The Effect of Derivative Assets on Information Acquisition and Price Behavior in a Rational Expectations Equilibrium

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This article shows that introducing derivative assets increases incentives to collect information about asset payoffs. The increase in information collection makes the price of the underlying asset more informative and causes the expected price to increase. Extending the model to a dynamic setting with multiple risky assets, we find that introducing derivative assets for one asset increases the expected prices of positively correlated assets and reduces price reaction to future earnings announcements. These findings are consistent with the bulk of the empirical evidence on the relationship between the introduction of derivative assets and the behavior of asset prices.

The number and scope of derivative markets have grown considerably in recent years. The effect of such growth on market volatility for the underlying assets has been a central issue in the empirical literature, especially since the stock market crash of 1987. It is popularly held that the creation of derivative assets contributes to the excessive market volatility. It has recently been proposed that additional regulation be introduced.[1] It is therefore important to understand how the introduction of derivative assets affects stock price volatility, market liquidity, and price efficiency.

In Black and Scholes (1973), option contracts are redundant assets and have no effects on the price of the underlying assets. However, when the assumption of *complete*, *competitive*, and *frictionless* markets is relaxed, the introduction of an option may affect the price of the underlying asset.

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[1] *Wall Street Journal*, May 20, 1994. The General Accounting Office (GAO) calls for the government to take more responsibility for the regulation of the derivatives market. GAO's comptroller general, Charles Bowsher, said that government is unprepared for a financial crisis with derivatives.

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When markets are incomplete, introducing options can change the span of security payoffs. Investors are not necessarily made better off by the introduction of options which may affect the prices of existing assets and thus the value of their endowments.[2] Detemple and Selden (1991) show that when the prices of the underlying asset and the derivative asset are jointly determined in a general equilibrium model with incomplete financial markets, the introduction of a call option will in general change the price of the underlying asset.

In the presence of asymmetric information, introduction of options in financial markets affects information revelation through prices. Grossman (1988) argues informally that a traded option contract allows aggregation of information about the extent to which investors are using dynamic hedging strategies in the market.[3] Since this aggregation of information is not available in the absence of options, the market may be more volatile without options. His model predicts that the introduction of options may reduce the volatility of the underlying asset. In a dynamic continuous-time trading model in which investors have nontraded income and private information about asset payoffs, Huang and Wang (1997) show that the addition of derivative securities affects the trading volume and prices of existing assets and the informational and allocational efficiency of the market.

Even when the market is dynamically complete in the absence of options, it may become incomplete when an option is traded and the arbitrage-based pricing formula may not apply in general. Back (1993) analyzes a continuous trading model of the type of Kyle (1985) with a single informed trader with perfect knowledge of the value of the underlying asset, noise traders who trade in both the option and the underlying asset, and a competitive risk-neutral market maker. The market maker sets the prices to be the expectation of the payoffs of the assets given his information. He shows that the introduction of options changes information the market receives. This change of the local state space precludes spanning of the option by the underlying asset and causes the underlying asset to have stochastic volatility. Unless the noise traders' demands in the option and the underlying asset are perfectly correlated, the option price will be different from the Black–Scholes price.

In a *noncompetitive* securities market, when the adverse selection problem is too severe, the market may break down so that no equilibrium exists.[4] Biais and Hillion (1994) show that the introduction of options can also make

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[2] See, for example, Ross (1976), Breeden and Litzenberger (1978), Hakansson (1978), John (1984), and Green and Jarrow (1987). Allen and Gale (1994) analyze the incentives for financial innovation and the optimal security design when it is costly to open new markets.

[3] When investors have heterogeneous beliefs, Kraus and Smith (1996) show that trading in a replicatable option can break a non-fully revealing equilibrium and thus affect the price process of the underlying asset.

[4] See, for example, Glosten (1989), Bhattacharya and Spiegel (1991), and Bhattacharya, Reny, and Spiegal (1995).

the market more liquid and avoid market breakdowns in a strategic trading model. In their model, derivative assets have an ambiguous effect on the informativeness of the price of the underlying asset.

In the presence of market frictions such as trading cost, the introduction of derivative assets can also facilitate the informed investors' participation in financial markets by reducing transaction cost and mitigating short-sale constraints. Black (1975) and Cox and Rubinstein (1985) argue that the cost of trading in options will often be lower than the cost of making a sequence of trades to replicate the payoffs of options. Diamond and Verrecchia (1987) show that restrictions on short sales can be mitigated by trading in options. Consequently the introduction of options may increase the informational content of underlying asset prices by attracting more informed investors into the market.[5]

In this article we study the effect of introducing derivative assets on the decision to acquire private information prior to trading in a noisy rational expectations model of the type of Hellwig (1980) and Admati (1985).[6] We show that when trading opportunities are improved by introducing a certain type of derivative asset, the incentive to acquire private information is increased.[7] As a result, when information acquisition is determined endogenously, the effect of the introduction of derivative assets is to increase the amount of information collected. This makes the price more informative. The increase in informational efficiency causes the price to be higher on average in the more complete market. In addition, price volatility is reduced. These theoretical predictions are generally consistent with the empirical evidence of the effects of option listings on volatility, price, and analyst interest in the underlying asset.

The model is further extended to a dynamic setting with multiple risky assets following Admati (1985). We show that introducing derivative assets on one asset will decrease the price reaction to public information. In addition it increases the price of assets whose payoffs are positively correlated with the underlying asset. Our results are consistent with empirical findings of Detemple and Jorion (1990).

The article is organized as follows: the next section describes the single-period model. Section 2 considers endogenous information acquisition in

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[5] See also Figlewski and Webb (1993) for an empirical study of the relationship between short sales and trading in options.

[6] In a single risky asset model, Brennan and Cao (1996) show that when the market is made effectively complete by the introduction of a quadratic derivative asset or a complete set of options, the price of the underlying asset will stay the same. However, they did not discuss endogenous information acquisition in their article.

[7] Boot and Thakor (1993) show that the capital structure of a firm can alter the incentive of investors to acquire private information. The expected revenue for a good firm is higher when its asset cash flows are partitioned into information sensitive and insensitive claims, since more investors will acquire information. They also argue informally that the introduction of derivatives stimulates information acquisition.

the dynamic trading model with multiple risky assets. Section 3 analyzes the welfare effects of the derivative asset when acquisition of information is endogenized. Section 4 compares the implications of the model and the results of the empirical literature. Section 5 concludes the article. All proofs are provided in the appendix unless otherwise mentioned.

1. The Economic Structure and Equilibrium

1.1 The basic model

The market is assumed to consist of a continuum of two groups of investors: informed and uninformed. Informed investors are indexed by i where $i \in [0, \lambda]$ and uninformed investors are indexed by u, where $u \in (\lambda, 1]$. There are two assets in the market: one riskless asset and one risky asset. We assume without loss of generality that individual endowments of the riskless asset are zero and that the riskless interest rate is zero. There are three times: -1, 0, 1. At time -1, investors receive their endowments of the risky asset and informed investors decide whether to acquire private signals. Informed investors receive their private signals and trade with uninformed investors at time 0. Consumption occurs at time 1. The risky asset pays off at time 1 an amount $\tilde{v}$,[8] where $\tilde{v}$ is normally distributed with mean $\bar{v}$ and precision h. Let a denote an investor who may be informed or uninformed. The endowment of the risky asset for investor a is denoted x_a. The per capita supply of risky assets, $\tilde{x}$, is normally distributed with mean $\bar{x}$ and precision ϕ. Before trading at time 0 each investor receives a private signal about the risky asset payoff,

$$\tilde{z}_a = \tilde{v} + \tilde{\epsilon}_a, \tag{1}$$

where $\tilde{\epsilon}_a$ is a normal random variable with mean 0 and precision ψ_a. We set $\psi_u = 0$, $\psi_i = \psi$ so that uninformed investors receive a degenerate signal and all informed investors receive a signal of the same precision. Information acquisition is costly and will be endogenized in Section 1.3. The variable $\tilde{\epsilon}_a$ is independently distributed.[9] Investor a has a negative exponential utility function defined over time 1 wealth w_a, $U(w_a) = -exp(-\rho w_a)$, where the risk aversion ρ is assumed to be positive and the same for all investors.[10]

We first derive the equilibrium taking informed investors' decision to acquire private signals as exogenous. (Information acquisition is endoge-

[8] We use a tilde to denote a random variable; the variable without the tilde denotes the realized value of the random variable.

[9] Note that we are assuming that investors' signal errors are independent. By the law of large numbers, we will have $\int_0^1 \tilde{\epsilon}_a da = 0$. For a more rigorous treatment, see He and Wang (1995). If signal errors are correlated, the equilibrium is qualitatively similar to that described below, except that the coefficients of the linear pricing function [Equation (2)] must be derived from the roots of a cubic equation [see Grundy and McNichols (1989)]; the assumption made here permits direct calculation of these coefficients.

[10] Adding heterogeneity in the risk aversion of the investors will not affect any of the results in the article.

nized in Section 1.3.) Let $\tilde{P}_0$ be the price of the risky asset, $\tilde{D}_{0a}$ be the demand of investor a for the risky asset at time 0, $\tilde{F}_{0a}$ denote the information set of investor a at time 0, and $\tilde{F}_0$ denote the public information set that includes the price at time 0. Investor a's time 1 wealth is given by $\tilde{w}_a = x_a \tilde{P}_0 + (\tilde{v} - \tilde{P}_0)\tilde{D}_{0a}$. It is well known that in this setting [Hellwig, (1980)] a linear equilibrium exists in which

$$\tilde{P}_0 = K_0^{-1}[(K_0 - \lambda\psi)\tilde{\mu}_0 + \lambda\psi\tilde{v} - \rho\tilde{x}], \tag{2}$$

$$\begin{aligned}\tilde{D}_{0a} \;=\; & K_{0a}[\tilde{\mu}_{0a} - \tilde{P}_0]/\rho = [\psi_a \tilde{z}_a - \lambda\psi\tilde{v} + \rho\tilde{x} \\ & -(\psi_a - \lambda\psi)\tilde{P}_0]/\rho,\end{aligned} \tag{3}$$

where

$$\tilde{\mu}_{0a} \equiv E(\tilde{v}|\tilde{F}_{0a}) = \frac{h\bar{v} + \psi_a \tilde{z}_a + \lambda^2\psi^2\phi\tilde{q}/\rho^2}{h + \psi_a + \lambda^2\psi^2\phi/\rho^2},$$

$$\tilde{\mu}_0 \equiv \tilde{\mu}_{0u} \equiv E[\tilde{v}|\tilde{F}_0] = \frac{h\bar{v} + \lambda^2\psi^2\phi\tilde{q}/\rho^2}{h + \lambda^2\psi^2\phi/\rho^2},$$

$$\tilde{q} = \tilde{v} - \rho(\tilde{x} - \bar{x})/\psi,$$

$$K_{0a} \equiv \mathrm{var}^{-1}[\tilde{v}|F_{0a}] = h + \psi_a + \lambda^2\psi^2\phi/\rho^2,$$

$$K_0 \equiv \int_0^1 K_{0a}da = h + \lambda\psi + \lambda^2\psi^2\phi/\rho^2.$$

Investor a's wealth at time 1, $\tilde{w}_a$, is a linear function of the risky asset payoff, $\tilde{v}$, and may be written as

$$w_a(\tilde{v}) = x_a \tilde{P}_0 + (\tilde{v} - \tilde{P}_0)\left[\psi_a(\tilde{z}_a - \tilde{P}_0) - \frac{h\tilde{P}_0}{1 + \lambda\psi\phi/\rho^2}\right]/\rho. \tag{4}$$

Following Hellwig (1980), the expected utility of investor a conditional on his endowment and before receipt of his private signal is

$$EU_a = \frac{-1}{\sqrt{LK_{0a}}}\exp\left(-\rho x_a\bar{v} + \frac{\rho^2 x_a^2}{2h} - \frac{\rho^2(x_a - \bar{x})^2}{2K_0^2 L_0}\right), \tag{5}$$

where

$$L_0 \equiv \mathrm{Var}(\tilde{v} - \tilde{P}_0) = K_0^{-2}(K_0 + \lambda\psi + \rho^2\phi^{-1}).$$

In this equilibrium $K_{0u} \equiv \mathrm{var}^{-1}[\tilde{v}|P_0]$ measures the informativeness of the price. Therefore we may interpret K_{0u} as a measure of how well the market price reveals the fundamental value of the risky asset.

The marginal rate of substitution for investor a between wealth contingent on $v = v_l$ and $v = v_k$ is given by

$$\begin{aligned} M_{kl}^a &= \frac{\exp\{-K_{0a}(v_k - \mu_{0a})^2/2\}\exp\{-\rho w_a(v_k)\}}{\exp\{-K_{0a}(v_l - \mu_{0a})^2/2\}\exp\{-\rho w_a(v_l)\}} \\ &= \exp\left\{-\frac{K_{0a}}{2}(v_k - v_l)(v_k + v_l - 2\mu_{0a})\right. \\ &\quad \left. -(v_k - v_l)K_{0a}(\mu_{0a} - P_0)\right\} \\ &= \exp\left\{-\frac{1}{2}K_{0a}(v_k - v_l)(v_k + v_l - 2P_0)\right\}. \end{aligned} \tag{6}$$

Since the marginal rate of substitution is investor specific through K_{0a}, the precision of the investor's posterior beliefs, this equilibrium in which allocations are constrained to be linear functions of the asset payoff leaves it open to the introduction of derivative assets to make the market more complete. This problem is addressed in the next section.

1.2 Introduction of derivative assets

How would introducing a derivative asset, for example, a call option, affect the price of the underlying asset? Unfortunately we do not have a definitive answer. In general, the introduction of options not only affects the trading opportunity of investors but also the information set of investors. In an incomplete market, the presence of an option can change the demand for the risky asset. The equilibrium price of the risky asset and the information it conveys will be different when an option is traded. In addition, the price of the option itself can reveal valuable information to investors. However, in the current setup, the introduction of certain derivative assets does not affect the price of the risky asset and the price of the option does not reveal any new information to the investors. We first analyze the effects of introducing these derivative assets and then discuss the effects of introducing general derivative assets.

Definition. A derivative asset $\tilde{G} = g(|\tilde{v} - \tilde{P}_0|)$ is called a generalized straddle if $g(\cdot)$ is a monotonic function. A derivative asset $\tilde{R} = r(|\tilde{v} - \tilde{P}_0|)\text{Sign}(\tilde{v} - \tilde{P}_0)$ is called a generalized spread if $r(\cdot)$ is a monotonic function.

Let P_G, P_R denote the prices of the derivative assets and D_{Ga}, D_{Sa} denote the demand for each asset by investor a. The effect of the introduction of these derivative assets is described in the following theorem:

Theorem 1. *When a generalized straddle $\tilde{G}$ is introduced to the market, there exists a partially revealing rational expectations equilibrium in which*

$$\tilde{P}_0 = K_0^{-1}[(K_0 - \lambda\psi)\tilde{\mu}_0 + \lambda\psi\tilde{v} - \tilde{\rho}x], \tag{7}$$

$$\tilde{D}_{0a} = K_{0a}[\tilde{\mu}_{0a} - P_0]/\rho = [\psi_a\tilde{z}_a - \psi_a\tilde{v} + \rho\tilde{x} - (\psi_a - \lambda\psi)\tilde{P}_0]/\rho, \tag{8}$$

$$\int_0^{\infty}(g(v) - P_G)\exp[-K_{0a}v^2/2 - \rho D_{Ga}g(v)]dv = 0, \tag{9}$$

$$\lambda D_{Gi} + (1-\lambda)D_{Gu} = 0. \tag{10}$$

The expected utility for investor a before the receipt of his private signal but after his endowment is realized is

$$EU_a^G = EU_a\sqrt{\frac{2}{\pi K_{0a}}}\int_0^{\infty}\exp[-K_{0a}v^2/2 - D_{Ga}(g(v) - P_G)\rho]dv. \tag{11}$$

When a generalized spread $\tilde{R}$ is introduced to the market, the demand and price of other assets are not affected and the demand and price for $\tilde{R}$ are both zero.

Remark 1. Notice that we can always write a call option at the money to be

$$\tilde{C} \equiv \max(\tilde{v} - \tilde{P}_0, 0) = (|\tilde{v} - \tilde{P}_0| + \tilde{v} - \tilde{P}_0)/2.$$

Buying two shares of call option at the money is equivalent to buying one share of a straddle at the money and one share of the underlying asset. Similarly if call options with strike prices distributed symmetrically around the underlying asset price are introduced to the market, investors will be effectively trading in a generalized straddle.[11] The result of Theorem 1 also applies in these cases.

Remark 2. A special example of the generalized straddle is a quadratic derivative asset with payoff function $\tilde{Q} = (\tilde{v} - P_0)^2$. In this case the price of the derivative asset and the demand of the quadratic derivative asset have closed-form solutions as shown in Brennan and Cao (1996).[12]

[11] Suppose two call options with strike prices distributed symmetrically around $\tilde{P}_0$, $\tilde{C}_1 = \max(\tilde{v} - K, 0)$ and $\tilde{C}_2 = \max(\tilde{v} - (2P_0 - K), 0)$ are introduced to the market. This is equivalent to trading with $\tilde{G} \equiv \tilde{C}_1 + \tilde{C}_2 - \tilde{v} + K = \max(|\tilde{v} - P_0| + K - P_0, 0)$ and $\tilde{S} \equiv \tilde{C}_1 - \tilde{C}_2 - \tilde{v} + K = \mathrm{Sign}(\tilde{v} - P_0)\min(-|\tilde{v} - P_0| - K + P_0, 0)$. $\tilde{G}$ is a generalized straddle, and $\tilde{S}$ is a generalized spread defined in the text. Since the demand for the generalized spread is zero in equilibrium, the economy with $\tilde{C}_1$, and $\tilde{C}_2$ is the same as if only $\tilde{G}$ is introduced to the market.

[12] There could exist a fully revealing equilibrium in which the risky asset and the derivative asset become riskless assets. But in this fully revealing equilibrium, it is unclear how price can incorporate information

Let P_Q denote the price of the quadratic derivative asset and D_{Qa} denote the demand for it by investor a. Then the price and demand for the derivative asset are given by

$$P_Q = K_0^{-1}, \tag{12}$$

$$D_{Qa} = \frac{P_Z^{-1} - K_{0a}}{2\rho} = \frac{\lambda\psi - \psi_a}{2\rho}. \tag{13}$$

Since the introduction of the derivative asset has no effect on the price of the underlying asset, it has no direct effect on price efficiency or price volatility. However, if the information is costly and the decision to acquire private information is endogenized, introduction of the derivative asset will affect the price of the underlying asset, price efficiency, and price volatility.

1.3 Endogenized information acquisition

In this section we discuss how the derivative asset changes investors' incentives to acquire costly information following Verrecchia (1982). We assume that the cost for informed investors to acquire a signal $\tilde{z}_i$ with precision ψ_i is a function of ψ_i alone and is the same function for all informed investors, which we write as $C(\psi_i)/(2\rho)$. In addition, we assume that $C(\cdot)$ is increasing, convex, and twice continuously differentiable and that the cost of not acquiring a signal is zero, that is, $C_i' \geq 0$, $C_i'' \geq 0$, $C(0) = 0$.

We consider a symmetric equilibrium in which all informed investors acquire private signals of the same precision. Investor i chooses precision of his private signal ψ_i to maximize his expected utility. In the case without the derivative asset, his expected utility with costly information acquisition is $e^{C(\psi_i)/2}$ times his expected utility without cost as expressed in Equation (5).

From Equation (5), ignoring the cost of information acquisition, the expected utility of investor i who acquires a signal of precision ψ_i, given that the average informed investor acquires a signal of precision ψ, is[13]

$$EU_i = \frac{-1}{\sqrt{L_0 K_{0i}}} \exp\left[-\rho x_i \bar{v} + \frac{\rho^2 x_i^2}{2h} - \frac{\rho^2 (x_i - \bar{x})^2}{2K_0^2 L_0}\right]. \tag{14}$$

Taking a monotonic transformation $V_i \equiv -[\ln(-EU_i)]/\rho$, the certainty equivalent value for an investor with a signal of precision ψ_i is

$$V_i = \frac{\ln[K_{0i}]}{2\rho} + \frac{\ln[L_0]}{2\rho} - x_i \bar{v} + \frac{\rho x_i^2}{2h} - \frac{(x_i - \bar{x})^2 \rho}{2K_0^2 L_0}.$$

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when investors' demands are independent of their private signals. A partially revealing equilibrium can be restored if we assume that introducing new derivative assets will introduce new noises in the pricing of the derivative assets through the introduction of new noise traders in the derivative assets.

[13] We have derived Equation (5) under the hypothesis that there are only two values for ψ_a, but it remains valid for an individual choosing any precision, provided that the average informed investor acquires a signal of precision ψ.

The marginal value of private information is

$$MV_i \equiv \frac{dV_i}{d\psi_i} = \frac{1}{2\rho K_{0i}}.$$

The marginal cost of private information is

$$MC_i \equiv \frac{dC(\psi_i)/(2\rho)}{d\psi_i} = \frac{C'(\psi_i)}{2\rho}.$$

In equilibrium, marginal value equals marginal cost:

$$\frac{1}{2\rho K_{0i}} = \frac{C'(\psi_i)}{2\rho}.$$

Therefore, in a symmetric equilibrium, all informed investors acquire signals of the same precision, and we have $\psi_i = \psi$. Informed investors will choose ψ^* given below:[14]

$$\psi^* = \max\{0, \psi | C'(\psi)(h + \psi + \lambda^2\psi^2\phi/\rho^2) = 1\}. \tag{15}$$

Next we turn to the economy with the derivative asset to analyze how the introduction of the derivative asset will affect informed investors' incentive to acquire information. In the presence of a generalized straddle, the marginal value of the private information is higher, hence informed investors acquire more precise signals, as shown below.

Proposition 1. *Introduction of a generalized straddle causes informed investors to acquire more precise signals.*

This proposition leads us to expect that information collection activities would increase after options are listed and to conjecture that, cross-sectionally, information collecting activities will be greater for firms with listed options. One way to collect more information is to hire more financial analysts.[15] Thus the model predicts that the number of analysts following a stock will increase after the stock is listed on an options exchange. Furthermore, the number of analysts following stocks with options listed should be higher than the number of analysts following stocks without options listed.

When informed investors acquire more precise private signals, the price becomes more informative about the payoff of the risky asset. The return of the asset is therefore less risky, and the expected price will increase as a result:

$$E[\tilde{P}_0] = \bar{v} - \rho\bar{x}/K_0.$$

[14] The existence problem and the second-order condition are addressed in Verrecchia (1982).

[15] We assume that the precision of an informed investor's private information is proportional to the number of financial analysts hired. This would be true if the noise terms in analysts' information are independent.

The expected price increases with K_0 and therefore with ψ when the expected per capita supply is positive.

To analyze the effect on volatility of introducing a derivative asset, we need to calculate the volatility of the price change. The volatility of the price change calculated from Equation (2) is given by

$$\mathrm{var}[\tilde{v} - \tilde{P}_0] = \frac{K_0 + \lambda\psi + \rho^2/\phi}{K_0^2}, \tag{16}$$

$$\frac{d\{\mathrm{var}[\tilde{v} - \tilde{P}_0]\}}{d\psi} = -\frac{2\lambda(\rho^2 + \lambda\psi\phi)(\lambda\psi\phi + \rho^2)}{\rho^2\phi K_0^3} < 0.$$

The effect of introducing a derivative asset on the price of the underlying asset is summarized as follows.

Proposition 2. *Introduction of a generalized straddle will increase the expected price and reduce price change volatility.*

In the next section we extend the preceding results to a setting with multiple trading periods and multiple risky assets. We then analyze the effects of introducing derivative assets on price reaction to earnings announcements and the price of related assets. Since the quadratic derivative asset gives closed form solutions, we will focus our analysis using this particular derivative asset in the next section.

2. Dynamic Trading with Multiple Risky Assets

In this section we analyze how the introduction of derivative assets on one asset affects the price of other assets. In addition, we analyze the effects of derivative assets on price reaction to public information in a dynamic setting. We first describe a dynamic trading model with multiple assets and no derivative assets. The model is a multiasset generalization of the model in Section 1, and the setup is similar to Admati (1985). As in Section 1, all random variables in this section have a jointly normal distribution. There is one riskless asset and m risky assets. The payoff of the risky assets is represented by an $m \times 1$ random vector $\tilde{V}$ with mean $\bar{V}$. Investor $a \in [0, 1]$ is endowed with risky assets described by the vector X_a. Investor a obtains a vector of private signals $\tilde{Z}_a$:

$$\tilde{Z}_a = \tilde{V} + \tilde{\epsilon}_a,$$

where $\tilde{\epsilon}_a$ and $\tilde{V}$ are independent, and $\tilde{\epsilon}_a$ has a mean of zero and is independent of $\tilde{\epsilon}_k, k \neq a$. The vector of aggregate per capita supply of the risky assets, $\tilde{X}$, has mean $\bar{X}$ and is independent of $\tilde{v}$ and $\tilde{\epsilon}_a$. We further assume that there are no new supply shocks in the market after time 0. Trading starts at time 0 and after that there are public announcements just before the market reopens at times $\tau_t = t/T, t = 1, \ldots, T-1$. The public signal at time τ_t is represented by $\tilde{Y}_t = \tilde{V} + \tilde{n}_t$. The random error $\tilde{n}_t$ has a mean of zero. The precision matrices of $\tilde{V}$, $\tilde{X}$, $\tilde{\epsilon}_a$, and $\tilde{n}_t$ are denoted by H, Φ, Ψ_a, and Ξ_t, respectively.[16] All the elements of Ψ_a are uniformly bounded. The population average of the precision matrix Ψ_a is given by

$$\bar{\Psi} \equiv \int_0^1 \Psi_a da.$$

The total number of trading sessions is T. At time 1, the risky asset payoff is realized and investors consume. Let $\tilde{P}_t$ be the price vector of the risky assets at time τ_t, and $\tilde{D}_{ta}$ be the demand vector for the risky assets by investor a, $\tilde{F}_t$ be the public information set including the prices at trading session t, and $\tilde{F}_{ta}$ be the information set of investor a at trading session t. With this general setup we first derive a rational expectations equilibrium without the derivative asset.

2.1 A dynamic model with multiple risky assets without the derivative asset

Theorem 2. *In a T trading session economy, there exists a partially revealing rational expectations equilibrium in which*[17]

$$\tilde{P}_t = K_t^{-1}[(K_t - \bar{\Psi})\tilde{\mu}_t + \bar{\Psi}\tilde{V} - \rho\tilde{X}], \tag{17}$$

$$\tilde{D}_{ta} = K_{ta}[\tilde{\mu}_{ta} - \tilde{P}_t]/\rho = [\Psi_a\tilde{Z}_a - \bar{\Psi}\tilde{V} + \rho\tilde{X} - (\Psi_a - \bar{\Psi})\tilde{P}_t]/\rho, \tag{18}$$

where

$$\tilde{\mu}_{t_a} \equiv E(\tilde{V}|\tilde{F}_{ta}) = K_{ta}^{-1}(H\bar{V} + \sum_{j=0}^{t} \Xi_j\tilde{Y}_j + \Psi_a\tilde{Z}_a + \bar{\Psi}\Phi\bar{\Psi}\tilde{Q}/\rho^2),$$

$$\tilde{\mu}_t \equiv E(\tilde{V}|\tilde{F}_t) = \left(K_t - S)^{-1}(H\bar{V} + \sum_{j=0}^{t} \Xi_j\tilde{Y}_j + \bar{\Psi}\Phi\bar{\Psi}\tilde{Q}/\rho^2\right),$$

$$\tilde{Q} = \tilde{V} - \rho\bar{\Psi}^{-1}(\tilde{X} - \bar{X}),$$

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[16] Similar to the single risky asset model, we let $\Xi_0 = \Xi_T^{-1} = 0$ to reflect the assumption that there is no public information at time 0 and all risky asset payoffs are realized at time 1.

[17] For notational simplicity, we define $\tilde{Y}_0 = 0$ since there is no public information at time 0. This model can be easily extended to the case with public information release at time 0.

$$K_{ta} \equiv \mathrm{var}^{-1}(\tilde{V}|\tilde{F}_{ta}) = H + \Psi_a + \sum_{j=0}^{t} \Xi_j + \bar{\Psi}\Phi\bar{\Psi}/\rho^2,$$

$$K_t \equiv \int_0^1 K_{ta} da = H + \bar{\Psi} + \sum_{j=0}^{t} \Xi_j + \bar{\Psi}\Phi\bar{\Psi}/\rho^2.$$

The expected utility of trader a is given by

$$\begin{aligned} EU_a &\\ = \prod_{t=1}^{T-1} &\sqrt{[\det|I + (\bar{\Psi} - \Psi_a)K_{ta}^{-1}(\bar{\Psi} - \Psi_a)K_t^{-1}K_{ta}K_{(t-1)a}^{-1}\Xi_t K_t^{-1}|]^{-1}} \\ &\times \frac{-1}{\sqrt{\det|L_0 K_{0a}|}} \exp[-\rho\bar{V}' X_a + \frac{\rho^2}{2}\{X_a' H^{-1} X_a \\ &- (X_a - \bar{X})' K_0^{-1} L_0^{-1} K_0^{-1}(X_a - \bar{X})\}], \end{aligned}$$

where

$$L_0 \equiv \mathrm{var}(\tilde{V} - \tilde{P}_0) = K_0^{-1}(K_0 + \bar{\Psi} + \rho^2\Phi^{-1})K_0^{-1}.$$

In the dynamic trading model, the prices act as sufficient statistics for all public information. In addition, neither the allocations nor the prices of the risky assets at time τ_t are affected by trading halts or addition of trading sessions before or after τ_t, as long as aggregate public information received up to time τ_t remains the same. Since prices are unaffected by trading halts, such halts serve only to reduce the opportunities for risk sharing. Therefore trading halts always make investors worse off, while the addition of trading sessions always makes investors better off. The trading strategy of the investors is given by

$$\tilde{D}_{(t+1)a} - \tilde{D}_{ta} = (\bar{\Psi} - \Psi_a)(\tilde{P}_{t+1} - \tilde{P}_t)/\rho.$$

When there is only a single risky asset, uninformed (informed) investors will buy (sell) on good news and sell (buy) on bad news. The intuition for the result is as follows. Investors take positions in the risky asset, based on their private information signals, past public signals, and current public signals. Given their holdings, a public signal leads investors to revise their priors. Most importantly, uninformed investors revise their means more drastically than do informed investors. This implies that if the public signal conveys good news about the payoff of the risky asset, uninformed investors increase their assessment of the expected payoff more than do informed investors; as a result, the price rises to clear the market, and uninformed investors purchase more from informed domestic investors; the reverse occurs if the news is bad and the price falls.[18]

2.2 Equilibrium in the presence of the derivative assets

Next we consider the introduction of derivative securities in the dynamic multiple risky asset setting. The derivative securities are represented by a derivative matrix $\tilde{Z} = (\tilde{V} - P_0)(\tilde{V} - P_0)'$. Each element in $\tilde{Z}$ represents a security, and $\tilde{Z}_{kl} = \tilde{Z}_{lk}$ represents the same security. The derivative securities are in zero net supply. Let the $m \times m$ matrix $\tilde{P}_{tZ}$ denote the prices of the derivatives and the $m \times m$ matrix $\tilde{D}_{taZ}$ denote the demand by investor i for the options at time t.[19] We need $m(m+1)/2$ derivative assets to make the market effectively complete in the first trading session. Since the market is effectively complete in the presence of derivative assets, no trade will occur when new private information or public information arrives in the absence of liquidity traders. Furthermore, the price reaction to public information is the same in both the incomplete market and the effectively complete market. The partially revealing rational expectations equilibrium is described in the following theorem.

Theorem 3. *There exists a partially revealing rational expectations equilibrium in the economy with the derivative assets in which prices and demands are given by*

$$\tilde{P}_t = K_t^{-1}[(K_t - \bar{\Psi})\tilde{\mu}_t + \bar{\Psi}\tilde{V} - \rho\tilde{X}], \tag{19}$$

$$\begin{aligned} \tilde{D}_{ta} &= K_{ta}[\tilde{\mu}_{ta} - \tilde{P}_t]/\rho - 2\tilde{D}_{taZ}(\tilde{P}_t - \tilde{P}_0) \\ &= [\Psi_a \tilde{Z}_a - \bar{\Psi}\tilde{V} + \rho\tilde{X} - (\Psi_a - \bar{\Psi})\tilde{P}_0]/\rho, \end{aligned} \tag{20}$$

$$\tilde{P}_{tZ} = (\tilde{P}_t - \tilde{P}_0)(\tilde{P}_t - \tilde{P}_0)' + K_t^{-1}, \tag{21}$$

$$\begin{aligned} \tilde{D}_{taZ} &= \frac{1}{2\rho}[\{\tilde{P}_{tZ} - (\tilde{P}_t - P_0)(\tilde{P}_t - \tilde{P}_0)'\}^{-1} - K_{ta}] \\ &= \frac{1}{2\rho}(\bar{\Psi} - \Psi_a). \end{aligned} \tag{22}$$

[18] Consistent with our results, Ippolito (1992) uses annual data to show that investors react to individual mutual fund performance by directing investment to mutual funds that have performed well in the recent past and directing investments away from poorly performing mutual funds.

[19] This means that, at trading session t, investor a's demand for $\tilde{Z}_{kl}$ is the klth element of D_{tZa} and the price of $\tilde{Z}_{kl}$ is the klth element in $\tilde{P}_{tZ}$.

The expected utility of investor a is given by

$$EU_a = \frac{\exp[\frac{1}{2}tr[(\bar{\Psi} - \Psi_a)K_0^{-1}] - \rho\bar{V}'X_a + \frac{\rho^2}{2}[X_a'H^{-1}X_a - (X_a - \bar{X})'K_0^{-1}L_0^{-1}K_0^{-1}(X_a - \bar{X})]]}{-\sqrt{\det|L_0K_0|}},$$

where $tr[A]$ is the sum of diagonal elements of square matrix A.

Notice that the investors' demands remain the same after the first round of trading. This is a direct application of Milgrom and Stokey's (1982) no-trade theorem. The incentive for investor a to trade in the underlying risky assets is removed by his holding of the derivative assets, as shown by his demand schedule for the risky assets.

In a multiple-asset model, the introduction of derivative assets affects the acquisition of private signals in a much more complex fashion than in the single-asset model. The simple results of the single-asset model do not, in general, apply in the multiple asset model. The results depend on the variance-covariance matrices of the asset payoffs, the noise in investors' private signals, and the per capita supply of the assets. It is possible that introducing derivative assets increases the quality of the signals for one asset while decreasing the quality of the signals for other assets. In these cases, it is impossible to compare the quality of signals of informed investors. Nevertheless, in the next section we endogenize information acquisition in a dynamic setting when there are two risky assets.

2.3 Endogenous information acquisition in a dynamic model

To simplify the analysis, we discuss the information acquisition problem in a two risky asset, two trading session economy. We assume that the payoffs of the two risky assets are positively correlated. Informed investors have access to information technology about the payoff of the first asset. We further assume that Ξ_1 and Ψ_i are diagonal, so that the errors in the public information signal are independent and the supply of each risky asset is independent. In addition, the mean supplies of the risky assets are positive. The precision matrices H, Ψ_i, Ξ_1, and Φ are represented as follows:

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}, \ \Psi_i = \Psi = \begin{pmatrix} \psi & 0 \\ 0 & 0 \end{pmatrix},$$
$$\Xi_1 = \begin{pmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{pmatrix}, \ \Phi = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}, \ h_{12} < 0.$$

The last inequality implies that the asset payoffs of the two assets are positively correlated.

We assume that the cost for information acquisition is $C(\psi_i)/[2\rho]$. We further assume that $C(\cdot)$ is increasing, convex, and twice continuously differentiable and that the cost of not acquiring a signal is zero. Proposition 3 describes how the introduction of derivative assets affects the decision to acquire information in the dynamic setting.

Proposition 3. *In a two risky asset, two trading session economy, introduction of derivative assets will cause informed investors to acquire more precise information.*

Introduction of derivative assets increases the incentive to collect information about asset payoffs. This, in turn, increases the informational content of the price of the underlying asset and causes the expected price to increase. When prices are more efficient, the price reaction to future public information should be smaller. Proposition 4 summarizes the effects of the introduction of derivatives on the expected prices of the risky assets and the price reaction to future public information.

Proposition 4. *In a two asset, two trading session economy, introducing derivative assets has the following effects:*
(i) The expected prices of both assets will increase.
(ii) The price reaction of both assets to the surprise in public information will be reduced.
(iii) The expected price changes and volatilities of price changes will decrease.

Intuitively, information about the payoff of the first asset also reveals some information about the payoff of the second risky asset. Increased information acquisition on the first asset also improves price efficiency of the second asset, which causes its price to increase. In addition, since more information is incorporated in the prices when the derivative assets are traded, the prices respond less to public signals such as earnings announcements.

3. Welfare Implications

Apparently the introduction of derivative assets makes both informed and uninformed investors better off. However, when the decision to acquire private information is endogenized, the derivative asset causes informed investors to acquire more precise information and therefore increases the risk of their endowments. In this sense, as analyzed by Diamond (1985), private information can make everybody worse off.[20]

[20] The welfare effects analyzed here are conditional on investors' endowments. Since in this model, the variance of investors endowments is infinite, the unconditional expected utility is not well defined. The model can be modified such that each investor receives an endowment of $\tilde{x}_a = \tilde{x} + \tilde{\omega}_a$, and $\tilde{\omega}_a$ is normally distributed and independent across investors. In this case, an investor can use his endowments to get some information about the per capita asset supply, and unconditional social welfare is well defined. In addition, the effect of the derivative asset on the behavior of the price of the underlying asset and social welfare is similar. However, such an exercise does not bring any new insight and introduces unnecessary complexity.

For simplicity, we assume that $\bar{x} = 0$. Let ψ^* be the precision of the private signals of informed investors; the utility of investor a in the absence of the derivative asset is given by

$$\begin{aligned} EU_a &= -\sqrt{\frac{1}{L_0 K_{0a}}} \exp\left(C(\psi_a)/2 - \rho \bar{v} x_a + \frac{\rho^2 x_a^2}{2h} - \frac{\rho^2 x_a^2}{2K_0^2 L} \right) \\ &= -\exp(-\rho\alpha_a - \rho\bar{v}x_a + \beta\rho^2 x_a^2/2), \end{aligned} \tag{23}$$

$$\alpha_a = \frac{1}{2\rho}[\ln[L_0 K_{0a}] - C(\psi_a)],$$
$$\beta = 1/h - 1/K_0^2 L_0.$$

Let $\hat{\psi}$ denote the precision of the private signals of informed investors. When a generalized straddle $\tilde{G}$ is introduced, the utility of investor a is given by

$$\begin{aligned} E\hat{U}_a &= -\sqrt{\frac{2}{\hat{L}_0 \pi}} \exp\left(\frac{C(\psi_a)}{2} - \bar{v} x_a \rho \right. \\ &\quad \left. + \frac{1}{2}\left(\frac{1}{h} - \frac{1}{\hat{K}_0^2 \hat{L}_0}\right)\rho^2 x_a^2 \right) \int_0^\infty \frac{\exp[-\hat{K}_{0a} v^2/2]}{\exp[\rho D_{Ga}(g(v) - P_G)]} dv \\ &= -\exp(-\rho\hat{\alpha}_a - \rho\bar{u}x_a + \hat{\beta}_a \rho^2 x_a^2/2), \end{aligned} \tag{24}$$

$$\begin{aligned} \hat{\alpha}_a &= \frac{1}{2\rho}\left[\ln\left[\hat{L}_0 \hat{K}_0\right] - C(\psi_a) \right. \\ &\quad \left. + \ln\left[\int_0^\infty \exp\left[-\frac{\hat{K}_{0a} v^2}{2} - \rho D_{Ga}(g(v) - P_G) \right] \right] dv \right], \end{aligned}$$
$$\hat{\beta} = 1/h - 1/\hat{K}_0^2 \hat{L}_0.$$

We have already shown that $\hat{\psi} > \psi^*$; it follows that $\hat{\beta} > \beta$. The term β is the certainty equivalent cost per unit of endowment risk. This indicates that when the derivative asset is introduced, investors with large endowments are worse off since more information about the value of their endowment is released before they can trade. If such signals were perfectly informative, the effect would be the same as if investors had been prevented from trading their endowments.

The term α_a measures the speculative gain an investor with zero endowment would make from trading. The effect of introducing derivatives on the welfare of an investor with zero endowment is indeterminate. We will show numerically that investors with zero endowment can be made better off or worse off depending on the value of the parameters. We consider a quadratic derivative asset and a quadratic cost function for private information, $C(\psi_i) = c\psi_i^2$. Figure 1 plots $\hat{\alpha}_a$ and α_a as a function of λ for informed and uninformed investors. Informed investors without endowments are made better off by the introduction of derivative assets when the proportion of informed investors is small. On the other hand, uninformed investors without endowments in the risky asset are made worse off when the proportion of informed investors, λ, is small. In addition, there exist parameters for which both informed and uninformed investors are made worse off by the introduction of the derivative asset. This shows that the introduction of derivative assets can make everybody worse off when the decision to acquire private information is endogenized. Trading in derivative assets itself increases risk sharing among investors and thus improves social welfare. However, it also increases the incentive to acquire private information, which causes the information conveyed through the price to be more precise. Since price is a public signal, the second effect reduces risk sharing among investors. When the second effect dominates the first effect, everybody is worse off.[21]

The preceding arguments provide an example of the time inconsistency of the benefit of introducing derivative assets. There are cases when, ex ante, before investors acquire their private signals, everyone would agree not to open the derivative market. However, ex post, after investors have acquired their private signal, everyone would agree to open the derivative market. In addition, the equilibrium with the introduction of the derivative asset is renegotiation-proof.

4. Implications and a Survey of the Empirical Literature

Our model has many testable empirical predictions about the effects of introducing derivative assets. However, it should be noted that any linkage between the theoretical implications and the empirical literature should be interpreted with caution. The theoretical results come from comparing two models of trading, one with derivatives and one without derivatives. On the other hand, the empirical findings compare the price behavior of the underlying asset before and after the introduction of derivatives. If the economy has changed over time or the introduction of derivatives is endogenously determined following a change in some parameter that makes it profitable to introduce the derivative, the empirical findings cannot be directly compared with the model's implications. With these reservations in mind, we review the empirical literature on derivatives trading and compare it with our theoretical predictions.

[21] The welfare results should be interpreted with caution since we do not include production in this model.

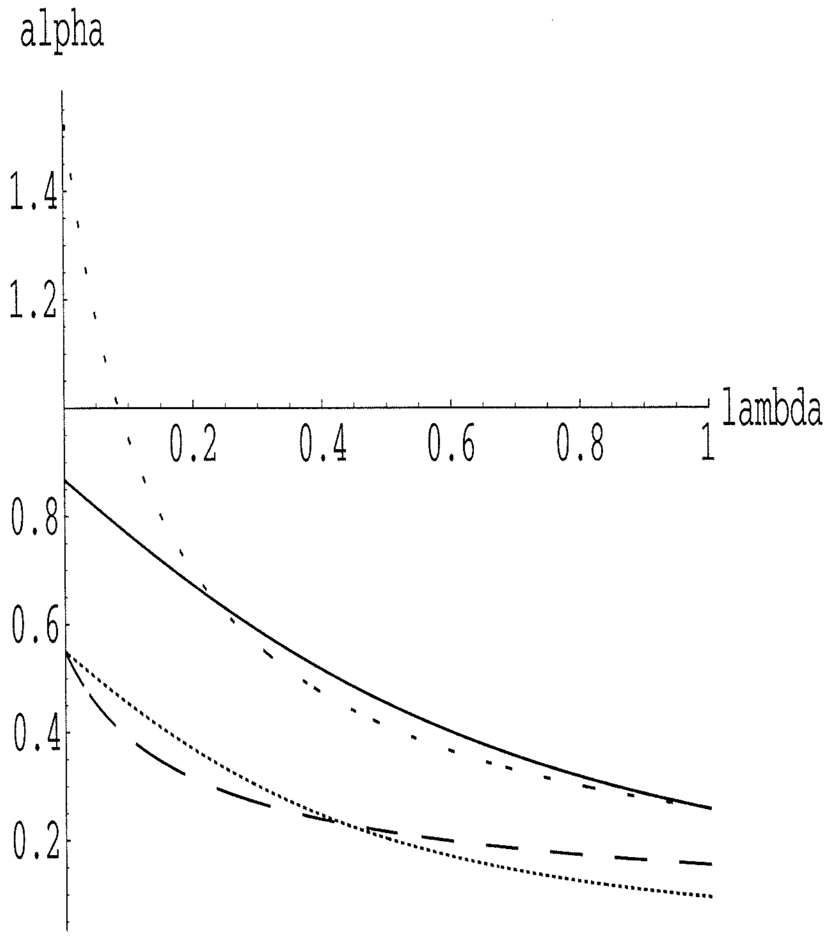

Figure 1
Welfare effects
The certainty equivalent gains from trading with and without the derivative asset, $\hat{\alpha}$ and α, of an investor with zero endowment of the risky asset are plotted with respect to the proportion of informed investors, λ. The top two curves are the certainty equivalent values for an informed investor; the bottom two curves are the certainty equivalent values for an uninformed investor with and without the derivative asset. For the parameters of the model we chose $h = 0.5, \rho = 1, \phi = 1, c = 1$. Solid curve: informed investor without the derivative asset. Dotted curve: uninformed investor without the derivative asset. Sparsely dashed curve: informed investor with the derivative asset. Long-dashed curve: uninformed investor with the derivative asset. For λ between 0.22 and 0.4, both informed and uninformed investors are better off in the absence of the derivative asset.

4.1 Derivatives trading and the price of the underlying asset

This model presented here predicts that, on average, the price of the underlying asset should increase after derivative assets are introduced. The risk in holding the underlying asset is reduced when informed investors collect more private information, thereby causing more information to be revealed about the asset payoff through trading. The empirical literature that examines this issue appears to be consistent with our predictions.

Early studies by Branch and Finnerty (1981) found a substantial price increase before the announcement date and around the listing date. By contrast, Whiteside, Dukes, and Dunne (1983) found no price impact during the announcement period. Conrad (1989) examined the price effect of option introduction from 1974 to 1980. Using a sample of 96 stock options, she found a risk-adjusted cumulative abnormal return (CAR) averaging 2.95% in days -10 to $+10$ of option listings. However, she did not find any price effect around the announcement dates. Detemple and Jorion (1990) confirmed her results in a study of a larger sample of 300 options listed between 1973 and 1986. They found a positive CAR averaging 2.9% in the two weeks around option listings. Consistent with Conrad's results, they found no announcement effect over the whole period 1973–1986. However, when the sample was divided into three subperiods, they found a significant increase of 2% during the three-day period around the announcement date in the second subperiod from 24 June 1975 to 26 March 1982.

In the two weeks surrounding the listing date, Detemple and Jorion (1990) also found a significant increase in the value of the market portfolio of the order of 1.1% for the value-weighted market index and 1.5% for the equally weighted market index. These results are consistent with our model with multiple positively correlated assets, since the average correlation of the optioned stock with the equally weighted market index is positive (0.444). The equal-weighted industry index increased by about 1.6%, which is more than the market as a whole but less than the stock itself.

Detemple and Jorion further showed that the effects of option introduction on the price of the underlying asset have become smaller in recent years.[22] Our model suggests that as the market becomes more complete with the introduction of options, additional new options trading will have less effect on the price of the underlying assets. In the two-asset model described in Section 2, introduction of options on one asset causes an increase in price of the second asset. If options on the second asset are introduced to the market afterward, it is likely that the price effect on the second asset will be reduced since the price is already elevated.

Finally, Detemple and Jorion have investigated the effects of delisting in a sample of 32 stocks, effects that should be opposite to those of listing. They found an abnormal return of -0.88% on the day of delisting. Although insignificant, the sign is opposite to that of the listing event.

[22] Sorescu (1995) found evidence consistent with option dealers manipulating underlying stock prices before 1981, which accounts for most of the positive price impact documented by prior studies.

4.2 Effects of derivative trading on price volatility

Our model predicts that the introduction of derivatives reduces price volatility as price becomes a less biased estimate of the asset payoff due to more information collection. This prediction is consistent with the bulk of empirical literature that examines the effects of derivative trading on price volatility.

Earlier studies by Nathan (1974) and Hayes and Tennenbaum (1979) found that the optioned stocks experienced a decrease in volatility compared to nonoptioned stocks. Similarly, Trennepohl and Dukes (1979) showed that the systematic risk of optioned stocks as measured by beta has a larger decline compared to nonoptioned stocks. Klemkovsky and Maness (1980) found mixed results on the effects of option listing on the average beta and volatility of the underlying stock, while Whiteside, Dukes, and Dunne (1983) found no statistically significant change in systematic risk after option listing.

A problem with these early studies is the small sample sizes and the fact that some study only the systematic risk measured by beta. Many recent studies have used much larger sample sizes and focused on the volatility of the returns of the underlying asset. Rao and Ma (1987) and Ma and Rao (1988) used a much larger sample of 251 stocks that have options listed over the period of 1973–1983. They concluded that 71 stocks had higher volatility, while 109 had lower volatility after option listing. Using multivariate analysis, they found that stocks with low returns, high risk, low trading volume, and low growth potential were likely to be stabilized, following the listing of options.

Nabar and Park (1988) looked at the effect of the introduction of options on 390 U.S. stocks over a single-year period. They used an event study technique, calculating pre- and postlisting variances in 30-day return periods before and after listing. They also used another sample of 340 stocks as a control group and found a small decline of 4% to 8% in volatility after options were introduced. Skinner (1989) worked with a sample of 304 stocks over the period 1973–1986 to conclude that the variance decreases by 10% to 20% in raw returns depending upon the time interval used. The decline was about 10% when the returns were adjusted for the market return each day. Conrad (1989) used a sample of 96 optioned stocks and studied the volatility of excess returns adjusted for the beta of the stock and found that the variance of daily excess returns declined after option listings. Neither Skinner nor Conrad found any significant change in the beta of the optioned stock. In a cross-sectional study with 255 optioned stocks and 176 nonoptioned stocks, Ho (1993) found that over the period 1980–1983, the median value of beta was significantly greater for the optioned stock than for the nonoptioned stock.

Bansal, Pruitt, and Wei (1989) used squared daily returns as a measure of volatility and found this measure was lower by 6.4% in the 100 days after option listings in a sample of 175 stocks during the period 1973–1980. Damodaran and Lim (1991) studied a sample of 200 stocks and concluded that the listing of options led to a significantly lower variance in the raw return of about 20% after option listings. The variance of excess return declined about 15% after option listings.

A caveat regarding these studies on volatility was that they do not control for marketwide volatility changes. Lamoureux (1991) found that the period in which options have been introduced was characterized by an economy-wide drop in residual variances. After controlling for marketwide change in volatility, he found no effects of the introduction of options on volatility. Freund, McCann, and Webb (1995) divided the sample into three subperiods and found that the variance of optioned stocks decreased only in the earliest introductions at a low level of confidence.

n summary, despite early studies showing that option listings reduce stock return volatility of the underlying assets, controlling for marketwide changes in volatility indicates that such reduction in volatility is small and occurs only for the earliest options introductions. However, it should be noted that none of the studies in this area has found evidence showing that option introduction destabilizes the underlying asset.

4.3 Price reaction to earnings announcements

The model presented here predicts that the price reaction to public announcements should be reduced when option trading is introduced, since information collection is more intensive before public announcement. As a result, the informational content of future earnings announcements decreases.

Using data from 1973–1986 for a sample of 214 stocks, Skinner (1990) found smaller abnormal returns on unexpected earnings surprises after option listings. In a cross-sectional study with 255 optioned stocks and 179 nonoptioned stocks, Ho (1993) found that the surprise associated with quarterly earnings announcements was greater for nonoptioned firms than optioned firms, and that the prices of optioned firms anticipated annual accounting earnings changes earlier than those of nonoptioned firms. Botosan and Skinner (1993) found that the price drift after earnings announcements is less significant after option listings and Ho (1993) found that the price drift was less for optioned firms than those of the nonoptioned firms.

Several researchers have examined the effects of option listings on the speed of price adjustment using time-series and cross-sectional studies. Jennings and Starks (1986) used a sample of 180 stocks during 1981–1982 and found that the prices of optioned stocks adjusted more quickly to earnings reports than did the prices of nonoptioned stocks of comparable size. In a sample of 200 stocks during the period 1973–1985, Damodaran and Lim (1991) found that the price of the underlying asset adjusted more quickly to information after options were listed.

These empirical results support the prediction that the price of the underlying asset is more informative after derivative assets are introduced. As a result, the informational content of subsequent public signals is reduced.

4.4 Financial analysts and institutional holdings

This model suggests that the number of analysts following a firm will increase after an option listing and that the optioned firm should be followed with larger numbers of financial analysts. In addition, if institutional investors are considered to be informed investors, the optioned firms should be associated with higher institutional holdings.[23]

In a sample of 214 firms, Skinner (1990) found that the average number of analysts following the stock increased from 11.75 twelve months before option listings to 15.39 twelve months after listing. His result was confirmed by Damodaran and Lim (1991), who found that the average number of analysts following stocks in the sample increased from 13.326 in the prelisting period to 15.839 in the postlisting period. In addition, the number of *Wall Street Journal* articles on the firm increased from 33.098 to 40.164 after the postlisting period. The differences cannot be explained by the market average change over the same period. In a cross-sectional study of 259 optioned firms and 176 nonoptioned firms, Ho (1993) found that the average number of analysts following optioned firms was 11.450 and 8.920 for nonoptioned firms and that the financial coverage in the *Wall Street Journal* was 37.140 articles for optioned firms and 22.990 for nonoptioned firms.

Damodaran and Lim (1991) also reported that the number of institutional holdings increased dramatically after the option listings, with the percentage holding increasing from 19.39% in the quarter before the listing to 32.56% after the listing. Ho (1993) found the average number of institutional investors for optioned firms is 250.99 versus 134.20 for nonoptioned firms. These cross-sectional and longitudinal studies support model predictions that information collection becomes more intensive after option listings.

4.5 Trading volume

The dynamic trading model suggests that the introduction of derivative assets with long maturity dates will reduce trading volume based on private information. However, this result should be interpreted with caution since the effect on trading volume in the underlying asset depends on the kind of derivative assets introduced in the market and the parameters of the model. When options with short maturity dates are introduced and the precisions in

[23] In this model the proportion of informed investors is fixed. It is straightforward to modify our model to endogenize the proportion of informed investors, and it can be shown that there are parameters such that the proportion of informed investors will increase after the introduction of derivative assets.

the private signals of informed investors are small, trading volume increases with the introduction of option trading.[24]

Using 43 stocks with option listings, Hayes and Tennenbaum (1979) found a statistically significant increase in trading volume after option listings. Another early study by Whiteside, Dukes, and Dunne (1983) using 71 stocks also found an increase in average trading volume. The average change in market-adjusted daily volume was only 0.25 shares, which is not statistically significant. Similar to studies of price volatility, early studies on trading volume suffer from small sample size. New studies with a large sample of optioned stocks also give ambiguous results. Skinner (1989) found an increase in the raw trading volume after the listing of options in a sample of 304 firms over the period 1973–1986. His result was confirmed by Damodaran and Lim (1991) in a study of 200 stocks between 1973 and 1986. However, Damodaran and Lim (1991) also found that the change is insignificant for the same period in market-adjusted trading volume. Bansal, Pruitt, and Wei (1989) divided their sample of 175 stocks over the period 1973–1986 into two groups and found that the market-adjusted volume increased after option listings before 1979 but not thereafter.

In a cross-sectional study of 255 optioned stocks and 176 nonoptioned stocks, Ho (1993) found that trading was much more active in optioned stocks. The median (mean) monthly volume for the optioned stocks was 2.50 (2.89) million shares. In contrast, the median (mean) monthly volume for the nonoptioned stocks was 0.77 (0.94) million shares. These differences are significant at the 0.01 level.

Rao, Tripathy, and Dukes (1991) studied 49 OTC-traded stocks with listed options and found a significant increase in the average daily stock trading volume in the post-option listing period. The average daily unadjusted trading volume exhibited a statistically significant increase of 10.2% and 74.3% for the short-term and long-term, respectively. These results persisted even after controlling for marketwide changes in trading activity. On a market-adjusted basis, the relative increase in trading volume for the short-term and long-term was 5.3% and 12.2%, respectively. However, only the latter was statistically significant using a nonparametric test.

In summary, these results seem to be consistent with a weak increase in the raw trading volume of the underlying asset. In some longitudinal studies it is found that the increase disappears after controlling for market volume increases during the same period.

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[24] The results with long-maturity options in Section 2 suggest that the recent introduction of long-term equity anticipation securities (LEAPs) should be associated with a decrease in trading volume. However, if short-term derivative assets, $\tilde{Z}_1 = (\tilde{P}_{11} - \tilde{P}_{10})^2$ and $\tilde{Z}_2 = (\tilde{v}_1 - P_{11})^2$, are traded in the market instead of $\tilde{Z}_{10} = (\tilde{v}_1 - \tilde{P}_{10})^2$, then there exist parameters such that trading volume may increase after the introduction of derivative assets. See also Cao (1995) and Brennan and Cao (1996).

4.6 Market liquidity

The model presented here predicts that the price aggregates more private information after the introduction of option trading. Consequently, price should be less sensitive to a noninformational supply shock, and market liquidity should increase after the opening of option trading.

There are no direct tests on market liquidity and option trading to our knowledge. However, an indirect measure of liquidity is the bid-ask spread. Fedenia and Grammatikos (1992) used a much larger sample of 419 firms over the period 1970–1988 to find that the average bid-ask spread decreases after the listing of options for 341 NYSE-traded stocks. However, the bid-ask spread increased for the 78 stocks in the sample that are traded over the counter. On the contrary, Rao, Tripathy, and Dukes (1991) found evidence that the bid-ask spread decreased on average for the 49 OTC stocks in their sample. The source of these discrepancies was unclear, but the small sample size may be a problem in the study of OTC stocks. Fedenia and Grammatikos (1992) also tried to dissect the change in bid-ask spread into three components: the market liquidity effect, the price volatility effect, and the effect on bid-ask spread alone. They found the component contributed by market liquidity was negative in both NYSE stocks and OTC stocks, implying that market liquidity increased after options are introduced. Damodaran and Lim (1991) used their sample of 200 stocks over the period 1973–1986 to measure the serial correlation for the bid-ask spread proposed by Roll (1984). They concluded that the bid-ask spread declined after the listing of options.

In summary, most of the results on bid-ask spread are consistent with a decrease in bid-ask spread and an increase in market liquidity, which is consistent with our model.

5. Conclusion

This study presents a theoretical model to analyze the interaction of derivative assets and the price of the underlying asset. We show that the presence of the derivative asset causes informed investors to acquire more precise information, since they can take advantage of their superior information in both the underlying asset market and the derivative market. The model explains several empirical findings concerning the introduction of derivatives: (i) the introduction of derivatives causes an increase in the price of the underlying asset; (ii) the introduction of derivatives causes an increase in the level of the market index; (iii) the introduction of derivatives causes a decrease in the response of the price of the risky asset to public information; (iv) the introduction of derivatives increases the number of analysts following the underlying asset. Moreover, this model also predicts that the introduction of option trading should reduce the price reaction of positively related assets to public information and increase market liquidity. The empirical evidence of a decrease in bid-ask spread after option trading provides indirect support for this prediction on market liquidity.

Detemple and Selden (1991) also showed theoretically that the price of the underlying asset increased when an option is introduced to the market. This happens because investors who believe that the stock is more risky than others hold long positions in the option. However, the intuition behind the results in their model and ours are very different. In our model, investors have asymmetric information, and the result is driven by the effect of the introduction of a derivative asset on investors' information acquisition decision, while in Detemple and Selden (1991) investors' exogenous disagreement on the downside risk of the stock plays the critical role. If the information acquisition decision of the informed investors is taken as given in our model, then, contrary to Detemple and Selden's (1991) model, introduction of derivative assets will have no effect on the price. Therefore this model indicates that even if the introduction of a derivative asset does not directly affect the asset price, it affects the investors' decision to acquire private information, which indirectly affects the asset price.

Following the conventional rational expectations literature, we assume that the state variables in this model are multinormally distributed and that investors have negative exponential utility. In addition, we restrict the type of derivative assets introduced to the market. These assumptions are clearly oversimplified. It would be interesting to relax them and analyze a model with more general utility functions, asset distributions, and derivative assets. However, these generalizations would make the model intractable and require numerical methods to find the equilibrium. In general the effect of an introduction of derivative assets on information acquisition depends on the trade-offs between three factors. First, the introduction of derivative assets expands the trading opportunities of investors and thereby increases the marginal value of information. Second, the introduction of derivative assets will also reveal information through their prices, thereby decreasing the marginal value of information. Third, the price of the underlying asset and the information it contains will be affected by the introduction of derivatives, which will also affect the marginal value of information.

It is possible that the introduction of derivative assets may reduce the incentive to collect information in some settings. Therefore we should be careful in applying our theoretical results here to public policy in regulating derivatives markets. Nevertheless, this model demonstrates that introducing derivatives will affect the incentive to collect information, the price of the underlying asset, and the price response to public information, and provides possible explanations for the facts found in empirical studies.

Appendix

Proof of Theorem 1. Let D_{0a} and D_{Ga} be the optimal demands for the risky asset and the generalized straddle and P_G be the price of the generalized straddle.

Let w_{0a} be the wealth of investor a at time 0; then the wealth of investor a at time 1 is

$$\tilde{w}_a = w_{0a} + D_{0a}(\tilde{v} - P_0) + D_{Ga}[g(|\tilde{v} - P_0|) - P_G].$$

Assuming that $g(\cdot)$ is measurable and satisfies a growth condition such that there exists a constant K and $g(v) < Kv^2, \forall v$, the first-order condition for investor a is given by

$$E_a[(\tilde{v} - P_0)U'(\tilde{w}_a)] = 0, \tag{25}$$

$$E_a[[g(|\tilde{v} - P_0|) - P_G]U'(\tilde{w}_a)] = 0. \tag{26}$$

Calculating the expectations explicitly, defining $u \equiv v - P_0$, and dropping irrelevant terms, Equations (25) and (26) become

$$\int_{-\infty}^{\infty} u \exp[-(D_{0a}\rho - K_{0a}[\mu_{0a} - P_0])u - K_{0a}u^2/2 - D_{Ga}g(|u|)\rho]du = 0, \tag{27}$$

$$\int_{-\infty}^{\infty} (g(u) - P_G) \exp[-(D_{0a}\rho - K_{0a}[\mu_{0a} - P_0])u \\ - K_{0a}u^2/2 - D_{Ga}g(|u|)\rho]du = 0. \tag{28}$$

When $D_{0a} = K_{0a}[\mu_{0a} - P_0]/\rho$, the linear term in the exponent term in Equation (27) disappears, the integrand in Equation (27) is an odd function of u, and the integral on the left-hand side of Equation (27) becomes zero. This gives the demand and the price for the risky asset in Theorem 1.

Given the optimal demand for the risky asset, Equation (28) simplifies to

$$\int_{-\infty}^{\infty} (g(u) - P_G) \exp[-K_{0a}u^2/2 - D_{0aG}(g(|u|) - P_G)\rho]du = 0. \tag{29}$$

The expression for the price of the derivative asset follows immediately. The optimal demand for the derivative asset only depends on K_{0a}, P_G. Since the aggregate demand for the derivative asset sums up to zero, we have

$$\int_0^1 D_{Ga}da = 0.$$

The introduction of a generalized spread can be treated similarly. Since the payoff of a generalized spread is orthogonal to the marginal utility of investors in the economy, the price and demand of a generalized spread are both zero.

Proof of Proposition 1. Without loss of generality, we assume that $g(\cdot)$ is monotonically increasing. We need to show that the marginal value of information increases after a generalized straddle is introduced. We first show that the demand for the generalized straddle is smaller for informed investors.

The derivative $d[D_{Ga}]/d[K_{0a}]$ can be determined from the implicit function [Equation (9)] as

$$\frac{d[D_{Ga}]}{d[K_{0a}]} = -\frac{\int_0^{\infty}(G(u) - P_G)u^2 \exp[-K_{0a}u^2/2 - D_{Ga}g(u)/r]du}{2\int_0^{\infty}(g(v) - P_G)^2 \exp[-K_{0a}u^2/2 - D_{Ga}f(u)/r]du}.$$

Define Z to be a positive random variable with probability density proportional to $\exp[-K_{0a}z^2/2 - D_{Ga}g(z)/r]$. We can rewrite $d[D_{Ga}]/d[K_{0a}]$ as

$$\frac{d[D_{Ga}]}{d[K_{0a}]} = -\frac{\mathrm{cov}[Z^2, g(Z)]}{2\mathrm{var}[g(Z)]} < 0.$$

The inequality comes from the monotonicity of $g(\cdot)$. Therefore the demand for the derivative asset is smaller for informed investors. Since the total supply of the derivative asset is zero, informed investors' demand for the derivative asset must be negative.

We can now determine the marginal value of information. In the presence of the derivative asset, the expected utility of informed investor i who acquires a signal ψ_i, given that all other informed investors acquire signals of precision ψ, is given by

$$EU_i^G = EU_i\sqrt{\frac{2}{\pi K_{0i}}}\int_0^{\infty} \exp[-K_{0i}u^2/2 - D_{Gi}(g(v) - P_G)\rho]du. \tag{30}$$

Dropping irrelevant terms, taking a log transformation, and differentiating with respect to ψ_i, investor i's marginal value of information, MV_i^G, is

$$MV_i^G = MV_i + \frac{\int_0^{\infty}(u^2 - 1/K_{0i})\exp[-K_{0i}u^2/2 - D_{Gi}(g(u) - P_G)\rho]du}{\rho\int_0^{\infty}\exp[-K_{0i}u^2/2 - D_{Gi}(g(u) - P_G)\rho]du}. \tag{31}$$

Define V to be a normally distributed random variable with mean 0 and variance K_{0i}^{-1}. We can rewrite Equation (31) as

$$MV_i^G - MV_i = \frac{\mathrm{cov}[V^2, \exp[-D_{Gi}g(|V|)\rho]]}{2\rho E[\exp[-D_{Gi}g(|V|)\rho]]} > 0.$$

The inequality comes from the negativity of D_{Gi} and the monotonicity of $g(\cdot)$. The marginal value of information increases in the presence of the derivative asset, and it follows that informed investors will acquire more precise information. This completes the proof of Proposition 1.

Proof of Theorem 2. We provide the proof for the two trading session economy. The equilibrium in the last trading session is equivalent to Admati's (1985) single trading session model, and the proof is omitted. We focus on the proof of the equilibrium in the first trading session. It is straightforward to prove that price function in the theorem clears the market. We need only to show that the demand of investor a in the first period is optimal given the public information and his private signal.

Given the equilibrium trading strategy in the second trading session, dropping constant terms, investor a's expected utility in the second trading session is

$$E[U_a|F_{0a}, \tilde{c}, \tilde{b}] = -\exp[-\rho D_0'\tilde{c} - \tilde{b}'K_{1a}\tilde{b}/2], \tag{32}$$

where

$$\tilde{c} = \tilde{P}_1 - P_0,\ \tilde{b} = \tilde{\mu}_{1a} - \tilde{P}_0.$$

A useful property is that

$$\tilde{c} = A(\tilde{v} - P_0 + \tilde{\delta}),$$

where $A = K_1^{-1}(K_1 - K_0)$, $\tilde{\delta} = (K_1 - K_0)^{-1}\Xi_1\tilde{\eta}_1$.

Defining $\Omega \equiv \text{var}^{-1}[\tilde{\delta}|F_{0a}]$, we have

$$\text{var}[\tilde{c}|F_{0a}] = A(K_{0a}^{-1} + \Omega_a^{-1})A.$$

Take the regression of $\tilde{b}$ on $\tilde{c}$, we can write $\tilde{b}$ as

$$\tilde{b} = B_a\tilde{c} + \tilde{\zeta}_a,$$

where

$$\text{cov}[\tilde{c}, \tilde{\zeta}_a|F_{0a}] = 0,$$

$$B_a = (K_{0a} + \Omega_a)^{-1}\Omega_a A^{-1} - I.$$

Define $\tilde{\mu}_{ba} \equiv E[\tilde{b}|F_{0a}, \tilde{c}]$, $\mu_{\zeta a} \equiv E[\tilde{\zeta}_a|F_{0a}]$, $\mu_{ca} \equiv E[\tilde{c}|F_{0a}]$, $\mu_{\zeta a} \equiv E[\tilde{\zeta}_a|F_{0a}]$, $\text{Var}[\tilde{\zeta}_a|F_{0a}] = \Gamma_a^{-1}$. Since ζ_a is the residual term of regression of $\tilde{b}$ on $\tilde{c}$, it is independent of $\tilde{c}$. We first calculate the expected utility conditional on $\tilde{b}$ and drop irrelevant terms. This gives the following expression:

$$\begin{aligned} E[U_a|F_{0a}, \tilde{c}] &= -\exp[-D_{0a}'\tilde{c}\rho - \tilde{\mu}_{ba}'K_{1a}\tilde{\mu}_{ba}/2 \\ &\qquad + \tilde{\mu}_{ba}'K_{1a}(K_{1a} + \Gamma_a)^{-1}K_{1a}\tilde{\mu}_{ba}/2] \\ &= -\exp[-D_{0a}'\tilde{c}\rho - \tilde{\mu}_{ba}'(K_{1a}^{-1} + \Gamma_a^{-1})^{-1}\tilde{\mu}_{ba}/2] \\ &= -\exp[-D_{0a}'\tilde{c}\rho - \tilde{\mu}_{ba}'(K_{0a} + \Omega_a)\tilde{\mu}_{ba}/2]. \end{aligned} \tag{33}$$

Taking the expectation with respect to $\tilde{c}$, we get

$$E[U_a|F_{0a}] = -\exp[-\Pi_a'\Delta_a\Pi_a - (\tilde{\mu}_{0a} - \tilde{P}_0)'K_{0a}(\tilde{\mu}_{0a} - \tilde{P}_0)], \tag{34}$$

where

$$\Pi_a = D_{0a}\rho + B_a'(K_{0a} + \Omega_a)\tilde{\mu}_{\zeta_a} - A^{-1}(K_{0a}^{-1} + \Omega_a^{-1})^{-1}A^{-1}\tilde{\mu}_{ca},$$

$$\Delta_a = [B_a'(K_{0a} + \Omega_a)B_a + A^{-1}\{K_{0a}^{-1} + \Omega_a^{-1}\}^{-1}A^{-1}]^{-1}.$$

Since Δ_a is positive definite, the first-order condition implies that

$$\Pi_a = 0. \tag{35}$$

Determination of conditional expectations $\tilde{\mu}_{\zeta_a}$ and $\tilde{\mu}_{\tilde{c}a}$ gives the following equalities:

$$\tilde{\mu}_{\zeta a} = (K_{0a} + \Omega_a)^{-1}K_{0a}(\tilde{\mu}_{0a} - \tilde{P}_0),$$

$$\tilde{\mu}_{ca} = A(\tilde{\mu}_{0a} - \tilde{P}_0).$$

With a little more algebra, we have the demand schedule for the first trading session:

$$\tilde{D}_{0a} = K_{0a}[\tilde{\mu}_{0a} - \tilde{P}_0]/\rho.$$

Proof of Theorem 3. We provide the proof for the single trading session case. The proof for the dynamic model is similar to that in Theorem 2 and is omitted here. Suppose investor a's demands for the risky asset and the derivative assets are $\tilde{d}_{0a}$, d_{aZ}, respectively. Direct calculation of the certainty equivalent of investor a's expected utility given his signal and the price is as follows:

$$\begin{aligned}\gamma_a \;=\; & w_a + \frac{1}{2\rho}\ln\left(\frac{\det|K_{0a} + 2d_{aZ}\rho|}{\det|K_{0a}|}\right) - tr(d_{aZ}P_Z) \\ & - (\tilde{d}_{0a} - \tilde{D}_{0a})'(2K_{0a} + 4d_{Za}\rho)^{-1}(\tilde{d}_{0a} - \tilde{D}_{0a})\rho \\ & + (\tilde{\mu}_{0a} - P_0)'K_{0a}(\tilde{\mu}_{0a} - P_0)/\rho.\end{aligned}$$

The first-order condition for d_{0a} clearly gives

$$\tilde{d}_{0a} = \tilde{D}_{0a}.$$

The first-order condition for the optimal demand, D_{aZ}, gives

$$D_{aZ} = \frac{1}{2\rho}(P_Z^{-1} - K_{0a}).$$

Proof of Proposition 3. The expected utility of informed trader i who acquires a signal in the first risky asset with precision ψ_i, given that other informed investors acquire signals of precision ψ, is given by

$$\begin{aligned}EU_i \;=\; & \sqrt{\det|I + (\bar{\Psi} - \Psi_i)K_{1i}^{-1}(\bar{\Psi} - \Psi_i)K_0^{-1}K_{1i}K_{0i}^{-1}\Xi_1 K_0^{-1}|^{-1}} \\ & \times \frac{-1}{\sqrt{\det|L_0K_{0i}|}} \\ & \times \exp\left[\frac{C(\psi_i)}{2} - \bar{V}'X_i\rho + \frac{\rho^2}{2}X_i'(H^{-1} - K_0^{-1}L_0^{-1}K_0^{-1})X_i\right].\end{aligned}$$

Ignoring the corner solutions and making the log transformation of the expected utility, we get the first-order condition for the optimal precision ψ^* in a symmetric equilibrium:

$$\left.\frac{\partial f(\psi_i, \psi)}{\partial \psi_i}\right|_{\psi_i=\psi=\psi^*} = C'(\psi^*),$$

where

$$\begin{aligned}& f(\psi_i, \psi) \\ & \quad = \ln[\det|(I + (\bar{\Psi} - \Psi_i)K_{1i}^{-1}(\bar{\Psi} - \Psi_i)K_0^{-1}K_{1i}K_{0i}^{-1}\Xi_1 K_0^{-1})L_0K_{0i}|].\end{aligned}$$

When derivative assets are traded, the expected utility of informed trader i is

$$EU_i = -\frac{\exp[\frac{\rho^2}{2}[X_i'H^{-1}X_i - X_i{\prime}K_0^{-1}L_0^{-1}K_0^{-1})X_i]]}{\sqrt{\det|L_0K_{ia}|}\exp[tr[(\Psi_a - \bar{\Psi})K_0^{-1}]/2 + \bar{V}'X_i\rho]}. \tag{36}$$

Let $\hat{\psi}$ denote the optimal solution for ψ in the economy with the derivative asset, then the first-order condition for the information acquisition problem in a symmetric problem is as follows:

$$\left.\frac{\partial g(\psi_i, \psi)}{\partial \psi_i}\right|_{\psi_i=\psi=\hat{\psi}} = C'(\hat{\psi}),$$

$$g(\psi_i, \psi) = (\psi_i - \lambda\psi)\frac{h_2 + \xi_2}{\det|K_0|}.$$

If $\hat{\psi} \leq \psi^*$, we have

$$\begin{aligned} C'(\hat{\psi}) &= \left.\frac{\partial g(\psi_i, \psi)}{\partial \psi_i}\right|_{\psi_i=\psi=\hat{\psi}} \\ &\geq \left.\frac{\partial g(\psi_i, \psi)}{\partial \psi_i}\right|_{\psi_i=\psi=\psi^*} \\ &> \left.\frac{\partial f(\psi_i, \psi)}{\partial \psi_i}\right|_{\psi_i=\psi=\psi^*} \\ &= C'(\psi^*). \end{aligned}$$

Since $C(\psi)$ is convex, this contradicts the assumption that $\hat{\psi} \leq \psi^*$. Therefore, we must have $\hat{\psi} > \psi^*$.

Proof of Proposition 4 From Equation (17), the expected prices of the two assets in the two-period economy is given below:

$$E[\tilde{P}_t] = \bar{V} - K_t^{-1}\bar{X}.$$

We have

$$\frac{\partial K_t^{-1}}{\partial \psi} = -K_t^{-2}\frac{\partial K_t}{\partial \psi}.$$

Since all elements of K_t^{-1} are positive and all elements of K_t increase with ψ, it follows that all elements of K_t^{-1} decrease with ψ. Proposition 3 indicates that investors will acquire more precise information after the introduction of derivative assets, and therefore Proposition 4 (i) holds.

From Equation (17), price response to the public information $\tilde{Y}_1$ is measured by[25]

$$\frac{\partial \tilde{P}_1}{\partial \tilde{Y}_1} = K_1^{-1}\Xi_1.$$

In this simple two asset economy, the prices of both assets exhibit a reduced response to public information when ψ increases. Similarly, it is straightforward to show that the expected price changes and the volatility of price changes of both assets will decrease with ψ:

$$E[\tilde{P}_1 - \tilde{P}_0] = K_1^{-1}\Xi_1 K_0^{-1}\bar{X},$$

$$\text{var}[\tilde{P}_1 - \tilde{P}_0] = K_1^{-1}\Xi_1 K_0^{-1} + K_1^{-1}\Xi_1 K_0^{-1}(\bar{\Psi} + \rho^2\Phi^{-1})K_1^{-1}\Xi_1 K_0^{-1}.$$

[25] Here we apply an implication of Theorem 3 that the introduction of the derivative asset does not affect the price process of the underlying asset given that the decision to acquire private information has been made.

$$\begin{aligned}\frac{\partial \mathrm{var}[\tilde{P}_1 - \tilde{P}_0]}{\partial \psi} &= -K_1^{-1}\Xi_1 K_0^{-1}[K_0^{-1}\frac{\partial(K_0-\bar{\Psi})}{\partial\psi} + \lambda K_1^{-1}] \\ &\quad + \frac{\partial[K_1^{-1}\Xi_1 K_0^{-1}]}{\partial\psi}(\bar{\Psi}+\rho^2\Phi^{-1})K_1^{-1}\Xi_1 K_0^{-1} \\ &\quad + K_1^{-1}\Xi_1 K_0^{-1}(\bar{\Psi}+\rho^2\Phi^{-1})\frac{\partial[K_1^{-1}\Xi_1 K_0^{-1}]}{\partial\psi}.\end{aligned}$$

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