Risk Spillovers and Required Returns in Capital Budgeting

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This article integrates strategic product market analysis with price-taking asset pricing theory. We demonstrate that a firm's market power can lead to scale-dependent and potentially infinite required returns. Scale dependency, which we relate to risk spillovers between expansionary and existing cash flows, reflects the divergence of incremental from existing required returns. The firm-specific nature of risk spillovers potentially destroys the concept of a common industry "risk class." Our analysis raises important questions regarding the validity of widely used "comparables" methods for determining risk-adjusted discount rates.

For decades, finance students and practitioners have been taught the technique of valuation by discounting cash flows. The standard approach takes an investment's estimated expected cash flows and discounts them at an estimated required rate of return. While project managers—with a comparative advantage in determining product market strategy, production, and revenue levels—estimate expected cash flows, it is common to have another party estimate required returns. This party typically examines the historical relationship between the firm's (or a comparable firm's) returns and factors deemed relevant for a specific asset pricing model. The two pieces are combined using a discounted cash flow technique to value a proposed investment.

This standard approach to project valuation typically involves the use of a constant discount rate across all production possibilities.[1] For exam-

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[1] For a review of the standard approach, see, for example, Copeland, Kuller, and Murrin (1996).

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ple, when using the capital asset pricing model (CAPM), beta is assumed constant across different production plans, cost scenarios, and competitor reactions. A project's risk characteristics, however, necessarily derive from fundamental demand and supply considerations in the product and factor markets. The profits of a project, if any, can more accurately be considered as the rents from market power in these markets. We demonstrate that a careful analysis of the risk characteristics of future economic rents will typically yield required returns which vary with scale. In fact, firms expanding their output alter (1) their own expected profit and the expected profits of all firms sharing the product and factor markets in which the firm has market power; and (2) their own required return and that applied to other firms sharing the affected markets. The erosion of existing revenue (own and competitor) is a basic concept introduced in the microeconomic theory of the firm. That one firm's production decision affects other firms' profitabilities through a "spillover" is a cornerstone of industrial organization theory. Our analysis concentrates on the class of less well-understood interactions which we term *risk spillovers*: expansion-induced changes in the risk and, therefore, the value of existing (own and competitor) production. In situations where market power introduces risk spillovers within or between firms, the use of scale-invariant, industry or historical required returns to value entry or expansion is generally inappropriate. Since most, if not all, firms exhibit some degree of non-price-taking (market power) in product or factor markets, the robustness of the standard valuation approach deserves closer scrutiny.[2]

Our analysis combines the strategic orientation of industrial organization theory with the widely accepted price-taking behavior of asset pricing theory. We introduce common and *endogenous* risk in product profits while maintaining a valuation framework which prices only relevant risks. In this context, valuation procedures which separate cash flow estimation from discount rate determination are shown to be conceptually flawed. Specifically, because expected cash flows *and* required returns are both driven by the industry's production configuration, there is no natural division of labor: the estimation of required returns (denominators) necessitates detailed knowledge of the product and factor markets; the selection of optimal scale and, therefore, expected cash flows (numerators) necessitates detailed knowledge of how required returns are determined. Product market strategists should have a model of equilibrium required returns as they contemplate various (own and other-firm) output levels.[3] Financial analysts estimating required returns need a model of imperfectly competitive product and factor

[2] Perfect product and factor market competition is a suspicious assumption for firms that have the wherewithal to conduct project evaluation. Perfectly competitive markets do not permit profits sufficient to recover the sunk costs associated with project evaluation.

[3] Our analysis shows that even when given the correct required return at optimal production, a strategist using that required return will not select the optimal level of production.

market equilibrium. Historical estimates of required returns which ignore variations in an imperfectly competitive firm's product and factor markets are therefore inadequate.

Work on this particular type of product and financial market interaction appears to be sparse.[4] However, the insight that product market power may relate to a firm's required returns is not entirely new. Thomadakis (1976) analyzes the role of monopoly power in determining a firm's returns under optimized output when its cost and (constant elasticity) demand functions are related to market movements.[5] Subrahmanyam and Thomadakis (1980) further develop this line of analysis by providing a structural model for a CAPM beta under related assumptions and conclude that the beta of an optimizing firm is a decreasing function of its market power as measured by the Lerner index. We begin with a very general specification of the product and securities markets which includes the Thomadakis (1976) and Subrahmanyam and Thomadakis (1980) product market model as a subset. After establishing that changes in required returns typically accompany changes in scale, we demonstrate that the magnitude of necessary adjustments depends directly on one's own scale and on that of competitors. Quantity-dependent revenue and risk spillovers within and between firms complicate the task of deriving required returns. For a given level of change in aggregate output, firms with different levels of existing output will, in general, have different hurdle rates for the change in output.

The article is structured as follows: Section 1 motivates scale dependencies in required returns within the traditional CAPM context and more generally. Section 2 presents specific CAPM examples. Section 3 extends the analysis to general asset pricing beyond the CAPM framework. Section 4 discusses the implications our findings have for calibrating the value of entering or expanding existing product markets and concludes.

1. Scale Dependencies in a CAPM World

It is commonplace for finance textbooks to emphasize the connection between systematic risk or beta and the general nature of a firm's product market. For example, it is widely understood that utilities and semiconductor firms have different levels of systematic risk and hence the discount rates used to evaluate projects in these industries necessarily differ. There is also widespread recognition that, when a firm is not a price taker in selling

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[4] Most of the recent product and financial market interaction literature has focused on cross-market signaling, a subject we do not address in this article. Earlier literature, which has been summarized by Baron (1979), considered stock market economies with production, endogeneity of the market portfolio and shareholder unanimity. We consider only the case of price-taking in the security market and thereby avoid the unanimity and efficiency questions that motivated this previous literature.

[5] We compare and contrast our analysis with that of Thomadakis in Section 2.1.5.

output or purchasing inputs, it should consider the effect of an incremental expansion on the price and cost of existing output. In most texts, the absence of price-taking in the sale of products is treated with a digression on the erosion of existing revenues. However, the typical discussion of expansion in the same industry widely assumes that the discount rate applied to the marginal cash flows can be obtained by estimating the systematic risk of existing cash flows.[6] Our analysis, in contrast to the typical treatment, focuses on the variation in discount rates due to the choice of scale. We document the loss of generality in applying constant discount rates when valuing different scales of operation in a given market or industry. We demonstrate that identifying risk classes by appealing to existing industry (comparable) required returns may be quite misleading.

Consider a firm involved in a multifirm investment and production game. We simplify and clarify the analysis of scale choice by addressing a firm facing residual demand and cost curves.[7] Let the state of the world at the time production is sold be represented by ω. Assume that the firm has invested I in fixed assets that support a range of possible output quantities q up to a capacity of $q_{max}(I)$, subject to incurring investment-contingent average costs of $c(q, \omega, I)$. The firm faces a residual demand curve denoted by $p(\omega, q, I)$ where dependence on I reflects the potential for different competing producers' responses to the firm's investment commitment (and capacity). The firm's profit in state ω under quantity strategy q is

$$\pi(\omega, q, I) = p(\omega, q, I)q - c(\omega, q, I)q. \tag{1}$$

Since our emphasis is imperfect product and factor—rather than security—markets, we assume that the firm is atomistic in the security market. That is, the firm ignores the effect its production strategy (and competitors' production strategies) has on the statistical properties of security market returns.[8] Accordingly, agents value a state-contingent future dollar at a totally exoge-

[6] See, for example, the treatment in Brealey and Myers (1996, pp. 279–282) in which the price externality on existing output is accounted for but the discount rate is assumed to be the same as that applicable to preexpansion cash flows.

[7] The residual demand and cost specifications are reduced forms for several versions of imperfect competition, including quantity and price competition with imperfect substitutes.

[8] It is well known that if firms consider their endogenous influence on security market investment opportunities, maximizing firm value may not be in the stockholders' best interests. An extensive literature exists on the appropriate objective function for a firm whose claims are traded in CAPM-conforming capital markets where firms consider security market endogeneity. See, for example, Fama (1972), Jensen and Long (1972), Stiglitz (1972), Ekern and Wilson (1974), Merton and Subrahmanyam (1974), and Grossman and Stiglitz (1977), and the survey by Baron (1979). However, this literature analyzes a distinctively different type of risk spillover from that which we consider. In these earlier articles, firms that can influence the return on the market portfolio through their output strategies impose risk spillovers on all other firms' shareholders through the (partially) endogenous statistical properties of market returns. In contrast, our firms are price takers in the securities market and do not consider their influence, if any, on other firms' shareholders through endogeneity in security market returns. The risk spillovers we analyze derive from the firms' strategic role in the markets related to producing and selling their products.

nous current price. Such a structure combined with the absence of arbitrage (and some regularity assumptions) yields the standard pricing kernel representation of value:

$$\begin{aligned} V[\pi(\omega, q, I)] &= E[\xi(\omega)\pi(\omega, q, I)]/R_f \\ &= E[\xi(\omega)\,(p(\omega, q, I)q - c(\omega, q, I)q)]/R_f, \end{aligned} \tag{2}$$

where $0 < R_f < \infty$ is the gross return on the risk-free asset and $\xi(\omega)$ is R_f times the ratio of the state price of ω to its probability density.[9] Note that $V[.]$ is the present value operator. Determining an optimal (subgame-perfect) postinvestment quantity requires finding a $q^* \in [0, q_{\max}(I)]$ such that $V[\pi(\omega, q, I)]$ is maximized at $q = q^*$. After the optimal quantity policy $q^*(I)$ is determined in this fashion, it is possible to back up and determine the optimal level of investment I which maximizes the "net present value" of $V[\pi(\omega, q^*(I), I)] - I$, given the optimal policy $q^*(I)$. Because our primary concern is characterizing the present value operator $V[.]$ for various choices of q, we will suppress functional dependence on I except where needed for clarity. By explicitly working with this postinvestment subgame, we differentiate our contribution from existing intuitions relating required returns to choices of technology, fixed versus variable costs and operating leverage. In addition, characterizing the postinvestment (present value) subgame is a prerequisite to a proper analysis of the more complex (net present value) game incorporating investment and scale interactions.[10] We indicate by a tilde those variables that depend on ω.

The derivation of $\tilde{\xi}$ in a CAPM world is straightforward and yields [see, e.g., Dybvig and Ingersoll (1982)]

$$\tilde{\xi} = 1 - \lambda\left(\tilde{R}_m - \overline{R}_m\right),$$

where $\overline{R}_m$ is the mean of the market return and $\lambda = (\overline{R}_m - R_f)/\operatorname{var}[\tilde{R}_m]$. Combining these elements we see that

$$\begin{aligned} V[\tilde{\pi}(q)] &= E[\tilde{\xi}\tilde{\pi}(q)]/R_f \\ &= \frac{1}{R_f}\left\{E[\tilde{\pi}(q)] - \lambda\operatorname{cov}[\tilde{\pi}(q), \tilde{R}_m]\right\}, \end{aligned} \tag{3}$$

which is the standard certainty equivalent formulation of the CAPM, except that dependence (or lack thereof) on q is explicit. By writing the standard

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[9] This normalized "price per unit probability" ratio is the relevant portion of a Radon–Nikodym derivative which transforms the true probability distribution of ω to its "risk-neutralized" distribution. In what follows, we use the fact that this transformation creates a new *probability* measure, namely $E[\xi(\omega)] = 1$.

[10] As is widely appreciated, investment strategy can alter demand and cost structure. Additional variation in discount rates introduced by interactions between investment and the product and factor market strengthens our main thesis that discount rates depend on scale.

CAPM equation for equilibrium required returns as

$$R(q) = \frac{E[\tilde{\pi}(q)]}{V[\tilde{\pi}(q)]} = R_f + \beta(q)(\overline{R}_m - R_f) \tag{4}$$

we can see that equilibrium beta can be written as

$$\beta(q) = \left[\frac{E[\tilde{\pi}(q)]}{V[\tilde{\pi}(q)]} - R_f\right] \Big/ \left(\overline{R}_m - R_f\right), \tag{5}$$

from which it is immediately apparent that beta will vary with q's if and only if equilibrium required returns $R(q)$ do.[11] Substituting for $V[\tilde{\pi}(q)]$ using Equation (3) gives

$$\beta(q) = \frac{\lambda \operatorname{cov}[\tilde{\pi}(q), \tilde{R}_m]}{E[\tilde{\pi}(q)] - \lambda \operatorname{cov}[\tilde{\pi}(q), \tilde{R}_m]} \left(\frac{R_f}{\overline{R}_m - R_f}\right). \tag{6}$$

1.1 Scale Dependency in Beta

While it is fundamental that beta varies with production quantity if and only if required returns do, a more interesting characterization of this scale invariance condition is given by:

Proposition 1. *Necessary and sufficient conditions for a nonzero scale-invariant beta are (i) there exists q such that* $\beta(q) \neq 0$*; and (ii) expected profits are proportional to the covariance of profits with market returns:*

$$E[\tilde{\pi}(q)] \propto \operatorname{cov}[\tilde{\pi}(q), \tilde{R}_m].$$

Proof. Equation (5) has already established that proportionality between $E[\tilde{\pi}(q)]$ and $V[\tilde{\pi}(q)]$ is necessary and sufficient for scale-invariant betas. Thus betas are scale invariant if and only if there exists κ such that

$$E[\tilde{\pi}(q)] = \kappa V[\tilde{\pi}(q)], \text{ for all } q.$$

Substitution for $V[\tilde{\pi}(q)]$ yields

$$E[\tilde{\pi}(q)] = \kappa \left[\frac{E[\tilde{\pi}(q)] - \lambda \operatorname{cov}[\tilde{\pi}(q), \tilde{R}_m]}{R_f}\right].$$

Since we have assumed that a q exists such that $\beta(q) \neq 0$, we know that $\kappa \neq R_f$, which permits us to display the linear relationship between

[11] While we are stressing the equilibrium nature of our claims, it is important to realize the practical nature of the question we ask: What is the equilibrium discount rate yielding a present value equal to the true (security market equilibrium) value of future cash flows? Calibrating this true value is the purpose of a valuation exercise.

expected profits and covariance as

$$E[\tilde{\pi}(q)] = \frac{\kappa\lambda}{\kappa - R_f}\,\mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m].$$

Thus $K = \kappa\lambda/(\kappa - R_f)$ is the proportionality constant relating expected profits to the covariance of profits with market returns.[12] ■

An economic intuition for the proportionality result is best obtained by recasting the result in the format of quantity elasticities. Differentiating Equation (6) with respect to q and rearranging gives

$$\frac{d\beta(q)}{dq} = \frac{\lambda\left(E[\tilde{\pi}(q)]\frac{d}{dq}\,\mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m] - \mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m]\frac{d}{dq}E[\tilde{\pi}(q)]\right)}{R_f(\overline{R}_m - R_f)V[\tilde{\pi}(q)]^2}.$$

For a q with nonzero expected profits and covariance we can define quantity elasticities by

$$\begin{aligned}
\epsilon(E[\tilde{\pi}(q)]) &\equiv \frac{q}{E[\tilde{\pi}(q)]}\frac{dE[\tilde{\pi}(q)]}{dq}\\
\epsilon(\mathrm{cov}[\tilde{\pi}(q)]) &\equiv \frac{q}{\mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m]}\frac{d\,\mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m]}{dq}.
\end{aligned}$$

We can write the derivative as

$$\begin{aligned}
\frac{d\beta(q)}{dq} &= \frac{\lambda E[\tilde{\pi}(q)]\,\mathrm{cov}[\tilde{\pi}(q), \tilde{R}_m]}{qR_f(\overline{R}_m - R_f)V[\tilde{\pi}(q)]^2}\\
&\quad \times \left(\epsilon(\mathrm{cov}[\tilde{\pi}(q)]) - \epsilon(E[\tilde{\pi}(q)])\right), \qquad (7)
\end{aligned}$$

which is zero at every feasible $q > 0$ if and only if the quantity elasticities of expected profits and covariance with market returns are everywhere (in q) equal. Alternatively, when the elasticity of covariance is greater (less) than that of expected profits, we would expect the marginal beta to be higher (lower) than the average beta.

In the presence of imperfect competition in the product or factor market, we would expect the elasticity of expected profits to change with quantity.

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[12] In the degenerate case where the firm is free of systematic risk at every q, beta is constant although expected profits are not proportional to the zero covariance with market returns. In this case, however, risk-adjusted discounting is not a interesting issue. In what follows, we refer to the nondegenerate cases as “risky firms.”

To impose the additional constraint of an identical quantity variation in the elasticity of covariance with market returns requires very restrictive assumptions about either the structure of the product and factor markets or the nature of risk. Before introducing examples we document that scale variation in required returns necessitates some quantity dependence in prices or costs. Specifically, consider $p(\omega, q) = \tilde{p}$ and $c(\omega, q) = \tilde{c}$:

Corollary 1. *A firm that takes unit costs and product prices as given and independent of the firm's production strategy has a beta that does not vary with scale.*

Proof. Note $\tilde{\pi}(q) = \tilde{p}q - \tilde{c}q = q(\tilde{p} - \tilde{c})$. The ratio of expected profits to covariance is

$$\frac{E[\tilde{\pi}(q)]}{\text{cov}[\tilde{\pi}(q), \tilde{R}_m]} = \frac{qE[\tilde{p} - \tilde{c}]}{q\,\text{cov}[\tilde{p} - \tilde{c}, \tilde{R}_m]} = \frac{E[\tilde{p} - \tilde{c}]}{\text{cov}[\tilde{p} - \tilde{c}, \tilde{R}_m]},$$

which does not depend on q. By Proposition 1, beta does not vary with q. ■

Scale-independence in prices and costs eliminates the need to consider scale variation in beta. A beta that varies with scale dictates some more basic dependence of price and cost on scale. Scale dependency in price reflects market power and the absence of price-taking. Scale dependency in costs derives from market power in the factor markets or from economies of scale in total costs. A more general condition that must be violated for scale variation in beta to occur involves the separability of uncertainty from scale:

Corollary 2. *A sufficient condition for scale-invariant betas is that the profit function be multiplicatively separable in ω and q.*

Proof. Under multiplicative separability there exist $\mu(\omega)$ and $n(q)$ such that $\pi(\omega, q) = \mu(\omega)n(q)$. The ratio of expected profits to covariance is

$$\frac{E[\tilde{\pi}(q)]}{\text{cov}[\tilde{\pi}(q), \tilde{R}_m]} = \frac{n(q)E[\tilde{\mu}]}{n(q)\,\text{cov}[\tilde{\mu}, \tilde{R}_m]} = \frac{E[\tilde{\mu}]}{\text{cov}[\tilde{\mu}, \tilde{R}_m]},$$

which does not depend on q. By Proposition 1, beta does not vary with q. ■

Each of the following examples fails to separate multiplicatively in ω and q due to market power in the product market.

2. CAPM Examples

2.1 Linear Demand

Consider the case of systematically risky linear product demand and constant (over q) average cost of the forms[13]:

$$\begin{aligned}
\tilde{p}(q) &= \tilde{a} - \tilde{b}q \\
&= \left[\overline{a} + \beta_a \left(\tilde{R}_m - \overline{R}_m\right) + \tilde{\epsilon}_a\right] \\
&\quad - \left[\overline{b} - \beta_b \left(\tilde{R}_m - \overline{R}_m\right) - \tilde{\epsilon}_b\right] q \\
\tilde{c}(q) &= \tilde{c} \\
&= \overline{c} + \beta_c(\tilde{R}_m - \overline{R}_m) + \tilde{\epsilon}_c,
\end{aligned}$$

where $\overline{a} = E[\tilde{a}] > 0$, $\overline{b} = E[\tilde{b}] > 0$, $\overline{R}_m = E[\tilde{R}_m] > R_f$, $\overline{c} = E[\tilde{c}] > 0$, and all ϵ terms are mean zero and orthogonal to market returns. Note that our use of security market returns in defining the product demand curve does not imply causality of any kind. We consider this functional relationship to be a reduced form of a more explicit model where factors drive all demand functions. Hence a realized higher stock market return is associated with a higher level of economic activity that is characterized by suitable demand shocks throughout the economy. For our linear product demand

$$E[\tilde{\pi}] = q(\overline{a} - \overline{c} - \overline{b}q) \tag{8}$$

$$\operatorname{cov}[\tilde{\pi}, \tilde{R}_m] = q \operatorname{var}[\tilde{R}_m]\,(\beta_a - \beta_c + \beta_b q) \tag{9}$$

and therefore from Equation (3)

$$V[\tilde{\pi}] = q\,[A - Bq]\,/R_f, \tag{10}$$

where

$$\begin{aligned}
A &= \overline{a} - \overline{c} - (\beta_a - \beta_c)(\overline{R}_m - R_f) \\
B &= \overline{b} + \beta_b(\overline{R}_m - R_f).
\end{aligned}$$

The return beta at profit $\tilde{\pi}$, $\beta_{\tilde{\pi}}$, is[14]

$$\beta_{\tilde{\pi}} = \frac{\operatorname{cov}[\tilde{\pi}, \tilde{R}_m]}{V[\tilde{\pi}]\operatorname{var}[\tilde{R}_m]} = \frac{[\beta_a - \beta_c + \beta_b q]\,R_f}{A - Bq}.$$

[13] While our examples emphasize (deemphasize) demand (cost) curve sources of variation in discount rates, it should be noted that we could have generated a wholly analogous set of examples by considering price influence on factor costs rather than on output revenue.

[14] Since we have considered the case of constant (over q) marginal costs, the formula for beta makes clear that our results are not being driven by operating leverage.

For discrete scale changes of Δq which induce change in cash flows of $\Delta\tilde{\pi}$, we can apply Equations (8) and (9) to the starting level q and the notional postexpansion level $q + \Delta q$ and take differences to get

$$E[\Delta\tilde{\pi}] = \Delta q \left(\overline{a} - \overline{c} - \overline{b}(2q + \Delta q)\right) \tag{11}$$

$$\mathrm{cov}[\Delta\tilde{\pi}, \tilde{R}_m] = \Delta q\, \mathrm{var}[\tilde{R}_m]\,(\beta_a - \beta_c + \beta_b(2q + \Delta q))$$

which leads to

$$\begin{aligned} V[\Delta\tilde{\pi}] &= \frac{1}{R_f}\left\{E[\Delta\tilde{\pi}] - \lambda\,\mathrm{cov}[\Delta\tilde{\pi}, \tilde{R}_m]\right\} \\ &= \frac{\Delta q}{R_f}\{(A - B(2q + \Delta q))\} \end{aligned} \tag{12}$$

and, if we denote the beta for the expansion by $\beta_{\Delta\tilde{\pi}}$ then

$$\beta_{\Delta\tilde{\pi}} = \frac{\mathrm{cov}[\Delta\tilde{\pi}, \tilde{R}_m]}{V[\Delta\tilde{\pi}]\,\mathrm{var}[\tilde{R}_m]} \tag{13}$$

$$= \frac{(\beta_a - \beta_c + \beta_b(2q + \Delta q))R_f}{A - B(2q + \Delta q)}. \tag{14}$$

To be even more concrete, consider the case of nonstochastic constant marginal costs. In particular let $\overline{a} = 5$, $\overline{b} = 1$, $\beta_a = 1$, $\beta_b = 3$, $R_f = 1$, $\overline{R}_m - R_f = .08$, $\overline{c} = 1$, and $\beta_c = 0$. From Equation (10) it can be easily shown that optimality is attained at

$$q^* = \frac{49}{31} \doteq 1.58065.$$

Together Equations (8) and (10) give $E[\tilde{\pi}(\frac{49}{31})] = \frac{3675}{961} \doteq 3.82414$, $\beta_{\frac{49}{31}} = 4450/1519 \doteq 2.92956$, $R_{\frac{49}{31}} - 1 \doteq 23.4\%$, $V[\tilde{\pi}(\frac{49}{31})] = \frac{2401}{775} \doteq 3.09806$.

Now consider a currently suboptimal output of 1.3, where $E[\tilde{\pi}(1.3)] = 3.51$, $\beta_{1.3} = 2.12305$, $R_{1.3} - 1 \doteq 17\%$, $V[\tilde{\pi}(1.3)] = 3.0004$, as compared with the level achieved after the notional 20% expansion to 1.56: $E[\tilde{\pi}(1.56)] = 3.8064$, $\beta_{1.56} = 2.8606$, $R_{1.56} - 1 \doteq 22.8\%$, $V[\tilde{\pi}(1.56)] = 3.09754$. The contemplated expansion would increase the systematic risk of all of the company's product market cash flows. It might appear that using a constant beta (at either 2.12306 or 2.8606) or risk-adjusted discount rate (at 17% or 22.8%) might be a good approximation for valuing the expansion project. However, this would overstate expansion value by around 150%. To see this we move to the incremental required returns. From Equations (11), (12), and (14) we see that $E[\Delta\tilde{\pi}] \doteq .2964$, $V[\Delta\tilde{\pi}] \doteq .09714$, $\beta_{\Delta\tilde{\pi}} \doteq 25.6424$, $R_{\Delta\tilde{\pi}} - 1 \doteq 205\%$. To see how poorly the historical dis-

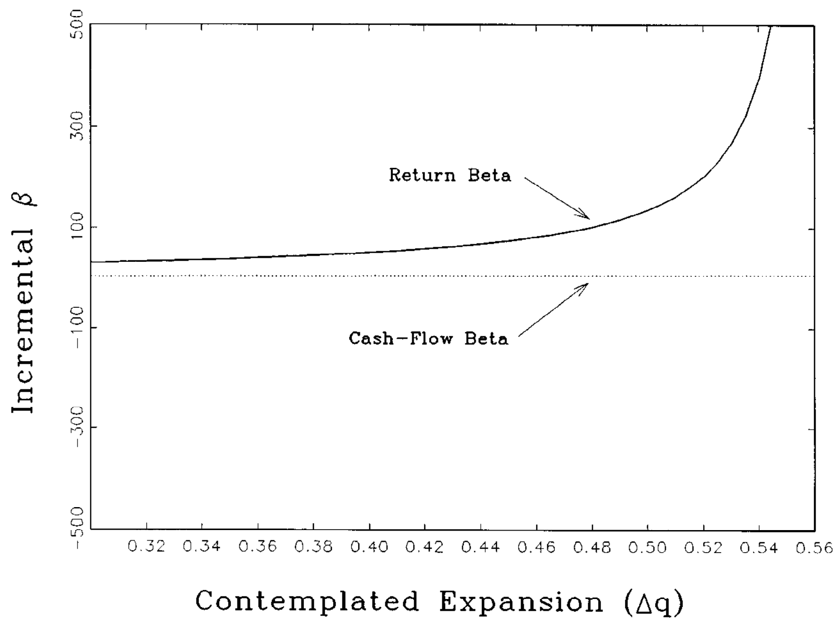

Figure 1
Variations in beta with increases in production

count rate performs, note that the ratio of calibrated to true value is

$$\frac{E[\Delta\tilde{\pi}]/R_{1.3}}{V[\Delta\tilde{\pi}]} = \frac{.2964/1.17}{.09714} \doteq 2.61.$$

Similarly, the new projected firm hurdle rate grossly overstates the value of expansion:

$$\frac{E[\Delta\tilde{\pi}]/R_{1.56}}{V[\Delta\tilde{\pi}]} = \frac{.2964/1.228}{.09714} = 2.49.$$

Figure 1 plots the behavior of $\beta_{\Delta\tilde{\pi}}$ for $q = 1.3$ as the planned expansion Δq varies. Note that expansions up to a total output of 1.8613—corresponding to Δq of .5613—will have positive present values ($V[.] > 0$) and betas.[15] The figure displays the potential for infinite required returns and vividly demonstrates that using historical required returns can provide an arbitrarily poor approximation of true incremental required returns.

[15] The value maximizing expansion is of course .28065. However, since expanding to this level creates value, the firm can expand beyond this level and still have a positive expansion value.

It is fairly easy to see that applying anything approximating the existing or pro forma overall required return (or beta) will result in serious overvaluation and the potential for misguided overexpansion.[16] In other linear examples, undervaluation and misguided contraction may occur. Since most firms using discounted cash flow employ some historical estimate of the assumed constant return beta they may be subject to gross valuation errors.

In contrast, Figure 1 also plots the much less volatile cash flow beta (β_{cf}) which can be used in conjunction with certainty-equivalent valuation [following Equation (3)]. However, the superiority of a cash flow beta is not assured. To see this we return to the numerical example which, at the existing output of 1.3, had a return beta of 2.12306. The related cash flow beta is the product of the return beta times the value at a quantity of 1.3, $\beta_{cf} = 2.12306 * 3.0004 = 6.3700$. Applying this cash flow beta to the 20% expansion project gives a calibrated value of

$$\frac{E[\Delta\tilde{\pi}] - (\overline{R}_m - R_f)\beta_{cf}}{R_f} = \frac{.2964 - (.08)(6.3700)}{1} = -.2132$$

as compared to the true expansion value of .09714. Using the historical cash flow beta underestimates the value of the expansion project because the marginal profit is diminishing rapidly and thereby contracting the cash flow beta to 2.4285 instead of the current 6.3700. The certainty equivalent adjustment is thus extremely overstated and the value correspondingly extremely understated at a factor of -2.19 times the true value. The cash flow beta estimation error of 3.9415 (and percent error of 162%) compares to the return beta estimation error of -23.5194 (and percent error of -91.72%). The valuation errors associated with the use of cash flow beta are as extreme and, at least in this example, if not recognized as such, could result in foregoing a value-increasing expansion. Therefore, using a historical cash flow beta does not solve the problem of differentiating marginal risk from average risk.

2.2 Constant Elasticity: The Thomadakis Case

Our linear demand example demonstrates the potentially extreme variations that can be encountered when considering the value of expansionary cash flows for firms with scale-dependent betas. As we mentioned in the introductory remarks, Thomadakis (1976) also considers the relationship of product market power to required returns. Using a constant elasticity demand function and scale invariant marginal cost, Thomadakis (1976) establishes that

[16] Of course we have structured this example so that the contemplated expansion has positive present value. In the absence of an increase in the required level of investment I, true present value is not destroyed by moderate expansion; it is merely misperceived when the historically estimated beta is used. This same inflated present value, however, can provide a falsely positive *net* present value when netted against an expense for capacity expansion (increased I).

required returns under optimal production vary with a market power parameter. While such a result would also emerge from our model, our primary focus is on the scale variation of betas for any *given level* of market power. Despite the difference in emphasis, the two models are closely related.

To see this, note that Thomadakis's formulation for revenue is of the form

$$\text{Revenue} = f(u, n)a(\omega)q^{1-n}$$

where $f(u, n) = (1 - un)/(1 - n)$; u is Thomadakis's market power parameter, a is the stochastic component of the demand curve, and n is the elasticity of demand. Cash flow is, therefore, given by

$$\pi(\omega, q, u, n) = f(u, n)a(\omega)q^{1-n} - c(\omega)q.$$

Thomadakis (1976) demonstrates that this form of cash flow dictates CAPM required returns that vary systematically with the market power parameter, u, when production quantities are determined optimally. Since he treats specific demand and cost functions, Thomadakis's analysis is not concerned with the precise conditions under which such dependence occurs. It is not hard to show, however, that, for Thomadakis's model, when β depends on market power u, it also depends on scale q. Put differently, whenever multiplicative separability obtains, we have shown that required rates of return do not depend on q. By implication, required returns also do not depend on u.[17] Thus the dependence of required return on market power is intimately related to its dependence on scale (for a given market power).

Thomadakis's analysis raises the point that required returns may depend on product market power. He argues that the identification of a risk class for an entire industry is problematic when different firms have different market power. We add to this the insight that even a single firm with a given market power faces different risk classes for different choices of its own scale. Our example demonstrates that scale-related variations in required returns can be substantial and therefore can have significant impact on the calibrated values used in capital budgeting decisions. We will return to these practical implications after we broaden the notion of risk class by extending our analysis to asset pricing models more general than the CAPM.

3. Beyond CAPM: Risk Spillovers

To extend our inquiry regarding scale dependencies in required returns beyond CAPM to other asset pricing models, we return to the general valuation

[17] This is easiest to see in the case of the trivial cost function $c(\omega) = 0, \forall\omega$. The profit function is, then, mutliplicatively separable in ω and q and the required return depends on neither u nor q.

representation of Equation (2):

$$V[\pi(\omega, q)] = \frac{E[\xi(\omega)\pi(\omega, q)]}{R_f}$$
$$= \frac{E[\tilde{\xi}\ (\tilde{p}(q)q - \tilde{c}(q)q)]}{R_f}.$$

Differentiating with respect to q, denoting the derivative of price and average cost in quantity by p_q and c_q, and suppressing the argument q yields

$$\frac{dV[\tilde{\pi}]}{dq} = \frac{E\left[\tilde{\xi}\left(\tilde{p} + q\tilde{p}_q - \tilde{c} - q\tilde{c}_q\right)\right]}{R_f}$$
$$= \frac{V[\tilde{\pi}]}{q} + \frac{qE\left[\tilde{\xi}(\tilde{p}_q - \tilde{c}_q)\right]}{R_f}. \tag{15}$$

The first term, $V[\tilde{\pi}]/q$ represents the value increase from marginal output produced and sold at preexpansion prices. This term represents the rate of value change for a price-taking product market participant facing constant marginal costs. The second term, $qE[\tilde{\xi}(\tilde{p}_q - \tilde{c}_q)]/R_f$, represents the additional net scale-dependent change in value due to non-price-taking in the product market and nonconstant marginal costs.

To examine the relationship between market power and scale dependency in required returns, consider the risk-adjusted discount rate $R(q)$:

$$R(q) = \frac{E[\tilde{\pi}]}{V[\tilde{\pi}]} = \frac{qE[\tilde{p} - \tilde{c}]}{V[\tilde{\pi}]}. \tag{16}$$

$R(q)$ is well behaved (finite) when the project has nonzero value, an assumption we maintain throughout this section. Differentiating equation (16) yields

$$\frac{dV[\tilde{\pi}]}{dq} = \frac{V[\tilde{\pi}]}{q} + \frac{qE\left[\tilde{p}_q - \tilde{c}_q\right]}{R(q)} - \frac{dR(q)}{dq}\frac{V[\tilde{\pi}]}{R(q)}. \tag{17}$$

The first term corresponds to that in Equation (15). The second term is the adjustment for erosion advocated in textbook treatments of valuation for firms having market power.[18] It is the value of eroded cash flows under the hypothesis of a constant discount rate. When $R(q)$ is constant, $dR/dq = 0$ in Equation (17) shows that the textbook adjustment is correct. When $R(q)$ varies with scale, as in our CAPM examples, the third term in Equation (17)

[18] Brealey and Myers (1996) and Ross, Westerfield, and Jaffe (1996), among others, provide traditional treatments of erosion which consider only changes in expected cash flows.

is nonzero and the textbook adjustment may not yield reasonable estimates of value enhancement or erosion.

Comparing Equation (17) with Equation (15) yields

$$V[\tilde{\pi}]\frac{dR(q)}{dq} = qE\left[\tilde{p}_q - \tilde{c}_q\right] - qE\left[\tilde{\xi}(\tilde{p}_q - \tilde{c}_q)\right]\frac{R(q)}{R_f}. \qquad (18)$$

It is clear once again from the two right-hand terms that, for firms of finite nonzero value, the textbook and true adjustments ($qE[\tilde{p}_q - \tilde{c}_q]/R(q)$ and $qE[\tilde{\xi}(\tilde{p}_q - \tilde{c}_q)]/R_f$, respectively) are equal if and only if $dR/dq = 0$. Equation (18) also provides the generalization of Corollary 1's CAPM result on the necessity of quantity-dependent product prices or unit costs for scale dependency in required returns. Taking $\tilde{p}_q = \tilde{c}_q = 0$ in Equation (18) directly implies that $dR/dq = 0$. Formally,

Proposition 2. *At a given q, scale dependency in required returns ($dR/dq \neq 0$ at q) necessarily requires a positive probability of product price influence or quantity-dependent unit costs* ($\Pr[\tilde{p}_q \neq 0$ *or* $\tilde{c}_q \neq 0] > 0$).

To consider the relationship between scale dependency and risk spillovers we decompose the correct adjustment into three terms using $\tilde{\Psi} = 1 - \tilde{\xi}$:

$$\frac{qE[\tilde{\xi}(\tilde{p}_q - \tilde{c}_q)]}{R_f} = q\frac{E[\tilde{p}_q - \tilde{c}_q]}{R_f} - q\frac{E[\tilde{\Psi}\tilde{p}_q]}{R_f} + q\frac{E[\tilde{\Psi}\tilde{c}_q]}{R_f}. \qquad (19)$$

$qE[\tilde{p}_q - \tilde{c}_q]/R_f$ is a risk-neutral producer's changes in value related to price influence and nonconstant costs. The producer's additional adjustment for revenue-related risk spillovers is $-qE[\tilde{\Psi}\tilde{p}_q]/R_f$. The producer's additional adjustment for cost-related risk spillovers is $qE[\tilde{\Psi}\tilde{c}_q]/R_f$. To confirm the notion that the second and third terms in Equation (19) are risk spillovers, note that if demand and supply slopes are not correlated with ξ, then these terms vanish since $E[\tilde{\Psi}] = 0$. Conversely, if the (random) marginal spread between revenue and cost, $\tilde{p}_q - \tilde{c}_q$, varies systematically with state prices (and therefore $\tilde{\Psi}$), the last two terms in Equation (19) are together nonzero (summed) and influence $dV[\tilde{\pi}]/dq$. Linear-in-q risk spillovers occur when the firm is currently producing at $q > 0$ and has a marginal spread that is correlated with state prices. An important point to note is that the risk spillover adjustment will differ for producers with different existing output levels q. This, in general, will lead to different hurdle rates and valuations for different producers considering the same scale change. We will return to this point in our concluding remarks.

When the firm is expected to have a net influence on the spread between its price and cost ($E[\tilde{p}_q - \tilde{c}_q] \neq 0$), we can substitute $\tilde{\Psi} = 1 - \tilde{\xi}$ into Equation (18) and rearrange to establish a necessary and sufficient condition for $dR/dq = 0$ in terms of risk spillovers:

$$\frac{R_f}{R(q)} = 1 - \frac{E\left[\tilde{\Psi}(\tilde{p}_q - \tilde{c}_q)\right]}{E\left[\tilde{p}_q - \tilde{c}_q\right]},$$

where $R(q) = R$ is the constant risky rate for the firm at all scales. When net risk spillovers $(E[\tilde{\Psi}(\tilde{p}_q - \tilde{c}_q)])$ are present, unless they are proportional to the revenue spillovers $(E[\tilde{p}_q - \tilde{c}_q])$ in a specific way (their ratio equals $1 - R_f/R(q)$ for $R(q)$ equal to the constant rate R), discount rates will vary with scale.[19] As we argued earlier, such proportionality conditions are not satisfied by many reasonable specifications of demand and cost. For a firm that influences prices, it does not seem reasonable to assume that discount rates remain constant over scale.

4. Concluding Remarks

Our analysis highlights two important points when using a "comparable" firm's returns (or beta) to set an entrant's or expansionist's required returns: (1) true incremental betas diverge in potentially significant ways from existing industry betas due to the divergence of marginal from average risk; (2) the risk spillover or externality engendered by expansion affects all who share the expanding firm's markets leading to complex endogenous product and factor market responses. While the first point is just an application of our general insights, the second is the related but distinct point that oligopolistic equilibrium depends on reactions which recognize risk externalities. While, for simplicity, we have suppressed the choices of other firms and considered only the residual demand curved faced by one firm, the insights easily carry through to cross-firm risk spillovers among multiple producers. Perhaps the easiest way to see this is in a totally undifferentiated product market where all producers face the price $p(\omega, Q)$ where $Q = \sum_j q_j$. For this functional form of p, it is clear that p_Q is a variable influenced by each and every firm. From the analogue of Equation (15) risk changes will, in general, spill over to other firms sharing the same imperfectly competitive product market. It is also clear from Equation (18) that each firm's discount rate will, in general, be affected by the product market strategy of all other firms sharing that product market.[20] Under the more complex case of monopolistic competition where producer i faces demand function $p_i(\omega, q_1, q_2, \ldots q_N)$, similar results apply. Even when an entrant adopts an incumbent's production scale and cost technology, the entrant cannot accurately gauge the required re-

[19] The uniform absence of net risk spillovers (either due to $\xi = 1$ or other causes) induces a constant discount rate of R_f.

[20] The general equilibrium implications of the risk interactions between the various producers are presented in greater detail in Bhattacharyya and Leach (1995).

turn on investment by using the incumbent's pure-play "comparable" beta. Doing so incorrectly calibrates the risk and value of entry (even if the entrant correctly recognizes expected cash flow spillovers to and from existing firms).[21] Perhaps more interestingly, a new firm has no existing output to be affected by a risk spillover and therefore will have a different hurdle rate than existing producers who are considering the same market expansion. We conclude that using existing required returns to calibrate the value of entering an existing product market involves potentially significant theoretical inconsistencies, in addition to any empirical challenges relating to finding "comparable" firm data.

That equilibrium depends on the nature of risk spillovers is not an insight found among the traditional results of industrial organization where risk issues are not emphasized. Unfortunately, we cannot simply take the traditional insights and adjust them for "risk class" considerations as our results show there is no well-defined risk class for expansionary investments. In addition, even with only minor cost differences, risk considerations can move equilibrium required returns in opposite directions at firms affected by a common advantageous cost shock.[22] Long-range product strategy is also affected as a time series of industrial structures and the associated risk spillover considerations result in a time series of (endogenous) scale-dependent discount rates.

An additional implication of our analysis is the failure of decentralized capital budgeting. As potential investment outlays are compared to present values (PVs) of resulting cash flows (to yield NPVs), evaluators must be able to calibrate the PVs, the process of which is our focus. As an example of the nature of the decentralization problem, suppose that a product manager is considering a project scale decision (optimizing q). Even if she were somehow given the correctly calculated equilibrium discount rate which applies to the project's optimized cash flows, when she takes this discount rate as given (independent of q), she will arrive at a suboptimal production level since her first order condition is misspecified. In addition, it is not clear how she would be provided such a discount rate. The rate depends on knowing the optimal q, the knowledge of which obviates the need for her to optimize (over q). Concisely, required returns are partially endogenous to the firm and competitors who share the product and related factor markets. Most likely, such required returns vary with production plans.

Summarizing, in our analysis of project valuation, we demonstrate that price influence creates a risk spillover from additional investment to exist-

[21] It seems that, in practice, managers tack on a premium (relative to the "comparable" rate) for projects which enter unfamiliar businesses. While this practice has been criticized as an adjustment for diversifiable risk related to new business, it may actually reflect a qualitative adjustment for the divergence of postentry from existing required returns, due to risk spillovers and any induced strategic response.

[22] See Bhattacharyya and Leach (1995) for a formal treatment of this point.

ing investment, driving a wedge between existing risk and incremental risk. Expansion hurdle rates based on historical required returns are thus biased. We find that cross-firm risk spillovers influence the equilibrium choice of quantity and create a divergence of the risk of entering an existing market from the risk faced by participants currently in the market. Any use of "comparable" betas for evaluating the profitability of entry is thus suspect. From a conceptual viewpoint, our aim has been to synthesize the strategic analysis of imperfectly competitive product and factor markets with traditional financial valuation in a way that incorporates market values and considers relevant risk. In analyzing financial and product market interactions related to a firm's production strategy (asset-side interactions), our work complements the literature addressing financial and product market interactions in capital structure and payout policy (liability-side interactions). Our synthesis emphasizes (1) the need to use the functional form for required returns rather than a historical proxy; (2) the potential problems of employing a single beta for different product market strategy configurations; (3) the inseparability of cash flow estimation from discount rate determination; and (4) the role of risk spillovers and the related strategic response to entering a new market or expanding an existing one.

References

Baron, D. P., 1979, "Investment Policy, Optimality, and the Mean-Variance Model," *Journal of Finance*, 34, 207–232.

Bhattacharyya, S., and J. C. Leach, 1995, "Product and Financial Market Interactions: Risk Spillovers and the Valuation of Rents," working paper, University of Colorado, Boulder.

Brealey, R. A., and S. C. Myers, 1996, *Principles of Corporate Finance* (5th ed.), McGraw Hill, New York.

Copeland, T. E., T. Kuller, and J. Murrin, 1996, *Valuation: Measuring and Managing the Value of Companies* (2d ed.), Wiley, New York.

Dybvig, P. H., and J. E. Ingersoll, Jr., 1982, "Mean-Variance Theory in Complete Markets," *Journal of Business*, 55, 233–251.

Ekern, S., and R. Wilson, 1974, "On the Theory of the Firm in an Economy with Incomplete Markets," *Bell Journal of Economics*, 5, 171–180.

Fama, E. F., 1972, "Perfect Competition and Optimal Production Decisions Under Uncertainty," *Bell Journal of Economics*, 3, 509–530.

Grossman, S. J., and J. E. Stiglitz, 1977, "On Value Maximization and Alternative Objectives of the Firm," *Journal of Finance*, 32, 389–402.

Jensen, M. C., and J. B. Long, Jr., 1972, "Corporate Investment Under Uncertainty and Pareto Optimality in the Capital Markets," *Bell Journal of Economics*, 3, 151–174.

Merton, R. C., and M. G. Subrahmanyam, 1974, "The Optimality of a Competitive Stock Market," *Bell Journal of Economics*, 5, 145–170.

Ross, S. A., R. W. Westerfield, and J. F. Jaffe, 1996, *Corporate Finance* (4th ed.), Irwin, Chicago.

Stiglitz, J. E., 1972, "On the Optimality of the Stock Market Allocation of Investment," *Quarterly Journal of Economics*, 86, 25–60.

Subrahmanyam, M. G., and S. B. Thomadakis, 1980, "Systematic Risk and the Theory of the Firm," *Quarterly Journal of Economics*, 94, 437–451.

Thomadakis, S. B., 1976, "A Model of Market Power, Valuation and the Firm's Returns," *Bell Journal of Economics*, 7, 150–162.