Payment System Settlement and Bank Incentives

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In this article, we consider the relative merits of net versus gross settlement of interbank payments. Net settlement economizes on the costs of holding non-interest-bearing reserves, but increases moral hazard problems. The "put option" value of default under net settlement can also distort banks' investment incentives. Absent these distortions, net settlement dominates gross, although the optimal net settlement scheme may involve a positive probability of default.

Interbank payment networks effect payment by exchange of deposit claims on banks in the network. Such payments do not become final until they are settled. Virtually all transfers in large-value, electronic networks are settled by transfer of an offsetting amount of central bank funds, that is, outside money. Under current institutional arrangements in most countries, this is the only method of settling large-value interbank claims.[1]

The act of settlement has economic meaning since it determines an ordering of claims on bank assets. Once a transfer of funds has been settled, remaining creditors

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[1] See Bank for International Settlements (1997, p. 3). In some countries, though not in the United States, the use of central bank funds is a legal requirement for interbank settlements.

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(including depositors) of the sending bank cannot (in most circumstances) recover the transferred funds should the bank fail. In the words of Corrigan (1990), "the [settled] money in question is 'good money' even if at the next moment the sending institution goes bust."

Various large-value interbank payment networks employ different rules for settling interbank payments.[2] Some networks, such as the Swiss Interbank Clearing (SIC) network, operate under real-time gross settlement (RTGS). That is, payment messages entered into the payments network are continuously cleared and settled by transfer of reserve funds from the paying bank to the receiving bank. Other networks, such as the Clearing House Interbank Payment Network (CHIPS),[3] operate under net settlement rules. That is, at the close of each business day, the value of all payments due to and due from each bank in the network is calculated on a net basis. Banks ending the day in a net debit position (banks whose due-tos exceed their due-froms) transfer reserves to the network. The network, in turn, transfers these reserve funds to net creditor banks.[4]

The allocation of intraday credit in large-value net settlement networks is of policy concern, given the very large flows associated with these networks. The average gross daily volume of payments over CHIPS, for example, is easily over $1.2 trillion, and Schoenmaker (1995, p. 21) puts average peak daily net debit positions on CHIPS at roughly $50 billion. To give some perspective to these numbers, consider that annual GDP for the United States is roughly $7.5 trillion, and that overnight reserve balances held by commercial banks at the Fed (i.e., the total of all noncurrency reserves) averages less than $15 billion.

In policy circles, it has often been argued that net settlement of interbank payments can reduce the riskiness of such systems.[5] Formal analyses of this question have concluded that the real result is not a reduction but a reallocation of risk, away from participating banks and toward either the government as guarantor of the payments system, deposit insurance facilities, or the public (see Emmons (1995b) and Freixas and Parigi (1996)). It follows that investigations of the relative merits of various settlement

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[2] For a survey of large-value payment systems in the G-10 countries, see Bank for International Settlements (1993, 1997) and Folkerts-Landau, Garber, and Schoenmaker (1996).

[3] CHIPS is operated by the New York Clearing House Association. Payments on CHIPS are most commonly associated with the dollar legs of foreign exchange transactions. See Federal Reserve Bank of New York (1991) and New York Clearing House Association (1995) for a discussion of the operation of CHIPS.

[4] Other sets of rules for clearing and settlement are possible. For example, the Federal Reserve's Fedwire system is nominally a real-time gross settlement system, since all payment messages immediately become liabilities of the Federal Reserve System and therefore equivalent to reserve money. However, the Fedwire system resembles a net settlement system in the sense that participating banks are allowed to overdraft their accounts and have historically had access to free daylight credit (subject to certain limits and, more recently, some modest charges). See Federal Reserve Bank of New York (1995) on the operation of Fedwire.

[5] See, for example, the "Lamfalussy Report" of the Bank for International Settlements (1990).

procedures should consider the effect that these procedures have on overall riskiness associated with a payments network, not on riskiness for a particular subset of the network participants.

In our analysis we show that changes in the overall riskiness of interbank payment networks will be tied to changes in the banks' behavior induced by various rules for settlement. In particular, we examine the effects of settlement rules on banks' tendencies to honor interbank commitments ("net due-tos") rather than default. Small variations in settlement rules can result in significant changes in bank incentives and ultimately bank behavior.

At the most basic level, the trade-off between net and real-time gross settlement can be characterized as a trade-off between two distortions.[6] Net settlement increases default probability and thereby the costs associated with potential defaults, while gross settlement increases the costs associated with holding reserves.[7] The relative merits of the two systems will depend on the relative size of these two costs. Different weightings of these costs as viewed by bank regulators and by banks can lead to different conclusions about optimality of particular settlement rules, and thus may explain the current lack of consensus on this issue.

We examine the trade-off between these two distortions using the continuous-time inventory model described in Harrison (1985). This framework is particularly convenient for this problem because it provides tractable probability distributions for banks' net positions (under net settlement) and liquidity demands (under RTGS). This framework also allows us to analytically calculate the effects of placing upper limits ("caps") on banks' net debit positions. This is a relevant calculation because net debit caps are employed in all real-world payment networks that settle on a net basis.

Our main conclusions are as follows. First, for the simplest case, where bank asset quality is fixed and bank assets can always be liquidated at book value, we find net settlement always dominates real-time gross settlement. However, the optimal net settlement scheme may be one which necessarily involves some probability of default. Second, when we examine the case where the quality of bank assets is a choice variable, we find that the potential costs of net settlement rise due to negative effects on bank asset quality.

These results are of course dependent on the particular assumptions of our model. The model will present some illustrative examples of the trade-off between net and gross settlement, taking certain institutional features of large-value payment systems as given. Our analysis simplifies in many important respects, particularly with respect to the completeness of the

[6] This view of net versus gross settlement is in accord with policy-related discussions on this issue and the related literature (see below). There is another view of net settlement which emphasizes the effects of replacing gross contractual obligations with netted obligations [i.e., "netting by novation;" see, e.g., Green (1997)].

[7] Baer, France, and Moser (1996) study a similar trade-off in the design of futures clearing arrangements.

market for reserves. Our setup also results in an incentive problem that causes banks to default too frequently. The setup is chosen because it is tractable and because it allows us to focus on certain essential trade-offs between the two settlement systems. The relative importance of the trade-offs will vary and further research will be necessary in order to take our results beyond the exploratory stage.

1. Literature Review

Angelini and Giannini (1994) present a systematic comparison of the trade-offs between the costs of liquidity versus the costs associated with defaults under net settlement. Under the assumption that the risk of a bank failure rises monotonically with the interval between settlements, they derive an optimal settlement interval for a net settlement system. Using a similar approach, Schoenmaker (1994, 1995) compares the costs associated with real-time gross and net settlement of interbank payments, based on the costs associated with settlement failure (bank defaults), the opportunity costs of collateral holdings by the banks in the network, and "gridlock" or payment delay costs associated with gross settlement or higher collateral requirements. Using "macro"-level data on payment flows through two large-value U.S. interbank payments networks (CHIPS and Fedwire), Schoenmaker concludes that for these networks, cost of real-time gross settlement (without free daylight overdrafts) would probably outweigh the reduction in the risks associated with bank failures. However, these analyses assume that the probability of default is exogenous to the structure and settlement rules of the network.

Furfine and Stehm (1996) analyze this issue from a different perspective. They consider the problem of a payments network that operates as an RTGS system, where intraday credit is extended by a central bank. The central bank's optimal credit policy is based on balancing the costs of stricter credit policies (gridlock and collateral costs) against the costs of more liberal credit policies (defaults), where these costs are incorporated into nonstochastic functions. The optimal credit policy then depends on the relative weights associated with each type of cost.

Emmons (1995b) builds a microeconomic underpinning for interbank net settlement systems by emphasizing cost savings resulting from minimization of the demand for non-interest-bearing reserves, and from the scale economies associated with costly state verification and delegated monitoring in the case of bank failures. He cautions, however, that the cost savings associated with each of these "natural monopolies" (i.e., in settling payments and in liquidation of failed banks) is unlikely to be realized under current institutional arrangements. Emmons (1995a) extends this framework to investigate the effects of net settlement in terms of risk shifting from other network participants to the deposit insurer and other bank creditors.

Freixas and Parigi (1996) analyze characteristics of net versus gross settlement by considering a model in which gross settlement corresponds to an interbank settlement in reserves and net settlement corresponds to settlement in debt claims. In the version of their model in which there is full information concerning the quality of bank assets, net settlement dominates. However, in the version of the model where there is private information concerning banks' asset values, net settlement can lead to the risk of "contagion," that is, the risk that a failure of one bank can spread to another.[8] The issue of contagion is also taken up in a related article by Rochet and Tirole (1996), who build a model of interbank lending in order to analyze the effects of government safety-net programs on banks' incentives to monitor each other's asset quality.

In contrast to the last two articles, we will abstract from issues of contagion in the analysis below. We argue that while contagion is interesting and important, for most present-day payment networks, the high likelihood of regulatory intervention in the event of potential settlement failures ensures that the probability of contagion will be virtually zero.[9] Thus it makes sense to concentrate first on the problem of the costs associated with individual defaults. Alternatively, one could interpret our analysis of "net settlement" as applying to RTGS settlement systems where central banks grant interest-free, uncollateralized daylight credit.

2. A Model of Interbank Settlement

Many of the critical differences between net and gross settlement systems can be illustrated with a simple discrete-time example. In this setup we will analyze the incentives of a representative bank[10] in an interbank payments network, where the bank exists only for a single "trading day."[11] The bank can hold three types of assets: A, "earning assets;" M, "reserves;" and payments due from other banks or "due-froms," DF; it holds two types of liabilities, payments due to other banks or "due-tos," DT; and C, "deposits."

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[8] Kahn and Roberds (1996) present a related model in which settlement via bank debt is only welfare enhancing to the extent such debt can be collateralized.

[9] In the case of CHIPS and other privately operated payment networks, it should be emphasized that there are no explicit guarantees of settlement from the Fed or any other central bank. However, the perception that such private networks can be "too big to fail" is a common one. See, for example, Eisenbeis (1987, p. 48), Brimmer (1989, p. 15), or Bernanke (1990, p. 150). See also Bank for International Settlements (1996, pp. 6–8), which describes the resolution of four recent potential liquidity crises associated with foreign exchange markets, in each case without incidence of contagion.

[10] Following, the term "bank" will refer to any settling participant of a payments network that settles on the books of a central bank.

[11] That is, the bank is liquidated at the end of the trading day. Hence a "default" by a bank simply changes the priority of end-of-day claims on bank assets. A more realistic setup would acknowledge the value of the bank as a going concern, which would tend to lessen the likelihood of default, ceteris paribus.

By assumption, reserves are non–interest bearing and due-to positions cannot be collateralized.[12]

A bank starts the day at time zero with only earning assets A and deposits C. For convenience, initial holdings of reserves are set to zero.[13] During the course of the day, due-froms and due-tos will accumulate exogenously according to the demands of depositors. No delay is permitted: as soon as a bank receives instruction from a depositor to make a payment, the payment message must be entered into the payment network, or the bank will be in default.[14]

We will take as a legal restriction that net due-to positions must be settled by transfer of reserves. There is no legal reserve requirement. Instead, reserves will be purchased and/or accumulate as needed according to the settlement rules of the payment system. Under real-time gross settlement, for example, banks continually pay off net due-tos as they are realized. Initially we assume that bank assets have constant value over the trading day and that they can be exchanged for reserves at book value.[15] However, the market for reserves is imperfect in the sense that reserves accumulated during the day cannot be exchanged for earning assets during the day, but must be held overnight without receiving interest. We use the assumption that reserves cannot be reinvested during the day as a convenient approximation for the nonexistence of a continuous market in reserves. The existence of such a market, combined with an effective peg of the short-term interest rate, would imply that banks could end each day holding zero net reserves. In the absence of such a market, a "musical chairs" situation is created whereby some banks end up holding reserves.[16] Note also that reserves accumulated during the day can be used to settle due-tos realized during the day.

We will take the "social cost" of holding reserves to be the seignorage cost or implicit tax of $r > 0$ per dollar of reserves held at the end of the

[12] Generally banks cannot pledge assets to private counterparties; see the discussion in Goodfriend (1990). Positions on CHIPS are de facto partially collateralized via a device known as "additional settlement obligations" (ASOs). See Federal Reserve Bank of New York (1991) and New York Clearing House Association (1995) on ASOs. Partial collateralization would not qualitatively change the incentive structure studied below.

[13] In practice, banks begin the day with some reserve holdings. Typically banks' reserve holdings are supplemented at some point during the day via overnight repos with the central bank. Reserves obtained in this fashion are returned the following morning. Above, we approximate this situation by setting initial reserve holdings to zero.

[14] Some of the studies cited above have analyzed the costs associated with the delay of payments. In our model, allowing for delay of payments would only introduce an additional dimension of moral hazard and would be unlikely to change the qualitative nature of our results.

[15] In other words, the bank in our example is forced to fund liquidity needs through "reverse repos." In practice, banks in the United States can also obtain reserves by "purchasing Fed funds," that is, by taking out an unsecured loan of reserves from another bank [markets for Fed funds and repos are discussed in, e.g., Stigum (1990)]. We rule out the existence of a Fed funds market for purposes of tractability.

[16] Lucas (1994) describes this outcome as follows: "[I]n a monetary economy, it is in everyone's private interest to try to get someone else to hold non-interest-bearing . . . reserves. But someone has to hold it all, so all of these efforts must simply cancel out."

day.[17] This specification in effect assumes that money is not superneutral, so that holding non-interest-bearing reserves entails some welfare cost. The nominal rate times reserve holdings serves as a first-order approximation for the welfare costs of inflation that could be derived in a more complete model of the payment system, for example, as in Freeman (1996) or Green (1997). We will also assume that seignorage costs are passed on to depositors. The assumption that the inflation tax can be passed on to depositors simplifies the analysis by allowing us to do separate calculations of banks' reserve holdings and their net worth. This assumption is consistent with a legal restriction against paying interest on demand deposits, coupled with a restriction that funds received during the day must be held overnight as demand deposits.

The finality of reserves transfers will be the key factor in reducing the bank's incentive to default. In other words, settling interbank claims by transferring reserves to other banks irreversibly commits the bank to favor interbank claims on its assets over all others. We will take this "irreversibility" feature of reserves transfers as a given institutional feature of the model environment.[18]

Below we will calculate the optimal size of net debit caps for various environments. These calculations assume that the level of initial asset holdings are observable and known at the time decisions are made concerning the size of net debit caps, and that the size of the caps cannot subsequently be changed during the trading day. This assumption has some basis in real-world practice. Net debit caps on CHIPS, for example, can be in most circumstances be changed only during a 20-minute interval before the opening of business [See New York Clearing House Association (1995, p. 26)]. In Section 4 we extend this model to situations in which the level of asset holdings by a bank is a random variable unknown to other parties at the instant in which debit caps are set.

A bank's decision to default is made on the basis of profit maximization. It will have an incentive to default more often than is socially optimal. Two sets of assumptions underlie this excessive default rate: (1) that default allows the bank to increase the priority of the equity-holders, at the expense of other participants in the network, and (2) that the social costs of default exceed the losses felt by the defaulting bank.

The net worth of a bank if it does not default is the value of its assets minus its liabilities, that is,

$$NW = A + DF + M - C - DT\,. \tag{1}$$

[17] Under the setup above, this cost vanishes in the case of a Friedman deflation, that is, if $r = 0$, or if interest is paid on reserves. However, our results do not qualitatively change as long as there is some positive opportunity cost of holding reserves.

[18] We note that this assumption is consistent with current U.S. institutions. Transfers of reserves via Fedwire immediately become liabilities of the Federal Reserve System; see Federal Reserve Bank of New York (1995, p. 11).

Notional net worth at time zero is simply $NW_0 \equiv A - C$, which we assume to be positive. If a bank defaults, we assume that it is forced into liquidation by a regulator. In this case its net worth is given by

$$\begin{aligned} NW &= \alpha(\text{assets}) - \alpha(\text{deposits}) - \beta(\text{net due-tos}) \\ &= \alpha(A - C) - \beta(ND), \end{aligned} \tag{2}$$

where $1 > \alpha > \beta > 0$ and $ND \equiv DT - DF$. In other words, the cost of liquidation procedures diminishes the value of a bank's assets, but it also allows the bank to partially shift priority away from other banks participating in the payments network. Under this assumption, liquidation disproportionately punishes holders of interbank claims, implying that default is a tempting option for banks with a large net debit position relative to their capital.[19]

End-of-day default under net settlement occurs if net worth after the default exceeds its net worth under normal settlement, which is equivalent to

$$\gamma\, ND(\text{end-of-day}) > NW_0, \text{ where } \gamma \equiv \frac{\alpha - \beta}{1 - \alpha}. \tag{3}$$

The social cost of default is Ξ, where $\Xi \geq (1 - \alpha)A$. That is, the cost of default is at least as great as the value of assets lost from the defaulting bank. However, the total cost of default may also include additional costs, such as the costs of payment system disruptions.

We now consider a discrete-time example in which the day is divided into three periods — 0 (morning), 1 (noon), and 2 (close of business). The intraday evolution of net due-tos is stochastic. We assume that there is an equal probability of depositors receiving $C/2$ in funds or wishing to send $C/2$ in funds in the morning and again in the afternoon. Thus

$$ND(1) = \begin{cases} C/2 \text{ with probability } 1/2 \\ -C/2 \text{ with probability } 1/2 \end{cases} \tag{4}$$

$$ND(2) = \begin{cases} C \text{ with probability } 1/4 \\ 0 \text{ with probability } 1/2 \\ -C \text{ with probability } 1/4. \end{cases} \tag{5}$$

We choose parameter values so that a bank with C in net due-tos finds it advantageous to default, but a bank with $C/2$ in net due-tos does not. This means that

$$\gamma C > NW_0 = (A - C) > \gamma(C/2). \tag{6}$$

[19] Note that banks need not literally have positive net worth in the default state in order for this incentive to exist. A similar incentive could exist if default favored a select group of bank creditors. Such inequities in priority could result from political considerations, for example, if the failing bank is based in one country while the payment network is based in another, or from statutory provisions favoring certain depositors over other creditors.

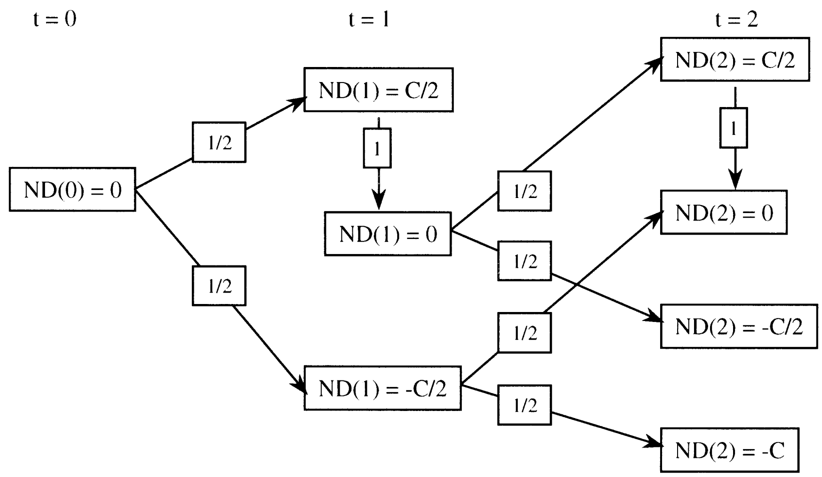

Figure 1
Evolution of bank's net debit position under real-time gross settlement
Numbers in boxes represent conditional probabilities.

Under RTGS banks pay off net due-tos as they are realized. They do so by selling earning assets in return for reserves. Hence, if the bank incurs a net due-to position of $C/2$ in period 1, it must immediately liquidate $C/2$ worth of earning assets. This implies that the bank's maximum net due-to position in period 2 will be $C/2$, which in turn implies that the bank will have no incentive to default in either period (the evolution of the bank's net position under RTGS is shown in Figure 1).

To verify this claim, consider the bank's situation if it arrives in period 2 holding $A - C/2$ in net assets and facing a due-to position of $C/2$. If it defaults its net worth is

$$\alpha(A - C/2) - \beta C/2, \tag{7}$$

which under Equation (1) is less than $A - C$, which is its net worth if it settles. Since its net worth is therefore $A - C$ in period 2, regardless of the due-tos that arrive in period 2, it has no interest in defaulting in period 1 if its due-to position is $C/2$.

Since there is no possibility of strategic default under real-time gross settlement, the expected social costs under RTGS are simply the seignorage costs times the expected reserve holdings at the end of period 2. To evaluate this cost, consider the four equally likely possibilities. If the bank pays out to other banks in each of the two periods, it holds no reserves in the final period. If the bank receives funds in each of the two periods, it holds reserves in an amount equal to C in the final period. If it receives funds in the first

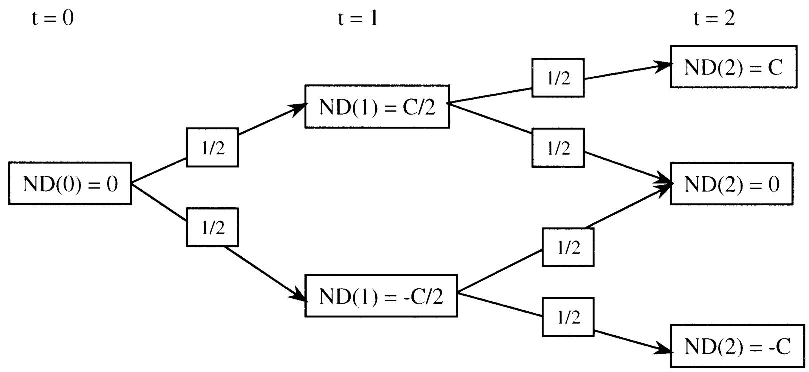

Figure 2
Evolution of bank's net debit position under net settlement
Numbers in boxes represent conditional probabilities.

period and pays in the second, it holds zero reserves. Finally, if it pays in the first period but receives funds in the second period, it holds reserves equal to $C/2$. Thus the expected level of reserve holdings is $(3/8)C$, and social costs are $(3/8)Cr$.

Under net settlement without net debit caps, the bank ends up with a period 2 net debit position of C with probability $1/4$ (see Figure 2). In this case the bank will default under Equation (6). With probability $1/4$ the bank ends up with a period 2 net credit position of C, and with probability $1/2$ the bank ends up with a zero net position. The social costs of this settlement system are therefore given by the social cost of default Ξ, times the probability of default $(1/4)$, plus the cost Cr of holding reserves at the end of the day times the probability of ending the day in a net credit position, that is, $1/4$. The total social cost of net settlement is therefore given by the sum of these two costs, that is, $Cr/4 + \Xi/4$.

In the last calculation, we assumed that the failure of the representative bank does not result in the failure of other banks participating in the payments network. In other words, failure of a bank does not lead to a problem of systemic risk. This could happen if either the failing bank's due-to position were spread over a large number of creditor banks, or if in the case of a default, the failing bank's liabilities were assumed by a private or governmental guarantor.

There are several points that can be made from this simple example. First, changing the rules from real-time gross to net settlement necessarily (though perhaps, weakly) increases the risk of a settlement failure. Second, the social costs of a net settlement system may be less than under gross settlement, even though net settlement can increase the risk of a default. This can happen because net settlement reduces the seignorage costs associated with settlement. Obviously if strategic default were unavailable, then net settlement would always dominate gross settlement, since net settlement would lessen seignorage costs without increasing default risk. We do not regard this as a telling rejoinder to the relevance of the model: strategic default as modeled here is merely the simplest form of moral hazard problem. More complicated forms of moral hazard, such as choice of riskier investment, offering depositors early payment, and the like, will generate similar costs, as long as interbank liability is limited in cases of default.

Finally, we note that a system of net settlement with a net debit cap $D > 0$ offers the possibility of some economization on reserve balances with reduced default probability. For this example it is easy to show that a net debit cap of $D^* = \gamma^{-1} NW_0$ decreases expected reserve holdings while eliminating default. Therefore net settlement with a net debit cap of D^* dominates gross settlement, and indeed if default costs are greater than some critical level, this is the optimal net debit cap. (Because of the discrete nature of this example, the optimal cap is constant for a wide range of parameter values. In the continuous models of subsequent sections, we will examine the effects of changing social cost parameters on optimal net debit caps.)

3. The Model with Continuous Payment Flows

We can better model the incentive problems faced by interbank payment networks if we modify the setup of the previous section to allow for smooth evolution of payment flows over the trading day. In this section we will index time as a continuous parameter t on the unit interval, where $t = 0$ indicates the start of the trading day and $t = 1$ indicates the close of business. For analytical convenience, we will assume that net payment flows $X(t)$ for a representative bank will evolve as a driftless standard Brownian motion [see, e.g., Harrison (1985, p. 1)] over the unit interval. This assumption is natural, if we regard each bank as small relative to the overall size of the network, and if we regard individual payment orders as small relative to the size of the bank. If either of these presumptions is grossly violated, a more complicated process will be needed to model net payments.[20]

We assume that initial net payments $X(0)$ are zero. Ex ante, for any time $t \in [0, 1]$, $X(t) \sim N(0, t)$. In the case of net settlement with no caps, the bank's net debit position at any time t is equal to $X(t)$. In the case of net settlement with net debit cap D, the net debit position $Z(t)$ at any time t is a *regulated* Brownian motion with upper control barrier at D [Harrison (1985,

[20] Also note that it would be feasible to bound the net payment process within finite "reflecting barriers," but that such a restriction would entail a considerable increase in the model's mathematical complexity. See, for example, the computations in Appendix B of Bertola and Caballero (1994).

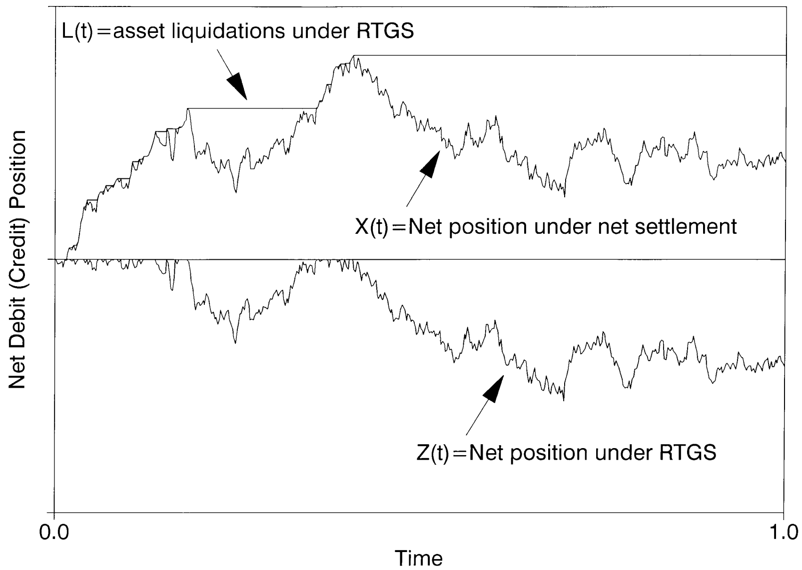

Figure 3
Evolution of net positions in the continuous-time model

p. 14)]. The process $Z(t)$ ranges over $(-\infty, D]$ and may be represented as

$$Z(t) = X(t) - L(t), \tag{8}$$

where $L(t)$ is defined as

$$L(t) = \sup_{0 \leq s \leq t} [\max\{X(s) - D, 0\}]. \tag{9}$$

The interpretation of $L(t)$ is the total cumulated quantity of earning assets that must be sold in order to keep the bank's net debit position $Z(s)$ below the cap D at all times $s \in [0, t]$. Note that L is a nondecreasing and continuous process and only increases when the cap is binding. Representative realizations of $X(t)$, $L(t)$, and $Z(t)$ are shown in Figure 3 for the case where $D = 0$.

We begin our analysis by considering the social costs of real-time gross settlement, which corresponds to a net debit cap $D = 0$. Under a net debit cap of zero it is never in the interest of the bank to default as long as its net worth is positive. Since the net worth of the bank remains constant as long as there is no cost to liquidating assets, no default occurs under gross settlement. Thus social costs of gross settlement are proportional to the seignorage costs associated with the level of reserves the bank holds at the end of the day. To calculate the level of reserve holdings at time 1, note that this level equals $-Z(t)$. Since $D = 0$, $-Z(t)$ is a regulated Brownian motion with lower reflecting barrier at zero. The random variable $-Z(1)$ has a distribution

$$F_1(x) = \Phi(x) - \Phi(-x) = 2\Phi(x) - 1, \qquad x \geq 0, \tag{10}$$

where Φ is the standard normal distribution.[21] Thus the expected social cost of gross settlement (per bank) is

$$SC_{GS} = r \int_0^\infty x\, dF_1(x). \tag{11}$$

To evaluate the social costs of net settlement, we again note that in this framework, there is never a reason to default before the end of the trading day. Since liquidation of assets imposes no penalty, there is no gain from early default. Early default is also costly in the sense that it eliminates the option value of a firm waiting until the end to discover if there is an influx of net due-tos to increase the firm's holdings. Thus for net settlement the social costs can be calculated by observing the costs associated with the terminal position of the bank. For simplicity, we begin by considering net settlement without debit caps. The social cost consists of two components — one proportional to the probability of default and the other proportional to the expected holdings of reserves at the end of the day. The expected holding of reserves is $\max\{-Z(1), 0\}$. As in the previous section, default occurs if $Z(1) > \gamma^{-1} NW_0$. Since there is no net debit cap, $Z(t) = X(t)$, implying that $Z(1)$, and therefore $-Z(1)$, is a standard normal variable. Thus

$$SC_{NS} = r \int_0^\infty x\, d\Phi(x) + \Xi\Phi(-\gamma^{-1} NW_0). \tag{12}$$

For general values of D, the social costs of net settlement involve identical considerations; however, the distribution of $-Z(1)$ is more complicated. The distribution is given by

$$F_2(x; D) = \begin{cases} \Phi(x) - \Phi(-x - 2D), & x \geq -D \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

Thus the social cost can in general be written as a function of D:

$$SC(D) = r \int_0^\infty x\, dF_2(x; D) + \Xi F_2(-\gamma^{-1} NW_0; D). \tag{14}$$

Note that when $D = 0$ this formula reduces to SC_{GS} and when $D = \infty$ it reduces to SC_{NS}.

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[21] Note that $-Z(1)$ can be thought of as a "precautionary demand" for reserves. See, for example, Sprenkle and Miller (1980).

In short, Equation (14) says that the total social cost of a settlement system is equal to the sum of expected seignorage costs and expected costs associated with default. Note that Equation (14) presumes that seignorage costs do not vary, depending on the fact of a default. There are two equivalent ways of interpreting this assumption: First, we measure seignorage costs as the long-run costs of diverting funding from return-bearing assets in order to make the payments system work. Second, we imagine that in the event of a default the bank is immediately taken over by the network and depositors are made whole but continue to bear the costs of the reserves used for the settling payments. This is consistent with our abstraction from systemic risk concerns.

As long as the debit cap is below the critical level $\gamma^{-1} NW_0$, there is zero probability of default. Moreover, increases in the debit cap reduce the expected seignorage cost. To see this note that the family of distributions $F_2(.; D)$ is ordered in the sense of first-order stochastic dominance as D increases, that is, $\partial F_2(x; D)/\partial D > 0$ for $x > -D$. Thus gross settlement is always dominated by net settlement for small levels of the net debit cap, and the social costs of the payment network unambiguously fall as D increases to $\gamma^{-1} NW_0$. Desirability of further increases in the net debit cap depends on the parameters of the model. The following results describe this dependence.

Proposition 1. *As Ξ/r increases, that is, as the cost of default increases relative to seignorage charges, the optimal value of D decreases.*

Proof. From the discussion above $SC(D)$ is minimized on $[\gamma^{-1} NW_0, \infty)$. Clearly the proposition holds (weakly) for either of the endpoints of this interval, so we restrict our attention to interior minima. Defining $SC^*(D) \equiv SC(D)/r$, first- and second-order conditions for the minimization of SC^* are given by

$$-\Phi(-2D) + (\Xi/r)\phi(\gamma^{-1} NW_0 - 2D) = 0 \qquad (15)$$

$$\phi(2D) + (\Xi/r)(\gamma^{-1} NW_0 - 2D)\phi(\gamma^{-1} NW_0 - 2D) > 0 \qquad (16)$$

where ϕ is the standard normal density. Straightforward comparative statics manipulations yield

$$\frac{\partial D}{\partial(\Xi/r)} = -\frac{\phi(\gamma^{-1} NW_0 - 2D)}{2\phi(2D) + 2(\Xi/r)(\gamma^{-1} NW_0 - 2D)\phi(\gamma^{-1} NW_0 - 2D)} < 0. \qquad (17)$$

■

Proposition 2. *As $\gamma^{-1} NW_0$ increases, that is, as we increase the critical level of net debit position for default to be tempting, the optimal value of D decreases.*

Proof. Again restricting our attention to interior minima, standard comparative statics calculations using Equation (15) yields

$$\frac{\partial D}{\partial(\gamma^{-1} NW_0)} = \frac{(\Xi/r)(\gamma^{-1} NW_0 - 2D)\phi(\gamma^{-1} NW_0 - 2D)}{2\phi(2D) + 2(\Xi/r)(\gamma^{-1} NW_0 - 2D)\phi(\gamma^{-1} NW_0 - 2D)} < 0. \tag{18}$$

■

Corollary. *If* $(\Xi/r) > 0$ *is sufficiently small, the optimal net debit cap implies a positive probability of default.*

Proof. The first derivative of $SC(D)$ is given by

$$SC'(D) = -2r\Phi(-2D) + 2\Xi\phi(\gamma^{-1} NW_0 - 2D). \tag{19}$$

For Ξ/r sufficiently close to zero, it follows that $SC'(\gamma^{-1} NW_0) < 0$. ■

Propositions 1 and 2 characterize the socially optimal choice of net debit cap D, where the socially optimal cap minimizes the sum of seignorage costs (borne by the bank's customers) and default costs (borne by unspecified parties). If some or all of the costs of default can be shifted to parties outside of the network, conflicts may arise concerning the proper level of net debit cap.

To see this last point, suppose that all default costs are borne by outside parties. It is then easy to show that the true expected net worth of the representative bank in the network is increasing in the net debit cap D. Denoting the "true" (as opposed to notional) net worth of the bank as $\hat{W}$, at time $t = 1$ this quantity is given by

$$\hat{W}(NW_0, Z(1)) = \begin{cases} NW_0, \text{ if } Z(1) \leq (\gamma^{-1} NW_0), \text{ i.e., if no default} \\ \alpha NW_0 + (\alpha - \beta)Z(1), \text{ if } Z(1) > (\gamma^{-1} NW_0), \\ \quad \text{i.e., if default.} \end{cases} \tag{20}$$

Hence if D is large enough so that defaults can occur, the "true" time $t = 0$ expected net worth of the bank can be calculated as

$$\begin{aligned} E_0\hat{W} &= NW_0(1 - F_2(-\gamma^{-1} NW_0)) \\ &\quad + \alpha NW_0 F_2(-\gamma^{-1} NW_0) - (\alpha - \beta) \int_{-D}^{-\gamma^{-1} NW_0} x \, dF_2(x), \end{aligned} \tag{21}$$

which simplifies to

$$E_0\hat{W} = NW_0 + (\alpha - \beta) \int_{-D}^{-\gamma^{-1} NW_0} F_2(x; D) \, dx. \tag{22}$$

Equation (22) says the true expected net worth of the bank is given by its notional net worth NW_0 plus the "option value" associated with defaults. This option value, which is similar to the option value associated with deposit insurance, derives from the bank's access to a default-free payment system.[22] The value of the option can be shown to increase with the net debit cap:

Proposition 3. *The true expected net worth of the bank $E_0\hat{W}$ is increasing in the net debit cap D.*

Proof. Restricting our attention to the nontrivial case where D is large enough to allow defaults, differentiate Equation (22) to obtain

$$\begin{aligned} \frac{\partial E_0\hat{W}}{\partial D} &= (\alpha - \beta)\int_{-D}^{-\gamma^{-1}NW_0}\left(\frac{\partial}{\partial D}\right) F_2(x; D)\,dx \\ &= -2(\alpha-\beta)(\Phi(\gamma^{-1}NW_0 - 2D) - \Phi(-D)) > 0 \end{aligned} \tag{23}$$

■

4. Extensions

4.1 Changes in the payment process

We can extend this model to consider the effects of changes in the volume of payment flows or changes in the intervals between settlement. Formally we do this by changing the variance parameter of the net payment process $X(t)$ to be σ^2, and by allowing the settlement time to be some arbitrary time $\tau > 0$. This implies that $X(\tau) \sim N(0, \sigma^2\tau)$, and that the distribution of net credit positions at settlement time $-Z(\tau)$ will be given by

$$F_3(x; D) = \begin{cases} \Phi\left(\dfrac{x}{\sigma\tau^{1/2}}\right) - \Phi\left(\dfrac{-x-2D}{\sigma\tau^{1/2}}\right), & x \geq -D \\ 0, & \text{otherwise.} \end{cases} \tag{24}$$

We interpret higher values of σ as an increase in payment volume through the network, and higher values of τ as an increase in the interval between settlements. Numerical experimentation suggests that the effects of increasing payment volume and/or settlement intervals on the optimal net debit cap depend in a complicated way on the relative costs of the inflation tax and default. The following analytical result describes this dependence for "small values" of the critical net debit position $\gamma^{-1}NW_0$.

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[22] See, for example, Merton (1977) on the option value of deposit insurance. Recent estimates of the option value provided by the "federal safety net" for U.S. banks — see Whalen (1997) for a survey — have been quite small or even negative. Such estimates have not attempted to model the "the payment system" option value discussed above, and hence may be somewhat downward biased.

Proposition 4. *For small values of $\gamma^{-1} NW_0$, the optimal net debit cap increases with increases in payment volume (σ) or with increases in the settlement interval (τ).*

Proof. Concentrating on interior minima, first- and second-order conditions for social cost minimization are given by

$$-\Phi\left(\frac{-2D}{\sigma\tau^{1/2}}\right)+(\Xi/r)\phi\left(\frac{\gamma^{-1}NW_0-2D}{\sigma\tau^{1/2}}\right)=0 \tag{25}$$

$$\phi\left(\frac{2D}{\sigma\tau^{1/2}}\right)+(\Xi/r)\left(\frac{\gamma^{-1}NW_0-2D}{\sigma\tau^{1/2}}\right)\phi\left(\frac{\gamma^{-1}NW_0-2D}{\sigma\tau^{1/2}}\right)>0. \tag{26}$$

For the purpose of comparative statics calculation, it is convenient to define payment "intensity" $\iota=\sigma\tau^{1/2}$. Applying standard comparative static methods yields

$$\frac{\partial D}{\partial\iota}=\frac{\frac{D}{\iota}\phi\left(\frac{2D}{\iota}\right)-\left(\frac{\Xi}{2r}\right)\left(\frac{\gamma^{-1}NW_0-2D}{\iota}\right)^2\phi\left(\frac{\gamma^{-1}NW_0-2D}{\iota}\right)}{\phi\left(\frac{2D}{\iota}\right)+\left(\frac{\Xi}{r}\right)\left(\frac{\gamma^{-1}NW_0-2D}{\iota}\right)\phi\left(\frac{\gamma^{-1}NW_0-2D}{\iota}\right)}. \tag{27}$$

For the limiting value $\gamma^{-1}NW_0=0$, Equation (27) reduces to

$$\frac{\partial D}{\partial\iota}=\frac{\frac{D}{\iota}\phi\left(\frac{2D}{\iota}\right)\left(1-\frac{2D\Xi}{r\iota}\right)}{\phi\left(\frac{2D}{\iota}\right)-\left(\frac{\Xi}{r}\right)\left(\frac{2D}{\iota}\right)\phi\left(\frac{2D}{\iota}\right)}>0. \tag{28}$$

It follows by continuity that $\partial D/\partial t>0$ for "small" positive values of $\gamma^{-1}NW_0$. ■

In words, Proposition 4 says that increased payment volume and/or longer settlement intervals imply higher optimal net debit caps, for the case where the critical net debit position is small.

4.2 Settlement rules and bank portfolio choice

We extend the model to consider the interaction of various settlement rules and bank portfolio decisions. We divide portfolios between risky loans and riskless bonds, and consider the effect of differences in the riskiness of the two sorts of assets.

Bonds are assumed to yield zero net return. That is, a portfolio of A_1 in bonds turns out to be worth A_1 with certainty. A loan portfolio of size A_2 will turn out to have one of two realizations. With probability $1-p$ the portfolio will turn out to be worthless. With probability p the portfolio will turn out to be worth $R(A_2)>1$, where the gross return function $R(\cdot)$ is strictly increasing and concave. In other words, the expected marginal value of an additional unit of the loan portfolio is $pR'(A_2) > 0$ and the portfolio is subject to diminishing marginal returns.

We assume that on the night before (at time $t = -1$) the bank allocates its total assets A between the loan and bond portfolios. It learns the realization of the return on the loan portfolio in the morning before trading begins. The managers of the payment network must set the rules of the settlement scheme without knowing either the bank's portfolio decision or the realization of the loan value.

In the absence of other considerations, the optimal size of the loan portfolio would be determined by the marginal condition $pR'(A_2) = 1$, which we assume is satisfied for $A_2^* \in (0, A)$. Given limited liability there is the possibility of the bank preferring to overinvest in the risky portfolio. The main result of this section is that the use of net settlement *increases* the temptation of firms to overinvest in risky portfolios.[23]

To see this, let us set parametric restrictions on the function R so that in the absence of any participation in the payments network, the bank would have an incentive to choose the efficient level of A_2. The following condition is necessary and sufficient for this to be the case:

$$pR(A_2^*) - A_2^* + A - C \geq p(R(\tilde{A}_2) - \tilde{A}_2 + A - C), \qquad (29)$$

where $\tilde{A}_2$ is defined by the condition $R'(\tilde{A}_2) = 1$, and by the assumption $\tilde{A}_2 \in (0, A)$.

Equation (29) says that, absent settlement considerations, the bank would choose a level of investment in loans A_2^* that would leave the bank with positive net worth under the bad investment outcome. By assumption, this level of investment is more profitable than the most profitable level of investment in loans, assuming that the bank has negative net worth under the bad outcome.

Propositions 5 and 6 describe the effect of net and gross settlement on the bank's investment decision.

Proposition 5. *Given Equation (29), under real-time gross settlement the bank chooses a loan portfolio of efficient size A_2^*.*

Proof. Since there is no incentive to default under RTGS (see the discussion of Section 3), participation in the payments network cannot change a bank's net worth. Hence, under Equation (29), the bank will choose a loan portfolio of efficient size. ■

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[23] The idea that the "put option" feature of equity creates a conflict between equity-holders and creditors of a firm is hardly new. For a general discussion of this idea and its relevance in banking environments, see Flannery (1994). In this section we show that to the extent that net settlement creates a new class of unsecured creditors, it also has the potential to create a new set of conflicts between equity-holders and these creditors.

Proposition 6. *Given Equation (29), and given a net debit cap D large enough to allow for default, under net settlement the bank chooses a loan portfolio of greater than efficient size, and the size of the bank's loan portfolio increases as the net debit cap increases.*

Proof. See Appendix A.

Corollary. *The possibility of asset substitution reduces the optimal debit cap*.

Proof. See Appendix A.

In words, Proposition 6 says that under net settlement, the option value of default can cause banks to overinvest in risky assets. From the individual bank's point of view, a poor investment outcome can sometimes be mitigated by forcing other banks (or their guarantors) to share in this loss. Proposition 5 says that under gross settlement, no such incentive exists. Finally, the Corollary says that the optimal net debit cap must fall as a result of these considerations.

5. Summary and Conclusion

Those whose primary concern is the leeway given to participants in interbank payment networks favor real-time gross settlement. However, RTGS is expensive for banks, who must retain large holdings of costly reserves in order to satisfy the liquidity demands of an RTGS system. This article outlines the trade-offs between the costs and benefits associated with net settlement relative to RTGS. The costs of net settlement result from allowing banks the "put option" of default on their interbank obligations, and from the resulting distortions in asset holding decisions, while the benefits of net settlement result from saving on the costs of liquidity.

In practice, default considerations represent a very small aspect of the decisions made by banks in a payment network during the course of their day-to-day operations. There are three justifications for concentrating on default as the decision that a bank controls. First, when defaults do occur, they are likely to be expensive. Second, the default decision is representative of many other decisions which a bank makes — for example, more general portfolio decisions, the decision to allow depositors access to uncleared funds and similar extensions of intraday credit, or the decision by a bank to delay the sending of payment messages to other banks — in which the actions are at best imperfectly monitored by the managers of the payment network or governmental regulators, but which have an impact on the overall risk associated with the network. For all of these decisions, the conflict between the interest of an individual bank and the "social" interest of the payments network are exacerbated by the bank's holding a large due-to position. Third, the default decision is very simple to analyze. The framework we have developed can be usefully extended to consider the more complex decisions which are of concern to both operators of private payment networks and to governmental regulators.

Our results demonstrate the difficulty of welfare comparisons between different settlement rules, even in a very restrictive model environment. In the simplest case, with no asset substitution and no liquidity costs, some form of net settlement dominates (Proposition 1). Increases in banks' net worth increase the optimal net debit cap in a nonlinear fashion (Proposition 2). Increases in payments flows and/or settlement times can increase the optimal net debit cap (Proposition 4). The possibility of asset substitution increases the likelihood of default, and therefore increases the costs associated with net settlement (Propositions 5 and 6).

Despite these ambiguities, our analysis does have some clear policy implications for the regulation of payment networks that settle on a net basis. First, Proposition 3 says that to the extent that default costs can be shifted to outside parties, payment network participants have an incentive to maximize this subsidy by setting as large a net debit as is feasible. Access to a payment system that directly settles on the books of a central bank may represent a form of subsidy analogous to that provided by deposit insurance. This result would therefore be consistent with the "Lamfalussy standards" set forth in the Bank for International Settlements (1990), which require payment networks to share in the costs of potential defaults by posting collateral sufficient to cover the maximum net debit of any single payment network member. Second, our analysis clearly shows that in comparing various settlement rules, the risk of a default is an endogenous variable and should not be taken parametrically. Finally, Propositions 5 and 6 show that the welfare costs associated with suboptimal settlement schemes are not limited to liquidity costs and/or the costs of default. When settlement rules distort banks' investment decisions, losses in allocational efficiency can also result.

5.1 Future Research

The results in this article were derived by examining the incentives of a single representative bank in an environment subject to numerous institutional constraints. The two natural directions for future research would thus emphasize (1) strategic interactions among participants in payment networks, and (2) relaxation of the numerous institutional constraints imposed in the analysis above.

Among these constraints, there are two that are most essential to our approach. The first is the constraint that finality is provided solely by transfer of costly central bank liabilities. The second is the constraint that the central bank on whose books the payment network settles will always act to prevent abnormal settlement, that is, that in some states the central bank will absorb some portion of the cost of bank defaults in order to prevent a systemic crisis. The first constraint is justified by its universal prevalence, whereas the second one can be justified as a subgame perfect outcome, conditional on the first constraint. That is, central banks cannot credibly commit to allowing settlement failures on large-value payment systems that settle on their books.[24]

As long as these two constraints are satisfied, the payment network and its associated central bank become, at least in some states of the world, creditors of the banks that participate in the network. The choice between net settlement and gross settlement is thus, as in the above environment, essentially a question of capital structure. Our conjecture is that more complicated environments which incorporate these two constraints will display trade-offs similar those studied above.

Deeper questions are posed if we examine environments without the constraint that settlement finality can only be provided by transfer of central bank liabilities. Analysis of such environments would require a more fundamental examination of the concept of "payment finality," its attendant costs and benefits, and the appropriate mechanisms for its provision.

Appendix A: Proof of Proposition 6 and its Corollary

Proof of Proposition 6. To show the first part of the proposition, we first show that the bank's time $t = 0$ true expected net worth $E_0\hat{W}$ grows less than proportionately with its notional net worth NW_0. Totally differentiating the expression for true net worth [Equation (22)], we obtain

$$\frac{dE_0\hat{W}}{d(NW_0)} = 1 - (1-\alpha)F_2(-\gamma^{-1}NW_0) \in (0,1). \tag{30}$$

Now consider the bank's time-zero notional net worth under successful and unsuccessful investment outcomes

$$NW_0 = \begin{cases} NW_0^s \equiv R(A_2) - A_2 + A - C, & \text{if successful} \\ NW_0^u \equiv -A_2 + A - C, & \text{if unsuccessful} \end{cases} \tag{31}$$

and define true time-zero expected net worth $E_0\hat{W}^s$ and $E_0\hat{W}^u$ (i.e., under success and failure) analogously. The bank's problem is to choose A_2 at time $t = -1$ to maximize true expected net worth $E_{-1}\hat{W}$. If the resulting value of A_2 is so large as to cause $NW_0^u < 0$, then the proposition holds trivially. So suppose that the optimal choice of A_2 implies $NW_0^u \geq 0$. Then

[24] For a formal argument along these lines, see Mengle (1990, pp. 168–172).

the first-order condition for maximization of $E_{-1}\hat{W}$ will be given by

$$\begin{aligned}\frac{d}{dA_2}\left(pE_0\hat{W}^s + (1-p)E_0\hat{W}^u\right) &= p\frac{dE_0\hat{W}^s}{d(NW_0^s)}\frac{d(NW_0^s)}{dA_2} \\ &\quad + (1-p)\frac{dE_0\hat{W}^u}{d(NW_0^u)}\frac{d(NW_0^u)}{dA_2} \\ &= 0, \end{aligned} \tag{32}$$

which is equivalent to

$$\frac{dE_0W^s}{d(NW_0^s)}pR'(A_2) = p\frac{dE_0W^s}{d(NW_0^s)} + (1-p)\frac{dE_0W^u}{d(NW_0^u)}. \tag{33}$$

Note that Equation (30) implies that the right-hand side of Equation (33) is in (0,1), implying (by concavity of R) that if A_2 satisfies Equation (33), it must be the case that $A_2 > A_2^*$. To verify that a solution to Equation (33) represents a unique maximum, consider the second-order condition

$$\begin{aligned} &pR''(A_2) + p\gamma(1-\alpha)F_2'(-\gamma^{-1}NW_0^s)(R'(A_2)-1) \\ &\quad - (1-p)\gamma F_2'(-\gamma^{-1}NW_0^u) < 0. \end{aligned} \tag{34}$$

The first and last terms of the left-hand side of Equation (34) are clearly negative, while the second term must be negative under the assumption that $NW_0^u \geq 0$.

To show the second part of the proposition, denote the solution to Equation (33) as $\hat{A}_2$. Straightforward manipulation of Equations (33) and (34) implies

$$\frac{\partial\hat{A}_2}{\partial D} = \frac{\begin{array}{c}-2p(1-\alpha)\phi(\gamma^{-1}NW_0^s - 2D)(R'(A_2)-1) \\ +(1-p)(1-\alpha)\phi(\gamma^{-1}NW_0^u - 2D))\end{array}}{\begin{array}{c}pR''(A_2) + p\gamma(1-\alpha)F_2'(-\gamma^{-1}NW_0^s)(R'(A_2)-1) \\ -(1-p)\gamma F_2'(-\gamma^{-1}NW_0^u)\end{array}} > 0. \tag{35}$$

■

Proof of the Corollary. If banks can invest in risky loans, then the social cost function of Equation (14) becomes

$$\begin{aligned} SC(D) &= r\int_0^\infty x\,dF_2(x;D) \\ &\quad + \Xi[p(F_2(-\gamma^{-1}NW_0^s;D)) + (1-p)(F_2(-\gamma^{-1}NW_0^u;D))] \end{aligned} \tag{36}$$

and the first- and second-order conditions of Equations (15) and (16) become

$$-\phi(-2D) + (\Xi/r)\left[p\phi(\gamma^{-1} NW_0^s - 2D) \right.$$
$$\left. + (1-p)\phi(\gamma^{-1} NW_0^u - 2D)\right] = 0 \qquad (37)$$

$$\phi(2D) + (\Xi/r)\left[p(\gamma^{-1} NW_0^s - 2D)\phi(\gamma^{-1} NW_0^s - 2D) \right.$$
$$\left. + (1-p)(\gamma^{-1} NW_0^u - 2D)\phi(\gamma^{-1} NW_0^u - 2D)\right] > 0. \qquad (38)$$

Comparative statics using Equations (37) and (38) yield

$$\frac{\partial D}{\partial p} = \frac{(\Xi/r)(\phi(\gamma^{-1} NW_0^s - 2D) - \phi(\gamma^{-1} NW_0^u - 2D))}{\begin{array}{c}[2\phi(2D) + 2(\Xi/r)[p(\gamma^{-1} NW_0^s - 2D)\phi(\gamma^{-1} NW_0^s - 2D) \\ + (1-p)(\gamma^{-1} NW_0^u - 2D)\phi(\gamma^{-1} NW_0^u - 2D)]]\end{array}}, \qquad (39)$$

which is negative for $p \in [0, 1)$ and approaches zero as $p \uparrow 1$. ■

Appendix B: Policy Analysis

In this appendix we show how the continuous-time model could be used to quantitatively analyze policy questions. We emphasize that these calculations are illustrative in nature, particularly since we make no claim that driftless Brownian motion is an empirically adequate model of net payment flows over any real-world payments network. We note, however, that given sufficient data, the approach used here could be easily adapted to more complex and presumably more realistic net payment processes.

4.1 Calibration of the model

As an example, we will apply the model to the CHIPS network, using some detailed data on the CHIPS network contained in Federal Reserve Bank of New York (1991). The data is based on an October 1990 survey of activity over CHIPS. To map the model into available data, suppose that at any time t during the trading day, where $t = 1$ corresponds to close of business, the cumulative flow of payments from bank i to bank j is given by $p_{ij}(t) \geq 0$. Volume over payment networks is usually cited in terms of a daily gross payment flow, that is,

$$\sum_i \sum_j p_{ij}(1) \qquad (40)$$

For example, for CHIPS, the average daily gross flow over CHIPS over this period was approximately \$800–\$900 billion. However, as the analysis above makes clear, the relevant measure of network size for purposes of risk analysis is the *net payment flow*, defined for time t as the absolute sum of all banks' net positions, divided by two in order to eliminate double counting, that is,

$$\frac{1}{2}\sum_i \left| \sum_j \left(p_{ij}(t) - p_{ji}(t)\right) \right| \equiv N(t) \tag{41}$$

For the period covered by the survey, the average end-of-day values for $N(t)$ was about \$18.3 billion, and the maximum recorded level of $N(t)$ was roughly \$39 billion.

To calibrate the model to these values, we first aggregate CHIPS into a "representative bank." In this case the close-of-business aggregate net position of the network $N(1)$ corresponds to $\frac{1}{2}|Z(1)|$, where $Z(1)$ is defined in Equation (8), since the network as a whole cannot hold negative reserves. We then choose a standard deviation σ for the representative bank's net payment process $X(t)$ so that its average close-of-business net position equals \$18.3 billion. That is, we require σ to satisfy, from Equation (24),

$$\begin{aligned} E[N(1)] &= \tfrac{1}{2}E|Z(1)| = \tfrac{1}{2}E|-Z(1)| \\ &= \tfrac{1}{2}\int_{-D}^{\infty} |x|\, dF_3(x; D) = \$18.3\text{ MM}. \end{aligned} \tag{42}$$

Finding a value of σ that satisfies Equation (42) requires an estimate of the actual net debit cap D for the entire network. Based on the survey information we put D at \$40 billion. Using this value of D, solving Equation (42) numerically via monte carlo integration yields

$$\sigma = \$106.06\text{ MM}. \tag{43}$$

4.2 Calculation of optimal net debit caps

To calculate the optimal net debit cap for the entire network, we require two more pieces of information: (1) the value of the critical net debit position $\gamma^{-1}NW_0$ at which default becomes tempting, and (2) the ratio of bankruptcy to liquidity costs Ξ/r. Since an abnormal settlement has never occurred on CHIPS, we consider values of the critical net debit position that imply a very small likelihood of default. For example, if a default were occur once every 10 years on average, we would choose $\gamma^{-1}NW_0$ such that

$$F_3(-\gamma^{-1}NW_0; D) = 1/2500 \tag{44}$$

for $D = \$40$ billion. We also consider a range of values for Ξ/r. An optimal net debit cap can then be calculated as the numerical solution to the first-order condition of Equation (25).

Table 1 shows that for values of Ξ/r less than 0.1, essentially no net debit cap is required. As the cost associated with default rises to half the level of liquidity costs, the net debit cap is finite, but at \$63.4 billion, considerably more generous than the approximately \$40 billion cap that was actually in

Table 1
Optimal Net Debit Caps

Bankruptcy:Liquidity Cost Ratio	Optimal System Net Debit Cap
$\Xi/r = 0.1$	\$162 MM
$\Xi/r = 0.4$	\$63 MM
$\Xi/r = 0.7$	\$40 MM

Source: Authors' calculations, using data from an October 1990 survey of CHIPS. Calculations assume that one default is expected every 10 years on average.

place over this period. For values of Ξ/r much in excess of one half, the optimal cap quickly falls to \$40 billion.

Our conclusion from this exercise is that the period covered by the survey, the amount of intraday credit allowed by CHIPS, was quite justifiable. This could result from either (1) CHIPS placing a high weight being on the costs of a default, relative to liquidity costs, or (2) its need to ensure against the effects of a sudden shock to participants' net worth, as described in Section 4.2 above.

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