Global Diversification, Growth, and Welfare with Imperfectly Integrated Markets for Goods

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In this article we examine the effect of the imperfect mobility of goods on international risk sharing and, through that, on the investment in risky projects, welfare, and growth. Our main result is that the welfare gain from integration of financial markets is not greatly reduced by the presence of goods market imperfections, modeled as a cost of transferring goods from one country to the other. We also find that the gain is nonmonotonic with respect to investors' risk aversion and the aggregate volatility of output growth. The policy implication to be drawn is that financial market integration is a worthwhile goal to pursue even when full goods mobility has not been achieved.

The benefits, along with the impact on growth, of free trade in goods are frequently contrasted with those of the free movement of capital around the world. While, on the one hand, the benefits of freer trade, provided competition is pure and perfect, are viewed as substantial and are opposed only by the need of some governments to raise revenues in the form of tariffs, unfettered capital flows on the other are regarded with more suspicion. The reasons for this suspicion may be valid ones (deviations from rational expectations, herding behavior, bubbles, crises, etc.), but they ought to be brought into play only after the impact of free capital flows under ideal conditions is well understood. It is undeniable, however, that the benefits of free capital flows under ideal conditions, and their merits relative to free trade, have not been the subject of as much research as free trade itself.

What seems to be needed is a study that would help decide to what extent, and in what sense, the degree of openness of goods markets and the degree

We are grateful to two anonymous referees and to Pierluiggi Balduzzi, Ming Huang, Urban Jermann, Maurice Obstfeld, Geert Rouwenhorst, Angel Serrat, Jiang Wang, and Tan Wang. We would also like to acknowledge suggestions from seminar participants at Carnegie Mellon University, Catholic University (Leuven), Dartmouth College, MIT, Northwestern University, Stanford University, University of California at Berkeley, University of Michigan, Wharton, the NBER Finance Lunch Seminar Series and Summer Institute, the meetings of the European Finance Association and the Western Finance Association in 1998, and the meetings of the American Finance Association in 2000. This article was completed while R. Uppal was visiting MIT; he acknowledges gratefully the hospitality extended to him by MIT during this visit. Address correspondence to Bernard Dumas, INSEAD, Finance Department, Boulevard de Constance, 77305 Fontainebleau Cedex, France, or e-mail: bernard.dumas@insead.fr.

The Review of Financial Studies Spring 2001 Vol. 14, No. 1, pp. 277–305

of openness of asset markets are substitutes or complements to each other. In the limit of full market segmentation, there is no question that they have to be regarded as complements: in the absence of any physical transfers between countries, the financial markets would be closed *de facto* because the payment (physical delivery) of dividends across countries would become impossible. So, in effect, a closure of the goods market acts as a closure of the asset market.[1] The unwillingness to trade in the asset market is inherently linked to the inability to transfer goods across borders because of the role of the financial market as a reflection of the future of the physical economy. But, in other ways, the two forms of openness are substitutes. We shall demonstrate that, at times at which it is not optimal to trade commodities, trade in financial securities can serve as a substitute. It is this link between asset trade and commodity trade that we wish to analyze.

In our view, developed economies today are characterized by financial markets that may or may not be integrated, but by commodity markets in which goods are not perfectly mobile. Evidence to that effect is presented in Section 1. In the current article, our objective is to measure the benefits of asset market integration when there exist costs to transferring (shipping) physical resources from one country to the other. Our focus is on the reduction of the welfare gain from integration, brought about by the imperfections of the real side of the world economy, while we assume that financial markets are complete, integrated, and frictionless.

Our analysis is closely related to Obstfeld (1994), where a model of international asset trade linking an economy's long-run output growth rate to its financial openness is developed. In his model, an economy that opens its asset market to trade may experience a substantial increase in growth (and welfare) because the ability to diversify risk allows a more intensive use of the riskier, higher expected return technologies. The assumption made by Obstfeld when opening the asset market to trade is that the market for goods is frictionless. That is, in contrast to our framework, in his model there are no impediments whatsoever to transferring physical resources from one country to the other. In comparison to our focus on the effect of segmented commodity markets, Subrahmanyam (1975), Errunza and Losq (1989), and Basak (1995), examine the welfare loss from capital controls. They study how the expansion in the set of financial claims, which one can invest in, affects welfare. Brennan and Solnik (1989) also provide an empirical estimate of the welfare gains from international financial flows.[2]

[1] Conversely, in a one-good model, a closing of the financial market amounts to complete autarky. Cole and Obstfeld (1991) have studied the opening of asset markets in a two-good, pure-exchange, frictionless economy with a logarithmic utility function and found that there are cases where the trading of financial securities is unnecessary.

[2] Obstfeld (1992) suggests that the gains from capital flows reported in Brennan and Solnik (1989) may have been overestimated.

Financial flows between nations arise from saving and investment decisions, which are made on the basis of intertemporal considerations. To account for the intertemporal nature of these decisions, it would not be realistic to examine a pure-exchange economy. Thus, we explicitly model the production decision. Obstfeld (1982) discusses why it is necessary to account for intertemporal considerations that arise in this context, and Cole and Obstfeld (1991) underscore the need to model the production decision explicitly. Also, for internal consistency, and because we are studying financial flows between large economies, our model is a general equilibrium one. That is, we do not make the small country assumption which is typically made in models analyzing capital flows.[3] Thus, our model allows for the intertemporal general equilibrium analysis of the linkages between the real and financial sectors of an open economy and extends the early work of Helpman and Razin (1978) on the relation between the financial and real sectors in international economies. Arndt and Richardson (1987) and Allen and Stein (1990) discuss why it is important to study the issues that arise in this context.

When we study only the extreme cases of zero transfer cost and infinite transfer cost, as we do in Section 3, the analysis is identical to a comparison between complete autarky and complete integration of both markets, as was done by Obstfeld (1994). But, in Section 4, we gradually increase transfer costs; as we move away from mobility in the market for goods, we maintain the assumption of full integration in the financial market. Investors remain free to modify costlessly their portfolio of domestic and foreign assets. In so doing, however, they take into account which country they live in; the physical resources currently trapped in their country act as a (temporarily) nontraded asset.

Our main finding is that the welfare gain from integration of financial markets is not greatly reduced by the presence of goods market imperfections, modeled as a cost of transferring goods from one country to the other. The policy implication of this result is that financial market integration is a worthwhile goal to pursue even when full goods mobility has not been achieved. The intuition underlying the above result is as follows. The ability to invest physically confers to goods asset characteristics, similar to those of storable goods. For this reason, physical frictions reduce the benefits of asset market integration comparatively little: the *optimal timing* of physical transfers mitigates the effect of transfer costs. At times at which it is not optimal to transfer goods, foreigners do not collect their payoffs. The financial market

[3] In particular, the interest rate in our model is determined endogenously and the equilibrium that we analyze is, in contrast to Lucas (1982), not a perfectly pooled one. The interest rate is endogenous in the following sense: when the decision is made (endogenously) to operate the riskless technology in a given country, the interest rate in that country is equal to the return on this riskless technology; on the other hand, when no investment is made in the riskless technology, then the interest rate is higher than the riskless technological rate and depends on the state variables of the model.

makes it possible for them simply to accumulate claims which are invested in the home production process and which will be redeemed later. Thanks to a well-functioning financial market, a good deal of shipping expenditures can be avoided.

The balance of the article is organized as follows. In Section 1, we present the evidence from earlier studies showing that the movements of goods around the world is far from free. In Section 2, we describe the model and indicate how the equilibrium can be obtained as a welfare maximization problem. In Section 3, we derive the effect of financial integration on welfare and growth. In Section 4, we obtain the equilibrium as one gradually increases transfer costs and we study a calibration based on stock market data, in order to gain some insight into the likely effect of integration in the real world. In Section 5, we examine the case with transfer costs as one changes the various parameter values. The appendix contains a description of the numerical procedure used to solve the problem.

1. Empirical Evidence on the International Mobility of Goods

In our view, developed economies today are characterized by financial markets that may or may not be integrated, but by commodity markets in which goods are not perfectly mobile. For example, investors in the United States can trade in Japanese financial claims quite easily, but incur significantly larger costs for trading goods with another country, say Japan. Similarly, it is much easier for investors within Western European countries to trade in foreign financial claims, but barriers to trade in goods still exist. The strong integration of international financial markets in developed economies is supported by the following observations documented in Frankel (1986): (i) the removal of capital controls following the end of the Bretton Woods agreement, (ii) the development of Euro-currency markets, and (iii) the absence of arbitrage opportunities in international financial markets. Halliday (1989) reports on the absence of capital controls in the financial markets of Austria, Belgium, Denmark, Germany, Ireland, Italy, Japan, Netherlands, the United Kingdom, and the United States. Wheatley (1988) and Gultekin, Gultekin, and Penati (1989) also provide evidence in support of integrated financial markets. To reflect the better integration of financial markets relative to commodity markets, in our model we will assume that financial markets are perfectly integrated.

However, as Gagnon (1989) shows, and as the importance attached to achieving world trade agreements attests, significant barriers to commodity trade still exist. Even though tariff levels have fallen in the last 20 years, this decrease has been more than offset by the increase in the magnitude of nontariff barriers. A comparison of the distortionary costs of quantitative restrictions in the three sectors (textiles, automobiles, and steel) with the distortionary costs of the remaining tariff protection reveals that the costs of quantitative restrictions are 10 times larger than the costs of tariffs [Melo and Tarr (1992, p. 12)]. Further details on the type of barriers to international trade that exist, for example, restrictive licenses and differences in technical regulations across countries, are provided in van Nunen (1990).

Deardorff and Stern (1990, Table A7) report that the average tariff in industrialized countries is about 6.6%. Deardorff (1989, Tables 4 and B5) reports, on the basis of several studies, the low and high estimates of *ad valorem* equivalents of nontariff barriers for the various traded goods in the major industrialized countries. For example, in the United States, the low (high) estimates range from 0% (0%) for machinery to 14.5% (20.5%) for food. For Japan, the low (high) estimates are 0% (0%) for machinery and 27.1% (58.1%) for food. Melo and Tarr (1992, pp. 200), based on their study of the textiles, automobile, and steel industries in the United States, estimate that the average tariffs that would generate the same welfare costs as those resulting from quotas in the three sectors is 49%.[4]

As has been documented by Marston (1990), an additional imperfection of the goods market, made possible by the existence of protections and transfer costs, arises from the fact that firms are able to price discriminate between markets.

As a result of these various imperfections, wide and persistent deviations from parity in the prices of goods between countries are pervasive. Engel and Rogers (1996) and many others have studied the deviations from the law of one price and the deviations from purchasing power parity (PPP) arising from it. Evidence on price discrepancies and costs for transporting goods across countries can also be found in Haskel and Wolf (1999) and Parsley and Wei (1999). When attempts have been made to fit an autoregressive model to PPP deviations, it has frequently been found that they follow a martingale behavior that would allow them randomly to drift away from zero indefinitely.

In the present article, as in Dumas (1992), we model the imperfection of commodities markets by introducing a proportional cost for exporting the physical good from one country to another. This specification implies the existence of two regimes prevailing at different times: one in which transfers of goods take place and cause price deviations quickly to revert to some definite level imposed by transfer costs, and one in which transfers of goods do not occur at the present time so that prices drift away from parity more freely. The recent introduction into the empirical literature on PPP deviations of threshold autoregressive statistical models (TAR) by Obstfeld and Taylor (1997) and O'Connell and Wei (1997) and of smooth transition autoregressive models (STAR) by Michael, Nobay, and Peel (1997) provides support for this specification.

[4] Erzan and Kuwahara (1989) report that the effective tariff rate is as high as 66% in Central America and 51% in South America, while the effective tariff rate in Africa is about 37%, in West Asia 5%, and in the rest of Asia 25%. A significant proportion of this effective tariff arises from nontariff barriers. For example, the percentage of the effective tariff that is a consequence of nontariff barriers is 100% in Central America, 85% in Africa, and 21% for Asia (other than West Asia).

In this article, the friction between international economies is modeled as a deadweight cost rather than as a tariff revenue that is available to the country collecting it. This is a conservative assumption because, even with the friction between economies being treated as a deadweight cost, its effect on welfare and growth is very small; thus putting back the "tariff" would only strengthen our results.

2. Social Choice in an Economy with Frictions

The economy is made up of two countries with perfectly integrated financial markets. The two countries are identified as the home and foreign country, and an asterisk (*) is used to denote all variables for the foreign country. The frictions in the goods market are modeled by introducing a proportional shipping cost for exporting/importing the physical good from one country to another.[5] One good exists in two versions, differentiated only by geographic location. The physical good that is produced in each country can be consumed, exported to the other country or reinvested in the production process.

The two countries are *ex ante* symmetric in all respects. Each country is populated by a large number of identical firms and consumers, the domestic and foreign representative consumers have identical preferences, and the production technologies in both countries are of the same form. The only factor that distinguishes one country from another is that production in each country is subject to a country-specific shock, and it is costly to export the physical good from one country to another. That is, each country is characterized by a unique innovation in its output process. Each representative investor is constrained to consume only from the stock of the good located in her own country: before one can consume from the foreign good, the good must be imported to the home country and the shipping cost must be paid.

The model we describe below differs from the one in Dumas (1992) in three respects: (i) preferences are given by recursive rather than expected utility functions, (ii) productivity shocks to the risky technology are allowed to be correlated across countries, and (iii) in addition to the risky technology, agents also have access to a riskless production technology. The riskless technology is introduced to allow for a second, indirect, and powerful effect on growth just as in Obstfeld (1994); that is, the two technologies are there as a parable for the choice between more risky and less risky projects.

2.1 The production set

Firms use as input the same good that they produce and this good is also the consumption good. It is assumed that all firms in a country have access to

[5] For a model where the transfer cost is a fixed amount rather than a proportional one, see Shrikhande (1992). Benninga and Protopapadakis (1988) model the shipping cost as one that arises because trade takes time. Dumas (1988, Appendix I) shows how the shipping cost in our model can be reinterpreted as one that arises because shipping takes time.

the same two production technologies: one is riskless with a rate of return r; the other is a constant returns to scale, risky production technology which is characterized by its instantaneous expected rate of return, μ, and instantaneous standard deviation, σ. We assume that $\mu > r$. The fraction of physical capital allocated to the risky technology is denoted w and the fraction allocated to the riskless one is $1 - w$. Adjustments in the allocation of capital to the two technologies *within* a country are instantaneous and costless. Nonpositive allocations are not feasible: $0 \leq w \leq 1$. There are no barriers to entry in an industry. With these assumptions, perfect competition obtains and the value of the domestic (foreign) industry is equal to the value of the existing stock of the domestic (foreign) good, $K(t)$ $(K^*(t))$. The instantaneous return on an investment in the domestic industry is

$$\begin{aligned} dY(t) &= [r + w(\mu - r)]K(t)dt + w\sigma K(t)dz(t), \\ K(0) &= K_0. \end{aligned}$$

The flow $dY(t)$ exceeds $dK(t)$, the formation of capital in the domestic industry, by the amount of domestic consumption and net exports. Similarly, the instantaneous return on an investment in the foreign industry is

$$\begin{aligned} dY^*(t) &= [r + w^*(\mu - r)]K^*(t)dt + w^*\sigma K^*(t)dz^*(t), \\ K^*(0) &= K_0^*. \end{aligned}$$

The two fundamental sources of uncertainty in this model are the production shocks in the home and foreign country. For symmetry, the technology shocks in the two countries have the same volatility, σ. In contrast to Dumas (1992), we allow the shocks to output in the two countries to be correlated, and we denote the correlation between the two shocks by η.

Exporting the good from one country to another entails a proportional cost. Let τ, where $\tau > 0$, denote the proportional (effective) cost for moving goods from one location to another. Thus, for every unit of good that is exported from one country, only $(1 - \tau)$ arrives in the other country. To account for the export of the good from one country to another, we introduce two nondecreasing processes, $X(t)$ and $X^*(t)$, with $X(0) = X^*(0) = 0$ which represent the cumulative amounts of the goods exported, since time 0, from the home to the foreign location, and from the foreign to the home location, respectively. These processes regulate the joint process for the stock of the domestic and foreign goods, $K(t)$ and $K^*(t)$.[6] The regulated processes for $K(t)$ and $K^*(t)$, adjusted for consumption $c(t)$ and exports/imports between the two countries,

[6] The theory of the instantaneous control of a Brownian motion, first examined in Benes, Shepp, and Witsenhausen (1980), is developed in Harrison and Taksar (1983) and Harrison (1985). For a rigorous treatment, see Davis and Norman (1990). For a simple exposition of the mathematical theory, see Dixit (1991) and Dumas (1991). Pindyck (1991) and Dixit (1992) provide a review of the applications of this theory to various economic problems.

are described by the following stochastic differential equations:

$$\begin{aligned} dK(t) &= [r + w(\mu - r)]K(t)dt + w\sigma K(t)dz(t) \\ &\quad - c(t)dt - dX(t) + (1-\tau)dX^*(t), \\ K(0) &= K_0; 0 \le w \le 1;\ \mu > r; \end{aligned} \tag{1}$$

$$\begin{aligned} dK^*(t) &= [r + w^*(\mu - r)]K^*(t)dt + w^*\sigma K^*(t)dz^*(t) \\ &\quad - c^*(t)dt - dX^*(t) + (1-\tau)dX(t); \\ K^*(0) &= K_0^*;\ 0 \le w^* \le 1;\ \mu > r. \end{aligned} \tag{2}$$

Given that the home and foreign investors are identical and that the production shocks have the same mean and variance, in an economy with frictionless commodity markets it would be optimal to diversify production risk by always maintaining equal stocks of the good at home and abroad. That is, it would be optimal to regulate the processes for $K(t)$ and $K^*(t)$ continuously so that $K(t) = K^*(t)$. However, in the presence of the shipping cost it is not optimal to correct an imbalance in the relative quantity of the good at home and abroad at each instant. This is because the benefits obtained by exporting or importing the good may be less than the cost incurred in doing so. Dumas (1988) and Davis and Norman (1990) have shown that, in the presence of transfer costs, there will exist a region of the state space within which it is optimal not to export or import the good. With proportional transfer costs and with isoelastic utility, this no-trade region is a cone in (K, K^*) space delineated by two rays from the origin. Further, given that the transfer costs are proportional, the optimal shipping policy is to trade the infinitesimal amount of physical goods just necessary to stay within the no-trade region.

2.2 Consumer preferences

Consumption of the domestic investor, at any instant t, is denoted $c(t)$ and that of the foreign investor by $c^*(t)$. Preferences of the representative consumer in the domestic country are given by an isoelastic recursive utility function, with the degree of relative risk aversion (RRA) equal to $1 - \alpha$ with $\alpha < 1$. Separately we specify the elasticity of intertemporal substitution (EIS) as being equal to $1/(1-\rho)$, where $\rho < 1$.[7] The special case of the time-additive expected lifetime utility function is obtained for $\alpha = \rho$. The distinction between these two aspects of preferences—risk aversion and intertemporal substitution—is made possible by the use of nontime additive, recursive utility proposed by Kreps and Porteus (1978), Epstein and

[7] The use of recursive utility also allows us to compare our results with those of Obstfeld (1994).

Zin (1989), and Duffie and Epstein (1992).[8] The preferences of the foreign representative investor are similarly defined. We also assume that the representative consumers in the two countries start out with identical endowments and have the same rate of impatience, β.

Geoffard (1996) has shown, under certainty, that the objective function of an investor-consumer with the aforementioned preferences may be written in a pseudo time-additive form; he calls this variational utility. El Karoui, Peng, and Quenez (1997) and Dumas, Uppal, and Wang (2000) show that Geoffard's deterministic formulation can be extended to a stochastic setting and is then equivalent to the stochastic recursive utility given in Duffie and Epstein (1992).[9] Using the results in Dumas, Uppal, and Wang, we can express the objective of a single investor as follows:

$$\begin{aligned} & \sup_{c} \inf_{\nu} E_t \left\{ \int_t^{\infty} x_t F(c_t, \nu_t) dt \right\}, \\ \text{s.t.} \ : \quad & dx = -\nu_t x_t dt, \\ \text{and} \ : \ & \text{usual budget or feasibility constraints,} \end{aligned} \tag{3}$$

where $F(c, \nu)$, called the "felicity function," is in our case equal to

$$F(c, \nu) = \beta \frac{c^{\alpha}}{\alpha} \left[-\frac{\rho - \alpha}{\alpha - \frac{\rho \nu}{\beta}} \right]^{\frac{\alpha}{\rho} - 1}. \tag{4}$$

Here, $\alpha, \rho < 1, \neq 0$, β is the rate of impatience, and x_t may be interpreted as a psychological discount factor applicable to utility. The discount factor is endogenous, however, since its dynamics are governed by the choice variable ν.

Note that there exists a close correspondence between Geoffard's "felicity function" and the "aggregator function" of Duffie and Epstein (1992). Denoting the latter by $f(c, v)$, one can show that the two functions are Legendre transforms of each other:

$$F(c, \nu) = \sup_{v} [f(c, v) + \nu v];$$

and

$$f(c, v) = \inf_{\nu} [F(c, \nu) - v \nu].$$

[8] When utility is isoelastic, risk aversion and elasticity of intertemporal substitution are intertwined with a third aspect of investor preferences: the preference for early or late resolution of uncertainty. It can be shown [see Kreps and Porteus (1978) and Skiadas (1998)] that $\rho < \alpha$ ($\rho > \alpha$) implies preference for late (early) resolution when $\alpha < 0$ ($\alpha > 0$). In the case of preference for early resolution, the inf operator in Equation (3) should be replaced by a sup operator.

[9] The reader must be warned that the existence assumptions stated in Dumas, Uppal, and Wang (1997) are not satisfied in the case of constant risk aversion and constant elasticity of intertemporal substitution which is hypothesized below as Equation (4). Hence, the existence of variational utility may be an issue. However, a recent article by Schroder and Skiadas (1997) provides a foundation for stochastic differential utility that would be valid for the case at hand.

The felicity function of Equation (4) is the Legendre transform of the aggregator (with variance multiplier equal to zero) proposed by Duffie and Epstein (1992, p. 367) for isoelastic, Kreps–Porteus preferences:

$$f(c, v) = \frac{\beta}{\rho} \frac{c^{\rho} - (\alpha v)^{\rho/\alpha}}{(\alpha v)^{\frac{\rho}{\alpha}-1}}.$$

2.3 Social choice

The equilibrium of the economy is solved for by means of the equivalent social planner's problem. The motivation for using this approach to obtain the solution, rather than solving directly the first-order conditions for each agent, is that it is much more convenient: one needs to determine only a single unknown function (the value function of the planner), which can be obtained by solving a single differential equation. On the other hand, if one used the direct approach, there would be several unknown functions that one would have to solve for—the value functions of the individual agents and also the asset prices—and identifying these would require solving multiple differential equations, which because of the equilibrium nature of the problem, would have to be solved simultaneously.

The results in Dumas, Uppal, and Wang (2000) show that the objective function of the central planner is equal to the sum of the individual objective functions of Equation (3). Using these results and the above formulation of the single-agent objective function allows a straightforward treatment of the welfare optimization problem by means of continuous-time, stochastic dynamic programming, which is a considerable simplification in comparison with extant methods.[10]

The central planner's optimization problem, for the times at which no shipment takes place, may be written as

$$\begin{aligned} V(K, K^*, x, x^*) \equiv & \sup_{c,\, c*,\, w,\, w^*} \inf_{\nu,\, \nu^*} E_t \left\{ \int_t^{\infty} \left[x F(c, \nu) + x^* F\left(c^*, \nu^*\right) \right] dt \right\}, \\ & \text{s.t. } : \\ & dK = ([r + w(\mu - r)]K - c)dt + w\sigma K dz;\ \ 0 \le w \le 1; \\ & dK^* = ([r + w^*(\mu - r)]K^* - c^*)dt + w^*\sigma K^* dz^*; \\ & \qquad 0 \le w^* \le 1; \\ & dx = -\nu x dt, \\ & dx^* = -\nu^* x^* dt, \end{aligned} \tag{5}$$

with initial conditions $K(0) = K_0$, $K^*(0) = K_0^*$, $x(0) = x^*(0) = 1$.

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[10] If one used the approach suggested in Duffie, Geoffard, and Skiadas (1994), one would need to solve a system of $2 \times N$ differential equations, where N is the number of agents. Thus, in the case of our economy, one would have to solve a system of four differential equations.

The Hamilton–Jacobi–Bellman differential equation for the value function V is

$$0 = \sup_{c,c*,w,w^*} \inf_{\nu,\nu^*} \Big\{ xF(c,\nu) + x^*F(c^*,\nu^*) - V_x \nu x - V_{x^*}\nu^* x^* + V_K\big([r + w(\mu - r)]K - c\big) + V_{K^*}\big([r + w^*(\mu - r)]K^* - c^*\big) + \frac{1}{2}\sigma^2 V_{KK} w^2 K^2 + \frac{1}{2}\sigma^2 V_{K^*K^*}(w^*K^*)^2 + \sigma^2 \eta V_{KK^*} wKw^*K^* \Big\}, \tag{6}$$

subject to boundary conditions at the points where shipment of goods takes place: $V_{K^*} = (1-\tau)V_K$ at one end, and $(1-\tau)V_{K^*} = V_K$ at the other [smooth-pasting boundary conditions; see Dumas (1992)].

After substituting in the first-order conditions for the $\inf_{\nu,\nu^*}$ problem, we get

$$0 = \sup_{c,c*,w,w^*} \Big\{ xf(c,V_x) + x^*f(c^*,V_{x^*}) + V_K\big([r + w(\mu - r)]K - c\big) + V_{K^*}\big([r + w^*(\mu - r)]K^* - c^*\big) + \frac{1}{2}\sigma^2 V_{KK} w^2 K^2 + \frac{1}{2}\sigma^2 V_{K^*K^*}(w^*K^*)^2 + \sigma^2 \eta V_{KK^*} wKw^*K^* \Big\}, \tag{7}$$

where f is, as we recall, the aggregator function of the Duffie–Epstein utility specification. The approach adopted here provides an elegant and straightforward derivation of the Bellman equation in terms of the aggregator f.

After substituting in the definition of the aggregator function f, the first-order conditions for all decision variables except the choices of technologies,[11] the differential equation for the value function V is

$$0 = \sup_{0\le w,w^*\le 1} \Bigg\{ \left(\frac{1}{\rho} - 1\right) V_K \left[\frac{V_K (\alpha V_x)^{\frac{\rho}{\alpha}-1}}{x\beta}\right]^{\frac{1}{\rho-1}} + \left(\frac{1}{\rho} - 1\right) V_{K^*} \times \left[\frac{V_{K^*} (\alpha V_{x^*})^{\frac{\rho}{\alpha}-1}}{x^*\beta}\right]^{\frac{1}{\rho-1}} - \frac{\beta}{\rho}\alpha V + V_K[r + w(\mu - r)]K + V_{K^*}[r + w^*(\mu - r)]K^* + \frac{1}{2}\sigma^2 V_{KK}(wK)^2 + \frac{1}{2}\sigma^2 V_{K^*K^*}(w^*K^*)^2 + \sigma^2 \eta V_{KK^*}(wK)(w^*K^*) \Bigg\}. \tag{8}$$

[11] $c = \left[\frac{V_K(\alpha V_x)^{\frac{\rho}{\alpha}-1}}{x\beta}\right]^{\frac{1}{\rho-1}}$; $c^* = \left[\frac{V_{K^*}(\alpha V_{x^*})^{\frac{\rho}{\alpha}-1}}{x^*\beta}\right]^{\frac{1}{\rho-1}}$.

Our task is to obtain the unknown function V from this partial differential equation with four arguments which, furthermore, incorporates a constrained optimization problem. Examining Equation (5), it is clear that V has some homogeneity properties: doubling K and K^* will double c and c^* and multiply V by 2^α, while doubling x and x^* will leave ν and ν^* unchanged and double V. Whenever $V(K, K^*, x, x^*)$ is a solution to the optimization problem, so are $\zeta^{-\alpha}V(\zeta K, \zeta K^*, x, x^*)$ and $\zeta^{-1}V(K, K^*, \zeta x, \zeta x^*)$ for any ζ. We will take advantage of these properties in solving for the value function, V.

3. Benefits to Financial Market Integration

The levels of welfare and growth achieved in the above economy can be calculated. As shown in Obstfeld (1994), the calculations can be done analytically for the extreme cases of perfect integration ($\tau = 0$) and total autarky ($\tau = 1$). The intermediate case of international commodity markets with frictions ($0 < \tau < 1$) is studied in the next section by solving numerically the welfare optimization problem of Equation (5).

3.1 Effect of integration on welfare and growth

Consider the two extreme cases in turn. First, *under autarky* ($\tau = 1$), the solution to Equation (5) exists provided that

$$\xi^{aut} > 0,$$

where

$$\frac{c}{K} = \xi^{aut} \equiv \frac{\rho}{1-\rho}\left[\frac{\beta}{\rho} - r - w^{aut}(\mu - r) + \frac{1}{2}(1-\alpha)(w^{aut})^2\sigma^2\right], \tag{9}$$

$$w^{aut} = \min\left[\frac{\mu - r}{(1-\alpha)\sigma^2}, 1\right]. \tag{10}$$

It is noteworthy that risk aversion, $1 - \alpha$, plays a role in the decision w to invest in risky versus riskless production, whereas only the elasticity of intertemporal substitution $1/(1-\rho)$ affects only the consumption decision c/K. After lengthy but simple calculations, the solution for the unknown function V is found to be

$$V^{aut}(K, K^*, x, x^*) = \kappa^{aut}[xK^\alpha + x^*(K^*)^\alpha], \tag{11}$$

where

$$\kappa^{aut} = \frac{1}{\alpha}\left[\left(\frac{1}{\beta}\right)^{\frac{1}{1-\rho}}\xi^{aut}\right]^{\frac{\alpha(\rho-1)}{\rho}}.$$

Under complete integration ($\tau = 0$), the solution exists provided that

$$\xi^{opt} > 0,$$

where

$$\frac{c+c^*}{K+K^*} = \xi^{opt} \equiv \frac{\rho}{1-\rho}\left[\frac{\beta}{\rho} - r - w^{opt}(\mu - r) + \frac{1}{4}(1-\alpha)(w^{opt})^2(1+\eta)\sigma^2\right], \tag{12}$$

$$w^{opt} = \min\left[\frac{2}{1+\eta}\frac{\mu - r}{(1-\alpha)\sigma^2}, 1\right] \geq w^{aut}. \tag{13}$$

The remark above, concerning the separate roles of risk aversion and elasticity of intertemporal substitution, also applies here. The solution for V is found using the observation that under integration perfect physical balance ($K = K^*$) is maintained at all times:

$$V^{opt}(K, K^*, x, x^*) = \kappa^{opt}(x + x^*)(K + K^*)^{\alpha}, \tag{14}$$

where

$$\kappa^{opt} = \frac{1}{\alpha}\left[\frac{1}{2}\left(\frac{2}{\beta}\right)^{\frac{1}{1-\rho}}\xi^{opt}\right]^{\frac{\alpha(\rho-1)}{\rho}}.$$

The levels of welfare and the rates of growth achieved under autarky and integration can now be compared. Our measure of the welfare cost, that is, compensating variation in initial endowments, is the same as that used in Obstfeld (1994). The compensating variation is the increase, π, in initial endowments (K and K^*) that is required in the economy with completely segmented real sectors so that the lifetime expected utility in this economy is increased to the same level as that in the economy with integrated markets.[12] That is, the fraction π is one that makes $V^{opt}[K, K^*, x, x^*] = V^{aut}[(1+\pi)K, (1+\pi)K^*, x, x^*]$. At the point $K = K^*$ and $x = x^*$, this compensating difference is given by

$$\pi = \left[\frac{\xi^{opt}}{\xi^{aut}}\right]^{-\frac{1-\rho}{\rho}} - 1.$$

Since in the two extreme cases ($\tau = 0$ and $\tau = 1$), the ratio of consumption over wealth is constant over time, expected growth rates of consumption $\left(\frac{1}{c}\frac{E[dc]}{dt}\right)$ and wealth are equal to each other and their values corresponding to the two extreme situations are given by

$$g^{aut} = w^{aut}(\mu - r) + r - \xi^{aut}$$
$$g^{opt} = w^{opt}(\mu - r) + r - \xi^{opt}.$$

[12] The calculation is done for the situation in which the countries under autarky have equal capital stocks: $K = K^*$.

Table 1
Results and parameter values for base case calibration

Scenario	$\xi = c/K$	Growth rate	w	π = welfare gain
Perfect segmentation: autarky	0.0266	0.0169	0.3750	—
Perfect integration: optimum	0.0272	0.0217	0.4826	0.13

The first row of this table gives the outcome for the calibration under autarky and the second row under perfect integration of commodity and financial markets. For each case, we report the consumption rate, the growth rate, and the share of total investment in risky technology. The number in the last column gives the welfare gain from perfect integration of commodity and financial markets relative to the case of autarky. Parameters values are $r = 0.02$, $\mu = 0.08$, $\sigma = 0.20$, $\eta = 0.554$, $\beta = 0.02$, $1 - \alpha = 4$, $1/(1-\rho) = 0.5$.

3.2 A calibration with imperfect mobility of goods

Obstfeld (1994) provides several example situations to illustrate the role played by the willingness to bear risk, which is increased under integration. His examples are meant only to highlight a number of analytical properties of the economy and not to provide a realistic assessment of the welfare gains brought about by integration, perfect or imperfect. However, he also provides two numerical calibrations representing the quantitative situation of several regions of the world. His first calibration is based on observed consumption behavior, the second on observed stock returns.

We unify our interpretation of the effects at play by considering a single calibration accompanied by some comparative statics exercises. We report the effect of shipping costs for the calibration of Obstfeld that is based on stock market data.[13] Given that our model has been constructed for a symmetric situation, we amend his numbers slightly to estimate welfare gains as they would be approximately when one integrates western Europe and northern America: $r = 0.02$, $\mu = 0.08$, $\sigma = 0.20$, and $\eta = 0.554$.

The quantities corresponding to this exercise are reported in Table 1. As explained by Obstfeld, there is both a direct and an indirect effect on welfare. The welfare gain does not simply arise from improved risk bearing; it also arises from the fact that because of the improved opportunities for risk sharing, investors' willingness to invest in risky projects increases from 0.375 under autarky to 0.4826 under integration. In total, the welfare impact, π, of integration is quite large: 13%.[14] Thanks to the increased willingness to bear risk under integration, the expected growth rate of consumption increases from 1.69% to 2.17%.

4. The General Case with Frictions

We now solve the optimization problem of Equation (5) for $0 < \tau < 1$. The only parameter that we need to specify, in addition to the ones specified in

[13] Because of some numerical difficulties which we have encountered, however, risk aversion is kept at 4, somewhat below Obstfeld's number which was 6.

[14] In some cases, Obstfeld found much higher welfare gains. The reason for the difference is that Obstfeld is able to accumulate the benefits of integrating many countries, whereas we are limited to two countries only.

the previous section, is the transfer cost (τ). On the basis of the evidence of Section 1 above, we aim to be conservative by considering values for τ between 0% and 25%. The appendix explains how one exploits the homogeneity, and then solves the differential equation numerically by successive interpolations over two variables: $\omega = \frac{K}{K+K^*}$ and $\lambda = \frac{x}{x+x^*}$.

The goal of our undertaking is to determine how rapidly welfare and growth decrease as one gradually introduces sand in the gear, that is, lets transfer costs impede the flow of goods between countries. Is it or is it not the case that moderate transfer costs would drastically reduce the welfare and growth gains of integration? We answer the question at the point of equal weights for the two countries, both in terms of welfare weights: $\lambda = 0.5$ and in terms of physical balance: $\omega = 0.5$. The main reason for performing the analysis at the point $\omega = 0.5$ is that this is the only point that is common across simulations for different parameters, and in particular, across all levels of shipping costs (0–100%). Thus if we picked some other point, for example, the boundary of the cone, then this point would be changing with the parameter values, making it difficult to compare across different cases.

Table 2 gives the decrease in the welfare gain in the numerical calibration as one varies the proportional shipping costs. The drop is not negligible. Nonetheless, for seemingly large transfer costs of 25%, the welfare gain over and above autarky is still as large as 10.77%, compared with the 13% gain from full integration. Evidently, as the flow of international physical transfers has been optimized, a compromise has been found between reaping the benefits of international diversification (arising both from risk sharing and from optimal investment) and saving on shipping costs. The flow of physical transfers has been sufficiently slowed down to economize on transfer costs and preserve the larger part of the welfare gains, and this has been made possible by the presence of financial markets. That is, in this production economy financial markets make it possible to store the physical resource while waiting to ship it at the optimal time so that the drop in welfare for reasonable levels of shipping cost is not precipitous. The comparative dynamics exercise conducted in the next section will verify that this conclusion is robust to changes in parameter values.

In Table 2 we also report the effect of shipping costs on the expected growth rate of consumption. Even with a 25% transfer cost, the expected growth rate of consumption is 1.99%, which represents a reduction by only a third of the gain arising from perfect integration.

Shipping costs act on growth in two interrelated ways. First, they represent, of course, a deadweight loss to society. For this reason alone, a reduction of shipping costs enhances growth. But, the size of this impact depends on shipping activity, which is dampened by the costs. Second, as shipping activity is reduced, the ability to diversify is hindered and with it the willingness to undertake risky projects. We saw that an effective reduction of the willingness to undertake risky projects produces a drop in expected growth. We illustrate these two effects by means of a comparative dynamics exercise.

Table 2
The effect of shipping costs on welfare and growth

Shipping cost	Welfare gain	Expected growth rate
0%	0.1300	0.02172
1%	0.1290	0.02163
2%	0.1281	0.02155
3%	0.1272	0.02146
4%	0.1263	0.02138
5%	0.1254	0.02130
10%	0.1209	0.02092
15%	0.1164	0.02058
20%	0.1121	0.02027
25%	0.1077	0.01999
Autarky	0	0.01690

This table presents the percentage increase, π, in initial capital stocks that is required to make the welfare level under autarky equal to that under financial integration with different levels of shipping costs. The table also presents the expected rate of growth of consumption with different levels of shipping costs. The case with zero shipping costs corresponds to that of perfect integration in both financial and commodity markets. The parameter values are as in Table 1: $r = 0.02$, $\mu = 0.08$, $\eta = 0.554$, $\beta = 0.02$, $1 - \alpha = 4$, $1/(1 - \rho) = 0.5$, $\sigma = 0.20$.

5. Comparative Dynamics

In this section we conduct a comparative analysis of the equilibrium under varying parameter values. First, we digress briefly to recall some basic principles concerning the comparative statics of savings behavior. These are useful in interpreting the comparison of the two extreme situations of autarky and integration, separately and in comparative analyses with each other.[15] Then, we consider the effect on our results of changes in the parameters for the calibration described in the previous section. In both these analyses, recursive utility allows us to distinguish between aversion to risk across states of nature and aversion to risk over time.

5.1 The comparative statics of savings behavior

Consider a simple two-period, deterministic consumption problem and consider a shift in the rate of interest. The usual Slutsky analysis of income and substitution effects indicates that an increase in the rate of interest unambiguously increases period 2 consumption. It reduces period 1 consumption (or increases savings) if and only if the substitution effect dominates the income effect, which happens when the elasticity of intertemporal substitution is greater than one. Irrespective of the effect on period one consumption, the growth rate of consumption between periods 1 and 2 is raised unambiguously.

The extension of these basic results to the case of continuous-time uncertainty is straightforward when the assumption of recursive, isoelastic utility function is made, for it allows, in many cases, a simple separation of the consumer's attitudes toward risk and toward time. A consumer endowed with this form of utility treats a risky return exactly as he would a riskless one

[15] Integration permits diversification of risk and is, therefore, equivalent to a decrease in the level of risk of an investor's portfolio.

equal to the risky return's certainty equivalent value.[16] Under that assumption, the certainty equivalent of a country's return under autarky is equal to [see Equation (9)]

$$r + w^{aut}(\mu - r) - \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2.$$

Under integration, given that capital is allocated equally to the two countries, the combined world certainty equivalent rate of return is [See Equation (12)]

$$r + w^{opt}(\mu - r) - \frac{1}{4}(1-\alpha)\left(w^{opt}\right)^2(1+\eta)\sigma^2.$$

We now consider the effect of expected return and risk on savings, the certainty equivalent growth rate of consumption, and the expected growth rate of consumption.

- As far as *savings* behavior is concerned, we can repeat what has been said for the case of certainty: A reduction in portfolio risk is then equivalent to an increase in expected return; only the certainty equivalent matters. An increase in the expected rate of return on the portfolio $r + w(\mu - r)$ (or a decrease in its variance) unambiguously reduces consumption (or increases savings) if and only if the substitution effect dominates the income effect, which happens when the elasticity of intertemporal substitution is greater than 1 ($\rho > 0$). This fact is immediately apparent in the factor ρ which is in front of the right-hand sides of Equations (9) and (12).
- The *certainty equivalent* of the growth rate of consumption is defined as

$$CEg \triangleq \frac{1}{c}\frac{E[dc]}{dt} - \frac{1}{2}(1-\alpha)\frac{[dc/c]^2}{dt}.$$

Under autarky, it is equal to

$$\begin{aligned} CEg^{aut} &= g^{aut} - \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2 \\ &= -\frac{1}{1-\rho}\left[\beta - r - w^{aut}(\mu - r) + \frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2\right], \end{aligned}$$

and under perfect integration it is

$$\begin{aligned} CEg^{opt} &= g^{opt} - \frac{1}{4}(1-\alpha)\sigma^2\left(w^{opt}\right)^2(1+\eta) \\ &= -\frac{1}{1-\rho}\left[\beta - r - w^{opt}(\mu - r) + \frac{1}{4}(1-\alpha)\left(w^{opt}\right)^2(1+\eta)\sigma^2\right]. \end{aligned}$$

[16] See Selden (1979). The recursive utility function, which we use here, is a special case of the ordinal certainty equivalent (OCE) utility proposed by Selden. Epstein and Zin (1989) have criticized OCE utility functions for generally being time inconsistent, the special case of recursive utilities being the only one leading to time consistency.

As in the case of certainty, it is increased unambiguously by an increase in the expected rate of return (or a drop in risk) of an investor's portfolio.

- Finally, we consider the *expected rate of growth of consumption* under autarky and perfect integration, which are obtained by adding back the variance of return to the certainty equivalent:

$$\begin{aligned} g^{aut} &= -\frac{1}{1-\rho}\left[\beta - r - w^{aut}(\mu - r)\right] \\ &\quad -\frac{\rho}{1-\rho}\frac{1}{2}(1-\alpha)\left(w^{aut}\right)^2\sigma^2, \end{aligned} \tag{15}$$

$$\begin{aligned} g^{opt} &= -\frac{1}{1-\rho}\left[\beta - r - w^{opt}(\mu - r)\right] \\ &\quad -\frac{\rho}{1-\rho}\frac{1}{4}(1-\alpha)\sigma^2\left(w^{opt}\right)^2(1+\eta). \end{aligned} \tag{16}$$

From the result above on the certainty equivalent of the growth rate of consumption, it follows that the expected rate of growth of consumption increases unambiguously with an increase in the expected rate of return. However, the expected rate of growth of consumption may increase or decrease with a drop in risk, depending on whether or not the decrease in the variance term (which has been added back to the certainty equivalent) overrides the increase in the certainty equivalent rate of return. In the absence of a portfolio shift (w held fixed), the expected rate of growth is increased if and only if the intertemporal elasticity of substitution is greater than 1 ($\rho > 0$) [see also Devereux and Smith (1994)]. But when the investor combines optimally a risky and a riskless asset and no constraints are binding, a decrease in the level of risk of the risky asset induces a portfolio shift. Substituting optimal portfolio decisions, from Equations (10) and (13), into Equations (15) and (16), respectively,

$$\begin{aligned} g^{aut} &= -\frac{1}{1-\rho}(\beta - r) + \frac{2-\rho}{1-\rho}\frac{1}{2}\frac{(\mu - r)^2}{(1-\alpha)\sigma^2} \\ g^{opt} &= -\frac{1}{1-\rho}(\beta - r) + \frac{2-\rho}{1-\rho}\frac{1}{1+\eta}\frac{(\mu - r)^2}{(1-\alpha)\sigma^2}. \end{aligned}$$

Then, the expected rate of growth of consumption is unambiguously increased by a decrease in risk.

These basic ideas apply exactly to our model in the extreme cases of full integration and full segmentation.[17] They will apply approximately for the intermediate case of imperfect mobility of goods.

[17] They are reflected exactly in Table 3 below, in the rows marked "zero shipping cost."

5.2 Analysis of the equilibrium for different parameter values

We now undertake a comparative analysis of the equilibrium for different parameter values. For the base case, we consider the calibration described in Section 3.2, in which the portfolio weights on the risky technology are less than unity. Our results are reported in Table 3.

First, consider in panel A of Table 3, a variation in the level of output risk, σ (the size of productivity shocks) from 0.1 to 0.2 (the base case). Row A1 of Table 3 shows that an increase in output risk raises the welfare gain of imperfect integration relative to autarky, for as long as the investment in the risky industry (the portfolio weight, row A7) is constrained at 1, but decreases it when the weight becomes free to move. This is exactly as under perfect integration (row A2).

The expected growth rate is, of course, reduced by the presence of shipment costs (compare rows A3 and A4). This is true for two reasons. First, shipment costs drain resources and their mere existence would reduce growth. The second reason is better understood if we first examine the effect of the level of risk on the opening of the cone (row A9): a higher level of risk causes the cone of no shipment in (K, K^*) space to widen. This is because a higher level of risk, for an unchanged position of the cone, would imply more frequent shipments and excessive costs.[18] But as the cone is widened, less diversification occurs and that induces agents to invest less in the risky technologies than they would under perfect integration (compare rows A7 and A8).[19] Because of this physical investment choice, the expected growth rate is further reduced relative to perfect integration.

Returning to the effect of risk on expected growth (row A3), we find here again a nonmonotonic variation, as under perfect integration (row A4). The maximum would generally occur at a different point than under full integration because the portfolio weights are lower, so that the $w \leq 1$ constraint becomes operative at a different level of risk. Finally, the rate of consumption out of wealth (row A5) drops monotonically as risk increases, because of the precautionary motive. This is true equally with or without shipping costs (rows A5 and A6).

Consider next a variation (panel B of Table 3) in the degree of risk aversion, $1 - \alpha$, from 0.5 to 4 (the base case). Examining the equations corresponding to the perfect integration and autarky situations (in Section 5.1), it is easy to verify in these two extreme cases that risk and risk aversion play a role only via the compound parameter $(1 - \alpha) \times \sigma^2$. Under imperfect integration, however, an increase in risk aversion does not produce exactly the same effect as an increase in risk. In fact, it causes the cone of no shipment to tighten (row B9) for as long as the investment in risky technology

[18] This effect overcomes the desire to tighten the cone arising from increased risk which enhances the benefits of good diversification.

[19] That is, whenever the weight is not bumping against the 100% constraint.

Table 3
The comparative statics for the calibration

Panel A: Varying volatility			$\sigma = .100$	$\sigma = .125$	$\sigma = .150$	$\sigma = .175$	*$\sigma = .200$*
A1	π, Welfare gain	10% shipping cost	0.1050	0.1948	0.1854	0.1494	*0.1209*
A2		zero shipping cost	0.1146	0.2112	0.2005	0.1602	*0.1300*
A3	Expected growth rate	10% shipping cost	0.0372	0.0414	0.0371	0.0274	*0.0209*
A4		zero shipping cost	0.0378	0.0421	0.0386	0.0284	*0.0217*
A5	Consumption rate	10% shipping cost	0.0210	0.0188	0.0163	0.0146	*0.0135*
A6		zero shipping cost	0.0211	0.0189	0.0164	0.0147	*0.0136*
A7	Portfolio weights	10% shipping cost	1.0	1.0	0.8444	0.6212	*0.4753*
A8		zero shipping cost	1.0	1.0	0.8580	0.6304	*0.4826*
A9	Cone opening	10% shipping cost	0.2558	0.2704	0.3440	0.4684	*0.5744*
A10		zero shipping cost	0.0	0.0	0.0	0.0	*0.0*
Panel B: Varying RRA			$1-\alpha=.5$	$1-\alpha=1.5$	$1-\alpha=2$	$1-\alpha=3$	*$1-\alpha=4$*
B1	π, Welfare gain	10% shipping cost	0.0324	0.1710	0.1883	0.1495	*0.1209*
B2		zero shipping cost	0.0502	0.2003	0.2173	0.1627	*0.1300*
B3	Expected growth rate	10% shipping cost	0.0326	0.0401	0.0405	0.0277	*0.0209*
B4		zero shipping cost	0.0339	0.0417	0.0434	0.0290	*0.0217*
B5	Consumption rate	10% shipping cost	0.0229	0.0189	0.0170	0.0147	*0.0135*
B6		zero shipping cost	0.0231	0.0192	0.0172	0.0148	*0.0136*
B7	Portfolio weights	10% shipping cost	1.0	1.0	0.9379	0.6322	*0.4753*
B8		zero shipping cost	1.0	1.0	0.9652	0.6435	*0.4826*
B9	Cone opening	10% shipping cost	0.4626	0.3666	0.3680	0.4850	*0.5744*
B10		zero shipping cost	0.0	0.0	0.0	0.0	*0.0*
Panel C: Varying EIS (with RRA = 4)			$\frac{1}{1-\rho}=.10$	$\frac{1}{1-\rho}=.30$	*$\frac{1}{1-\rho}=.50$*	$\frac{1}{1-\rho}=1.1$	$\frac{1}{1-\rho}=1.2$
C1	π, Welfare gain	10% shipping cost	0.0995	0.1092	*0.1209*	0.1767	0.1912
C2		zero shipping cost	0.1077	0.1178	*0.1300*	0.1883	0.2036
C3	Expected growth rate	10% shipping cost	0.0152	0.0181	*0.0209*	0.0295	0.0309
C4		zero shipping cost	0.0159	0.0188	*0.0217*	0.0304	0.0318
C5	Consumption rate	10% shipping cost	0.0164	0.0150	*0.0135*	0.00929	0.00857
C6		zero shipping cost	0.0165	0.0151	*0.0136*	0.00928	0.00855
C7	Portfolio weights	10% shipping cost	0.4744	0.4749	*0.4753*	0.4766	0.4768
C8		zero shipping cost	0.4826	0.4826	*0.4826*	0.4826	0.4826
C9	Cone opening	10% shipping cost	0.5660	0.5700	*0.5744*	0.5918	0.5950
C10		zero shipping cost	0.0	0.0	*0.0*	0.0	0.0
Panel D: Varying EIS (with RRA = 1.5)			$\frac{1}{1-\rho}=.10$	$\frac{1}{1-\rho}=.30$	$\frac{1}{1-\rho}=.50$	$\frac{1}{1-\rho}=1.1$	$\frac{1}{1-\rho}=1.2$
D1	π, Welfare gain	10% shipping cost	0.1218	0.1424	0.1710	0.4172	0.5455
D2		zero shipping cost	0.1433	0.1670	0.2003	0.4940	0.6525
D3	Expected growth rate	10% shipping cost	0.0261	0.0331	0.0401	0.0613	0.0648
D4		zero shipping cost	0.0270	0.0343	0.0417	0.0637	0.0673
D5	Consumption rate	10% shipping cost	0.0261	0.0225	0.0189	0.0082	0.0064
D6		zero shipping cost	0.0265	0.0228	0.0192	0.0082	0.0063
D7	Portfolio weights	10% shipping cost	1.0	1.0	1.0	1.0	1.0
D8		zero shipping cost	1.0	1.0	1.0	1.0	1.0
D9	Cone opening	10% shipping cost	0.3554	0.3610	0.3666	0.3828	0.3854
D10		zero shipping cost	0.0	0.0	0.0	0.0	0.0

Shipping costs are set at 10%. All quantities (except for the cone size) are measured at the center point where $x/(x+x^*) = 0.5$, $\omega \equiv K/(K+K^*) = 0.5$. The "Expected growth rate" is the expected growth rate of consumption. The "Consumption rate" is the consumption of one country relative to the total capital stock in both countries, $c/(K+K^*)$. The "Portfolio weights" are the share of each country's capital stock invested in the risky technology, w. The number for the "Cone opening" is the size of the no-trading region, which is given by the difference between $\bar{\omega}$ and $\underline{\omega}$. Base case numbers are displayed in italics.

is constrained. This is because increased risk aversion enhances the benefits of diversification. Admittedly, an increase in risk has this effect as well, but an increase in risk, as we saw, has the additional effect of increasing the frequency of shipments, which risk aversion does not. However, whenever the investment into risky technology is not constrained (as is the case for sufficiently high risk aversion), an increase in risk aversion widens the cone (row B9 again). In short, because of the portfolio-shift effect, the effect of risk aversion on the cone is not monotonic. Nonetheless, the effects of risk aversion on the welfare gain, the expected growth rate, and the consumption rate (rows B1–B6) are qualitatively similar to those of risk, and the direction of these effects depends entirely on whether the portfolio weight on risky projects is constrained or not (rows B7 and B8).

Finally, in panels C and D of Table 3, we turn to the elasticity of intertemporal substitution (EIS), varying from 0.1 to 1.2 (0.5 being the base case), for two values of risk aversion: one low and one high. The lower risk aversion of 1.5 is introduced in order to examine a situation in which the $w \leq 1$ constraint is binding.[20] All the effects of EIS are monotonic and they are under imperfect integration as under perfect integration: welfare and growth increase with EIS and the rate of consumption declines. As one increases EIS, society is more willing to displace consumption between different points in time. As a result, the cone of no shipment widens slightly (rows C9 and D9). The EIS has very little effect on the share invested into the risky technologies (row C7). This is to be contrasted with the perfect integration and autarky cases in which it has *no* impact whatsoever on it, this being a manifestation of the separation between EIS, which affects intertemporal choices, and risk aversion, which affects attitude to risk. In the presence of frictions, the separation is somewhat lost: the widening of the cone which is induced by an increase in EIS, is accompanied by a small increase in the willingness to invest in risky projects.

6. Conclusion

In this article we have studied, in a general equilibrium setting, the impact on welfare and growth of the imperfect mobility of goods. We have modeled a two-country, one-good economy in which financial markets are complete and frictionless, but the markets for real goods are partially segmented so that the single good exists really in two versions, differentiated by location. The imperfection of the goods market is modeled by introducing a proportional cost for exporting goods from one country to another. In each country, two technologies are present: one risky and the other riskless.

We have shown that the welfare gain from financial market integration is not drastically reduced by the presence of imperfect goods mobility, modeled as a cost of transferring goods from one country to the other. Similarly, the quantitative impact of goods market imperfection on growth is not very large. These results obtain even though the friction in commodity markets is treated

[20] This allows us to verify whether the effect of EIS differs depending on whether the constraint is binding or not. We find that it does not.

as a deadweight cost; if this friction was assumed to be a tariff whose revenue could be used to increase consumption, the welfare loss would be even smaller. While the welfare gain is not monotonic with investors' risk aversion and the aggregate volatility of output growth, qualitatively speaking, most of the effects obtained under perfect goods mobility are preserved under imperfect mobility. This is because the *optimal timing* of physical transfers mitigates the effect of transfer costs. At times when it is not optimal to transfer goods, foreigners do not collect their payoffs. The financial market makes it possible for them to simply accumulate claims which will be redeemed later.

The policy implication to be drawn is that financial market integration is a worthwhile goal to pursue even at a time when goods market integration has not been achieved. Even though trade agreements such as GATT and NAFTA have not removed all barriers to trade in goods and services, policymakers could still facilitate the flow of financial capital. Furthermore, if financial markets behave in accordance with our model and mitigate the imperfections in the goods market, the further welfare gains to be reaped by bringing down tariffs from their current level, which is about 6% among developed countries, seem trivial.

The setting that we have chosen, which featured a risky constant return to scale production economy with a single good which is differentiated only by location and which is investible, was meant to give the central role to the asset market. If all goods were perishable, it is clear that financial markets would not provide the same benefits that we have identified. Nonetheless, even in a pure exchange economy with only perishable goods, endowment shocks with sufficient persistence would still give a *raison d'être* to inter-country state-contingent consumption loans. In future research, other settings ought to be investigated in an attempt to measure the relative importance of welfare benefits arising from goods market openness and asset market openness, depending on the nature and number of the goods, on the nature of the stochastic processes for endowments and/or productivity shocks, and on the number and degree of asymmetry of the countries making up the world.

Appendix A: Characterizing the Planner's Problem

In this appendix we modify the differential equation that characterizes the problem in Equation (5), and show how one can use the homogeneity of the problem to reduce the state space. Then, we describe the boundary conditions for this differential equation. The numerical procedure used to solve this problem is described in Appendix B.

A.1 Deriving the general differential equation for $H(\omega, \lambda; \tau)$

After substituting in the first-order conditions for all six decision variables, the differential equation for the value function V is

$$0 = \left(\frac{1}{\rho} - 1\right) V_K \left[\frac{V_K (\alpha V_x)^{\frac{\rho}{\alpha}-1}}{x\beta}\right]^{\frac{1}{\rho-1}} - x\frac{\beta}{\rho}\alpha V_x$$

$$+ \left(\frac{1}{\rho} - 1\right) V_{K^*} \left[\frac{V_{K^*} (\alpha V_{x^*})^{\frac{\rho}{\alpha}-1}}{x^*\beta}\right]^{\frac{1}{\rho-1}} - x^*\frac{\beta}{\rho}\alpha V_{x^*}$$

$$+ V_K[r + w(\mu - r)]K + V_{K^*}[r + w^*(\mu - r)]K^*$$
$$+ \frac{1}{2}\sigma^2 V_{KK}(wK)^2 + \frac{1}{2}\sigma^2 V_{K^*K^*}(w^*K^*)^2 + \sigma^2\eta V_{KK^*}(wK)(w^*K^*), \tag{A1}$$

where the portfolio weights are optimized under the constraint that they must lie between 0 and 1:

$$\{w, w^*\} \equiv \begin{cases} \{0, 1\} & \text{if } w_1 < 0 & \text{and } w_0^* > 1 \\ \{w_1, 1\} & \text{if } 0 < w_1 < 1 & \text{and } w_\cap^* > 1 \\ \{1, 1\} & \text{if } w_1 > 1 & \text{and } w_1^* > 1 \\ \{0, w_0^*\} & \text{if } w_\cap < 0 & \text{and } 0 < w_0^* < 1 \\ \{w_\cap, w_\cap^*\} & \text{if } 0 < w_\cap < 1 & \text{and } 0 < w_\cap^* < 1 \\ \{1, w_1^*\} & \text{if } w_\cap > 1 & \text{and } 0 < w_1^* < 1 \\ \{0, 0\} & \text{if } w_0 < 0 & \text{and } w_0^* < 0 \\ \{w_0, 0\} & \text{if } 0 < w_0 < 1 & \text{and } w_\cap^* < 0 \\ \{1, 0\} & \text{if } w_0 > 1 & \text{and } w_1^* < 0, \end{cases} \tag{A2}$$

and where

$$w_\cap(K, K^*, x, x^*) \equiv -\frac{\mu - r}{K\sigma^2}\frac{V_K V_{K^*K^*} - V_{K^*}V_{KK^*}\eta}{V_{KK}V_{K^*K^*} - (V_{KK^*}\eta)^2}$$
$$w_1(K, K^*, x, x^*) \equiv -\frac{V_K(\mu - r) + \sigma^2\eta V_{KK^*}K^*}{\sigma^2 V_{KK}K}$$
$$w_0(K, K^*, x, x^*) \equiv -\frac{V_K(\mu - r)}{\sigma^2 V_{KK}K},$$

while $w_\cap^*$, w_1^* and w_0^* are defined analogously.

Taking advantage of the homogeneity properties of V, we define the function H:

$$V(K, K^*, x, x^*) \equiv (K + K^*)^\alpha(x + x^*)H(\omega, \lambda), \tag{A3}$$

where

$$\omega \equiv \frac{K}{K + K^*}, \qquad \lambda \equiv \frac{x}{x + x^*}.$$

Substituting this expression into Equation (A1), we get a differential equation for H:

$$0 = \left(\frac{1}{\rho} - 1\right)\left[\alpha H + (1 - \omega)H_\omega\right]^{\frac{\rho}{\rho-1}}\left[\frac{(\alpha(H + H_\lambda(1 - \lambda)))^{\frac{\rho}{\alpha}-1}}{\lambda\beta}\right]^{\frac{1}{\rho-1}}$$
$$+ \left(\frac{1}{\rho} - 1\right)[\alpha H - \omega H_\omega]^{\frac{\rho}{\rho-1}}\left[\frac{\left(\alpha\left(H - \lambda H_\lambda\right)\right)^{\frac{\rho}{\alpha}-1}}{(1 - \lambda)\beta}\right]^{\frac{1}{\rho-1}}$$
$$- \frac{\beta}{\rho}\alpha H + \alpha H\left[r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}\right],$$

where

$$\mathbf{r} \equiv \begin{bmatrix} (1 + (1 - \omega)H_\omega/(\alpha H)) \times (\mu - r) \\ (1 - \omega H_\omega/(\alpha H)) \times (\mu - r) \end{bmatrix}$$
$$\mathbf{w} \equiv \begin{bmatrix} w\omega \\ w^*(1 - \omega) \end{bmatrix}. \tag{A4}$$

Here, $\{w, w^*\}$ is as defined in Equation (A2), but with

$$\begin{bmatrix} w_\cap \omega \\ w_\cap^*(1-\omega) \end{bmatrix} = -\mathbf{\Sigma}^{-1}\mathbf{r},$$

$$\begin{bmatrix} w_1 \omega \\ w_1^*(1-\omega) \end{bmatrix} = \begin{bmatrix} -\frac{\mu - r + \Sigma_{12}(1-\omega)}{\Sigma_{11}} \\ -\frac{\mu - r + \Sigma_{21}\omega}{\Sigma_{22}} \end{bmatrix},$$

$$\begin{bmatrix} w_0 \omega \\ w_0^*(1-\omega) \end{bmatrix} = \begin{bmatrix} -\frac{\mu - r}{\Sigma_{11}} \\ -\frac{\mu - r}{\Sigma_{22}} \end{bmatrix},$$

where

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix},$$

$$\Sigma_{11} = \sigma^2(\alpha-1)\big[1 + 2(1-\omega)H_\omega/(\alpha H) + (1-\omega)^2 H_{\omega\omega}/(\alpha(\alpha-1)H)\big]$$

$$\Sigma_{12} = \sigma^2\eta(\alpha-1)\big[1 + (1-2\omega)H_\omega/(\alpha H) - \omega(1-\omega)H_{\omega\omega}/(\alpha(\alpha-1)H)\big]$$

$$\Sigma_{22} = \sigma^2(\alpha-1)\big[1 - 2\omega H_\omega/(\alpha H) + \omega^2 H_{\omega\omega}/(\alpha(\alpha-1)H)\big].$$

Finally, dividing through by αH:

$$\begin{aligned} 0 = {} & \left(\frac{1}{\rho} - 1\right)\left[1 + (1-\omega)\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{\left(1 + (1-\lambda)\frac{H_\lambda}{H}\right)^{\frac{\rho}{\alpha}-1}}{\lambda\beta}\right]^{\frac{1}{\rho-1}} \\ & + \left(\frac{1}{\rho} - 1\right)\left[1 - \omega\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{\left(1 - \lambda\frac{H_\lambda}{H}\right)^{\frac{\rho}{\alpha}-1}}{(1-\lambda)\beta}\right]^{\frac{1}{\rho-1}} \\ & - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}, \end{aligned} \tag{A5}$$

where $\mathbf{w}$ is given by Equation (A4).

A.2 Deriving the boundary conditions for the PDE for $H(\omega, \lambda; \tau)$

We now take care of shipments which occur at the lower edge, $\omega = \underline{\omega}$, and the upper edge, $\omega = \overline{\omega}$, of the cone of no action.

We start by obtaining the boundary conditions for $H(\omega, \lambda)$ at $\underline{\omega}$. That is, we wish to determine the level of the value function at the lower edge, $\omega = \underline{\omega}$, where $V_{K^*} = (1-\tau)V_K$ [smooth-pasting boundary condition; see Dumas (1992)]. At that point the most convenient way of imposing the boundary condition is to write the value function locally as

$$H(\underline{\omega}, \lambda) = \Gamma(\underline{\omega}, \lambda)[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha, \tag{A6}$$

implying that

$$H_\omega(\underline{\omega}, \lambda) = \alpha H \frac{\tau}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}, \tag{A7}$$

$$H_{\omega\omega}(\underline{\omega}, \lambda) = \alpha(\alpha-1) H \frac{\tau^2}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^2}. \tag{A8}$$

To use Equations (A6) and (A7) as boundary conditions, we need to determine $\Gamma(\underline{\omega}, \lambda)$. This can be done by substituting the expressions for H_ω and $H_{\omega\omega}$ into Equation (A5), the PDE for $H(\omega, \lambda)$. Doing this gives

$$\begin{aligned}
0 = & \left(\frac{1-\rho}{\rho}\right)\left[1+\frac{\tau(1-\omega)}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} \left(\alpha\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha}\right)^{\frac{\rho/\alpha}{\rho-1}} \\
& \times (\lambda\beta)^{\frac{1}{1-\rho}}\left(\left[1+\frac{(1-\lambda)H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha}}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \\
& + \left(\frac{1-\rho}{\rho}\right)\left[1-\frac{\tau\omega}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} (\alpha\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha})^{\frac{\rho/\alpha}{\rho-1}} \\
& \times ((1-\lambda)\beta)^{\frac{1}{1-\rho}}\left(\left[1-\frac{\lambda H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha}}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} \\
& - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}.
\end{aligned}$$

Noting the following two simplifications,

$$\left[1+(1-\omega)\frac{\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}\right] = \frac{1}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}, \tag{A9}$$

$$\left[1-\omega\frac{\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}\right] = \frac{1-\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}, \tag{A10}$$

and collecting terms allows us to write the differential equation as

$$\begin{aligned}
0 = & \left(\frac{1-\rho}{\rho}\right)[\alpha\Gamma(\underline{\omega},\lambda)]^{\frac{\rho}{\alpha(\rho-1)}}\beta^{\frac{1}{1-\rho}} \\
& \times \left\{\lambda^{\frac{1}{1-\rho}}\left(\left[1+\frac{(1-\lambda)H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha}}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}} + (1-\tau)^{\frac{\rho}{\rho-1}}\right. \\
& \left. \times (1-\lambda)^{\frac{1}{1-\rho}}\left(\left[1-\frac{\lambda H_\lambda}{\Gamma(\underline{\omega},\lambda)[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^{\alpha}}\right]^{\frac{\rho}{\alpha}-1}\right)^{\frac{1}{\rho-1}}\right\} \\
& - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w},
\end{aligned} \tag{A11}$$

where $\mathbf{w}$ is as defined before, and is evaluated at $\underline{\omega}$. Using the results in Equations (A6), (A7), and (A8), we also have that at $\underline{\omega}$,

$$\begin{aligned}
\Sigma_{11} &= \sigma^2(\alpha-1)\left[1+2(1-\underline{\omega})\frac{\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}+(1-\underline{\omega})^2\frac{\tau^2}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^2}\right], \\
\Sigma_{12} &= \eta\sigma^2(\alpha-1)\left[1+(1-2\underline{\omega})\frac{\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}-\underline{\omega}(1-\underline{\omega})\frac{\tau^2}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^2}\right], \\
\Sigma_{22} &= \sigma^2(\alpha-1)\left[1-2\underline{\omega}\frac{\tau}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]}+\underline{\omega}^2\frac{\tau^2}{[\underline{\omega}+(1-\tau)(1-\underline{\omega})]^2}\right].
\end{aligned}$$

Equation (A11) defines $\Gamma(\underline{\omega}, \lambda)$ implicitly; thus, $\Gamma(\underline{\omega}, \lambda)$ can be obtained by solving this equation.

To obtain the BC for $H(\omega, \lambda)$ at $\bar{\omega}$, we repeat the above analysis at the point where $\omega = \bar{\omega}$.

Appendix B. Details of the Numerical Procedure

We wish to obtain the solution, $H(\omega, \lambda = 0.5)$, for the partial differential equation (PDE) given in Equation (A5). Note that this differential equation includes $H_\lambda(\omega, \lambda)$, a term for the first derivative with respect to λ, but no terms for the second derivative, $H_{\lambda\lambda}$, or the cross-derivative, $H_{\lambda\omega}$. We take advantage of this by treating the partial differential equation for $H(\omega, \lambda)$ as an ordinary differential equation, where we approximate the term $H_\lambda(\omega, \lambda)$ with the finite difference:

$$H_\lambda(\omega, \lambda = 0.5) = \frac{H(\omega, \lambda = 0.5 + \Delta) - H(\omega, \lambda = 0.5 - \Delta)}{2\Delta}.$$

We then reduce the size of Δ until we have a good approximation of the derivative. The steps in our numerical procedure are as follows:

1. Start by choosing $\Delta = 0.5$. Thus, to approximate $H_\lambda(\omega, \lambda = 0.5)$, we need to solve the differential equations for $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0)$. The differential equation for $H(\omega, \lambda = 1)$ is given below, in Equation (B1).
   - Determine the two boundary conditions for the differential equation for $H(\omega, \lambda = 1)$ at $\underline{\omega}$ and at $\bar{\omega}$. The ones for $\underline{\omega}$ are given in Equations (A6) and (A7), and the ones at $\bar{\omega}$ are analogous.
   - To solve the differential equation for $H(\omega, \lambda = 1)$, use the "shooting technique:" start with a guess for the initial value for $\underline{\omega}$, and use the boundary conditions in Equations (A6) and (A7) at $\underline{\omega}$ to solve this differential equation. To determine whether the initial guess for $\underline{\omega}$ is optimal, use the boundary conditions at $\bar{\omega}$ to check if these are satisfied by the solution. Revise the guess for $\underline{\omega}$ until boundary conditions at both extreme points are satisfied.
2. From the solution for $H(\omega, \lambda = 1)$, infer the solution for $H(\omega, \lambda = 0)$. That is, $H(1 - \omega, \lambda = 0) = H(\omega, \lambda = 1)$. Interpolate from $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0)$ the value of $H_\lambda(\omega, \lambda = 0.5)$; use this to solve the original differential equation, $H(\omega, \lambda = 0.5)$
3. Now decrease Δ to 0.25. Solve for $H(\omega, \lambda = 0.75)$ with the derivative $H_\lambda(\omega, \lambda = 0.75)$ approximated from the solution of the differential equations for $H(\omega, \lambda = 1)$ and $H(\omega, \lambda = 0.5)$. Continue decreasing the size of Δ until convergence.

To obtain the differential equation $H(\omega, \lambda = 1; \tau)$, start with the differential equation that we already have for $H(\omega, \lambda)$ in Equation (A5). Setting $\lambda = 1$ yields

$$0 = \left(\frac{1}{\rho} - 1\right)\left[1 + (1-\omega)\frac{H_\omega}{\alpha H}\right]^{\frac{\rho}{\rho-1}} (\alpha H)^{\frac{\rho/\alpha}{\rho-1}} \left[\frac{1}{\beta}\right]^{\frac{1}{\rho-1}} - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}. \tag{B1}$$

To obtain the boundary conditions for $H(\omega, \lambda = 1)$ at $\underline{\omega}$ we take advantage of equations (A6)–(A8). To use Equations (A6) and (A7) as boundary conditions, we need to determine the constant $\Gamma(\underline{\omega}, \lambda = 1)$. This can be done by substituting the expressions for H_ω and $H_{\omega\omega}$ into Equation (B1), the differential equation for $H(\omega, \lambda = 1)$. Doing this gives,

$$0 = \left(\frac{1-\rho}{\rho}\right)\left[1 + \frac{\tau(1-\omega)}{[\underline{\omega} + (1-\tau)(1-\underline{\omega})]}\right]^{\frac{\rho}{\rho-1}} (\alpha\Gamma(\underline{\omega})[\underline{\omega} + (1-\tau)(1-\underline{\omega})]^\alpha)^{\frac{\rho/\alpha}{\rho-1}} \beta^{\frac{1}{1-\rho}}$$
$$- \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}.$$

Using the result in Equation (A9), we can write the above differential equation as

$$0 = \left(\frac{1-\rho}{\rho}\right)\beta^{\frac{1}{1-\rho}} (\alpha\Gamma(\underline{\omega}))^{\frac{\rho/\alpha}{\rho-1}} - \frac{\beta}{\rho} + r + \mathbf{w}'\mathbf{r} + \frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}.$$

Simplifying we get

$$\Gamma(\underline{\omega}, \lambda = 1) = \frac{1}{\alpha}\left\{\beta^{\frac{1}{\rho-1}}\left(\frac{\rho}{1-\rho}\right)\left[\frac{\beta}{\rho} - r - \mathbf{w}'\mathbf{r} - \frac{1}{2}\mathbf{w}'\mathbf{\Sigma}\mathbf{w}\right]\right\}^{\frac{\alpha(\rho-1)}{\rho}}, \tag{B2}$$

where $\mathbf{w}$ is evaluated at $\underline{\omega}$ as before.

To obtain the boundary conditions for $H(\omega, \lambda = 1)$ at $\bar{\omega}$, we repeat the above analysis at the point where $\omega = \bar{\omega}$.

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