# The Impact of Debt Financing on Entry and Exit in a Duopoly

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This article investigates the interaction between market entry, company foreclosure, and capital structure in a duopoly. We find that the order in which firms foreclose is determined not only by differences in firm-specific factors, but also by common economic factors, such as the interest rate and the market profit volatility. We extend the exit model by allowing financially distressed firms to renegotiate their debt contracts through a one-off debt exchange offer. We find that firms with high bankruptcy costs or with prospects of profit improvement can get bigger reductions on their debt repayments. Investigating market entry, we find that financial vulnerability of the incumbent induces earlier entry.

Firms come and go, and the death of one firm may help the survival of another. One can almost regard this as a natural law of economics. Corporate bankruptcies, not surprisingly, have led to extensive research into the causes of business failure. Generally speaking, these causes can be categorized into aggregate economic factors and firm-specific factors.

Altman (1971) examines the influence of aggregate economic factors on business failure during the period 1947–1970. He finds that a firm's propensity to fail is heightened during periods of reduced economic growth, poor stock market performance, and unfavorable money supply conditions.

There are many different firm-specific factors that can cause businesses to fail, but the vast majority are, in one way or another, linked to inefficiency, competition, or the firm's financial structure. Inefficiencies result from poor management decisions and they adversely affect operating profits. Dun and Bradstreet (1980b) report that more than 90% of all failures result, directly or indirectly, from one type of inefficiency or another. Competition is often mentioned as another cause of bankruptcy. For example, an inefficient monopolist that is protected by a patent may be less likely to go bankrupt than a firm facing cutthroat competition. The market structure in which the

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firm operates is therefore an important determinant of firm entry and exit. Competition between firms weeds out inefficient and poorly managed companies and creates space for new and more innovative firms. More recently, the finance literature has stressed the importance of financial variables such as high leverage, the lack of financial resources, and liquidity problems. As such, an efficient firm can still fail, simply because of bad financial management. Brigham and Gapenski (1997) note that a recent Dun and Bradstreet compilation attributes 47% of business failures to financial factors. In conclusion, on a firm-specific level, business failure is linked to a firm's real decisions, competitive environment and financing decisions.

The difficulty is that, in reality, business failure is rarely caused by one single economic or firm-specific factor. More often, failure is the result of a complex interaction between various causes. It is only recently, however, that researchers have tried to unravel the intricacies of this interaction. For instance, several recent empirical studies seem to indicate that there exists a negative relationship between leverage and investment (or output levels), that is, product-market competition becomes "softer" when leverage increases. Chevalier (1995), examining exit decisions in the supermarket industry, finds that unleveraged firms are more likely to open stores and less likely to exit in markets where competitors have recently experienced a leveraged buyout. Kovenock and Philips (1997) extend previous studies by including both market structure and plant-level efficiency as determinants of investment and plant closing decisions, and find both variables to be highly significant. They find that highly leveraged firms in industries with high concentration are more likely to close plants and are less likely to increase investment. Rival firms are less likely to close plants and more likely to invest when the market share of leveraged firms is higher. Zingales (1998) studies the survival of trucking companies (during the 8 years after the Carter deregulation) as a function of their economic efficiency (fitness) and their financial resources (fatness). He finds evidence that more efficient firms are more likely to survive after deregulation, but he also discovers that their leverage at the beginning of the deregulation period has a negative impact on the probability of survival 8 years later. Therefore not only the "fittest" but also the "fattest" firms survive. Finally, Dun and Bradstreet (1980a,b) find that young firms have a more vulnerable capital structure and are more prone to bankruptcy than older more established firms.

In parallel to the empirical literature, a number of theoretical articles have investigated the link between a firm's real (dis)investment decisions, financing decisions, and market structure. One strand of articles [eg., Dotan and Ravid (1985), Brander and Lewis (1986, 1988), Fudenberg and Tirole (1986a), Maksimovic (1988), Poitevin (1989), and Bolton and Scharfstein (1990), among others] ties in with the microfinance literature [see Ravid (1988) for a review] and has been investigating the impact of market structure on the interaction between financing and production decisions.[1,2] Some of these articles suggest that product-market competition becomes "softer" when leverage increases [e.g., Fudenberg and Tirole (1986a), Poitevin (1989), Bolton and Scharfstein (1990), and Faure-Grimaud (1998)], while other articles argue that leverage changes managerial and shareholder incentives in a way that makes product-market competition "tougher" [e.g., Brander and Lewis (1986), Maksimovic (1988), Fulghieri and Nagarajan (1996)]. It should be noted that these models are essentially static one-period models, where production decisions are made at the beginning of the period and profits are realized at the end of the period. Entry and exit decisions of firms are not explicitly modeled.[3] Furthermore, since these models are static, it is not obvious how they could be used for pricing purposes.

Another strand of articles has focused on modeling investment and bankruptcy decisions within a dynamic contingent claims analysis framework. Brennan and Schwartz (1984) and Mello and Parsons (1992) study contingent claims models in which real investment and closure decisions interact with debt valuation. Dixit (1989) studies entry and exit decisions under all equity financing. Fisher, Heinkel, and Zechner (1989) analyze a firm's optimal dynamic financing policy in the presence of recapitalization costs. Mauer and Triantis (1994) analyze the interaction between a firm's dynamic investment, operating, and financing decisions in a model with operating adjustments and recapitalization costs. Others model firm foreclosure and its impact on optimal capital structure and bond prices [e.g., Leland (1994) and Leland and Toft (1996) for infinite and finite maturity debt, respectively]. One common feature of all these models is that they implicitly assume that the firm operates in a monopolistic environment. Fries, Miller, and Perraudin (1997) study the other polar case of entry and exit under perfect competition and its implication for the pricing of corporate debt. The model has similarities with the Leahy (1993) model, which studies competitive entry and exit under all equity financing.

Surprisingly the intermediate oligopoly case has been little studied, despite the fact that it is the most common market structure.[4] The study of business failure in an oligopoly differentiates itself from the other two polar

[1] Fulghieri and Nagarajan (1996) focus on entry rather than on the production decision. They examine the strategic role of high leverage and bankruptcy threats in deterring entry into monopolistic markets.

[2] A number of game-theoretical articles model firm exit under all equity financing within a deterministic framework, where time is the state variable and assuming either complete [Ghemawat and Nalebuff (1985, 1990)] or incomplete information ('war of attrition' models) [Fudenberg and Tirole (1986b)]. The exit game developed in this article relates most closely to Ghemawat and Nalebuff (1985).

[3] The usual assumption is that firms default if operating profits are below the debt obligations [e.g., Brander and Lewis (1986, 1988)]. This ignores the important fact that the equity holders have the option to keep the firm going by injecting cash. Empirical observation confirms that firms which make losses often continue to operate for quite a long time.

[4] Smets (1991) and Grenadier (1996) present Stackelberg leader-follower models of entry under all equity financing, and do not consider bankruptcy.

cases in that it requires the explicit modeling of strategic behavior *between* firms. Although strategic behavior has recently been introduced into contingent claims models of corporate default, the focus has been on modeling conflicts of interests between different stakeholders *within* a financially distressed firm. The motivation for these models lies in the empirical observation [see Franks and Torous (1989, 1994)] that financially distressed firms often manage to renegotiate their claims and that it is not unusual for equity holders or junior creditors to obtain concessions from senior creditors. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) present a model of strategic debt service by equity holders, while Mella-Barral (1999) develops a model of dynamic debt restructuring.

This article presents a dynamic contingent claims model of strategic entry and exit in a duopoly. When determining the entry and exit thresholds, strategic behavior between the firms is explicitly modeled. The model allows us to analyze the order in which firms go bankrupt within a given industry, and the extent to which this order is influenced by aggregate economic factors and firm-specific factors. In addition, we allow financially distressed firms to decrease their debt through take-it-or-leave-it debt exchange offers made by the equity holders to the bank. Our model therefore also allows for strategic behavior between the stakeholders of a given firm. We analyze whether the terms of the debt exchange (i.e., the size of the concessions made by the bank) depend on the market structure in which the firm operates. We also examine whether debt reorganizations can influence the order in which firms foreclose and hence a firm's prospects of survival. Answers to these questions may provide us with some understanding of why debt reorganizations help some firms to survive while for other firms they are merely a delay of execution.

We obtain the following main results. In the case of market exit, we find that higher operating profits, lower debt repayments, larger incremental gains from becoming a monopolist, and higher bankruptcy costs give a firm a competitive advantage versus its competitors in the battle for survival. Moreover, the extent to which these firm-specific determinants can be substituted for one another depends on factors common to all firms, such as the interest rate, profit growth, and profit volatility. Changes in the general economic environment can therefore reverse the order in which firms are expected to foreclose. Turning to market entry, we find that a follower's need to borrow money tends to delay its entry, whereas any financial vulnerability of the incumbent firm tends to speed up entry. The latter finding supports those articles which argue that leverage makes product-market competition softer.

The structure of the article is as follows. In Section 1, I develop a benchmark case by analyzing the valuation of a monopolistic firm which has an option to leave the market if profits decline. In Section 2, I focus on the duopoly case, and derive an equilibrium condition that allows us to determine the order in which firms foreclose when market conditions deteriorate.

Section 3 extends the model to the oligopoly case. In Section 4, I allow financially distressed firms to get debt relief through a one-off debt exchange offer made by the equity holders to the bank. I analyze how the option to reduce debt affects the firms' exit strategies. Section 5 completes the model by analyzing the decision of a firm to enter into a market that has one active firm. We investigate how the timing of entry is influenced by the cost of entry and by the incumbent's and the entrant's capital structure. We try to shed some new light on whether increased leverage makes product-market competition "tougher" or "softer." We conclude by examining whether the youngest (entrant) or the oldest (incumbent) firm dies first whenever the market declines.

## 1. Firm Valuation in a Monopoly

In this section we model the foreclosure decision of a levered monopolist who operates in a market with uncertain operating profits. The model is similar to the one developed by Fries, Miller, and Perraudin (1997), Mella-Barral (1999), and Mella-Barral and Perraudin (1997), among others, and it will serve as a simple benchmark for our model of exit in a duopoly, developed in Section 2.

Consider a firm that generates a net operating income of $\Pi x_t$, per period, where $\Pi$ is a strictly positive constant and $x_t$ follows a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dB_t, \tag{1}$$

for constants $\mu < r$, $\sigma > 0$, $\mu < \frac{\sigma^2}{2}$, and a standard Brownian motion, $B_t$.[5] The state variable $x_t$ represents the impact of aggregate economic factors on operating profits, while $\Pi$ is a firm-specific profit parameter that depends on the firm's efficiency and the market structure.

Next, suppose that the firm has entered into a debt contract under which bondholders receive a perpetual flow of coupon payments, $b$, until the firm goes bankrupt. Assume further that investors are risk neutral and that there exists a risk-free asset yielding a constant interest rate, $r$, at which investors may borrow or lend freely. Under these conditions the value of the firm's equity and debt must satisfy, respectively, the following equations:

$$rV_t = (\Pi x_t - b) + \frac{d}{d\Delta}\mathrm{E_t} V_{t+\Delta}\Big|_{\Delta=0}, \tag{2}$$

$$rL_t = b + \frac{d}{d\Delta}\mathrm{E_t} L_{t+\Delta}\Big|_{\Delta=0}. \tag{3}$$

[5] The condition $\mu < \frac{\sigma^2}{2}$ ensures that the expected time to hit the bankruptcy threshold, $\underline{x}$, is finite. This implies, for example, that in the deterministic case the profit growth rate needs to be negative.

Under the assumption that $V_t$ and $L_t$ are twice continuously differentiable functions of $x_t$, application of Ito's lemma gives

$$rV(x_t) = (\Pi x_t - b) + \mu x_t V'(x_t) + \frac{\sigma^2}{2} {x_t}^2 V''(x_t),$$

$$rL(x_t) = b + \mu x_t L'(x_t) + \frac{\sigma^2}{2} {x_t}^2 L''(x_t). \tag{4}$$

The solution for $V$ and $L$ can then be determined by imposing the relevant boundary conditions. Some boundary conditions specify the value of the equity and debt in case the firm goes bankrupt. A first value-matching condition states that the equity has no value when the equity holders file for bankruptcy, that is, $V(\underline{x}) = 0$. Next, bankruptcy stops the flow of interest repayments, but the bank receives instead $X(x_t)$, the residual value of the firm in liquidation, and consequently the following value-matching condition needs to be satisfied: $L(\underline{x}) = X(\underline{x})$. Finally, bankruptcy is triggered when the equity holders do not wish to cover losses any longer. Indeed, since equity holders could always inject capital to cover losses, the closure trigger, $\underline{x}$, is effectively their choice, and it will be set at the level which maximizes the equity value. The optimal foreclosure point, $\underline{x}$, satisfies the following "smooth-pasting" condition: $V'(\underline{x}) = 0$. By combining the above boundary conditions and imposing the "no-bubble" condition, it is straightforward to prove the following proposition.

***Proposition 1.*** *The nonstrategic value of the firm's equity and debt are, respectively,*

$$V(x_t, b, \Pi, \underline{x}) = \left\{ \frac{\Pi x_t}{r-\mu} - \frac{b}{r} + \left[ \frac{b}{r} - \frac{\Pi \underline{x}}{r-\mu} \right] \left( \frac{x_t}{\underline{x}} \right)^{\lambda} \right\} \tag{5}$$

$$L(x_t, b, \Pi, \underline{x}) = \frac{b}{r} - \left[ \frac{b}{r} - X(\underline{x}) \right] \left( \frac{x_t}{\underline{x}} \right)^{\lambda} \tag{6}$$

*where $\lambda$ is the negative root of the characteristic equation $\lambda(\lambda-1)\sigma^2/2 + \mu\lambda = r$. Bankruptcy occurs the first time when $x_t$ falls below $\underline{x} = \frac{-\lambda b(r-\mu)}{(1-\lambda)\Pi r}$.*

The equations for the value of the equity and the debt have simple intuitive interpretations. The equity price equals the discounted value of all future operating profits, minus the discounted value of all future interest payments, plus an option to foreclose the company. This latter option ensures that the equity price remains positive, which reflects the limited liability of the equity holders. The value of the risky debt equals the value of the risk-free debt (i.e., the discounted value of a perpetual stream of interest payments), minus a risk premium which reflects the fact that, when the company goes bankrupt, all future coupon payments are canceled and the bank acquires the liquidation value of the firm instead. The total firm value is then given by $W(x_t, b, \Pi, \underline{x}) \equiv V(x_t, b, \Pi, \underline{x}) + L(x_t, b, \Pi, \underline{x})$.

## 2. Firm Valuation in a Duopoly

We now proceed to the more general case where there are two competing firms in the market. In order to value both firms, we will have to address the following three questions. First, which firm will leave the market first? Second, when should the loser optimally leave the market, and when should the monopolist follow suit? Third, what are the values of the losing and winning firms? We solve these questions in reverse order. We first determine the firm values when their exit strategies are exogenously given. Next, we derive the optimal foreclosure times when the roles of the winner and the loser are exogenously assigned. Third, we determine endogenously which firm will leave the market first.

Let us first specify the conditions under which the firms are operating by introducing the following assumptions.

***Assumption 1.*** *There are two firms in the market that each receive profits after interest repayments of $\pi_k x_t - b_k$ $(k = i, j)$. If one of the firms leaves the market (exit is irreversible), then the remaining firm receives per period a total net monopoly profit of $\Pi_k x_t - b_k$ until it leaves the market, where $\pi_k \leq \Pi_k$.*

***Assumption 2.*** *If both firms decide to default at the same time, then each firm either acquires the right to become a monopolist, with probability* $1/2$, *or else, has to leave the market.*

***Assumption 3.*** *Both firms have complete information with respect to all model parameters (including their opponent's).*

Assumption 1 states that firms are better off operating in a monopoly than in a duopoly. Assumption 2 is merely a decision rule to resolve ties. Assumption 3 means that both agents know everything about themselves and each other. Next, we introduce the following assumption about the exit strategy space of the firms.

***Assumption 4.*** *Each firm is restricted to a single exit trigger strategy, such that the firm stays in the market if the state variable $x_t$ is above this trigger, and leaves the market if the state variable is below (or at) this trigger. At the start of the game, the state variable is high enough to ensure that both firms want to stay in the market.*[6]

The above assumption greatly simplifies the analysis, as it rules out mixed strategy equilibria or strategies where the firm can abandon the market in an upward movement of the state variable. Our choice of single trigger strategies can be justified within the context of this article on at least two grounds.

[6] This means more specifically that $x_0 > \max[\underline{x}_{jd}, \underline{x}_{id}]$, where $\underline{x}_{jd}$ and $\underline{x}_{id}$ are the firms' duopoly exit thresholds as defined below.

First, they are very simple strategies, requiring firms to have only a very low level of rationality. Second, we also want to examine in this article the impact of debt reorganization and market entry on firm foreclosure in a duopoly. Extensions of this type would become intractable if we do not restrict the analysis to simple exit strategies.

The value of each firm will depend on whether it ultimately ends up as a monopolist or whether it forecloses first. In what follows, we denote the firm in the former (latter) case as the "winner" ("loser"), and we use the corresponding subscript "$w$" ("$l$"). Let us assume, for the moment, that firm $j$ leaves the market first at an exogenously given trigger $\underline{x}_j$, while $i$ follows afterwards at $\underline{x}_i$, where by assumption $\underline{x}_i < \underline{x}_j$. Hence firm $j$ receives the duopoly profits $\pi_j x_t - b_j$ per period until bankruptcy at $\underline{x}_j$, at which point the firm is liquidated. This corresponds exactly to the scenario in the previous section, and the value of the firm is therefore similar to the one given in Proposition 1, except for the fact that firm $j$'s bankruptcy trigger, $\underline{x}_j$, still needs to be determined.

Firm $i$ receives the duopoly profits $\pi_i x_t - b_i$ until the state variable $x_t$ drops to $\underline{x}_j$. From then onward, firm $i$ receives the monopoly profits $\Pi_i x_t - b_i$ until it leaves the market at $\underline{x}_i$. The proposition below gives the equity and debt values for the winning and losing firm when the exit triggers are exogenously given.

***Proposition 2.*** *If firm $j$ is the duopoly firm that is expected to foreclose first, then the value of its equity and debt are given by*

$$\begin{aligned} V_{jl}(x_t, b_j, \pi_j, \underline{x}_j) &= V(x_t, b_j, \pi_j, \underline{x}_j), \\ L_{jl}(x_t, b_j, \pi_j, \underline{x}_j) &= L(x_t, b_j, \pi_j, \underline{x}_j). \end{aligned} \tag{7}$$

*If firm $i$ is the duopoly firm that is expected to foreclose last, then the value of its equity and debt are given by*

$$\begin{aligned} V_{iw}(x_t, b_i, \pi_i, \underline{x}_i, \underline{x}_j) = {} & \left\{ \left( \frac{\pi_i x_t}{r-\mu} - \frac{b_i}{r} \right) + \left[ \frac{(\Pi_i - \pi_i)\underline{x}_j}{r-\mu} \right.\right. \\ & \left.\left. + \left( \frac{b_i}{r} - \frac{\Pi_i \underline{x}_i}{r-\mu} \right) \left( \frac{\underline{x}_j}{\underline{x}_i} \right)^{\lambda} \right] \left( \frac{x_t}{\underline{x}_j} \right)^{\lambda} \right\} \qquad \text{for } x_t \geq \underline{x}_j, \end{aligned}$$

$$V_{iw}(x_t, b_i, \pi_i, \underline{x}_i, \underline{x}_j) = \left[ \frac{\Pi_i x_t}{r-\mu} - \frac{b}{r} + \left( \frac{b_i}{r} - \frac{\Pi_i \underline{x}_i}{r-\mu} \right) \left( \frac{x_t}{\underline{x}_i} \right)^{\lambda} \right] \qquad \text{for } x_t < \underline{x}_j,$$

$$L_{iw}(x_t, b_i, \Pi_i, \underline{x}_{im}) = \frac{b_i}{r} \left[ 1 - \left( \frac{x_t}{\underline{x}_i} \right)^{\lambda} \right] + X_i(\underline{x}_i) \left( \frac{x_t}{\underline{x}_i} \right)^{\lambda}. \tag{8}$$

*Firm $j$ leaves the market first. This happens when the state variable falls to $\underline{x}_j$. Firm $i$ defaults afterwards when $x_t$ falls further to $\underline{x}_i$ (with $\underline{x}_j > \underline{x}_i$). The functions $V$ and $L$ are as defined in Proposition* 1.

Next, we derive the optimal trigger values for $\underline{x}_j$ and $\underline{x}_i$, at which firms $j$ and $i$ exit first and second, respectively, under the assumption that the roles of the winner and the loser are exogenously assigned. The optimal value for $\underline{x}_i$ can be found be maximizing $V_{iw}(x_t, b_i, \pi_i, \underline{x}_i, \underline{x}_j)$ with respect to $\underline{x}_i$, or equivalently, by applying the smooth-pasting condition: $\frac{\partial V_{iw}(x_t, b_i, \pi_i, \underline{x}_i, \underline{x}_j)}{\partial x_t}|_{x_t=\underline{x}_i} = 0$. Irrespective of whether $x_t$ is smaller or greater than $\underline{x}_j$, this gives

$$\underline{x}_i = \frac{-\lambda b_i(r-\mu)}{(1-\lambda)\Pi_i r} \equiv \underline{x}_{im}. \tag{9}$$

Note that the trigger is time consistent (i.e., the trigger is optimal for $x_t \geq \underline{x}_j$, as well as for $x_t < \underline{x}_j$) and independent of firm $j$'s exit trigger. Furthermore, the winning firm leaves at the same time as it would do if it were a monopolist. This should not come as a surprise, as firm $i$'s decision to leave the market is made *after* its rival has left the market.

We will show below that if $j$ has to leave the market first, it will do so at the trigger which optimizes its duopoly equity value $V_{jl}$. The intuitive explanation is that, under complete information, there is no rational reason for $j$ to back out at a later point, given that it cannot beat its rival. Combining the results above leads to the following proposition.

***Proposition 3.*** *If firm $j$ leaves the market before $i$, then $j$ leaves as soon as the profit variable $x_t$ falls below its duopoly exit trigger*

$$\underline{x}_{jd} \equiv \frac{-\lambda b_j(r-\mu)}{(1-\lambda)\pi_j r}, \tag{10}$$

*while firm $i$ leaves when $x_t$ falls below its monopoly exit trigger*

$$\underline{x}_{im} \equiv \frac{-\lambda b_i(r-\mu)}{(1-\lambda)\Pi_i r}. \tag{11}$$

Finally, we need to determine which firm forecloses first in equilibrium. We first determine the firms' optimal reaction functions. The optimal reaction function for firm $i$, for example, states whether, for a given exit threshold of firm $j$, $\underline{x}_j$, firm $i$ prefers either to leave first at $\underline{x}_{id}$ or else to outlive its rival. The equity value of staying ($V_{iw}$) is strictly increasing in the rival's exit trigger and is plotted in Figure 1 for decreasing values of $\underline{x}_j$. The solid line represents the equity value of firm $i$ as a monopolist. The equity value when leaving first at $\underline{x}_{id}$ ($V_{il}$) is represented by the double-dotted line and is zero for $x_t \leq \underline{x}_{id}$. If $i$'s rival is expected to leave early (i.e., $\underline{x}_j$ is high), then firm $i$ will find it worthwhile to postpone its foreclosure, if necessary, in order to safeguard a more profitable monopoly position. If, on the other hand, firm $j$ is due to leave only in the very distant future (i.e., $\underline{x}_j$ is low), then firm $i$ will prefer to leave first, as its equity holders will not wish to cover losses for such a long time. Given that $V_{iw}$ is monotonically increasing in $\underline{x}_j$ and

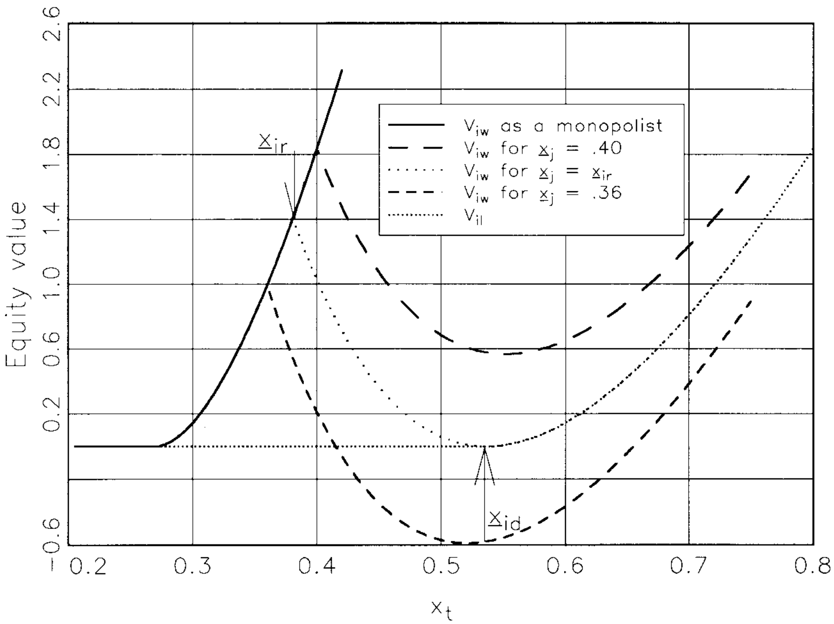


**Figure 1**
**Firm $i$'s equity value under varying exit scenarios**
The three U-shaped curves represent firm $i$'s equity value ($V_{iw}$) in a duopoly as a function of the profit variable ($x_t$) for various levels of the rival's exit trigger ($\underline{x}_j$). The double dotted line represents firm $i$'s equity value ($V_{il}$) if it leaves the market before its rival, on the first occasion when the profit variable falls below $\underline{x}_{id}$. The solid line is firm $i$'s equity value as a monopolist. The reservation threshold ($\underline{x}_{ir}$) is the value of $\underline{x}_j$ for which firm $i$ is indifferent between leaving first at $\underline{x}_{id}$ and waiting until the rival leaves. The parameter values for this figure are $\mu = 0.0$, $r = 0.05$, $\sigma = 0.2$, $b_i = 1$, $\pi_i = 1$, and $\Pi_i = 2$.

since $V_{iw}$ and $V_{il}$ have the same polynomial form, then, using a continuity argument, there must be a threshold, $\underline{x}'_j$, for which $i$ is indifferent between leaving first or becoming a monopolist at $\underline{x}'_j$, that is, $\exists \underline{x}'_j$ such that

$$\forall x_t \geq \underline{x}_{id} : V_{il}(x_t, b_i, \pi_i, \underline{x}_{id}) = V_{iw}(x_t, b_i, \pi_i, \Pi_i, \underline{x}_{im}, \underline{x}'_j)$$
$$\text{with} \quad \underline{x}_{id} \geq \underline{x}'_j \geq \underline{x}_{im} (i, j = 1, 2; i \neq j). \tag{12}$$

Substituting for $V_{il}$ and $V_{iw}$ in Equation (12) and simplifying gives

$$\underline{x}'_j = \left[ \frac{(r-\mu)b_i}{(\Pi_i - \pi_i) r (1-\lambda)} \left( \underline{x}_{id}{}^{-\lambda} - \underline{x}_{im}{}^{-\lambda} \right) \right]^{\frac{1}{1-\lambda}} \equiv \underline{x}_{ir}. \tag{13}$$

By construction, agent $i$ is indifferent between leaving first at $\underline{x}_{id}$ or becoming a monopolist at $\underline{x}_{ir}$. Hence $\underline{x}_{ir}$ can be interpreted as firm $i$'s "reservation" threshold, as it gives a measure of how long agent $i$ would be willing to make losses and to hold out in order to become a monopolist.[7]

It is now straightforward to deduce firm $i$'s reaction function. First, for all $\underline{x}_j$ greater than $\underline{x}_{ir}$, it is optimal for firm $i$ to stay in the market until firm $j$ leaves. This strategy is time consistent as it remains optimal when $x_t$ falls below $\underline{x}_{jd}$. Indeed, $\forall x_t < \underline{x}_{id}$ : $V_{iw}(x_t, b_i, \pi_i, \Pi_i, \underline{x}_{im}, \underline{x}_j) > V_{il}(x_t, b_i, \pi_i, \underline{x}_{id}) = 0$ (in fact, Figure 1 shows that $V_{iw}$ *increases* when $x_t$ falls to $\underline{x}_j$ as the probability of $i$ becoming a monopolist goes up). Next, $\forall \underline{x}_j < \underline{x}_{ir}$ : $V_{iw}(x_{il}, b_i, \pi_i, \Pi_i, \underline{x}_{im}, \underline{x}_j) < V_{il}(\underline{x}_{id}, b_i, \pi_i, \underline{x}_{id}) = 0$, and hence it is optimal for firm $i$ to leave at $\underline{x}_{id}$. Finally, if $\underline{x}_j = \underline{x}_{ir}$, then, strictly speaking, firm $i$ is indifferent between leaving first at $\underline{x}_{id}$ or becoming, *with probability one*, a monopolist at $\underline{x}_{ir}$. However, if $\underline{x}_j$ equals $\underline{x}_{ir}$, then there is a tie in which case firm $i$ becomes a monopolist only with probability 1/2 (according to Assumption 2). Consequently, firm $i$ will strictly prefer to leave first. In summary, for a *given* exit trigger $\underline{x}_j$ for firm $j$, firm $i$'s optimal response is as follows: if $\underline{x}_j \leq \underline{x}_{ir}$, then firm $i$ strictly prefers to leave first at $\underline{x}_{id}$, while if $\underline{x}_j > \underline{x}_{ir}$ then firm $i$ will wait till firm $j$ leaves first.

The reaction functions allow us to identify the Nash equilibria of the game. We can consider three cases. In the first case, firm $i$ "strictly dominates" firm $j$. This means either that $j$'s monopoly tenure is too short to deter $i$ from staying (i.e., $\underline{x}_{ir} < \underline{x}_{jm}$) or else firm $i$'s duopoly tenure is long enough to deter firm $j$ from staying in the market (i.e., $\underline{x}_{id} \leq \underline{x}_{jr}$). In either case there is a unique Nash equilibrium which involves $j$ leaving first. Indeed, when $\underline{x}_{ir} < \underline{x}_{jm}$, then, even if $j$ were to stay till $\underline{x}_{jm}$, it is still optimal for $i$ to stay till $\underline{x}_{im}$. Hence, since $\underline{x}_{im} < \underline{x}_{jr}$, it is optimal for $j$ to cut its losses and to leave at $\underline{x}_{jd}$. Similarly, if $\underline{x}_{id} \leq \underline{x}_{jr}$, then $i$ is, at the earliest, going to leave at $\underline{x}_{id}$. Consequently $j$ should leave first at $\underline{x}_{jd}$. An analogous reasoning can be made for the second case where firm $j$ is strictly dominant and $i$ leaves first, that is, when $\underline{x}_{jr} < \underline{x}_{im}$ or if $\underline{x}_{jd} \leq \underline{x}_{ir}$. In the third and intermediate case, neither firm is strictly dominant, that is, $\underline{x}_{jm} \leq \underline{x}_{ir} < \underline{x}_{jd}$ and $\underline{x}_{im} \leq \underline{x}_{jr} < \underline{x}_{id}$. In this case there are two Nash equilibria: $j$ leaving first at its duopoly threshold, and $i$ staying, and vice versa. Indeed, if $i$ could precommit to stay till its monopoly threshold ($\underline{x}_{im}$) is hit, then it is optimal for $j$ to leave first, while if $j$ could precommit to stay till $\underline{x}_{jm}$, then the optimal Nash response for $i$ is to leave first. The two Nash equilibria may, however, not be equally appealing as one of the equilibria may involve the "weaker" firm forcing out the "stronger." Moreover, if the stronger firm somehow fails to leave immediately at its duopoly exit trigger, the weaker firm's plan to stay may no longer be optimal. We therefore refine the Nash equilibrium concept by imposing the requirement of subgame perfection. This condition

[7] It is important to note that $\underline{x}_{ir}$ is not really a trigger as no agent necessarily acts at that point. Note also that $\underline{x}_{ir}$ is the value for $\underline{x}_j$ for which the value function $V_{iw}$ equals $V_{il}$ and therefore smooth pastes to the zero value line at $\underline{x}_{id}$, that is, $\underline{x}_{ir}$ is also the solution to $\frac{\partial V_{iw}(x_t, b_i, \pi_i, \Pi_i, \underline{x}_{im}, \underline{x}_{ir})}{\partial x_t}\Big|_{x_t = \underline{x}_{id}} = 0$.

dictates that the equilibrium needs to be Nash in all possible subgames. Using a modified version (see appendix) of the backward induction argument used by Ghemawat and Nalebuff (1985), we show that only one of the two Nash equilibria is subgame perfect. Similar to the deterministic exit game in Ghemawat and Nalebuff (1985), we find that, for our stochastic exit game, the subgame perfect equilibrium corresponds to the firm with the lowest monopoly exit threshold leaving last.[8] We can summarize the above results in the following proposition:

***Proposition 4.*** *If firm $i$ strictly dominates firm $j$ (i.e., $\underline{x}_{ir} < \underline{x}_{jm}$ or $\underline{x}_{id} \leq \underline{x}_{jr}$), then there is a unique Nash equilibrium whereby firm $j$ leaves first. If neither firm strictly dominates the other (i.e., $\underline{x}_{jm} \leq \underline{x}_{ir} < \underline{x}_{jd}$ and $\underline{x}_{im} \leq \underline{x}_{jr} < \underline{x}_{id}$), then there are two Nash equilibria: firm $i$ $(j)$ leaving first at $\underline{x}_{id}$ $(\underline{x}_{jd})$ and firm $j$ $(i)$ leaving second. Only one of the two Nash equilibria is subgame perfect, namely the equilibrium which involves the firm with the highest monopoly exit threshold leaving first.*

Combining the above cases, simplifying and expressing them in terms of the underlying model parameters, one can characterize the unique equilibrium described in the above proposition by the following decision rule:

***Proposition 5.*** *Firm $j$ leaves the market first if and only if ($\underline{x}_{id} \leq \underline{x}_{jr}$) or ($\underline{x}_{im} < \underline{x}_{jm}$ and $\underline{x}_{ir} < \underline{x}_{jd}$), or equivalently if*[9]

$$\frac{b_i}{b_j} < \max\left[\pi_i\left(\frac{\pi_j{}^{\lambda}-\Pi_j{}^{\lambda}}{-\lambda(\Pi_j-\pi_j)}\right)^{\frac{1}{1-\lambda}}, \min\left[\frac{\Pi_i}{\Pi_j}, \frac{1}{\pi_j}\left(\frac{-\lambda(\Pi_i-\pi_i)}{\pi_i{}^{\lambda}-\Pi_i{}^{\lambda}}\right)^{\frac{1}{1-\lambda}}\right]\right] \tag{14}$$

$$\equiv \max\left[\kappa_1, \min[\kappa_2, \kappa_3]\right] \equiv \Theta(\pi_i, \pi_j, \Pi_i, \Pi_j). \tag{15}$$

*Furthermore, since $\kappa_1 \leq \kappa_3$, the three possible regimes for $\Theta$ arise as follows: $\Theta = \kappa_1 \iff \kappa_2 \leq \kappa_1 \leq \kappa_3$, $\Theta = \kappa_2 \iff \kappa_1 \leq \kappa_2 \leq \kappa_3$ and $\Theta = \kappa_3 \iff \kappa_1 \leq \kappa_3 \leq \kappa_2$. Hence, if $\frac{b_i}{b_j} < \kappa_1$, then $i$ strictly dominates, while if $\frac{b_i}{b_j} > \kappa_3$, then $j$ strictly dominates. If $\kappa_1 < \frac{b_i}{b_j} < \kappa_3$ then neither firm strictly dominates.*

From proposition 5 we can deduce that the order in which firms foreclose is determined not only by differences in firm-specific factors such as operating profits, debt levels, and the incremental gain of becoming a monopolist, but (through $\lambda$) also by economic factors which both firms have in common, such as the interest rate, profit growth rate, and the profit volatility.

[8] If the firms have the same monopoly exit trigger (i.e., $\underline{x}_{im} = \underline{x}_{jm}$) and neither firm strictly dominates, then there are two subgame perfect equilibria, corresponding to one or the other firm leaving first. For clarity of exposition, we do not consider this knife-edge case in the main body of the article. The interested reader is referred to the appendix.

[9] The equivalence is not entirely exact as we do not account for the case where $\frac{b_i}{b_j} = \Theta$. In this case the outcome depends on which constraint is binding: $i$ wins if $\underline{x}_{id} = \underline{x}_{jr}$, $j$ wins if $\underline{x}_{ir} = \underline{x}_{jd}$ and for $\underline{x}_{im} = \underline{x}_{jm}$ there are two equilibria if neither agent strictly dominates. We leave these borderline cases out of the proposition for clarity of exposition.

Let us first concentrate on the firm-specific factors. It is clear that, everything else being equal, the firm with the highest coupon level will default first. In other words, the firm with the most financial resources survives, that is, survival of the "fattest." In the case of the duopoly profit parameters, one can prove that the three terms which make up $\Theta$ are nondecreasing in $\pi_i$ and therefore $\frac{\partial\Theta(\pi_i,\pi_j,\Pi_i,\Pi_j)}{\partial\pi_i} \geq 0$.[10] Hence increases in firm $i$'s duopoly profits strengthen its position. This is in line with the conventional wisdom that more efficient firms are more resistant to default, that is, survival of the "fittest." The above two results are consistent with the empirical finding of Zingales (1998) that the "fittest" and the "fattest" firms are most likely to survive.

Next, there is the monopoly profit parameter. Note that $\Pi_i - \pi_i$ is a measure for the incremental benefit of becoming a monopolist. One can again prove that $\frac{\partial\Theta(\pi_i,\pi_j,\Pi_i,\Pi_j}{\partial\Pi_i} \geq 0$, which implies that as firm $i$'s benefit of becoming a monopolist increases, it is prepared to hang on longer in the market, if necessary. We could refer to this as the survival of the "greediest" firm. Furthermore, $\lim_{\Pi_i\to+\infty} \underline{x}_{ir} = 0 = \lim_{\Pi_i\to+\infty} \underline{x}_{im}$, and $\lim_{\Pi_i\to\pi_i} \underline{x}_{ir} = \underline{x}_{id}$, that is, when the second mover's advantage becomes infinitely big ($\Pi_i \to \infty$) then firm $i$ will be willing to stay in the market forever, although when there is no incremental gain ($\Pi_i \to \pi_i$) strategic considerations no longer play a role, and firm $i$ will simply behave as if the other firm is not there and leave at its nonstrategic trigger $\underline{x}_{id}$.[11]

The following property is very useful to give us a feel for the overall relative importance of the firm-specific factors.

***Property 1.*** $\forall a, b > 0 : \Theta(a\pi_i, b\pi_j, a\Pi_i, b\Pi_j) = \frac{a}{b}\Theta(\pi_i, \pi_j, \Pi_i, \Pi_j)$.

Property 1 implies that the firm which enjoys a larger *proportional* increase in its profits than its rival becomes stronger and more likely to survive. A firm cannot, however, improve its relative position by increasing all its firm-specific parameters—$\pi_i$, $\Pi_i$, *and* $b_i$—by a constant factor. In other words: firms which are identical up to a scale factor in their profit and coupon parameters are equally strong.

Next, Figure 2 shows how the relative duopoly market share $\frac{\pi_j}{\pi_i}$, and the incremental benefit of becoming a monopolist $\left(\frac{\Pi_i}{\pi_i}\right)$, determine $\Theta$. The plot is constructed for the special (but plausible) case where the monopoly profits are the same for both firms ($\Pi_i = \Pi_j = \Pi$), and hence $\kappa_2 = 1$. The solid

[10] Since $\pi_j < \Pi_j$, it follows that $\kappa_1 \geq 0$, and since $\kappa_1$ is linear in $\pi_i$, it follows that $\frac{\partial\kappa_1}{\partial\pi_i} \geq 0$. Next $\frac{\partial\kappa_2}{\partial\pi_i} = 0$, and finally one can show that $\frac{\partial\kappa_3}{\partial\pi_i} \geq 0 \Leftrightarrow f\left(\frac{\Pi_i}{\pi_i}\right) \equiv \left(\frac{\Pi_i}{\pi_i}\right)^\lambda - \lambda\left(\frac{\Pi_i}{\pi_i}\right) + \lambda - 1 \geq 0$. Since $f(1) = 0$, $f(+\infty) = +\infty$ and $\forall y > 1 : f'(y) > 0$, it follows that $\frac{\partial\kappa_3}{\partial\pi_i} \geq 0$.

[11] An interesting example is, for instance, the case where we have a duopoly consisting of a very big (say 90% market share) firm and a very small firm, and where, upon market exit, the remaining firm cannibalizes a fraction of the defaulted firm's market share. Our model predicts that strategic considerations are unlikely to be important for the big firm, as the smaller firm is just "too small to matter."

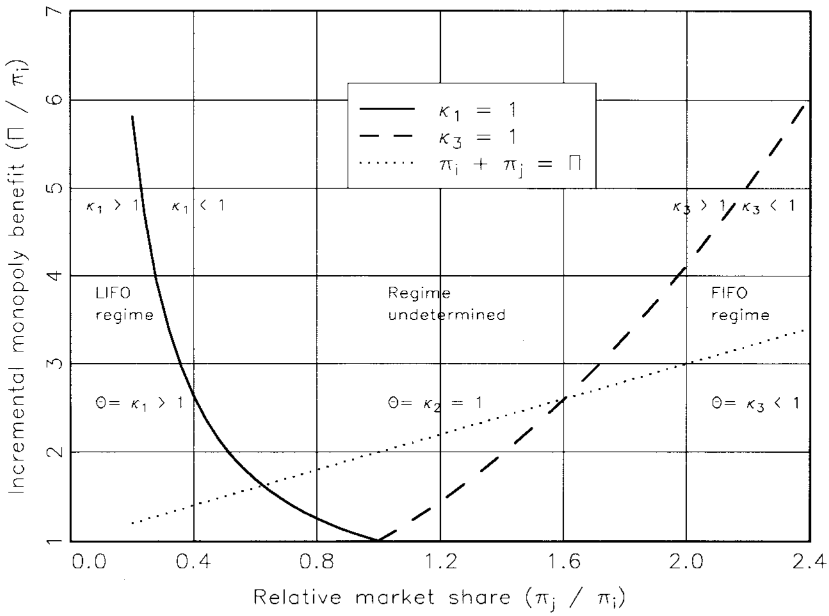


**Figure 2**
**Entry and exit regimes as a function of market shares**
The figure plots the critical coupon ratio ($\Theta$) and the corresponding long-run entry and exit regime as a function of the firms' relative market share and the incremental benefit of becoming a monopolist. The $\Theta$ space is divided into three zones: $\Theta > 1$ (area to the left of the solid line), $\Theta = 1$ (area between the solid and the dashed line), and $\Theta < 1$ (area to the right of the dashed line). In the first zone the "last-in-first-out" regime prevails, in the second zone either regime could occur and in the third zone the "first-in-first-out" regime dominates. The area above the dotted line represents the economically relevant profit combinations where a monopolist performs better than two duopolists. The baseline parameter values for this figure are $\mu = -0.01$, $r = 0.025$, $\sigma = 0.1$, and $\pi_i = 1$.

line ($\kappa_1 = 1$) and the dashed line ($\kappa_3 = 1$) divide the $\Theta$ space into three zones. The first zone, $\Theta = \kappa_1 > 1$, corresponds to the case where $i$ has a much larger market share than $j$, which allows firm $i$ to win despite having a relatively higher coupon than firm $j$. Moreover, if $\frac{b_i}{b_j} < \kappa_1$, then $i$ not only wins, but also strictly dominates $j$ (since $\underline{x}_{id} \leq \underline{x}_{jr}$). In a second zone, $\Theta = \kappa_2 = 1$, the market shares are somewhat more equally distributed, and the firm with the lowest coupon (and hence lowest monopoly trigger) wins. Of interest, in this zone, a slightly larger monopoly tenure (through a lower debt coupon) is sufficient to offset a relatively large disadvantage in terms of duopoly market share. For example, for $\frac{\Pi}{\pi_i} = 3$, and for any relative market share $\left(\frac{\pi_j}{\pi_i}\right)$ lying between 0.36 and 1.7, the firm with the lowest coupon wins. This rather strong result is similar to the one described in Ghemawat and Nalebuff (1985). They show that the firm with the longer monopoly tenure (i.e., lower monopoly trigger) has a substantial competitive advantage that can only be offset if it has a disproportionately shorter duopoly tenure

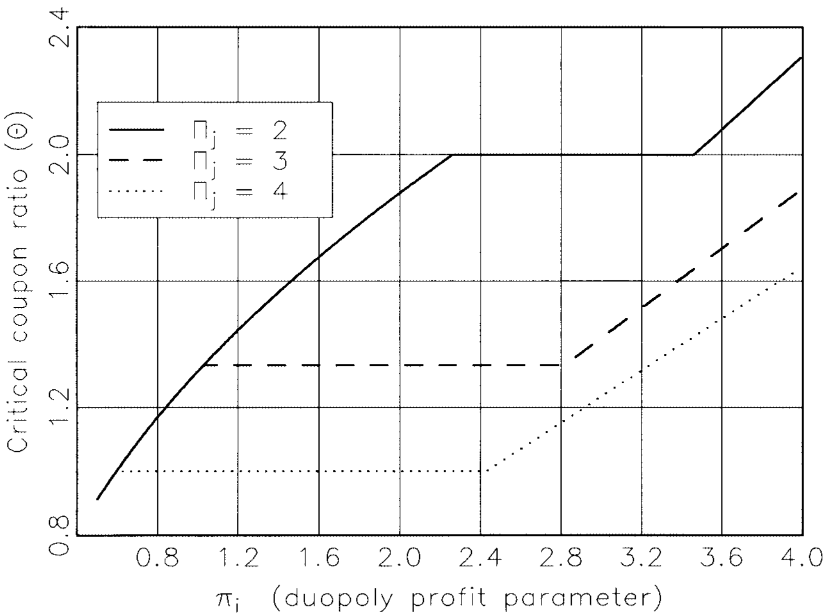


**Figure 3**
**The critical coupon ratio (Θ) as a function of firm *i*'s duopoly profit parameter ($\pi_i$)**
The critical coupon ratio (Θ) is plotted as a function of firm *i*'s duopoly profit parameter for varying values of firm *j*'s monopoly profits, that is, $\Pi_j$ = 2 (solid line), 3 (dashed line), and 4 (dotted line). The parameter values for this figure are $\mu = -0.01$, $r = 0.025$, $\sigma = 0.1$, $\pi_j = 1.5$, and $\Pi_i = 4$.

than its rival.[12] Note that as the incremental monopoly benefit $\left(\frac{\Pi}{\pi}\right)$ increases, larger discrepancies in the market shares are allowed without affecting the outcome. The reason is that both firms have an increasing incentive to hang on longer, making strict dominance only possible through larger market share or coupon asymmetry. Finally, in a third zone, $\Theta = \kappa_3 < 1$, firm *i* has a substantially smaller market share. It can only win by having a substantially smaller coupon (i.e., $\frac{b_i}{b_j} < \kappa_3 < 1$), and it will otherwise be strictly dominated by firm *j*. We will come back to this figure at the end of this article, when we discuss entry and exit regimes.

Figure 3 gives us some insight into the relative importance of individual firm-specific factors by plotting out the critical coupon ratio (Θ) as a function of the duopoly profit parameter ($\pi_i$) for varying values of $\Pi_j$. As previously mentioned, Θ is nondecreasing in $\pi_i$, that is, as $\pi_i$ increases, firm *i* is able to survive longest for increasing coupon ratios. The curve also clearly illustrates

[12] Note that this result depends crucially on the assumption that equity holders have no capital constraints. Under this assumption, since firms are forward looking (sunk losses are irrelevant), the firm with the longest monopoly tenure has the longer "breath" and can credibly justify incurring losses longer. The firm with the higher monopoly threshold knows that its rival has an incentive to hold out longer and therefore finds it optimal to back out. If, however, there were to be a constraint on the amount of cash equity holders can inject then results may change quite dramatically. We leave this as an interesting topic for future research.

the previously discussed three regimes ($\kappa_1, \kappa_2$, and $\kappa_3$) which determine $\Theta$. For low values of $\pi_i$, $\Theta$ equals $\kappa_3$,[13] and hence firm $i$ wins if it is not strictly dominated by $j$. Next, there is a constant region, $\Theta = \kappa_2$,[14] where the firm with the lowest monopoly trigger wins. Note again that the outcome remains unaffected for large variations in $\pi_i$. Finally, for high values of $\pi_i$, $\Theta$ equals $\kappa_1$[15] and firm $i$ strictly dominates firm $j$ if $\frac{b_i}{b_j} < \Theta$.

We now proceed to the impact on business failure of economic factors which both firms have in common, that is, $\sigma, r$, and $\mu$. Analytical solutions are unfortunately not always available, as $\kappa_1$ and $\kappa_3$ are quite complicated functions of $\lambda$, but extensive numerical simulations indicate the following: $\frac{\partial\kappa_1}{\partial\sigma} < 0$, $\frac{\partial\kappa_3}{\partial\sigma} > 0$, $\frac{\partial\kappa_1}{\partial\mu} > 0$, $\frac{\partial\kappa_3}{\partial\mu} < 0$, $\frac{\partial\kappa_1}{\partial r} > 0$, and $\frac{\partial\kappa_3}{\partial\sigma} < 0$.[16] All derivatives of $\kappa_2$ with respect to $\mu$, $\sigma$, and $r$ are trivially equal to zero. A first conclusion is that common economic factors can affect the order in which firms default, provided the firms are sufficiently different from one another. When firms are, however, rather similar (i.e., situated in regime $\kappa_2$ where neither firm can strictly dominate), then economic shocks will affect those firms in the same way and not influence the order of exit. When firms are, however, sufficiently different, then volatility increases are to the relative advantage of the firm with the lower profit and higher coupon parameters, while increases in the profit growth and interest rates are to the relative advantage of the firm having the higher profit and lower coupon parameters.

Finally, we examine the impact of the incremental benefit of becoming a monopolist on the *value* of the equity. One can show that

$$\frac{\partial V_{iw}}{\partial \Pi_i} = \left[\frac{\underline{x}_{jd}}{r-\mu} + \frac{\lambda b_i}{\Pi_i r(1-\lambda)}\left(\frac{\underline{x}_{jd}}{\underline{x}_{im}}\right)^{\lambda}\right]\left(\frac{x_t}{\underline{x}_{jd}}\right)^{\lambda}.$$

$$> 0 \iff \underline{x}_{jd} > \underline{x}_{im} \tag{16}$$

Hence firm $i$'s equity value increases with its monopoly profits, but the increase is weighted by the discount factor, $\left(\frac{x_t}{\underline{x}_{jd}}\right)^{\lambda}$, to account for the time it will take for firm $j$ to leave the market. This is also reflected in the following relative pricing measure, $RP(x_t) \equiv \frac{V_{iw}(x_t)-V_{il}(x_t)}{V_{il}(x_t)}$, as $\frac{\partial RP(x_t)}{\partial x_t} < 0$, $\frac{\partial^2 RP(x_t)}{\partial {x_t}^2} > 0$ and $\lim_{x_t\to+\infty} RP(x_t) = 0$, which means that the strategic issues discussed in this article start playing less of a role as the likelihood of firms defaulting becomes smaller.

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[13] Indeed, it follows from $\kappa_1 \le \kappa_3$, $\lim_{\pi_i\to 0}\kappa_1 = 0$ and $\lim_{\pi_i\to 0}\kappa_3 = 0$ that $\kappa_1 \le \kappa_3 < \kappa_2$ and hence that $\Theta = \kappa_3$ for low $\pi_i$.

[14] Since $\frac{\partial\kappa_3}{\partial\pi_i} > 0$, $\frac{\partial\kappa_2}{\partial\pi_i} = 0$, and $\kappa_1 \le \kappa_3$, it follows that, as $\pi_i$ increases starting from 0, at some point we will obtain a situation where $\kappa_1 < \kappa_2 < \kappa_3$ and hence $\Theta = \kappa_2$.

[15] For high $\pi_i$, we get $\kappa_2 < \kappa_1 < \kappa_3$ and hence $\Theta = \kappa_1$.

[16] From Proposition 5 we know that $\kappa_1$ and $\kappa_3$, respectively, correspond to the constraints $\underline{x}_{id} \le \underline{x}_{jr}$ and $\underline{x}_{ir} < \underline{x}_{jd}$ Hence, the derivatives of $\kappa_1$ and $\kappa_3$ are really determined by the relative sensitivities of the reservation and duopoly thresholds with respect to $\sigma$, $\mu$, and $r$.

In conclusion, we have shown that the surviving firm is the one with the highest operating profits ("fittest"), lowest debt repayments ("fattest"), and the highest incremental benefit of becoming a monopolist ("greediest"). It is, however, more important to be "fat and greedy" than to be "fit," as there is a substantial advantage to the firm which has the longest monopoly tenure.

## 3. Firm Valuation in an Oligopoly

By applying Proposition 5 iteratively, it is straightforward to extend the model to the case where there are more than $n$ firms in the market. In order to find out which firm will leave first, we can adopt the following algorithm. Take any two firms and (assuming we know the profit parameter for each firm after its competitor has left) apply Proposition 5. Remove the winning firm from the list (as this firm is no longer a candidate for leaving first). After repeating this procedure $n-1$ times, only the weakest firm remains and we have singled out the firm which is first to leave the market. One can then iteratively apply the previous algorithm to markets of decreasing size and, by doing so, completely determine the order in which firms are to leave the market.

## 4. Leverage Reductions Through Debt Exchange Offers

Thus far we have assumed that when firms get into financial distress, they cannot get relief through a reduction in their interest repayments.[17] Franks and Torous (1994), comparing financial recontracting in distressed exchanges and Chapter 11 reorganizations, find that distressed firms often manage to renegotiate their debt obligations. In particular, in the case of informal workouts, the majority of payments to bank debt are in the form of new bank debt involving either an extension of maturity or a reduction of interest or principal. They argue that creditors are willing to write down their claims in exchange for equity holders relinquishing the option to delay repayment of the creditors' claims. The option to delay may take the form of threatening to enter Chapter 11 or, once in Chapter 11, delaying the firm's formal reorganization. It is the costly nature of any formal reorganization which explains the huge bargaining power equity holders have in informal workouts.

The formal modeling of debt reorganizations has received extensive and thorough attention in recent articles by Hart and Moore (1989), Gertner and Scharfstein (1991) and Mella-Barral (1995, 1999), among others. In this article, we restrict debt relief to one particular type of out-of-court reorganization, namely debt exchange offers. In a debt exchange, bondholders swap their old debt contract for a new one with a reduced coupon repayment. Debt exchanges have a double effect on the value of the debt. On the one hand,

[17] Note that since there are no taxes, there is no incentive for firms to increase their leverage over time.

the value of all future coupon repayments, if the firm fully recovers, will be lower. On the other hand, the coupon reduction will increase the chances of the firm recovering. If the latter effect dominates, a debt exchange increases the market value of the debt and bondholders have an incentive to accept the debt exchange offer.

We use a simplified version of a debt exchange model developed by Mella-Barral (1995). He examines the polar cases where debt is either held by a single bank (private debt) or by many bondholders (public debt) and finds that under the often criticized exit-consent provision, optimal public debt exchange offers are ex ante the most efficient. The scope of this section is much narrower: we illustrate the flexibility of our model by integrating debt exchanges into our duopoly framework. In addition, assuming that all debt is held by a bank, we investigate how competition across distressed firms affects the outcome of debt exchange offers, and also how those debt reorganizations in turn affect firm exit in a duopoly. We leave efficiency and bankruptcy law implications as an interesting topic for future research.

In the remainder of this section we first present a simplified version of the Mella-Barral (1995) debt exchange model for a monopolistic firm. We then extend the model to allow for debt exchanges within a duopoly and show how, under close competition, equity holders can get larger debt reductions in the presence of a rival firm. In particular, we show how a firm with high bankruptcy costs can, through a debt exchange, acquire a relative competitive advantage over its competitor with lower bankruptcy costs, and thereby influence the order in which firms are expected to foreclose. Finally, we consider some comparative statics.

### 4.1 Debt exchange offer in a monopoly

In order to model the debt exchange offer, we need to make an assumption about the form of the renegotiation process and the distribution of the bargaining power across the stakeholders of the firm. Legal evidence [see Warfield (1994, p. 173] indicates that, at least in the initial stages of reorganizations, debtors control the renegotiation and sometimes even have exclusive rights to submit proposals. Empirical evidence on workouts also stresses the bargaining power of equity holders. Therefore, in line with Hart and Moore (1989), Aghion and Bolton (1992), and Mella-Barral (1995), among others, it is not unreasonable to assume that the owner manager has all the bargaining power in the renegotiation which takes the form of a take-it-or-leave-it debt exchange offer.[18] In particular:

***Assumption 5.*** *Debt reorganization happens through a one-off take-it-or-leave-it debt exchange offer made by the owner manager.*

[18] Note that reducing the bargaining power would decrease the debtor's share in the surplus arising from the debt exchange, but it would not change the qualitative nature of our results. See Fan and Sundaresan (1997) for a model where bargaining power is split between both parties.

We do not allow for multiple debt exchange offers in order to keep the extension tractable and concise. Moreover, one could argue that multiple reorganizations are costly and may be unacceptable to the bank.

A first issue to be addressed is the timing of the debt exchange. Intuitively one would expect that if the debt exchange takes place at an early stage when the firm is not (or only a little) financially distressed, then no or only a very small concession can be obtained from the bank. By delaying till the company is more acutely distressed, the owner will be able to obtain bigger concessions but the cash flows will have to be discounted at a higher discount rate, as they come at a later point in time. Below we will prove that for a monopolistic firm the incentive to keep delaying the debt exchange offer is always dominating. Mella-Barral (1995), however, argues persuasively that, in practice, once profits drop below the firm's liquidation point, equity holders may become unwilling to inject cash any longer, especially if they feel uncertain about the outcome of the exchange offer beyond that point. This idea is captured in the following assumption:

***Assumption 6.*** *The owner manager fears that workouts proposed in the future could be unsuccessful and is therefore unwilling to postpone debt exchange offers beyond the point where direct liquidation of the firm becomes optimal.*

We can now derive the value of the equity and the debt when the firm has a debt exchange option available. In general, a debt exchange will occur when the profit state variable $x_t$ falls below some level $\underline{x}_e$, and consists of the existing debt contract being swapped for a new one with a coupon $b^*$ which is lower than the original coupon $b$. For a given pair $(\underline{x}_e, b^*)$ it is straightforward to derive the value of the equity and the debt prior to and after the debt exchange.

***Proposition 6.*** *At the first occasion when the profit variable falls below a level $\underline{x}_e$, the old debt contract with coupon b is exchanged for a new contract with reduced coupon $b^*$. The value of the equity and the debt prior to reorganization are then respectively given by*

$$\begin{aligned} &V_e\big(x_t, \underline{x}_e, b, b^*, \Pi, \underline{x}(b^*, \Pi)\big) \\ &\quad = \frac{\Pi x_t}{r-\mu} - \frac{b}{r} + \left[V\big(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi)\big) - \left(\frac{\Pi x_t}{r-\mu} - \frac{b}{r}\right)\right]\left(\frac{x_t}{\underline{x}_e}\right)^{\lambda}, \\ &L_e\big(x_t, \underline{x}_e, b, b^*, \Pi, \underline{x}(b^*, \Pi)\big) \\ &\quad = \frac{b}{r} + \left[L\big(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi)\big) - \frac{b}{r}\right]\left(\frac{x_t}{\underline{x}_e}\right)^{\lambda}, \end{aligned} \tag{17}$$

*where the values of the equity and the debt right after the debt exchange are respectively given by $V(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi))$ and $L(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi))$ and* *where* $\underline{x}(b^*, \Pi) = \frac{-\lambda b^*(r-\mu)}{(1-\lambda)r\Pi}$ *is the point at which the company is ultimately liquidated.*

Next, we need to determine the optimal timing of the debt exchange and the corresponding coupon on the new debt contract. Assuming that debt exchanges happen through "take-it-or-leave-it" offers made by the owner, she will choose $(\underline{x}_e, b^*)$, the timing and the size of the exchange offer, in such a way that her equity value is maximized. In doing so, she is, however, constrained by the fact that the offer should be acceptable to the bank, that is, the value of the new debt contract should be no less than the value of the old debt contract. Given that the exchange offer happens within a non-cooperative setting where the owner has all power, she will extract all of the surplus and make an offer which makes the bank indifferent between accepting or rejecting the offer. This means that $(\underline{x}_e, b^*)$ must satisfy the constraint $L(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi)) = L(\underline{x}_e, b, \Pi, \underline{x}(b, \Pi))$, where the left-hand side of the constraint denotes the value of the debt after a successful debt exchange, and the right-hand side represents the value after the offer has been turned down. The owner's constrained optimization problem can then be written as

$$\begin{aligned} &\max_{\underline{x}_e, b^*} V_e\big(x_t, \underline{x}_e, b, b^*, \Pi, \underline{x}(b^*, \Pi)\big) \\ \text{s.t. } &\frac{b^*}{r} + \left[X\big(\underline{x}(b^*, \Pi)\big) - \frac{b^*}{r}\right]\left(\frac{\underline{x}_e}{\underline{x}(b^*, \Pi)}\right)^{\lambda} \\ &= \frac{b}{r} + \left[X\big(\underline{x}(b, \Pi)\big) - \frac{b}{r}\right]\left(\frac{\underline{x}_e}{\underline{x}(b, \Pi)}\right)^{\lambda}. \end{aligned} \tag{18}$$

Rearranging the constraint, it is possible to solve for $\underline{x}_e$ in terms of $b^*$ and one can show that $\frac{\partial \underline{x}_e}{\partial b^*} > 0$. In particular, as $\underline{x}_e \to \infty$ then $b^* \to b$. The reason is that when exchange offers are made at very high profit levels, the probability of the firm defaulting in future is almost zero, and the bank will not be motivated to make any significant reduction in interest repayments. On the other hand, as $\underline{x}_e$ falls, the probability of default starts going up dramatically. While coupon reductions for distressed firms lower the discounted value of future interest repayments, should the firm fully recover, they also lower the probability of the firm defaulting. The owner will set $b^*$ in such a way that both effects exactly offset one another, so that the new debt contract has the same value as the old one.

Thus far we have not made any explicit assumptions about the functional form of the firm's liquidation value, $X(x_t)$. When the owner defaults on her debt repayments, the bank acquires what is left of the company after bankruptcy. The following assumption of proportional bankruptcy costs allows us to obtain simple analytical solutions:

***Assumption 7.*** *The residual value of the firm following bankruptcy , $X(x_t)$, equals a fraction $\alpha$ of the unlevered firm value, that is,*

$$X(x_t) = \frac{\alpha \Pi x_t}{r - \mu} \text{ where } 0 \leq \alpha \leq 1. \tag{19}$$

The simplest way to solve the owner's optimization problem [Equation (18)] is by substituting the constraint into the objective function, hereby eliminating the variable, $\underline{x}_e$. Optimizing with respect to $b^*$, we find that the equity value is decreasing in $b^*$ and is maximized for the lowest possible value of $b^*$. Since $\frac{\partial \underline{x}_e}{\partial b^*} > 0$, it is therefore optimal to delay reorganization as long as possible, which means that Assumption 6 is binding, and that reorganization takes place when the owner wishes to foreclose. This leads to the following proposition:

***Proposition 7.*** *A firm which faces no competition has an incentive to postpone the debt exchange offer as long as possible. Under Assumptions* 5, 6*, and* 7 *this means that the debt exchange happens when the firm would find it optimal to default, that is, at* $\underline{x}(b, \Pi)$. *The corresponding coupon after restructuring, $b^*$, as a fraction of the original coupon, b, is the solution to*

$$-\left(1 + \frac{\alpha\lambda}{1-\lambda}\right)\left(\frac{b^*}{b}\right)^{1-\lambda} + \frac{b^*}{b} + \frac{\alpha\lambda}{1-\lambda} + 0. \tag{20}$$

We can conclude that the write-down in interest repayments is determined not only by the profit level ($\underline{x}_e$) at which the debt exchange takes place, but also by the recovery rate ($\alpha$) and the parameter $\lambda$ which in its turn is a function of the interest rate, the growth rate, and volatility of operating profits. This is consistent with the findings of Franks and Torous (1994), which showed that the cross-sectional variation in write-downs was explained by the detrimental effect of asset sales on firm value and by the firm's deteriorating operating performance. It may appear somewhat paradoxical that the firms with the lowest recovery rates (e.g., least tangible assets) get the largest debt relief and are therefore least likely to go bankrupt after the debt exchange offer. We will return to this issue when we present a more detailed analysis of the determinants of $\frac{b^*}{b}$ in Section 4.3.

## 4.2 Debt exchange offer in a duopoly

Contrary to the monopoly case, a debt exchange in a duopoly can potentially improve the competitive position of a firm in the market, and reverse the order in which it is expected to leave. If this is the case, then the debt exchange causes the firm's default probability to decrease and the quality of its debt to improve. We will show that, for this reason, the bank will be prepared to make a bigger percentage coupon write-down than it would do for a monopolist or, for that matter, a duopoly firm which does not improve its competitive position through a debt exchange.

Let us consider in a systematic way the various possible outcomes. In general, we can distinguish three cases. The first (second) case is the one where the duopoly firm $i$ is so weak (strong), compared to its rival, that it will leave the market first (second), whether or not it goes through a reorganization. This case resembles closely the monopolist's (nonstrategic) debt exchange offer (see previous section), and one can show again that the firm will want to delay the debt exchange as long as possible, that is, until $\underline{x}(b_i, \pi_i)$ ($\underline{x}(b_i, \Pi_i)$) when the firm would otherwise be liquidated. Analogous to the monopoly case, the coupon after reorganization, $b^*{}_i$, is the solution to $L(\underline{x}(b_i, \pi_i), b^*{}_i, \pi_i, \underline{x}(b^*{}_i, \pi_i)) = L(\underline{x}(b_i, \pi_i), b_i, \pi_i, \underline{x}(b_i, \pi_i))$ $(L(\underline{x}(b_i, \Pi_i), b^*{}_i, \Pi_i, \underline{x}(b^*{}_i, \Pi_i)) = L(\underline{x}(b_i, \Pi_i), b_i, \Pi_i, \underline{x}(b_i, \Pi_i)))$ and is the same as in the monopoly case, due to its independence of the profit scale parameter (see Proposition 7).

In the third case, the competitors are of more comparable strength: firm $i$ leaves the market second if it is granted debt relief, whereas it leaves first if the debt exchange offer is turned down. Hence a debt exchange can make a crucial difference to the future competitive position of the firm, and consequently also to the future quality of its debt. Intuitively one can predict that in such cases the bank will be willing to make larger proportional reductions in the coupon than in the two previous cases. The size of the new coupon, $\hat{b}_i$ (we use a different notation to differentiate from the above $b^*{}_i$), will depend again on the timing of the exchange, that is, $\hat{b}_i(\underline{x}_{ie})$ (with again $\frac{\partial \hat{b}_i(\underline{x}_{ie})}{\partial \underline{x}_{ie}} > 0$) and is the solution to $L(\underline{x}_{ie}, \hat{b}_i, \Pi_i, \underline{x}(\hat{b}_i, \Pi_i)) = L(\underline{x}_{ie}, b_i, \pi_i, \underline{x}(b_i, \pi_i))$. Analogous to the monopoly case, the incentive to delay the debt exchange still exists. However, contrary to the monopoly case, the equity holder may now prefer to make the debt exchange offer before the liquidation point [i.e., before $\underline{x}(b_i, \pi_i)$], and this for strategic reasons which we will explain below. Under the best-case scenario, however, the firm will be able to reorganize when it would otherwise foreclose, and it will then obtain the lowest possible coupon $\hat{b}_i(\underline{x}(b_i, \pi_i)) \equiv \underline{\hat{b}}_i$. The results can be summarized in the following proposition:

***Proposition 8.*** *When a firm i faces competition, the size of its coupon reduction depends on whether or not the debt exchange will improve the competitive position of the firm. If the order in which the firm leaves the market does not depend on the occurrence of a debt exchange, then the coupon $b^*{}_i$ after the debt exchange is the solution to*

$$-\left(1 + \frac{\alpha_i \lambda}{1-\lambda}\right)\left(\frac{b^*{}_i}{b_i}\right)^{1-\lambda} + \frac{b^*{}_i}{b_i} + \frac{\alpha_i \lambda}{1-\lambda} = 0. \tag{21}$$

*If, however, acceptance of the debt exchange offer means that the firm could leave second, while otherwise it would have had to leave first, then it can* *get a larger percentage coupon write-down. The value of the new coupon, $\hat{b}_i$, depends on the timing of the debt exchange, but the lower bound for it is realized if the debt exchange offer can be made at the firm's nonstrategic liquidation point. This lower bound is defined as $\underline{\hat{b}}_i \equiv \hat{b}_i(\underline{x}(b_i, \pi_i))$ and is the solution to*

$$-\left(1+\frac{\alpha_i\lambda}{1-\lambda}\right)\left(\frac{\Pi_i}{\pi_i}\right)^{\lambda}\left(\frac{\underline{\hat{b}}_i}{b_i}\right)^{1-\lambda}+\frac{\underline{\hat{b}}_i}{b_i}+\frac{\alpha_i\lambda}{1-\lambda}=0. \tag{22}$$

*One can show that $b^*{}_i \geq \hat{b}_i \geq \underline{\hat{b}}_i$.*

The proposition indicates that the bank has an incentive to make a bigger repayment concession if this leads to an *improvement* in the strategic position of the firm within the market [as reflected in Equation (22) by $\frac{\Pi_i}{\pi_i}$, the ratio of the monopoly profits to the duopoly profits]. Bankruptcy costs again play an important role in determining the terms of the debt exchange. In particular, higher bankruptcy costs lead *ex post* to larger coupon reductions. In summary, according to our model, banks are prepared to make bigger concessions to firms which are too costly to liquidate, and to firms for which debt relief guarantees a better future profit performance.

The above proposition also implies that the availability to each firm of a one-off debt exchange offer can influence the order in which firms abandon the market. The intuition for this is that, while previously only the equity holders were determining the companies' survival, now the bondholders will also play a role. In particular, when a firm's owner gets tired of injecting cash, the bank has the power to keep the firm going by reducing the coupon repayments. Survival within a duopoly therefore also depends on how low each firm's bank is prepared to set the coupon repayments. From the previous proposition we know that the bank would be willing to reduce its coupon down to $\underline{\hat{b}}_i$ if necessary to secure the firm's victory. Consequently it follows from the exit decision rule $\left[\frac{b_i}{b_j} < (>)\Theta\right]$ that asymmetric reductions in firms' coupons can reverse the order in which firms are expected to leave. In particular, given that banks have strong incentives to rescue firms with high bankruptcy costs or with high incremental benefits of becoming a monopolist (see Section 4.3 for detailed comparative statics), it follows from the above proposition that these firms are at a comparative advantage in times of distress. Combining this with our previous findings, we can conclude that there will be survival of the fittest (i.e., highest $\pi_i$, $\Pi_i$), the fattest (lowest $b_i$, $\underline{\hat{b}}_i$), the greediest $\left(\text{highest } \frac{\Pi_i}{\pi_i}\right)$, and the least tangible (lowest $\alpha_i$) firms.

The final issue to be addressed is the optimal timing of the debt exchange offer within a duopoly framework. While for the monopolist it is optimal to delay the debt exchange as long as possible, this is not necessarily the case for the stronger duopolist whose debt derives value from the expectation that it will become a monopolist. By refusing to inject capital any longer and by making an early debt exchange offer, the owner can, under certain conditions, put the future monopoly growth option of the company at stake and hereby cause the debt value to degrade in a way similar to bankruptcy costs. If the loss in debt value is big enough, the bank will have an incentive to restore the firm's original competitive position by accepting the coupon reduction proposed by the owner. The owner's power to extract concessions from the bondholder diminishes, therefore, after its competitor has left the market or gets into financial distress. This can induce the stronger firm to strategically propose a debt exchange offer just before the competing firm defaults or reorganizes.[19] A detailed treatment of the timing of debt exchange offers in a duopoly is, however, beyond the scope of this article.

### 4.3 Comparative statics

In this section we study the determinants of the ratio of the coupon after reorganization to the coupon before reorganization. We know already that this ratio varies from $\frac{b^*_i}{b_i}$ down to $\frac{\hat{b}_i}{b_i}$. In what follows, we restrict the analysis to the lower bound, as the upper bound is merely a special case of the lower bound where $\Pi_i$ equals $\pi_i$. Totally differentiating Equation (22) with respect to the recovery rate $\alpha_i$, and using the result that $\left(\frac{x_e}{\underline{x}(\hat{b}_i, \Pi_i)}\right)^\lambda < \frac{1}{1-\lambda+\alpha_i\lambda}$ (see appendix), one finds that $\left(\partial \frac{\hat{b}_i}{b_i}/\partial\alpha_i\right) > 0$, that is, lower recovery rates lead to larger coupon write-downs. In the limiting case where recovery rates are zero, the bank is indifferent between liquidating the firm and relieving it from all future repayments, that is, $\lim_{\alpha_i\to 0} \frac{\hat{b}_i}{b_i} = 0$.

Analogously, one can show that $\left(\partial \frac{\hat{b}_i}{b_i}/\partial \frac{\pi_i}{\Pi_i}\right) > 0$. This means that *if* a firm can acquire a future monopoly position thanks to a restructuring, then bondholders will be willing to make bigger concessions as the incremental profit from becoming a monopolist increases. Hence the future reward of becoming a monopolist not only makes equity holders stay in the market, but also encourages bondholders to make bigger sacrifices as long as these result in improved profit prospects for the firm. One can easily show that $\lim_{\frac{\pi_i}{\Pi_i}\to 0} \frac{\hat{b}_i}{b_i} = \frac{r}{b_i} X_i(\underline{x}(b_i, \pi_i)) = \frac{-\lambda\alpha_i}{1-\lambda}$. This reflects the choice bondholders face: either the firm will be foreclosed, in which case they receive the scrap value $X_i(\underline{x}(b_i, \pi_i))$, or they can accept the debt exchange and get risk-free debt for the same value $\frac{\hat{b}_i}{r}$. The debt has become (quasi) risk free in the latter case as monopoly profits are so high that the firm is unlikely ever to default as a monopolist. These results are illustrated in Figure 4 (A) which plots the coupon write-down, $\frac{\hat{b}_i}{b_i}$, as a function of bankruptcy costs for three different levels of incremental monopoly gains: $\frac{\Pi_i}{\pi_i}$ equals 1, 1.2, and 1.5. For a plau-

[19] A firm may find it advantageous to make a debt exchange offer right before its opponent leaves the market or reorganizes, thereby forcing the bank to make bigger coupon concessions than it would make after the rival has left the market. Indeed, by spending her debt exchange option prematurely, the owner is putting her firm on the brink of defeat and confronts the bank with the choice between turning down the firm's offer, in which case the firm loses, or making a debt concession (which ex ante was not necessary for survival) and thereby ensuring that the company will become a monopolist.

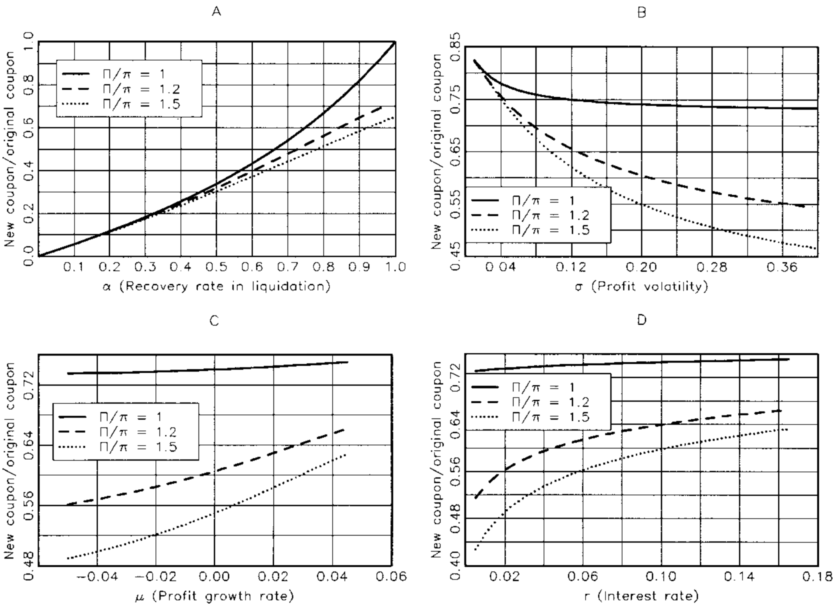


**Figure 4**
**Sensitivity analysis of the coupon write-down**
The plot shows the impact of changes in, respectively, (A) the recovery rate, (B) profit volatility, (C) profit growth rate, and (D) interest rate on the coupon write-down resulting from a debt exchange offer. The ratio of the new contractual coupon to the original coupon is plotted for various degrees of the incremental benefit of becoming a monopolist, that is, $\frac{\Pi}{\pi} = 1$ (solid line), 1.2 (dashed line), and 1.5 (dotted line). The parameter values for this figure are $\mu = 0.0$, $r = 0.05$, $\sigma = 0.2$, and $\alpha_i = 0.85$.

sible level of total bankruptcy costs (say 20%) this corresponds to coupon writedowns of 32%, 42%, and 49%, respectively.[20]

The sensitivities of $\frac{\hat{b}_i}{b_i}$ with respect to $\mu, \sigma$, and $r$ are only investigated numerically, as the derivative of the coupon ratio with respect to $\lambda$ (although closed form) is rather intractable. Figure 4B shows that the size of the coupon write-down increases with business risk. The reason is that increased volatility pushes up (through $\lambda$) the default probability. Figures 4C and D show that the opposite effect occurs for higher growth and interest rates.

## 5. Debt Financing and Firm Entry

In this section we extend our duopoly model to investigate the impact of debt financing on firm entry into a market where a levered monopolist is active. Upon entry of the follower, the situation reverts to the duopoly exit game (see Sections 2 and 4). If, after entry, market conditions sufficiently

[20] For comparison, Franks and Torous (1994) found percentage write-downs for bank debt of 13.4% and 13.6% for distressed exchanges and Chapter 11 reorganizations, respectively. For senior debt the percentages were 21.5% and 53.0%, respectively.

deteriorate, then two scenarios are possible: the leader forecloses first, and the follower leaves second, that is, first in first out (FIFO), or alternatively, the follower abandons first, while the leader leaves second, that is, last in first out (LIFO). Note that the first scenario also includes the special case whereby entry by the follower induces *immediate* exit by the leader. We will discuss this case in detail below.

We assume that the duopoly profits for the leader and the follower are given by $\pi_L$ and $\pi_F$, respectively, while the monopoly profits for both firms are, for simplicity, assumed to be the same, that is, $\Pi_L = \Pi_F \equiv \Pi$. Upon entry, the follower pays a nondepreciable sunk cost ($K_F$), and enters into a perpetual debt contract with coupon $b_F$. The coupon of the monopolist is denoted by $b_L$. In line with recent corporate finance articles [Hart and Moore (1989) and Aghion and Bolton (1992) among others], debt can be justified on the basis that firms are not wealthy enough to finance their entire activities themselves, and require external financing.[21] Finally, in order to keep the notation and exposition simple, note that we do not consider, in this section, debt reductions after entry. The effect of debt reorganization was already analyzed in the previous section and one could incorporate the results here as well, if necessary.

The aim of this section is to address the following questions. When does the follower enter the market, and how is entry influenced by the cost of entry and the way the leader and the follower are financed? And, which exit regime is likely to prevail (LIFO or FIFO) if, through entry and exit, the market structure can keep alternating between a duopoly and a monopoly?

For given values of the firms' profit and coupon parameters, we can (using Proposition 5) determine whether the follower is going to leave first or second. It is then straightforward to derive the value for the follower's option to enter under the LIFO or FIFO regime.[22]

***Proposition 9.*** *The value of the follower's option to enter the market under the LIFO regime is given by*

$$OW_{Fl}(x_t, \bar{x}_{Fl}) = \left( \frac{\pi_F \bar{x}_{Fl}}{r-\mu} + \frac{(\alpha-1)\pi_F \underline{x}(b_F, \pi_F)}{r-\mu} \left( \frac{\bar{x}_{Fl}}{\underline{x}(b_F, \pi_F)} \right)^{\lambda} - K_F \right) \left( \frac{x_t}{\bar{x}_{Fl}} \right)^{\beta}. \quad (23)$$

[21] Alternatively, the traditional trade-off theory of capital structure argues that the optimal debt-equity mix strikes a balance between the tax deductibility of interest payments and the cost of financial distress. In this article I assume that there are no taxes in order to keep the entry and exit model as transparent as possible.

[22] We assume for analytical convenience that the incumbent cannot leave the market *before* entry has happened. In a previous version I relaxed this assumption by allowing the market to die before entry could take place, which has the implication that the incumbent's exit threshold before entry is higher than its monopoly exit threshold when entry is no longer possible. The reason is that the threat of entry lowers the expected profits of the incumbent which makes it less attractive to hang on when profits are low. We found, however, that in economic terms the effect is of second order when the entry threshold and the incumbent's exit threshold are not too close. Hence we ignore this case for simplicity as it complicates the solutions substantially without adding much additional insight.

*The optimal entry trigger, $\bar{x}_{Fl}$ is the solution to $\frac{\partial OW_{Fl}(x_t,\bar{x}_{Fl})}{\partial \bar{x}_{Fk}}=0$. Furthermore, one can show that $\frac{\partial \bar{x}_{Fl}}{\partial K_F}>0$, $\frac{\partial \bar{x}_{Fl}}{\partial \Pi}=0$, $\frac{\partial \bar{x}_{Fl}}{\partial b_F}>0$ and $\frac{\partial \bar{x}_{Fl}}{\partial \underline{x}_{Lw}}=0$.*
*The value of the follower's option to enter the market under the FIFO regime is given by*

$$\begin{aligned} OW_{Fw}(x_t,\bar{x}_{Fw}) &= \left(\frac{\Pi\bar{x}_{Fw}}{r-\mu}+\frac{(\alpha-1)\Pi\underline{x}(b_F,\Pi)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda}-K_F\right)\left(\frac{x_t}{\bar{x}_{Fw}}\right)^{\beta} \\ &\qquad \text{if} \quad \bar{x}_{Fw}<\underline{x}_{Ll} \\ &\equiv \underline{OW}_{Fw}(x_t,\bar{x}_{Fw}) \\ OW_{Fw}(x_t,\bar{x}_{Fw}) &= \left(\frac{\pi_F\bar{x}_{Fw}}{r-\mu}+\frac{(\alpha-1)\Pi\underline{x}(b_F,\Pi)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda}\right. \\ &\qquad \left. +\frac{(\Pi-\pi_F)\underline{x}(b_L,\pi_L)}{r-\mu}\left(\frac{\bar{x}_F}{\underline{x}(b_L,\pi_L)}\right)^{\lambda}-K_F\right)\left(\frac{x_t}{\bar{x}_{Fw}}\right)^{\beta} \\ &\qquad \text{if} \quad \bar{x}_{Fw}\geq\underline{x}_{Ll} \\ &\equiv \overline{OW}_{Fw}(x_t,\bar{x}_{Fw}) \end{aligned} \tag{24}$$

*The entry strategy, $\bar{x}_{Fw}$, depends on the cost of entry, $K_F$ and the incumbent's foreclosure trigger, $\underline{x}_{Ll}\equiv\underline{x}(b_L,\pi_L)$:*
*if $K_F<\underline{K}$ then $\bar{x}_{Fw}<\underline{x}_{Ll}$ and $\bar{x}_{Fw}$ is the solution to $\frac{\partial \underline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}}=0$,*
*if $\underline{K}\leq K_F\leq\overline{K}$ then $\bar{x}_{Fw}=\underline{x}_{Ll}$ and*
*if $\overline{K}<K_F$ then $\bar{x}_{Fw}>\underline{x}_{Ll}$ and $\bar{x}_{Fw}$ is the solution to $\frac{\partial \overline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}}=0$,*
*where $\underline{K}$ and $\overline{K}$ are the solution to respectively*

$$\frac{(\alpha-1)\Pi}{r-\mu}(\lambda-\beta)\left(\frac{\underline{x}_{Ll}}{\underline{x}(b_F,\Pi)}\right)^{\lambda-1}+\frac{\beta\underline{K}}{\underline{x}_{Ll}}+\frac{\Pi(1-\beta)}{r-\mu}=0 \tag{25}$$

$$\frac{(\alpha-1)\Pi}{r-\mu}(\lambda-\beta)\left(\frac{\underline{x}_{Ll}}{\underline{x}(b_F,\Pi)}\right)^{\lambda-1}+\frac{\beta\underline{K}}{\underline{x}_{Ll}}+\frac{\pi(1-\lambda)+\Pi(\lambda-\beta)}{r-\mu}=0. \tag{26}$$

*If $K_F\leq\overline{K}$ then entry induces immediate exit of the incumbent. One can show that*
*If $\bar{x}_{Fw}<\underline{x}_{Ll}$ then $\frac{\partial \bar{x}_{Fw}}{\partial K_F}>0$, $\frac{\partial \bar{x}_{Fw}}{\partial \Pi}<0$, $\frac{\partial \bar{x}_{Fw}}{\partial b_F}>0$ and $\frac{\partial \bar{x}_{Fw}}{\partial \underline{x}_{Lw}}=0$.*
*If $\bar{x}_{Fw}=\underline{x}_{Ll}$ then $\frac{\partial \bar{x}_{Fw}}{\partial K_F}=0$, $\frac{\partial \bar{x}_{Fw}}{\partial \Pi}=0$, $\frac{\partial \bar{x}_{Fw}}{\partial b_F}=0$ and $\frac{\partial \bar{x}_{Fw}}{\partial \underline{x}_{Lw}}>0$.*
*If $\bar{x}_{Fw}>\underline{x}_{Ll}$ then $\frac{\partial \bar{x}_{Fw}}{\partial K_F}>0$, $\frac{\partial \bar{x}_{Fw}}{\partial \Pi}<0$, $\frac{\partial \bar{x}_{Fw}}{\partial b_F}>0$ and $\frac{\partial \bar{x}_{Fw}}{\partial \underline{x}_{Lw}}<0$.*

Let us first concentrate on the LIFO case. As would be expected, higher costs of entry slow market entry. Entry is further delayed if the firm has to rely on debt financing (i.e., $\frac{\partial \bar{x}_{Fl}}{\partial b_F}>0$). The reason is that the value of the firm is a decreasing function of leverage since there are bankruptcy costs but no taxes in our model. Totally differentiating the first-order condition for $\bar{x}_{Fl}$ with respect to $\alpha$, one can show that higher recovery rates (or equivalently, lower bankruptcy costs) speed up entry (i.e., $\frac{\partial \bar{x}_{Fl}}{\partial \alpha} < 0$). From the real options literature [see for instance Dixit and Pindyck (1994) for proofs], it is well known that more uncertainty, higher interest rates, and higher entry costs delay entry (i.e., $\frac{\partial \bar{x}_{Fl}}{\partial \sigma} > 0$, $\frac{\partial \bar{x}_{Fl}}{\partial r} > 0$ and $\frac{\partial \bar{x}_{Fl}}{\partial K_F} > 0$). Finally, as the follower cannot win, his entry decision is not influenced by potential monopoly profits (i.e., $\frac{\partial \bar{x}_{Fl}}{\partial \Pi} = 0$), nor by the incumbent's foreclosure time (i.e., $\frac{\partial \bar{x}_{Fl}}{\partial \underline{x}_{Lw}} = 0$).

Moving on to the FIFO case, the main difference now is that the entrant will experience an increase in profits if the incumbent leaves. A direct consequence of this is that the entry threshold (which partially incorporates these potential monopoly profits) is reduced. Moreover, the probability of the incumbent defaulting goes up at an increasing rate as the time to entry decreases. In the limit, for $\bar{x}_{Fw} \leq \underline{x}_{Ll}$, entry leads to immediate crowding out of the incumbent. Since the expected duration of the duopoly period is increasing in the entry threshold, the follower has an additional incentive to enter sooner rather than later. The combined effect can lead to situations where the entrant invests earlier than if it were to get monopoly profits from the start (see below).

The proposition shows how the entry strategy depends on the cost of entry. If the cost is low ($K_F < \underline{K}$), entry happens at such low profit levels that the incumbent is not able to survive the competition and is crowded out. Entrants with a cost ranging from $\underline{K}$ to $\overline{K}$ all enter at the same time, that is, the first time when $\underline{x}_{Ll}$ is hit, and drive out the incumbent immediately as well. The underlying intuition for this result is that the entrant prefers to eliminate the incumbent rather than fully exploiting its option value of waiting to invest, because the latter would increase the prospect of competition.[23] This leads to the surprising result that the entrant invests earlier than if it were to receive monopoly profits from the start and that the incumbent defaults when profits are actually rising.[24] For costs of entry above $\overline{K}$, it is no longer feasible for the entrant to crowd out the incumbent immediately, and we get a situation where both firms share the market. The time to entry is strictly increasing with $K_F$. Moreover, the incentive to speed up entry becomes weaker as the probability of the monopolist ever leaving, $\left(\frac{\bar{x}_{Fw}}{\underline{x}_{Ll}}\right)^{\lambda}$, decreases. In the limit, as $K_F \to \infty$, the entry threshold under the FIFO regime converges to the nonstrategic entry threshold under the LIFO regime.

Considering the comparative statics for $\bar{x}_{Fw}$, the results are fairly standard and comparable to the FIFO regime for $\bar{x}_{Fw} < \underline{x}_{Ll}$. When $\bar{x}_{Fw} = \underline{x}_{Ll}$, the results are less standard as parameter changes only affect entry in as far as they directly influence the *incumbent's* exit threshold, $\underline{x}_{Ll}$. Paradoxically

[23] A standard result from real options theory is that uncertainty combined with irreversibility leads to investment beyond the break-even point, reflecting the value of the option to delay the investment.

[24] The technical explanation is that even though $OW_{Fw}(x_t, \bar{x}_{Fw})$ is continuous in $\bar{x}_{Fw}$, there is a kink at $\bar{x}_{Fw} = \underline{x}_{Ll}$ (i.e., there is a discontinuity in the derivative). In particular, $\forall K_F \in (\underline{K}, \overline{K}) : \frac{\partial \underline{OW}_{Fw}(x_t, \bar{x}_{Fw})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}} > 0 > \frac{\partial \overline{OW}_{Fw}(x_t, \bar{x}_{Fw})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}}$ (see appendix for further details).

a lower exit threshold leads to earlier entry, and hence also earlier exit by the incumbent! Consequently (as $\frac{\partial \underline{x}_{Ll}}{\partial \sigma} < 0$) an increase in volatility leads to earlier entry and earlier exit, which is diametrically opposed to the standard result in the real options literature that uncertainty delays entry and exit. Finally, considering the case where both firms share the market, the interesting result is the inverse relationship between the follower's entry and the incumbent's exit threshold (i.e., $\frac{\partial \bar{x}_{Fw}}{\partial \underline{x}_{Ll}} < 0$): earlier exit (and therefore factors which influence earlier exit, such as high leverage) by the incumbent leads to earlier entry by the follower. This result combined with the previous finding that increased leverage delays a firm's entry and speeds up its exit gives support to the stance that leverage makes product-market competition softer [see Fudenberg and Tirole (1986a), Poitevin (1989), Bolton and Scharfstein (1990), and Faure-Grimaud (1998)]. It is consistent with the empirical findings of Chevalier (1995) and Kovenock and Philips (1997) that firms which are highly levered, or increase leverage, experience more aggressive investment behavior from their rivals. Note that volatility now has the more standard effect of delaying exit ($\frac{\partial \underline{x}_{Ll}}{\partial \sigma} < 0$) and entry ($\frac{\partial \bar{x}_{Fw}}{\partial \sigma} > 0$).

To conclude this section, it is interesting to examine which regime (LIFO or FIFO) is likely to prevail in the long run if the market structure can keep alternating between a monopoly and a duopoly.[25] We assume in our analysis that the profit parameters ($\pi_L$, $\pi_F$, and $\Pi$) remain unchanged over time, and we try to identify the restrictions on the coupon levels of the entrants over time which lead to a consistent cyclical behavior. This means that we look for cyclical behavior according to either the LIFO or the FIFO regime. Under a recurrent LIFO regime, the firm entering first acquires a first mover's advantage that allows it to stay in forever. It shares the market with another firm when the economy is doing well and it operates as a monopolist in bad times. Under a recurrent FIFO regime, the oldest firm is the first one to die (death can happen suddenly in a rising market or some time after entry in a declining market). Each firm therefore has a similar life cycle: it starts off as an entrant in a duopoly, then it becomes a monopolist, followed by being the incumbent in a duopoly, after which it dies. The proposition below gives the conditions under which a cyclical LIFO and FIFO regime occur.

***Proposition 10.*** *If the market structure alternates between a monopoly and a duopoly and the coupon level of the ith entrant is denoted by $b_{Fi}$ then*
*LIFO is the long run regime if $\forall i : b_L < b_{Fi}\Theta(\pi_L, \pi_F, \Pi, \Pi)$,*
*FIFO is the long run regime if $\forall i : b_L > b_{Fi}\Theta(\pi_L, \pi_F, \Pi, \Pi)^i$.*
*In the former case the incumbent stays in forever, while in the latter case the incumbent is continuously being replaced by an entrant.*

An important question now is which regime is likely to prevail. It follows from the proposition that this is mainly determined by whether $\Theta$ is bigger

25 I thank the referee for pointing this out.

or smaller than unity. Indeed, if $\Theta > 1$, then the FIFO regime is unlikely to persist in the long run as this would imply that, each round, the coupon of the follower has to decrease by a factor $\frac{1}{\Theta}$, on average.[26] The LIFO regime is therefore more likely to prevail. On the other hand, if $\Theta < 1$, then there is bias toward the FIFO regime (provided that the debt requirements of each subsequent entrant do not grow too rapidly). If $\Theta$ equals unity, then there is no bias toward one regime or the other, and the outcome depends entirely on the realization of the future entrants' coupon levels $b_{Fi}$.

Figure 2 (previously discussed in Section 2) plots the prevailing entry and exit regime as a function of the firms' relative market share and the incremental monopoly benefit (for firm $i$ being the leader and firm $j$ being the follower). A common economic assumption that is implemented in the plot is that a monopolist can do at least as well as two duopolists, that is, $\pi_i + \pi_j \leq \Pi$. This constraint is represented by the dotted line and implicitly eliminates all combinations below this line. We conclude from the plot that the entry and exit regime is largely determined by the relative market share. When there is a first mover's advantage (and the incremental monopoly profit ratio, $\Pi/\pi_L$, is below, say, 2), then the LIFO regime is likely to be the economically prevailing one. For very large second mover's advantages, the FIFO regime dominates, while for all intermediate cases either regime could occur. Considering, however, the fact that incumbent firms often have an advantage in terms of market share, and eliminating all cases for which $\pi_L + \pi_F < \Pi$, the plot seems to indicate that the LIFO regime is the one more likely to prevail for most standard economic assumptions with respect to the profit and coupon parameters. This is in line with the empirical evidence that more established firms are more likely to survive than younger firms.

## 6. Conclusion

This article has examined the impact of capital structure on the investment and foreclosure decision of firms in a duopoly. Recognizing that the future gain of becoming a monopolist gives a firm an added incentive to stay in the market as long as possible, we derived a condition that allows us to determine the order in which firms leave the market. Consistent with empirical evidence by Zingales (1998), I found that (everything else being equal) the firm foreclosing last is the one with relatively high operating profits, lower debt repayments and a larger incremental gain from becoming a monopolist, that is, survival of the "fittest, fattest, and greediest" firm. I found that there is a strong bias towards survival of the firm with the longest monopoly tenure (i.e., lowest monopoly exit threshold), which implies that it is more important to be "fat and greedy" than to be "fit." An interesting finding is that

[26] Although entry costs (and therefore the need for external financing) sometimes go down through time (e.g., in the case where entry requires the adoption of new technology), this is unlikely to continue indefinitely.

the extent to which higher operating profits can be substituted for lower debt repayments (and vice versa) depends on factors common to both firms, such as the interest rate, profit growth, and volatility. The important implication of this is that a change in these common economic factors may reverse the order in which firms are expected to foreclose.

Next, I extended the exit model by allowing financially distressed firms to reduce their debt repayments through a one-off "take-it-or-leave-it" debt exchange offer made by the equity holder. Bankruptcy occurs when both the equity holder and the bondholder refuse to support the firm any longer. We showed that equity holders have an incentive to delay the exchange offer as long as possible, as bigger concessions can be obtained when the firm is more distressed. We also found that firms with higher bankruptcy costs, or with higher incremental monopoly benefits, can get bigger reductions on their debt repayments. Allowing for debt exchange offers can therefore reverse the order in which firms go bankrupt.

Next, we investigated the impact of debt financing on market entry. We found that, in the presence of bankruptcy costs and the absence of taxes, the need to borrow money will delay a firm's entry. Of interest, the follower's time of entry also depends on the incumbent's capital structure if the follower is destined to outlive the incumbent. A higher coupon level set by the leader induces earlier entry by the follower, as the latter is keen to drive out the financially vulnerable opponent as soon as possible. This predatory behavior is in line with the empirical evidence presented by Chevalier (1995) and Kovenock and Philips (1997) that firms which are highly levered experience more aggressive investment behavior from their rivals. As such, the findings in this article are consistent with those theoretical articles on capital structure and product-market competition which suggest that a firm's product-market competition becomes "softer" when its leverage increases. I also showed that, for low entry costs, entry happens at such low profit levels that it leads to an instant crowding out of the incumbent firm. Comparative statics with respect to the entry trigger can, under those circumstances, be very different from the predictions of standard real options theory. Finally, we analyzed entry and exit when the market structure can keep alternating between a monopoly and a duopoly. I found that for the economically plausible case where there is a first mover's advantage in terms of profits or market share, there is a bias toward the youngest firm defaulting first and the established firm surviving over time.

The article could be extended in various interesting ways. One could, for instance, endogenize the production decision and analyze the impact of capital structure on output decisions of firms. Within our duopoly framework, production increases could be used strategically to speed up the opponent's market exit, whenever it gets into financial distress (similar to the predatory entry behavior described in this article). Another extension could be to introduce incomplete information into the model. This could, for example, imply that both firms know the exact realization of their own profit parameter, but not of their opponent's. Consequently firms would have to make their exit decision without knowing their rival's foreclosure trigger.

## Appendix

*Proof of Proposition 1.* The general solutions to the differential Equations (2) and (3) are

$$V(x_t) = A_0 + A_1 x_t + A_2 x_t{}^{\lambda} + A_3 x_t{}^{\beta},$$
$$L(x_t) = B_0 + B_1 x_t + B_2 x_t{}^{\lambda} + B_3 x_t{}^{\beta}.$$

Substituting the solutions into Equations (2) and (3) and equating coefficients on like terms yields $A_0 = -(1-\tau)\frac{b}{r}$, $A_1 = \frac{(1-\tau)\Pi}{r-\mu}$, $B_0 = \frac{b}{r}$, and $B_1 = 0$.

As $x_t \to \infty$, the possibility of bankruptcy goes to zero and $V$ and $L$ must therefore satisfy the following no-bubbles conditions:

$$\lim_{x_t\to\infty} V(x_t) = \mathrm{E}_t\left[\int_t^{\infty} (\Pi x_t - b)\exp(-r(s-t))ds\right] = \frac{\Pi x_t}{r-\mu} - \frac{b}{r},$$
$$\lim_{x_t\to\infty} L(x_t) = \mathrm{E}_t\left[\int_t^{\infty} b\exp(-r(s-t))ds\right] = \frac{b}{r}, \tag{27}$$

which implies that $A_3 = B_3 = 0$, since $\beta > 0$.

At the foreclosure trigger, $\underline{x}$, in the absence of arbitrage the following value-matching conditions must be satisfied: $V(\underline{x}) = 0$ and $L(\underline{x}) + X(\underline{x})$, which allow us to determine the constants $A_2$ and $B_2$. Finally, the optimal foreclosure trigger, $\underline{x}$, satisfies the following "smooth-pasting condition" $V'(\underline{x}) = 0$.

*Proof of Proposition 2.* $V_{iw}$ and $L_{iw}$ are the solutions to the same differential Equations (2) and (3) and the general solution is therefore the same as given in the proof of Proposition 1. The no-bubble conditions remain the same as well. The value-matching and smooth-pasting conditions differ, however.

At $x_t = \underline{x}_j$, firm $j$ leaves the market and the following value-matching conditions have to be satisfied:

$$V_{iw}(\underline{x}_j) = V_{il}(\underline{x}_j, b_i, \Pi_i, \underline{x}_i)$$
$$= \frac{\Pi_i \underline{x}_{jd}}{r-\mu} - \frac{b}{r} + \left(\frac{b}{r} - \frac{\Pi_i \underline{x}_i}{r-\mu}\right)\left(\frac{\underline{x}_j}{\underline{x}_i}\right)^{\lambda},$$
$$L_{iw}(\underline{x}_j) = L_{il}(\underline{x}_j, b_i, \Pi_i, \underline{x}_i), \tag{28}$$

where $V_{il}$ and $L_{il}$ are the nonstrategic claims as defined in Proposition 1. The value-matching condition determines the remaining constants $A_2$ and $B_2$.

*Proof of Proposition 3.* The proof is as described in the main text preceding the proposition. The optimal triggers $\underline{x}_{jd}$ and $\underline{x}_{im}$ are the solutions to the smooth-pasting condition $\frac{\partial V_{jl}(x_t, b_j, \pi_j, \underline{x}_{jd})}{\partial x_t}|_{x_t=\underline{x}_{jd}} = 0$ and $\frac{\partial V_{iw}(x_t, b_i, \pi_i, \underline{x}_{im}, \underline{x}_{jd})}{\partial x_t}|_{x_t=\underline{x}_{im}} = 0$.

*Proof of Proposition 4.* Case 1. Firm $i$ strictly dominates firm $j$.

a) $\underline{x}_{ir} < \underline{x}_{jm}$

The unique Nash equilibrium consists of $i$ staying until $\underline{x}_{im}$ and $j$ exiting at $\underline{x}_{jd}$, hereafter for short: $(\underline{x}_{im}, \underline{x}_{jd})$. Indeed, if $i$ plays $\underline{x}_{im}$, then (since $\underline{x}_{im} < \underline{x}_{jm} < \underline{x}_{jr}$) $j$ should play $\underline{x}_{jd}$. If $j$ plays $\underline{x}_{jd}$, then (since $\underline{x}_{jd} > \underline{x}_{ir}$) it is optimal for $i$ to play $\underline{x}_{im}$ and hence the equilibrium is Nash.
$(\underline{x}_{id}, \underline{x}_{jm})$ is not a Nash equilibrium: if $j$ plays $\underline{x}_{jm}$, then it is optimal for $i$ to play $\underline{x}_{im}$, and hence $j$ should play $\underline{x}_{jd}$.

b) $\underline{x}_{id} < \underline{x}_{jr}$

$(\underline{x}_{im}, \underline{x}_{jd})$ is again the unique Nash equilibrium: if $i$ plays $\underline{x}_{im}$ ($< \underline{x}_{jr}$), then it is optimal for $j$ to play $\underline{x}_{jd}$. If $j$ plays $\underline{x}_{jd}$, then it is optimal for $i$ to play $\underline{x}_{im}$.
$(\underline{x}_{id}, \underline{x}_{jm})$ is not a Nash equilibrium: if $i$ leaves at $\underline{x}_{id}$ then (since $\underline{x}_{id} \leq \underline{x}_{jr}$), $j$ should play $\underline{x}_{jd}$ and not $\underline{x}_{jm}$.

Case 2. Neither firm strictly dominates the other, that is, $\underline{x}_{jm} \leq \underline{x}_{ir} < \underline{x}_{jd}$ and $\underline{x}_{im} \leq \underline{x}_{jr} < \underline{x}_{id}$.

a) $\underline{x}_{im} < \underline{x}_{jm}$

In this case there are two Nash equilibria. We show that only one of the two is subgame perfect. Given the Markovian structure of the problem and the fact that there is complete information, the relevant history of the game is captured entirely by the level of the state variable $x_t$ and the action of the players (i.e., whether or not they are still in the market). A proper subgame starts at the end of every partial history in the node originating with $x_t$. Hence an equilibrium is subgame perfect if it is a Nash equilibrium in *all* possible subgames. In what follows $V_{jm}(x_t)$ denotes the firm value for firm $j$ as a monopolist, while $V_{jw}(x_t, \underline{x}_i)$ is a shorthand notation for $V_{jw}(x_t, b_j, \pi_j, \underline{x}_{jm}, \underline{x}_i)$.

*We first prove that $(\underline{x}_{im}, \underline{x}_{jd})$ is subgame perfect: $\forall x_t \in [\underline{x}_{im}, \underline{x}_{id}] : V_{iw}(x_t, \underline{x}_{jd}) \geq 0$ since $\underline{x}_{jd} > \underline{x}_{ir}$, and hence, if $j$ leaves at $\underline{x}_{jd}$, it is optimal in all subgames for $i$ to stay till $\underline{x}_{im}$. Conversely, $\forall x_t \in [\underline{x}_{jm}, \underline{x}_{jd}] : V_{jw}(x_t, \underline{x}_{im}) < 0$ since $\underline{x}_{im} < \underline{x}_{jm}$, and hence it is optimal for $j$ in all subgames to play $\underline{x}_{jd}$.

*Next, using a modified version of Ghemawat and Nalebuff's (1985) backward induction argument, we prove that $(\underline{x}_{id}, \underline{x}_{jm})$ is not subgame perfect. Suppose that $j$ plays $\underline{x}_{jm}$, then $\exists y_1 > \underline{x}_{jm}$ such that $i$ is willing to sustain losses over $[\underline{x}_{jm}, y_1]$ in order to reap monopoly profits over $[\underline{x}_{im}, \underline{x}_{jm}]$, that is,: $\forall x_t \leq y_1$: $i$'s expected value of staying until $\underline{x}_{im}$ equals $V_{iw}(x_t, \underline{x}_{jm}) \geq 0$ and $\forall x_t > y_1$: $V_{iw}(x_t, \underline{x}_{jm}) < 0$. Hence if $x_t < y_1$, $i$'s optimal strategy is to stay. We now want to prove that there exists a subgame in which it is optimal for firm $i$ not to abandon, even if firm $j$ threatens to play $\underline{x}_{jm}$. Suppose that $x_t \in (\underline{x}_{jm}, y_1)$ and that firm $i$ were to abandon. The earliest time at which this could possibly be optimal is if $x_t$ rises to $y_1$, which allows us to construct an upperbound for firm $j$'s value function, denoted by $\overline{V}_j(x_t, \underline{x}_{jm}, y_1)$.[27] This value function is again the solution to the differential Equation (2) subject to a number of boundary conditions (see proof of Proposition 1). The value-matching conditions are now, however, $\overline{V}_j(\underline{x}_{jm}, \underline{x}_{jm}, y_1) = 0$ and $\overline{V}_j(y_1, \underline{x}_{jm}, y_1) = V_{jm}(y_1)$, that is, $j$ leaves the market first at $\underline{x}_{jm}$ and becomes a monopolist at $y_1$. The value function $\overline{V}_j$ is plotted in Figure 5 and is situated below $j$'s monopoly value $\forall x_t \in (\underline{x}_{jm}, y_1]$. Hence, since $\frac{\partial V_{jm}(x_t)}{\partial x_t}|_{x_t=\underline{x}_{jm}=0}$, it must be the case that $\frac{\partial \overline{V}_j(x_t)}{\partial x_t}|_{x_t=\underline{x}_{jm}} < 0$. Furthermore, since $V_{jm}(y_1) > 0$, using a continuity argument,

[27] Note that a more complicated exit strategy for firm $i$ would involve not restricting it to one exit trigger (here $\underline{x}_{im}$) but allowing it to also optimally choose an upper exit barrier $z_1$ ($> \underline{x}_{im}$) such that the value function smoothpastes at $\underline{x}_{im}$ and $z_1$. It is easy to see that actually $z_1 > y_1$. However, as mentioned in Assumption 4, we construct the value functions under the assumption that firms stick to a single trigger point strategy and leave more complicated strategies as a topic for future research.

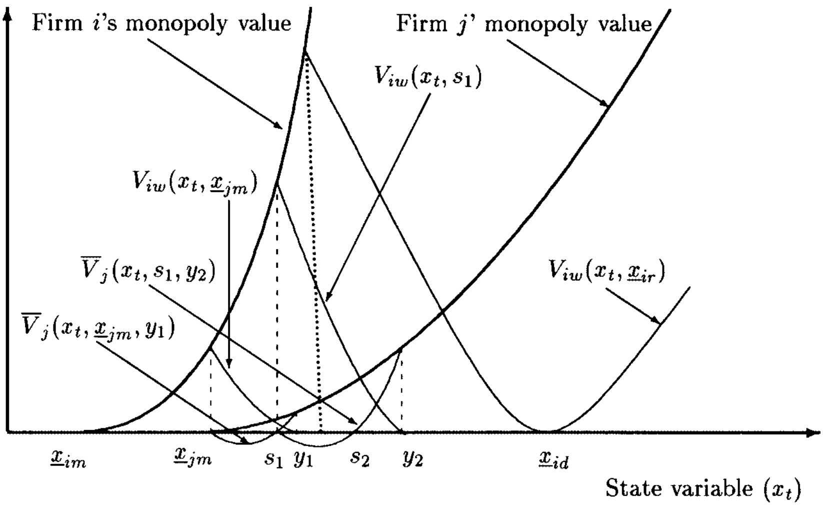


**Figure 5**
**Verification of subgame perfection through backward induction**
The figure illustrates the backward induction argument which proves that the Nash equilibrium, where the firm with the longer monopoly tenure exits first, is not subgame perfect. Hence, the equilibrium where firm $i$ exits first at $\underline{x}_{id}$ and firm $j$ exits last at $\underline{x}_{jm}$ is not subgame perfect. $V_{iw}(x_t, \underline{x}_j)$ denotes firm $i$'s value, when the firm is a duopolist, but becomes a monopolist if $x_t$ falls below $\underline{x}_j$. $\overline{V}_j(x_t, a, b)$ denotes the value of firm $j$, when the firm is a duopolist, but where it leaves the market if $x_t$ falls below $a$, or becomes a monopolist if $x_t$ rises above $b$. The dotted line represents firm $i$'s reservation threshold, $\underline{x}_{ir}$.

$\exists s_1 \in (\underline{x}_{jm}, y_1) : \overline{V}(s_1, \underline{x}_{jm}, y_1) = 0$. Consider now any subgame starting at $x_t \in (\underline{x}_{jm}, s_1)$, whereby $i$ and $j$ are still both in the market. Since $\overline{V}(x_t, \underline{x}_{jm}, y_1) < 0$, it is optimal for $j$ to leave. On the other hand, since $V_{iw}(x_t, \underline{x}_{jm}) > 0$, it is optimal for $i$ to stay. Hence the Nash equilibrium in all these subgames is that $j$ drops out immediately and $i$ stays in the market. Hence the equilibrium whereby $j$ stays till $\underline{x}_{jm}$ is not subgame perfect.

Ghemawat and Nalebuff (1985) prove not only that, in the deterministic case, there is a unique subgame perfect equilibrium, but also that there is no subgame in which the weaker firm can credibly force out the stronger firm. To illustrate that the argument above is consistent with Ghemawat and Nalebuff (1985), we now extend the argument by recursion and derive a sufficient condition under which $i$ can force out $j$ for all possible subgames. Given that $j$ should leave at $s_1$ if $i$ is still in the market at that point, there exists a point $y_2$ such that firm $i$ is willing to sustain losses over the interval $[s_1, y_2]$ in order to make monopoly profits over $(\underline{x}_{im}, s_1)$. We can again construct an upper bound for firm $j$'s value function over $(s_1, y_2)$, this time using the value matching conditions $\overline{V}_j(s_1, s_1, y_2) = 0$ and $\overline{V}_j(y_2, s_1, y_2) = V_{jm}(y_2)$. Define similarly $s_2$ such that $\overline{V}_j(s_2, s_1, y_2) = 0$. Hence, upon reaching $s_2$, firm $i$ will find it worthwhile to stay and therefore firm $j$ will leave. The argument continues by calculating a sequence $s_k$ until we reach $s_n > \underline{x}_{ir}$. The Ghemawat and Nalebuff (1985) argument implies that $j$ will leave (if it has not left before) the market upon reaching $s_n$ and given that $s_n > \underline{x}_{ir}$, it is optimal for $i$ to stay until $\underline{x}_{im}$ and hence $j$ should leave the market at $\underline{x}_{jd}$. It is important to note that in order to complete the recursion beyond $\underline{x}_{ir}$, the slope of $\overline{V}_j(x_t)$ at each point $x_t \in [\underline{x}_{jm}, \underline{x}_{ir}]$ should be negative, or equivalently:

$$\left.\frac{\overline{V}_j(x_t, z_k, z_{k+1})}{\partial x_t}\right|_{x_t = z_k} < 0 \ \forall \ z_k \in \left[\underline{x}_{jm}\underline{x}_{ir}\right]. \tag{29}$$

As shown before, the condition is always satisfied at $x_t = \underline{x}_{jm}$, and therefore the equilibrium $(\underline{x}_{id}, \underline{x}_{jm})$ is not subgame perfect over $[\underline{x}_{jm}, s_1]$. One can, however, numerically show[28] that if $\sigma$ is large, $\mu$ positive or only slightly negative, and $i$ and $j$ are not too asymmetric, then it can happen that Equation (29) is not satisfied for all $z_k \in [\underline{x}_{jm}, \underline{x}_{ir}]$. The intuition is that when there is a lot of uncertainty and the state variable decreases slowly, then firm $i$ cannot be guaranteed that its duopoly losses will be offset by future monopoly losses (as $x_t$ may float around for too long in the loss-making region). The consequence of this is that, while the equilibrium $(\underline{x}_{id}, \underline{x}_{im})$ is not subgame perfect, it could nevertheless occur if the game were to evolve off the equilibrium path, that is, if firm $j$ were to stay "accidently" in the market beyond $\underline{x}_{jd}$. If, however, the downward trend is sufficiently dominant then Equation (29) will be satisfied and the Ghemawat and Nalebuff (1985) backward recursion argument goes through all the way. Indeed, if $\mu < 0$, then as $\sigma \to 0$, $\forall k : s_k \to y_k$, and we get a series $s_k$ which converges to the one described in Ghemawat and Nalebuff (1985).

b) $\underline{x}_{im} = \underline{x}_{jm}$

In this case there are two subgame perfect equilibria. Indeed $\forall x_t \in [\underline{x}_{jm}, \underline{x}_{jd}]$: $V_{jw}(x_t, \underline{x}_{im}) \leq 0$, and hence $j$ should leave at $\underline{x}_{jd}$. On the other hand, since $\underline{x}_{jd} > \underline{x}_{ir}$, $\forall x_t \in [\underline{x}_{im}, \underline{x}_{id}]$: $V_{iw}(x_t, \underline{x}_{jd}) > 0$, and hence it is optimal for $i$ to stay. Subgame perfection of the other equilibrium can be shown analogously by relabeling the firms. Note that this knife-edge case was left out of the main body of the article for clarity of exposition.

*Proof of Proposition 5.* We first prove that Equation (14) is a necessary Condition ($\Longrightarrow$):

If $i$ wins then it follows from Proposition 4 that $i$ strictly dominates (call this case 1) or neither firm strictly dominates and $i$ has the lowest monopoly trigger (call this case 2).

$$\text{Case 2} \iff (\underline{x}_{jr} \geq \underline{x}_{im} \text{ and } \underline{x}_{jd} > \underline{x}_{ir} \text{ and } \underline{x}_{im} < \underline{x}_{jm}) \iff (\underline{x}_{jd} > \underline{x}_{ir} \text{ and } \underline{x}_{im} < \underline{x}_{jm})$$
$$\text{Case 1} \iff (\underline{x}_{ir} < \underline{x}_{jm} \text{ or } \underline{x}_{id} \leq \underline{x}_{jr}).$$

However, $\underline{x}_{ir} < \underline{x}_{jm} \Longrightarrow \underline{x}_{im} \leq \underline{x}_{ir} < \underline{x}_{jm} \leq \underline{x}_{jd} \Longrightarrow (\underline{x}_{ir} < \underline{x}_{jd})$ and $(\underline{x}_{im} < \underline{x}_{jm})$. Hence, if case 1 or case 2 are satisfied, this implies that $\underline{x}_{id} \leq \underline{x}_{jr}$ or $(\underline{x}_{ir} < \underline{x}_{jd}$ and $\underline{x}_{im} < \underline{x}_{jm})$. Substituting the thresholds for the underlying model parameters gives Equation (14). That Equation (14) is a sufficient condition ($\Longleftarrow$) follows directly from Proposition 4.

Finally, we prove that $\kappa_1 \leq \kappa_3$: $\underline{x}_{id} \leq \underline{x}_{jr}$ implies that $\underline{x}_{ir} \leq \underline{x}_{id} \leq \underline{x}_{jr} \leq \underline{x}_{jd}$, which implies $\underline{x}_{ir} \leq \underline{x}_{jd}$. Hence, if $\frac{b_i}{b_j} \leq \kappa_1$, then $\frac{b_i}{b_j} \leq \kappa_3$ and therefore $\kappa_1 \leq \kappa_3$.

*Proof of Proposition 6.* The proof is analogous as in Proposition 1, the main difference being the value-matching conditions which are now for the equity and the debt, respectively, given by $V_e(\underline{x}_e, \underline{x}_e, b, b^*, \Pi, \underline{x}(b^*, \Pi)) = V(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi))$ and $L_e(\underline{x}_e, \underline{x}_e, b, b^*, \Pi, \underline{x}(b^*, \Pi)) = V(\underline{x}_e, b^*, \Pi, \underline{x}(b^*, \Pi))$.

[28] Although Equation (29) seems to be a condition in the two variables $z_k$ and $z_{k+1}$, one can substitute $z_k$ using the condition $V_{iw}(z_{k+1}, z_k) = 0$, which can be written as

$$z_k + \left( \left[ \frac{b_i}{r} - \frac{\pi_i z_{k+1}}{r-\mu} - \left( \frac{b_i}{r} - \frac{\Pi_i \underline{x}_{im}}{r-\mu} \right) \left( \frac{z_{k+1}}{\underline{x}_{im}} \right)^{\lambda} \right] \frac{(r-\mu) z_{k+1}^{-\lambda}}{\Pi_i - \pi_i} \right)^{\frac{1}{1-\lambda}} \equiv z_k(z_{k+1}) \tag{43}$$

One can then easily verify Equation (29) by varying $z_{k+1}$ over the interval $[\underline{z}, \bar{z}]$ such that $z_k(\underline{z}) = \underline{x}_{jm}$ and $z_k(\bar{z}) = \underline{x}_{ir}$.

*Proof of Proposition 7.* The constraint can be rewritten as

$$\underline{x}_e = \left[ \frac{b - b^*}{-\left(\frac{\alpha\lambda}{1-\lambda} + 1\right)\left(b^* \underline{x}(b^*, \Pi)^{-\lambda} - b\underline{x}(b, \Pi)^{-\lambda}\right)} \right]^{\frac{1}{\lambda}}. \tag{30}$$

We first establish the result that $\frac{\partial x_e}{\partial b^*} > 0$. Differentiating $\underline{x}_e$ with respect to $b^*$, and substituting for $\underline{x}(b, \Pi)$ and $\underline{x}(b^*, \Pi)$ one finds that the derivative is positive $\forall b^* \in [0, b)$ if

$$f(b^*) \equiv \lambda \left(\frac{b^*}{b}\right)^{1-\lambda} + (1-\lambda)\left(\frac{b^*}{b}\right)^{-\lambda} - 1 < 0. \tag{31}$$

Evaluating $f(b^*)$ at $b$ and 0 gives $f(b) = 0$ and $f(0) = -1$. Differentiating $f(b^*)$ we find that $f'(b^*) > 0$ over the relevant interval and hence $f(b^*) < 0$.

Next, differentiating $\underline{x}_e$ with respect to $b^*$, and rearranging finds that the positivity of the derivative is equivalent to

$$\left[b^* \underline{x}(b^*, \Pi)^{-\lambda} - b\underline{x}(b, \Pi)^{-\lambda}\right] + \left[(1-\lambda)\underline{x}(b^*, \Pi)^{-\lambda}(b - b^*)\right] < 0. \tag{32}$$

Substituting out the first bracketed term using the constraint and rearranging gives $(\frac{x_e}{\underline{x}(b^*, \Pi)})^{\lambda} < \frac{1}{1-\lambda+\alpha\lambda}$. Next, substituting Equation (30) for $\underline{x}_e$ into $V_e(x_t, \underline{x}_e, b, b^*, \pi, \underline{x}(b, \pi))$ gives a function to be optimized with respect to $b^*$ only. Differentiating with respect to $b^*$ and simplifying leads to

$$\lambda x_t{}^{\lambda} b^{*-\lambda} \left(\frac{\pi}{r-\mu}\right)^{\lambda} r^{\lambda-1} \left(\frac{\lambda}{-1+\lambda}\right)^{-\lambda} (1-\alpha) < 0. \tag{33}$$

The equity value is therefore optimized for as low a debt coupon as possible. It follows from Assumption 6 that we have a corner solution where the debt exchange offer takes place at the firm's foreclosure point. Substituting $x_e = \underline{x}(b, \Pi)$ gives Equation (20), which can be solved for the new contractual coupon, $b^*$.

*Proof of Proposition 8.* Let us denote $\tilde{b}$ the coupon after reorganization. We can distinguish the following three cases:

Case 1. Firm $i$ leaves first (independent of a debt exchange). In this case the optimization problem becomes

$$\max_{\underline{x}_e, \tilde{b}} V_e\left(x_t, \underline{x}_{ie}, b_i, \tilde{b}_i, \pi_i, \underline{x}(\tilde{b}_i, \pi_i)\right)$$

$$= \frac{\pi_i x_t}{r-\mu} - \frac{b_i}{r} + \left[V\left(\underline{x}_{ie}, \tilde{b}_i, \pi_i, \underline{x}(\tilde{b}_i, \pi_i)\right) - \left(\frac{\pi_i \underline{x}_{ie}}{r-\mu} - \frac{b_i}{r}\right)\right]\left(\frac{x_t}{\underline{x}_{ie}}\right)^{\lambda} \tag{34}$$

$$\text{subject to } L\left(\underline{x}_{ie}, \tilde{b}_i, \pi_i, \underline{x}(\tilde{b}_i, \pi_i)\right) = L\left(\underline{x}_{ie}, b_i, \pi_i, \underline{x}(b_i, \pi_i)\right). \tag{35}$$

Case 2. Firm $i$ leaves second (independent of a debt exchange). In this case the optimization problem becomes

$$\max_{\underline{x}_e, \tilde{b}} V_{ew}(x_t, \underline{x}_{ie}, b_i, \tilde{b}_i, \pi_i, \Pi_i, \underline{x}_{jd})$$

$$= \frac{\pi_i x_t}{r-\mu} - \frac{b_i}{r} + \left[V_e\left(\underline{x}_{jd}, \underline{x}_{ie}, b_i, \tilde{b}_i, \Pi_i, \underline{x}(\tilde{b}_i, \Pi_i)\right) - \left(\frac{\pi_i \underline{x}_{jd}}{r-\mu} - \frac{b_i}{r}\right)\right]\left(\frac{x_t}{\underline{x}_{jd}}\right)^{\lambda} \tag{36}$$

$$\text{subject to } L\left(\underline{x}_{ie}, \tilde{b}_i, \Pi_i, \underline{x}(\tilde{b}_i, \Pi_i)\right) = L\left(\underline{x}_{ie}, b_i, \Pi_i, \underline{x}(b_i, \Pi_i)\right). \tag{37}$$

Case 3. Firm $i$ leaves second or first (dependent on whether or not a debt exchange takes place). In this case the optimization problem becomes

$$\max_{\underline{x}_e,\tilde{b}} V_{ew}(x_t, \underline{x}_{ie}, b_i, \tilde{b}_i, \pi_i, \Pi_i, \underline{x}_{jd})$$
$$= \frac{\pi_i x_t}{r-\mu} - \frac{b_i}{r} + \left[V_{iw}(\underline{x}_{ie}, \tilde{b}_i, \pi_i, \Pi_i, \underline{x}_{jd}) - \left(\frac{\pi_i \underline{x}_{ie}}{r-\mu} - \frac{b_i}{r}\right)\right]\left(\frac{x_t}{\underline{x}_{ie}}\right)^{\lambda} \tag{38}$$

$$\text{subject to } L\big(\underline{x}_{ie}, \tilde{b}_i, \Pi_i, \underline{x}(\tilde{b}_i, \Pi_i)\big) = L\big(\underline{x}_{ie}, b_i, \pi_i, \underline{x}(b_i, \pi_i)\big). \tag{39}$$

Analogous to Proposition 7, one can again rewrite the constraint and substitute for the variable $\underline{x}_{ie}$ in the objective function. Differentiating the objective function with respect to $\tilde{b}$, one finds again that equity holders have an incentive to bargain for as low a coupon as possible, and therefore to postpone the reorganization as long as possible. Using Assumption 6 this means that in Cases 1 and 2, $\underline{x}_{ie}$ equals $\underline{x}(b,\pi)$ and $\underline{x}(b,\Pi)$, respectively. Substituting this into the constraints leads in either case to Equation (21). In case 3 the equity holder can postpone at best until $\underline{x}(b_i,\pi_i)$. Substituting $\underline{x}_{ie} = \underline{x}(b_i,\pi_i)$ into the constraint gives Equation (22).

Finally we need to show that $b_i^* \geq \underline{\hat{b}}_i$. First, it follows from Equations (21) and (22) that $b_i^* \to \underline{\hat{b}}_i$ as $\frac{\Pi_i}{\pi_i} \to 1$. Second, totally differentiating Equation (22) with respect to $\frac{\Pi_i}{\pi_i}$ and using the result that $\left(\frac{x_{ie}}{\underline{x}(\hat{b}_i,\Pi)}\right)^{\lambda} < \frac{1}{1-\lambda+\alpha_i\lambda}$ one finds that $\frac{\partial \hat{b}_i}{\partial \frac{\pi}{\Pi}} > 0$. Hence combining these two results, it follows that $\underline{\hat{b}}_i \leq \hat{b}_i \leq b_i^*$.

*Proof of Proposition 9.* We present the proof for $OW_{Fw}$ only. The proof for $OW_{Fl}$ is analogous. $OW_{Fw}$ satisfies the ordinary differential equation:

$$rOW_{Fw}(x_t) = \mu x_t OW'_{Fw}(x_t) + \frac{\sigma^2}{2} x_t^2 OW''_{Fw}(x_t). \tag{40}$$

The general solution to this differential equation is given by $A_i x_t^{\lambda} + B_i x_t^{\beta}$, where, as in Proposition 1, $\lambda$ and $\beta$ are the negative and positive root the characteristic equation, respectively. Again, the no-bubble condition for $x_t \to 0$ implies that $A_i = 0$ $(i = 1, 2)$. The value-matching conditions under the FIFO regime for $\bar{x}_{Fw} < \underline{x}_{Ll}$ and $\bar{x}_{Fw} \geq \underline{x}_{Ll}$ are, respectively, given by

$$\begin{aligned}
OW_{Fw}(\bar{x}_{Fw}, \bar{x}_{Fw}) &= B_1 \bar{x}_{Fw}^{\beta} = \frac{\Pi \bar{x}_{Fw}}{r-\mu} + \frac{(\alpha-1)\Pi \underline{x}(b_F,\Pi)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda} - K_F \quad \bar{x}_{Fw} < \underline{x}_{Ll}\\
&\equiv \underline{OW}_{Fw}(\bar{x}_{Fw}, \bar{x}_{Fw})\\
OW_{Fw}(\bar{x}_{Fw}, \bar{x}_{Fw}) &= B_2 \bar{x}_{Fl}^{\beta} = \frac{\pi_F \bar{x}_{Fw}}{r-\mu} + \frac{(\alpha-1)\Pi \underline{x}(b_F,\Pi)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda}\\
&\quad + \frac{(\Pi-\pi_F)\underline{x}_{Ll}}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}_{Ll}}\right)^{\lambda} - K_F \quad \bar{x}_{Fw} \geq \underline{x}_{Ll}\\
&\equiv \overline{OW}_{Fw}(\bar{x}_{Fw}, \bar{x}_{Fw})
\end{aligned} \tag{41}$$

and allow us to determine $B_1$ and $B_2$. Optimizing $OW_{Fw}$ with respect to $\bar{x}_{Fw}$ gives the first-order condition

$$\frac{\Pi(1-\beta)}{r-\mu} + \frac{(\alpha-1)\Pi(\lambda-\beta)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda-1} + \frac{\beta K}{\bar{x}_{Fw}} = 0 \qquad \bar{x}_{Fw} < \underline{x}_{Ll}$$
$$\frac{\pi_F(1-\beta)}{r-\mu} + \frac{(\alpha-1)\Pi(\lambda-\beta)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda-1} + \frac{\beta K}{\bar{x}_{Fw}}$$
$$+ \frac{(\Pi-\pi_F)(\lambda-\beta)}{r-\mu}\left(\frac{\bar{x}_{Fw}}{\underline{x}_{Ll}}\right)^{\lambda-1} = 0 \qquad \bar{x}_{Fw} \geq \underline{x}_{Ll}$$

It follows that $\frac{\partial \underline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}} > \frac{\partial \overline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}}$ since $\Pi > \pi_F$. Using the first-order conditions, define now $\underline{K}$ and $\overline{K}$ as the solutions to, respectively, $\frac{\partial \underline{OW}_{Fw}(x_t,\bar{x}_{Fw};\underline{K})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}} = 0$ and $\frac{\partial \overline{OW}_{Fw}(x_t,\bar{x}_{Fw};\overline{K})}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}} = 0$. Hence, $\forall K_F \in (\underline{K}, \overline{K}) : \frac{\partial \underline{OW}_{Fw}(x_t,\bar{x}_{Fw};K_F)}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}} > 0 > \frac{\partial \overline{OW}_{Fw}(x_t,\bar{x}_{Fw};K_F)}{\partial \bar{x}_{Fw}}|_{\bar{x}_{Fw}=\underline{x}_{Ll}}$ and the optimal entry threshold, $\bar{x}_{Fw}$ is therefore at $\underline{x}_{Ll}$. For $K_F < \underline{K}$ we have an interior maximum at $\bar{x}_{Fw} < \underline{x}_{Ll}$, which is the solution to $\frac{\partial \underline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}} = 0$. One can verify that the second-order condition is always satisfied. For $K_F > \overline{K}$ we have an interior maximum at $\bar{x}_{Fw} > \underline{x}_{Ll}$, which is the solution to $\frac{\partial \overline{OW}_{Fw}(x_t,\bar{x}_{Fw})}{\partial \bar{x}_{Fw}} = 0$. The second-order condition is given by

$$\frac{(\alpha-1)\Pi(\lambda-1)(\lambda-\beta)}{(r-\mu)\bar{x}_{Fw}}\left(\frac{\bar{x}_{Fw}}{\underline{x}(b_F,\Pi)}\right)^{\lambda-1} - \frac{\beta K_F}{\bar{x}_{Fw}^2} + \frac{(\Pi-\pi_F)(\lambda-1)(\lambda-\beta)}{(r-\mu)\bar{x}_{Fw}}\left(\frac{\bar{x}_{Fw}}{\underline{x}_{Ll}}\right)^{\lambda-1} < 0. \tag{42}$$

The second-order condition is satisfied as long as the third term does not become too big, which requires that $\underline{x}_{Ll}$ does not become too high as this would cause the incumbent to be driven out upon entry and would lead us back into one of the previous cases. Of interest, one can substitute the first-order condition into the second-order conditions to get $\bar{x}_{Fw} > \frac{-\lambda\beta K_F(r-\mu)}{(1-\lambda)(\beta-1)\pi_F} \equiv \frac{-\lambda}{1-\lambda}\bar{x}_{FE}$, where $\bar{x}_{FE}$ is the entry threshold for an all equity financed perpetual duopoly firm. The intuition is again that $\bar{x}_{Fw}$ ($\underline{x}_{Ll}$) should be high (low) enough in order to avoid the situation where extreme predatory behavior means that entry leads to immediate exit.

The comparative statics for $K_F < \underline{K}$ and $K_F > \underline{K}$ can straightforwardly be derived by totally differentiating the appropriate first-order condition with respect to the various parameters. Imposing the second-order condition then leads to the appropriate signs. For $\underline{K} \leq K_F \leq \overline{K}$ one merely needs to investigate the comparative statics with respect to $\underline{x}_{Ll} = \frac{-\lambda b_L(r-\mu)}{(1-\lambda)\Pi r}$.

*Proof of Proposition 10.* The proposition can be obtained by repeatedly applying Proposition 5:

$$\begin{aligned}
\text{LIFO is the long run regime} &\iff \frac{b_L}{b_{F1}} < \Theta \wedge \frac{b_L}{b_{F2}} < \Theta \wedge \frac{b_L}{b_{F3}} < \Theta \wedge \cdots \\
&\iff \forall i : b_L < b_{Fi}\Theta. \\
\text{FIFO is the long run regime} &\iff \frac{b_L}{b_{F1}} > \Theta \wedge \frac{b_{F1}}{b_{F2}} > \Theta \wedge \frac{b_{F2}}{b_{F3}} > \Theta \wedge \cdots \\
&\iff \forall i : b_{Fi} < \frac{b_L}{\Theta^i}.
\end{aligned}$$

**References**


Aghion, P., and P. Bolton, 1992, "An Incomplete Contracts Approach to Financial Contracting," *Review of Financial Studies*, 59, 473–494.

Altman, E., 1971, *Corporate Bankruptcy in America*, Heath Lexington Books, Lexington, MA.

Anderson, R. W., and S. Sundaresan, 1996, "Design and Valuation of Debt Contracts," *Review of Financial Studies*, 9, 37–68.

Bolton, P., and D. Scharfstein, 1990, "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 80, 93–106.

Brander, J., and T. Lewis, 1986, "Oligopoly and Financial Structure," *American Economic Review*, 76, 956–970.

Brander, J., and T. Lewis, 1988, "Bankruptcy Costs and the Theory of Oligopoly," *Canadian Journal of Economics*, 21, 221–243.

Brennan, M. J., and E. S. Schwartz, 1984, "Valuation of Corporate Claims," *Journal of Finance*, 39, 593–607.

Brigham, E. F., and L. C. Gapenski, 1997, *Financial Management: Theory and Practice*, Dryden Press, Fort Worth, TX.

Chevalier, J., 1995, "Debt and Product Market Competition: Local Market Entry, Exit, and Expansion Decisions of Supermarket Chains," *American Economic Review*, 85, 415–435.

Dixit, A., 1989, "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 97, 620–638.

Dixit, A., and R. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ.

Dotan, A., and S. A. Ravid, 1985, "On the Interaction of Real and Financial Decisions of the Firm under Uncertainty," *Journal of Finance*, 40, 501–517.

Dun and Bradstreet, 1980a, *Business Failure Record*, New York.

Dun and Bradstreet, 1980b, *Etudes sur les Faillites Commerciales au Canada*, Toronto.

Fan, H., and S. Sundaresan, 1997, "Debt Valuation, Renegotiations and Optimal Dividend Policy," mimeo, Columbia University.

Faure-Grimaud, A., 1998, "Product Market Competition and Optimal Debt Contracts: The Limited Liability Effect Revisited," mimeo, London School of Economics.

Fisher, E. O., R. Heinkel, and J. Zechner, 1989, "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance*, 44, 19–40.

Franks, J. R., and W. N. Torous, 1989, "An Empirical Investigation of U.S. Firms in Chapter 11 Reorganization," *Journal of Finance*, 44, 747–767.

Franks, J. R., and W. N. Torous, 1994, "A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations," *Journal of Financial Economics*, 35, 349–370.

Fries, S. M., M. Miller, and W. R. M. Perraudin, 1997, "Debt Pricing in Industry Equilibrium," *Review of Financial Studies*, 10, 39–68.

Fudenberg, D., and J. Tirole, 1986a, "A 'Signal-Jamming' Theory of Predation," *Rand Journal of Economics*, 17, 366–376.

Fudenberg, D., and J. Tirole, 1986b, "A Theory of Exit in Duopoly," *Econometrica*, 54, 943–960.

Fulghieri, P., and S. Nagarajan, 1996, "On the Strategic Role of High Leverage in Entry Deterrence," *Journal of Banking and Finance*, 20, 1–23.

Gertner, R., and D. Scharfstein, 1991, "A Theory of Workouts and the Effects of Reorganization Law," *Journal of Finance*, 46, 1189–1222.

Ghemawat, P., and B. Nalebuff, 1985, "Exit," *Rand Journal of Economics*, 16, 184–194.

Ghemawat, P., and B. Nalebuff, 1990, "The Devolution of Declining Industries," *Quarterly Journal of Economics*, 105, 184–186.

Grenadier, S., 1996, "The Strategic Exercise of Options: Development Cascades and Overbuilding in Real Estate Markets," *Journal of Finance*, 51, 1653–1679.

Hart, O., and J. Moore, 1989, "Default and Reorganization: A Dynamic Model of Debt," Working Paper 520, MIT.

Kovenock, D., and G. Phillips, 1997, "Capital Structure and Product Market Behavior: An Examination of Plant Exit and Investment Decisions," *Review of Financial Studies*, 10, 767–803.

Leahy, J. V., 1993, "Investment in Competitive Equilibrium: the Optimality of Myopic Behavior," *Quarterly Journal of Economics*, 108, 1105–1133.

Leland, H. E., 1994, "Risky Debt, Bond Covenants and Optimal Capital Structure," *Journal of Finance*, 49, 1213–1252.

Leland, H. E., and K. B. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987–1019.

Maksimovic, V., 1988, "Capital Structure in Repeated Oligopolies," *Rand Journal of Economics*, 19, 389–407.

Mauer, D. C., and A. J. Triantis, 1994, "Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework," *Journal of Finance*, 49, 1253–1277.

Mella-Barral, P., 1995, "Optimal Debt Exchange Offers," mimeo, University of Cambridge.

Mella-Barral, P., 1999, "The Dynamics of Default and Debt Reorganization," *Review of Financial Studies*, 12, 535–578.

Mella-Barral, P., and W. R. M. Perraudin, 1997, "Strategic Debt Service," *Journal of Finance*, 52, 531–556.

Mello, A. S., and J. E. Parsons, 1992, "The Agency Costs of Debt," *Journal of Finance*, 47, 1887–1904.

Poitevin, M., 1989, "Financial Signalling and the 'Deep-Pocket' Argument," *Rand Journal of Economics*, 20, 26–40.

Ravid, S. A., 1988, "On Interactions of Production and Financial Decisions," *Financial Management*, 17, 87–99.

Smets, F., 1991, "Exporting versus FDI: The Effect of Uncertainty, Irreversibilities and Strategic Interactions," mimeo, Yale University.

Warfield, D., 1994, "Creditor Strategies in Small Business Bankruptcies," in J. A. Pusateri, K. S. Kressin, and J. J. O'Malley (eds.), *Small Business Bankruptcy Reorganizations*, John Wiley & Sons, New York.

Zingales, L., 1998, "Survival of the Fittest or the Fattest? Exit and Financing in the Trucking Industry," *Journal of Finance*, 53, 905–938.