

The Design of Bank Loan Contracts

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The unique characteristics of bank loans emerge endogenously to enhance efficiency in a model of renegotiation between a borrower and a lender in which there is the potential for moral hazard on each side of the relationship. Firm risk is endogenous and renegotiated interest rates on the debt need not be monotone in firm risk. The initial terms of the debt are not set to price default risk but rather are set to efficiently balance bargaining power in later renegotiation. Loan pricing may be nonlinear, involving initial transfers either from the borrower to the bank or from the bank to the borrower.

Empirical work strongly suggests that bank loans are different from corporate bonds.¹ This evidence has spawned a number of hypotheses about exactly what banks do to make themselves valuable. These theories have stressed various kinds of screening and monitoring of borrowers. In this article we argue that the interesting and valuable functions of banks occur between the time they make a loan and collect repayment. We focus on banks' ability to renegotiate credit terms with borrowers, and on the tight link between that renegotiation and monitoring. Our model shows how the unique characteristics of bank loans emerge endogenously to enhance efficiency. These characteristics include seniority (i.e., the bank has first claim on the assets of the borrower in the event of default); an option for the bank to liquidate the loan at any time (perhaps in the form of very tight covenants); and an initial loan rate set not to price the risk of default, but to minimize subsequent costs associated with moral hazard and renegotiation. As a consequence of this last feature, initial loan pricing may involve transfers—either from the

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¹ For example, James (1987) finds a positive and significant abnormal stock response to firms announcing the signing of bank loan agreements. Also see Lummer and McConnell (1989). Hoshi, Kashyap, and Scharfstein (1990) find that Japanese firms in financial distress that are members of a "main-bank" coalition (*keiretsu*) invest and sell more after the onset of distress than do distressed firms that are not members of a bank coalition. Other evidence includes Gilson, John, and Lang (1990) and Slovin, Sushka, and Polonchek (1993).

borrower to the bank (in the form of up-front fees, compensating balance requirements, or the purchase of other bank services), or from the bank to the borrower with the bank providing underpriced services to the borrower. The model also predicts that firm risk is endogenous and state dependent, and that renegotiated interest rates on the debt need not be monotone in firm risk.

While loans and bonds are both debt contracts, we take the defining feature of bank loans to be that they are typically held by a single creditor. We will argue that this feature makes “monitoring” of the borrower both feasible and desirable. Bonds, on the other hand, are (by definition) held by dispersed creditors for whom monitoring and renegotiation are relatively costly because of free-rider problems.² We incorporate this distinction in a model of renegotiation between a borrower and a bank lender. Renegotiation of the contract terms is triggered by the arrival of new information that may lead the borrower to add inefficient risk to the project (i.e., asset substitution) absent changes in the terms of the loan. There is also the potential for moral hazard on the part of the bank since the bank may “hold up” the borrower by (credibly) threatening to liquidate the borrower’s project, thereby extracting a higher interest rate.

The interplay between the two moral hazard problems leads to a number of outcomes to renegotiation. The bank may liquidate the project, raise the interest rate, forgive some of the debt, or stay with the status quo. “Monitoring” the borrower can be interpreted to mean liquidating inefficient projects and renegotiating lower interest rates to prevent borrower risk taking. But we show that in renegotiation the bank is not always successful in preventing the borrower from taking on additional risk. Sometimes the bank allows a borrower to continue with the project even though the borrower chooses to add risk to the project. In equilibrium, the variance of the value of the borrowing firm is therefore endogenously time and state dependent. Because the bank can only succeed in preempting risky behavior in the moderately distressed cases by writing off some of the debt or lowering the rate, renegotiation results in renegotiated interest rates that are not monotone in borrower quality: the healthiest borrowers are left alone, the moderately distressed are granted concessions, while the most distressed are forced to submit to harsher terms.

The contract design problem involves a number of considerations, each of which we address. First, there is the question of whether renegotiation is desirable. In other words, is it efficient for the borrower to obtain funds from a bank, as opposed to obtaining funds from agents who cannot renegotiate? Answering this question involves comparing the outcomes of obtaining funds

² Our model is consistent with any secured debt-holder who has sufficient bargaining power to renegotiate with a borrower (and we do not take a stand on how large a position this requires). Typically banks are single lenders, making renegotiation practical. Kahan and Tuckman (1993) argue that firms do have mechanisms at their disposal to negotiate with decentralized bondholders, but they are potentially costly to shareholders.

from a single lender, such as a bank, to the alternative of issuing bonds to dispersed lenders. Issuing bonds commits the firm and its creditors not to renegotiate. The second design issue concerns the contract with the bank, if funds are obtained from a bank. Here the question is whether the contract should include a provision which allows the bank to ask for the collateral prior to maturity of the loan (even if the borrower has not missed a payment). We assume that the contract can feasibly include the liquidation option which allows the bank to “call the loan” at any time, and we ask whether it is optimal to include this provision.

If the liquidation option is included, then the third contract design consideration involves the specification of the initial contract form, considering that both parties know that at an interim date the contract can be renegotiated upon the arrival of new information. While we assume that if the project continues at the interim date it must do so under a debt contract that matures at a final date, this does not determine the optimal form of the initial contract, since the borrower and the lender know that any initial contract will subsequently be renegotiated. The outcome of the renegotiation has efficiency considerations, since some projects will be liquidated by the bank, while others will become riskier (when borrowers add risk). The social gain from bank loans comes from the enhanced ability to thwart inefficient risk taking and to liquidate bad projects. Because the bank may liquidate too frequently, however, the net value of bank loans rests on the costs of excessive liquidation being small relative to the costs of excessive continuation. We show how the terms of the initial contract affect the renegotiation outcome by allocating bargaining power between borrower and lender to minimize inefficient risk taking.

Our model identifies a unique role for bank loans that is independent of pricing default risk. The initial equilibrium interest rate on loans does not primarily reflect a default premium. Rather it is the rate that results *ex ante* in minimal expected asset substitution by borrowers following renegotiation. The loan is certain to be renegotiated, and the outcome of bargaining between the two parties is partly determined by the bank’s threat to liquidate. But the credibility of this threat depends in part on the amount owed to the bank. Intuitively, the amount owed must be high enough so that the bank will not be overly tempted to hold up the borrower for higher payments and thereby induce excessive risk taking, but not so high that the bank would be insufficiently willing to forgive some of the debt in order to discourage excessive risk taking. Given such considerations, there is no guarantee that the loan rate that minimizes these expected agency costs will result in zero expected profits for lenders. Consequently, competition by banks can result in nonlinear pricing arrangements for loans such as origination fees or cross-subsidization with other products, as are often observed. Previous explanations of the structure of bank loan pricing have relied on screening in asymmetric information environments [e.g., Thakor and Udell (1987)].

Our results are related to the literature on the role of banks, including Sharpe (1990), Rajan (1992), and Detragiache (1994). In the models of Sharpe and Rajan, banks learn private information about borrowers and are able to exploit this information to hold up borrowers. We include this moral hazard on the part of the banks and, in addition, include moral hazard by the borrower. In Detragiache's model renegotiation is beneficial, but can lead to ex ante risk taking by the borrower. Her focus is on alternative bankruptcy regimes.

Our article is also related to the literature on the role of banks as ex post monitors, which views banks' primary role as verifying reported (and otherwise unobservable) output in settings with costly state verification [e.g., Diamond (1984)]. This theory cannot explain observed interaction between banks and borrowers during the life of the contract. Moreover, the role of banks as ex post monitors suggests that banks should be junior claimants (and perhaps equity claimants) because their incentive to monitor would then be strongest.³ Fama (1985) argues that this is the case. But in fact, banks are typically senior, secured claimants. It seems difficult to reconcile this feature of bank loans with the bank's role as ex post monitor. Our model addresses this issue.

The model is specified in Section 1. Section 2 provides preliminary results and definitions of payoffs. Section 3 looks at the renegotiation and liquidation decisions predicted by the model. Section 4 examines the initial pricing of the loan and the role of debt. Section 5 discusses the results, and Section 6 contains some final remarks.

1. The Borrowing and Lending Environment

There are four dates, $t = 0, 1, 2, 3$, in the model economy and two representative risk-neutral agents: a borrowing firm and a lender (which we will call the "bank"). A summary of the model is as follows. The borrowing firm has a project which requires some external financing: at date $t = 0$ the firm obtains funding from a competitive bank. The funding is governed by a contract that matures at date $t = 2$. At $t = 1$, before the contract matures, some news arrives about the firm's future project payoffs. The new information is observed by both the bank and the borrower, but it is not verifiable. Based on this information, and in particular if there is bad news, the borrower may choose to take a costly risk-increasing action. The contract may allow the bank to demand the collateral at this time (or, synonymously, the project liquidation value) instead of waiting for the contract to mature at date $t = 2$. Also at $t = 1$ the two parties may renegotiate the terms of the contract. Whether the borrower expends resources to add risk to the project, or whether instead the bank ends the contract early by seizing the collateral, depends on the outcome of renegotiation. Finally, if the project is not

³ In costly state verification models the value of the borrower is not known until monitoring takes place. Thus, even if the bank's junior claim is worthless, the bank does not know this until it monitors.

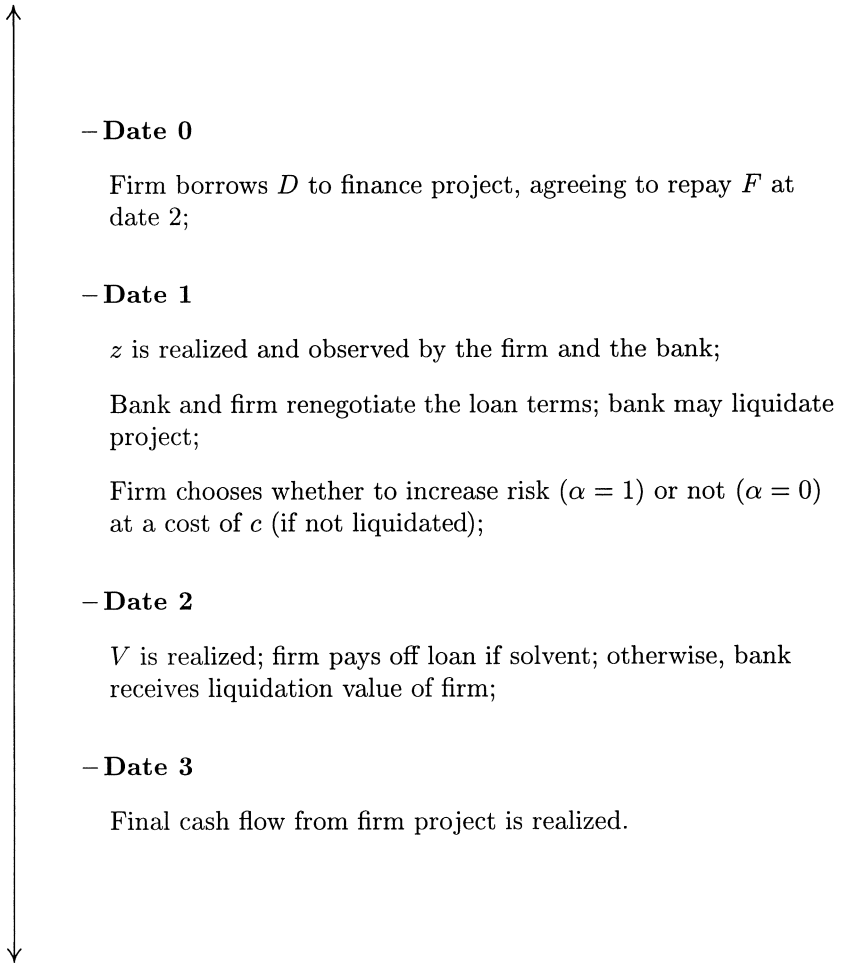


Figure 1

Sequence of Events

liquidated at $t = 1$, then at $t = 2$ the borrower repays the loan or is liquidated. If the borrower's project is not liquidated at $t = 2$, then a final payoff is received at $t = 3$. Figure 1 shows the timing of the model and Table 1 provides a concise summary of notation and definitions for future reference.

1.1 Detailed assumptions of the model

1.1.1 Projects and borrowers. The borrower's project requires a fixed scale of investment which, without loss of generality, we will set to one. The borrower has an amount $1 - D$ available to invest, but must obtain the

Table 1

Summary of Some Notation

$y_2(z)$	Cash flow from the project at $t = 2$ for borrower type z
x_t	Scale of the project at time t ($t = 0, 1$; $x_0 = 1$)
D	Initial amount borrowed
F_0	Initial face value of the debt
V	Value of the project to the entrepreneur as of $t = 2$
L_t	Liquidation value of the project to the lender ($t = 1, 2$)
F^N	Renegotiated face value of debt at $t = 1$
α	Indicator variable for switching to riskier project, $\alpha \in \{0, 1\}$
c	Cost of switching to the riskier project
π^T	Total expected value of the project at $t = 1$
π^R	Expected bank profit with renegotiation
π^U	Expected bank profit absent renegotiation
z^*	$\inf\{z \Gamma(z) \geq 0, \text{ given } F\}$, that is, threshold for switching to risky project given initial contract
z^{**}	$\inf\{z \Gamma(z) \geq F^N\}$, that is, threshold for switching projects given renegotiated contract
z_{RN}	$\inf\{z \pi^U \geq L_1, \text{ for all } F^N \leq F\}$, threshold for liquidation to be a credible threat
z_{EL1}	$\inf\{z \pi^T \geq L_1, \alpha = 0\}$, threshold for efficient liquidation absent switching projects
z_{EL2}	$\inf\{z \pi^T \geq L_1, \alpha = 1\}$, threshold for efficient liquidation given switching to the riskier project
z_{IL}	$\inf\{z \pi^R \geq L_1\}$, threshold for liquidation to be profit maximizing for the bank
$F^{++}(z)$	value of $F^N(z)$ that maximizes $\pi^R(F^{++}, z, \alpha = 1)$
$F^-(z)$	value of $F^N(z) > F$ that maximizes π^R given $\alpha = 1$

remainder, D , externally. The project generates cash flow realizations at dates $t = 2$ and $t = 3$ of $y_2(z)$, and $V(z)$, respectively, where z is the borrower type realized at $t = 1$. We refer to V as the project value, ignoring any liquidation possibilities (see below), and usually suppressing the dependence on z . For simplicity we assume a required rate of return of zero. The value V has a probability distribution given by $G(V; z, \alpha)$, where z , interpreted as “news” or borrower “type,” is a random variable whose value is realized at $t = 1$, and where α indexes the project that the borrower selects at $t = 1$ (i.e., whether risk is added to the project). $G(V; z, \alpha)$ is continuous and differentiable in V and z and has bounded support, $[V_l, V_h]$. We assume that:

Assumption 1. Higher values of z represent “good news” in the sense that the conditional distribution of $f(z|V)$ exhibits the monotone likelihood ratio property (MLRP), that is, $f(z|V)/f(z|V^*)$ is monotone in z , increasing if $V > V^*$, and decreasing otherwise [see Milgrom (1981)].

The random variable z , realized at $t = 1$, has density $h(z)$ and support $[z_l, z_h]$. We will refer to z as the borrower “type.”

1.1.2 Liquidation values. The project value as of $t = 2$, V , is to be interpreted as the net present value of the project when it is in the hands of the borrower who is assumed to have some special expertise relative to the bank. If the bank becomes the owner of the project, then it is worth a different value, the “liquidation” value or “collateral” value. Liquidation at date t means that the project yields L_t at that date in lieu of any future payoffs subsequent to the liquidation date. For simplicity, we assume:

Assumption 2. *Liquidation is all or nothing; liquidation values are certain and verifiable by both parties. Also, $D > L_1 > L_2$.*⁴

The last part of Assumption 2 says that the project requires outside financing in an amount that exceeds its liquidation value at any point in time so fully secured debt is not feasible.⁵

1.1.3 Asset substitution by the borrower. At $t = 1$ the borrower having received news, z , has the ability to unilaterally add risk to his project at a cost to the expected project return of c : adding risk reduces both V and L_2 by the amount c .⁶ Adding risk, referred to as “asset substitution,” is denoted by the discrete variable α (which equals 1 if the additional risk is taken and 0 otherwise).

Assumption 3. *Additional riskiness takes the form of a mean preserving spread:*

$$V_1 = V_0 + \epsilon$$

where V_α is the value of the project given choice α , and where $E(\epsilon|V_0) = 0$.

We denote the distribution of ϵ by $K(\epsilon)$ and the density by $k(\epsilon)$. The support of ϵ is $[\epsilon_l, \epsilon_h]$.

Assumption 4. $V_0 + \epsilon_l \geq c$.

Assumption 4 says that adding risk is always feasible; the borrower can always pay the cost c out of the project value when $\alpha = 1$ is chosen.

1.1.4 The contracting environment. The contracting environment is as follows:

Assumption 5. *The following are observable, but not verifiable: the borrower's project choice at $t = 1$, α ; the project value, V ; the realization of the borrower type, z ; and the realized cash flow $y_2(z)$.*

Assumption 5 means that contracts can only be made contingent on the $t = 1$ liquidation value and payments by the borrower to the lender. These variables are observable by all parties, in particular, third-party contract enforcers.

⁴ The assumption that liquidation is all or nothing is without loss of generality since partial liquidation is never optimal in any case. We prove this in Gorton and Kahn (1992). Also, note that if $L_1 = L_2$, then the bank can never be worse off by allowing the project to continue at $t = 1$ and, thus, will never liquidate the project at that date. The assumption that $L_1 > L_2$ implies that at earlier stages of the project liquidation is less costly, that is, more can be recovered.

⁵ We also assume that there is no choice concerning collateral; the borrower uses all the collateral that the project provides and has no other collateralizable resources.

⁶ The assumption that the liquidation value, L_2 , is also reduced by the amount c if risk is added ($\alpha = 1$) is not necessary, but appears (to us) to be realistic.

1.1.5 Contracts and renegotiation. A “bank” is distinguished from other providers of funds by:

***Assumption 6.** Among possible funds providers, only banks can renegotiate at $t = 1$.*

According to Assumption 6, a bond blockholder who could carry out renegotiation is labeled a “bank” for our purposes.⁷ Other fund providers are viewed as dispersed and incapable of coordinating renegotiating efforts. However, while we assume that an agent must be a bank in order to renegotiate, whether the contract includes the right to seize collateral prior to maturity is a separate issue.

***Assumption 7.** A contract can include a provision allowing for the lender to seize the borrower’s collateral at will at $t = 1$.*

We will call this contract provision the “liquidation option.” Since the lender must decide when to seize the borrower’s collateral, only banks would consider including this provision. This contract provision may be thought of as a reduced form for sufficiently detailed covenants that when violated allow the bank to demand collateral.⁸ Exercising the liquidation option is infeasible for other creditors because, by assumption, other lenders cannot renegotiate and hence cannot initiate liquidation. Combining Assumption 6 and Assumption 7 means that there are three distinct securities to consider: corporate bonds (dispersed holders who cannot renegotiate), and bank loan contracts with and without the liquidation option.

In order to most simply characterize the renegotiation outcomes at $t = 1$, we assume that:

***Assumption 8.** The bank can credibly make a take-it-or-leave-it offer at $t = 1$.*

***Assumption 9.** Borrowers have no alternative source of financing at the date of renegotiation, $t = 1$.*

The outcome of renegotiation at $t = 1$ will either be liquidation of the project or a contract specifying a payment to be made at $t = 2$ (either on new terms or at the status quo ante). Because cash flows are not verifiable, they

⁷ Thus the term “bank” is intended to apply to any agent who is the sole (or sufficiently large) lender to the borrowing firm and lends according to the contract we specify in the model. We do not intend the term to strictly apply to institutions chartered by the government, but rather to a broader class of agents, including so-called nonbank banks such as insurance companies, firms such as General Motors Acceptance Corporation, and agents who hold blocks.

⁸ In the United States, bank loan contracts contain detailed covenants which are easily violated, triggering the bank’s right to demand collateral even if the borrower has not missed a payment on the loan. In other countries, such as Japan, the loan contract is more straightforward in stating that the bank has the right to demand collateral any time.

can be consumed by the borrower; they cannot be seized by outside lenders, such as the bank, but may be handed over voluntarily by the borrower. In this setting Kahn (1992) shows that debt is an optimal contract.⁹ For the purposes of this article we assume that:

Assumption 10. *Debt is the optimal contract from $t = 1$ to $t = 2$. Failure to repay the debt at $t = 2$ triggers liquidation, that is, the parties are committed to liquidation if there is a default.*

In order to avoid liquidity problems, we assume that the cash flow at $t = 2$, $y_2(z)$, is sufficiently high, for all z , so that it is feasible to repay the lender at $t = 2$ if the borrower so chooses.

1.1.6 Opportunism by the bank. When the bank has the opportunity to threaten liquidation early (because this contract provision has been included) it may use this threat to simply extract surplus from the borrower. We will call this “opportunism.” Bank opportunism will sometimes have efficiency considerations. Let $\pi^R(F^N, z, \alpha)$ be the expected profits of the bank as of $t = 1$ after renegotiation has resulted in a new face value for the debt of F^N . (α is a function of F^N and z , but for clarity we include it as an argument of the expected profit function.) F^N could be higher or lower than the initial face value. If the bank can succeed in obtaining a higher rate, it faces a choice: raise the rate to maximize expected profit, accepting that the borrower will choose $\alpha = 1$ (call this rate F^{++}); or raise the rate to the highest level so that the borrower just chooses $\alpha = 0$ (call this rate F^+).

Assumption 11. $\pi^R(F^{++}, z, \alpha = 1) > \pi^R(F^+, z, \alpha = 0)$, for all z .

Assumption 11 means that bank opportunism has efficiency considerations since, if it can, the bank will renegotiate an interest rate which is so high that the borrower will add risk, even if the borrower would not add risk at the initial interest rate. Assumption 11 is not the only case in which bank opportunism will have efficiency considerations. Furthermore, it is not necessary for the analysis, but it is the most interesting case. The alternative assumptions are discussed further below and results for these cases are given in Appendix C.

1.1.7 Parameter restrictions. Appendix A details three further assumptions concerning parameters of the model. Assumption 12 ensures that adding risk always results in a positive probability of solvency. This assumption simply makes the problem interesting since it says that when risk is added there is always some chance for the borrower to benefit. Assumption 13 guarantees that the bank always prefers that the borrower not add risk. Again, this is the

⁹ Bolton and Scharfstein (1990) and Hart and Moore (1998) show the optimality of debt in similar settings.

interesting problem since otherwise the bank would not want to prevent asset substitution. Finally, Assumption 14 ensures that bank profits are increasing in the economically relevant range of F . The assumption allows us to ignore this issue of debt forgiveness (which has no efficiency considerations).

1.2 Discussion of the model

Renegotiation occurs when news, z , arrives and is observed by both parties to the contract. Bank loans include covenants which require the firm to supply regular accounting information and provide the bank with an opportunity to investigate the firm.¹⁰ Thus we view it as reasonable that the bank can observe z , which should be interpreted as new information about the firm's prospects that is not freely available to (or easily interpretable by) the public.

The timing of the model assumes that news (z) arrives before the cash flows. This is for simplicity. Since the loan cannot mature before sufficient cash flows from the project are realized, there is always potential for renegotiation during the course of the loan. We simply label the arrival of news and the consequent renegotiation as $t = 1$, but in principle these events can occur at any time prior to maturity, provided the borrower has time to add risk if he so chooses.

Renegotiation at $t = 1$ is complicated by two moral hazard problems. The first moral hazard problem concerns the borrower. The borrower can threaten to add risk to the project in order to transfer value from the bank. Adding risk is costly because it reduces the project value, V , and the liquidation value, L_2 , by c . This can be interpreted as a transaction cost; the borrower must pay to modify the existing project so as to increase riskiness.¹¹ We will show below that our assumptions restrict attention to cases where the added risk is inefficient. Obviously if the additional risk is in the interest of both parties, then such an action should, and will, be taken and we do not concern ourselves with it (by Assumption 13).

The other moral hazard problem is bank opportunism. The bank may opportunistically threaten to liquidate in order to extract surplus from the borrower once news, z , has arrived. If the bank has the power to threaten liquidation and can thereby extract surplus from the borrower, it may behave inefficiently. Indeed, Assumption 11 says that the bank will behave this way if it has a credible liquidation threat. Of importance, this opportunism has efficiency considerations since the borrower will choose to add risk ($\alpha = 1$) when the bank behaves opportunistically.

The credibility of this threat by the bank depends on the design of the contract. The contract design problem involves the considerations discussed

¹⁰ Zimmerman (1975), Quill, Cresci, and Shuter (1977), and Morsman (1986) describe real-world covenants. Rajan and Winton (1995) discuss the theoretical rationale for their existence.

¹¹ At $t = 1$ we assume that costless, or extremely inexpensive, ways of adding risk can be prevented costlessly by the bank through covenant restrictions.

in the introduction. First, is renegotiation desirable? If it is, then should the contract with the bank include a provision that allows the bank to ask for the collateral prior to maturity of the loan? We assume that the contract can feasibly include the liquidation option which allows the bank to “call the loan” at $t = 1$ if it so wishes, and we ask whether it is optimal to include this provision.¹² If the liquidation option is included, then the third contract design consideration involves the specification of the initial ($t = 0$) contract form. Knowing that any contract will be renegotiated at $t = 1$, what contract should be signed at $t = 0$? Our analysis attacks this question by asking: What is the gain to specifying the face value of the debt to be paid at $t = 2$, denoted F_0 , at $t = 0$? In our analysis it is feasible for the parties to specify $F_0 = D$ at $t = 0$ (or, for that matter, $F_0 = \infty$). For example, specifying $F_0 = D$ would be tantamount to an initial agreement under which the lender essentially says to the borrower: “Here’s an amount of money, D . I have the right to liquidate at $t = 1$, at which time we’ll work out the details of the contract.” This specification of the initial contract says that the bank can threaten to liquidate all borrower types at $t = 1$, receiving L_1 , unless borrowers agree to the bank’s offer of F^N at that date. We will show how renegotiation outcomes are affected by the specification of F_0 at $t = 0$ even though it is common knowledge that renegotiation will occur. The range of borrower types for which the liquidation threat is credible depends on the initial specification of F_0 . The size of F_0 will lead to efficiency considerations via its ability to influence the bank’s bargaining power at $t = 1$. The costs and benefits of allocating power to the bank will determine the initial F_0 .

Since we have assumed that the borrower has no alternative financing source at $t = 1$, the borrower cannot threaten to refinance from other sources. It will also turn out that the bank’s ability to threaten the borrower is limited. Thus it is not obvious how the surplus at $t = 1$ will be split. We have assumed that the bank can credibly make a take-it-or-leave-it offer at $t = 1$ and hence can obtain all the surplus. Since banking is competitive at $t = 0$, the possibility of extracting surplus at date $t = 1$ will be priced *ex ante*. The surplus will be split differently if other bargaining games are allowed, but this will not effect our results concerning efficiency.

2. Definitions and Preliminary Lemmas

In this section we provide preliminary definitions and results. We prove two lemmas to build understanding of the model. First, we analyze the borrower’s

¹² The interpretation of this is that while borrower type, z , is not verifiable, a contract can contain verifiable provisions (covenants) which are always triggered by the arrival of the news, z . Loan covenants are written in terms of variables measurable according to accounting procedures, for examples, net worth, leverage, etc., and consequently are verifiable, though violations may be forgiven by the bank. See Zimmerman (1975), Quill, Cresci, and Shuter (1977), and Morsman (1986). Bank loan contracts are written with a large number of covenants so that small deviations of the state of the firm trigger covenant violations, allowing the firm to “call” the loan. Sometimes the bank excuses such violations. Because of these covenants, the option to “call” is best viewed as always verifiably being “in the money” for bad borrowers.

decision at date $t = 1$ concerning adding risk. This defines a critical borrower type z^* below which the borrower will add risk in the absence of any bank action. Then we show that adding risk is inefficient. We then define the payoffs relevant to the subsequent analysis. Finally, we outline the possible renegotiation outcomes and provide some intuition before the formal analysis.

2.1 News arrival, the borrower's project choice at $(t = 1)$, and efficiency

At $t = 1$ the borrower and lender observe the realization of borrower type, z . The realization of a low z means that the borrower's equity is worth less than it was ex ante. In this situation, as is well known, the borrower may have an incentive to switch projects to add risk ("asset substitution"). Borrowers who receive bad news (low z realizations) will be tempted to switch from their initial project, $\alpha = 0$, to a higher risk project, $\alpha = 1$. By increasing the variance of the project, the value of the firm's equity can be increased at the expense of the bank. But since it is costly to take this action, only firms with sufficiently bad "news" will choose $\alpha = 1$, as the following lemma shows.

Lemma 1. *Given F_0 , there exists some z^* such that setting $\alpha = 1$ is profitable for the borrower if and only if $z < z^*$. Furthermore, z^* is increasing in F_0 .*

Proof. See Appendix B. ■

The lemma establishes that there is a critical borrower type, z^* , below which borrowers choose to add risk to their projects. Define the gain to the bank from the borrower of type z adding risk to be $\Gamma_B(z; F)$; see Appendix B. Then z^* is defined by $\Gamma(z^*; F_0) = 0$. We refer to $z < z^*$ as "bad" borrowers, and to $z > z^*$ as "good" borrowers. Also, eventually, we solve for F_0 , the initial face value of the debt. In this regard, it is important to know how z^* depends on F_0 , since lenders will take adverse incentive affects of higher F_0 into account initially and during any renegotiation. As the lemma shows, the dependence is intuitive: the higher the borrower's debt burden, the more likely it is that asset substitution will be appealing.

Lemma 1 shows that borrowers of type $z < z^*$ will, ceteris paribus, add risk. Our focus is on situations where the risk taking by the borrower is unprofitable for the bank and socially inefficient. The next lemma shows that, under our assumptions, this is ensured.

Lemma 2. *The addition of risk by the borrower ($\alpha = 1$) is unprofitable for the bank.*

Proof. See Appendix B. ■

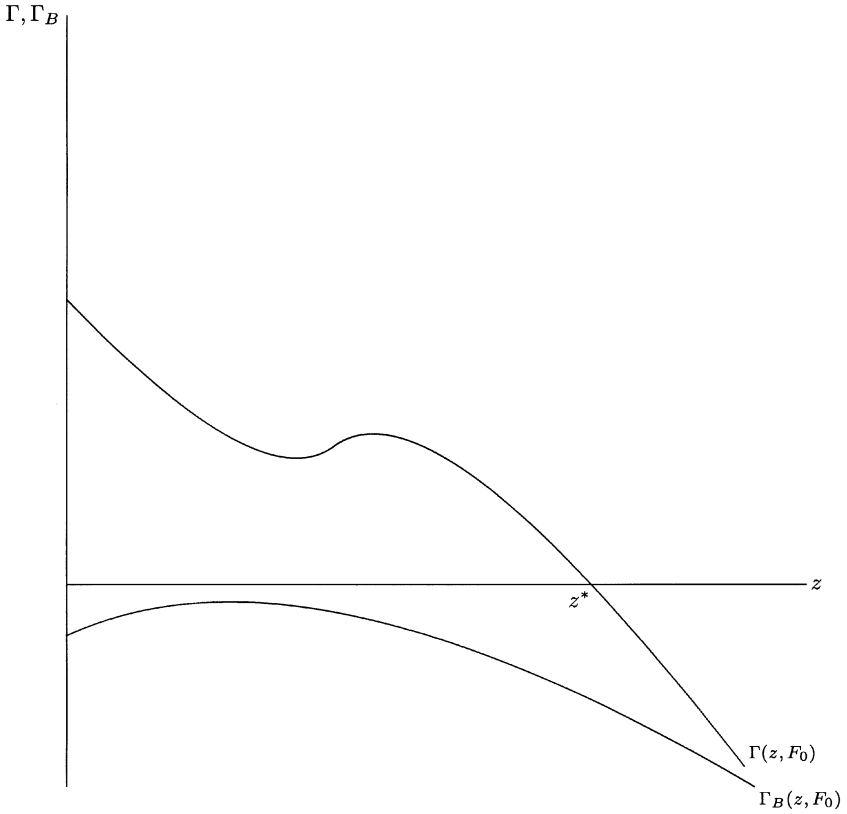


Figure 2

It follows immediately that since asset substitution by the borrower is always bad for the bank, it is socially harmful on the margin. That is, for some range of $z < z^*$, a borrower of type z^* is indifferent to adding risk while the bank strictly prefers that risk not be added. Figure 2 depicts typical “gain” functions for the borrower and lender. Lemmas 1 and 2 only say that the gain for the borrower crosses zero somewhere from above, while the gain for the lender is always negative under our assumptions. Thus the sum of the two gains (which represents the net social gain from asset substitution) will cross zero to the left of z^* . This implies that there is a range of z values to the left of z^* such that asset substitution is inefficient yet is in the private interest of the borrower absent preemptive action by the bank.

2.2 Payoffs

At $t = 1$ the bank may liquidate the project or renegotiate the interest rate. Let F^N be the new (i.e., renegotiated) face value for the debt to be paid

at $t = 2$. In general, F^N will depend on z , but this notation is usually suppressed.

Define the total expected payoff to the project as of $t = 1$ for given z and choice of α , $\pi^T(F^N, z, \alpha)$, as follows:

$$\begin{aligned} \pi^T(F^N, z, \alpha) &\equiv L_2 G(F^N(z) | z, \alpha) \\ &+ \int_{F^N(z)}^{V_h} V g(V | z, \alpha) dV + y_2(z) - \alpha c. \end{aligned} \quad (1)$$

Note that this is not the first-best total expected value, but the second best.¹³ Define unrenegotiated bank profit, $\pi^U(F_0, z, \alpha)$, to be expected bank profit as of $t = 1$, from a borrower of type z , when evaluated at the initial face value of the debt, F_0 , given that the borrower chooses α according to whether $z < z^*$:

$$\pi^U(F_0, z, \alpha) \equiv (L_2 - \alpha c) G(F_0 | z, \alpha) + F_0 [1 - G(F_0 | z, \alpha)], \quad (2)$$

where α is a function of F_0 and z .

To facilitate discussion of liquidation define:

$$\begin{aligned} z_{EL1} &= \inf\{z: \pi^T(F^N, z, \alpha = 0) = L_1\}; \\ z_{EL2} &= \inf\{z: \pi^T(F^N, z, \alpha = 1) = L_1\}; \\ z_{IL} &= \inf\{z: \max[\pi^R(F^N, z, \alpha = 1), \pi^U(F_0, z, \alpha = 1)] = L_1\}. \end{aligned}$$

The point z_{IL} is defined as the lowest borrower type at which the best the bank can do under any renegotiation strategy (including not renegotiating) is just equal to the liquidation value of the project. As will become clear, the subscript “EL1” denotes first-best efficient liquidation because the value of projects of type lower than z_{EL1} is expected to be less than the liquidation value of the project even if the borrower does not add risk. The subscript “EL2” denotes second-best efficient liquidation, indicating that the value of projects of type $z_{EL1} < z < z_{EL2}$ is expected to be less than the liquidation value *only* if the borrower chooses to add risk. If the borrower does not add risk, then these projects should not be liquidated (from the point of view of a social planner). Note that $z_{EL1} < z_{EL2}$. The reason for this inequality is that switching to $\alpha = 1$ reduces the expected return because it costs c to switch projects. The subscript “IL” denotes inefficient or excessive liquidation because, as will be seen, some projects of type $z > z_{EL2}$ may be liquidated. z_{IL} is defined with respect to the bank’s expected profit and thus will define when liquidation occurs. Consequently, z_{IL} may or may not coincide with z_{EL2} , as seen below.

¹³ The payoff is second best because sometimes L_2 is obtained due to default on the debt. Under first best this would not happen.

2.3 Renegotiated interest rates

If the bank does not liquidate the borrower's project, it may seek to renegotiate the interest rate on the loan.¹⁴ In this subsection we outline the possible renegotiation outcomes (to be analyzed subsequently) and provide some intuitive explanation. The intuition follows the ordering of the z -cutoff points shown in Figure 3.

¹⁴ In fact, even absent the moral hazard problem of asset substitution, it would be in the bank's interest to change F upon learning z simply to increase expected payoffs. We postpone discussion of this until later.

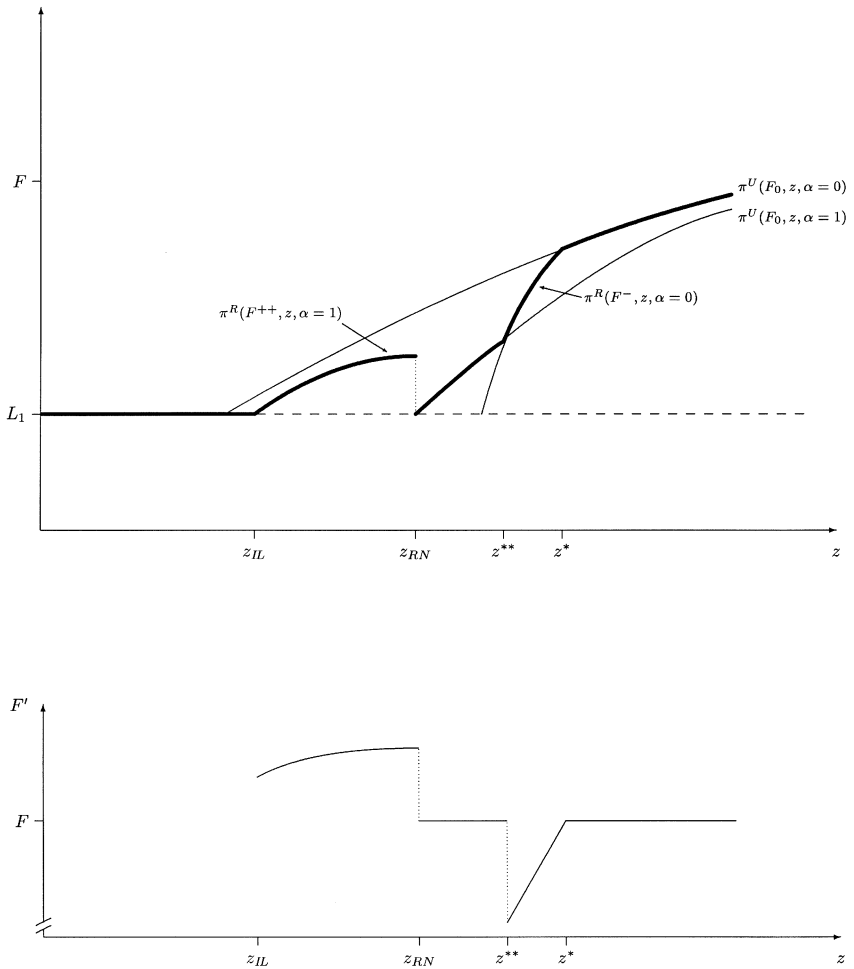


Figure 3

Define renegotiated bank profits at $t = 1$, when a new interest rate $F^N(z)$ has been agreed to as follows:

$$\begin{aligned} \pi^R(F^N, z, \alpha) \equiv & (L_2 - \alpha c)G(F^N(z) | z, \alpha) \\ & + F^N(z)[1 - G(F^N(z) | z, \alpha)]. \end{aligned} \quad (3)$$

Again, α is the same function of F and z . Renegotiated bank profit is the return the bank expects to receive from the project of a borrower of type z , where the borrower of type z chooses project α , and promises to repay $F^N(z)$, the new interest rate agreed upon at date $t = 1$.

One possible renegotiation outcome would be a *lower interest rate*. For example, if the borrower type is such that the gain to switching projects is positive, that is, $z < z^*$, then the bank may forgive part of the debt by lowering the interest rate to induce the borrower not to add risk (switching to $\alpha = 1$). Consider a borrower of type just worse (i.e., lower) than z^* . Such a borrower will choose to add risk, $\alpha = 1$, but is near indifference. If the value of the borrower's equity were a little higher, then $\alpha = 0$ would be chosen so the cost c would not be borne. The bank may find it profitable to raise the value of the borrower's equity by forgiving some debt. While this lowers the face value of what the borrower contracts to repay, the bank's expected profits may rise because the borrower, with reduced leverage, chooses not to add risk. (In fact, it is possible that the bank would want to forgive debt even for some $z > z^*$, simply because it improves expected profit.) In any case, define $F^-(z)$ to be the highest value of F such that $\alpha = 0$ solves the borrower's $t = 1$ problem of maximizing the (expected) gain to adding risk.

It need not be the case that $F^-(z) < F_0$. But if $F^-(z^*) < F_0$, then for some range of borrowers in the interval $z_1 < z < z^*$, the bank may want to forgive debt. But at some point, for sufficiently low z , lowering the interest rate to induce the borrower not to switch projects will reduce the bank's expected profit below what it would earn if it maintained the initial contract (F_0) and allowed the borrower to add risk ($\alpha = 1$). Define z^{**} to be the borrower type at which the bank is indifferent between these two choices: $\pi^R(F^-, z = z^{**}, \alpha = 0) = \pi^U(F_0, z = z^{**}, \alpha = 1)$, where $\pi^R(F^-)$ is the bank's expected profit as of $t = 1$ when the renegotiated interest rate is decreased [$\pi^R(F^+)$ will indicate expected bank profit when the renegotiated interest rate is increased]. Note that by definition it is always the case that $z^{**} < z^*$; the borrower would only be tempted to choose $\alpha = 1$ if $z < z^*$, that is, when the gain to switching projects is positive ($\Gamma(z) > 0$). Thus z^{**} is the threshold value of z below which (even with renegotiation) the borrower chooses $\alpha = 1$. See Figure 3.

Since z^{**} defines the point at which borrowers add risk, it will be important to know how this point varies with F_0 . The answer is given by:

Lemma 3. z^{**} is increasing in F_0 .

Proof. Note that $\pi^R(F^N, z, \alpha)$ is independent of F_0 , but $\partial\pi^U/\partial F_0 > 0$ for $F_0 < F^\#$, by Assumption 13. Since z^{**} is defined as the point where $\pi^R(F^-, z = z^{**}, \alpha = 0) = \pi^U(F_0, z = z^{**}, \alpha = 1)$ the lemma follows. ■

If forgiving debt to induce the borrower to choose $\alpha = 0$ is not profitable, then the bank may seek to raise the interest rate, provided it has a credible (i.e., subgame perfect) threat to liquidate. Define z_{RN} to be the solution to $\max[\pi^U(F_0, z_{RN}, \alpha), \pi^R(F^-, z_{RN}, \alpha)] = L_1$ and if $\pi^U > L_1$, for all z , then $z_{RN} = z_1$. For $z < z_{RN}$ the bank expects its (unrenegotiated) profit to be less than the current liquidation value and hence has a credible threat to liquidate. The subscript “RN” denotes renegotiation since for $z < z_{RN}$ the bank can credibly threaten the borrower and demand a higher interest rate. If the bank can credibly threaten the borrower, then the higher interest rate is given by:

$$F^{++}(z) = \text{Argmax}_{F^N} (L_2 - \alpha c)G(F^N | z, \alpha) + F^N[1 - G(F^N | z, \alpha)]. \quad (4)$$

Recall that under Assumption 11, the bank’s expected profit is higher if it raises the interest rate so much that the borrower adds risk, as opposed to raising it to $F^+(z)$ and receiving $\pi^R(F^+(z), z, \alpha = 0)$.

As shown in Figure 3, as the type of the borrower declines, there comes a point where raising the interest rate cannot raise the expected value of the loan to the bank above the liquidation value, L_1 . As defined above, at z_{IL} , $\pi^R(F^{++}, z_{IL}, \alpha = 1) = L_1$. Again, however, it is good to keep in mind that there can be other cases where the bank can profitably raise the interest rate.

As with the other critical z -values, z_{RN} depends on F_0 .

Lemma 4. z_{RN} is decreasing in F_0 .

Proof. When $F_0 < F^\#$, $\partial\pi^U/\partial F_0 > 0$, by Assumption 13. ■

2.4 Definition of equilibrium at $t = 1$ and specification of cases

At $t = 1$ the bank and the borrower know L_1 , observe the realization of z , and choose a new contract, F^N , or liquidation, subject to constraints imposed by the existing contract, F_0 . The existing contract and the borrower’s type determine $\pi^U(F_0, z, \alpha)$, that is, the unrenegotiated expected bank profit. An equilibrium at $t = 1$ is (1) a choice of α by a z -type borrower which maximizes the borrower’s expected profits, given the new contract, $F^N(z)$ (assuming liquidation does not occur); and (2) a choice of (new) interest rate, $F^N(z)$, or liquidation, by the bank, given the borrower’s type, z , and choice of α , which maximizes the bank’s expected profit. The resulting bank profit function, which we will denote by $\pi^B(F^N, z, \alpha)$, is the upper envelope of the four profit functions based on the different renegotiation outcomes, that is,

$$\pi^B(F^N, z) = \max\{\pi^R(F^{++}, z, \alpha), \pi^R(F^-, z, \alpha), \pi^U(F_0, z, \alpha), L_1\},$$

given the optimal choice of risk, α , by the borrower as a function of z .

The precise pattern of renegotiation outcomes as a function of z depends on the location of z_{RN} relative to z^{**} and z^* . These in turn depend on F_0 and L_1 . We treat L_1 as fixed and let F_0 trace out all of the possibilities, although of course ultimately F_0 will be determined by equilibrium conditions. Lemmas 1, 3, and 4 imply that there are three scenarios to consider (as depicted in Figure 4) corresponding to low, intermediate, and high values of F_0 ¹⁵. At low values of F (the bottom panel of Figure 4 the bank has a credible threat to renegotiate over a wide range of z , so $z_{RN} > z^*$ and there is never any issue of forgiving debt. The bank just “holds up” everyone with $z \leq z_{RN}$, even knowing that they will add risk as a consequence.

At intermediate and high values of F_0 we have $z_{RN} < z^*$, so there is a range of forgiveness. The difference between the two is that with high F_0 , $z_{RN} < z^{**}$ so there is a range in which risk taking occurs because it is not in the bank’s interest to forgive. The loan is still profitable though, so the bank has no credible threat that would allow it to increase F either, and it just leaves it at F_0 . In the intermediate case the forgiveness range runs into the “hold-up” range, so risk taking coincides with the bank’s increasing F .

It will turn out that the equilibrium value of F_0 corresponds to the boundary between the intermediate and high F_0 cases, with $z^{**} = z_{RN}$. This is because, as Figure 4 makes clear, the range of risk taking (which occurs for $z < \max[z^{**}, z_{RN}]$) is thereby minimized. In the next two sections we will go into more detail on the high F_0 case, and relegate the other cases to Appendix C.

3. Results: Renegotiation and Liquidation Decisions at $t = 1$

Having dispensed with the preliminaries, we can now turn to the actual predictions of the model. The real point of interest in the model is at $t = 1$, when all the important decisions get made. At that point project-specific information has arrived, and the borrower and lender have to decide whether to continue the project and if so, on what terms. At $t = 2$, behavior is mechanical—the borrower repays the loan at whatever the prevailing terms are if the project is solvent, or he does not, and the project is liquidated. At $t = 0$ all borrowers are identical, so the only problem is to determine the initial face value of the loan, a problem we turn to in Section 4.

3.1 The liquidation decision

What triggers liquidation? By definition of z_{IL} , projects of borrowers of type $z < z_{IL}$ are liquidated. In the high F_0 case, liquidation begins at the point where $\pi^R(F^+, z_{IL}, \alpha = 1) = L_1$. If $\pi^T(z_{IL}, \alpha = 1) = \pi^R(F^{++}, z_{IL}, \alpha =$

¹⁵ In Figure 4 the curve labeled $\pi^R(F^-)$ is $\pi^R(F^+)$ for $z < z^*$ and $\pi^R(F^-)$ for $z < z^*$. To avoid complicating the figure we only include one label.

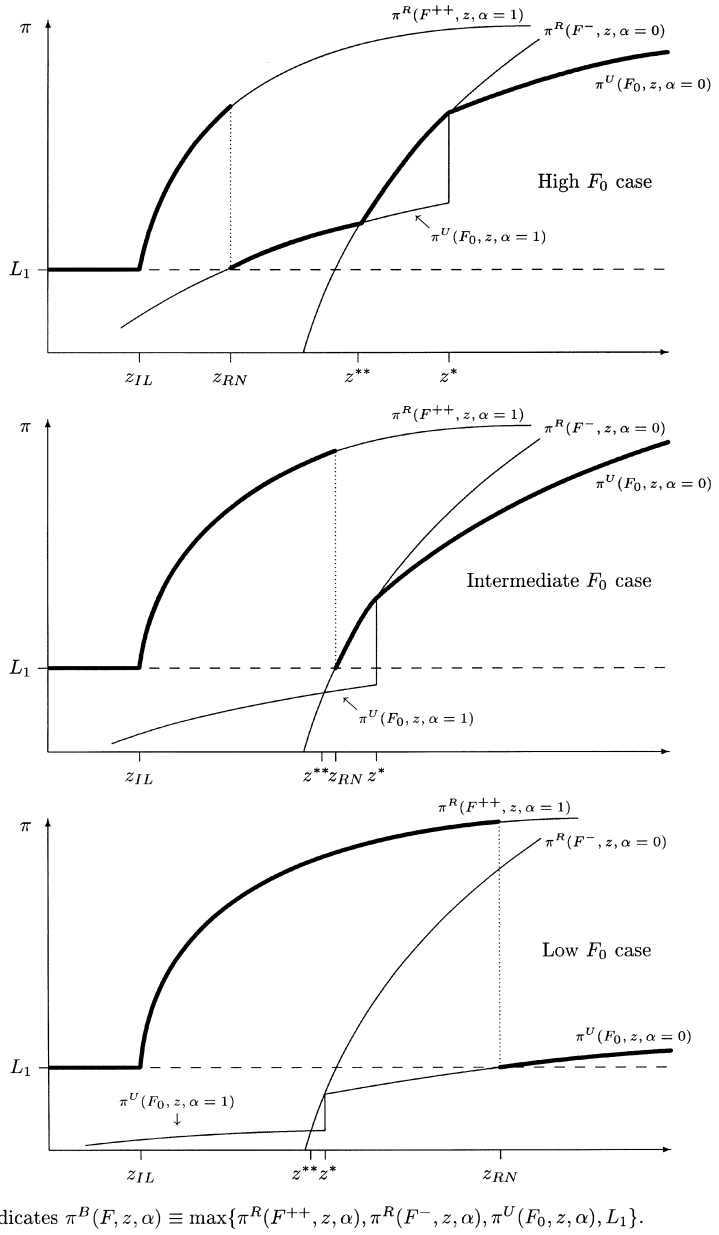


Figure 4

1) = L_1 , (i.e., $z_{EL2} = z_{IL}$), then the projects liquidated in the range $z_{EL1} < z < z_{IL}$ are second-best liquidated since total expected profits are positive if the borrower did not choose $\alpha = 1$. However, if $\pi^T(z_{IL}, \alpha = 1) > \pi^R(F^+, z_{IL}, \alpha = 1) = L_1$, then $z_{IL} > z_{EL2}$, and even more projects are liquidated, inefficient (or excessive) liquidation ("IL") beyond the second best. This inefficient liquidation (relative to second best) can happen because there is no way for the bank to overcome the incentive the borrower has to choose more risk. Forgiveness does not increase the bank's expected profit by enough, nor does raising the interest rate. (We discuss the issue of side payments below.)

Liquidation of socially wasteful projects will be an important role for the bank to play. But by giving the bank the power to liquidate there is also the possibility that the bank liquidates projects inefficiently. This cost will have to be weighed against the benefits of liquidating efficiently.

3.2 Renegotiation outcomes

We now turn to renegotiation with borrowers who are not liquidated, maintaining the focus on the high F_0 case. Renegotiation outcomes, as a function of borrower type, are characterized by the bank choosing the outer envelope of four expected profit curves: renegotiated profit when the interest rate is raised, $\pi^R(F^{++}, z, \alpha = 1)$; renegotiated profit when debt is forgiven (i.e., the interest rate is lowered), $\pi^R(F^-, z, \alpha = 0)$; unrenegotiated profit, $\pi^U(F_0, z, \alpha)$; and liquidation. Figure 3 graphically portrays the four bank profit curves in the high F_0 case. The next proposition formalizes the intuition that the bank will choose the outer envelope of these profit curves subject to its ability to extract surplus from the borrowers.

Proposition 1. *In the high F_0 case, renegotiation results in*

- (i) $F^N(z) = F_0$ for all $z > z^*$, that is, no change in the interest rate. The borrower chooses $\alpha = 0$.
- (ii) $F^N(z) = F^-(z) < F_0$ for all $z \in [z^{**}, z^*]$, that is, forgive debt (lower the rate) so that the borrower chooses $\alpha = 0$.
- (iii) $F^N(z) = F_0$ for all $z \in [z_{RN}, z^{**}]$, that is, no change in the interest rate. The borrower chooses $\alpha = 1$.
- (iv) $F^N(z) = F^{++}(z) > F_0$ for all $z \in [z_{IL}, z_{RN}]$, that is, raise the interest rate and let the borrower choose $\alpha = 1$.

Proof. See Appendix B. ■

Intuitively the proposition says the following: Upon arrival of news at $t = 1$, there are four potential outcomes in addition to immediate liquidation:

1. With favorable news, the status quo obtains, as the borrower is not interested in asset substitution and the bank has no credible threat to

- liquidate the project and thereby extract a higher interest rate through renegotiation.
2. With moderately unfavorable news, the bank will choose to forgive some of the debt (i.e., lower the interest rate) in order to induce the borrower not to engage in costly asset substitution.
3. With more unfavorable news, however, the bank will not be able to preclude asset substitution by offering debt forgiveness. Instead, the asset substitution will occur and the project will become more risky.
4. Finally, with the most unfavorable news, asset substitution will occur but the bank will be able to extract a higher interest rate through renegotiation because the project's prospects are so poor that the bank has a credible threat to liquidate.

Thus the bank is unable always to preclude asset substitution and the resulting endogenous increase in project risk. It will turn out that in equilibrium cases 3 and 4 above coincide; that is, the bank will either forgive some of the debt to preempt asset substitution, or it will concede the substitution and extract a higher interest rate. The status quo is never the best option once bad news arrives.

The proposition can also be understood with reference to Figure 3. Starting with the highest type borrowers, those with $z > z^*$ unrenegotiated bank profits are given by $\pi^U(F_0, z, \alpha = 0)$ since these borrowers do not switch projects. The bank cannot credibly threaten these borrowers to extract a higher rate because in this range, $\pi^U(F_0, z, \alpha = 0) > L_1$ (that is, $z_{RN} < z^*$). The bank may or may not forgive debt for these borrower types (we assume that there is no forgiveness by Assumption 13), but in any case these borrowers choose $\alpha = 0$. Therefore these borrowers continue their projects and the bank maintains the initial interest rate F_0 . This is shown in the lower panel of the figure.

Borrowers with types below z^* will choose to add risk to their projects, *ceteris paribus*. But the bank is not in a position to threaten all of these borrowers with liquidation because the point at which the bank can credibly threaten and force renegotiation, z_{RN} , is below z^* ($z_{RN} < z^*$). However, by providing debt forgiveness to some of these borrowers they can be induced to not add risk. Debt forgiveness raises the value of the borrower's equity by just enough to make taking the costly, risk-increasing, action unprofitable. The question is whether this is profitable for the bank. In the figure it can be seen that the bank's expected profit when debt is forgiven (that is, the interest rate is lowered to $F^-(z) < F_0$) is higher than unrenegotiated bank profits given that borrowers choose $\alpha = 1$. (The interval $[z^{**}, z^*]$ may not exist.)

Debt forgiveness is optimal as long as $\pi^R(F^-, z, \alpha = 0) > \pi^U(F_0, z, \alpha = 1)$, that is, until the bank must forgive so much debt that it prefers to stay with the initial contract and allow the borrower to add risk. At the point

$z^{**}, \pi^R(F^-, z^{**}, \alpha = 0) = \pi^U(F_0, z^{**}, \alpha = 1)$, so debt forgiveness is only provided for borrowers of type $z^{**} < z < z^*$ since they can be induced to not add risk, which is in the bank's best interest. For borrowers in the range $z_{RN} < z < z^{**}$ there is no change in the interest rate since these borrowers cannot be threatened to get a higher rate and debt forgiveness is not profitable. Consequently, borrowers of type $z_{RN} < z < z^{**}$ are allowed to add risk and continue under the old contract. This is shown in the bottom panel of the figure where these borrowers continue with an interest rate of F_0 .

For borrowers of type $z_{IL} < z < z_{RN}$ it is not profitable for the bank to forgive debt (since $z^{**} > z_{RN}$), but the project is worth continuing. The bank can force the borrower to pay a higher interest rate because the threat of liquidation is credible for these borrower types [since $\pi^U(F_0, z, \alpha = 1) < L_1$ in this range]. Finally, at z_{IL} , $\pi^R(F^{++}, z_{IL}, \alpha = 1) = L_1$, so borrowers of lower type than this are liquidated.

Proposition 1 covers the case assumed by Assumption 11, that it is always more profitable for the bank to raise the rate to F^{++} and let the borrower add risk, if the bank can credibly threaten liquidation. Appendix B analyzes the alternatives to Assumption 11 as well as the high F_0 and low F_0 cases.

3.3 Discussion

Two features of Proposition 1 are worth noting. First, the bank is not entirely successful in controlling risk. Borrowers of type $z_{IL} < z < z^{**}$ choose to add risk and are allowed to continue their projects. Thus, in equilibrium, borrower risk varies endogenously. Second, renegotiated interest rates are not monotonic in borrower type as can be seen in the lower panel of Figure 3. Starting from z^* , the bank first lowers the interest rate to forgive debt (until z^{**} is reached), then maintains the initial rate (until z_{RN} is reached), and then raises the rate (until z_{IL} is reached) after which projects are liquidated.

We have allowed for the possibility that the bank may increase F if it has a credible threat to liquidate, regardless of whether the borrower will choose to add risk or not. We have postponed until now the possibility of debt forgiveness simply as the result of new information being received at $t = 1$, namely z . Even absent any moral hazard problem, the bank may be able to increase its expected profits by lowering F for some borrowers. This possibility would only change the shape of the π^U functions monotonically without qualitatively changing Figure 3 or any of the results described above. In particular, without the moral hazard problem, these reoptimized interest rates would introduce no new nonmonotonicity in the pattern of renegotiated interest rates as a function of borrower type z .

4. Initial Loan Pricing and the Role of Debt

The renegotiation outcomes at $t = 1$ were determined above assuming that the contract contained the liquidation option and assuming a given F_0 that

had been determined earlier at $t = 0$. If the liquidation option is not included in the contract, then the bank, being a single agent, can renegotiate, but cannot threaten liquidation. Before considering the optimality of the liquidation option, which is done in Section 6, we turn to the determination of F_0 in the case where the liquidation option is included in the contract. In this case, both parties to the contract know that renegotiation can occur. Then, what role does F_0 play? Why bother specifying F_0 , at all, given that it is renegotiated after news arrives? To answer these questions we proceed in two steps. First, we demonstrate how efficiency considerations determine F_0 by affecting the bargaining power of the bank. This will determine the F_0 that is socially optimal (in the second-best sense). Then we inquire as to how the (second-best) efficient F_0 can be implemented when lenders act competitively and earn zero expected profits.

4.1 The socially optimal initial interest rate

The socially optimal (second-best) F_0 , call it F_0^* , will minimize inefficient risk-taking subject to the moral hazards. To determine F_0^* we first need to decide which of the three cases defined above, high F_0 , low F_0 , or intermediate F_0 , is most efficient. We can summarize the analysis so far, with respect to which borrowers will add risk to their projects, by combining the results of Proposition 1 with the results in Appendix B:

Low F_0 case: For $z_{IL} < z < z_{RN}$, $\alpha = 1$, while for $z_{RN} \leq z \leq z_h$, $\alpha = 0$.

Intermediate F_0 case: For $z_{IL} < z < z_{RN}$, $\alpha = 1$, while for $z_{RN} \leq z \leq z_h$, $\alpha = 0$.

High F_0 case: For $z_{IL} < z < z^{**}$, $\alpha = 1$, while for $z^{**} \leq z \leq z_h$, $\alpha = 0$.

In the intermediate and low F_0 cases, the inefficient risk taking begins at z_{RN} , while in the high F_0 case it begins at z^{**} . The next two lemmas show how these risk-taking ranges vary with F_0 .

Lemma 5. *In the high F_0 case, the risk-taking range is shrinking as F_0 decreases.*

Proof. By Lemma 3, $\partial z^{**}/\partial F_0 > 0$. ■

Lemma 6. *In the intermediate and low F_0 cases, the risk-taking range is increasing as F_0 decreases.*

Proof. By Lemma 4, z_{RN} is rising as F_0 decreases. ■

As F_0 decreases, the risk-taking range decreases in the intermediate case, but increases in the high and low cases. It is immediate that the optimal F_0 is on the boundary between the high and intermediate cases:

Proposition 2. *The constrained socially optimal F_0 is such that $z^{**} = z_{RN}$.*

Figure 5 depicts the optimal configuration. The proposition results from the fact that any reduction in asset substitution brought about through renegotiation is welfare improving. Since the bank forgives over the range $[z^{**}, z^*]$, that range of borrowers is discouraged from inefficiently adding risk. Any higher value of F_0 would make it more costly on the margin for the bank to forgive sufficiently to prevent asset substitution. This would have the effect of raising z^{**} and thereby increasing the range of asset substitution. Any lower value of F_0 would increase z_{RN} , that is, it would provide the bank with a credible threat to liquidate for the marginal borrower. The effect would be a transfer to the bank at the cost of a decline in project quality, as the bank's

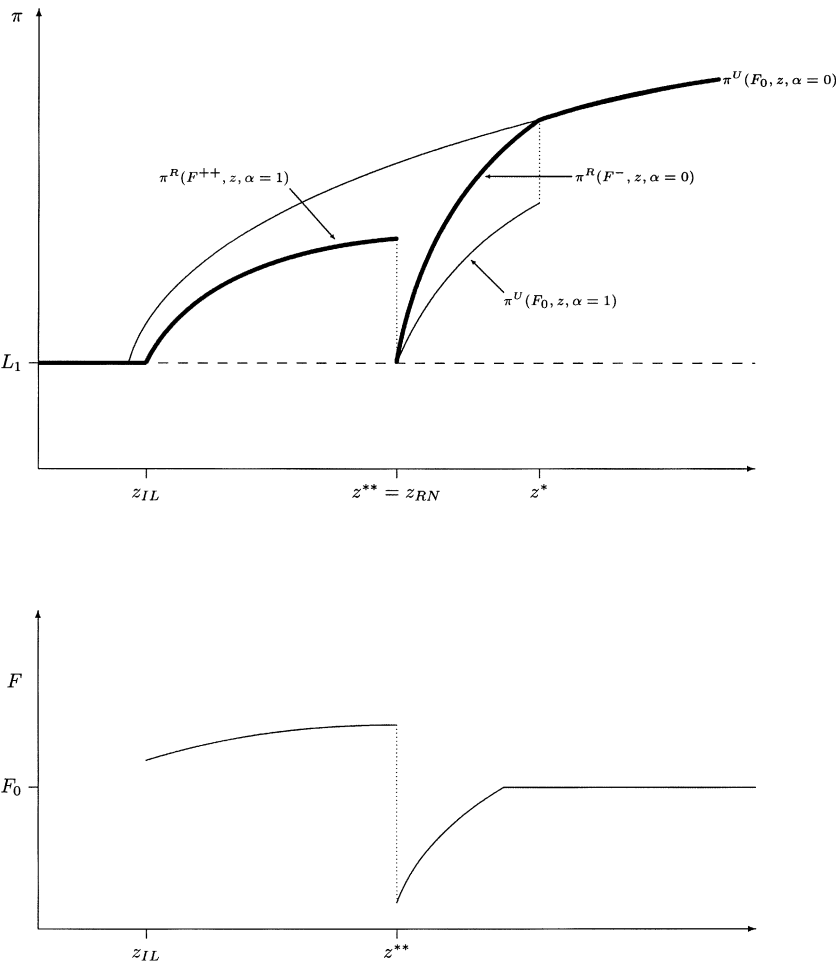


Figure 5

ability to behave opportunistically entices it to abandon its antiasset substitution measures. Thus the equilibrium F_0 optimally balances off the two moral hazard problems.

While we have yet to discuss how the socially optimal F_0 of Proposition 2 will be implemented, we stress the importance of the proposition. The face value of the debt serves a critical role in allocating bargaining power between borrowers and lenders. It would only be a complete coincidence if that face value bore any relation to default risk. Consequently there is no reason to expect the equilibrium F_0 to imply zero profits. The next section addresses this last issue.

4.2 Implementation of the socially optimal F_0

Let F_0^* denote the optimal value of F_0 . Given the nature of bank loans, it should be clear that linear pricing is not necessary. Thus if F_0^* implied that banks would make positive profits, competitive banks could still price loans at F_0^* and compete by offering other goods or services for free, up to the point that they make zero profits on the whole package. This is the case depicted in Figure 6. While it might seem odd that expected profits for the bank are declining at $F_0 = F_0^*$, the intuition should be clear: a lower value of F_0 would be regarded with suspicion by borrowers, who would foresee that the bank would be more likely to hold them up in the interim. Borrowers would thus prefer the slightly higher F_0 because it is more credible. Also, note that bank profits are the same at extreme values of F_0 because the range of risk taking is broad and the renegotiated F would be the same in either case. The point is that a very high or very low value of F_0 is ignored, as both sides know it will be reset in the interim.

On the other hand, it is possible that at $F_0 = F_0^*$ banks would make negative expected profits. In this case, competitive banks could charge origination fees to make up the difference, if that were feasible. Under our assumptions, however, the borrower has no surplus liquidity at $t = 0$, so competitive banking cannot implement the social optimum. In this case, the bank would have to lend the borrower additional money to cover the origination fee. But this would be tantamount to charging a higher F_0 . Thus if F_0^* did imply negative profits, and there were no way to extract origination fees from the borrower without effectively increasing the borrower's leverage, then competition would drive F_0 to the zero-profit point (as the figure makes clear, there would likely be more than one) that had the highest total profits. This would be inefficient relative to the scenario depicted in Figure 6, but would be the best the system could accomplish.

Our result that bank loans will generally involve nonlinear pricing is consistent with the observation that the loan rate is only one component of pricing bank loans. In addition to the interest rate, banks also use a variety of fees and, at least in the past, tied lending to other services. Booth and Chua (1995) discuss the prevalence of, and different types of fees in, bank

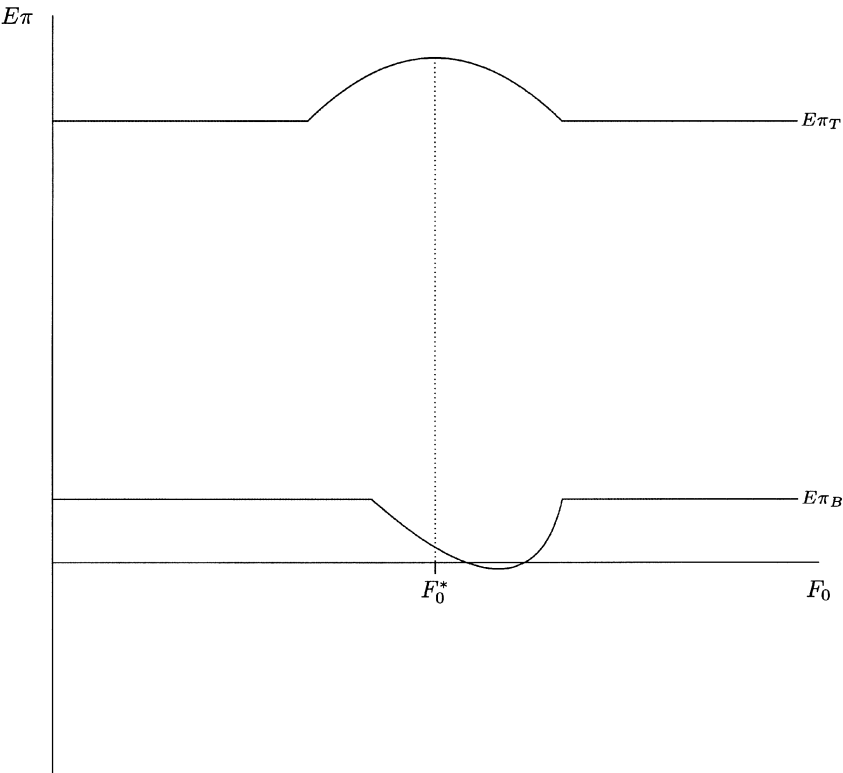


Figure 6

loan contracts. For example, Booth and Chua find that an up-front fee is charged in 45% of the sample loan contracts examined. Other fees are not mutually exclusive and are also common. Overall, Booth and Chua show that substantial heterogeneity exists in the pricing of loan contracts. Our explanation for the presence of such pricing structures differs considerably from the existing literature. To explain this structure of bank loan pricing, the previous literature has focused on the presence of informational asymmetries related to the credit risk of the borrower. In Thakor and Udell (1987) borrowers reveal their default characteristics based on their choice of contract terms. In Berlin (1987), borrowers self-select across contract types based on their probability of borrowing.

5. Discussion

What makes bank loans valuable? Why are bank loans senior? In this section we discuss how our model addresses these questions.

5.1 Bank loans, the option to liquidate, and corporate bonds

The features of bank loans that distinguish them from conventional corporate bonds are the bank's ability to renegotiate the terms of existing loans and to call in or "liquidate" them if that is desirable. Thus, as mentioned above, there are really three distinct securities to consider: corporate bonds, bank loans with the liquidation option, and bank loans without the liquidation option. It should be immediately clear that the bank loan without the liquidation option dominates corporate bonds. Banks by assumption have the ability to renegotiate, which leads to more efficient outcomes in some states of the world. Otherwise there is no difference, so the gain in efficiency is unambiguous. Literally interpreted, this result would turn the question of bank loan's value on its head and raise the question of why corporate bonds are valuable. This result does not, however, immediately extend to junior corporate bonds issued in addition to bank loans (see discussion below). Moreover, in practice firms have begun to have corporate bonds mimic the forgiveness feature of bank loans by utilizing exchange offers. Typically such exchange offers are a device for the borrowing firm to initiate forgiveness and reduce its debt obligations [see Asquith, Mullins, and Wolff (1989)]. This is discussed by Gertner and Scharfstein (1991).

So the remaining question concerns the value of the liquidation option. Although we regard the liquidation option as virtually intrinsic to bank loans, it is still useful to analyze why such an option would be valuable. The value of this option hinges entirely on the range of projects in which liquidation occurs, that is, $[z_1, z_{IL}]$. We know that in general there is (at least in the absence of side payments) inefficient liquidation over the range $[z_{EL2}, z_{IL}]$. On the other hand, in the absence of the liquidation option there would be inefficient *continuation* over the range $[z_1, z_{EL2}]$.

Clearly, the desirability of the liquidation option is in general ambiguous. It will depend on the shape of the density of z over these ranges as well as the mapping from z to expected payoffs. The fact that bank loans almost invariably do contain a liquidation option (usually implicitly through covenants) suggests that the excessive liquidation costs may in practice be relatively small—perhaps because of side payments, but also because the range or magnitude of inefficient liquidations is simply not very large in comparison to the problem of excessive continuation in the absence of banks' ability to liquidate.

5.2 Junior debt and related concerns

Our model does not explicitly include junior debt. There is little loss in generality though, because everything in the article carries through conditional on the presence of a fixed amount of junior debt associated with the project. Since bank loan covenants would generally specify limits on junior debt, the bank can simply consider junior debt as part of the borrower's project, and whatever agency problems may be associated with junior debt can be

thought of as already accounted for in the probability distribution over project payoffs.

Even though we do not treat junior debt explicitly, our model nevertheless sheds light on a puzzle that emerges from the existing literature on financial intermediation: Why should banks as senior claimants engage in monitoring the behavior of borrowers more closely than junior claimants do? Junior claimants would seem to have a greater incentive to monitor (in a costly state verification setting), as Fama (1985) has argued.¹⁶ Our view is that in addition to their ability to act unilaterally, banks' status as senior claimants puts them in the position to gain the most in the event of liquidation. Certainly junior creditors can force a borrower into bankruptcy, but then they risk getting little or nothing because of their junior status. Banks, as senior claimants, have an incentive to force liquidation, possibly excessively so, as we have seen. If the likelihood of excessive liquidation can be reduced via prepayment options, then bank loans dominate other forms of debt because the prospect of relatively efficient liquidation raises the value of the firm *ex ante* by lowering the cost of debt.^{17, 18} If senior creditors were decentralized they would find it costly to undertake the efficiency-enhancing renegotiation process to avoid asset substitution and inefficient liquidation.

The presence of decentralized junior debt could make it more difficult for the bank to preclude asset substitution through renegotiation, but there are ways around that. The difficulty is that the temptation to take on risk is a function of the debt:equity ratio. The bank would have to forgive more debt in order to counter the borrower's incentive to add risk if there are junior debtholders, and some of the benefits would spill over to them. Moreover, even if it is in the collective interest of the junior debtholders to participate in the forgiving, there is a free-rider problem, as each debtholder would try to hold out and let the others bear the burden.

One mechanism a bank has at its disposal to deal with the free-rider problem works as follows. The bank can say to the firm: "We will forgive $x\%$ of the debt provided you can get the junior debtholders to do so as well." The firm can appeal to the junior debtholders through a con-

¹⁶ Fama (1985) argues that the benefits of banks' monitoring activities spill over into the corporate debt market as the presence of bank debt on a corporation's balance sheet functions as a sort of "seal of approval" that enables it to issue debt directly. The problem with this scenario is that bank debt is senior to corporate debt. Consequently banks should have less incentive to monitor borrowers' subsequent behavior than the junior creditors would have. Yet firms often have both bank loans and publicly issued and traded bonds.

¹⁷ Prepayment is another contract feature that we did not consider, but that works in favor of bank loans. A prepayment option allowing the borrower to prepay debt at date $t = 1$ can reduce the cost of excessive liquidation by the bank, increasing the benefits of loans over bonds. Then, as shown in Gorton and Kahn (1994), inefficient liquidation can be reduced or eliminated and borrowers might never want to add risk.

¹⁸ As junior claimants banks could still forgive debt, while as senior claimants they would not forgive since subordinated debtors would be the beneficiaries. Thus when junior debt is present, and banks are senior lenders, banks are not likely to forgive principal. This corresponds to the findings of Asquith, Gertner, and Scharfstein (1991) who study distressed junk bond issuers and find that the banks rarely forgave principal, but did defer principal and interest payments.

sent solicitation that amounts to a “coercive exchange offer” [see Kahan and Tuckman (1993)], which effectively plays off the junior debtholders against each other to get them to do what is in their collective interest. Kahan and Tuckman find that even though such consent solicitations involve apparent redistributions of wealth from bondholders to stockholders, they are typically associated with positive abnormal bondholder returns. This is consistent with the spirit of our analysis which argues that such renegotiations are efficiency enhancing. Of course the ability of firms to induce renegotiation with decentralized junior debtholders suggests that such renegotiation is not impossible, as we have assumed, but merely more costly than with banks.

6. Final Remarks

We summarize our key findings as follows:

1. Since the key advantage of bank loans arises from banks’ ability to monitor and renegotiate in order to mitigate moral hazard problems, it is not surprising that the key determinant of bank loan pricing is also the mitigation of moral hazard. Specifically, we find that the equilibrium interest rate on loans does not primarily reflect a default premium. Rather, it is the rate that results *ex ante* in minimal expected asset substitution by borrowers. Since there is no guarantee that this rate results in zero profits, competition by banks will result in nonlinear (in the amount borrowed) pricing arrangements for loans.
2. The volatility of corporate securities is endogenous and variable. The firm sometimes has an incentive to increase volatility. The outside claimant that is in a position to prevent this, the bank, only imperfectly controls borrower risk-taking. The bank interacts with the borrower during the course of the contract. It is in a position to do this because by assumption it is a single agent and so can renegotiate higher interest rates, liquidate, or forgive debt. The bank controls risk in two ways: it may liquidate the project or it may change the borrower’s incentive to add risk by debt forgiveness. But, importantly, there are borrower types for which the bank cannot prevent risk from being added, but whose projects are allowed to continue. This means that the variance of the value of the firm (and the mean) depend, in equilibrium, on the borrower type and, in particular, is not constant.
3. The social value of bank loans relative to other instruments presumes that excessive liquidation costs are small relative to excessive continuation costs, that is, that banks do not, in effect, “throw the baby out with the bath water” in the course of monitoring and liquidating projects.

Appendix A

Parameter restrictions

The following assumptions involve an endogenous variable, F , and therefore must be handled with care. Their role is only to ensure that the parameters of the problem are such that the model behaves reasonably. It turns out that for extreme values of F the characterizations of outcomes in the paper are not complete. These additional cases are either implausible or economically uninteresting, and would only burden the article with additional complexity. The essence of the assumptions is to show that these outcomes can be ruled out by appropriate (and mutually compatible) parameter restrictions.

Let F_0 denote the amount initially specified by the contract to be repaid at $t = 0$. Clearly, F_0 must be in the range $[D, V_h + \epsilon_h]$. At $t = 1$ a different amount, F^N , may be negotiated. Let F denote either of these values. Then:

Assumption 12. $\epsilon_h > c + F$.

In other words, the upper bound of the support of ϵ is sufficiently large that adding risk always results in a positive probability of solvency. This assumption simply makes the problem interesting since it says that when risk is added there is always some chance for the borrower to benefit.

Assumption 13. $L_2 + c/[1 - K(c)] > F$.

(Recall that $K(\epsilon)$ is the distribution function for ϵ .) This assumption says that c is sufficiently large and/or the distribution of ϵ is sufficiently skewed that for a given F , the bank always prefers that the borrower not add risk. Again, this is the interesting problem since otherwise the bank would not want to prevent asset substitution.

Let $F^\#(\alpha, z) \equiv \arg \max_F (L_2 - \alpha c)G(F|V; z, \alpha) + F(1 - G(F|V; z, \alpha))$. This is the value of F that maximizes the bank's expected profit as of $t = 1$ for a borrower of type z . Let $F^\# = \inf\{F^\#(\alpha, z)\}$. Then:

Assumption 14. $F^\#$ is larger than any F_0 or F^N that the bank would consider.

This assumption ensures that bank profits are increasing in F over the relevant range. It is straightforward to extend the results of this article to the case where F_0 or F^N is larger than $F^\#$. Lenders can always forgive debt at $t = 1$ in order to ensure that they are on the upward sloping portion of the bank profit function. The assumption allows us to ignore this issue of forgiveness (which has no efficiency considerations). To avoid burdening the article with additional complexity, in what follows we will always assume that any F under consideration is less than $F^\#$.

Assumptions 12, 13, and 14 ensure that, whatever the equilibrium F turns out to be, we can choose parameters that are consistent with the characterizations in the analysis.

Appendix B

Proof of Lemma 1. The first part of the lemma says that there exists a trigger value of z , which we denote z^* , such that the borrower chooses $\alpha = 1$ if and only if $z \leq z^*$. That is, the moral hazard problem is more severe for those who get bad news. In the following discussion we use the notation $E_x[w(x, y)]$, where w is a function of random variables x and y , to indicate that the expectation is with respect to x alone. We first provide the following lemma.

Lemma A1. Let V and z be two random variables with joint distribution $G(V, z)$ and assume that the conditional distribution of z given V has the MLRP property. Let $\psi: R \rightarrow R$ be some continuous function that crosses zero only once, and from above. Then the function $\xi: R \rightarrow R$, $\xi(z) = E_V[\psi(V, F)|z]$ crosses zero at most once, and from above.

Proof. See Karlin (1968).

Recall that $\psi(V, F_0) \equiv E_\epsilon[\pi^F(V + \epsilon - c, F_0) - \pi^F(V, F_0)]$, where $\pi^F(w) = \max[w - F_0, 0]$ is the profit to the borrowing firm. We denoted the expected gain to a borrower of type z from switching from project $\alpha = 0$ to $\alpha = 1$ by $\Gamma(z)$. Hence $\Gamma(z) = E_V[\psi(V, F_0)|z]$. At $t = 1$, having observed z , the borrower chooses α to maximize profits. To prove Lemma 2 we apply Lemma A1 and need only show that $\psi(V, F_0)$ crosses zero only once, and from above. By Assumption 15, the upper bound of the support of ϵ is greater than $c + F_0$. We have

$$\psi(V, F_0) = \int_{c+F_0-V}^{\epsilon_h} [\epsilon - (c + F_0 - V)]h(\epsilon)d\epsilon - \max[V - F_0, 0].$$

We know that $V \leq F_0$ implies $\psi(V, F_0) > 0$. Further, since for $V > F_0$

$$\begin{aligned} \psi(V, F_0) &= \int_{c+F_0-V}^{\epsilon_h} \epsilon h(\epsilon)d\epsilon \\ &\quad - (c + F_0 - V)(1 - H(c + F_0 - V)) - (V - F_0), \end{aligned}$$

we have

$$\lim_{V \rightarrow \infty} \psi(V, F_0) = \lim_{V \rightarrow \infty} -VH(c + F_0 - V) - c < 0.$$

We also have, for $V > F_0$,

$$\frac{\partial \psi}{\partial V} = -H(c + F_0 - V) \leq 0.$$

Therefore, ψ has the desired properties, and we have proven the proposition.

We now turn to proving the second part of the lemma, that is, that z^* is increasing in F_0 . We have $\Gamma(z^*, F_0) = 0$ implicitly defining $z^*(F_0)$. To prove that z^* is increasing in F_0 , it suffices to show that

$$-\frac{\partial \Gamma}{\partial F_0} \bigg/ \frac{\partial \Gamma}{\partial z} > 0$$

evaluated at z^* and F_0 . By the proof of Lemma 1, we already know that $\partial \Gamma(z^*)/\partial z < 0$, since at z^* the function Γ crosses zero from above. So it remains to show that $\partial \Gamma(z^*, F_0)/F_0 > 0$. For this we need to see how $\psi(V, F_0)$ depends on F_0 . We have from before,

$$\psi(V, F_0) = \int_{c+F_0-V}^{\epsilon_h} [\epsilon - (c + F_0 - V)]h(\epsilon)d\epsilon - \max[V - F_0, 0],$$

which we now want to consider as a function of F_0 holding V fixed. But it is straightforward to verify that $\partial \psi/\partial F_0 > 0$. Hence $\partial \Gamma(z^*, F_0)/\partial F_0 = E[\partial \psi(V, F_0)/\partial F_0|z^*] > 0$. ■

Proof of Lemma 2. Define the gain to the bank from the borrower of type z adding risk to be $\Gamma_B(z; F) = E_V[\omega(V)|z]$, where

$$\begin{aligned} \omega(V, F) &= -c + [1 - H(F + c - V)](F - L_2) & \text{if } V < F \\ &= -H(F + c - V)(F - L_2 + c) & \text{if } V \geq F. \end{aligned}$$

$\omega(V)$ is discontinuous at $V = F$. Also $\omega(V)$ can be positive for $V < F$ in the vicinity of F . But, for given F , $\omega(V) < 0$, for all V , if $F < L_2 + c/[1 - H(c)]$. This cannot be true for all possible values of F , but for any given value it suffices that c or $H(c)$ be sufficiently large. But Assumption 15 states that $\epsilon_h > c + F$, and Assumption 16 states that $L_2 + c/[1 - H(c)] > F$. Thus $\omega(V)$ is assured of lying everywhere below zero. Recalling that ψ is the gain to the borrower, we have shown that $\psi + \omega$, which is the social gain, lies everywhere below ψ . ■

Proof of Proposition 1. We take the cases in reverse order. Part 4: First, we must show that $[z_{IL}, z_{RN}]$ exists. For $z > z_{IL}$, $\Gamma(z) > 0$ implies $\Pr(V > F_0) > 0$, that is, $\pi^T(z, \alpha = 1) > L_1$. That implies $\pi^U(F_0, z, \alpha = 1) > 0$. As $z \rightarrow z_{IL}$, $\pi^T(z, \alpha = 1) \rightarrow L_1$ and $\pi^U(F_0, z, \alpha = 1) < L_1$. Thus $[z_{IL}, z_{RN}]$ exists. In the interval $[z_{IL}, z_{RN}]$, $\pi^T(F^N, z, \alpha = 1) > L_1$, so the project should not be liquidated, but $\pi^U(F_0, z_{RN}, \alpha = 1) < L_1$, that is, at the unrenegotiated contract the bank would be better off liquidating the project. Thus, $F^N = F_0$ is not optimal. The fact that $z_{RN} < z^{**}$ means that $\pi^R(F^-, z, \alpha = 0) < \pi^U(F_0, z, \alpha = 1)$. Therefore, forgiving some of the debt by lowering the interest rate cannot be optimal. Hence, the project is profitable even if the borrower chooses $\alpha = 1$, and the bank sets $F^N = F^{++}(z)$, that is, raises the interest rate. Part 3: The borrower will choose $\alpha = 1$ because $z < z^*$, but the bank cannot raise the interest rate because it has no credible threat since $z > z_{RN}$. $\pi^R(F^-, \alpha = 0, z) < \pi^U(F_0, \alpha = 1, z)$ because $z < z^{**}$, so debt forgiveness is not optimal. Since $\pi^U(F_0, \alpha = 1, z) > L_1$, the best the bank can do is maintain the current contract. Part 2: In this range borrowers choose to add risk, $\alpha = 1$, since $z < z^*$, but the bank has no credible liquidation threat since $z_{RN} < z^{**}$. However, assuming the interval $[z^{**}, z^*]$ exists, lowering the interest rate results in $\pi^R(F^-, z, \alpha = 0) > \pi^U(F_0, z, \alpha = 1)$. Part 1: Borrowers in this range do not add risk and the bank has no credible threat. Thus the best the bank can do is maintain the initial contract. ■

Appendix C

Renegotiation outcomes for the intermediate F_0 case

The intermediate F_0 case is the situation where $z^{**} < z_{RN} < z^*$. Liquidation occurs for $z < z_{IL}$.

Proposition B1. If $z^{**} < z_{RN} < z^*$, then renegotiation results in:

- (i) $F^N(z) = F^+(z) > F_0$, for all $z \in [z_{IL}, z_{RN}]$; that is, raise rate; borrower chooses $\alpha = 1$.
- (ii) $F^N(z) = F^-(z) < F_0$, for all $z > z_{RN}$; that is, forgive debt; borrower chooses $\alpha = 0$.

Proof. Part 1: For $z \in [z_{IL}, z_{RN}]$ the borrower will choose $\alpha = 1$, ceteris paribus. Liquidation is not optimal for these borrowers since $z > z_{IL}$. Also, because $z < z_{RN}$, $\pi^U(F, z, \alpha = 1) < L_1$, so maintenance of the initial contract is not optimal. Since $z < z_{RN}$ the bank can credibly threaten the borrower. By Assumption 11, $\pi^R(F^{++}, z, \alpha = 1) > \pi^R(F^+, z, \alpha = 0)$, that is, it is more profitable for the bank to raise the rate by so much that the borrower chooses $\alpha = 1$, rather than raise the rate to the point where the maximum surplus is extracted and the borrower chooses $\alpha = 0$. So the bank raises the interest rate and the borrower chooses $\alpha = 1$. Part 2: For $z > z^*$ the project is profitable and the borrower will choose $\alpha = 0$, ceteris paribus. The bank cannot threaten the borrower since $z_{RN} < z^*$, so the initial contract is maintained. ■

Renegotiation outcomes for the low F_0 case

The low case is the situation where $z^{**} < z^* < z_{RN}$, that is, unrenegotiated bank profits are less than the liquidation value starting at borrower types higher than the type at which there is an incentive to switch projects and add risk. In this situation the bank can credibly threaten to liquidate borrowers who have no intention of switching projects (in addition to those who do).

Proposition B2. If $z^{**} < z^* < z_{RN}$, then renegotiation results in the following outcomes:

- (i) $F^N(z) = F^+(z) > F_0$ for all $z \in [z_{IL}, z_{RN}]$; that is, raise rate; borrower chooses $\alpha = 1$;
- (ii) $F^N(z) = F_0$ for all $z > z_{RN}$; that is, no change; borrower chooses $\alpha = 0$.

Proof. Similar to Proposition B1. ■

Alternatives to Assumption 11

Assumption 11 assumed that $\pi^R(F^N, z, 1) > \pi^R(F^N, z, 0)$ for all z and F . We now briefly reconsider Propositions 1, B1, and B2, when Assumption 11 is not assumed. The first alternative to Assumption 11, subcase 1, occurs when $\pi^R(F^N, z, 1)$ cuts $\pi^R(F^N, z, 0)$ from above at a point $\hat{z}$ such that $z_{IL} < \hat{z} < z_{RN}$. For this case:

Proposition B3. *If $z_{RN} < z^{**} < z^*$, and subcase 1, then renegotiation results in:*

- (i) $F^N(z) = F^{++}(z) > F_0$, for all $z \in [z_{IL}, \hat{z}]$; that is, raise rate; borrower chooses $\alpha = 1$.
- (ii) $F^N(z) = F^+(z) < F_0$, for all $z \in [\hat{z}, z_{RN}]$; that is, raise the rate but such that the borrower chooses $\alpha = 0$.
- (iii) $F^N(z) = F_0$, for all $z \in [z_{RN}, z^{**}]$; that is, no change; borrower chooses $\alpha = 1$.
- (iv) $F^N(z) = F^-(z)$, for all $z \in [z^{**}, z^*]$; that is, forgive debt; borrower chooses $\alpha = 0$.
- (v) $F^N(z) = F_0$, for all $z > z^*$; that is, no change; borrower chooses $\alpha = 0$.

Proof. Part 1: For $z \in [z_{IL}, \hat{z}]$ the borrower is choosing $\alpha = 1$. Liquidation is not optimal since $z > z_{IL}$. Since $\hat{z} < z_{RN}$, $\pi^U(F, z, \alpha = 1) < L_1$, so maintenance of the initial contract is not optimal. By the definition of subcase 1, $\pi^R(F^{++}, z, \alpha = 1) > \pi^R(F^+, z, \alpha = 0)$ so the bank raises the interest rate. Part 2: As above, neither liquidation nor maintenance of the initial contract is optimal. But, in this range, by the definition of subcase 1, $\pi^R(F^{++}, z, \alpha = 1) < \pi^R(F^+, z, \alpha = 0)$ so the bank raises the rate as far as possible while maintaining the incentive for the borrower to choose $\alpha = 1$. Part 3: In this range the bank can no longer credibly threaten the borrower so raising the rate is not feasible. Forgiveness is not profitable for the bank (by definition of z^{**}). So the rate does not change and the borrower chooses $\alpha = 1$. Part 4: Now it is profitable to forgive debt so that the borrower chooses $\alpha = 0$. Part 5: In this range the borrower will choose $\alpha = 0$, ceteris paribus. The bank has no credible threat to liquidate and cannot raise the rate. The rate stays the same and the borrower chooses $\alpha = 0$. ■

Subcase 2 is the situation where $z_{RN} < \hat{z} < z^{**} < z^*$. In this case, the result is the same as above since the bank cannot threaten to liquidate borrowers of type $\hat{z} \in [\hat{z}, z_{RN}]$. Subcase 3 is $z_{RN} < z^{**} < \hat{z} < z^*$. Again, there is no change, for the same reason. The same is true for the case where $z_{RN} < z^{**} < z^* < \hat{z}$. The final possibility is the case where $\pi^R(F^N, z, 1) < \pi^R(F^N, z, 0)$ for all z and F , the opposite assumption of Assumption 11. In this case, it can easily be shown that the borrower never adds risk, since it is always profitable for the bank to forgive rather than raise the rate.

For the intermediate and low F_0 cases there are similar, straightforward variations when we deviate from Assumption 11. These are omitted for the sake of space.

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