common knowledge to all the agents and the information structure is hierarchically nested.

[4] For example, consider the services provided by companies such as Trimtabs or AMGData, which sell data on the daily net inflows into mutual funds for commercial forecasting purposes.

### 1.2 Rational expectations equilibrium

***Proposition.*** *There exists a stationary rational expectation equilibrium, where the prices of the assets are a linear function of the underlying state variables. In particular, the price of the stock is*

$$P_S = \phi_S + p^*_{SD} D + p^*_{S\pi}\pi + p_{S0} + p_{S\theta}\boldsymbol{\theta} + \mathbf{p}_{S\Delta}\boldsymbol{\Delta}, \tag{6}$$

*and the price of the derivative is*

$$P_F = \phi_F + p^*_{FD} D + p^*_{F\pi}\pi + \mathbf{p}^*_{F\theta}\boldsymbol{\theta} + p_{F0} + \mathbf{p}_{F\theta}\boldsymbol{\theta} + \mathbf{p}_{F\Delta}\boldsymbol{\Delta}, \tag{7}$$

*where the coefficients* $p_{SD}, p_{S\nu}, p_{S\Delta}, p^*_{SD}, p^*_{S\pi}, p_{S0}, p_{S\theta}, p^*_{FD}, p^*_{F\pi}, p^*_{F\theta}, p_{F0}$, $p_{F\theta}$, *and* $p_{F\Delta}$ *are defined below.*

The proposition characterizes the standard rational expectation equilibrium previously treated in the literature [Grossman (1976), Wang (1993, 1994)]. In order to determine it, first, we conjecture a functional form of asset prices as linear combinations of all the state variables. Then we solve the agents' learning and optimization problems. Finally, we use the asset market clearing conditions to calculate the equilibria.

**1.2.1 Definition of the value of the stock.** Stock prices are a function of both the primary state variables $(D, \boldsymbol{\nu})$ and the estimated state variables of each agent in the economy, $\hat{\boldsymbol{\nu}}^j = (\hat{\pi}^j, \hat{\theta}^j_1, \hat{\theta}^j_2)'$, where the superscript $j$ refers to the $j$th agent. Alternatively, asset prices are a function of the true state variables and of agents' estimation errors $(\boldsymbol{\Delta}^j = (\Delta^j_\pi, \Delta^j_{\theta_1}, \Delta^j_{\theta_2})')$. Thanks to the assumption of the CARA utility function and to the linearity of the laws of motion of $D$ and $\boldsymbol{\nu}$, we can conjecture that the relationship between asset prices and state variables is linear. Using the Campbell and Kyle (1993) approach, stock prices are postulated as

$$\begin{aligned} P_S &= const + p_{SD} D + p_{S\nu}\boldsymbol{\nu} + \mathbf{p}_{S\Delta}\boldsymbol{\Delta} \\ &= \phi_S + p^*_{SD} D + p^*_{S\pi}\pi + p_{S0} + p_{S\theta}\boldsymbol{\theta} + \mathbf{p}_{S\Delta}\boldsymbol{\Delta}, \end{aligned} \tag{8}$$

where $\phi_S = \frac{a_\pi p^*_{S\pi}\bar{\pi}}{r}$, $p^*_{SD} = \frac{1}{r+k}$, $p^*_{S\pi} = \frac{p^*_{SD}}{r+a_\pi}$, $p_{S0}$ is a constant, and the vectors $\mathbf{p}_{S\theta} = (p_{S\theta_1}, p_{S\theta_2})$, $\mathbf{p}_{S\Delta^i} = (p_{S\Delta^i_\pi}, p_{S\Delta^i_{\theta_1}}, p_{S\Delta^i_{\theta_2}})$, and $\mathbf{p}_{S\Delta^u} = (p_{S\Delta^u_\pi}, p_{S\Delta^u_{\theta_1}}, p_{S\Delta^u_{\theta_2}})$. Here $\mathbf{p}_{S\Delta}\boldsymbol{\Delta}$ is equal to $\mathbf{p}_{S\Delta^i}\boldsymbol{\Delta}^i + \mathbf{p}_{S\Delta^n}\boldsymbol{\Delta}^u$ in the first scenario, and to $\mathbf{p}_{S\Delta^i}\boldsymbol{\Delta}^i$ in the second scenario. Indeed, in the first scenario, where both classes of agents are filtering, the learning errors of both agents become relevant ($\boldsymbol{\Delta}^i$ for the information purchasing agents and $\boldsymbol{\Delta}^n$ for the fully uninformed ones). In the second scenario, on the contrary, only the errors of the information purchasing agents ($\boldsymbol{\Delta}^i$) affect prices, as the informed agents are fully informed.

**1.2.2 Definition of the value of the derivative.** We consider a financial derivative written on the stock as a contract of instantaneous maturity that delivers a gross payoff in the next instant equal to the asset it is written on. This is a standard continuous-time representation.[5] Unlike the stock, the payoff of the derivative is a function of an existing underlying asset and not a function of a dividend process of its own. This implies that the introduction of the new asset reduces the degree of stochastic uncertainty in the economy. The specification of the value of the dividend allows us to define the price of the derivative on the basis of an equilibrium relationship. The dividend ($D_F$) follows a diffusion process equal to the change in the price of the underlying asset:

$$dD_F = dP_s = [\eta_0\bar{\pi} + \eta_D + \eta_\pi \pi + \boldsymbol{\eta}_\nu \boldsymbol{\nu} + \mathbf{p}_{S\Delta}\boldsymbol{\Delta}]\,dt + \mathbf{b}_{D_F}d\mathbf{z}, \tag{9}$$

where $\eta_0 = a_\pi p^*_{S\pi}$, $\eta_D = -\alpha p^*_{SD}k$, $\boldsymbol{\eta}_\nu = (\alpha p^*_{SD}\beta - a_\pi p^*_{S\pi}; -\alpha \mathbf{p}_{S\theta}\mathbf{a}_\theta)$, and $\mathbf{b}_{D_F} = \mathbf{b}_{P_S}$. As is the case for the stock, the value of the derivative can be calculated as a linear combination of both the primary and the induced state variables,

$$P_F = \phi_F + p^*_{FD}D + p^*_{F\pi}\pi + \mathbf{p}^*_{F\theta}\boldsymbol{\theta} + p_{F0} + \mathbf{p}_{F\theta}\boldsymbol{\theta} + \mathbf{p}_{F\Delta}\boldsymbol{\Delta}, \tag{10}$$

where $\phi_F = \frac{\eta_0}{r} + [\beta\frac{a_\pi\eta_D}{(r*(r+a_\pi)(r+k))} + \frac{a_\pi\eta_\pi}{(r*(r+a_\pi))}]\bar{\pi}$, $p^*_{FD} = \frac{\eta_D}{r+k}$, $p^*_{F\pi} = \frac{p^*_{SD}}{r+a_\pi}\eta_D + \frac{p^*_{S\pi}}{r+a_\pi}\eta_\pi$, $\mathbf{p}^*_{F\theta} = (\frac{1}{r+a_{\theta_1}}\boldsymbol{\eta}_\nu[2], \frac{1}{r+a_{\theta_2}}\boldsymbol{\eta}_\nu[3])$, where $p_{F0}$ is a constant and $\boldsymbol{\eta}_\nu[n]$ refers to the $n$th component of the vector $\boldsymbol{\eta}_\nu$.[6] Therefore the law of motion of the generic $x$th asset (stock as well as derivative) is

$$dP_x = [p^*_{xD}(\beta\pi - kD) + p^*_{x\pi}a_\pi(\bar{\pi} - \pi) + \mathbf{a}_\theta\mathbf{p}_{x\theta}\boldsymbol{\theta} + \mathbf{p}_{x\Delta}\boldsymbol{\Delta}]\,dt + \mathbf{b}_{P_x}d\mathbf{z}. \tag{11}$$

Each agent perceives a different set of state variables that he wants to insure against. Hence each agent $j$ has a different perception of the law of motion of the $x$th asset, that is

$$dP^j_x = \mathbf{p}^j_x * \boldsymbol{\psi}^j\,dt + \mathbf{b}^j_{P_x}d\hat{\mathbf{z}}^j, \tag{12}$$

where $\mathbf{p}^j_x$ and $\mathbf{b}^j_{P_x}$ are vectors of constants and $d\hat{\mathbf{z}}^j$ is the vector of the stochastic components of the induced state variables (see Appendix A). $\boldsymbol{\psi}^j$ represents the vector of states the $j$th agent wants to insure against. The values of

[5] Breeden (1984) defines this contract further: "If $I_t$ is the price of corn, then this futures contract would have a gross payoff that is equal to the price of corn in the next instant, just as a standard futures contract at maturity can be regarded as having a gross payoff equal to the value of the underlying commodity." This definition is also analogous to the discrete time representation of a perpetual derivative contract on an index which entitles to a perpetual stream of payments equal to the change of value of the underlying index [Shiller (1993)]. An alternative, intuitive way of seeing this asset is to consider it as a closed-end fund.

[6] We have that $\mathbf{p}_{F\Delta}\boldsymbol{\Delta} = \mathbf{p}_{F\Delta^i}\boldsymbol{\Delta}^i + \mathbf{p}_{F\Delta^n}\boldsymbol{\Delta}^n$ in the first scenario, and $\mathbf{p}_{F\Delta}\boldsymbol{\Delta} = \mathbf{p}_{F\Delta^i}\boldsymbol{\Delta}^i$ in the second scenario.

its elements in the four subcases are given in Appendix A. Its law of motion follows

$$d\boldsymbol{\psi}^j = \mathbf{a}_{\psi^j}\,dt + \mathbf{b}_{\psi^j}\boldsymbol{\psi}^j\,d\hat{\mathbf{z}}^j. \tag{13}$$

We can therefore define the law of motion of the excess rate of return of the $x$th asset as perceived by the $j$th agent ($Q_x^j$) as

$$dQ_x^j = (D_x - rP_x^j)dt + dP_x^j = \mathbf{e}_{Q_x}^j\boldsymbol{\psi}^j\,dt + \mathbf{b}_{Q_x}^j\,d\hat{\mathbf{z}}^j, \tag{14}$$

where $\mathbf{e}_{Q_x}^j$ and $\mathbf{b}_{Q_x}^j$ are described in Appendix A.

**1.2.3 Agents' learning problem.** Assets provide agents with signals about the underlying states. In particular, the price of the $x$th asset can be written as $P_x = \Sigma_x + \aleph_x$, where $\aleph_x$ is a component that is a function of observable states (dividends) and $\Sigma_x$ is a component that is a function of unobservable states ($\Sigma_x = \mathbf{p}_{x\nu}\boldsymbol{\nu}$). $\Sigma_x$ represents a signal of the underlying unknown processes ($\boldsymbol{\nu}$). It follows that the vector of market signals is $(D, \Sigma_S)$ in the one-asset case and $(D, \Sigma_S, \Sigma_F)$ in the two-asset case. Agents use the market signals as well as the outside signals they are allowed to purchase $(\varphi_\pi, \varphi_{\theta_1}, \varphi_{\theta_2})$ in order to infer the underlying states $(\pi, \theta_1, \theta_2)$.

***Theorem 1.*** *Agents use the signals to make estimates of the underlying state variables. The optimal filtering for the jth agent generates a set of estimates* $(\hat{\pi}^j, \hat{\theta}_1^j, \hat{\theta}_2^j)$. *The estimated state variables follow the law of motion*:

$$\begin{bmatrix} d\hat{\pi}^j \\ d\hat{\theta}_1^j \\ d\hat{\theta}_2^j \end{bmatrix} = \begin{bmatrix} a_\pi(\bar{\pi} - \hat{\pi}) \\ -a_{\theta_1}\hat{\theta}_1^j \\ -a_{\theta_2}\hat{\theta}_2^j \end{bmatrix} dt + \mathbf{H}^j(\mathbf{b}_s\mathbf{b}_s')^{\frac{1}{2}}\,d(\hat{\mathbf{z}}^j). \tag{15}$$

*Proof.* See Appendix A.

The information variance-covariance matrix ($\mathbf{H}^j$) and the new induced stochastic processes ($d\hat{\mathbf{z}}^j$) are defined by the parameters of the learning process and depend on the number of signals available. The fact that there are fewer market signals ($D$, $P_S$, and $P_F$) than states ($D$, $\boldsymbol{\nu}$, $\boldsymbol{\varepsilon}$) keeps the information set of all the non-fully informed agents (i.e., the information purchasing agents as well as the fully uninformed ones) far from being complete. The equilibrium is not fully revealing, even after the introduction of the derivative. We can measure agents' learning errors as the square forecasting error ($SFE = E[(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu})(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu})']$).

### 1.2.4 Solving agents' optimization problem

***Assumption 7.*** *Each agent j chooses the optimal amount to consume* ($c^j$) *and the optimal fraction of total wealth* ($W^j$) *to invest in the stock* ($y_S^j$) *and in* *the derivative* $(y_F^j)$, *where* $\mathbf{F}^j = (y_S^j, y_F^j)'$. *The agents who are not purchasing information* (*the fully informed and the fully uninformed*) *solve the standard portfolio problem*,

$$\max_{c^j, \mathbf{y}^j} E_0\left[-\int_0^\infty e^{-\zeta^j s - \gamma^j c^j}\, ds\right] \tag{16}$$

$$s.t.\, dW^j = (rW^j - c^j)\, dt + \mathbf{F}^{j'} d\mathbf{Q}^j. \tag{17}$$

The information purchasing agents, instead, first decide the amount of information they want to buy and, conditional on it, the optimal portfolio-consumption choice. That is, they solve

$$\max_{\Upsilon}\left\{\max_{c^j, \mathbf{y}^j|\Upsilon}, E_0\left[-\int_0^\infty e^{-\zeta^j s - \gamma^j c^j}\, ds\right]\right\} \tag{18}$$

$$s.t.\, dW^j = (rW^j - \Upsilon * \varkappa - c^j)\, dt + \mathbf{F}^{j'} d\mathbf{Q}^j, \tag{19}$$

where $\mathbf{Q}^j = (Q_S^j, Q_F^j)'$ is the vector of asset excess rates of return over the riskless asset, as it is estimated by the $j$th agent, and $\varkappa$ is the unitary cost of the information purchased $(\Upsilon)$.

In order to solve this problem we work backwards. First, we define the optimal consumption and investment rule for a generic amount of information and solve for equilibrium. This equilibrium would be the one that would prevail if the agents were constrained in terms of the amount of information they can purchase. Then we solve for the optimal amount of information, that is, the one that generates the highest utility.

The first step is identical for both non-information purchasing agents and information purchasing agents. The only difference is the information set the two classes of agents have available.

***Theorem 2.*** *For the generic jth agent, the optimal consumption rule* $(c^j)$ *is*

$$c^j = rW^j + \frac{1}{2}\boldsymbol{\psi}^{j'}\mathbf{v}^j\boldsymbol{\psi}^j - \log(r), \tag{20}$$

*while the optimal investment policy* $(F^j)$ *is*

$$\mathbf{F}^j = \left(r\gamma^j \mathbf{b}_Q^j \mathbf{b}_Q^{j'}\right)\left(\mathbf{e}_Q^j - \mathbf{b}_Q^j \mathbf{b}_\psi^{j'} \mathbf{v}^j\right)\boldsymbol{\psi}^j, \tag{21}$$

*where* $\mathbf{v}^j$ *is a component of the indirect utility function, as specified in Appendix A, and* $\mathbf{F}^j = (y_S^j, y_F^j)' = (\mathbf{f}_S^j\boldsymbol{\psi}^j, \mathbf{f}_F^j\boldsymbol{\psi}^j)'$, *where* $\mathbf{f}_S^j = (f_{S0}^j, f_{S\theta_1}^j, f_{S\theta_2}^j, f_{S\theta_F}^j, f_{S\Delta}^j)$ *and* $\mathbf{f}_F^j = (f_{F0}^j, f_{F\theta_1}^j, f_{F\theta_2}^j, f_{F\Delta}^j)$.

**1.2.5 Determining the equilibria.** To find the competitive solution, we impose the following market clearing conditions:

$$(\omega)\mathbf{F}^i + (1-\omega)\mathbf{F}^n = \mathbf{I} + \mathbf{\Theta}, \tag{22}$$

where $i$ represents the information purchasing agents and $n$ represents the non-information purchasers. The latter are the uninformed agents in the first scenario and the fully informed ones in the second scenario. Also, $\mathbf{I} = (1, 0)$ and $\mathbf{\Theta} = (\theta_1 + \theta_2, \theta_1 + \theta_2)$.

Matching coefficients turn the above equations into a system of nonlinear equations in the asset prices' coefficients ($p_{S0}, p_{S\pi}, p_{S\theta_1}, p_{S\theta_2}, p_{F0}, p_{F\pi}, p_{F2\theta_1}, p_{F\theta_2}, \ldots$). These equations, together with the systems of equations necessary to solve for the value functions of the informed and uninformed agents, fully specify the equilibrium, conditional on a certain amount of information purchased.

The next step is to solve for the optimal purchase of information. The information purchasing agents keep purchasing information up to the point when they become indifferent to the purchase of additional information. In order to implement this we consider successively increasing amounts of information ($\Upsilon_h$ and $\Upsilon_{h+\Delta}$). For each level of purchased information we solve Equations (18) and (19) and calculate the corresponding utility. We then consider the differential between the utility before (at $h$) and after ($h+\Delta$) the purchase of information for each level of information. The optimal amount of information corresponds to the point where the incremental utility is zero. That is,

$$E_{0|\Upsilon_1}\left[-\int_0^\infty e^{-\zeta^j s - \gamma^j c^j} ds\right] - E_{0|\Upsilon_0}\left[-\int_0^\infty e^{-\zeta^j s - \gamma^j c^j} ds\right] = 0. \tag{23}$$

This way of determining the value of information by comparing the ex ante expected payoff function calculated at different levels of purchased information is in the spirit of Admati and Pfleiderer (1987, 1988). Given that we are dealing with atomistic agents in competitive markets, where each single agent is neglecting the impact of his decision on equilibria, we solve the maximization problem of the information purchasing agents, keeping the distribution of prices invariant when we determine the incremental value of information for the information purchasing agents.

Therefore, in order to solve the model we need to iteratively solve the system of nonlinear equations defined by the market clearing conditions [Equation (22)] and the value functions for the two classes of agents. These determine the equilibria, conditional on the purchase of a fixed amount of information. Then, for each equilibrium, we have to calculate the incrementals value of an additional unit of information by comparing information purchasers' utility function before and after the purchase of information, experimenting with increasing amounts of information purchased up to the level where Equation (23) is satisfied. Finally, prices, volatilities, volumes, and the correlations between volume and volatility can be found as closed-form functions of the coefficients that we have solved for (See Appendix A).

**1.2.6 Calibration.** The fact that the model not only does not allow for a closed form solution, but also requires an iterative procedure in order to identify the equilibrium characterized by the optimal purchase of information implies that only a numerical solution can be derived. However, numerical solutions may rely heavily on the choice of the structural parameters of the model. Therefore, in order to provide a solution that is reasonably close to real data, we calibrate the model to some of the empirical evidence.

Calibration is performed by selecting parameters that yield returns (risk-free rate and risk premium) and volatilities that are close to those found in the empirical literature. It is worth noting that, given that the purpose of the artcle is the description of two different scenarios, matching with the real data can be done on either scenario (i.e., either starting with the first scenario one-asset case or starting with second scenario one-asset case). We choose to calibrate the model on the basis of the second scenario. A calibration based on the first scenario (not reported) delivers similar comparative numeric results, consistent with the ones derived by a calibration based on the second scenario. All the numerical results below are derived from the model calibrated in this way.

The parameters generated by the calibration for the benchmark case and details of the calibration are reported in Appendix B. The model is also solved for various specifications where some of the benchmark parameters are changed (different degrees of risk aversion of the two classes of agents, different number of informed and uninformed agents) for different scenarios before and after the introduction of the derivative. Furthermore, in order to assess the robustness of the results and their sensitivity to a change in the parameters, we solve the model for different sets of alternative parameters. Similar comparative results are obtained.

## 2. Effects of Financial Innovation on Asset Allocations

Both the change of the informational structure of the market and the endogenization of the purchase of information affect the way the introduction of a derivative impacts the underlying assets. We first focus on the role played by the type of market informational structure. We then consider how the endogenization of the purchase of information interacts with it. All the following results are derived through simulation.

***Result 1.*** *The introduction of the derivative induces the more informed agents to increase their holdings of the underlying assets, regardless of the market informational structure. This happens in both scenarios and for all the cases considered.*

The effects of financial innovation differ depending on the type of informational structure of the market where the new asset is introduced. A main direct effect is reallocation of the holdings of the risky asset among differentially informed agents. Comparing the one-asset case to the two-asset case, panel A of Table 1 shows that, in both scenarios, the introduction of a derivative induces the relatively more informed agents to hold more of the stock and the less informed ones to hold less of it. However, the more informed agents are not always those who purchase information. In the first scenario, some agents purchase information in order to gain an informational advantage: they are the ones who increase their holdings of the underlying asset. In the second scenario, some agents purchase information in order to reduce their informational disadvantage: they then reduce their holdings

**Table 1**
**Asset holdings**

| | First scenario | | Second scenario | |
|---|---|---|---|---|
| | Information purchasers | Uninformed | Fully informed | Information purchasers |
| **Panel A** | | | | |
| **One-asset case** | | | | |
| Benchmark* | 0.993 | 1.007 | 1.325 | 0.675 |
| $\omega = 0.65$ | 0.997 | 1.005 | 1.543 | 0.707 |
| $\omega = 0.35$ | 0.981 | 1.010 | 1.190 | 0.647 |
| $\gamma^i = 1.4$ | 0.792 | 1.208 | 1.468 | 0.532 |
| $\gamma^u = 1.4$ | 1.148 | 0.852 | 1.212 | 0.788 |
| **Two-asset case** | | | | |
| Benchmark* | 1.155 | 0.845 | 1.915 | 0.085 |
| $\omega = 0.65$ | 1.058 | 0.891 | 2.706 | 0.080 |
| $\omega = 0.35$ | 1.075 | 0.859 | 1.478 | 0.111 |
| $\gamma^i = 1.4$ | 0.910 | 1.090 | 1.910 | 0.090 |
| $\gamma^u = 1.4$ | 1.351 | 0.649 | 1.909 | 0.091 |
| **Panel B. Constrained equilibria where the amount of information is exogenously imposed** | | | | |
| **One-asset case** | | | | |
| Benchmark* | 0.944 | 1.055 | 1.424 | 0.574 |
| $\omega = 0.65$ | 0.963 | 1.068 | 1.652 | 0.659 |
| $\omega = 0.35$ | 0.927 | 1.038 | 1.211 | 0.607 |
| $\gamma^i = 1.4$ | 0.791 | 1.209 | 1.539 | 0.461 |
| $\gamma^u = 1.4$ | 1.144 | 0.856 | 1.296 | 0.704 |
| **Two-asset case** | | | | |
| Benchmark* | 3.96 | −1.96 | 1.917 | 0.083 |
| $\omega = 0.65$ | 2.60 | −1.98 | 2.706 | 0.081 |
| $\omega = 0.35$ | 6.19 | −1.79 | 1.484 | 0.100 |
| $\gamma^i = 1.4$ | 3.90 | −1.90 | 1.924 | 0.076 |
| $\gamma^u = 1.4$ | 3.42 | −1.42 | 1.908 | 0.092 |

Panels A and B report the holdings of the risky asset by the different classes of agents in the two scenarios before and after the introduction of the derivative (one-asset case and two-asset case). We consider the benchmark case and the cases where we vary either the fraction of information purchasers ($\omega$), the degree of risk aversion of the information purchasers ($\gamma^i$), or the degree of risk aversion of the non-information purchasing agents ($\gamma^u$). The holdings are the holdings for the individual agent. The benchmark case is defined according to the following set of parameters: $\omega = 0.5$, $\gamma^i = \gamma^u = 1$, $\zeta^i = \zeta^u = 0.2$, $a_\pi = 0.6$, $a_{\theta 1} = 0.587$, $a_{\theta 2} = 0.5$, $b_\pi = 1.28$, $b_{\theta 1} = 1.9$, $b_{\theta 2} = 2.9$, $b_D = 0.9$, $k = 1$, $\beta = 1$, $\bar{\pi} = 1.35$. In Panel A, equilibria are determined at the point where agents optimally choose the amount of information to purchase. In Panel B, equilibria are determined for an exogenously imposed amount of information (10 units).

of the underlying asset. We will see that this difference drives most of the following results. Let's now try to explain what causes it.

The change in the incentive to hold stocks depends on the improved hedging opportunities provided by the derivative. The demand for stocks ($y_S^j = \mathbf{f}_S^j \hat{\boldsymbol{\psi}}^j$) is a function of the agents' perception of the underlying states ($\hat{\boldsymbol{\psi}}^j$) as well as of their sensitivity to them ($\mathbf{f}_S^j \equiv f(J^j)$). Here $J^j$ represents the value function of the $j$th agent (Appendix A). Any change in either of these factors (risk or perception of it) affects the agents' incentives to invest in the risky assets by changing the agents' degree of informativeness. The derivative reduces both the exposure to risk, by providing a way of hedging against it, and the perception of it (learning risk). Let's look at these two elements separately.

The derivative provides a good hedge against both fundamental risk and the risk due to asymmetry of information. The more an agent is exposed to risk, the more he alters his portfolio to hedge against it. Given that a derivative provides a good way to hedge risk, the introduction of a derivative alters the optimal portfolio allocation. Clearly this affects mostly the less informed agents, as they are the ones who are more exposed to asymmetric information risk. But the direction of the effect on their portfolio allocation is ambiguous. On the one hand, the less informed agents would tend to increase their demand of the derivative proportionately more than the more informed agents do. This would reduce their stock holdings. On the other hand, the availability of the derivative, by making their global portfolio less risky, would increase their appetite for riskier assets. This would induce them to increase their holdings of stocks more than the informed agents do. The net result depends on many elements, such as the starting portfolio composition, the original allocation between informed and less informed agents, and so on.

What differentiates the two types of informational structures is the fact that the introduction of the derivative also reduces the learning risk. By modifying the way agents dynamically hedge "learning risk," the change in the amount of information alters agents' propensity to invest in the risky asset. Let's see how this takes place.

Learning risk is due to imperfect knowledge. It arises when the investor attempts to protect himself against the risk of "learning bad news: that the mean (return) is low" [Brennan (1998)]. Higher uncertainty, by inducing higher learning risk, prompts the investor to invest more in risky assets in order to hedge the increased risk.[7] Conversely, more information, by decreasing learning risk, lowers the need to hedge against it, and therefore the

[7] In particular, Brennan (1997), in the case of the CRRA utility, and in a conditional Gaussian framework, shows that if relative risk aversion is less than zero, the investor invests a smaller fraction of wealth in the risky security, whereas if the relative risk aversion is greater than zero, the fraction of wealth invested in the risky asset is higher. In our model, we keep the crucial assumption of conditional Gaussian distribution, which lets the result depend only on the first moment of the unknown variables ($\boldsymbol{\nu}$), but we use a CARA utility function. This does not allow us to characterize a priori the direction of the learning effect. In our model, this effect is captured by $\mathbf{b}_{\Psi_j}\mathbf{v}_j$, that is, the sensitivity of the agents to the states ($\mathbf{v}_j$) and the estimation errors ($\mathbf{b}_{\Psi_j}$) themselves.

demand for risky assets. The introduction of a derivative, by providing the less informed agents with an additional signal and improving their ability to learn, reduces their learning risk. Lower learning risk implies a decrease in agents' hedging demand for risky assets. Given that the less informed agents are the ones more exposed to learning risk, they are also the ones more affected. The ensuing reduction in their stock holdings, coupled with the increase in their position in derivatives due to risk hedging motives, determines a reallocation of stocks from the portfolios of the less informed agents to the portfolios of the more informed ones.

## 3. Effects of Financial Innovation on Financial Variables

We now consider how the reallocation of holdings among differentially informed agents affects financial variables (risk premia, asset prices, volume, volatility, correlation between volume and volatility, and market informational efficiency), depending on the informational structure. In particular, we show that the introduction of a new asset produces some effects that are invariant to the change of the market informational structure and some that depend on it. The effects that are common across informational structures involve risk premia, volatility, asset prices, and agents' forecasting errors. The ones that depend on the market informational structure include trading volume, the correlation between volume and volatility and the degree of market informational efficiency.

***Result 2.*** *The introduction of the derivative reduces risk premia and increases asset prices, regardless of the informational structure. This happens in both scenarios and for all the cases considered.*

Table 2 (Panel A, columns 4 and 7) shows that the introduction of the derivative reduces the risk premia of the underlying asset in both scenarios. Indeed, the derivative improves risk sharing and increases the information publicly observable through prices. Higher risk sharing reduces the price of risk and therefore risk premia. More available information, by reducing the informational disadvantage of the less informed agents, reduces the component of risk premia due to asymmetric information.

Prices are a function of the long-term risk premia as well as of short-term fluctuations due to noise shocks and agents' learning errors. Table 3 (column 1) shows that the introduction of the derivative, by reducing risk premia and the sensitivity of prices to the underlying states, in both scenarios, also increases prices.

***Result 3.*** *The introduction of the derivative decreases volatility, regardless of the informational structure. This happens in both scenarios and for all the cases considered.*

**Table 2**
**Information purchased, learning errors, and risk premia**

| | First scenario | | | | Second scenario | | |
|---|---|---|---|---|---|---|---|
| | Information purchased | $SFE^i$ | $SFE^n$ | Risk premia | Information purchased | $SFE^i$ | Risk premia |
| **Panel A** | | | | | | | |
| **One-asset case** | | | | | | | |
| Benchmark* | 5.3 | 0.606 | 0.724 | 0.0107 | 1.2 | 0.716 | 0.0658 |
| $\omega = 0.65$ | 4.9 | 0.644 | 0.747 | 0.0108 | 1.2 | 0.716 | 0.0321 |
| $\omega = 0.35$ | 6.1 | 0.590 | 0.724 | 0.0109 | 1.2 | 0.716 | 0.1925 |
| $\gamma^i = 1.4$ | 10.2 | 0.526 | 0.724 | 0.0136 | 1.2 | 0.716 | 0.0908 |
| $\gamma^u = 1.4$ | 7.1 | 0.572 | 0.724 | 0.0137 | 1.2 | 0.709 | 0.0417 |
| **Two-asset case** | | | | | | | |
| Benchmark* | 1.2 | 0.161 | 0.167 | 0.0011 | 4.1 | 0.003 | 0.0004 |
| $\omega = 0.65$ | 1.2 | 0.160 | 0.167 | 0.0011 | 13.5 | 0.006 | 0.0006 |
| $\omega = 0.35$ | 1.2 | 0.160 | 0.167 | 0.0011 | 1.4 | 0.002 | 0.0004 |
| $\gamma^i = 1.4$ | 1.2 | 0.164 | 0.167 | 0.0011 | 3.3 | 0.003 | 0.0004 |
| $\gamma^u = 1.4$ | 1.2 | 0.160 | 0.167 | 0.0012 | 11.5 | 0.005 | 0.0006 |
| **Panel B. Constrained equilibria where the amount of information is exogenously imposed** | | | | | | | |
| **One-asset case** | | | | | | | |
| Benchmark* | 10 | 0.526 | 0.723 | 0.0126 | 10 | 0.520 | 0.0102 |
| $\omega = 0.65$ | 10 | 0.523 | 0.723 | 0.0129 | 10 | 0.460 | 0.0112 |
| $\omega = 0.35$ | 10 | 0.527 | 0.722 | 0.0123 | 10 | 0.520 | 0.0083 |
| $\gamma^i = 1.4$ | 10 | 0.529 | 0.724 | 0.0151 | 10 | 0.528 | 0.0108 |
| $\gamma^u = 1.4$ | 10 | 0.529 | 0.724 | 0.0150 | 10 | 0.455 | 0.0139 |
| **Two-asset case** | | | | | | | |
| Benchmark* | 10 | 0.079 | 0.181 | 0.0017 | 10 | 0.002 | 0.0007 |
| $\omega = 0.65$ | 10 | 0.079 | 0.181 | 0.0016 | 10 | 0.006 | 0.0009 |
| $\omega = 0.35$ | 10 | 0.077 | 0.177 | 0.0018 | 10 | 0.001 | 0.0006 |
| $\gamma^i = 1.4$ | 10 | 0.080 | 0.182 | 0.0018 | 10 | 0.005 | 0.0007 |
| $\gamma^u = 1.4$ | 10 | 0.079 | 0.180 | 0.0017 | 10 | 0.002 | 0.0010 |

Panels A and B report the amount of information purchased, the learning errors of the information purchasers ("$i$") and uninformed agents ("$n$") and the risk premia in the two scenarios before and after the introduction of the derivative (one-asset case and two-asset case, respectively). We consider the benchmark case and the cases where we vary either the fraction of information purchasers ($\omega$), the degree of risk aversion of the information purchasers ($\gamma^i$), or the degree of risk aversion of the non-information purchasing agents ($\gamma^u$). The benchmark case is defined according to the following set of parameters: $\omega = 0.5$, $\gamma^i = \gamma^u = 1$, $\zeta^i = \zeta^u = 0.2$, $a_\pi = 0.6$, $a_{\theta 1} = 0.587$, $a_{\theta 2} = 0.5$, $b_\pi = 1.28$, $b_{\theta 1} = 1.9$, $b_{\theta 2} = 2.9$, $b_D = 0.9$, $k = 1$, $\beta = 1$, $\bar{\pi} = 1.35$. In Panel A, equilibria are determined at the point where agents optimally choose the amount of information to purchase. In Panel B, equilibria are determined for an exogenously imposed amount of information (10 units).

Analogous to prices, volatility is affected by agents' learning errors (Table 3, column 2). In particular, unconditional volatility can be defined as $\sigma_{P_x} = [p^{*2}_{xD}\sigma^2_D + p_{x\nu}\sigma_\nu\sigma'_\nu p'_{x\nu} + p_{x\Delta}\sigma_{\nu\Delta}\sigma'_\Delta p'_{x\Delta} + p_{xD}\sigma_D p_{x\nu}\sigma_\nu + p_{xD}\sigma_D p_{x\Delta}\sigma_\Delta + p_{x\nu}\sigma_\nu\sigma'_\Delta\, p'_{x\Delta}]^{\frac{1}{2}}$. The derivative, by improving the possibilities of hedging and increasing the amount of information, lowers the sensitivity of asset prices to the underlying uncertainty and therefore reduces volatility. This takes place in both scenarios.

***Result 4.*** *Trading volume increases considerably in the scenario characterized by lower informational completeness and asymmetry and decreases in the scenario characterized by higher informational completeness and asymmetry.*

Trading volume is defined as the change in holdings, or *Trade* $= |\mathbf{f}^j_x d\boldsymbol{\psi}^j|$. Investors trade to hedge risk, as well as to exploit their informational advan-

**Table 3**
**Stock prices, volatility, volume, and correlation volume-volatility**

| | Price | $\sigma$ | Total V. | $V^i$ | corr($V^i$, $\sigma$) | $V^n$ | corr($V^n$, $\sigma$) |
|---|---|---|---|---|---|---|---|
| **Panel A. First scenario** | | | | | | | |
| **One-asset case** | | | | | | | |
| Benchmark* | 19.67 | 0.169 | 0.016 | 0.009 | 0.066 | 0.008 | 0.066 |
| $\omega = 0.65$ | 19.69 | 0.173 | 0.017 | 0.009 | 0.066 | 0.008 | 0.066 |
| $\omega = 0.35$ | 19.65 | 0.169 | 0.016 | 0.009 | 0.066 | 0.008 | 0.066 |
| $\gamma^i = 1.4$ | 18.60 | 0.181 | 0.015 | 0.007 | 0.084 | 0.008 | 0.084 |
| $\gamma^u = 1.4$ | 18.67 | 0.180 | 0.015 | 0.010 | 0.085 | 0.005 | 0.084 |
| **Two-asset case** | | | | | | | |
| Benchmark* | 24.25 | 0.158 | 0.366 | 0.207 | 0.015 | 0.159 | 0.017 |
| $\omega = 0.65$ | 24.25 | 0.158 | 0.344 | 0.195 | 0.015 | 0.149 | 0.016 |
| $\omega = 0.35$ | 24.25 | 0.159 | 0.393 | 0.223 | 0.015 | 0.171 | 0.107 |
| $\gamma^i = 1.4$ | 24.21 | 0.158 | 0.329 | 0.144 | 0.016 | 0.184 | 0.017 |
| $\gamma^u = 1.4$ | 24.20 | 0.158 | 0.312 | 0.208 | 0.016 | 0.104 | 0.017 |
| **Panel B. Second scenario** | | | | | | | |
| **One-asset case** | | | | | | | |
| Benchmark* | 11.53 | 0.810 | 0.029 | 0.027 | 0.005 | 0.002 | 0.007 |
| $\omega = 0.65$ | 16.17 | 0.609 | 0.038 | 0.035 | 0.001 | 0.003 | 0.009 |
| $\omega = 0.35$ | 5.57 | 1.030 | 0.024 | 0.022 | 0.009 | 0.002 | 0.006 |
| $\gamma^i = 1.4$ | 9.43 | 0.837 | 0.028 | 0.027 | 0.005 | 0.001 | 0.007 |
| $\gamma^u = 1.4$ | 14.42 | 0.570 | 0.029 | 0.026 | 0.001 | 0.003 | 0.007 |
| **Two-asset case** | | | | | | | |
| Benchmark* | 25.98 | 0.160 | 0.026 | 0.017 | 0.0002 | 0.009 | 0.0004 |
| $\omega = 0.65$ | 26.78 | 0.161 | 0.035 | 0.026 | 0.0004 | 0.009 | 0.0011 |
| $\omega = 0.35$ | 26.07 | 0.160 | 0.023 | 0.013 | 0.0001 | 0.010 | 0.00004 |
| $\gamma^i = 1.4$ | 25.97 | 0.161 | 0.027 | 0.016 | 0.0002 | 0.011 | 0.0004 |
| $\gamma^u = 1.4$ | 25.78 | 0.160 | 0.026 | 0.017 | 0.0004 | 0.009 | 0.0010 |

Table 3 reports prices (Price), volatility ($\sigma$), total trading volume (Total V.), trading volume of the information purchasers ($V^i$), correlation between volatility and trading volume of the information purchasers (corr($V^i$, $\sigma$)), trading volume of the non-information purchasers ($V^n$), and correlation between volatility and trading volume of the non-information purchasers (corr($V^n$, $\sigma$)) in the two scenarios before and after the introduction of the derivative (one-asset case and two-asset case, respectively). We consider the benchmark case and the cases where we vary either the fraction of information purchasers ($\omega$), the degree of risk aversion of the information purchasers ($\gamma^i$), or the degree of risk aversion of the non information purchasing agents ($\gamma^u$). The benchmark case is defined according to the following set of parameters: $\omega = 0.5$, $\gamma^i = \gamma^u = 1$, $\zeta^i = \zeta^u = 0.2$, $a_\pi = 0.6$, $a_{\theta 1} = 0.587$, $a_{\theta 2} = 0.5$, $b_\pi = 1.28$, $b_{\theta 1} = 1.9$, $b_{\theta 2} = 2.9$, $b_D = 0.9$, $k = 1$, $\beta = 1$, $\bar{\pi} = 1.35$. Equilibria are determined at the point where agents optimally choose the amount of information to purchase.

tage. Financial innovation, by increasing informational completeness and lowering informational asymmetry, reduces uncertainty and lowers hedge trading. At the same time, it also provides additional informational trading opportunities. Trade, unlike volatility, is influenced by both the risk-hedging and informational components. It therefore reacts to the introduction of a derivative in a way different from volatility.

Table 3 (columns 3, 4, and 6) shows that, in the first scenario, financial innovation, by stimulating informational trade of both classes of agents, increases overall trade. In the second scenario, on the contrary, the increase in the volume of trade of the information purchasing agents is offset by the reduction in trading activity of the fully informed agents. The latter increase their holdings of stocks and reduce their turnover. Also, the higher level of information does not benefit the fully informed agents, as they see their informational advantage reduced. This further decreases their incentive to trade.

These combined effects, by reducing trading of the fully informed agents, account for the overall reduction of trading volume in the second scenario.

***Result 5.*** *The introduction of the derivative reduces the correlation between trading volume and volatility in both scenarios. The entity of the reduction is greater in the scenario characterized by higher informational completeness and asymmetry.*

Also, the correlation between volume and volatility (corr($Volatility_x$, $Volume_x$) = corr($|dP_x|$, $|\mathbf{f}_x^j d\boldsymbol{\psi}^j|$)) is affected. Volatility is mainly a function of risk, while trading volume is affected by both informational and risk-hedging motives. The introduction of the derivative, by reducing riskiness, raises the informational component of trade and therefore reduces the correlation between trading volume and volatility. As it is shown in Table 3 (columns 5 and 7), this happens in both scenarios.

The reduction of the correlation between volume and volatility is greater in the second scenario than in the first one. Indeed, the fact that the reduction in the risk premia is greater in the second scenario than in the first one signals that in the second scenario there is a stronger reduction in the risk-hedging part of trading as opposed to the part due to the informational motive. If the informational component of trading prevails over the risk-hedging one, the correlation between volume and volatility (more linked to risk) should be lower.

These results are robust for the different agents. Also, the introduction of the derivative affects the correlation between volatility and trading volume in the same way for both more informed and less informed agents.

***Result 6.*** *The introduction of the derivative affects the degree of market efficiency differently in the two scenarios, depending on the type of market informational structure.*

We use as a proxy for efficiency the autocorrelation over time of the asset abnormal return, that is the return over the average long-run rate (Appendix A). The more efficient the market is, the lower the degree of predictability of asset return and therefore the lower the autocorrelation over time.

In the first scenario, the introduction of the derivative increases the predictability of stock prices in the short run and reduces it in the long run. In particular, as Figure 1 shows, the ratio of the autocorrelation of returns before and after the introduction of the derivative is greater than one for all the cases considered and increases with the number of lags up to 10 lags and then decreases. For a number of lags greater than 15, the introduction of the derivative significantly reduces stock autocorrelation and return predictability.

In the second scenario, on the contrary, the introduction of the derivative produces two competing effects. On one side, by increasing the amount of information collected, it increases price informativeness; on the other side, by reducing trading by the more informed agents, it lowers it. The net effect

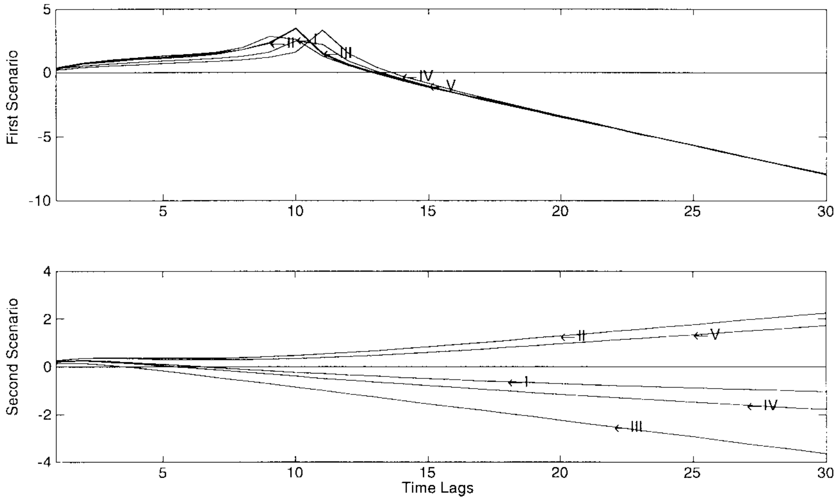


**Figure 1**
**Autocorrelation of stock returns over time**
The graphs report the logarithms of the ratios of the autocorrelations over time of the stock's abnormal rates of return before and after the introduction of the derivative in the two scenarios. We report the benchmark case (I), the case where the fraction of information purchasers is equal to 65% (II), the case where the fraction of information purchasers is equal to 35% (III), the case where the degree of risk aversion of the information purchasers is equal to 1.4 (IV), and the case where the degree of risk aversion of the non-information purchasing agents is equal to 1.4 (V).

depends on the ratio between informed and less informed agents and on agents' willingness to act on the basis of their information (degree of risk aversion). In the benchmark case and in the cases where either there are more fully informed agents ($\omega = 0.35$), or these are more risk tolerant ($\gamma^i = 1$), the introduction of the derivative, by amplifying the existing information, increases the informativeness of prices and reduces returns forecastability. On the contrary, in the other cases, the reduction in trading by the fully informed agents more than offsets the additional informational completeness of the market. This increases the degree of return predictability and reduces market efficiency. Therefore we have the counterintuitive result that more information (due to the introduction of the derivative) can reduce informational efficiency.

## 4. Effects of Financial Innovation on Information Purchase

Up to now we have focused on the equilibria which differ in terms of both the type of informational structure and the amount of information agents optimally purchase. We now want to analyze what role is played by the endogenous purchase of information in the definition of these equilibria. We will do this by comparing these equilibria to "constrained equilibria," that is, equilibria where the amount of information agents purchase is exogenously specified.

***Result 7.*** *The introduction of a derivative changes the incentive to purchase information. The direction of the effects depend on the type of informational structure.*

The introduction of the derivative not only increases the informativeness of prices in general, but it also changes the incentive to purchase information. Table 2 (panel A, columns 1 and 5) shows that the introduction of the derivative reduces the incentive to purchase information in the first scenario and increases it in the second scenario. In the first scenario, before the introduction of the derivative, there is little information and the purchase of information confers a high advantage on the more informed agents over the less informed ones. Furthermore, little of the purchased information "leaks" to the other agents through prices. The fact that information generates an informational advantage and that the information purchasers can appropriate most of its value makes the purchase of information highly valuable. Conversely, in the second scenario, the value of information is limited by the existence of a class of fully informed agents. Their trading makes prices informative enough and reduces the incentive for the uninformed agents to purchase information. Therefore the amount of information purchased at equilibrium is lower than in the first scenario.

We saw that the introduction of the derivative affects the equilibrium in two ways. First, by providing the uninformed agents with an additional signal, it allows them to appropriate, at no cost, part of the information held by the more informed agents. Second, it shifts the allocation of the stock away from the portfolios of the less informed agents to those of the more informed.

In the first scenario, both these effects concur to increase price informativeness and to allow the less informed agents to directly extract information from the joint observation of stock and derivative prices. The ability of the less informed agents to "free ride" at the expense of information purchasers reduces the incentive to purchase information. This is the same line of reasoning that lies behind the Grossman and Stiglitz result.

Conversely, in the second scenario, the benefits of additional purchases of information accrue to the information purchasing agents who can entirely appropriate the value of the incremental purchase of information. Therefore the information that is embedded in the derivative positively interacts with that which is purchased and there is no negative externality for the purchaser. This increases the incentive to purchase information. We can therefore say, using Admati's terminology, that the information purchased is a *complement* and not a *substitute* to that derived from asset prices.

The two informational effects produced by the introduction of the derivative generic increase in informativeness of prices and change in the amount of information purchased at equilibrium are not observable separately. Their roles can be inferred, however, by comparing the changes in risk premia at the unconstrained equilibria (Table 2, panel A) with those at the constrained equilibria (Table 2, panel B). In the latter case, we can control for the additional effect due to a variation in the amount of information purchased by imposing it to be identical before and after the introduction of the derivative, and consider only the effects due to a generic increase in the price informativeness. While at the constrained equilibria, the size of the reduction in risk premia is not very different between the two scenarios (Table 2, panel B, columns 4 and 7), at the unconstrained equilibria, the reduction of risk premia is stronger in the second scenario than in the first one (Table 2, panel A, columns 4 and 7). This is indeed the scenario where the derivative not only increases the informational content of prices, but also increases the amount of information purchased at equilibrium.[8] Moreover, if the derivative acts as a signal that magnifies the existing information, its effects should be stronger in the second scenario, where there is a class of fully informed agents.

The results above provide an empirically testable experiment. In the accounting literature there is robust empirical evidence that the number of analysts following a company is directly proportional to the level of disclosure of the company. That is, the amount of information purchased is a negative function of the degree of asymmetry of information (first scenario). At the same time, in the finance literature [Damodaram and Lim (1991)] it has been ascertained that the introduction of a derivative on an asset, in general, increases the number of analysts following such an asset. It would be interesting to see if this result holds also at a disaggregated level. That is, does the introduction of a derivative always increase the number of analysts following a company, or does it depend on the type of informational structure? The informational structure would correspond to the degree of disclosure of the company. Does the way information is distributed among agents affect the change in the number of analysts who follow an asset once a derivative on it has been introduced?

***Result 8.*** *The change in the incentive to purchase information affects the "hedging" and "learning risk" effects.*

This can be clearly seen by comparing constrained and unconstrained equilibria. If we consider the constrained equilibria, the introduction of the derivative reduces the stock holdings of the less informed agents more in the first scenario than in the second one (Table 1, panel B, columns 2 and 4). The reason is that the first scenario is characterized by higher informational uncertainty than the second one. Indeed, the only information contained in prices

[8] Indeed, if we consider the absolute change in risk premia (that is, the difference between risk premia before and after the introduction of the derivative), the drop is slightly greater in the first scenario. If, however, we consider the percentage change (that is, the ratio between the absolute change and the level of risk premia before the introduction of the derivative), the drop scenario is slightly greater in the second scenario than in the first.

is the one purchased by the information purchasing agents, while in the second scenario the existence of a class of fully informed agents makes the degree of price informativeness less dependent on the amount of information purchased. In order to hedge the higher uncertainty, the less informed agents tilt their portfolio composition towards the derivative more heavily in the first scenario than in the second scenario. This induces a more significant shift in holdings of the risky assets from the less informed agents to the more informed agents.

The opposite is true at the unconstrained equilibria, where the portfolio reallocation of the stock away from the less informed agents to the more informed ones is greater in the second scenario than in the first scenario. The reason is that the introduction of the derivative also changes the information acquired in equilibrium, increasing it in the second scenario and reducing it in the first scenario. This reduces the information purchasing agents' need to hold stocks in order to hedge learning risk in the second scenario, while increasing it in the first scenario (Table 1, panel A, columns 2 and 4).

***Result 9.*** *The introduction of the derivative changes the ordering in the degree of informativeness of the information purchasing agents across different information structures.*

We measure the degree of informativeness of an agent as the inverse of his square forecasting errors (SFEs). The difference between the SFEs of the information purchasing agents at the constrained equilibria and their SFEs at the unconstrained equilibria help us to quantify the effects due to the existence of a market for information. The results are shown in Table 2, panels A, B (columns 2, 3 and 6).

Before the introduction of the derivative, a comparison of equilibria with the same amount of information (constrained equilibria) shows that the second scenario is characterized by lower agents' errors than the first scenario (Table 2, panel B). This is expected, as the existence of a class of fully informed agents should make it easier for the less informed ones to filter information and the less informed agents can infer information by observing prices. If, however, we focus on the unconstrained equilibria, the learning errors are lower in the first scenario than in the second scenario (Table 2, panel A). This apparently puzzling result is due to the fact that more information is purchased in the first scenario than in the second one. This makes the information purchasing agents better informed in the first scenario than in the second one.

The introduction of the derivative changes all this. Not only does it reduce agents' learning errors, but it also inverts the ordering of the degree of informativeness across informational structures. The reduction in agents' learning errors holds across different informational structures, as the derivative, by providing an additional signal, improves agents' learning ability. However, the effect is stronger in the second scenario than in the first.

In the second scenario, both the increase in the holding of risky assets by the fully informed agents and the greater amount of information purchased at equilibrium concur to help the information purchasing agents improve their learning.

In the first scenario, on the contrary, the shift in holdings of risky assets away from the less informed agents to the more informed ones is partially offset by the reduction in the equilibrium level of purchased information. The fact that these two effects go in the same direction in the second scenario and partially offset each other in the first one induces a stronger reduction of agents' learning errors in the second scenario than in the first. This switches the ordering in the degree of informativeness of the agents.

***Result 10.*** *The introduction of the derivative changes the relative size of the risk premia in the two scenarios.*

Before the introduction of the derivative, risk premia at the unconstrained equilibria are puzzlingly lower in the scenario characterized by higher informational uncertainty (first scenario) (Table 2, panel A, columns 4 and 7). The reason is that in the second scenario the existence of a class of fully informed agents would both reduce the component of risk premia due to informational uncertainty and increase the one due to informational asymmetry. The fact that little information is purchased at equilibrium in the second scenario makes the informational asymmetry particularly strong and allows it to more than offset the reduction in risk premium due to lower informational uncertainty.

We can verify this relationship contrapositively by controlling for the purchase of information: risk premia should then be higher in the first scenario. Comparing constrained equilibria, in which information purchased is constant over scenarios (Table 2, panel B, columns 4 and 7), reveals that this is indeed the case. This is because in the second scenario the effect due to more information offsets the one due to informational asymmetry.

The introduction of the derivative, by increasing the amount of information purchased in the second scenario and reducing it in the first scenario, reduces informational asymmetry and modifies the direction of the informational effect in the two scenarios. This makes risk premia in the first scenario higher than those in the second scenario, both at constrained and unconstrained equilibria.

## 5. Conclusion

The main contribution of this article is to show how the effects of the introduction of a derivative depend on the market informational structure. We showed that most of the puzzling empirical results found by the financial microstructure literature can be fully explained in terms of differences in the market informational structure.

In particular, we showed that the introduction of a derivative, by changing the amount of available information, affects the propensity to invest in the risky asset of the agents, increasing the holdings of the risky underlying assets of the more informed agents and reducing the ones of the less informed agents. Also, if agents optimally choose both the assets to invest and the information to purchase, the introduction of a derivative increases the incentive to purchase information in markets characterized by high informational completeness and high asymmetry of information among the agents. Conversely, it reduces it in markets characterized by low informational completeness and low asymmetry of information.

The introduction of a derivative affects the underlying assets in different ways. It produces both effects that are invariant across different types of market information structures and effects that are different depending on the market-specific informational structure. The effects that are invariant across types of markets coincide with the few empirical results that are robust in the empirical literature, while the latter are the ones we have defined as discordant. In particular, financial innovation increases prices and reduces risk premia and volatility, while the effects on trading volume and on correlation between trading volume and asset correlation over time, depend on the market informational structure.

## Appendix A

*Proof of Theorem 1.* Let $\mathbf{m}$ represent a vector of states and $\mathbf{s}$ a vector of signals and let them follow the law of motion,

$$d(\mathbf{m}) = [\mathbf{a}_{m0} + \mathbf{a}_{m,m}\mathbf{m} + \mathbf{a}_{m,s}\mathbf{s}]\,dt + \mathbf{b}_m\,d\mathbf{z} \tag{24}$$

$$d(\mathbf{s}) = [\mathbf{a}_{s0} + \mathbf{a}_{s,s}\mathbf{s} + \mathbf{a}_{s,m}\mathbf{m}]\,dt + \mathbf{b}_s\,d\mathbf{z}, \tag{25}$$

where $\mathbf{z}$ is a vector of normal Wiener processes and $\mathbf{a}_{s0}$, $\mathbf{a}_{s,s}$, $\mathbf{a}_{s,m}$, $\mathbf{a}_{m0}$, $\mathbf{a}_{m,m}$, $\mathbf{a}_{m,s}$, $\mathbf{b}_m$, and $\mathbf{b}_s$ are matrices of constant coefficients. The induced state variables ($\hat{m}$) can be calculated as

$$d(\hat{\mathbf{m}}) = [\mathbf{a}_{m0} + \mathbf{a}_{m,m}\hat{\mathbf{m}} + \mathbf{a}_{m,s}\mathbf{s}]\,dt + [\mathbf{y}\mathbf{a}'_{mm} + \mathbf{q}_{ms}]\mathbf{q}_{ss}^{-1}\,d\hat{\mathbf{z}}, \tag{26}$$

where $\mathbf{q}_{m,m} = \mathbf{b}_s\mathbf{b}'_s$, $\mathbf{q}_{s,s} = \mathbf{b}_s\mathbf{b}'_s$, and $\mathbf{q}_{m,s} = \mathbf{b}_m\mathbf{b}'_s$, and $\mathbf{y}$ is the conditional square forecasting error of the filtering agent, equal to $\mathbf{y} = E[(\hat{\mathbf{m}} - \mathbf{m})(\hat{\mathbf{m}} - \mathbf{m})']$. It can be derived as the solution of the Riccati equation,

$$\dot{\mathbf{y}} = \mathbf{a}_{m,m}\mathbf{y} + \mathbf{y}\mathbf{a}'_{m,m} + \mathbf{q}_{m,m} - (\mathbf{y}\mathbf{a}'_{s,m} + \mathbf{q}_{m,s})\mathbf{q}_{s,s}^{-1}(\mathbf{y}\mathbf{a}'_{s,m} + \mathbf{q}_{m,s})' = 0$$

[see Lipster and Shiryayev (1984:370) and Wang (1993)].

In our case, the vector of states is $\mathbf{m} = \boldsymbol{\nu}$, while the signals are $\mathbf{s} = (D, \Sigma_S, \varphi_\pi, \varphi_{\theta_1}, \varphi_{\theta_2})$ in the one-asset case and $\mathbf{s} = (D, \Sigma_S, \Sigma_F, \varphi_\pi, \varphi_{\theta_1}, \varphi_{\theta_2})$ in the two-asset case. We focus on the steady-state solution ($\dot{\mathbf{y}} = \mathbf{0}$). Let's consider the two-asset case; the one-asset case is identical except that it has one signal fewer. Applying the optimal filtering rule we find that Equation (26) is, in our case, equal to

$$\begin{bmatrix} d\hat{\pi} \\ d\hat{\theta}_1 \\ d\hat{\theta}_2 \end{bmatrix} = \begin{bmatrix} a_\pi(\bar{\pi} - \hat{\pi}) \\ -a_{\theta_1}\hat{\theta}_1 \\ -a_{\theta_2}\hat{\theta}_2 \end{bmatrix} dt + \mathbf{H}(\mathbf{b}_s\mathbf{b}_s')^{\frac{1}{2}} d\hat{\mathbf{z}}, \tag{27}$$

where

$$d\hat{\mathbf{z}} = (\mathbf{b}_s\mathbf{b}_s')^{-\frac{1}{2}} \begin{bmatrix} dD - (\beta\pi - kD)\,dt \\ d\Sigma_S - [p_{S\pi}a_\pi(\bar{\pi} - \hat{\pi}) - p_{S\theta_1}a_{\theta_1}\hat{\theta}_1 - p_{S\theta_2}a_{\theta_2}\hat{\theta}_2]\,dt \\ d\Sigma_F - [p_{F\pi}a_\pi(\bar{\pi} - \hat{\pi}) - p_{F\theta_1}a_{\theta_1}\hat{\theta}_1 - p_{F\theta_2}a_{\theta_2}\hat{\theta}_2]\,dt \\ [d\varphi_\pi - a_\pi(\bar{\pi} - \hat{\pi})\xi_\pi]\,dt \\ [d\varphi_{\theta_1} + a_{\theta_1}\hat{\theta}_1\xi_{\theta_1}]\,dt \\ [d\varphi_{\theta_2} + a_{\theta_2}\hat{\theta}_2\xi_{\theta_2}]\,dt \end{bmatrix} \tag{28}$$

and

$$\mathbf{H} = \begin{bmatrix} h_{\pi D} & h_{\pi\Sigma_S} & h_{\pi\Sigma_F} & h_{\pi\varphi_\pi} & h_{\pi\varphi_{\theta_1}} & h_{\pi\varphi_{\theta_2}} \\ h_{\theta_1 D} & h_{\theta_1\Sigma_S} & h_{\theta_1\Sigma_F} & h_{\theta_1\varphi_\pi} & h_{\theta_1\varphi_{\theta_1}} & h_{\theta_1\varphi_{\theta_2}} \\ h_{\theta_2 D} & h_{\theta_2\Sigma_S} & h_{\theta_2\Sigma_F} & h_{\theta_2\varphi_\pi} & h_{\theta_2\varphi_{\theta_1}} & h_{\theta_2\varphi_{\theta_2}} \end{bmatrix} \tag{29}$$

is the matrix that filters the signals $(D, \Sigma_S, \Sigma_F, \varphi_\pi, \varphi_{\theta_1}, \varphi_{\theta_2})$.

**Defining the excess rate of return**

Each $j$th agent has a different perception of the law of motion of the excess return of the generic asset $x$ on the riskless bond $(Q_{jx}^j)$,

$$dQ_{jx}^j = \mathbf{e}_Q^j\boldsymbol{\psi}^j\,dt + \mathbf{b}_Q^j d\hat{\mathbf{z}}^j, \tag{30}$$

where $\hat{\mathbf{z}}^j$ is defined in Equation (28), depending on the agents' information set. Agents have different vectors of perceived state variables $(\boldsymbol{\psi}^j)$ due to their different information sets. We use the apex $f$ to define the fully informed agents, $n$ to define the non-information purchasing agents, and $i$ to define the information purchasing agents.

Each class of agents has different expectations about the states. For instance, defining $\Im^j$, the information set of the $j$th agent, in the first scenario one-asset case: $E^i_{|\Im^i}[\Delta^i_\pi] = E^i_{|\Im^i}[\Delta^i_{\theta_2}] = 0$, $E^n_{|\Im^n}[\Delta^n_\pi] = E^n_{|\Im^n}[\Delta^n_{\theta_2}] = 0$, $E^n_{|\Im^n}[\Delta^i_\pi] = E^n_{|\Im^n}[\Delta^i_{\theta_2}] = 0$. Also, $E^i_{|\Im^i}[\Delta^n_\pi] = E^i_{|\Im^i}[E^n_{|\Im^n}(\pi) - \pi] = E^n_{|\Im^n}(\pi) - E^i_{|\Im^i}(\pi)$, because $E^i_{|\Im^i}[E^n_{|\Im^n}(\pi)] = E^n_{|\Im^n}(\pi)$ and $\Im^n \subset \Im^i$. Analogously, $E^i_{|\Im^i}[\Delta^n_{\theta_2}] = E^i_{|\Im^i}[E^n_{|\Im^n}(\theta_2) - \theta_2] = E^n_{|\Im^n}(\theta_2) - E^i_{|\Im^i}(\theta_2)$, because $E^i_{|\Im^i}[E^n_{|\Im^n}(\theta_2)] = E^n_{|\Im^n}(\theta_2)$ and $\Im^n \subset \Im^i$.

Therefore it can be easily shown that $\boldsymbol{\psi}^i = (1, \hat{\theta}^i_1, \hat{\theta}^i_2, \hat{\pi}^n - \hat{\pi}^i, \hat{\theta}^n_2 - \hat{\theta}^i_2)$ and $\boldsymbol{\psi}^n = (1, \hat{\theta}^n_1, \hat{\theta}^n_2)$. Analogously it can be shown that in the first scenario two-asset case, $\boldsymbol{\psi}^i = (1, \hat{\theta}^i_1, \hat{\theta}^i_2, \hat{\pi}^n - \hat{\pi}^i)$ and $\boldsymbol{\psi}^n = (1, \hat{\theta}^n_1, \hat{\theta}^n_2)$; in the second scenario one-asset case, $\boldsymbol{\psi}^f = (1, \theta_1, \theta_2, \Delta^i_\pi, \Delta^i_{\theta_2})$ and $\boldsymbol{\psi}^n = (1, \hat{\theta}^i_1, \hat{\theta}^i_2)$, and in the second scenario two-asset case, $\boldsymbol{\psi}^f = (1, \theta_1, \theta_2, \Delta^i_\pi)$ and $\boldsymbol{\psi}^n = (1, \hat{\theta}^i_1, \hat{\theta}^i_2)$.

Therefore, in the first scenario two-asset case, by following Wang (1993), we have that

$$dQ^i_x = (D_x - rP_x)\,dt + dP_x = \left[e_{x0} + e_{x\theta_1}\hat{\theta}^i_1 + e_{x\theta_2}\hat{\theta}^i_2 + e_{x\Delta^n_\pi}(\hat{\pi}^n - \hat{\pi}^i)\right]dt + \mathbf{b}^i_x\,d\hat{\mathbf{z}}^i \tag{31}$$

for the information purchasing agents and

$$dQ^n_x = (D_x - rP_x)\,dt + dP_x = \left[e_{x0} + e_{x\theta_1}\hat{\theta}^n_1 + e_{x\theta_2}\hat{\theta}^n_2\right]dt + \mathbf{b}^n_x\,d\hat{\mathbf{z}}^n \tag{32}$$

for the fully uninformed agents, where $e_{x0} = -rp_{x0}$, $e_{x\theta_1} = -(r + a_{\theta_1})p_{x\theta_1}$, $e_{x\theta_1} = -(r + a_{\theta_2})p_{x\theta_2}$, and $e_{x\Delta_\pi^n} = -(r - a_{\Delta_\pi\Delta_\pi})p_{x\Delta_\pi^n}$, and $\mathbf{b}_x^i$, and $\mathbf{b}_x^n$ are easily derived using Equations (7), (27), (28), and (29) [see Wang (1993, Appendix A)]. In this case, $\boldsymbol{\psi}^j$ follows $d\boldsymbol{\psi}^j = \mathbf{a}_{\psi^j}\boldsymbol{\psi}^j\,dt + \mathbf{b}_{\psi^j}^j\,d\hat{\mathbf{z}}^j$, where

$$\mathbf{a}_{\psi^i} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -a_{\theta_1} & 0 & 0 \\ 0 & 0 & -a_{\theta_2} & 0 \\ 0 & 0 & 0 & a_{\Delta_\pi^n} \end{bmatrix}$$

and

$$\mathbf{b}_{\psi^i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ h^i_{\theta_1 D} & h^i_{\theta_1\Sigma_S} & h^i_{\theta_1\Sigma_S} & h^i_{\theta_1\varphi_\pi} & h^i_{\theta_1\varphi_{\theta_1}} & h^i_{\theta_1\varphi_{\theta_2}} \\ h^i_{\theta_2 D} & h^i_{\theta_2\Sigma_S} & h^i_{\theta_2\Sigma_S} & h^i_{\theta_2\varphi_\pi} & h^i_{\theta_2\varphi_{\theta_1}} & h^i_{\theta_2\varphi_{\theta_2}} \\ h^{n|i}_{\pi D} - h^i_{\pi D} & h^{n|i}_{\pi\Sigma_S} - h^i_{\pi\Sigma_S} & h^{n|i}_{\pi\Sigma_S} - h^i_{\pi\Sigma_S} & h^{n|i}_{\pi\varphi_\pi} - h^i_{\pi\varphi_\pi} & h^{n|i}_{\pi\varphi_{\theta_1}} - h^i_{\pi\varphi_{\theta_1}} & h^{n|i}_{\pi\varphi_{\theta_2}} - h^i_{\pi\varphi_{\theta_2}} \end{bmatrix}$$

for the information purchasing agents and

$$\mathbf{a}_{\psi^n} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -a_{\theta_1} & 0 \\ 0 & 0 & -a_{\theta_2} \end{bmatrix}$$

and

$$\mathbf{b}_{\psi^i} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ h^n_{\theta_1 D} & h^n_{\theta_1\Sigma_S} & h^n_{\theta_1\Sigma_S} & h^n_{\theta_1\varphi_\pi} & h^n_{\theta_1\varphi_{\theta_1}} & h^n_{\theta_1\varphi_{\theta_2}} \\ h^n_{\theta_2 D} & h^n_{\theta_2\Sigma_S} & h^n_{\theta_2\Sigma_S} & h^n_{\theta_2\varphi_\pi} & h^n_{\theta_2\varphi_{\theta_1}} & h^n_{\theta_2\varphi_{\theta_2}} \end{bmatrix}$$

for the fully uninformed agents. Stacking up the excess returns of the two assets and the asset demand of the $j$th agent, we define $\mathbf{e}_Q^j = (e_S^j, e_F^j)'$, $\mathbf{b}_Q^j = (b_S^j, b_F^j)'$, and $\mathbf{F}^j = (y_S^j, y_F^j)'$. Analogous considerations apply to the other cases.

*Proof of Theorem 2.* We can write the Hamilton–Jacobi–Bellman (HJB) equation for the generic agent $j$ as $\max_{c,y,y_F}, [L - \zeta J] = 0$, where, for notational simplicity, we avoid carrying over the superscript $j$. Also $L = -e^{-\zeta t - \gamma c} + J_W(rW - \Upsilon * \varkappa - c + \mathbf{F}'\mathbf{e}_Q\boldsymbol{\psi}) + \frac{1}{2}\mathbf{F}'\mathbf{b}_Q\mathbf{b}_Q'\mathbf{F}J_{WW} + \mathbf{F}'\mathbf{b}_Q\mathbf{b}_\psi'\mathbf{J}_{W\psi} + (\mathbf{a}_\psi\boldsymbol{\psi})'\mathbf{J}_\psi + \frac{1}{2}tr(\mathbf{b}_\psi\mathbf{J}_{\psi\psi}\mathbf{b}_\psi')$. We solve the value function by conjecturing a functional form for it ($J(W, \boldsymbol{\psi}, t) = -e^{-\zeta t - r\gamma W - V(\boldsymbol{\psi})}$) and substitute it in the HJB equation evaluated at the optimal control values for $c$ and $\mathbf{F}$. $V(\boldsymbol{\psi}) = \frac{1}{2}\boldsymbol{\psi}'\mathbf{v}\boldsymbol{\psi}$ and $\mathbf{v}$ represents the sensitivity of the agents to the states. It fully characterizes the agents' value functions. After substitution, the HJB equation can be rewritten as a continuous-time Riccati equation (CARE) of the type

$$\mathbf{L} = \mathbf{v}'\mathbf{A}\mathbf{v} + \mathbf{B}\mathbf{v} + \mathbf{v}'\mathbf{B}' + \mathbf{C} = \mathbf{0}, \tag{33}$$

where $\mathbf{A} = \mathbf{b}_\psi\mathbf{b}_\psi' - \mathbf{b}_\psi\mathbf{b}_Q'(\mathbf{b}_Q\mathbf{b}_Q')^{-1}\mathbf{b}_Q\mathbf{b}_\psi'$, $\mathbf{B} = -\mathbf{a}_\psi + \mathbf{e}_Q'(\mathbf{b}_Q\mathbf{b}_Q')^{-1}\mathbf{b}_Q\mathbf{b}_\psi' + \gamma\frac{r}{2}\mathbf{eye}$, $\mathbf{C} = \mathbf{e}_Q'(\mathbf{b}_Q\mathbf{b}_Q')^{-1} \times \mathbf{e}_Q + \boldsymbol{\Omega}_2$, and **eye** is an identity to maintain conformability. The solution of Equation (33) delivers the value function for the specific agent. The optimal consumption rule is $c = r\gamma W + \frac{1}{2}\boldsymbol{\psi}'\mathbf{v}\boldsymbol{\psi} - \log(\gamma r)$, while the optimal investment policy is $\mathbf{F} = (r\gamma\mathbf{b}_Q\mathbf{b}_Q')(\mathbf{e}_Q - \mathbf{b}_Q\mathbf{b}_\psi'\mathbf{v})\boldsymbol{\psi}$.

**Information equilibrium**

The present value of utility ($E_{0|\Upsilon_1}[-\int_0^\infty e^{-\zeta s - \gamma c}\,ds]$) is computed using the value function, as derived from the solution of the Riccati equations that define the HJB. The equations of motions of the states are simulated as $\mathbf{X}_{t+1} = \mathbf{X}_t + \mathbf{\Lambda}0 + \mathbf{\Lambda}1\, dt + \sqrt{dt}\, \mathbf{\Lambda 2 stoch}_t$, where $\mathbf{X}_t = (\pi_t, D, \theta_{1t}, \theta_{2t}, \varphi_{\pi t}, \varphi_{\theta_{1t}}, \varphi_{\theta_{2t}})$, $\mathbf{\Lambda 0} = (a_\pi \bar{\pi}, 0, 0, \xi a_\pi(\bar{\pi}), 0, 0)$, $\mathbf{\Lambda 2} = (\mathbf{b}_\pi, \mathbf{b}_D, \mathbf{b}_{\theta_1}, \mathbf{b}_{\theta_2},\ \xi_\pi \mathbf{b}_\pi + \frac{\mathbf{b}_{\varepsilon\pi}}{\sqrt{\Upsilon}}, \xi_{\theta_1}\mathbf{b}_{\theta_1} + \frac{\mathbf{b}_{\varepsilon\theta_1}}{\sqrt{\Upsilon}}, \xi_{\theta_2}\mathbf{b}_{\theta_2} + \frac{\mathbf{b}_{\varepsilon\theta_2}}{\sqrt{\Upsilon}})$, and

$$\mathbf{\Lambda 1} = \begin{pmatrix} -a_\pi & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{\theta_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{\theta_2} & 0 & 0 & 0 \\ -\xi a_\pi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\xi a_{\theta_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\xi a_{\theta_1} & 0 & 0 & 0 \end{pmatrix}.$$

The stochastic shocks ($\mathbf{stoch}_t$) are generated for each simulation as random draws from a normal distribution with zero mean and unit variance. For each source of uncertainty there are $T * N$ random entries, where $T$ is the length of the simulation period and $N$ is the number of alternative stochastic paths we want to generate, that is, the number of realizations over which we average. We set $T$ equal to 64 and $N$ equal to 10,000. $N$ is chosen so as to minimize the Monte Carlo error in the estimation. We choose a sample sufficient to provide us with Monte Carlo standard errors small enough to guarantee the robustness of the comparative results across different equilibria and market specifications. For each set of solutions of the model we use the same stochastic shocks, so as to be able to compare alternative solutions keeping the outside source of uncertainty equal across solutions. The equilibria are calculated at the point where $Udif^j <= abs(1.0e-008)$.

**Empirical statistics**

We now describe how the main variables are calculated. Again, we consider as an example the first scenario two-asset case. The other cases are solved analogously. The asset unconditional variance ($\sigma^2_{P_x}$) is $\sigma^2_{P_x} = p^{*2}_{xD}\sigma^2_D + p^{*2}_{x\pi}\sigma^2_\pi + p^2_{x\theta_1}\sigma^2_{\theta_1} + p^2_{x\theta_2}\sigma^2_{\theta_2} + p^2_{x\Delta^i_\pi}\sigma^2_{\Delta^i_\pi} + p^2_{x\Delta^n_\pi}\sigma^2_{\Delta^n_\pi} + p^2_{x\Delta^n_{\theta_2}}\sigma^2_{\Delta^n_{\theta_2}} + 2p^*_{xD}p^*_{x\pi}\sigma^2_\pi + 2p^*_{xD}p_{x\Delta^i_\pi}\sigma_{D_x\Delta^i_\pi} + 2p^*_{xD}p_{x\Delta^n_\pi}\sigma_{D\Delta^n_\pi} + 2p^*_{x\pi}p_{x\Delta^i_\pi}\sigma_{\pi\Delta^i_\pi} + 2p^*_{x\pi}p_{x\Delta^n_\pi}\sigma_{\pi\Delta^n_\pi} + 2p_{x\theta_1}p_{x\Delta^i_\pi}\sigma_{\theta_1\Delta^i_\pi} + 2p_{x\theta_1}p_{x\Delta^n_\pi}\sigma_{\theta_1\Delta^n_\pi} + 2p_{x\theta_2}p_{x\Delta^i_\pi}\sigma_{\theta_2\Delta^i_\pi} + 2p_{x\theta_2}p_{x\Delta^n_\pi}\sigma_{\theta_2\Delta^n_\pi} + 2p_{x\Delta^i_\pi}p_{x\Delta^n_\pi}\sigma_{\Delta^i_\pi\Delta^n_\pi} + 2p_{x\Delta^i_{\theta_2}}\ p_{x\Delta^n_\pi}\sigma_{\Delta^i_{\theta_2}\Delta^n_\pi}$, where $\sigma_{xy}$ is the covariance between $x$ and $y$. The conditional volatility of the main parameters ($\sigma^2_{Dx}, \sigma^2_\pi, \sigma^2_{\theta_1}, \sigma^2_{\theta_2}, \sigma^2_{\Delta^i_\pi}$, and $\sigma^2_{\Delta^n_\pi}$) and their cross-correlations ($\sigma_{\pi\Delta^i_\pi}$, $\sigma_{\theta_1\Delta^i_\pi}$, $\sigma_{\theta_2\Delta^i_\pi}$, $\sigma_{\pi\Delta^n_\pi}$, $\sigma_{\theta_1\Delta^n_\pi}$, and $\sigma_{\theta_2\Delta^n_\pi}$) are calculated [Åström (1970:65–68)] by solving the Riccati equation

$$\mathbf{AK} + \mathbf{KA}' + \mathbf{BB}' = \mathbf{0} \tag{34}$$

for $\mathbf{K}$, where $\mathbf{B} = (\mathbf{b}_D, \mathbf{b}_\pi, \mathbf{b}_{\theta_1}, \mathbf{b}_{\theta_2}, \mathbf{b}_{\Delta^i_\pi}, \mathbf{b}_{\Delta^n_\pi})'$ and

$$\mathbf{A} = \begin{bmatrix} -k & \beta & 0 & 0 & 0 & 0 \\ 0 & -a_\pi & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_{\theta_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_{\theta_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{\Delta^i_\pi} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{\Delta^n_{\theta_2}} \end{bmatrix}.$$

The autocovariance matrix of the abnormal excess returns is calculated analogously. The law of motion of the abnormal excess return for asset $x$ ($dQ^a_x = dQ_x - e_{x0}\, dt$) is Gaussian stationary. The autocovariance matrix as $E[\mathbf{e}(s)\mathbf{e}(t)'] = \mathbf{e}e^{\mathbf{C}(s-t)\mathbf{D}}\mathbf{e}'\ \ s \geq t \geq t0$, where $\mathbf{e}$ is the vector of stacked abnormal excess returns over the riskless rate, and **C** and **D** are matrices made of constants. **D** is calculated solving the Riccati equation

$$\mathbf{CD}+\mathbf{DC}'+\mathbf{EE}'=\mathbf{0}, \tag{35}$$

where

$$\mathbf{C}=\begin{bmatrix} -a_{\theta_1} & 0 & 0 & 0 & 0 \\ 0 & -a_{\theta_2} & 0 & 0 & 0 \\ 0 & 0 & a_{\Delta^i_\pi} & 0 & 0 \\ 0 & 0 & 0 & a_{\Delta^n_\pi} & 0 \end{bmatrix}$$

and $\mathbf{e}=[-p_{x\theta_1}(r+a_{\theta_1}), -p_{x\theta_2}(r+a_{\theta_2}), -p_{x\Delta^i_\pi}(r-a_{\Delta^i_\pi\Delta^i_\pi}), -p_{x\Delta^n_\pi}(r-a_{\Delta^n_\pi\Delta^n_\pi})]'$, $\mathbf{E}=[\mathbf{b}_D, \mathbf{b}_\pi, \mathbf{b}_{\theta_1}, \mathbf{b}_{\theta_2}, \mathbf{b}_{\Delta^i_\pi}, \mathbf{b}_{\Delta^n_\pi}]'$. For the $x$th asset, Trade is defined as the change in holdings, or $|\mathbf{f}^j_x\, d\boldsymbol{\psi}^j|$, while the correlation between volume and volatility can be expressed as $\mathrm{corr}(Volatility_x, Volume_x)=\mathrm{corr}(|dP_x|, |df\, \boldsymbol{\psi}^j|)=(1-\frac{2}{3.14})(1-\sqrt{1-\mathrm{corr}(dP_x, \mathbf{f}^j_x\, d\boldsymbol{\psi}^j)^2})\succeq 0$ [see Wang(1993, 1994)]. We use the definition of risk premium provided by Wang (1993). Let's define the risk premium as $\varepsilon=\frac{e}{P}$, where $e$ is the expected excess return of the stock. It is equal to $e=e_0+\mathbf{e}_{\boldsymbol{\nu}}\boldsymbol{\nu}+\mathbf{e}_\Delta\boldsymbol{\Delta}$, where $\boldsymbol{\Delta}=(\boldsymbol{\Delta}^i, \boldsymbol{\Delta}^u)$ in the first scenario and $\boldsymbol{\Delta}=(\boldsymbol{\Delta}^i)$ in the second scenario. If we define the long-run stationary level of $e$ and $P$ as $\bar{e}=e_0$ and $\overline{P}=(\phi+p_0)+(\frac{p^*_D}{k}+p^*_\pi)\bar{\pi}$, we have that the "long-run level" $\bar{\varepsilon}=\frac{\bar{e}}{\bar{P}}$. That is, long-run value of the risk premium can be calculated as the ratio between the long-run stationary level of the excess rate of return over the riskless asset ($e$) and long-run stationary level of the asset price. That is, $RiskPremium=\frac{r*\bar{\pi}}{\bar{\pi}+r*k*p_0}-r$.

## Appendix B: Model Computational Solution

The model is calibrated to empirical evidence and the benchmark parameters are chosen in order to yield returns and volatilities that look like the ones found in the empirical literature. This implies to calibrate on real data the specification of the model that describes the market before the introduction of the derivative and then to use the calibrated model to study the effects due to the introduction of the derivative. We consider the relevant empirical microstructure literature. Given that the article deals with a stylized derivative similar to a futures contract written on an index, we focus on the literature that analyzes the effects of the introduction of index futures on a stock market index. The articles we concentrate on are Harris (1989), Edwards (1988a, b), and Galloway and Miller (1997).

Regarding volatility, Harris (1989) reports an average daily return volatility for the sample of stocks belonging to the S&P 500 equal to 0.76 for the period 1975–1983 (Table 1). Edwards (1988a, b) reports a daily return volatility on the S&P 500 index equal to 0.8559 for the entire period 6/1/73–4/20/82 and equal to 0.8311 for the same period with the exclusion of the period 1979–1982. Given that restricting ourselves to the S&P 500 could be potentially misleading, we have also considered the S&P 400 index. Galloway and Miller (1997) report an average daily stock return volatility for the firms belonging to the MidCap 400 Index equal to 0.705 for the 250 trading days prior to the index futures trading. These results seem to suggest that the average daily return volatility should be between 0.75 and 0.85. In particular, we calibrate the model around a value equal to 0.81.

Regarding the expected rate of return, we separately calibrate it by assuming an annualized riskless rate equal to 5% and a risk premium equal to 6.5%. The choice of the riskless rate is based on the results of Campbell and Kyle (1993) and Veronesi (1999). They find an instantaneous riskless rate equal to 0.0041 and an annualized riskless rate equal to 5.09%. Regarding the risk premium, there is a huge literature on the topic and it has been found (depending on the sample period and the estimate) in general to be between 6% and 7%. We therefore set it to be equal to 6.5%. Finally, given that we are dealing with a futures written on an index, we assume that the index represents the market and set $\beta$ equal to one.

In the benchmark case, we consider $\gamma^i = \gamma^n = 1$. However, it is worth noticing that in a framework with CARA, the measure of risk aversion depends on the amount of wealth and must be defined jointly with it. In practice, what is really required is not so much $\gamma$, as the product between $\gamma$ and the level of wealth. Given that we standardized the amount of stocks in the economy to be equal to one unit, we implicitly determined the level of risk aversion. That is, $\gamma$ is the unitary risk aversion *per unitary level of wealth*. Therefore, $\gamma = 1$ is conditional on wealth being equal to \$1. The parameters that characterize the benchmark case are $\omega = 0.5$, $\gamma^i = \gamma^n = 1$, $\zeta^i = \zeta^n = 0.2$, $a_\pi = 0.6$, $a_{\theta_1} = 0.587$, $a_{\theta_2} = 0.5$, $b_\pi = 1.28$, $b_{\theta_1} = 1.9$, $b_{\theta_2} = 2.9$, $b_D = 0.9$, $k = 1$, $\beta = 1$, and $\bar{\pi} = 1.35$, where $i$ and $n$ stand for information purchasers and non-information purchasing agents, respectively. In order to test the robustness of the results and to analyze the effects of changes in risk aversion or the degree of concentration in the market, we also experiment with different parameters. In particular, we consider four alternative cases where either the degree of risk aversion of the agents or the degree of market concentration are varied, such that $\omega^i = 0.35$ and $\omega^n = 0.65$, $\omega^i = 0.65$ and $\omega^n = 0.35$, $\gamma^i = 1.4$ and $\gamma^n = 1$, and $\gamma^i = 1$ and $\gamma^n = 1.4$. We use a nonlinear equation solver that checks convergence using as a criterion the sum of absolute errors across equations. The criterion is set to $1e - 8$ to calculate the final values of the parameters. To check the robustness of the estimations we also perturb the solution for the benchmark case, randomly changing some starting values, to see if we would land on the same solution. In all the cases, we recover the original solutions.

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