

Debt Valuation, Renegotiation, and Optimal Dividend Policy

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The valuation of debt and equity, reorganization boundaries, and firm's optimal dividend policies are studied in a framework where we model strategic interactions between debt holders and equity holders in a game-theoretic setting which can accommodate varying bargaining powers to the two claimants. Two formulations of reorganization are presented: debt-equity swaps and strategic debt service resulting from negotiated debt service reductions. We study the effects of bond covenants on payout policies and distinguish liquidity-induced defaults from strategic defaults. We derive optimal equity issuance and payout policies. The debt capacity of the firm and the optimal capital structure are characterized.

Covenants that define default are an essential feature of debt contracts in both corporate and bank loan markets. A commonly observed covenant is the requirement that the borrower must pay contractual payments (such as coupons, principal, or sinking fund payments) to the lender at periodic intervals specified in the debt contract. *Any missed or delayed disbursement of interest or principal is treated as default* [Moody's Investors Service (1998)]. Moody's analyzed all defaults during 1982–1997. They report that about half of long-term public bond defaults resulted in bankruptcy. Of the defaults, 43% were accounted for by missed payments and 7% by distressed exchanges. In the context of such evidence, the value of placing a covenant requiring the timely payment of contractual obligations appears to be crucial. Modeling of such covenants is critical in the debt valuation literature. Merton (1974), Black and Cox (1976), and Leland (1994) are examples of articles that study the valuation consequences of such debt covenants. More recently, a number of articles have taken into consideration strategic issues and renegotiation in the context of debt valuation. [See the articles by Anderson and

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Sundaresan (1996) and Mella-Barral and Perraudin (1997)]. The role of cash flow-based covenants in such a context has been examined by Anderson and Sundaresan (1996), among others. The presence of such covenants may induce the borrower to alter its dividend policies as well as to issue equity to finance contractual debt service in order to avoid costly and premature renegotiation or liquidation. Further, such covenants may well affect the reorganization boundary at which the borrowers and lenders decide to renegotiate the contract.

Unfortunately the literature on debt valuation has tended to assume that dividend policies are *exogenous* to the valuation problem. In the existing models, dividend policy is not explicitly considered. Firms either simply do not have a policy with respect to dividends [Leland (1994)] or they pay out all residual cash flows as dividends [Kim, Ramaswamy, and Sundaresan (1993), Anderson and Sundaresan (1996), Anderson, Sundaresan, and Tychon (1996) (AST, hereafter), Leland and Toft (1996)]. Further, existing articles do not distinguish between the value of the assets of the firm and the value of the firm as an ongoing entity in formulating strategic debt renegotiation. This is because the debt valuation literature with strategic renegotiation has tended to assume zero corporate taxes. Absent corporate taxes there is no distinction between the value of the assets of the firm and the value of the firm itself. This in turn implies that bargaining over the value of the assets of the firm at any time (under the threat of liquidation) is also equivalent to bargaining over the value of the firm. In the presence of corporate taxes, however, the value of assets may differ from the value of the firm and it may be a matter of some significance as to whether the claimants bargain over the value of the assets of the firm or over the value of the firm as an ongoing entity. The presence of corporate taxes and the presumed tax advantage of debt are quite critical to the use of debt by borrowers in this model. *In the presence of taxes, the value of the firm is the object of bargaining and this itself depends on optimal reorganization policies and must be endogenously determined.* We characterize the endogenous process followed by the firm's value and show how the optimal default boundaries are determined when claimants bargain over the firm's value.

The thrust of our article is to explicitly draw out the implications of *relative bargaining power of claimants* on optimal reorganization and debt valuation. Our formulation nests as special cases several previous contributions which have tended to give the bargaining power exclusively to equity holders. We also examine nonnegotiable cash flow-based covenants and how they might impact the bargaining process and the resulting valuation of debt and equity. In particular, we explore how such a covenant might affect the dividend policy. The framework of our article draws on the insights of Hart and Moore (1989, 1994). We consider an economy in which the borrowers (equity holders) have exclusive access to a project that provides a continuous stream

of cash flows.¹ They prefer to finance the project with debt which has a tax advantage, but which could also result in potentially inefficient liquidation and financial distress. The driving force behind strategic behavior in our model is the presence of proportional and fixed costs of liquidation. We endogenize *both* the reorganization boundary and the optimal sharing rule between equity and debt holders upon default. We illustrate our framework with a Nash solution to the bargaining problem between lenders and borrowers which is the limit of take-it or leave-it offers.

We characterize two bargaining formulations. In the first formulation, the borrower and the lender bargain over the value of the assets of the firm. Since liquidation of assets is inefficient, debtors settle for a debt-equity swap in which the lenders exchange their claims for equity. Under this scheme we assume that all future tax benefits are lost and hence there is no distinction between the value of the assets and the value of the firm. (To keep the analysis simple we have abstracted from dynamic recapitalization.) We study debt-equity swaps primarily to compare our results with previous contributions in the literature where the object of bargaining is the value of the assets of the firm. The terms of the exchange and the trigger point at which the debt-equity swap occurs are found endogenously. In the second formulation, the borrower and the lender bargain over the value of the firm. When an endogenously determined trigger point is reached borrowers offer a debt service that is less than the contractual amount as an equilibrium outcome of the bargaining process. During the period of such strategic debt service there is no tax benefit. But when the fortunes improve and the contractual debt service payments are resumed, the tax benefit is assumed to be restored. Under this formulation the presence of potential future tax benefits will drive a wedge between the value of assets and the value of the firm. We characterize this difference and the bargaining solution where the ongoing firm value is the object of the bargaining problem. Our formulation generalizes the recent approaches in the literature to integrate debt valuation with developments in theoretical corporate finance in the presence of corporate taxes.

We show that without the cash flow-based covenant, it is optimal for equity holders to receive maximal residual cash flow as dividend even in the absence of strategic debt service. However, with the cash flow-based covenant, they would rather sacrifice the current dividend and reinvest them as retained earnings to avoid premature liquidation. When the borrowers service debt in a strategic manner, they reinvest the minimal amount such that the trigger point for strategic debt service is reached *before* the bond covenant becomes a binding constraint. *An implication of our analysis is that both claimants may benefit by the presence of such bond covenants.* Our results in this context are consistent with the empirical evidence documented by Healy and Palepu (1990) and DeAngelo and DeAngelo (1990). Healy and Palepu (1990)

¹ We abstract from agency problems in our formulation.

provide evidence that accounting-based dividend constraints are an effective way for the bondholder to restrict a firm's dividend policies. They show that firms cut dividends to circumvent dividend restriction and the magnitude is proportional to the tightness of the dividend constraint. DeAngelo and DeAngelo (1990) document that firms in financial distress make a large and prompt cut in dividends to avoid dividend constraints from becoming binding. We also characterize the circumstances under which equity will be issued to finance contractual coupon payments. This allows us to distinguish between *strategic defaults* and *liquidity defaults* and characterize the conditions under which each of them may occur. We are thus able to examine the value of placing such covenants in debt contracts to both equity holders and debtholders. Finally, we characterize the debt capacity and the optimal capital structure of the firm.

The remainder of the article is organized as follows: Section 2 derives the basic valuation model for risky debt with debt-equity swap and strategic debt service in the form of a bargaining game. Numerical results for finite maturity debt are obtained and the term structure of default premiums are characterized in Section 3. Section 4 allows the equity holders to choose optimal dividend policies (or equivalently, payout ratio) to maximize the value of their claims. Some empirical implications are discussed in Section 5. Section 6 concludes. All technical developments are in the appendix.

1. Debt Renegotiation

In this section, we develop a model of renegotiation. We then apply this model for debt valuation under two reorganization schemes: debt-equity swap and strategic debt service. The model is set in a continuous-time framework. The following assumptions underlie the model:

- (1) There is a firm which has equity and a single issue of perpetual debt which promises a flow rate of coupon c per unit time. Finite maturity debt is considered in Section 3. Admitting more than one layer of debt significantly complicates our analysis.²
- (2) To focus attention on default risk, we assume that the default-free term structure is flat and that the instantaneous riskless (default-free) rate is r per unit time.
- (3) When the firm pays its contractual coupon c , it is entitled to a tax benefit of τc ($0 \leq \tau < 1$). This is in fact the only motive for issuing debt in our model. During the default period, the tax benefits are lost.
- (4) Asset sales for dividend payments are prohibited.

² Fan (2000) generalizes this approach to model multiple layers of debt in the firm's capital structure.

- (5) The firm can be liquidated only at a cost. The proportional cost is α ($0 \leq \alpha \leq 1$) and the fixed cost is K ($K \geq 0$). The debt holders have strict absolute priority upon bankruptcy: suppose liquidation happens when the value of assets of the firm reach V_B , the bankruptcy trigger point, a cost of $\min(V_B, \alpha V_B + K)$ is taken away by outsiders; debt holders receive the remaining $\max[0, (1-\alpha)V_B - K]$; and equity holders receive nothing. This supplies the motivation for renegotiation and optimal dividend policies in the presence of a cash flow-based bond covenant. [At equilibrium, we will show that debt holders accept less than the contractual coupon and still permit equity holders to run the firm. This results in deviations from absolute priority.]
- (6) The asset value of the firm, denoted by V , follows the lognormal diffusion process

$$dV = (\mu - \beta)V dt + \sigma V dB_t, \quad (1)$$

where μ is the instantaneous expected rate of return on the firm gross of all payout, σ^2 is the instantaneous variance of the return on the firm and B_t is a standard Brownian motion. The cash payout at any time is βV and $\beta \leq \mu$, where β is the firm's cash payout ratio.³ The main implication of this assumption is that the investment policy is fixed. This assumption precludes us from studying the asset substitution issue.⁴ The specification also assumes that there is an exogenous reinvestment that takes place continuously but the free cash flows available for the payment of dividends and debt service is restricted to βV . There is no need to assume that the value of the assets of the firm is freely traded in the market. As long as equity issued by the firm is freely traded we can still apply the contingent claims pricing results in our model. This is shown in Ericsson and Reneby (1999). In the absence of taxes there is no distinction between the value of the assets of the firm and the value of the firm, which we denote as $v(V)$.

Liquidations are typically very costly when one takes into account direct and indirect costs associated with liquidation. The current institutional arrangements for reorganization provide ample opportunities for the borrower and the lender to avoid costly liquidations. Debt workouts often lead to exchange offers that result in deviations from absolute priority. Bankruptcies are often resolved using exchange offers of different types. They include

³ Instead of specifying the asset values exogenously, we could instead specify the cash flows exogenously and derive the equation above as the dynamics of the asset value endogenously.

⁴ One way to study the asset substitution effect is to allow the equity holders to select the risk level of the firm, σ .

delayed or missed interest or principal payments, extension of maturity, debt-equity swap, debt holidays, etc. Recall that delayed or missed payments accounted for 43% of defaults during 1982–1997. Essentially all these distressed exchanges and delayed payments can be considered as a value redistribution between equity and debt holders. We model such exchanges and delayed payments with a simple Nash bargaining game under the assumption that borrowers and lenders can renegotiate debt at no cost.⁵ Two cases are discussed explicitly next. One is *debt-equity swap*. At an endogenously determined lower reorganization boundary, debt holders are offered a proportion of the firm's equity to replace their original debt contract. This is best thought of as a distressed exchange. The other is *strategic debt service* (temporary coupon reduction). When the firm's asset value falls below a certain threshold, denoted as V_s , borrowers stop making the contractual coupon and start servicing debt strategically until the firm's asset value goes back above the threshold again. When this happens they resume contractual coupon payments. We begin by exploring debt-equity swaps. It is to be noted that in the absence of taxes there is no difference between these two formulations.

1.1 Debt-equity swap

We now consider our first formulation of reorganization where the claimants negotiate at an endogenously determined trigger point and decide not to operate the firm as an ongoing entity. They sell their stake to outsiders who pay them the value of the assets of the firm. A way to visualize this as follows: debt holders swap their debt for equity. Then they sell the equity to potential buyers. The equity value should reflect the tax benefits associated with future recapitalization. But we assume that the equity is priced so that the expected future tax benefits are offset by the costliness of the future renegotiation process with the outsiders. We do not model these costs. While they forego any potential tax benefits in the future (associated with keeping the firm alive), they are able to cash in their negotiated share of the assets for alternative uses. It is assumed that the private valuation of such alternatives exceed the present value of tax benefits from keeping the firm alive. The debt holders can simply sell the shares to willing buyers to cash out. In this manner they can avoid costly liquidation of assets. While it could be argued that the debt holders may be better off by engaging in strategic debt service to capture the future tax benefits, we nonetheless feel that debt-equity swaps are of interest for two reasons: first, they are used widely in emerging markets debt restructuring. Second, much of the academic literature in debt valuation has used the asset value as the object of bargaining or restructuring. In our formulation,

⁵ Empirical studies have shown that the costs of debt restructuring are significantly less than the costs of liquidation. We abstract from costly renegotiation in this framework to keep the analysis simple.

we can make the debt-equity swap to be the preferred alternative by incorporating costly negotiations and relatively low tax benefits. For simplicity we do not treat this extension.⁶

1.1.1 The bargaining game and the Nash solution. When reorganization results in a debt-equity swap, the firm becomes an all-equity firm. Since there are no tax benefits or the possibility of bankruptcy in the future,⁷ the total value of the firm is exactly the asset value V . As a consequence, the claimants are assumed to bargain over the value of the assets at the optimally chosen reorganization boundary V_S . We determine the optimal sharing rule between both parties at the trigger point V_S as

$$E(V_S) = \theta V_S, \quad D(V_S) = (1 - \theta)V_S,$$

where $E(\cdot)$ and $D(\cdot)$ are the values of equity and debt, respectively, θ is a parameter that reflects the sharing rule for the value of residual assets. In our model, θ is a variable between 0 and $\alpha + K/V_S$. It is to be stressed that V_S and θ are *endogenous* and may depend on α and K among other parameters of the model.

Let us denote η as the equity holders' bargaining power, and $1 - \eta$ is the debt holders' bargaining power. We solve for Nash solution θ^* as follows: the incremental value for equity holders by continuing as opposed to liquidating is $\theta V_S - 0$.⁸ The incremental value to debt holders by accepting the debt-equity swap instead of forcing liquidation is $(1 - \theta)V_S - \max[(1 - \alpha)V_S - K, 0]$. The Nash solution is characterized as:

$$\begin{aligned} \theta^* &= \arg \max_{0 \leq \theta \leq \alpha + K/V_S} \{\theta V_S - 0\}^\eta \{(1 - \theta)V_S - \max[(1 - \alpha)V_S - K, 0]\}^{1-\eta} \\ &= \min\left(\eta \frac{\alpha V_S + K}{V_S}, \eta\right). \end{aligned}$$

The solution to such a bargaining game can be characterized by the sharing rule θ which, in general, is a function of the bargaining power η . The following examples are of special interest since they pertain to previous contributions in the literature.

Example 1. By assigning all the bargaining power to equity holders, that is, $\eta = 1$, we get the Anderson and Sundaresan (1996) outcome as a special case. The strategic debt service game modeled in Anderson and Sundaresan (1996) suggests that debt renegotiation results in deviations from absolute

⁶ We thank the referee for noting that the strategic debt service with the firm's value as the object of bargaining is always superior within the assumptions of our model.

⁷ We abstract from dynamic capital structure issues to keep our analysis simple.

⁸ See Luce and Raiffa (1957, Chapter 6) for detailed definitions and properties.

priority and reduces wasteful liquidation. In their model, the stockholders propose take-it or leave-it offers to the bondholder at the trigger point of debt-equity swap, which we denote as V_{NC} . By assuming that the stockholders extract all the benefits from such an exchange offer, V_{NC} is determined as the creditors' indifferent point of liquidation and accepting equity holders' strategic offer. The associated outcome is shown below:

$$E(V_{NC}) = \alpha V_{NC} + K, \quad D(V_{NC}) = (1 - \alpha)V_{NC} - K.$$

However, if debt holders refuse the proposal of the exchange offer, equity holders will be much worse off. Thus the debt holders can threaten the equity holders by denying the proposal so that the equity holders have to increase their offer. Hence as long as the debt holders have any bargaining power, $\eta < 1$, the debt-equity swap trigger point would be lower than the indifference point of the debt holders.

Example 2. Leland (1994) does not include the possibility of debt renegotiation. Liquidation occurs at the endogenous reorganization trigger point V_B , which is the indifference point for equity holders to liquidate or to keep running the firm. The reorganization boundary turns out to be independent of the liquidation costs, as does the equity value.⁹ In Leland's model, the fixed cost is zero ($K = 0$), and so we have

$$E(V_B) = 0, \quad D(V_B) = (1 - \alpha)V_B.$$

In Leland's formulation, equity holders control the firm until liquidation. It might be in the interest of debt holders to forgive part of the debt service payments if in the process they can avoid the wasteful liquidations with the understanding that the savings can be shared by the two claimants. Thus the following sharing rule may prove to be superior:

$$E(\widehat{V}_B) = \alpha \widehat{V}_B, \quad D(\widehat{V}_B) = (1 - \alpha)\widehat{V}_B.$$

In this formulation, renegotiation occurs at the endogenous reorganization trigger point $\widehat{V}_B$.

Example 3. If the bargaining power is balanced equally between borrowers and lenders we have $\eta = 1/2$. This can be thought of as a reduced form representation of the alternate offers in the discrete-time game of negotiation postulated by Rubinstein (1982) as the time interval decreases to zero.¹⁰

In Figure 1 we illustrate the bargaining outcomes. Note that the take-it or leave-it outcome (Example 1, $\eta = 1$) can be compared explicitly with the outcome in which the power is equally balanced (Example 3, $\eta = 0.5$).

⁹ Leland (1994) considers exogenous deviations from absolute priority, which makes the reorganization boundary and the equity value depend upon the liquidation costs.

¹⁰ See Chapter 3 of Osborne and Rubinstein (1990) for a rigorous proof.

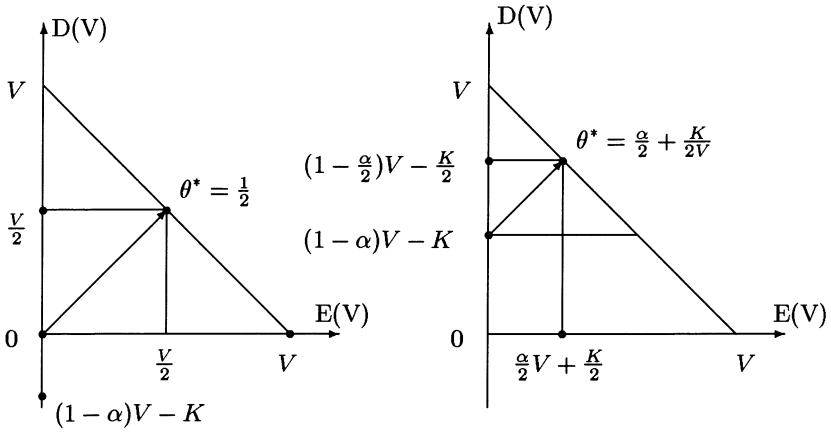


Figure 1

The bargaining outcome with equal bargaining power ($\eta = 0.5$)

The left figure shows the results when negotiation happens at such a low point V_S that it is not sufficient to cover the bankruptcy cost. $(0, 0)$ is the outcome if debt and equity holders decide to liquidate; $(V/2, V/2)$ is the optimal outcome. Similarly, the right figure indicates $[\alpha V/2 + K/2, (1 - \alpha)V/2 - K/2]$ as the outcome of the game and $[0, (1 - \alpha)V - K]$ as the suboptimal result arising from liquidation.

It is not surprising that the bargaining power of equity holders depends upon the liquidation cost $\alpha V_S + K$. The larger the liquidation costs are, the more benefits equity holders can obtain through debt renegotiation, and the earlier the trigger is going to be reached. The presence of a fixed liquidation cost K causes θ^* to depend on the trigger V_S , as the impact of fixed cost would be different at various levels of V_S . With $K = 0$, the optimal sharing rule θ^* would have collapsed to a constant $\eta\alpha$.

1.1.2 Valuation. The next step is to determine the lower debt renegotiation boundary V_S and solve for the equity and debt values. To keep the exposition clear and to illustrate the economic intuition better, we focus on the case where $K = 0$. (The complete treatment for $K > 0$ can be found in the technical appendix.) It is easy to show that the equity value $E(\cdot)$ satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 E_{VV} + (r - \beta)V E_V - rE + \beta V - c(1 - \tau) = 0. \quad (2)$$

As the value of the asset V approaches infinity, debt becomes riskless and hence the equity value must satisfy

$$\lim_{V \uparrow \infty} E(V) = V - \frac{c(1 - \tau)}{r}. \quad (3)$$

The lower boundary conditions follow from the results of the bargaining game described earlier (note that $\theta^* = \eta\alpha$):

$$\lim_{V \downarrow V_S} E(V) = \eta\alpha V_S, \text{ and } \lim_{V \downarrow V_S} E_V(V) = \eta\alpha.$$

Proposition 1. (i) The trigger point for debt-equity swap is

$$V_S = \frac{c(1-\tau)}{r} \frac{-\lambda_-}{1-\lambda_-} \frac{1}{1-\eta\alpha},$$

where

$$\lambda_- = \left[0.5 - \frac{(r-\beta)}{\sigma^2} \right] - \sqrt{\left[0.5 - \frac{(r-\beta)}{\sigma^2} \right]^2 + \frac{2r}{\sigma^2}} < 0;$$

(ii) The equity value is given by

$$E(V) = V - \frac{c(1-\tau)}{r}(1-P_S) + \eta\alpha V_S P_S - V_S P_S,$$

where $P_S = (V/V_S)^{\lambda_-}$ is the risk-neutralized probability of default.

(iii) The debt value is given by

$$D(V) = \frac{c}{r}(1-P_S) + (1-\eta\alpha)V_S P_S.$$

Note that the total value of the firm $v(V) = E(V) + D(V) = V + \frac{\tau c}{r}(1-P_S)$. The value of the firm is simply the value of the assets of the firm plus the expected value of future tax shields. In a debt-equity swap, the bargaining process assumes that the claimants are interested in dividing the assets rather than continuing to operate the firm. This may be motivated in part by the desire of the claimants to “cash in” their stakes to invest in other attractive opportunities that are not explicitly modeled here. The parameter λ_- has a special interpretation in this context. It is the elasticity of the probability of default with respect to the value of the assets of the firm. As such, it is negative and increasing with the volatility of the assets of the firm. In our model, debt holders will be able to extract some of the value of the firm under these conditions.

Corollary 1. The trigger point for debt-equity swap V_S always falls between the debt holders’ indifference point, $V_{NC} = \frac{c(1-\tau)}{r} \frac{-\lambda_-}{1-\lambda_-} \frac{1}{1-\alpha}$ [found by Anderson, Sundaresan, and Tychon (1996)] and the lower reorganization boundary $V_B = \frac{c(1-\tau)}{r} \frac{-\lambda_-}{1-\lambda_-}$ [found by Leland (1994)] which reflects the equity holders’ chosen point of liquidation.

Table 1
Comparative statics with debt-equity swaps

	Payout ratio β	Volatility σ^2	Liquidation cost α, K	Tax benefit τ
Trigger point $V_S = \left[\frac{c(1-\tau)}{r} + \eta K \right] \frac{-\lambda_-}{1-\lambda_-} \frac{1}{1-\eta\alpha}$	—	—	+	—
Default probability $P_S = (V/V_S)^{\lambda_-}$	[1]	[4]	+	—
Debt value at V_S $D(V_S) = (1 - \eta\alpha)V_S - \eta K$	—	—	[7]	—
Debt value at $V > V_S$ $(1 - P_S)c/r + P_S D(V_S)$	[2]	[5]	—	+
Risk premium at $V > V_S$ $c/D(V) - r$	[3]	[6]	+	—
Equity value at $V > V_S$ $V - (1 - P_S) \frac{c(1-\tau)}{r} - P_S D(V_S)$	+	+	—	+

This table describes the properties of the trigger point, probability of departure from contractual payment, and debt and equity values with proportional and fixed liquidation costs under debt-equity swaps.

1.1.3 Comparative statics. The comparative statics are listed in Table 1. We provide a discussion of highlights of Table 1 below: In the tables a “+” sign indicates that the variable on the corresponding row has a positive first derivative with respect to the parameter in the corresponding column.

- [1] As the payout ratio β increases, the firm’s growth potential drops. This results in a higher probability for debt renegotiation when firm value V is high. When the firm approaches the trigger point V_S which decreases with increasing β , there is less possibility to engage in debt renegotiation for a high cash payout firm.
- [2] For the same reason as in [1], debt issued by a high (low) cash payout firm has a high (low) value when the firm’s asset value is high (leverage is low). When the firm’s asset value is low (leverage is high), the liquidity problem becomes more severe for low cash payout firms. Therefore debt renegotiation happens more often, which leads to lower debt values.
- [3] It is easy to see that whenever the debt value increases (decreases), the risk premium goes down (up). Figure 2 visualizes the effect of payout ratio on the risk premium. As the payout ratio increases, dividends to equity holders will increase when the firm is solvent. Since the trigger point goes down with payout ratio, equity holders will get more dividends over the life of the firm, thereby reducing the value of debt.

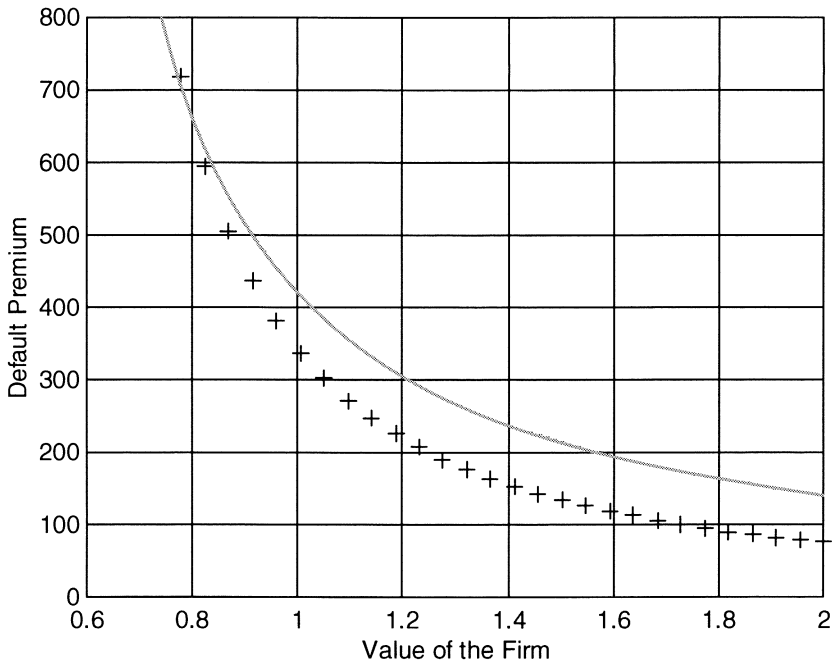


Figure 2

Default premium with various payout ratios

The lines plot the default premium at different levels of the firm's asset value V , for high and low payout ratios of 3% ('+' line) and 7% (solid line). It is assumed that the risk-free interest rate is 7.5%, the volatility is 25%, the proportional and fixed bankruptcy costs are both 0.2, the bargaining power of equity holders is 0.5, and the corporate tax benefit is 35%.

- [4] The higher the underlying volatility, the higher the chance of default for a low leverage firm (or V is high). On the contrary, higher volatility means more upside potential for firms with low asset value. Hence the probability of debt renegotiation is lower.
- [5] When volatility increases (decreases), the debt value declines (rises) at high level of the asset value of the firm (or low leverage). This result reverses when the asset value of the firm is close to debt renegotiation.
- [6] Risk premium behaves in precisely the opposite way to the debt value. Figure 3 plots the risk premium with the asset value of the firm for high (25%) or low (10%) volatilities. The higher the volatility of the firm, the greater is the risk premium in general. At low values of the firm, the effect goes the opposite way as documented by other articles in the literature.
- [7] Debt value at V_S is independent of the proportional liquidation cost α , but it decreases when the fixed liquidation cost K goes up.

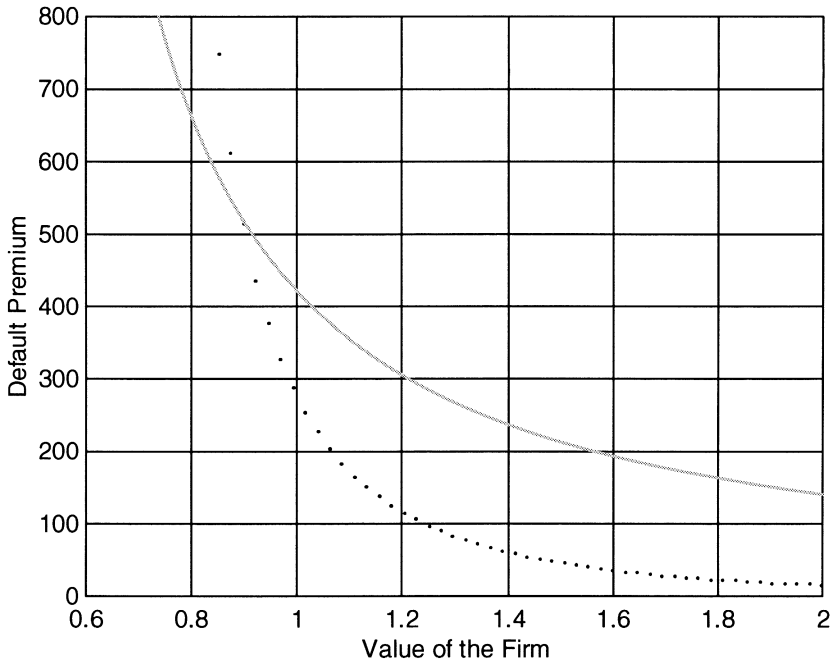


Figure 3
Default premium with various volatilities

The lines plot the default premium as the value of the firm's asset value varies, for two levels of volatility, 10% (dotted line) and 25% (solid line). It is assumed that the coupon level is 10%, the risk-free interest rate is 7.5%, the payout ratio is 7%, the proportional and fixed bankruptcy costs are both 0.2, the bargaining power of equity holders is 0.5, and the corporate tax benefit is 35%.

1.1.4 Cash flow-based bond covenant. As reported in Gilson, John, and Lang (1990), more than 50% of the firms under financial distress did not succeed in private debt restructuring of their debt and went into Chapter 11 bankruptcy procedure. Also documented by other empirical studies, forced liquidations are observed in the market.

Some of the financial distress and liquidation are triggered by various bond covenants written in the debt indenture. In the formulation of bargaining, we explored a debt contract with no covenants. In Proposition 2 we examine the effect of the "cash flow covenant," where when the firm cannot generate sufficient cash flow to meet the interest payment, the debt holders will take over or liquidate the firm. We treat the covenant in this context as a nonnegotiable condition. This is to be interpreted as a metaphor for costly negotiation if the covenant becomes binding. In this sense we are treating the cash flow covenant as a *hard covenant*. There is much empirical support in the accounting literature for this view. DeAngelo and DeAngelo (1990) and Healy and Palepu (1990) document that dividend constraints in the covenants

are respected by stockholders in choosing their dividend policies. For example, Healy and Palepu (1990) note that dividends are cut when dividend constraints are expected to become binding. Likewise, DeAngelo and DeAngelo (1990) show that dividends are cut by stockholders to avoid covenants from becoming binding. Of course, equity holders will recognize that they need to renegotiate the contract *before* the covenant becomes a binding constraint if it is in their interest. If the cash flow covenant becomes a binding constraint before the trigger point for debt renegotiation is reached then forced liquidations can occur. On the other hand, if the trigger point is reached before the cash flow covenant becomes binding, then forced liquidation will not occur. By combining the bond covenants with debt renegotiation, we are able to distinguish between strategic default leading to reorganizations and liquidity default leading to forced liquidations. Proposition 2 also shows how more liquid firms more likely avoid forced liquidations through private debt renegotiation. This proposition also foreshadows the dividend policies discussed in Section 4, where we show that dividend cuts are undertaken by stockholders to avoid cash flow-based covenants. This result is very much consistent with the recorded empirical evidence cited above.

Proposition 2. *With cash flow covenant in the bond contract, (i) If $V_S \geq c/\beta$, debt-equity swap will happen at V_S . The equity and debt values are the same as obtained in Proposition 1;*

(ii) If $V_S < c/\beta$, in other words, the liquidation cost is not sufficiently large to trigger a debt-equity swap, debt holders take over the firm at c/β , triggering liquidity-induced default. The default probability, equity, debt values, and their comparative statics are listed in Table 2, with highlights discussed below.

- [1] Notice that the implications on default probability by the payout ratio β are mixed: (i) High cash payout firms are less restricted by the cash flow covenant; (ii) A given reorganization boundary V_B will be reached less often for a low cash payout firm. When the asset value of the firm is low (or equivalently the firm leverage is high), the default probability declines as the payout ratio β increases because the first effect is more severe. When the firm value is high (leverage is low), the chance of default decreases with β as the second effect becomes dominant.
- [2] As the cash payout ratio β decreases, debt holders can take over the firm earlier, although they receive less contractual payments. If the bankruptcy costs are sufficiently low (high), they are better (worse) off by (not) being able to recover the debt value. Therefore the debt value goes up (down) if bankruptcy costs are low (high).
- [3] Risk premium behaves in exactly the opposite fashion to the debt values.

Table 2
Comparative statics with cash flow covenant

	Payout ratio β	Volatility σ^2	Liquidation cost α, K	Tax benefits τ
Bankruptcy trigger point $V_B = c/\beta$	—	/	/	/
Default probability at $V > V_B$ $P_B = (\beta V/c)^{\lambda_-}$	[1]	+	/	/
Debt value at default $D(V_B) = (1 - \alpha)c/\beta - K$	—	/	—	/
Debt value at $V > V_B$ $(1 - P_B)c/r + P_B D(V_B)$	[2]	—	—	/
Risk premium at $V > V_B$ $c/D(V) - r$	[3]	+	+	/
Equity value at $V > V_B$ $V - (1 - P_B)\frac{c(1-\tau)}{r} - P_B V_B$	—	—	/	+

This table describes properties of the bankruptcy trigger point, default probability, debt and equity values when there is a cash flow covenant in the debt contract, and $V_S < c/\beta$. For notational simplicity, assume $0 \leq (1 - \alpha)c/\beta - K < c/r$.

There are more opportunities for debt renegotiation for firms with a high cash payout ratio. This is due to the fact that they are able to avoid forced liquidations more successfully. Cash flow-based covenants may not necessarily protect debt holders. The risk premium is in fact higher with covenants when liquidation costs are high. In large part this is due to the presumed dividend policy that all residual cash flows are paid out as dividends. Later in Section 4, we show that cash flow covenants can restrict equity holders to adopt a no-dividend policy. This may actually benefit debt holders.

1.2 Strategic debt service

We now consider our second formulation of reorganization where the claimants negotiate at an endogenously determined trigger point to accept a reduced level of debt service (which is temporary until the fortunes improve) but continue to operate the firm. This enables them to get potential tax benefits in the future and the present value of such tax benefits are included in the bargaining process.¹¹ In this sense, the object of bargaining between debt holders and stockholders is in itself endogenous. We will first derive the value of the firm and then use it in characterizing negotiated debt service payments.

1.2.1 The bargaining game and the Nash solution. If the equity and debt holders can successfully reach an agreement of temporary coupon reduction when the firm is in financial distress, the firm will not lose its potential

¹¹ We are very grateful to Hong Liu for pointing this out.

or future tax benefits associated with having debt. At the trigger point for strategic debt service $\tilde{V}_S$, both parties will bargain the total value of the firm, denoted by $v(V)$, instead of the asset value of the firm V in the debt-equity swap case.

Lemma 1. *Given the trigger point for strategic debt service $\tilde{V}_S$, the total value of the firm is*

$$v(V) = \begin{cases} V + \frac{\tau c}{r} - \frac{\lambda_+}{\lambda_+ - \lambda_-} \frac{\tau c}{r} \left(\frac{V}{\tilde{V}_S}\right)^{\lambda_-}, & \text{when } V > \tilde{V}_S; \\ V + \frac{-\lambda_-}{\lambda_+ - \lambda_-} \frac{\tau c}{r} \left(\frac{V}{\tilde{V}_S}\right)^{\lambda_+}, & \text{when } V \leq \tilde{V}_S. \end{cases} \quad (4)$$

Note here the total value of the firm $v(V)$ is always larger than the asset value of the firm V . In other words, the whole pie that equity holders and debt holders bargain over is larger in this case. For any $V \leq \tilde{V}_S$,

$$\tilde{E}(V) = \tilde{\theta}v(V), \quad \tilde{D}(V) = (1 - \tilde{\theta})v(V).$$

The Nash solution to the bargaining game can be characterized as

$$\begin{aligned} \tilde{\theta}^* &= \arg \max \{ \tilde{\theta}v(V) - 0 \}^\eta \{ (1 - \tilde{\theta})v(V) - \max[(1 - \alpha)V - K, 0] \}^{1-\eta} \\ &= \min \left[\eta - \eta \frac{(1 - \alpha)V - K}{v(V)}, \eta \right]. \end{aligned}$$

It is obvious that equity holders will get a bigger share and that the debt holders will get a lesser proportion of the firm. But as our analysis below will show, *both claim holders* will be better off under this formulation.

1.2.2 Valuation. Once again we focus on the case of $K = 0$ in the rest of this section for illustrative purposes. (The case of $K > 0$ is discussed in the technical appendix.) Using standard techniques as in Dixit and Pindyck (1994), it can be shown that the equity value satisfies the following differential equations:

$$\frac{1}{2}\sigma^2 V^2 \tilde{E}_{VV} + (r - \beta)V \tilde{E}_V - r\tilde{E} + \beta V - c(1 - \tau) = 0, \quad \text{when } V > \tilde{V}_S;$$

$$\frac{1}{2}\sigma^2 V^2 \tilde{E}_{VV} + (r - \beta)V \tilde{E}_V - r\tilde{E} + \beta V - S(V) = 0, \quad \text{when } V \leq \tilde{V}_S,$$

where $S(V)$ is the value paid to debt holders under strategic debt service. Note that in the region $V > \tilde{V}_S$, the contractual coupon is paid and hence the tax shield is in effect. However, in the region $V \leq \tilde{V}_S$ the equity holders are strategically servicing the debt which may vary according to cash flow generated by the firm, and hence they lose the tax shield. This fact may be interpreted as debt holders agree to forgive some debt and the Internal Revenue Service (IRS) suspends tax benefits until contractual payments are

resumed.¹² Both the strategic debt servicing amount $S(V)$ and the trigger level $\tilde{V}_S$ are determined endogenously. Debt holders know ex ante that there will be periods in the future when contractual payments will not be made. They will naturally reflect that in the pricing of debt securities.

Parallel to the debt-equity swap case, the economics of the problem lead to the following boundary conditions:

$$\begin{aligned}\lim_{V \uparrow \infty} E(V) &= V - \frac{c(1-\tau)}{r}, \\ \lim_{V \downarrow \tilde{V}_S} E(V) &= \eta \left(\alpha \tilde{V}_S - \frac{\lambda_-}{\lambda_+ - \lambda_-} \frac{\tau c}{r} \right), \\ \lim_{V \downarrow \tilde{V}_S} E_V(V) &= \eta \left(\alpha - \frac{\lambda_+ \lambda_-}{\lambda_+ - \lambda_-} \frac{\tau c}{r \tilde{V}_S} \right).\end{aligned}$$

In Proposition 3 we characterize the trigger point for renegotiation and the valuation of debt and equity under negotiated debt service reductions.

Proposition 3. (i) *The trigger point for strategic debt service becomes*

$$\tilde{V}_S = \frac{c(1-\tau+\eta\tau)}{r} \frac{-\lambda_-}{1-\lambda_-} \frac{1}{1-\eta\alpha};$$

(ii) *The equity value is*

$$\tilde{E}(V) = \begin{cases} V - \frac{c(1-\tau)}{r} + \left[\frac{c(1-\tau)}{(1-\lambda_-)r} - \frac{\lambda_-(1-\lambda_+)\eta}{(\lambda_+-\lambda_-)(1-\lambda_-)} \frac{\tau c}{r} \right] \left(\frac{V}{\tilde{V}_S} \right)^{\lambda_-}, & \text{when } V > \tilde{V}_S; \\ \eta v(V) - \eta(1-\alpha)V, & \text{when } V \leq \tilde{V}_S. \end{cases}$$

(iii) *The debt value can be obtained as*

$$\tilde{D}(V) = v(V) - E(V).$$

(iv) *The strategic debt service amount when $V \leq \tilde{V}_S$, is given by*

$$S(V) = (1-\eta\alpha)\beta V.$$

It is to be noted that when $\tilde{V}_S < c/\beta < V$ cash flows are insufficient to finance contractual coupon and equity will be issued to finance coupons. They will do so until the optimal reorganization boundary is reached.

By writing the strategic debt service amount as $S(V) = \eta\beta V + \eta(1-\alpha)\beta V$ for $V \leq \tilde{V}_S$, we can get a better intuition: strategic debt service involves payment of a proportion of the firm's cash flows as well as the same proportion

¹² Our results can be easily extended to a case where the tax benefits accrue even during the strategic debt service period. The trigger point $\tilde{V}_S$ will decrease in such a case.

of the imputed payout from the residual value $(1 - \alpha)V$. And the proportion η characterizes the bargaining power of equity holders.

With strategic debt service, debt holders only experience a temporary coupon reduction. Both equity and debt holders can share the potential tax benefits when the firm gets out of the financial distress. The liquidation outcome for both equity and debt holders remains the same as in the debt-equity swap case. Equity holders benefit more as they receive a greater share of the pie. Moreover, the total pie for bargaining is larger as well. Though debt holders receive a smaller proportion of the pie, the actual amount they receive increases as well. *In this sense, the bargaining outcome with strategic debt service dominates debt-equity swap. But if the claimants have alternative uses for their funds that are more attractive than the potential tax benefits that they may be able to capture, and perceive that continued renegotiation can be costly, then they may prefer debt-equity swap.* In the next corollary, we explicitly compare the sharing rule, equity and debt values, and the trigger point for strategic debt service with those for debt-equity swap.

Corollary 2. (i) *Equity holders receive a bigger share under strategic debt service, than under debt-equity swap. Debt holders receive less.*

$$\tilde{\theta}^* \geq \theta^*, \quad (1 - \tilde{\theta}^*) \leq (1 - \theta^*).$$

(ii) *Both equity and debt holders are better off through strategic debt service.*

$$\tilde{E}(V) \geq E(V), \quad \tilde{D}(V) \geq D(V).$$

(iii) *Strategic debt service will be triggered earlier than debt-equity swap,*

$$\tilde{V}_S \geq V_S.$$

1.2.3 Cash flow-based bond covenants. Similar to the debt-equity swap case, when considering a cash flow-based bond covenant, whether the covenant becomes binding depends upon the relative level of the trigger point for strategic debt service $\tilde{V}_S$ and c/β . As pointed out in Corollary 2(iii), strategic debt service happens prior to debt-equity swap. As a result, there is a higher probability to avoid the cash flow-based covenant through strategic debt service.

We now turn our attention to the valuation of finite maturity debt. We show the results for the debt-equity swap case in the rest of this article because all our conclusions will hold qualitatively for strategic debt service as well, though the results are much more complicated. For clarity we have chosen to present just the debt-equity swap formulation alone.

2. Finite Maturity Debt

Although perpetual debt allows us to obtain closed-form solutions, it is important to examine debt contracts with finite maturities. Finite maturity debt contracts are most commonly issued by firms and traded around the world. The essence of this important modification is that the debt values are no longer time homogeneous. The fundamental valuation equation will explicitly depend on time, given the maturity at time T (or equivalently time to maturity). This renders the valuation equation to be a partial differential equation,

$$E_t + \frac{1}{2}\sigma^2 V^2 E_{VV} + (r - \beta)V E_V - rE + \beta V - c(1 - \tau) = 0, \quad (5)$$

with the following terminal conditions which can be motivated as before,

$$E(V, T) = \max[V - P, \eta(\alpha V + K)],$$

where P is the principal amount of the debt and T is the maturity time. Denote $p(t, T)$ as the risk-free bond value with maturity T at time t . The boundary conditions become

$$\lim_{V \uparrow \infty} E(V, t) = V + \frac{\tau c}{r} [1 - e^{-r(T-t)}] - p(t, T)$$

$$\lim_{V \downarrow V_S(t)} E(V, t) = \eta[\alpha V_S(t) + K],$$

where the trigger point $V_S(t)$ is the free boundary to be endogenously determined by the smooth pasting condition:

$$\lim_{V \downarrow V_S(t)} E_V(V, t) = \eta\alpha.$$

Debt value is obtained as the difference between the total value of the firm (V plus the expected tax benefits) and the value of equity. An analytical solution to this problem is no longer available. We have solved the problem using numerical methods.¹³ Figure 4 plots the term structure of risky debt. For debt with the same maturities, high leverage or high volatility leads to high-risk premiums. As time to maturity increases, risk premiums rise and then decline. This effect, which is consistent with empirical findings, is more evident for high-leverage firms.

Figure 5 plots the strategic debt service region for the AST model and ours with different liquidation cost structures (fixed, linear, and proportional). AST's model puts all the bargaining power to equity holders, while debt holders behave passively. Equity holders stop making contractual payment earlier as compared to our model. The risk premium is consequently overstated.

¹³ Anderson and Tu (1998) have documented the numerical procedures for solving the AST model in detail.

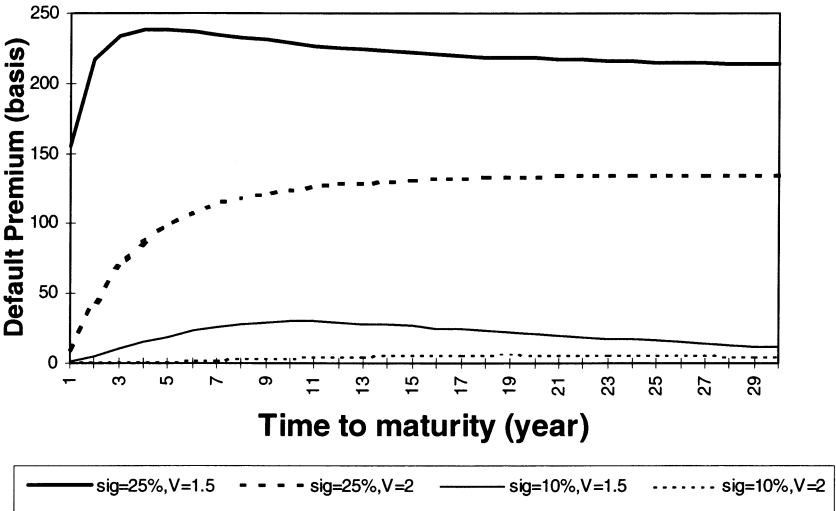


Figure 4
Default premium with finite maturity debt
This figure plots the default premiums for two levels of volatility (10% and 25%) and asset value of the firm ($V = 1.5$ and 2), as the maturity of the debt varies from 1 to 30 years. It is assumed that the coupon is 10%, the risk-free interest rate is 7.5%, the proportional and fixed liquidation costs are both 0.2, the equity holders' bargaining power is 0.5 and the corporate tax benefit is 0.

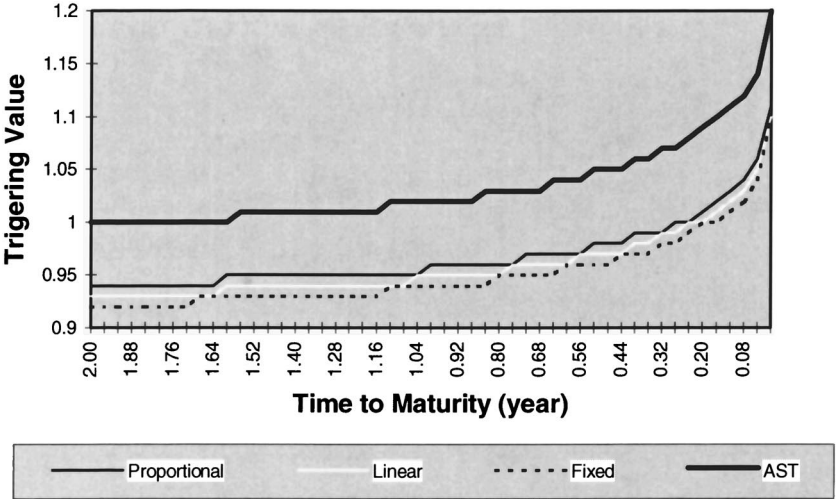


Figure 5
Trigger points for strategic debt service with finite maturity debt
The lines plot the regions for strategic debt service obtained by our model with a different liquidation cost structure (fixed cost 0.2, linear cost $K = 0.1$, $\alpha = 0.1$, and linear cost $\alpha = 0.2$) and by AST with fixed cost $K = 0.2$ only. It is also assumed that the coupon rate is 10%, the risk-free interest rate is 7.5%, the volatility is 17.3%, the bargaining power of equity holders is 0.5 and the corporate tax benefit is 0.

Table 3
Risk premiums of debt with finite maturities

Maturity	Volatility	Asset value	AST $K = 0.2$	$\alpha = 0$ $K = 0.2$	$\alpha = 0.2$ $K = 0.2$	$\alpha = 0.2$ $K = 0$
$T = 2$	$\sigma^2 = 0.03$	0.5	7285	5511	6324	4805
		1.0	1491	924	1491	888
		1.5	150	70	158	69
		2.0	12	4	13	59
		2.5	1	1	2	1
		∞	0	0	0	0
	$\sigma^2 = 0.10$	0.5	7285	5511	6324	4805
		1.0	1850	1350	1755	1251
		1.5	580	394	563	371
		2.0	199	126	197	120
		2.5	73	43	73	41
		3.0	28	16	29	16
		3.5	12	7	12	6
		∞	0	0	0	0
$T = 30$	$\sigma^2 = 0.03$	0.5	2584	1753	2109	1476
		1.0	507	372	508	366
		1.5	169	131	169	129
		2.0	84	66	85	65
		2.5	49	39	49	38
		3.0	31	24	31	24
		∞	0	0	0	0
	$\sigma^2 = 0.10$	0.5	2584	1753	2109	1476
		1.0	642	517	604	481
		1.5	355	296	337	279
		2.0	238	201	227	190
		2.5	170	145	162	137
		3.0	122	105	117	100
		3.5	85	73	82	70
		∞	0	0	0	0

This table lists risk premiums, in units of basis points, obtained in Anderson, Sundaresan, and Tychon (1996) and our model with different liquidation cost structure. It is assumed that in the base case the debt principal $P = 1$, the coupon level $c = 10\%$, the risk-free rate $r = 7.5\%$, and the payout ratio $\beta = 7\%$, and the tax benefit $\tau = 0$.

How well can we explain the risk premium observed in the market? Table 3 illustrates the risk premiums obtained by different models. The risk premiums are much higher than those obtained by Merton (1974) with similar volatilities and maturities. When the firm value is high, debt with shorter maturity tends to have a lower premium; when the firm value is low, longer term debt has a relatively lower premium. The implication of different liquidation cost structures is that fixed cost contributes more to the risk premium, especially when the firm's asset value is low. It produces a higher risk premium that is closer to empirical findings in the literature than proportional cost.

3. Optimal Dividend Policy

Most models in the literature tend to assume that the residual cash flows are simply paid out as dividends. In this section, dividends, or equivalently the total payout ratio, denoted by δ , are treated as a control variable in the firm's cash flow generating process. In our formulation, stockholders will choose

their dividend policies by acting to maximize their equity value. This will be fully anticipated by the debt holders (given full information in our model). They will therefore discount the value of debt, if necessary, to incorporate any adverse effects such policies may have on the debt value. In order to address this issue systematically, we begin with Leland's model and show that the implied dividend policy assumed by Leland is in fact optimal. Then we take up the optimal dividend policy in the presence of bond covenants and strategic debt service.

3.1 Endogenous dividends with no bond covenant

When cash payout βV exceeds the promised coupon rate c , stockholders have a decision to make: they can pay all the residual cash flows as dividends to themselves, or they can reinvest a fraction into the firm. The motivation for such an action is simple: by foregoing current dividends, the stockholders can avoid costly liquidations that may arise in the future. This feature is modeled in the following manner.

Whenever $\beta V \geq c$, the firm has no cash flow constraint and we refer to this state as a "good" state. We assume that the dynamics of the firm's value is given by

$$dV = (\mu - \beta)V dt + \sigma V dB_t + (\beta - \delta)V dt,$$

where δV denotes the aggregate payout. It can be a function of the firm's asset value and other deep parameters in the model. This may be written as

$$dV = (\mu - \delta)V dt + \sigma V dB_t.$$

Among the total cash flow βV , the retained earnings, $(\beta - \delta)V$, are reinvested back into the firm's value-generating activity. The total payout δV includes the coupon payment c and the dividends $\delta V - c$. And we constrain $c(1 - \tau)/V \leq \delta \leq \beta$, since the payout has to at least cover the debt obligations and no more than the total cash flows available. The idea is the following: with excess cash flows, stockholders may choose a value of δ (subject to the constraint noted above) so as to maximize their equity value.

Whenever $\beta V < c$, the firm is under a liquidity constraint and we refer to this state as a "bad" state. We assume that the dynamics of the firm's value is given by

$$dV = (\mu - \beta)V dt + \sigma V dB_t.$$

No dividend can be paid as the bond covenant specified in Assumption 4 stipulates, so that equity holders are not allowed to pay themselves a dividend by selling the firm's assets.¹⁴

¹⁴ With the possibility of external financing, the equity holders might manage to pay themselves a dividend in the "bad" state.

In the good state, the equity value, denoted by E^G , satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 E_{VV}^G + (r - \delta)V E_V^G - r E^G + \delta V - c(1 - \tau) = 0, \quad \text{when } \beta V \geq c. \quad (6)$$

In the bad state, the equity value, denoted by E^B , satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 E_{VV}^B + (r - \beta)V E_V^B - r E^B + \beta V - c(1 - \tau) = 0, \quad \text{when } \beta V < c. \quad (7)$$

The boundary conditions and the value-matching and smooth pasting conditions are similar to those specified in Section 2 and are presented in the appendix. The pricing of debt and equity depends in a significant manner on the threshold level, V_B , of the asset value of the firm at which debt holders take over the firm. For a given dividend policy or equivalently, for a given δ , we can prove the existence of an endogenous lower reorganization boundary V_B . This is shown in the appendix (Lemma 2).

Figure 6 shows that, as the payout ratio δ increases from $c(1 - \tau)/V$ to β , the value at which bankruptcy is triggered increases, as does the firm's equity value E^G . Intuitively, if the residual cash flows are invested back as retained earnings, they become accessible by the debt holders upon bankruptcy. It would be optimal for the equity holders to pay all the cash flows available as dividend, that is, $\delta^* = \beta$. We prove this conjecture next.

Proposition 4. *When $\beta V \geq c$ (the firm is in a “good” state), it is optimal to pay all the residual cash flows as dividends. When $\beta V < c$ (the firm is in a “bad” state), it is optimal to pay no dividends.*

$$E^G(V; \delta) \leq E^G(V; \beta), \quad \forall c(1 - \tau)/V \leq \delta \leq \beta. \quad (8)$$

If equity holders are value maximizing, the endogenous liquidation value and the debt value are the same as found by Leland (1994). We have thus provided a validation of the dividend policy and the valuation results of Leland in the context of our model.

Since equity holders control dividends and have the power to precipitate bankruptcy, they would choose maximum residual cash flow as dividends. This parallels the results of Radner and Shepp (1996), called the “take the money and run” strategy. On the other hand, this also increases the chance of bankruptcy and its cost since the trigger point would be higher than a lower dividend policy. The risk premium goes up, as shown in Figure 7. Since the debt contract must be valuable enough to raise the necessary level of investment at time 0, it is important to stress that such dividend policies may not be pursued at equilibrium.

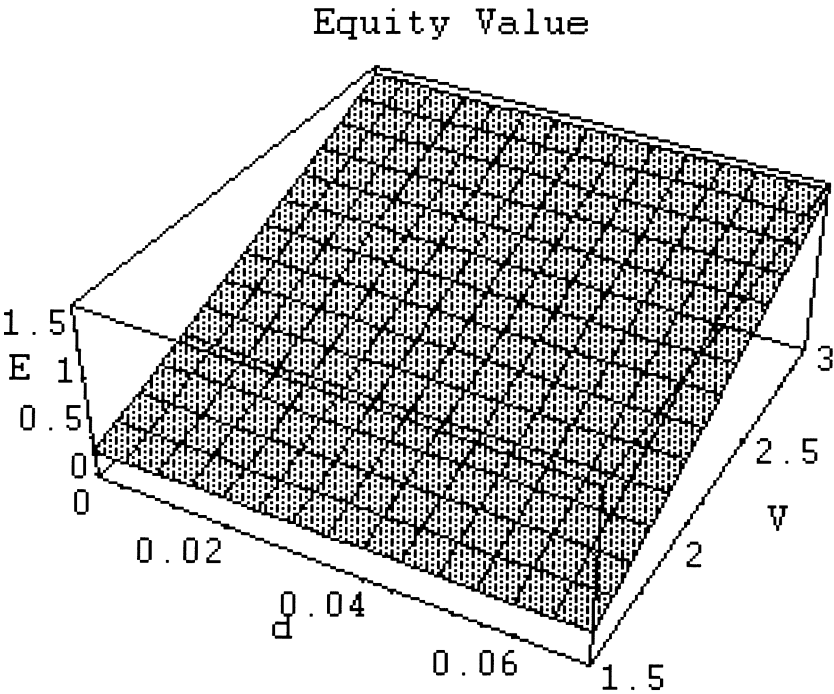


Figure 6
Equity value for various asset values and payout ratios
This figure shows how equity value varies as the firm's asset value and payout ratio change at the same time. It is assumed that the coupon level is 10%, the risk-free interest rate is 7.5%, the volatility is 17.3%, the equity holders bargaining power is 0.5, and the corporate tax benefit is 0.

3.2 Optimal dividend policy with cash flow covenant

What is the effect of bond covenants on optimal dividend policy? Given the danger of facing forced liquidation at a certain point and the need to raise a certain level of investment at time 0, will the equity holders still choose the maximal payout ratio? Intuition suggests that they may have an incentive not to pay out all the available cash flows to reduce the chance that the cash flow constraint becomes binding. This way they can avoid forced liquidations and improve the valuation of both debt and equity. The optimal dividend policy is stated next.

Proposition 5. *Under the cash flow covenant, if $\beta < r/(1 - \tau)$, the equity value-maximizing policy is to pay no dividends.*

Proposition 5 links bond covenants with dividend policies. It suggests that cash flow-based (hard) covenants actually lead to more conservative payout policies. The resulting increase in the future value of collateral will be shared in the event of financial distress by appropriate bargaining rules. It is not

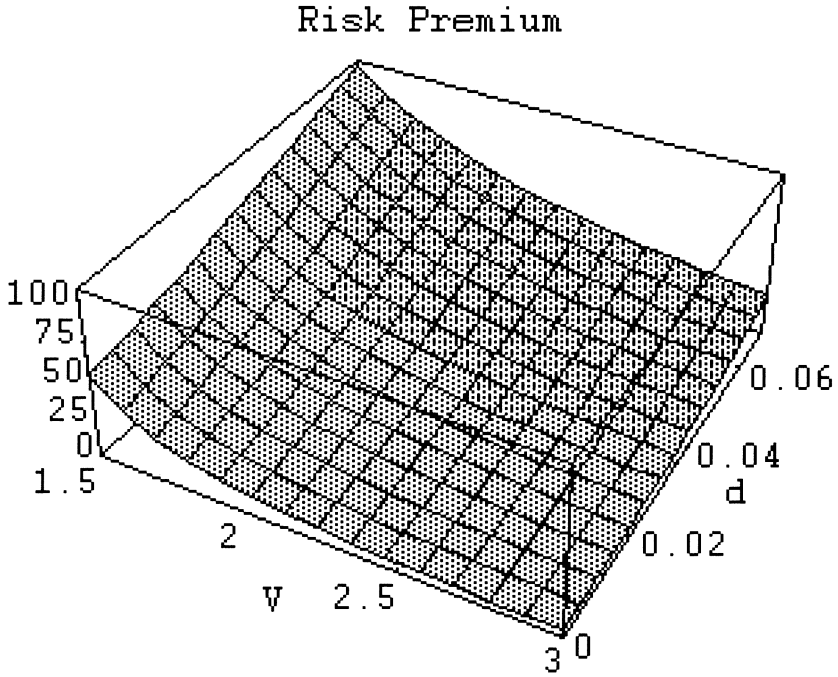


Figure 7

Interaction of risk premium on firm's asset value and payout ratio

This figure describes the interaction of risk premium (in units of basis points) on the firm's asset value and payout ratio. It is assumed that the coupon rate is 10%, the risk-free interest rate is 7.5%, the volatility is 17.3%, the equity holders' bargaining power is 0.5, and the corporate tax benefit is 0.

surprising that equity holders deny themselves dividends in order to prevent potential forced liquidation. The optimal asset value process is now

$$dV = (\mu V - c)dt + \sigma V dB_t. \quad (9)$$

The differential equations that firm's equity and debt values satisfy become the same as those in Merton (1974), subject to the appropriate boundary conditions.¹⁵ The debt value can be solved as

$$D(V; \cdot) = \frac{c}{r}(1 - P_D) + \left[(1 - \alpha) \frac{c}{\beta} - K \right] P_D, \quad (10)$$

where P_D is the risk-neutralized probability of hitting the binding covenant,

$$P_D = \left(\frac{V}{c/\beta} \right)^{-\frac{2r}{\sigma^2}} \frac{M(\frac{2r}{\sigma^2}, 2 + \frac{2r}{\sigma^2}, -\frac{2c}{\sigma^2 V})}{M(\frac{2r}{\sigma^2}, 2 + \frac{2r}{\sigma^2}, -\frac{2\beta}{\sigma^2})}, \quad (11)$$

¹⁵ See the appendix for details.

and $M(\cdot, \cdot, \cdot)$ is the confluent hypergeometric function. Debt value and the total firm value are both maximized by choosing minimum payout. The “cash flow covenant” is able to eliminate the conflict between debt and equity holders on dividend policy.

3.3 Dividend policy and debt renegotiation

When equity holders engage in debt renegotiation, they have the power to exploit the debt holders further. If there is no cash flow constraint in the bond indenture, the trigger point V_S will always be reached before the endogenous reorganization point V_B of Leland. Obviously they would choose to pay themselves all the residual cash flows as dividend. What happens if debt holders are protected with the cash flow covenant? With a different payout ratio, δ , the trigger point of strategic debt service changes in the following way:

$$V_S(\delta; \cdot) = \frac{-\gamma_-}{1 - \gamma_-} \left[\frac{c(1 - \tau)}{r} + \eta K \right] \frac{1}{1 - \eta\alpha},$$

where

$$\gamma_- = 0.5 - \frac{(r - \delta)}{\sigma^2} - \sqrt{\left[0.5 - \frac{(r - \delta)}{\sigma^2} \right]^2 + \frac{2r}{\sigma^2}} < 0. \quad (12)$$

When $V_S(\delta, \cdot) < c/\beta$ for all $c(1 - \tau)/V \leq \delta \leq \beta$, forced liquidation will result before debt negotiations are triggered. Therefore the optimal dividend policy and the associated security values are the same as in Proposition 4. In other words, when the liquidation cost is not big enough, equity holders do not have sufficient power to threaten debt holders. On the other hand, when $V_S(\delta, \cdot) > c/\beta$ for all $c(1 - \tau)/V \leq \delta \leq \beta$, or the liquidation cost is so severe that the equity holders can always start paying the creditors less coupon before the firm value reaches the forced liquidation value c/β , the equity holders will pay themselves maximal residual cash flow as a dividend. The creditors are less protected even with the cash flow-based covenant in this case. If there exists a number δ_0 within the interval $[c(1 - \tau)/V, \beta]$ such that $V_S(\delta_0, \cdot) = c/\beta$, then an interesting dividend policy emerges. If equity holders do not retain any earnings in “good” states, creditors would rather take over the firm whenever it is possible. Thus the potential liquidation costs are reflected in the stock and debt values. Therefore equity holders are better off to avoid forced liquidation by receiving less dividend. This might explain one of the empirical findings in DeAngelo and DeAngelo (1990) that some of the lower dividend payment is to enhance firm’s bargaining position. As the payout ratio δ goes up from $c(1 - \tau)/V$ to β , the trigger point $V_S(\delta, \cdot)$ decreases and the equity value increases. As long as the payout ratio is between $c(1 - \tau)/V$ and δ_0 , forced liquidation can be avoided. Equity holders will choose the highest value of δ_0 if they are value maximizing. An interior dividend policy emerges as stated next.

Proposition 6. *If there exists a $\delta_0 \in [c(1 - \tau)/V, \beta]$ such that $V_S(\delta_0, \cdot) = c/\beta$, then the optimal payout ratio is δ_0 . For $V \geq c/\beta$, the debt and equity values are*

$$D(V; \delta_0) = \frac{c}{r} + \left[(1 - \eta\alpha) \frac{c}{\beta} - \eta K - \frac{c}{r} \right] \left(\frac{V}{c/\beta} \right)^{\gamma_0};$$

$$E(V; \delta_0) = V - \frac{c(1 - \tau)}{r} + \left[\frac{c(1 - \tau)}{r} + \eta \left(K + \alpha \frac{c}{\beta} \right) \right] \left(\frac{V}{c/\beta} \right)^{\gamma_0}$$

where $\gamma_0 = 0.5 - (r - \delta_0)/\sigma^2 - \sqrt{[0.5 - (r - \delta_0)/\sigma^2]^2 + 2r/\sigma^2}$.

Proposition 6 describes a situation where an interior dividend policy is optimal. For example, for the following parameters— $c = 0.1$, $r = 0.075$, $\sigma^2 = 0.01$, $\tau = 0$, $\alpha = 0.2$, $K = 0.2$, $\eta = 0.5$ —the optimal payout ratio can be solved as $\delta_0 = 0.0351$. This gives an example where equity holders are willing to choose an interior dividend policy without modeling asymmetric information. The incentive for them is to enhance their bargaining position upon debt restructuring. As the fixed cost of liquidation increases, the optimal dividend yield decreases to lower the probability of a forced liquidation. As the volatility of the underlying firm's value increases, the optimal dividend payout decreases. These properties are shown in Figure 8.

Conflicts between equity and debt holders on dividend policy and the associated welfare loss still exist, since the risk premium is minimized when equity holders receive no dividend. The difference is shown in Figure 9.

However, without the “cash flow covenant” equity holders would rather collect all the residual cash flows as dividends. Hence by giving equity holders incentive to retain part of the residual cash flows, the covenant performs a useful disciplinary role.

4. Empirical Implications

Our model has several implications that sharply differ from those of previous contributions. These predictions concern (1) reorganization boundary and probabilities of default, (2) recovery ratio, (3) default premium, and (4) debt capacity and optimal capital structure.

4.1 Reorganization boundary and default probability

The reorganization boundaries and default probabilities derived from different models are presented in Table 4. We present the results with the fixed cost K to demonstrate how it affects our empirical implications.

The probabilities of default implied by our model can be easily contrasted with those of Leland and AST. All models have the same elasticity λ_- of probability of default with respect to the asset value of the firm. But our model predicts that the lower reorganization boundary depends on the

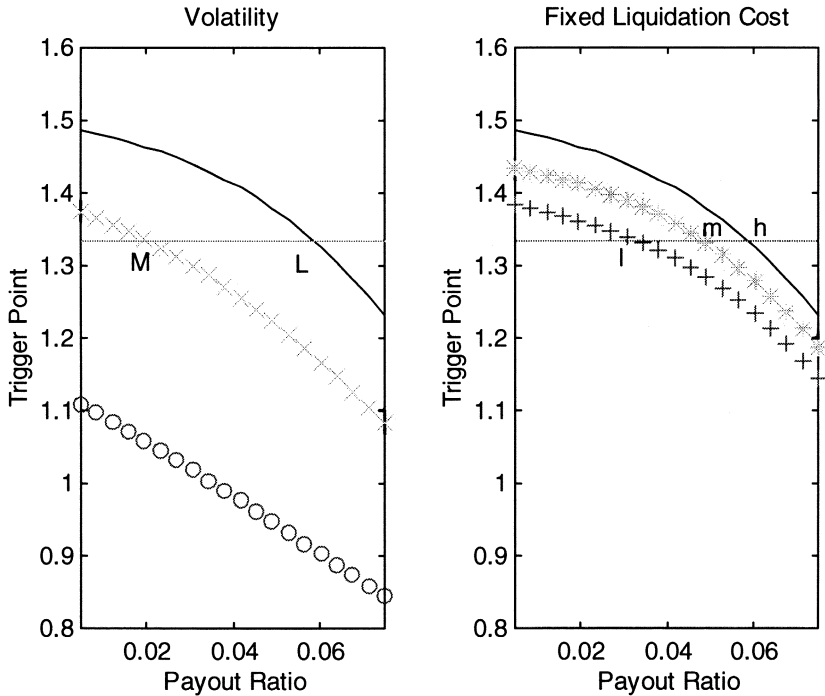


Figure 8
Strategic debt service trigger point as a function of payout ratio
The lines in the left panel of this figure plot how V varies as the firm's payout ratio increases for three levels of volatility: 10% (solid line), 15% ("x" line) and 25% ("o" line). The horizontal solid line is the threshold c/β . "M" and "L" are the points where optimal payout ratios are reached for medium and low level volatilities, respectively. The right panel plots V as a function of payout ratio for three levels of fixed liquidation cost: 0.2 (solid line), 0.1 ("*" line) and 0 ("+" line). "h", "m" and "l" are the points where the optimal payout ratios are reached for high, medium, and low levels of K , respectively.

liquidation costs, as does the AST model. The effect of liquidation costs on the probability of default depends on the bargaining game. These implications contrast with Leland's model, where the lower reorganization boundary is independent of liquidation costs. This implies that the probability of default in Leland's model is independent of liquidation costs. In our model, probability of default depends on the costs of liquidation. As we have shown earlier in Corollary 1, $V_B < V_S < V_{NC}$; the probability of default predicted in AST is higher than that in our model. Leland (1994) predicts the lowest probability of liquidation.

4.2 Recovery ratio

In the context of our model, let us define the recovery ratio as the amount debt holders will recover upon default divided by the risk-free debt value,

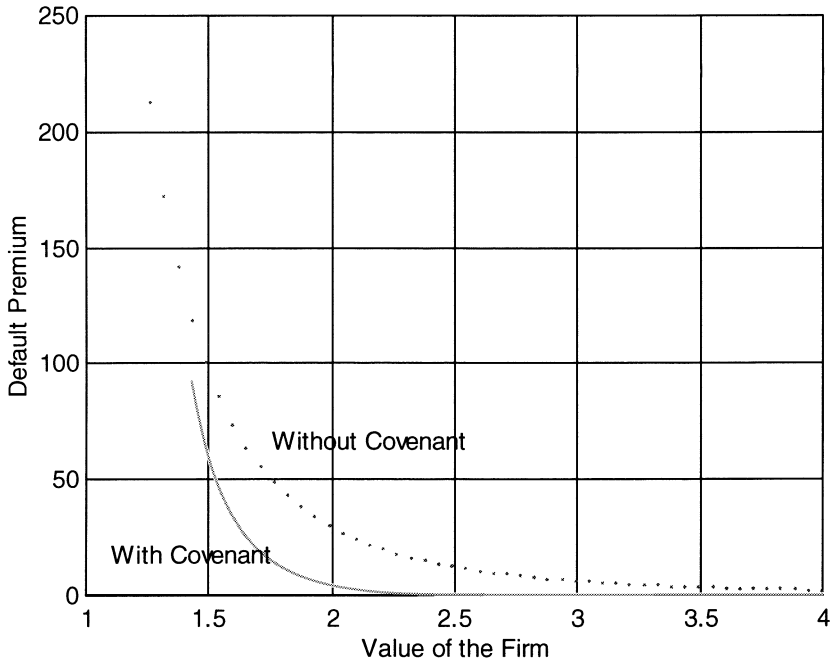


Figure 9

The effect of covenant on risk premium

The lines here plot the risk premiums of debt with and without the cash flow covenant at various firm values. It is assumed that the coupon level is 10%, the risk free interest rate is 7.5%, the volatility is 10%, the tax benefit is 0, the fixed and linear liquidation costs are both 0.2, and the bargaining power of equity holders is 0.5. The optimal payout ratios are 3.51% with covenant and 7% without covenant.

Table 4
Comparison of existing models

	Leland	AST	This model	With $c/\beta \leq V_S$	Covenant $c/\beta > V_S$
Reorganization boundary	V_B	V_{NC}	V_S	V_S	c/β
Sharing rule θ	0	$\frac{\alpha V_{NC} + K}{V_{NC}}$	$\frac{\alpha V_S + K}{2V_S}$	$\frac{\alpha V_S + K}{2V_S}$	0
Absolute priority	Yes	No	No	No	Yes
Forced liquidation	No	No	No	No	Yes
Default probability	$(\frac{V}{V_B})^{\lambda-}$	$(\frac{V}{V_{NC}})^{\lambda-}$	$(\frac{V}{V_S})^{\lambda-}$	$(\frac{V}{V_S})^{\lambda-}$	$(\frac{V}{c/\beta})^{\lambda-}$
Recovery ratio	RR_L	RR_{NC}	RR_S	RR_S	RR_B

This table compares several aspects of existing models in Leland (1994), Anderson, Sundaresan, and Tychon (1996), and our model without the cash flow-based covenant or with the bond covenant.

that is,

$$RR_S = \left[(1 - \eta\alpha) \left[\frac{c(1 - \tau)}{r} + \eta K \right] \frac{-\lambda_-}{1 - \lambda_-} \frac{1}{1 - \eta\alpha} - \eta K \right] / \frac{c}{r} \quad (13)$$

$$= \left[(1 - \tau) + \frac{\eta K}{c/r} \right] \frac{-\lambda_-}{1 - \lambda_-} - \frac{\eta K}{c/r}; \quad (14)$$

The recovery rate implied by our model is independent of the proportional liquidation cost α . This is due to the fact that the presence of liquidation costs is already optimally reflected in the choice of lower reorganization boundary by the borrowers. Similarly the recovery ratios implied by Leland and AST when $K = 0$ can be defined, respectively, as

$$RR_L = \left[(1 - \alpha) \frac{c(1 - \tau)}{r} \frac{-\lambda_-}{1 - \lambda_-} \right] / \frac{c}{r} = (1 - \alpha) \frac{-\lambda_-(1 - \tau)}{1 - \lambda_-}, \quad (15)$$

$$RR_{NC} = \left[(1 - \alpha) \frac{c(1 - \tau)}{r} \frac{-\lambda_-}{1 - \lambda_-} \frac{1}{1 - \alpha} \right] / \frac{c}{r} = \frac{-\lambda_-(1 - \tau)}{1 - \lambda_-}. \quad (16)$$

In AST model the recovery rate is the same as RR_S in our model. In general, when the fixed liquidation cost is negligible, the recovery rates predicted by our model are independent of the equity holders' bargaining power η . However, when one takes into consideration the fixed cost K , recovery rates depend on liquidation costs, though its importance is tempered by the nature of the game. In contrast, Leland's model, where liquidation happens, consistently predicts a lower recovery rate than the debt renegotiation models where no dead weight costs are actually incurred.

With the cash flow-based covenant in the bond indenture, the recovery rate is defined as the following when the covenant becomes binding:

$$RR_B = (1 - \alpha) \frac{c}{\beta} / \frac{c}{r} = (1 - \alpha) \frac{r}{\beta}. \quad (17)$$

4.3 Default premium

The combining effect of default probabilities and recovery rates determines the default premiums observed in the market. Since AST produces higher default probabilities than our model, while maintaining the same recovery rates, it always produces larger default premiums than our model. This is due to the fact that we assign equal bargaining power to both equity and debt holders, while in AST, debt holders extract more surpluses and this results in a higher value of debt and consequently a lower default premium. Figure 10 plots the risk premiums obtained from different models with a 35% tax rate. With the same leverage ratio and same level of liquidation costs, our model predicts a lower risk premium than the Anderson and Sundaresan (1996) model and its extension in AST. Leland's model predicts systematically lower

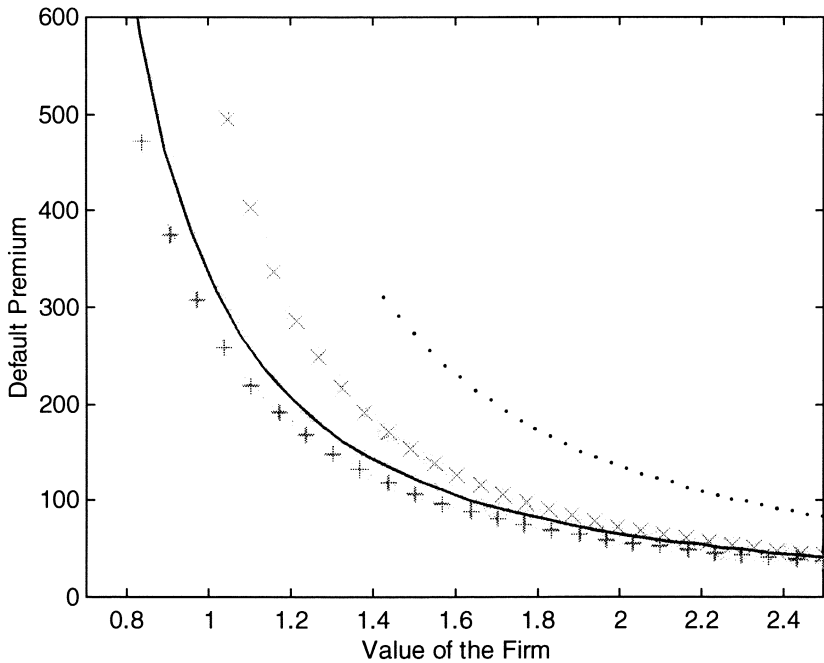


Figure 10
Risk premium predicted by existing models with 35% tax

The lines plot risk premiums as a function of the firm's asset value V , for different models: AST ("x" line), Leland ("+" line), and our model with cash flow covenant (dotted line) and without (solid line). It is assumed that the coupon rate is 10%, the risk-free interest rate is 7.5%, the payout ratio is 7%, the bargaining power of equity holders is 0.5, and the corporate tax benefit is 35%.

default probabilities and higher recovery rates, so the risk premiums it generates are mixed. It produces higher risk premium when liquidation costs and tax benefits are not that significant. Debt is less risky in our model with a high level of tax benefits. This is due to the fact that with high taxes, the trigger level goes down so that more contractual coupon payments are made. This effect can be verified in Figure 11 where the tax is only 15%.

4.4 Debt capacity and optimal capital structure

The debt capacity refers to the maximum level of credit that the lenders are willing to extend. The optimal capital structure refers to the level of debt that maximizes the total firm value (which is the sum of equity and debt). We adapt the definitions of Leland in this section. To make direct comparisons with Leland, we work with the same linear structure of liquidation costs for all models. Tables 5 and 6 list the debt capacity and optimal capital structure with respect to the total value of the firm.

The debt capacity in our model is always higher than in the AST model, since debt holders have the ability to bargain for more. In Leland's model,

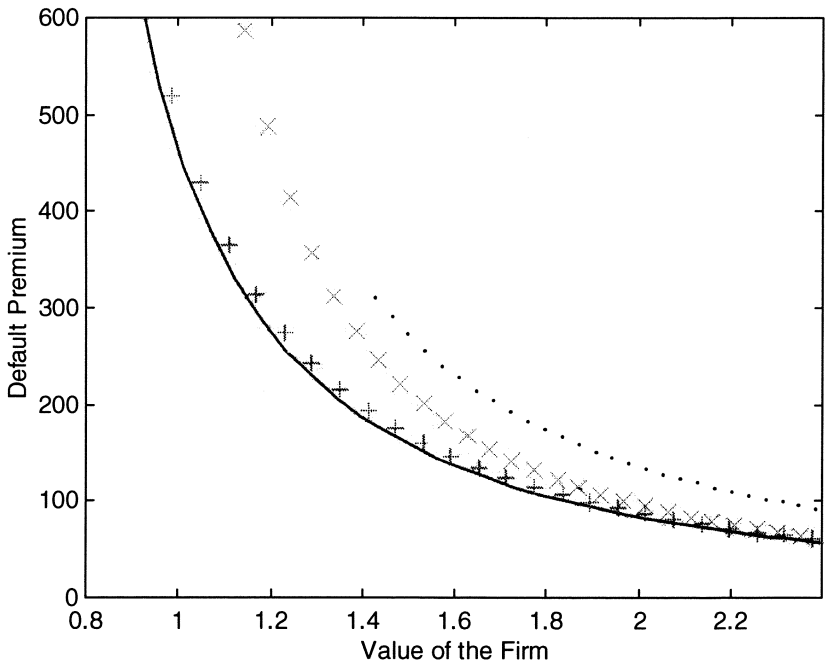


Figure 11
Risk premium predicted by existing models with 15% tax
The lines plot risk premiums as a function of the firm's asset value for different models: AST ("x" line), Leland ("+" line), and our model with cash flow covenant (dotted line) and without (solid line). It is assumed that the coupon rate is 10%, the risk-free interest rate is 7.5%, the payout ratio is 7%, the bargaining power of equity holders is 0.5, and the corporate tax benefit is 15%.

debt capacity is even higher due to the presence of strict absolute priority. The higher the liquidation costs, the wider is the scope for bargaining between the equity and debt holders, and hence the larger are the differences between the three models. Figure 12 plots debt capacities obtained by different models as liquidation costs vary.

Table 5
Debt capacity

V	r	β	σ^2	α	K	τ	Leland	AST	This model
2.0	7.5	7.0	0.03	0.2	0.2	35	90.0	68.0	81.4
1.5	7.5	7.0	0.03	0.2	0.2	35	89.7	64.6	79.7
2.0	7.0	7.0	0.03	0.2	0.2	35	89.9	68.1	81.5
2.0	7.5	6.5	0.03	0.2	0.2	35	90.0	67.9	81.4
2.0	7.5	7.0	0.10	0.2	0.2	35	90.0	68.5	81.9
2.0	7.5	7.0	0.03	0.5	0.0	35	86.1	52.3	74.3
2.0	7.5	7.0	0.03	0.0	0.0	35	94.0	94.0	94.0
2.0	7.5	7.0	0.03	0.2	0.2	15	92.5	69.2	83.6

This table compares the debt capacity in terms of the leverage ratio (in percentage), which is the market value of debt over the total firm value for various parameters among models of Leland, Anderson–Sundaresan–Tychon (AST), and ours without the cash flow covenant.

Table 6
Optimal capital structure

V	r	β	σ^2	α	K	τ	Leland	AST	This model
2.0	7.5	7.0	0.03	0.2	0.2	35	74.8	65.2	69.9
1.5	7.5	7.0	0.03	0.2	0.2	35	74.1	52.5	67.9
2.0	7.0	7.0	0.03	0.2	0.2	35	73.9	56.5	69.9
2.0	7.5	6.5	0.03	0.2	0.2	35	75.4	56.8	70.1
2.0	7.5	7.0	0.10	0.2	0.2	35	69.4	56.0	69.9
2.0	7.5	7.0	0.03	0.5	0.0	35	70.2	46.6	65.6
2.0	7.5	7.0	0.03	0.0	0.0	35	82.3	82.3	82.3
2.0	7.5	7.0	0.03	0.2	0.2	15	59.8	50.9	65.0

This table compares the optimal capital structure in terms of the leverage (in percentage) for various parameters among models of Leland, Anderson-Sundaresan-Tychon (AST), and ours without the cash flow covenant.

Tax is a crucial issue in determining the optimal capital structure. The interactions with liquidation costs for existing models are shown in Figure 13. Both AST and our model can prevent liquidations through debt renegotiation. Optimal capital structure is only an issue of how to maximally enjoy the tax benefits. On the one hand, the firm can receive more tax benefits by

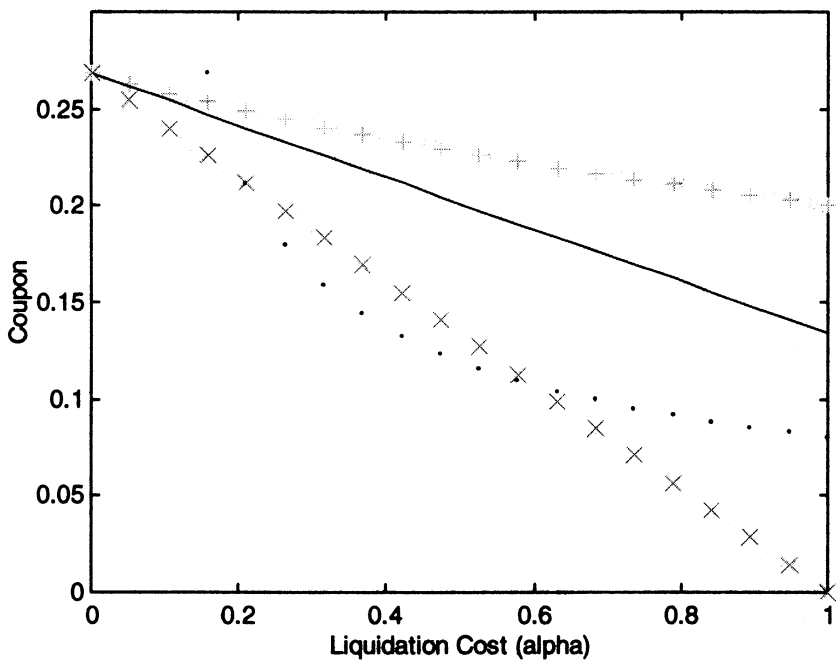


Figure 12
Debt capacity

The lines here plot the debt capacity at varying levels of bankruptcy cost (proportional cost only, i.e., $K = 0$), for different models: AST ("x" line), Leland ("+" line), and our model with cash flow covenant (dotted line) and without (solid line). It is assumed that the firm's asset value is 2, the coupon level is 10%, the risk-free interest rate is 7.5%, the payout ratio is 7%, the volatility is 17.3%, the bargaining power of equity holders is 0.5, and the corporate tax benefit is 35%.

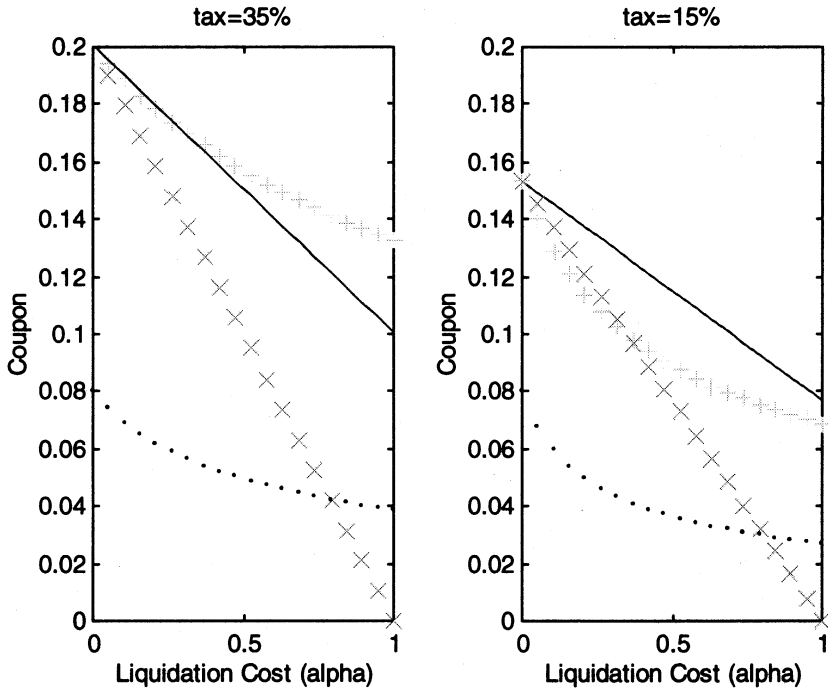


Figure 13
Optimal capital structure

The lines plot the optimal capital structure at varying levels of bankruptcy cost (proportional cost only) with two levels of tax benefit for different models: AST ("x" line), Leland ("+" line), and our model with the cash flow covenant (dotted line) and without (solid line). It is assumed that the firm's asset value $V = 2$, the coupon value $c = 10\%$, the risk-free interest rate is 7.5% , the payout ratio is 7% , the volatility is 17.3% , the bargaining power of equity holders is 0.5 , and the corporate tax benefit is 35% in the left panel and 15% in the right panel.

bearing more debt. On the other hand, higher leverage leads to earlier debt renegotiation where tax benefits are lost also. Since the trigger point in our model is always less than in the AST model, we predict a higher optimal leverage than the AST model due to the fact that tax benefits are availed for a longer period in our model.

Liquidations cannot be avoided in the Leland (1994) model with or without the cash flow covenant. The trade-off with more debt is between more tax benefits and higher potential liquidation costs. The bankruptcy value is higher with the cash flow covenant than without. It then results in lower optimal leverage in the presence of the cash flow-based covenant.

5. Conclusions

We provide a framework of debt renegotiation in which the bargaining powers of equity holders and debt holders can be varied to examine their effects on

reorganization boundaries, payout policies, and spreads. We explicitly considered two formulations of debt renegotiation: debt-equity swaps and strategic debt service (at a certain threshold, equity holders will strategically service the debt by paying less than the contractual coupon). The threshold value and the interest reduction amount are endogenously solved. We also allow dividend reinvestment to the firm's value-generating process. Equity holders always pay themselves the maximal available cash flows as dividends when they can optimally default on the debt. In the presence of a cash flow-based bond covenant, equity holders may voluntarily cut the dividend payment just enough to avoid hitting the covenant inefficiently. An innovation in our article is the fact that the object of bargaining is the value of the firm, which is in itself endogenous due to future reorganizations which results in uncertain tax shields.

Our framework might be extended to address the following questions: What happens when equity holders can reinvest the retained earnings into a new project? This risk shifting is clearly a topic of much interest. How does the bargaining game change when there are multiple classes of debt such as bank loan and public debt? What would be the dividend policy when considering the possibility of external financing and the signaling effect of dividend reduction? Debt valuation models based on the firm's asset value are often criticized on the grounds that they cannot be implemented since the firm's underlying asset value and its volatility are not observable. This criticism is no longer as effective since firms such as KMV already use versions of value-based models to estimate probabilities of default. In this context, estimation of default probabilities, recovery rates, and default premiums of risky debt contracts would be of great interest to practitioners. In addition, the modeling of a stochastic term structure of risk-free interest rates is clearly another possible area for further research.

Appendix

Debt-equity swap with fixed cost

The lower boundary conditions now become more complicated:

$$E(V_S) = \begin{cases} \eta(\alpha V_S + K), & \text{when } V_S \geq K/(1 - \alpha); \\ \eta V_S, & \text{when } V_S < K/(1 - \alpha). \end{cases} \quad (18)$$

$$E_V(V_S) = \begin{cases} \eta\alpha, & \text{when } V_S \geq K/(1 - \alpha); \\ \eta, & \text{when } V_S < K/(1 - \alpha). \end{cases} \quad (19)$$

The trigger point V_S of debt renegotiation can be summarized next. There are three cases of trigger points corresponding to (i) $(1 - \alpha)V_S > K$, (ii) $(1 - \alpha)V_S < K$, and (iii) $(1 - \alpha)V_S = K$.

Result 1. [Case 1] When the residual value at the trigger level is positive, that is,

$$V_S^H := \frac{-\lambda_-}{1-\lambda_-} \left[\frac{c(1-\tau)}{r} + \eta K \right] \frac{1}{1-\eta\alpha} \geq \frac{K}{1-\alpha},^{16}$$

where

$$\lambda_- = \left[0.5 - \frac{(r-\beta)}{\sigma^2} \right] - \sqrt{\left[0.5 - \frac{(r-\beta)}{\sigma^2} \right]^2 + \frac{2r}{\sigma^2}} < 0, \quad (20)$$

contractual coupon payments are stopped being paid at $V_S = V_S^H$ for all $V < V_S^H$.

[Case 2] When the residual value at the trigger level is not strictly positive, that is, when $V_S^H < K/(1-\alpha)$ and

$$V_S^L := \frac{-\lambda_-}{1-\lambda_-} \frac{c(1-\tau)}{\eta r} \leq \frac{K}{1-\alpha},$$

the trigger point for debt-equity swap is $V_S = V_S^L$.

[Case 3] The third case occurs when $V_S^H < K/(1-\alpha) < V_S^L$, and the trigger point becomes $V_S = K/(1-\alpha)$.

Proof of Result 1. Equation (2) incorporating the upper boundary condition [Equation (3)] has the following general solution:

$$E(V) = V - \frac{c(1-\tau)}{r} + AV^{\lambda_-},$$

where A is a constant to be determined. Suppose $V_S \geq K/(1-\alpha)$, then applying the value-matching condition $E(V_S) = \eta(\alpha V_S + K)$ and smooth pasting condition $E_V(V_S) = \eta\alpha$, we obtain that V_S and A satisfy the following equations:

$$\begin{aligned} V_S - \frac{c(1-\tau)}{r} - AV_S^{\lambda_-} &= \eta(\alpha V_S + K), \\ 1 - A\lambda_- V_S^{\lambda_- - 1} &= \eta\alpha. \end{aligned} \quad (21)$$

Denote the solution as

$$V_S^H = \frac{-\lambda_-}{1-\lambda_-} \left[\frac{c(1-\tau)}{r} + \eta K \right] \frac{1}{1-\eta\alpha}. \quad (22)$$

When $V_S^H \geq K/(1-\alpha)$, denoted as in Case 1, the trigger point is $V_S = V_S^H$. The corresponding equity value is

$$\begin{aligned} E(V) &= V - \frac{c(1-\tau)}{r} - \left[(1-\eta\alpha)V_S - \eta K + \frac{c(1-\tau)}{r} \right] \left(\frac{V}{V_S} \right)^{\lambda_-} \\ &= V - \frac{c(1-\tau)}{r} \left[1 - \left(\frac{V}{V_S} \right)^{\lambda_-} \right] - [(1-\eta\alpha)V_S - \eta K] \left(\frac{V}{V_S} \right)^{\lambda_-}, \end{aligned}$$

which can be interpreted as

$$E(V) = V - \frac{c(1-\tau)}{r} (1 - P_S) - D(V_S) P_S,$$

where $P_S = (V/V_S)^{\lambda_-}$ is the risk-neutralized probability of default. In fact, as the trigger point V_S changes, this formula still holds all the other cases discussed next.

¹⁶ A sufficient condition for this is $K \leq \frac{-2\lambda_-}{2-\lambda_-} \frac{c(1-\tau)}{r}$.

If $V_S^H < K/(1 - \alpha)$, suppose that $V_S < K/(1 - \alpha)$, then $E(V_S) = \eta V_S$, $E_V(V_S) = \eta$. We obtain the following:

$$\begin{aligned} V_S - \frac{c(1 - \tau)}{r} - A V_S^{\lambda_-} &= \eta V_S, \\ 1 - A \lambda_- V_S^{\lambda_- - 1} &= \eta. \end{aligned} \quad (23)$$

The solution is:

$$V_S^L := \frac{-\lambda_-}{1 - \lambda_-} \frac{c(1 - \tau)}{\eta r}. \quad (24)$$

If $V_S^L \leq K/(1 - \alpha)$, then the trigger point for debt-equity swap becomes $V_S = V_S^L$. This is called Case 2.

If $V_S^H < K/(1 - \alpha) < V_S^L$, then $V_S = K/(1 - \alpha)$. Because at $K/(1 - \alpha)$, the trigger point in the “high-value” region V_S^H cannot be reached, while the “low-value” region trigger point V_S^L has already been passed by, and the equity holders best response is to start paying less coupon right away. Let’s denote this as Case 3. Here it is sufficient to determine the equity value by the differential equation and the value-matching condition. The smooth pasting condition is not satisfied.

Note that when the fixed liquidation cost is $K = 0$, everything collapses to Case 1, since $V_S^H \geq 0$ is always satisfied. Cases 2 and 3 matter only when the fixed cost is significantly high. Result 1 provides a more balanced view of corporate debt in comparison to Leland (1994) and Anderson and Sundaresan (1996). Leland’s model does not allow for private debt workout and allows equity holders to remain in control until liquidation occurs. In contrast, our model suggests that liquidation will not occur and debt holders will work out their claims at a much earlier stage. The model of Anderson and Sundaresan assigns all the bargaining power to the equity holders and the outcome is based on a take-it or leave-it behavior. In their model, when the residual liquidation value is zero, all cash flows are paid as dividends and no coupons are paid to debt holders for the region $V \in (0, K)$. In this region, equity holders still retain control; though usually no dividend is distributed when the firm is under financial distress.

To solve for the debt value, we can write down the differential equations with boundary conditions similar to those for equity value. Analytical valuation formulas for debt are shown next.

Result 2. The value of debt is given by:

[Case 1] When $V_S = V_S^H$,

$$D(V) = \begin{cases} \frac{c}{r} - \left(\frac{c}{r} + \eta K \right) \frac{1 - \tau \lambda_-}{1 - \lambda_-} \left(\frac{V}{V_S^H} \right)^{\lambda_-}, & \text{when } V > V_S^H; \\ \eta V + \eta \max[0, (1 - \alpha)V - K], & \text{when } V \leq V_S^H. \end{cases}$$

[Case 2] When $V_S = V_S^L$, the debt value is

$$D(V) = \begin{cases} \frac{c}{r} - \frac{c}{r} \frac{1 - \tau \lambda_-}{1 - \lambda_-} \left(\frac{V}{V_S^L} \right)^{\lambda_-}, & \text{when } V > V_S^L; \\ \eta V, & \text{when } V \leq V_S^L. \end{cases}$$

[Case 3] When $V_S = K/(1 - \alpha)$ the debt value is given by

$$D(V) = \begin{cases} \frac{c}{r} - \left[\frac{c}{r} - \frac{\eta K}{(1 - \alpha)} \right] \left[\frac{V}{K/(1 - \alpha)} \right]^{\lambda_-}, & \text{when } V > K/(1 - \alpha); \\ \eta V, & \text{when } V \leq K/(1 - \alpha). \end{cases}$$

Note that when the value of the firm is below the trigger level V_S^H in Case 1, the debt value is simply η times the sum of the firm's asset value and any residual value. In the region where the value of the firm is above the trigger level, the value of debt is the expected value of the cash flows using the endogenous default probabilities as shown: $D(V) = (1 - P_S)c/r + P_S D(V_S^H)$, where $P_S = (V/V_S^H)^{\lambda_-}$. Case 1 is the usual situation since the fixed liquidation costs are rarely sufficiently high.

In Case 2 we encounter a very high fixed cost of liquidation. Once again, the interpretation of the debt value is simple: when the value of the firm is below the threshold level V_S^L , the residual value is zero and it is optimal for the debt holders and the equity holders to divide up the firm equally if they have equal bargaining power. When the firm value improves beyond the threshold level V_S^L , the value of debt is simply the expected value of the discounted cash flows as shown next: $D(V) = (1 - P_S)c/r + P_S D(V_S^L)$ where the default probability is $P_S = (V/V_S^L)^{\lambda_-}$ and the debt value upon default is $D(V_S^L) = \eta V_S^L$.

In Case 3, the trigger occurs when the liquidation value reaches zero for the first time. Below the trigger, the cash flows are equally shared and above the trigger, contractual payments are made. The debt values in the two regions do not satisfy the smooth pasting condition in Case 3.

Note that the game proposed here implies that in times of financial distress, debt holders make some concessions to equity holders, which has the flavor of deviations from absolute priority rule.

Proof of Lemma 1. Let us denote the total value of the firm as $v(V)$. Since in equilibrium, liquidation can always be avoided through costless renegotiation, the total value of the firm satisfies the following differential equations for any given renegotiation trigger V_S :

$$\frac{1}{2}\sigma^2 V^2 v_{VV} + (r - \beta)V v_V - rv + \beta V + \tau c = 0, \text{ when } V > V_S;$$

$$\frac{1}{2}\sigma^2 V^2 v_{VV} + (r - \beta)V v_V - rv + \beta V = 0, \text{ when } V \leq V_S,$$

with boundary conditions

$$\lim_{V \uparrow \infty} v(V) = V + \frac{\tau c}{r} \quad (25)$$

$$\lim_{V \downarrow V_S} v(V) = \lim_{V \uparrow V_S} v(V) \quad (26)$$

$$\lim_{V \downarrow V_S} v_V(V) = \lim_{V \uparrow V_S} v_V(V) \quad (27)$$

$$\lim_{V \downarrow 0} v(V) = 0 \quad (28)$$

From the differential equations and boundary conditions of Equations (25) and (28), we have

$$v(V) = \begin{cases} V + \frac{\tau c}{r} + A_1 \left(\frac{V}{V_S} \right)^{\lambda_-}, & \text{when } V > V_S; \\ V + A_2 \left(\frac{V}{V_S} \right)^{\lambda_+}, & \text{when } V \leq V_S, \end{cases} \quad (29)$$

where A_1 and A_2 are constants to be determined; λ_+ and λ_- are given next:

$$\lambda_{\pm} = 0.5 - \frac{r - \beta}{\sigma^2} \pm \sqrt{\left(\frac{r - \beta}{\sigma^2} - 0.5\right)^2 + \frac{2r}{\sigma^2}} < 0. \quad (30)$$

Substituting Equation (29) into the boundary conditions of Equations (26) and (27),

$$\begin{aligned} \frac{\tau c}{r} + A_1 &= A_2, \\ A_1 \lambda_- &= A_2 \lambda_+, \end{aligned}$$

and solving for A_1 and A_2 , we obtain $v(V)$, as shown in Equation (4).

Strategic debt service with fixed liquidation cost

When $V \leq V_S$, equity and debt holders bargain over the total value of the firm, $v(V)$. Assume $\tilde{\theta}$ is the share that equity holders will receive upon renegotiation, then $1 - \tilde{\theta}$ is the share for debt holders, that is,

$$E(V) = \tilde{\theta}v(V), \quad D(V) = (1 - \tilde{\theta})v(V).$$

The Nash solution to the bargaining game can be characterized by the sharing rule:

$$\begin{aligned} \tilde{\theta}^* &= \arg \max \left\{ \tilde{\theta}v(V) - 0 \right\}^\eta \left\{ (1 - \tilde{\theta})v(V) - \max[(1 - \alpha)V - K, 0] \right\}^{(1-\eta)} \\ &= \min \left[\eta - \eta \frac{(1 - \alpha)V - K}{v(V)}, \eta \right]. \end{aligned}$$

The next step is to solve the trigger point $\tilde{V}_S$ and the strategic debt service amount $S(V)$. Equity value satisfies the following equations:

$$\frac{1}{2}\sigma^2 V^2 \tilde{E}_{VV} + (r - \beta)V \tilde{E}_V - r \tilde{E} + \beta V - c(1 - \tau) = 0, \quad \text{when } V > \tilde{V}_S; \quad (31)$$

and

$$\frac{1}{2}\sigma^2 V^2 \tilde{E}_{VV} + (r - \beta)V \tilde{E}_V - r \tilde{E} + \beta V - S(V) = 0, \quad \text{when } V \leq \tilde{V}_S, \quad (32)$$

with boundary conditions

$$\lim_{V \uparrow \infty} \tilde{E}(V) = V - \frac{c(1 - \tau)}{r}, \quad (33)$$

$$\lim_{V \downarrow \tilde{V}_S} \tilde{E}(V) = \eta \left(\alpha \tilde{V}_S + K + \frac{-\lambda_-}{\lambda_+ - \lambda_-} \frac{\tau c}{r} \right), \quad (34)$$

$$\lim_{V \downarrow \tilde{V}_S} \tilde{E}_V(V) = \eta \left(\alpha + \frac{-\lambda_+ \lambda_-}{\lambda_+ - \lambda_-} \frac{1}{\tilde{V}_S} \right). \quad (35)$$

Based on Equations (31) and (33),

$$\tilde{E}(V) = V - \frac{c(1 - \tau)}{r} + AV^{\lambda_-}, \quad (36)$$

where A is a constant. Substituting the boundary conditions of Equations (34) and (35) into Equation (36),

$$\begin{aligned}\tilde{V}_S - \frac{c(1-\tau)}{r} + A\tilde{V}_S^{\lambda_-} &= \eta \left(\alpha \tilde{V}_S + K + \frac{-\lambda_-}{\lambda_+ - \lambda_-} \frac{\tau c}{r} \right) \\ 1 - A\lambda_- \tilde{V}_S^{\lambda_-} &= \eta \left(\alpha + \frac{-\lambda_+ \lambda_-}{\lambda_+ - \lambda_-} \frac{1}{\tilde{V}_S} \right)\end{aligned}$$

we obtain

$$\tilde{V}_S = \frac{-\lambda_-}{1 - \lambda_-} \left[\frac{c}{r} (1 - \tau + \eta\tau) + \eta K \right] \frac{1}{1 - \eta\alpha}.$$

To solve for the strategic debt service amount $S(V)$, we substitute the equity value obtained from debt renegotiation into Equation (32),

$$S(V) = \begin{cases} (1 - \eta\alpha)\beta V - \eta r K, & \text{when } K/(1 - \alpha) \leq V \leq \tilde{V}_S, \\ \eta\beta V, & \text{when } V < K/(1 - \alpha). \end{cases} \quad (37)$$

Note here we assume $\tilde{V}_S > K/(1 - \alpha)$ for illustrative purpose. Other scenarios can be discussed similarly to the case of debt-equity swap with fixed cost.

Payments to debt holders will be partitioned into three regions: (1) For $V > \tilde{V}_S$ the contractual payment will be made. (2) As the value first reaches $\tilde{V}_S$ from above, strategic debt service is triggered. The amount of $(1 - \eta\alpha)\beta V - \eta r K$ is paid to debt holders as long as the residual value after liquidation remains positive. (3) For values of the firm which do not leave any, equity holders and debt holders divide up the cash flows equally between coupon and dividend payments so that $S(V) = \eta\beta V$. In a special case when $K = 0$, the noncontractual payment would be $S(V) = (1 - \eta\alpha)\beta V$ for $V \leq \tilde{V}_S$. In other scenarios with extremely high fixed costs of liquidation, no residual value is left for both equity and debt holders when the threshold level $\tilde{V}_S$ is reached and the strategic debt service $S(V) = \eta\beta V$. Eventually when the threshold level $\tilde{V}_S$ is reached again, the contractual coupon will be resumed.

Lemma 2. *There exists a solution, which is the endogenous reorganization boundary V_B , to the following nonlinear relationship, for any given payout ratio $\delta \leq \beta$,*

$$(\lambda_- - \gamma_-) \left(\frac{c}{\beta} \right)^{\lambda_-} V_B^{\lambda_+ - 1} \left[V_B(\lambda_+ - 1) - \frac{c(1-\tau)}{r} \lambda_+ \right] \quad (38)$$

$$- (\lambda_+ - \gamma_-) \left(\frac{c}{\beta} \right)^{\lambda_+} V_B^{\lambda_- - 1} \left[V_B(\lambda_- - 1) - \frac{c(1-\tau)}{r} \lambda_- \right] = 0, \quad (39)$$

where

$$\lambda_{\pm} = 0.5 - \frac{r - \beta}{\sigma^2} \pm \sqrt{\left(\frac{r - \beta}{\sigma^2} - 0.5 \right)^2 + \frac{2r}{\sigma^2}} > 0,$$

$$\gamma_- = 0.5 - \frac{r - \delta}{\sigma^2} - \sqrt{\left(\frac{r - \delta}{\sigma^2} - 0.5 \right)^2 + \frac{2r}{\sigma^2}} < 0.$$

Proof of Lemma 2. Following the same approach as in Section 2, differential Equations (6) and (7) with their boundary conditions yield

$$E^G(V) = V - c(1 - \tau)/r + A_- V^{\gamma_-},$$

when $V \geq \frac{c}{\beta}$,

$$E^B(V) = V - c(1 - \tau)/r + B_+ V^{\lambda_+} + B_- V^{\lambda_-},$$

when $V_B \leq V < \frac{c}{\beta}$,

where γ_- is given in Equation (12). A_- , $B_\pm$, and V_B satisfy the following equations:

$$V_B - c(1 - \tau)/r + B_+ V_B^{\lambda_+} + B_- V_B^{\lambda_-} = 0, \quad (40)$$

$$1 + B_+ \lambda_+ V_B^{\lambda_+-1} + B_- \lambda_- V_B^{\lambda_- -1} = 0, \quad (41)$$

$$A_-(c/\beta)^{\gamma_-} = B_+(c/\beta)^{\lambda_+} + B_-(c/\beta)^{\lambda_-}, \quad (42)$$

$$A_- \gamma_- (c/\beta)^{\gamma_- -1} = B_+ \lambda_+ (c/\beta)^{\lambda_+ -1} + B_- \lambda_- (c/\beta)^{\lambda_- -1}. \quad (43)$$

Solving $B_\pm$ in terms of V_B by Equations (41)–(43),

$$B_\pm = \frac{\pm(\lambda_\mp - \gamma_-)(c/\beta)\lambda_\mp}{\Delta}, \quad (44)$$

where

$$\Delta = (\lambda_+ - \gamma_-)\lambda_- V_B^{\lambda_- -1} (c/\beta)^{\lambda_+} - (\lambda_- - \gamma_-)\lambda_+ V_B^{\lambda_+ -1} (c/\beta)^{\lambda_-}. \quad (45)$$

Substituting $B_\pm$ back into Equation (40) gives the result.

To show the existence of V_B , define functions $f(\cdot)$ and $g(\cdot)$ as

$$\begin{aligned} f(V_B) &= (\lambda_- - \gamma_-) \left(\frac{c}{\beta}\right)^{\lambda_-} V_B^{\lambda_- -1} \left[V_B(\lambda_+ - 1) - \frac{c(1 - \tau)}{r} \lambda_+ \right], \\ g(V_B) &= (\lambda_+ - \gamma_-) \left(\frac{c}{\beta}\right)^{\lambda_+} V_B^{\lambda_+ -1} \left[V_B(\lambda_- - 1) - \frac{c(1 - \tau)}{r} \lambda_- \right]. \end{aligned} \quad (46)$$

For all $\delta < \beta$, $\lambda_- - \gamma_- > 0$. Notice $\lambda_+ > 1$ and

$$\lim_{V_B \uparrow \infty} f(V_B) = +\infty, \quad \lim_{V_B \downarrow 0} f(V_B) = 0, \quad (47)$$

$$\lim_{V_B \uparrow \infty} g(V_B) = 0, \quad \lim_{V_B \downarrow 0} g(V_B) = +\infty. \quad (48)$$

Since $f(\cdot)$ and $g(\cdot)$ are both continuous functions, there exists some positive V_B satisfying Equation (39).

Proof of Proposition 4. Solving the optimal dividend policy is actually finding the $\delta(\cdot)$ that solves the following problem:

$$\max_{c(1-\tau)/V \leq \delta \leq \beta} E^G(V; \delta(\cdot)). \quad (49)$$

It is equivalent to $\max_{c(1-\tau)/V \leq \delta \leq \beta} L$, where

$$L = \frac{1}{2} \sigma^2 V^2 E_{VV}^G + (r - \delta) V E_V^G - r E^G + [\delta V - c(1 - \tau)]. \quad (50)$$

Since $\partial L / \partial \delta = V(1 - E_V^G)$, it is sufficient to show that $E_V^G < 1$ for all $V > c/\beta$. From the results in the previous lemma, $E_{VV}^G = A_- \gamma_- (\gamma_- - 1)$, we know that E_V^G is monotonic in V . As $\lim_{V \uparrow \infty} E_V^G = 1$, $\lim_{V \downarrow c/\beta} E_V^G = \lim_{V \uparrow c/\beta} E_V^B$, and $\lim_{V \downarrow V_B} E_V^B = 0$, it is sufficient to show that $\lim_{V \downarrow c/\beta} E_V^G < 1$.

If there exists a δ such that $\lim_{V \downarrow c/\beta} E_V^G > 1$, then $E_V^G(V; \delta) < c/\beta < E_V^G(V; \beta)$. δ would not be optimal. Therefore $E_V^G < 1$ for all $V > c/\beta$. We can conclude that E_V^G is monotonically increasing in δ . $\delta^* = \beta$.

When the firm is in a "bad" state, equity holders are not allowed to receive any dividend.

Proof of Equation (10). The differential equation is the same as in Merton (1974) and Black and Cox (1976):

$$\frac{1}{2}\sigma^2 V^2 D_{VV} + (rV - c)D_V - rD + c = 0. \quad (51)$$

With upper boundary condition $\lim_{V \uparrow \infty} D(V) = c/r$ and lower boundary condition $\lim_{V \downarrow c/\beta} D(V) = (1 - \alpha)c/\beta - K$, define $Z = 2c/\sigma^2 V$, $R = 2r/\sigma^2$, and $D(V) = c/r + Z^R e^{-Z} h(Z)$. Equation (51) reduces to Kummer's equation:

$$Zh_{ZZ} + (2 + R - Z)h_Z - 2h = 0,$$

which has a general solution of the form

$$C_1 M(2, 2 + R, Z) + C_2 Z^{-1-R} M(1 - R, -R, Z),$$

where C_1 and C_2 are constants to be determined. By the property $e^{-Z} \times M(a, b, Z) = M(b - a, b, -Z)$ of the hypergeometric function,

$$\begin{aligned} D(V) &= c/r + C_1 (2c/\sigma^2) V^{-2r/\sigma^2} M(2r/\sigma^2 V, 2 + 2r/\sigma^2, -2c/\sigma^2 V) \\ &\quad + C_2 (\sigma^2 V/2c) M(-1, -2r/\sigma^2 V, -2c/\sigma^2 V). \end{aligned}$$

By the upper boundary condition, $C_1 = 0$. The lower boundary condition determines C_2 , which results in Equation (10).

Proof of Proposition 5. For a given firm's payout ratio δ ,

$$E(V; \delta) = V - \frac{c(1 - \tau)}{r} + \left[\frac{c(1 - \tau)}{r} - \frac{c}{\beta} \right] \left(\frac{c}{\beta V} \right)^{-\gamma_-}, \quad (52)$$

where γ_- , which depends on δ , is given in Equation (12). Now $c(1 - \tau)/r - c/\beta < 0$, and maximizing equity value by choosing a feasible payout ratio δ between $c(1 - \tau)/V$ and β is equivalent of minimizing γ_- when δ falls in the same interval. And

$$\frac{\partial \gamma_-}{\partial \delta} = \frac{1}{\sigma^2} \left[1 - \frac{\sigma^2/2 - r + \delta}{\sqrt{(\sigma^2/2 - r + \delta)^2 + 2r\sigma^2}} \right] > 0. \quad (53)$$

The optimal payout ratio is $c(1 - \tau)/V$, or equivalently it is not optimal to pay dividend.

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