

Agency and Brokerage of Real Assets in Competitive Equilibrium

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Brokerage contracts for many categories of real assets are characterized by a common, constant commission rate payable upon sale, exclusive agency, and contractual asking prices. For a large market in steady state, these conventional contracts produce in equilibrium no agency problem between a broker and his clients. Each broker spends the same time or effort selling each client's asset as the broker would spend on his own assets. As in standard agency problems, extra effort by a broker generates first-order stochastically dominant distributions of bids by potential buyers. Unlike standard agency problems, each broker can allocate his time or effort between selling the assets of his multiple clients and searching for new clients in competition with other brokers. Because brokers' time spent searching for new sellers is dissipative, entry by brokers is excessive in equilibrium.

Brokerage contracts for real assets have several standard characteristics. Generally, a broker's contractual compensation is a constant percentage of an asset's sale price, payable upon sale. For example, with residential real estate in the United States, the commission

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is commonly about 6% of the sale price.¹ Typically, a broker obtains an exclusive contract to market a seller's asset. With residential real estate this contract is called an exclusive listing. Often the contract includes a private asking price with the following characteristic. If a seller receives and rejects an offer from a potential buyer at or above the contractual price, then he must pay the commission to the exclusive broker.² In turn, the exclusive agent of the seller can subcontract with other brokers who provide potential buyers. Although the seller can have only one broker, his broker can represent multiple sellers. If a broker has multiple clients, then he can split his time or effort, as he chooses, between their assets. Simultaneously, he can spend time searching for new clients. A seller who hires an exclusive broker retains relatively little contractual control.

These stylized facts are not easily explained by standard agency models with one principal and one agent.³ Perhaps the most perplexing puzzle is proportional brokerage commissions. In standard models extra effort by the agent generates first-order stochastically dominant distributions of payoffs to the principal. In this application to real assets, brokers are agents; sellers are principals; effort by an agent is time spent marketing a seller's asset; and payoffs to the principal are bids by potential buyers. In standard models if a broker is risk neutral, then he is induced to make a first-best or Pareto-optimal effort by a net brokerage contract in which he is the residual claimant. With a net contract the seller realizes the minimum of the sale price and a fixed contractual payment, while the broker receives the residual. Proportional commissions, by contrast, induce the agent to expend too little effort. In standard agency models the broker always has only one seller and never needs to search for new sellers; hence he always allocates all his time between leisure and selling the asset of his single client. Since the marginal value of leisure does not depend on the brokerage fee, a percentage commission less than 100% induces the broker to consume excessive leisure and thereby to expend insufficient effort.

Within this model of competitive equilibrium for brokers, conventional contracts produce no agency problem between brokers and their current clients. Here each broker is assumed to acquire from each client an exclusive contract with a common proportional com-

¹ Within a local market the commission rate is identical for all brokers and all residences in a category. Across markets it is clustered around 6%. It tends to be lower for larger houses and new construction. See Carney (1982).

² In most jurisdictions a "full-price" offer cannot include contingencies, such as financing from third parties.

³ Important early articles include Holmstrom (1979) and Grossman and Hart (1983).

mission payable upon sale and a contractual asking or listing price. In the resulting equilibrium each broker is then shown to spend the same time or effort selling each client's asset as he would spend on his own assets. Also, each client chooses the same reservation price for his asset that the broker would select for his own assets. In this sense, the standard contract with a proportional commission, identical for all assets, and a private asking price produces no agency problem between a broker and his current clients. This surprising result follows from a standard model of agency with the following modifications. Brokers can spend time both selling the assets of current clients and searching for new clients. Each broker has a constant marginal productivity of time spent searching for new clients because competition among brokers equates the marginal productivity of search per unit of time. Also, all time spent searching for new clients is purely dissipative; it has a private value to each broker but no social value to either brokers as a group or their clients. The problem is then solved in steady state with independent Poisson arrivals of both buyers and new sellers and competitive entry by brokers.

The equilibrium without agency is not Pareto optimal. Two separate sources of externalities may preclude a first-best solution. The average arrival rate of buyers at each asset can depend on the average time allocated by all brokers to all other assets. In this case, the time allocated by each broker to each asset in the competitive equilibrium without agency differs from the first-best allocation of time selected by a social planner. Also, the time spent searching for new sellers is purely dissipative in this model with identical brokers. If all brokers spend proportionally more time searching for new clients, then no seller benefits and each broker incurs a personal cost. In the partial equilibrium without entry by brokers, the total dissipative loss from both externalities decreases in the constant commission rate. In the equilibrium with competitive entry, the total loss therefore decreases in the common cost of entry for each broker.

Previous articles on agency and brokerage of real assets appear exclusively in the literature on real estate.⁴ None has the combination of critical characteristics that produce the main results of this model: multiple brokers, possibly multiple sellers per broker, costly search for both buyers and new sellers, and a competitive equilibrium among brokers. Instead, almost all models match one broker with one seller, often with competitive entry by brokers. A few permit multiple sell-

⁴ Articles on agency and brokerage of real estate include Zorn and Larsen (1986), Wachter (1987), Carroll (1989), Anglin and Arnott (1991), Bagnoli and Khanna (1991), Geltner, Kluger, and Miller (1991, 1992), Miceli (1991), Arnold (1992), Anglin (1994), and Yavas (1995a,b, 1996). Other theoretical articles on brokerage of real estate are cited and discussed in Mantrala and Zabel (1995).

ers per broker, but none require brokers to search for new sellers. Here the critical characteristic is not the number of sellers per broker, but brokers' need to search competitively for new sellers. Nor is there a similar model in the larger literature on agency.⁵ Such a model would have multiple agents with multiple principals, costly search for new principals, and a competitive equilibrium. This is surprising if only because the broader problem is common to most classes of professionals. Obvious examples are accountants, attorneys, consultants, dentists, and doctors.

This article is organized as follows. The model and its major results are sketched in Section 1. The formal analysis then starts in Section 2. In the initial model each broker is delegated the decision to accept or reject offers for each client's asset. The solution to this initial problem is characterized in surprising detail in Section 3. Its unique solution is shown to eliminate in equilibrium the agency problem between a broker and his current clients. In Section 4 the initial problem is modified in two steps. First, the decision to accept or reject any offer is retained by the seller. A Markov perfect equilibrium is identified in which each seller's reservation price is excessive and each broker's resulting effort is insufficient relative to the previous equilibrium with full delegation. Next, the brokerage contract is modified such that the broker is paid his commission upon receipt of any offer not less than a private asking price. The optimal contractual price and the resulting reservation price for each seller are shown to be identical and equal to the optimal reservation price of each broker whenever the commission rate is sufficiently large in equilibrium. As a result, the agency problem again disappears in equilibrium. Extensions of the model are discussed in Section 5, and all major results are summarized in Section 6. All proofs appear in the Appendix.

1. Overview

In this section the critical characteristics of the model, its main results, and the economic intuition are described in more detail. Readers interested in the specifics of the model and its solution can skip to Section 2.

⁵ The most closely related model in Bernheim and Whinston (1986) has one agent, multiple principals, and a very different focus. Their agent neither searches for new principals nor competes with other agents, and each strategic principal can preclude the agent's participation. In other aspects their model is more general: their agent need not be risk neutral nor must their contract be linear. There is also a related literature with one principal, one or more agents, and multiple tasks for each agent. In Holmstrom and Milgrom (1987, 1991), the optimal sharing rule is linear in part because the agent's utility function is exponential while the principal's preferences are exponential in the first article and linear in the second.

The initial model presented in the next section has the following properties. Infinitely many identical brokers compete for clients in a market for a category of real assets. Each identical client offers one asset for sale and signs with one broker an exclusive agency agreement. By contrast, brokers may have multiple clients. Each broker works to sell the assets of his clients and receives after each sale a commission proportional to the realized price. The constant commission rate in equilibrium is constrained to be identical for all assets and all brokers. All brokers and sellers regard the standard brokerage contract with its common, constant commission rate as fixed in equilibrium, not subject to negotiation. In each period each broker allocates his time between labor and leisure. His working time or effort he can split between selling the assets of current clients and searching for new clients. In the initial model each seller delegates to his broker the decision to accept or reject offers from buyers. This temporarily eliminates the dual agency problem between a broker and his client in which each acts as an agent for the other. In the subsequent models sellers set their own reservation prices.

Buyers arrive in independent Poisson processes to inspect assets for sale. All assets are indistinguishable by buyers before inspections; after an inspection there is no private information between the broker, buyer, or seller. Similarly, new sellers arrive in independent Poisson processes to sign contracts with brokers. All brokers, all contracts, and all sellers are identical. The average arrival rate of buyers at a seller's asset depends on his broker's current allocation of time or effort to that asset and the concurrent average allocation by all brokers. Also, the arrival rate of a broker's new clients is proportional to his current time or effort spent searching for new clients measured relative to the concurrent average search by all brokers. Here two characteristics of this search for new clients are critical: for each broker the marginal productivity of time is constant, and for all brokers the total time spent is purely dissipative. The constant marginal productivity is generated by competition among brokers and a production function with labor as its only input. Also, the search for new clients must be purely dissipative because both brokers and clients are identical. Once a potential buyer arrives at an asset, he makes one offer for the asset drawn from an independent distribution. This offer can be interpreted as the solution to a bilateral bargaining problem with repeated offers and counteroffers between a symmetrically informed buyer and seller. The average arrival rates and distribution of offers are assumed to satisfy two technical conditions. There are no taxes or transaction costs. Also, all brokers and sellers are risk neutral with identical discount rates. Each broker acts to maximize the expected present value of his business. To enter the industry, each broker incurs a common cost

of entry. After a transaction each seller leaves the market. All novel assumptions are motivated in Section 2.

The initial equilibrium is identified in Section 3. It has the following properties. In each period the representative broker first chooses between labor and leisure. To each client he then allocates the same time or effort. Both choices are independent of his current number of clients. Finally, any remaining time he spends searching for new clients. For each client the broker selects the same reservation price, independent of his current number of clients. Surprisingly, each broker spends the same time or effort selling each client's asset and selects the same reservation price as he would for his own assets. In other words, the standard contract with a common, proportional commission produces no agency problem in equilibrium between a broker and his current clients. If there are no externalities from the efforts of competing brokers, then this effort by each broker and the associated reservation price are first best or Pareto optimal. In this case, the same solution is selected by both brokers operating competitively and a social planner enforcing the cooperative solution. With externalities, however, brokers' competitive solution is not first best. Also, the time or effort spent searching for new sellers is never optimal. Because brokers are identical, their search for new sellers is purely dissipative.

This limited optimality of conventional brokerage contracts in the initial model is shown as follows. Each broker acts to maximize the expected present value of his business. As a result, each broker equates his marginal productivity of time or effort spent with the asset of each seller to his marginal productivity of time spent or effort searching for new sellers to his marginal value of leisure. This property holds both in equilibrium with competitive entry by brokers and in partial equilibrium with no entry and an exogenous, constant commission rate. In the partial equilibrium the first two marginal productivities are proportional to the commission rate, so that the optimal time allocated to each client is independent of the commission rate. Since this solution does not depend on the broker's fractional fee, it must hold for a fee of 100%. In this case, the broker effectively represents his own assets and allocates accordingly his time or effort to all his current clients. An identical argument applies to the broker's choice of reservation prices. For this reason, no broker has in equilibrium an agency problem with his current clients. Still, the standard proportional commission does affect each broker's total time spent working and thereby his residual time spent searching for new sellers. However, the marginal productivity of the latter search is assumed to be constant, so that the optimal time spent searching for current clients is determined independently of the optimal time spent searching for new clients. In the partial equilibrium with no entry by brokers and an exogenous commission

rate, a higher rate raises each broker's total time at work and thereby his residual time looking for new clients. In the full equilibrium with competitive entry and an endogenous commission rate, a higher cost of entry then increases not only each broker's total time at work and his residual time with new clients, but also the commission rate and the average number of clients per broker.

In Section 4 sellers set their own reservation prices. Without contractual constraints on their reservation prices or some other mechanism to ensure cooperation between brokers and their clients, each seller exploits his role as agent in the dual agency relationship with his broker. After a broker has spent time or effort on a client's asset and thereby produced a potential buyer, the seller then ignores his broker's cost of subsequent search and sets his reservation price too high. Rational brokers anticipate this choice and spend too little time or effort. Consistent with the previous results, this second-best solution is independent of the constant commission rate in partial equilibrium without entry and thereby independent of the common cost of entry in equilibrium. Next, each seller sets his own reservation price, constrained by a contractual asking price. Whenever a seller receives from a potential buyer an offer at or above his private asking price, the seller must pay his broker the contractual commission even if he rejects the offer. If the fractional fee is sufficiently large in equilibrium, then the optimal asking price for both the broker and his client is equal to the broker's optimal reservation price. Brokers are again induced to spend the same time or effort and to select the same reservation price for clients and themselves. Again, this eliminates in equilibrium the agency problem between a broker and his current clients.

As argued in Section 5, the model is easily extended to include heterogeneous assets and brokers, buyers' brokers, and other technologies for finding new clients. As long as all participants are risk neutral, no information is private, and no experimentation with contractual terms is permitted, the central conclusion is unchanged. Conventional brokerage contracts with a constant percentage commission, identical for all assets, eliminate in equilibrium the agency problem between brokers and their current clients. This conclusion may not hold, however, if brokers or their clients can experiment with commission rates or other contractual terms. This potential loss in efficiency from experimentation may motivate rules, prevalent in practice, that restrict brokers to conventional contracts with fixed fractional fees.

2. Initial Model

The local market has a countable infinity of brokers or salespeople. Each broker acts as an agent for one or more sellers of a representative

category of real assets, such as single-family houses within the local market. Each broker obtains from each client an exclusive contract to sell his single asset. Period t is the interval of time between time t and time $t + \Delta t$. Later each time period is made arbitrarily small: $\Delta t \rightarrow 0$. Suppose that broker i has during period t the number of assets, clients, or sellers: $n_{it} = n$. For broker i with n clients, the j th asset, client, or seller is identified by the index ijn , for $j = 1, \dots, n$. This notation reflects the subsequent focus on a stationary solution to the broker's problem that depends not on calendar time t , but only his current number of clients n . Each broker can allocate his working time between both current and new clients. Given n clients, broker i allocates to asset ijn the time $x_{ijn}\Delta t$, for $j = 1, \dots, n$. This time is spent showing the asset to potential buyers. Simultaneously the broker spends the time $y_{in}\Delta t$ searching for new clients. Thereby he allocates to all his working activities the current fractions of time: $x_{in} = (x_{i1n}, \dots, x_{inn}) \geq 0$ and $y_{in} \geq 0$. Alternatively these activities can be interpreted as effort, assuming that allocations of effort are also mutually exclusive and exhaustive. In this model brokers spend no time or effort selling assets under contract to other brokers. This greatly simplifies the subsequent problem of agency to the conflict of interest between sellers as principals and brokers as agents. The more complicated problem with separate brokers for buyers and sellers and potential conflicts of interest between brokers is discussed in Section 5.

Time not spent working is consumed as leisure. With n clients, broker i allocates to work the total time $w_{in}\Delta t$ and to leisure the residual time $(1 - w_{in})\Delta t$. The current fraction of time spent at work is $w_{in} = \sum_{j=1}^n x_{ijn} + y_{in}$. Time spent working has the personal or human cost $\theta H(w_{in})\Delta t$, with the parameter, $\theta > 0$. The part played by the parameter θ is identified in Section 3. As in other agency models, the cost function H is twice continuously differentiable, increasing, and strictly convex. It has the convenient normalization, $H(0) = 0$. To preclude a corner solution with either no work, $w_{in} = 0$, or no leisure, $w_{in} = 1$, the function H also satisfies two corner conditions: $H'(0) = 0$ and $H'(1) = \infty$. For notational simplicity the agent's only variable cost is his time spent working. As in other agency models, the time or effort spent by a broker on an asset can be observed by its seller but not third parties, like courts. Consequently, contracts cannot be enforced conditional on the agent's allocations of time or effort.

Buyers arrive in independent Poisson processes to inspect the assets of sellers. During each period of length Δt , at most one new potential buyer arrives to inspect the asset of some seller, selected at random. Across all brokers the current average allocation of time to all current clients is $\bar{x}$. For asset ijn one buyer currently arrives

with the probability $F(x_{ijn}, \bar{x})\Delta t + o(\Delta t)$; no buyer arrives with the probability $1 - F(x_{ijn}, \bar{x})\Delta t + o(\Delta t)$; while two or more buyers arrive with the probability $o(\Delta t)$ of smaller order than Δt . The average arrival rate of buyers $F(x_{ijn}, \bar{x})$ depends on two variables: the broker's current allocation of time to the asset x_{ijn} and the concurrent average allocation by all brokers to all assets $\bar{x}$. The function F is everywhere twice continuously differentiable and homogeneous, and both increasing and strictly concave in its first argument with the two corner conditions: $F(0, \cdot) = 0$ and $F_1(0, \cdot) = \infty$. With the homogeneity the elasticity of a broker's own effort is constant along a ray from the origin. This property makes possible the nearly explicit solution in the first proposition. Without it, the subsequent solution is more complicated but still characterized by the same properties, as explained in Section 3. The monotonicity and concavity are standard assumptions. If a broker allocates more time to selling a client's asset, then more potential buyers learn of the asset and arrive to inspect it. This extra effort has diminishing marginal returns because the production process for sales has a fixed input of capital. An example is the multiple listing service for residential real estate.⁶ The second argument $\bar{x}$ is motivated in more detail below.

In general, the arrival rate of buyers at one asset depends on the fractions of time allocated by all brokers to all assets. For example, if other brokers spend more time marketing assets of their sellers, then buyers may allocate more of their scarce time available for inspecting potential purchases to those assets and less to the asset in question. In this case, the arrival rate at one asset is nonincreasing in the allocation of time to any other asset. In large markets, with many brokers and thereby many assets under contract, this interaction between the sale of one asset and the marketing of another is minimal. Moreover, this interaction is identical among all assets, given ex ante identical assets, identical brokers, and identical sellers. This notion of minimal, identical interactions in large markets is made more precise in the Appendix. As shown in the Appendix, these properties produce for a very large market with infinitely many brokers the above average arrival rate F with the two arguments, x_{ijn} and $\bar{x}$.

New sellers also arrive in independent Poisson processes. Each new seller signs a contract and thereby becomes a client of some broker, also selected at random. Across all brokers the current average allocation of time to new clients is $\bar{y}$. Also across all brokers the concurrent average number of clients per broker is $\bar{n}$. For broker i ,

⁶ Data on the production function for sales of residential real estate appear in Carney (1982) and Zumpano, Elder, and Crellin (1993).

one new asset, client, or seller currently arrives with the probability $\alpha \bar{n}(y_{in}/\bar{y})\Delta t + o(\Delta t)$; no new seller arrives with the probability $1 - \alpha \bar{n}(y_{in}/\bar{y})\Delta t + o(\Delta t)$; while two or more sellers arrive with the residual probability $o(\Delta t)$. The arrivals of buyers and sellers are independent Poisson events. For new sellers the average arrival rate is $\alpha \bar{n}(y_{in}/\bar{y})$. It reflects the technology of matching brokers and new sellers, as measured by the exogenous constant, $\alpha > 0$. More importantly, it is proportional to both the average number of assets per broker $\bar{n}$ and the broker's relative allocation of time to new clients, $y_{in}/\bar{y}$. The first proportionality is consistent with the above arrival rate of buyers. In this model of steady state, the average arrival rates of both buyers and new sellers in the market are proportional to the aggregate number of assets for sale. Thereby the average arrival rate of buyers at each asset is independent of the number of assets for sale, while the average arrival rate of new sellers at each broker is proportional to the average ratio of assets per broker $\bar{n}$. The second proportionality is consistent with equilibrium in the brokerage market, as explained below.

Brokers must compete for new sellers. Their perfectly competitive search for new sellers has only labor as an input—unlike the search for buyers which has a fixed input of capital. If the time spent searching for new sellers were more profitable during some parts of each period of length Δt than others, then more brokers would search at those times and less would search at other times. Competition among brokers would then equalize the marginal productivities per unit of time across all parts of each period. Alternatively, if each identical broker's marginal productivity of work diminishes proportionally in all activities, due to diminishing personal energy, then the fractions of time, x_{in} and y_{in} , can be rescaled in units of equal marginal personal productivity. For these reasons, the above average arrival rate of new clients for broker i is linear in the allocation of time y_{in} . Because brokers are competitive, this average arrival rate for each broker is also nonincreasing in the corresponding allocations of time by all other brokers. In large markets with many identical brokers, this interaction should be both minimal and nearly identical for all other brokers. Again, this notion is made more precise in the Appendix. By precisely the previous argument, the average arrival rate of new clients for broker i then depends on other brokers' efforts only through the concurrent average allocation of time $\bar{y}$. Also, because brokers are identical, their time spent searching for new sellers is assumed to be purely dissipative. Neither the number of new sellers in the market nor the speed at which sellers select brokers depends on the total time spent by all brokers searching for new clients. In this case, the arrival of new clients for broker i is homogeneous of degree zero in the two

variables, y_{in} and $\bar{y}$. Given the above linearity in y_{in} , the arrival rate must then be linear in the ratio, $y_{in}/\bar{y}$. Finally, if no new clients are acquired when no time is spent searching, $y_{in} = 0$, then the average arrival rate must be proportional to the relative time spent searching, $y_{in}/\bar{y}$. This proportional production function for new clients and its generalizations are discussed in Sections 3 and 5.

After a potential buyer arrives to inspect the asset of a specific seller, he identifies the quality of the match, if any, between his preferences and the asset's attributes. Depending on the match, the buyer then negotiates with the seller. Elsewhere this negotiation is modeled as a bilateral bargaining game between the potential buyer and seller.⁷ Here the focus is different, and the outcome of this negotiation is represented simply as the buyer's final offer to purchase the seller's asset at some price p_{ijn} . With no match there is no final offer: $p_{ijn} = 0$. The buyer's final offer is drawn from the probability distribution G . The random variable $\tilde{p}_{ijn}$ is independent of the arrivals of both buyers and sellers, independent across assets, and independent over time. Its conditional distribution G has a density g with the finite support $[0, 1]$. The upper bound 1 is merely a notational convenience. Also, the hazard function, $g/(1 - G)$, is nondecreasing everywhere. This familiar condition is satisfied by many standard distributions.

This simple, exogenous distribution of final offers requires more motivation. The distribution G can be induced by the distribution of reservation prices for buyers and sellers in a model of sequential search, followed by bilateral bargaining between matched buyers and sellers and subsequent search after failed negotiations.⁸ With symmetric information between potential buyers and sellers—as in this model of pure agency—sellers have no incentive to post public asking or listing prices.⁹ For this reason, the sale of asset ijn depends at most on the brokers' allocations of effort, x_{ijn} and $\bar{x}$. Given the previously assumed dependency of the average arrival rate F on these allocations, no economic insights are lost by specifying the distribution G as independent of both variables. Finally, the distribution G is independent of the asset's identity ijn because all assets are indistinguishable ex ante before inspections by potential buyers. The assets

⁷ There is a large literature on bargaining and search, both with symmetric and asymmetric information. For example, see Samuelson (1990) and the references cited therein.

⁸ For example, in Williams (1995) the buyer's final offer p_{ijn} is the Nash bargaining solution between a matched buyer and seller, conditional on their reservation prices. These reservation prices are determined in an equilibrium with costly search.

⁹ With private information about their type, sellers may have an incentive to signal with published listing prices. The signals can be credible if rejecting full-price offers is costly. Most notably, rejected offers may induce lawsuits for specific performance. Siedmann (1990) has a model of list pricing for interfirm trade with some similar characteristics.

cannot be identical ex post after inspections, since search would then be superfluous.

A buyer's final offer must then be accepted or rejected. In the initial model each seller delegates this decision to accept or reject a final offer to his broker. To decide, the broker must determine his reservation price for the asset. Conditional on his current number of clients n , broker i selects, simultaneously with all other brokers, his current reservation prices r_{ijn} for all assets ijn with $j = 1, \dots, n$. For broker i with n clients, this produces the vector of reservation prices, $r_{in} = (r_{i1n}, \dots, r_{inn})$. If the buyer's final offer p_{ijn} exceeds or equals the broker's current reservation price r_{ijn} , then the broker accepts the offer and the buyer acquires the asset. Alternatively, if $p_{ijn} < r_{ijn}$, then the broker rejects the final offer and the buyer departs. Thereby the reservation price r_{ijn} defines a stopping rule: stop looking for a buyer and sell if and only if $p_{ijn} \geq r_{ijn}$. This stopping rule is optimal under the conditions that are identified below.

Under the above assumptions, the agent's current number of clients changes through time in a generalized birth-death process. From period t to period $t + \Delta t$, the number of clients of broker i changes from n to $n + \Delta n$. Three possible increments, $\Delta n = +1, 0, -1$, occur with the probabilities:

$$\Delta n = \begin{cases} +1 & \alpha \bar{n}(y_{in}/\bar{y})\Delta t + o(\Delta t) \\ 0 & 1 - \alpha \bar{n}(y_{in}/\bar{y})\Delta t - \sum_{j=1}^n F(x_{ijn}, \bar{x})[1 - G(r_{ijn})]\Delta t + o(\Delta t) \\ -1 & \sum_{j=1}^n F(x_{ijn}, \bar{x})[1 - G(r_{ijn})]\Delta t + o(\Delta t). \end{cases} \quad (1)$$

All other possible increments, $\Delta n = \pm 2, \pm 3, \dots$, occur with a probability of smaller order than Δt . The probability $o(\Delta t)$ is associated with the simultaneous arrival of one buyer and one seller or multiple buyers and/or multiple sellers. The simple specification of Equation (1) is possible only because each arrival is an independent event with probabilities of order Δt . For example, one new seller and no potential buyers arrive during the current period, $\Delta n = 1$, with the probability: $[\alpha \bar{n}(y_{in}/\bar{y})\Delta t + o(\Delta t)][1 - \sum_{i=1}^n F(x_{ijn}, \bar{x})[1 - G(r_{ijn})]\Delta t + o(\Delta t)] = \alpha \bar{n}(y_{in}/\bar{y})\Delta t + o(\Delta t)$.

Upon the sale of an asset, the seller pays his broker a contractual commission. Throughout this article each broker's commission, bp , is proportional to the sale price p . The constant commission rate, $0 \leq b \leq 1$, is correctly regarded by all brokers, buyers, and sellers as fixed in equilibrium, not subject to experimentation by either brokers or sellers. In practice, this could reflect collusion among brokers

through their local real estate boards.¹⁰ Also, as in other agency problems, third parties, like courts, cannot observe agents' efforts. Consequently the commission cannot depend on a broker's current or past allocations of effort or time. Finally, the constant b cannot depend on the identity ijn of the asset or its seller, nor can it depend on calendar time t . This simple specification precludes both price discrimination among sellers and brokerage contracts with finite maturity. The former condition is consistent with common practice in many markets, whereas the latter is not. For example, with residential real estate exclusive listing contracts are commonly standardized within metropolitan markets, but maturities of more than 6 months are uncommon. However, this simple, stationary commission bp is only a notational convenience. With the symmetric information in this model, proportional contracts of all maturities eliminate in equilibrium the agency problem between brokers and their clients. In other words, the model has no predictions about the optimal maturity of brokerage contracts. For such predictions the model must be modified, as discussed in Section 5 [Miceli (1989)].

To simplify further the subsequent notation, transactions are treated as starkly as possible. Each seller realizes no direct benefit from his asset, like dividends, operating income, or implicit rent, while waiting for a transaction. Each seller is the sole owner of his asset, and no asset is encumbered by debt. All taxes and closing costs, other than brokerage commissions, are ignored. Thereby each risk-neutral seller has a portfolio with a terminal payoff equal to the net proceeds from the sale of his asset. If a transaction occurs at the price p , then the broker receives the commission bp , and the seller realizes the residual, $(1 - b)p$. After a transaction the seller leaves the market. His utility from departing is normalized at zero.

To make the problem tractable, both brokers and sellers are assumed to be risk neutral. Also all participants in the market have an infinite time horizon with the same discount rate per unit of time. Measured per unit of time, this discount rate is the positive constant ι . It can be interpreted as the constant, riskless rate of interest, measured per unit of time. Given the time-homogeneous Markov process in Equation (1), each participant in the market then faces a stationary problem, thereby eliminating calendar time t as a possible state variable. To preclude as a possible state variable the average number of assets, clients, or sellers per broker $\bar{n}$, the subsequent problem is solved in steady state. In steady state each broker i has a stationary

¹⁰ The optimality of the subsequent equilibrium without price experimentation may motivate the common practice among many local real estate boards of enforcing standard commissions and contracts.

distribution for his current number of clients $\tilde{n}$. With a stationary distribution the mean $E(\tilde{n})$ is constant, independent of calendar time t . In a market with infinitely many brokers, the average number of assets per broker $\tilde{n}$ equals the expected number of assets per broker $E(\tilde{n})$.¹¹ For this very large market, the average number of assets per broker $\tilde{n}$ is then constant in steady state. This constant is computed as part of the subsequent solution.

Under the above assumptions each broker has no limit on the length of time during which he can search for either buyers or sellers. In each period broker i first observes his current state n . Next he allocates his time among both current and new clients and simultaneously selects for all his sellers his current reservation prices. These choices maximize the present value of his business. He makes these choices, simultaneously with all other brokers, anticipating that both he and all other brokers will act optimally in all subsequent periods. For broker i in state n , the resulting optimal reservation prices and allocations of time are $r_{in}^* = (r_{i1n}^*, \dots, r_{inn}^*)$, $x_{in}^* = (x_{i1n}^*, \dots, x_{inn}^*)$, and y_{in}^* . For all brokers the corresponding average optimal allocations to current and new clients are $\bar{x}^*$ and $\bar{y}^*$, respectively. In a very large market with infinitely many competitive brokers, each broker correctly regards the average allocations, $\bar{x}^*$ and $\bar{y}^*$, as fixed, independent of his own allocations. In this market each broker also regards the commission rate b^* and average number of assets per broker $\tilde{n}^*$ as fixed in the subsequently defined competitive equilibrium, independent of his own actions. For broker i in state n , the optimal choices then solve the following problem:

$$\begin{aligned}
 V(n) = \max_{r_{in}, x_{in}, y_{in}} e^{-t\Delta t} & \left\{ \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) \Delta t \int_{r_{ijn}}^1 b^* p dG(p) \right. \\
 & - \theta H \left(\sum_{i=1}^n x_{ijn} + y_{in} \right) \Delta t + \alpha \tilde{n}^* (y_{in}/\bar{y}^*) \Delta t V(n+1) \\
 & + \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) [1 - G(r_{ijn})] \Delta t V(n-1) \\
 & + \left[1 - \alpha \tilde{n}^* (y_{in}/\bar{y}^*) \Delta t - \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) [1 - G(r_{ijn})] \Delta t \right] V(n) \Big\} \\
 & + o(\Delta t), \tag{2}
 \end{aligned}$$

¹¹ Consider a sequence of markets distinguished solely by the numbers of identical brokers m . In each market m both buyers and new sellers arrive in independent Poisson processes, as previously specified. In increasingly larger markets, $m \rightarrow \infty$, the average number of assets per broker $\tilde{n}$ converges almost surely to the expected number of assets per broker $E(\tilde{n})$.

subject to $0 \leq r_{in} \leq 1$, $x_{in}, y_{in} \geq 0$, and $\sum_{j=1}^n x_{ijn} + y_{in} \leq 1$ for $j = 1, \dots, n$ and $i, n = 0, 1, \dots$.

The maximand in Equation (2) can be motivated as follows. In Equation (2) the present value of the broker's business $V(n)$ in state n equals the expected future value, discounted over the interval of time Δt at the rate ι . During this interval at most one buyer arrives at the asset of some seller and pays a price p sufficiently high to induce a sale, $p \geq r_{ijn}$. This sale has two effects in Equation (2). It generates the expected commission shown in the first summation and reduces the number of clients by one with the probability shown in the second summation. During the same interval at most one seller arrives and raises the number of clients by one with the probability $\alpha \bar{n}^*(y_{in}/\bar{y}^*)\Delta t$. In the remaining event no buyer or seller arrives and the broker's number of clients n does not change. As indicated in Equation (2), these three events are mutually exclusive and exhaustive to order Δt . Implicit in Equation (2) is the reservation pricing rule or stopping rule: sell asset ijn whenever $p \geq r_{ijn}$. This rule is optimal for all possible constant commission rates: $0 \leq b \leq 1$.

In this initial model no seller selects his own reservation price. Instead, each seller's utility depends only on his current state and the decisions by all brokers. Seller ijn has the utility:

$$\begin{aligned}
 U(n) = e^{-\iota \Delta t} & \left\{ F(x_{ijn}^*, \bar{x}^*) \Delta t \int_{r_{ijn}^*}^1 (1 - b^*) p dG(p) \right. \\
 & + \alpha \bar{n}^*(y_{in}^*/\bar{y}^*) \Delta t U(n+1) \\
 & + \sum_{j' \neq j} F(x_{ij'n}^*, \bar{x}^*) [1 - G(r_{ij'n}^*)] \Delta t U(n-1) \\
 & + \left[1 - \alpha \bar{n}^*(y_{in}^*/\bar{y}^*) \Delta t - \sum_{j=1}^n F(x_{ijn}^*, \bar{x}^*) [1 - G(r_{ijn}^*)] \Delta t \right] U(n) \Big\} \\
 & + o(\Delta t), \tag{3}
 \end{aligned}$$

also for $j = 1, \dots, n$ and $i, n = 0, 1, \dots$. In Equation (3) the seller's present value or utility $U(n)$ in state n equals his expected future utility, discounted over the interval Δt , at the rate ι . During this interval at most one buyer arrives to inspect asset i with the probability $F(x_{ijn}^*, \bar{x}^*) \Delta t$. This potential buyer then pays a price p sufficiently high to induce a sale, $p \geq r_{ijn}^*$, with the probability $G(r_{ijn}^*)$. After the broker's commission, this sale generates the net cash inflow $(1 - b^*)p_{ijn}$. Otherwise no sale occurs and the seller retains his asset. His resulting utility depends on his broker's subsequent number of assets. The probabilities in Equation (3) of the subsequent states $n+1$, $n-1$, and n match the probabilities in Equation (2) with one

exception. The probability in Equation (2) of at least one sale with the subsequent state, $n - 1$, is replaced in Equation (3) by the probability of at least one sale of another asset, $j' \neq j$, with the same subsequent state.

The present value for the representative broker must satisfy two constraints. In this model the supply of potential new brokers is infinitely elastic. A countable infinity of identical individuals can enter the industry and become brokers. Each entrant incurs the same fixed cost, $\gamma > 0$. After entry each new broker begins with no clients and hence has the present value $V(0)$. Given no constraints on entry, the present value upon entry $V(0)$ must then equal the cost of entry:

$$V(0) = \gamma. \quad (4)$$

The constant γ includes both the direct costs of entry, like licensing and training, and all opportunity costs of entry, such as the present value of income foregone from the next best, alternative occupation. In practice, the direct costs of entry may be small.¹² Also, to preclude bubbles or explosive growth in present values with progressively larger numbers n of assets, clients, or sellers, the broker's incremental present value must be bounded above by some constant, $0 < \delta < \infty$:

$$\lim_{n \rightarrow \infty} V(n+1) - V(n) \leq \delta. \quad (5)$$

This inequality is the limiting or transversality condition that is familiar from many models in financial economics.

Sellers must be induced to hire brokers in equilibrium. In this model brokers are distinguished only by their current number of clients. Suppose that a new seller enters the market and immediately selects some broker with $n - 1$ clients. That broker then has n clients, and each client has the present value or utility $U(n)$ from Equation (3). Alternatively, the same owner can offer his asset for sale without a broker and immediately realize the exogenous present value or utility, $u_0 > 0$. This value is identical for all sellers, who are identical by previous assumption. In this situation a seller chooses to hire a broker with $n - 1$ clients if and only if the broker adds value: $U(n) \geq u_0$. This inequality is satisfied if brokers are sufficiently more productive than sellers without brokers, as reflected in the functions, F and G , for sales with brokers and the constant u_0 for sales without brokers.

The above problem is a repeated game with complete information. In each period the representative broker i first observes his current state n . Next he allocates his time among new and current clients, x_{in}

¹² With real estate licensing does not greatly restrict entry. See Manatrala and Zabel (1995) and the references cited therein. Related legal issues are discussed in Epley and Armbrust (1978), Erxleben (1981), and Johnson and Loucks (1986).

and y_{in} , and selects his sellers' reservation prices r_{in} . He acts simultaneously with all other brokers, conditional on his correct conjecture about the average optimal actions of all other brokers. The broker's state n then follows the Markovian process in Equation (1), conditional on brokers' current actions. In a Markov perfect equilibrium, these profiles of Markov strategies generate a Nash equilibrium in every proper subgame. This equilibrium—a solution to problems (2) through (5)—is characterized in Section 3. For the representative broker i , it includes the optimal allocations, x_{in}^* and y_{in}^* , and reservation prices r_{in}^* in each state n . It also includes the commission rate b^* and the ratio of assets per broker $\bar{n}^*$ in equilibrium. The latter two variables are the price and relative supply of brokerage services in equilibrium.

In these sequentially perfect equilibria, all players' strategies are Markov. Markov strategies depend on the history of previous plays only through Markov state variables that summarize the effect of history on the current environment. Previous actions influence current strategies only through their direct effect in Equation (1) on payoffs from the continuation game. Also, with Markov strategies reputation has no role. Past actions are not interpreted as signals of future intentions. Reputation is moot in this initial model where all decisions are delegated to brokers. A more complicated, more realistic model in which reservation prices are set by sellers is presented in Section 4.

3. Equilibrium with Delegation

In this section the previous problem is solved for a large market with many brokers in steady state. Problems (2) through (5) are solved first without the seller's participation constraint: $U(n) \geq u_0$. The unique solution—an equilibrium with full delegation of all decisions to brokers—is characterized in surprising detail. Most importantly, a proportional brokerage commission is shown to eliminate in equilibrium the agency problem between brokers and their clients. Each broker selects for each seller the same reservation price and same allocation of time that he would select for his own asset.

The previous problem is solved in the Appendix with two restrictions on the parameters. The first restricts the production function for new clients F . As previously assumed, F is homogeneous and increasing along any ray from the origin. In this case, the elasticity of F with respect to the broker's own allocation of time x_{ijn} is a positive constant: $x_{ijn}F_1(x_{ijn}, \bar{x})/F(x_{ijn}, \bar{x}) = \eta > 0$, for all $0 < x_{ijn} \leq 1$. In the subsequent solution each broker's optimal reservation price is the same for each asset. This reservation price is positive if and only if $\eta < 1$. Since sellers realistically do reject some offers in equilibrium, the subsequent solution focuses on the case: $0 < \eta < 1$. For notational

convenience, define the new parameter: $\pi \equiv \frac{\eta\iota}{\eta\iota + (1-\eta)\alpha}$. This new parameter is a probability, $0 < \pi < 1$, under the above restriction on the parameter η . The second restriction involves the parameter θ in the broker's personal cost function θH . For arbitrary values of this parameter, $\theta > 0$, the subsequent solution has an integer constraint that considerably complicates the exposition but adds no economic insights. As shown in the Appendix, the integer constraint is not binding for the feasible value: $\theta^* \approx 1$. This value and the resulting solution without the integer constraint are presented in Proposition 1.

Proposition 1. *In steady state problems (2) through (5) have a unique solution. With the parameters, $0 < \eta < 1$ and $\theta = \theta^*$, its solution uniquely satisfies the conditions:*

$$b^* = \frac{1 - \eta}{\eta\iota} \frac{x^*}{r^*} \theta^* H'(w^*), \quad (6)$$

$$\bar{n}^* = \pi \frac{w^*}{x^*}, \quad (7)$$

$$r_{ijn}^* = r^*, \quad \frac{\alpha}{\iota} (1 - \eta) = \frac{r^* [1 - G(r^*)]}{\int_{r^*}^{\infty} [1 - G(p)] dp}, \quad (8)$$

$$x_{ijn}^* = x^*, \quad \alpha = F(x^*, x^*) [1 - G(r^*)], \quad (9)$$

$$y_{in}^* = w^* - nx^*, \quad \gamma\iota = w^* \theta^* H'(w^*) - \theta^* H(w^*), \quad (10)$$

$$U(n) = \left[1 + \frac{\eta\iota}{(1 - \eta)(\alpha + \iota)} \right] (1 - b^*) r^*, \quad (11)$$

$$V(n) = \gamma + nb^* r^*, \quad (12)$$

for $i = 1, \dots; j = 1, \dots, n$; and $n = 0, 1, \dots, \frac{w^*}{x^*}$. The parameter θ^* has the bounds: $1 \leq \theta^* < 1 + \frac{w^* x^*}{\gamma\iota} H''(w^*) + O(x^{*2})$. The representative broker's current number of clients $\tilde{n}$ has a binomial distribution with the mean, $E(\tilde{n}) = \bar{n}^*$ in Equation (7):

$$\Pr\{\tilde{n} = n\} = \binom{w^*/x^*}{n} \pi^n (1 - \pi)^{(w^*/x^* - n)}, \quad (13)$$

for $n = 0, 1, \dots, \frac{w^*}{x^*}$. The remaining states, $n = \frac{w^*}{x^*} + 1$ and $\frac{w^*}{x^*} + 2, \dots$, are never attained.

Equations (6) through (13) uniquely characterize the solution to problems (2) through (5). For a large market in steady state, all brokers choose for all assets the same reservation price, $r^* > 0$, uniquely

satisfying the right side of Equation (8). Also, all brokers allocate to work the same fraction of time, $0 < w^* < 1$, uniquely satisfying the right side of Equation (10). The optimal allocation of time to each asset by each broker, $0 < x^* < 1$, is then uniquely determined by Equation (9). In turn, these optimal choices by brokers determine in equilibrium the commission rate, b^* in Equation (6), and the average number of assets per broker, $\bar{n}^*$ in Equation (7). The resulting utility or present value for the representative seller and broker are identified in Equations (11) and (12), respectively. The critical value of the parameter θ for which the integer constraint is not binding is approximated below Equation (12). At this value θ^* , the ratio w^*/x^* is an integer. Finally, the distribution in steady state for the each broker's current number of clients is shown in Equation (13). As a binomial distribution it has the variance $(1 - \pi)E(\bar{n})$.

In equilibrium all reservation prices and all allocations of time to all current clients are identical. Because all assets are identical *ex ante*, all reservation prices in Equation (8) and all allocations of time in Equation (9) are identical for all assets of each broker. Surprisingly this solution is the same for all brokers. In Proposition 1 each broker equates the marginal productivity of his time allocated to each asset to the marginal productivity of time spent searching for new sellers to the marginal cost of his total time at work. Because the marginal productivity of searching for new sellers is constant, independent of the time spent searching, both the time spent with each current client, x^* in Equation (9), and the total time at work, w^* in Equation (10), are independent of the broker's current number of clients n . In other words, each broker first chooses between labor and leisure and thereby determines his total time at work w^* . Next, he always allocates to each current client the same fraction of time x^* . Finally, he allocates to new clients any residual working time, $w^* - nx^*$. By precisely the same argument, each broker optimally selects for each seller the same reservation price r^* , independent of his current number of clients n . This solution holds if and only if the present value of the broker's business $V(n)$ is linear in n , as indicated in Equation (12).

The proportional brokerage commission, identical for all assets, produces no agency problem in equilibrium between a broker and his current clients. This can be seen by focusing on brokers' behavior out of equilibrium. To this end, temporarily ignore the competitive entry condition for brokers in Equation (4). For the resulting partial equilibrium, the broker's problem in Equation (2) has a unique solution that depends on the exogenous commission or price b . Again, each broker equates the marginal productivity of his time with each current client to the marginal productivity of his time spent searching for new clients. Both marginal productivities are proportional to the

constant b , and the latter marginal productivity is constant, independent of the broker's time spent searching for new clients. The broker's optimal allocation of time x^* is therefore independent of the exogenous commission rate b . By precisely the same argument, the optimal reservation price r^* is independent of the constant b . Details appear in the Appendix. Clearly, conflicts of interest cannot occur if brokers represent their own assets: $b = 1$ in problems (2) through (5). Because brokers select the same reservation price, r^* in Equation (8), and the same selling effort, x^* in Equation (9), for all constant commission rates, $0 < b \leq 1$, no agency problem between a broker and his current clients is produced by a proportional commission, $0 < b < 1$, that is identical for all assets.

In the partial equilibrium with an exogenous brokerage commission, only the total time at work, the residual time spent searching for new clients, and thereby the average number of clients per broker depend on the commission rate. In this partial equilibrium all three variables increase in the commission rate b . For the equilibrium, Equations (6) through (13), all three variables then increase in each broker's cost of entry γ . By contrast, both the optimal reservation price r^* and the optimal time or effort x^* are independent in equilibrium of the common cost γ . This distinct or separate determination of the optimal activities for current clients, r^* in Equation (8) and x^* and Equation (9), versus the residual allocation of time to new clients, y_n^* in Equation (10), depends critically on the assumed constant marginal productivity of searching for new clients. To see this, suppose instead that some broker has a diminishing marginal productivity. With a bigger commission rate b in partial equilibrium, the broker would then spend more time searching for new clients. This would depress his marginal productivity of searching for new clients and thereby induce him to spend more time searching for current clients. It may also induce him to modify his reservation price for each current client. With a linear production function for new clients, this interaction is absent. With a nearly linear function, it is minimal.

This perfectly competitive equilibrium without an agency problem is not first best. Two separate sources of externalities can produce two types of dissipative costs. First, the average arrival rate of buyers at each asset ijn can depend on the average allocation of time by all brokers to all assets $\bar{x}$. With this first externality the time allocated to each asset by each broker x^* is not first best.¹³ With negative externalities extra effort by one broker decreases the average arrival rate of buyers

¹³ Externalities in search models are studied in Mortensen (1982) and Hosios (1990). Other issues specific to real estate are discussed in Micelli (1992).

at other assets: $F_2 < 0$. Relative to the first-best solution selected by a social planner, both competitive brokers and sellers without brokers then choose a lower reservation price r^* and a higher allocation of time x^* . Alternatively, with positive externalities, $F_2 > 0$, the reservation price r^* is higher and the allocation of time x^* is lower than the first-best solution. Second, the time spent searching for new sellers by each identical broker y^* is purely dissipative in this model. The latter time has not only no benefit to new sellers but also a personal cost to each broker. The latter cost is also independent of the proportional brokerage commission. Again, consider the partial equilibrium in steady state produced by Equations (2), (3), and (5), without the competitive entry of brokers in Equation (4). As shown in the Appendix, both brokers' allocation of time to current clients x^* and their average allocation of time to new sellers $\bar{y}^*$ are independent in this partial equilibrium of the exogenous, common, constant commission rate b . Thereby, the dissipative cost from each externality, measured per broker, is independent of b . However, the average number of assets per broker in equilibrium n^* is increasing in each broker's cost of entry γ . Consequently, the total dissipation from costly search is decreasing in the cost of entry γ for the equilibrium without agency in Equations (6) through (13).¹⁴

The above, relatively simple statement of Proposition 1 depends in part on four analytically convenient assumptions: the homogeneous average arrival rate of buyers $F(x_{it}, \bar{x})$, the MLRP for the distribution of offers G , the specially selected parameter, $\theta^* \approx 1$, in the broker's cost of effort θH , and the proportional average arrival rate of new sellers $\alpha \bar{n}(y_{in}/\bar{y})$. None of these details are essential for the main results described above. Without the first assumption the elasticity η in Equation (8) depends on each broker's optimal allocation of time to each current client x^* . Otherwise the above solution is unchanged. Without the second assumption, the right side of Equation (8) may not have a unique solution r^* . Without the third assumption, the ratio w^*/x^* below Equation (13) is not an integer. Equations (8) and (9) are then satisfied only for the states n not exceeding this ratio and the binomial distribution of Equation (13) is replaced by a more complicated, compound distribution.¹⁵ Finally, the fourth assumption can be relaxed slightly. With a linear inhomogeneous arrival of new sellers, $\alpha_0 + \alpha_1 \bar{n}(y_{in}/\bar{y})$, Equations (8) and (9) are again satisfied for all initial

¹⁴ In this model with identical brokers, all implications of different abilities are ignored. See Leland (1979).

¹⁵ The largest attainable state n is then the smallest integer larger than the ratio w^*/x^* . In this state the broker allocates all available time to his current clients. The allocation to each client in this state is still the same but strictly less than x^* .

states, and Equation (13) is again replaced by a compound distribution. None of these modifications alter the main message of Proposition 1: with full delegation of all decisions to brokers, no agency problem exists in equilibrium between brokers and their current clients. The more important part played by the linearity of sellers' arrival rate is discussed in Section 5.

Sellers must be induced in equilibrium to hire brokers. In the previous equilibrium each seller attains the utility in Equation (11). Insert Equation (11) into the seller's participation constraint: $U(n) \geq u_0$. This generates an upper bound on the competitive commission rate of Equation (6):

$$0 \leq b^* \leq 1 - \frac{(\alpha + \iota)(1 - \eta)}{\iota + \alpha(1 - \eta)} \frac{u_0}{r^*} \equiv \beta_1, \quad (14)$$

with $\beta_1 \leq 1$. If $\beta_1 = 1$, then Equation (14) is always satisfied. Not only is $\beta_1 = 1$ if either $\eta = 1$ or $u_0 = 0$, but also β_1 is increasing in η and decreasing in u_0 . More proficient brokers have higher elasticities η of the average arrival rate of buyers with respect to a broker's own effort, while less skilled sellers without brokers have lower reservation utilities u_0 . In this sense, if brokers are sufficiently skilled relative to sellers, then all sellers are induced in equilibrium to hire brokers. In practice, brokers can have superior skills or access to a superior technology, like a multiple listing service for residential real estate. Henceforth Inequality (14) is assumed to hold.

The seller's participation constraint in Inequality (14) is critical for the previous results. When Inequality (14) holds, each seller has an incentive in equilibrium to hire a broker: each seller's utility in equilibrium with a broker, $U(n)$ from Equation (11), exceeds or equals his reservation utility without a broker, u_0 . This reservation utility u_0 reflects both the reservation price selected by an owner without a broker and his personal time spent selling his asset. These optimal choices by a nonbroker-owner may differ from a broker's optimal values, r^* in Equation (8) and x^* in Equation (9). The difference may reflect an amateur's less efficient production function for potential buyers. Alternatively, an owner not employed as a broker may have a different opportunity cost of time. Realistically, he may earn income at a marginal rate that differs from a broker's opportunity cost of time in equilibrium—his time not spent searching for new clients. Nevertheless, when Inequality (14) holds, each seller chooses in equilibrium to hire a broker and thereby accepts Equations (8) and (9). For this reason, a broker's optimal reservation price and allocation of time to each current client are constrained, first best if and only if the seller's participation constraint in Inequality (14) is satisfied.

4. No Delegation

In the previous problem sellers delegate all decisions to their brokers, whereas in practice sellers set their own reservation prices. For this reason the previous problem is now modified in two steps as follows. First, each seller is assumed to set his own reservation price, simultaneously with all sellers, after which all brokers simultaneously allocate their time among alternative activities. The resulting problems of both brokers and their sellers are solved in a Markov perfect equilibrium for a large market in steady state. Next, each seller is constrained by a contractual asking or listing price. With this contractual constraint the problems of brokers and sellers are again solved. The latter solution is shown to match the initial solution, Equations (6) through (13), whenever the competitive commission rate of Equation (6) is sufficiently large.

The first modification is as follows. To match the previous problem, the current state of each broker and each of his clients is again restricted to the broker's current number of clients n . Conditional on n , broker i solves Equation (2) with the following modification. The broker no longer chooses the current reservation prices r_{in} of all his sellers. Instead, he chooses, concurrently with all other brokers, only his current allocations of time, x_{in} and y_{in} . He does so anticipating correctly that each seller ijn subsequently selects, simultaneously with all other sellers, his preferred reservation price r_{ijn} . The solution to this problem—henceforth called modified problem I—is a Markov perfect equilibrium for a large market in steady state. The problem is solved in the Appendix, and its solution is stated in Proposition 2.

Proposition 2. *In steady state modified problem I has a unique solution. With the parameter θ^* the solution satisfies the conditions:*

$$r_{ijn}^* = r^{**} > r^*, \quad x_{ijn}^* = x^{**} < x^*, \quad w_{in}^* = w^*, \quad (15)$$

for $i = 1, \dots; j = 1, \dots, n; \text{ and } n = 0, 1, \dots, \frac{w^*}{x^*}$.

In this new equilibrium all reservation prices and all allocations of time to all assets are again equal. However, each seller now sets his reservation price above the price preferred by his broker: $r^{**} > r^*$. Anticipating this, each broker spends less time with each seller than in the previous solution: $x^{**} < x^*$. Thereby, brokers spend less time or effort on each client's asset than they would on their own assets. Finally, each broker spends the same time working w^* in both Propositions 1 and 2. When sellers set their own reservation prices, each broker with at least one client then spends more time searching for new sellers: $y_{in}^* = w^* - nx^{**} > w^* - nx^*$, for all $n \geq 1$. Because additional clients are less valuable to brokers in the new equilibrium,

prospective clients are also less valuable. Fewer brokers are then induced to enter the industry, and the new equilibrium has more clients per broker: $\bar{n}^{**} > \bar{n}^*$. Moreover, the second-best reservation price r^{**} and allocation of time x^{**} in Proposition 2 have properties much like the corresponding, constrained, first-best values in Proposition 1. Most notably, these second-best values also hold for all exogenous commission rates b in the partial equilibrium without competitive entry by brokers in Equation (4). Thereby the second-best values, r^{**} and x^{**} , are also independent in equilibrium of brokers' common cost of entry γ .

This conflict of interest in equilibrium between brokers and their clients can be explained as follows. When setting his reservation price, each seller ignores his broker's personal cost of searching for subsequent buyers. The broker incurs this cost when he diverts time from either leisure or his search for new clients. Additional costs of searching for buyers are more likely when the seller's reservation price is higher. Anticipating a higher price, the broker then allocates less time to the asset and thereby reduces his personal cost. The second-best solution reflects the dual agency problem between each broker and seller in which each acts as an agent for the other. Here, each broker acts as an agent for his seller when setting his time or effort, while each seller acts as an agent for his broker when selecting his reservation price. As such, the dissipative cost from the second-best solution is unrelated to the exogenous commission rate in partial equilibrium without competitive entry by brokers and thereby is unrelated to the cost of entry in competitive equilibrium. Not surprisingly, such behavior by brokers and sellers is observed in practice. Again, consider residential real estate. If a seller rejects a competitive offer, then his broker may regard him as unrealistic or unreasonable. The broker may then expend less effort and no longer actively market the seller's house.

Reputation has no role in this equilibrium. Because each broker allocates his time conditional only on his current number of clients, he ignores all past actions by his sellers. This includes all rejections of previous bids from potential buyers. Anticipating this, no seller worries about his broker's possible reaction if he rejects a price p in the range, $r^* \leq p < r^{**}$. Alternatively, the broker could condition his current effort on the client's previous rejections of offers at or above the broker's preferred reservation price r^* . Moreover, there might exist a subgame perfect equilibrium in which sellers select the optimal reservation price, r^* in Equation (8), and brokers respond with the optimal allocation of time, x^* in Equation (9). This equilibrium might involve some non-Markovian, "tit-for-tat" strategy in which brokers expend less effort for some period of time on an asset to punish a seller for previously rejecting an offer in the range r^* to r^{**} .

Alternatively, brokers and sellers could negotiate a contractual asking or listing price. Consider an asking price a_{ijn} , negotiated for the current period between broker i and seller ijn with the following property. If seller ijn receives a current offer p_{ijn} at or above the asking price a_{ijn} , then he must pay the commission bp_{ijn} to his broker even if he rejects the offer. This asking or listing price is a private, contractual matter between a broker and his client. As such, it is distinguished from the public or posted listing price that is common with residential real estate. A private asking price is now introduced into modified problem I, and the result is called modified problem II. In this second modification each seller sets, simultaneously with all other sellers, his current reservation price, subject to the above asking price. Anticipating this choice, each broker allocates, along with all other brokers, his time between current and new clients. Modified problem II is solved in the Appendix for a large market in steady state, and the solution is stated in the next proposition.

Proposition 3. *In steady state modified problem II has a unique solution. Pick the asking price from Equation (8): $a_{ijn} = r^*$, for all ijn . With the parameter θ^* the solution uniquely satisfies Equations (6) through (13) if*

$$b^* \geq \frac{\eta\iota}{\iota + \alpha(1 - \eta)} \equiv \beta_0, \quad (16)$$

with $0 < \beta_0 < 1$.

If the constant commission rate is sufficiently large in the equilibrium without agency, then an asking price equal to the broker's preferred reservation price is optimal for both the broker and each of his sellers. This asking price, $a_{ijn} = r^*$, induces each seller to select the same reservation price as both the asking price and the broker's preferred reservation price, r^* from Equation (8). Thus the broker is paid his commission in equilibrium if and only if the asset is sold. This property produces the equilibrium, Equations (6) through (13), whenever the resulting constant commission rate, b^* in Equation (6), satisfies (16). Inequality (16) holds, together with the seller's participation constraint in Equation (14), so that $\beta_0 \leq b^* \leq \beta_1$, whenever both the elasticity η and the utility u_0 are sufficiently small. In other words, neither brokers nor sellers can be too proficient at producing potential buyers. This equilibrium without an agency problem is first best among all feasible solutions to problems (2) through (5) for a large market with many brokers in steady state. Therefore the asking price in Equation (8) can be reached by bargaining between the broker and the seller. Of course, this equilibrium without agency may not be first best among all equilibria that can be produced by a social

planner. The first-best solution requires coordination between brokers and sellers. Coordination may be infeasible in a market with infinitely many brokers and sellers.

5. Extensions

5.1 Heterogeneous assets

In this model all assets are identical *ex ante* before inspections by potential buyers. For these assets the average arrival rate F and the distribution of offers G are identical. Heterogeneous assets with different arrival rates and different distributions of offers are easily introduced into the model. Suppose for simplicity that brokers represent two classes of assets, each with its own arrival rate and distribution of offers. In this case, the representative broker has two state variables: the number of his assets, clients, or sellers in each of his two classes. In steady state the equilibrium is then characterized by two ratios of assets per broker, with one for each class of assets. This more complicated problem also has a unique solution. For each asset in a class, the optimal reservation price and optimal allocation of time from each broker are constant, independent of both the common commission rate and the broker's current number of clients of each type. The reservation prices and allocations of time are identical within each class and different across the two classes. As a result, the agency problem is again eliminated in the equilibrium with full delegation and thereby in the equilibrium with contractual asking prices, one for each class. The same solution also holds with homogeneous assets and heterogeneous sellers. For example, some owners may have a more urgent need to sell their assets than others and hence have a higher discount rate.

5.2 Heterogeneous brokers

Realistically, brokers are not identical. In practice, their heterogeneity is reflected in the cross-sectional dispersion of their incomes, which may be due to different specializations, skills, or preferences for leisure. With heterogeneous brokers the search for new sellers is no longer purely dissipative, as in the model. Because new sellers now have an incentive to find more productive brokers, the search can be at most partly dissipative. Again, this modification does not alter the main results of the model. Suppose for simplicity that brokers come in two types, distinguished by their productivity. With competition among many brokers of each type and no fixed inputs into the production process for new clients, each broker again has a constant marginal productivity of search for new sellers, independent of his time spent searching. More productive brokers may have a higher,

constant marginal productivity than less productive brokers. If all brokers in each class are paid a common, constant commission rate, then each broker's reservation price and allocation of time for each asset is again a constant, independent of his constant commission rate and his current number of clients. Again, this solution can differ across the two classes of brokers. Again, the agency problem is eliminated in equilibrium.

5.3 Buyers' brokers

In practice, some brokers represent both buyers and sellers, while others specialize in either buyers or sellers.¹⁶ Brokers can be attached to buyers without altering the main results in the model. Again, a proportional commission, split equally between the two brokers, supports an equilibrium without an agency problem. An equal split may be necessary if both buyers and sellers are to have brokers in equilibrium. The problem becomes more complicated if brokers must also search for new buyers. Each broker then has two state variables: his current numbers of buyers and sellers as clients. Two aggregate ratios, the average numbers of buyers and sellers per broker, must then be determined in equilibrium for a large market in steady state. Once again, the optimal allocation of time by each broker to each client is constant, independent of both the constant commission rate and the broker's current number of clients of each type. With this solution the agency problem is again eliminated in equilibrium.

5.4 Other production functions for new clients

In the model each broker's average arrival rate of new clients is proportional to his own allocation of time. This assumption is reasonably plausible if brokers' time or effort is the sole input into the production process for new clients and cooperation among brokers is precluded. In fact, advertising by brokerage firms can reasonably be regarded as a fixed input into each broker's production function. If this search by identical brokers for new sellers is purely dissipative, then each broker's average arrival rate of new clients is linear inhomogeneous in his relative allocation of time, $y_{in}/\bar{y}$. In this case, the previous solution again applies with several minor modifications. Most importantly, all states or numbers of clients n above some integer n^* that is analogous to the ratio w^*/x^* below Equation (13) are now attainable, and the binomial distribution in Equation (13) is then replaced by a more complicated, compound distribution. Alternatively, brokers may be able

¹⁶ In most jurisdictions buyers' brokers are subagents of the seller's broker. As subagents they have a legal duty to disclose to the seller's broker private information revealed to them by their buyers.

to cooperate somewhat in their search for new clients—for example, by allocating all walk-in customers at brokerage firms to brokers with allotted floor time.¹⁷ This cooperation would reduce the total time spent searching for new clients and thereby the dissipative costs in equilibrium. Otherwise the previous solution would again apply.

5.5 Previous efforts by brokers

In the model the average arrival rate of potential buyers at an asset depends only on the current effort by the broker and the concurrent average effort by competitive brokers. Realistically the arrival rate depends on both current and past efforts by the broker and his competitors, with some attenuation or depreciation of past efforts over time. For asset ijn the current arrival rate F could depend on two state variables: z_{ijn} for broker i and $\bar{z}$ for the average across all brokers. The change in the first state variable could equal the broker's current allocation of time x_{ijn} minus depreciation δz_{ijn} , at the constant rate δ : $dz_{ijn}/dt = x_{ijn} - \delta z_{ijn}$. The change in the second state variable would be similar. For a large market in steady state, the optimal allocation x_{ijn}^* then has one of two values: $x_{ijn}^* = x_1^*$ with $y_{ijn}^* > 0$ for $n = 0, 1, \dots, n^*$, and $x_{ijn}^* = x_2^*$ with $x_1^* > x_2^*$ and $y_{ijn}^* = 0$ for $n = n^* + 1, \dots$. In turn, these two values induce in steady state a distribution for the state variable z_{ijn}^* with the mean $\bar{z}^*$. With this modification the principal properties of the previous solution would again apply with one minor modification. The binomial distribution in Equation (13) would again be replaced by a more complicated, compound distribution.

5.6 Maturity of brokerage contracts

In this model with identical brokers and infinite time horizons, the agency problem is eliminated in equilibrium by contracts of infinite maturity. In practice, brokerage contracts have a limited range of finite maturities. With residential real estate, maturities of 3 to 6 months are common. These standard maturities are compromises between two competing concerns. Because brokers are not identical and their productivities are not entirely evident to sellers before contracts are signed, some contracts must be terminated in equilibrium. This induces sellers to demand short maturities. Very short maturities are precluded by the agency problem between brokers and sellers. If, as argued above, the average arrival rate of potential buyers at an asset depends on both current and past efforts by the broker and his

¹⁷ Typically such cooperation occurs only in brokerage firms where brokers split their commissions with the firm, rather than the more recent innovations of 100% houses. In the latter firms agents rent their desks, buy services from the firm, and retain all their commissions at the margin.

competitors, then the length or maturity of the brokerage contract becomes important. With shorter contracts the broker has less time to benefit from his current effort and therefore expends less effort in equilibrium. To eliminate this agency problem, brokers must have contracts that expire only with the sale of the asset—exactly as in this model. The optimal maturity is a compromise between these two conflicting objectives. It is shorter if brokers' private productivities are more widely dispersed or their previous efforts are less important for current sales. It is longer if the weight attached to previous sales is a slowly decaying function of time or commissions can be guaranteed from sales after maturity to buyers that were procured before maturity.¹⁸

5.7 Price taking

Previously, all perfectly competitive brokers and sellers were assumed to accept passively the common commission rate paid to all brokers. The competitive commission in Equation (6) is then determined in equilibrium by the interaction between demand and supply. In fact, sellers may benefit by defecting from this equilibrium and paying their brokers higher percentage commissions. To see this, suppose that seller $i1n$, acting in isolation, offers his broker i a constant commission rate b_1 above the competitive rate b^* . Suppose that no broker solicits and no other seller offers rates different from Equation (6). This isolated offer may then induce broker i to allocate more time or effort to asset $i1n$ and less to other assets ijn : $\frac{\partial x_{i1n}^*}{\partial b_1}|_{b^*} > 0 > \frac{\partial x_{ijn}^*}{\partial b_1}|_{b^*}$ with $x_{ijn}^* = x_{i2n}^*$ for $j = 2, \dots, n$. In turn, this would induce broker i to reduce his reservation price for asset $i1n$ and raise his reservation price on all other assets ijn : $\frac{\partial r_{i1n}^*}{\partial b_1}|_{b^*} < 0 < \frac{\partial r_{ijn}^*}{\partial b_1}|_{b^*}$, with $r_{ijn}^* = r_{i2n}^*$ for $j \geq 2$. Given a sufficiently small competitive commission, $0 \leq b^* < B(n)$, seller $i1n$ would then benefit at the expense of all other sellers ijn : $\frac{\partial}{\partial b_1} U_{i1}(n)|_{b^*} > 0 > \frac{\partial}{\partial b_1} U_{ij}(n)|_{b^*}$ with $U_{ij}(n) = U_{i2}(n)$ for $j \geq 2$. Thereby seller $i1n$ is compensated for his slightly higher commission rate by his broker's extra effort. If broker i has more clients, then he can allocate more time away from other assets to asset $i1n$. Thus the bound B should be increasing in the broker's current number of sellers n . Clearly this defection would destroy the previous, perfectly competitive equilibrium.¹⁹ Any new equilibrium must then have either

¹⁸ Typically the listing broker has a contractual right to a commission on a sale to a buyer that he, another broker, or the seller procured during the listing period. The sale must occur within a limited period after the expiration of the listing contract.

¹⁹ This conjecture is verified in Anglin (1993) for the standard agency problem with no search by agents for new principals. Given a fixed number of clients per broker n and a linear search technology, $F(x_{ijn}, \bar{x}) \propto x_{ijn}$, each broker allocates all his time or effort to the client who offers the

a sufficiently high commission rate b to preclude possible defections or a regulatory constraint on the maximum feasible rate. This new equilibrium may have the same reservation price and same allocation of time, Equations (8) and (9), as the perfectly competitive equilibrium without an agency problem.²⁰ However, a higher commission rate in equilibrium would require a correspondingly higher cost of entry γ with its higher dissipative costs of searching for new sellers.

5.8 Net contracts

Sellers may also have an incentive to defect from an equilibrium with proportional commissions. The argument can be sketched as follows. Again, assume that all sellers offer their brokers one constant commission rate. In this case, seller i , again acting in isolation, can offer his broker a net contract. Upon the sale of asset ij , its seller receives the prior payment $\min(p^*, p_{ijn})$. Concurrently broker i receives the residual commission $\max(0, p_{ijn} - p^*)$. The contractual price p^* is set so that the expected payment to the broker is the same under both the net contract and the original proportional contract. Again, this new contract induces the broker to allocate additional time to the seller's asset and away from other assets. Again, this improves the seller's welfare. As a result, all sellers are again induced to defect to an equilibrium with net contracts. In the new equilibrium brokers' efforts for their current clients may no longer be first best because no seller can be induced to set his asking price above the contractual price p^* . In this case, efficiency can be improved by precluding net contracts. In some jurisdictions net contracts are precluded by law.

5.9 Risk aversion

In the model both brokers and owners are assumed to be risk neutral. In fact, only institutions with well-diversified beneficiaries or owners are likely to be effectively risk neutral. Examples are life insurance companies, nonprofit endowments, pension funds, public corporations, and REITs. Other participants in the market are likely to be risk averse—some more than others. For example, brokers of residential real estate in the United States may average about five to six transactions per year with an average commission below \$5000. By contrast, homeowners have far fewer transactions per year on average and far more equity per transaction. Moreover, the median homeowner in the United States invests most of his financial wealth in his home.

highest commission rate. This property precludes any equilibrium with experimental commission rates and multiple clients per broker, $n > 1$.

²⁰ This possibility is suggested by the results in Bernheim and Whinston (1986).

In 1986 American households held 41.3% of their net worth in their primary homes, with a median value of \$40,597, an additional 4.4% of their wealth in secondary homes, and only 4.0% in stocks, including pensions.²¹ With respect to transactions of residential real estate, brokers may then be risk averse, but less so than homeowners. Given an agency problem, any optimality of standard residential brokerage contracts with proportional commissions is then an open issue.

5.10 Private information

In both the model and all extensions discussed in this section, private information is precluded. Once a potential buyer arrives and inspects an asset, both the buyer and the seller have common knowledge. Realistically, both the buyer and the seller have private information about the asset, other competing assets, or their personal preferences. For example, suppose that only the seller knows his reservation price. A public asking or listing price may then convey valuable information to buyers. This posted price has the following property: whenever the owner receives an offer at or above the asking price, he must sell his asset to the bidder or face a lawsuit for specific performance that will force a sale. In this case, an asking price can convey credible information to potential buyers that induces them to incur the cost of inspecting the asset and negotiating with the seller. A seller and his broker must then choose an asking price that reflects the trade-off between the arrival rate of potential buyers and the distribution of their bids, conditional on arrival. Unlike the private, contractual asking price in Proposition 4, the public or posted asking price would exceed the optimal reservation price of the broker and seller. To be consistent with the empirical evidence from residential real estate, the optimal asking price must exceed the average transaction price by about 10 to 15%. Also, to match the evidence, such a model must explain why transaction prices more often exceed posted asking prices in hot markets with high demand than cold markets with low demand. In other words, why do sellers seemingly set their public asking prices too low in hot markets and too high in cold markets?

6. Summary

Brokerage contracts for real assets are commonly characterized by proportional commissions, payable upon sale at an identical rate for all assets in a category. Often the commission must be paid to the exclusive listing broker whenever an offer at or above a contractual,

²¹ See the U.S. Bureau of the Census, 1987, *Current Population Survey*, pp. 70–77.

private listing price is received from a potential buyer, even if the offer is rejected by the seller. As shown in this article, these conventional contractual terms eliminate in equilibrium the agency problem between a broker and his clients for a large market with many brokers in steady state. The broker's optimal time or effort spent selling each asset and its seller's reservation price are the same in equilibrium for brokers representing clients' assets or their own assets. To support this solution, the broker and his client optimally agree to a private listing price that matches the optimal reservation price. This solution is first best or Pareto optimal among all solutions that can be achieved by competitive brokers. Only in the absence of externalities among the selling efforts of competing brokers is it first best among all cooperative solutions that can be enforced by a social planner. In no case is a broker's search for new clients efficient. In equilibrium brokers spend excessive time or effort on their dissipative search for new clients. In partial equilibrium without competitive entry by brokers, the total dissipative cost from both selling assets of current clients and searching for new clients does not depend on the exogenous commission rate. Therefore, in equilibrium with competitive entry and an endogenous commission rate, the total dissipation does not depend on the cost of entry for each broker.

Appendix

Derivation of average arrival rates

In the model of Section 2, the market has a countable infinity of identical brokers. This market is the limit of a sequence of markets. Market m has m identical brokers: $m = 1, \dots$. In market m all assets other than ijn have the allocations: $x_{-ijn} = (x_{11n}, \dots, x_{ij-1n}, x_{ij+1n}, \dots, x_{mnn})$. The average arrival rate of buyers at each asset has the general form $\bar{F}(x_{ijn}, x_{-ijn})$. The nonnegative, real-valued function $\bar{F}$ is everywhere twice continuously differentiable, homothetic, and increasing along a ray from the origin; increasing and strictly concave in its first argument; and symmetric in its remaining $m\bar{n} - 1$ arguments. Restrict $\bar{n}$ to a constant, independent of calendar time t . For asset ijn and all other distinct assets, $i'j'n$ and $i''j''n$, with $(i, j) \neq (i', j') \neq (i'', j'')$, the continuous partial derivative $\frac{\partial \bar{F}}{\partial x_{i'j'n}}$ has the uniform bounds:

$$\left| \frac{\partial \bar{F}}{\partial x_{i'j'n}} \right| \leq \beta'_m, \quad \left| \frac{\partial \bar{F}}{\partial x_{i'j'n}} - \frac{\partial \bar{F}}{\partial x_{i''j''n}} \right| \leq \beta''_m, \quad (A1)$$

with the two constants β'_m and β''_m . These bounds have the respective orders: $\frac{1}{m}$ and $o\left(\frac{1}{m}\right)$. The latter bound can be rewritten equivalently as follows. In the $m\bar{n}$ vector x_{-ijn} , transpose the two allocations: $x_{i'j'n}$

and $x_{i'j'n}$, for $(i', j'n) \neq (i'', j''n)$, and represent the result by the $m\bar{n}$ vector x'_{-ijn} . By the symmetry of the function $\bar{F}$, the latter inequality is equivalent to

$$\left| \frac{\partial}{\partial x_{i'j'n}} \bar{F}(x_{ijn}, x_{-ijn}) - \frac{\partial}{\partial x_{i''j''n}} \bar{F}(x_{ijn}, x'_{-ijn}) \right| \leq \beta''_m.$$

Define the function: $F(x_{ijn}, \bar{x}) \equiv \bar{F}(x_{ijn}, \bar{x}, \dots, \bar{x})$. Consider the intermediate value, $z_{-ijn} = (z_{11n}, \dots, z_{ij-1n}, z_{ij+1n}, \dots, z_{mnn})$, satisfying the inequalities, $\min(x_{ijn}, \bar{x}) \leq z_{ijn} \leq \max(x_{ijn}, \bar{x})$, for all assets ijn . By the intermediate value theorem, there exists an intermediate value z_{-ijn} , dependent on x_{ijn} , such that

$$\begin{aligned} & |\bar{F}(x_{ijn}, x_{-ijn}) - F(x_{ijn}, \bar{x})| \\ &= \left| \sum_{j' \neq j} \frac{\partial}{\partial x_{ij'n}} \bar{F}(x_{ijn}, z_{-ijn})(x_{ij'n} - \bar{x}) \right. \\ &\quad \left. + \sum_{i' \neq i} \sum_j \frac{\partial}{\partial x_{i'jn}} \bar{F}(x_{ijn}, z_{-ijn})(x_{i'jn} - \bar{x}) \right| \\ &\leq m\beta'_m \left| \frac{1}{m} \sum_{j' \neq j} (x_{ij'n} - \bar{x}) + \frac{1}{m} \sum_{i' \neq i} \sum_j (x_{i'jn} - \bar{x}) \right| + \beta''_m(m\bar{n} - 1) \\ &= \beta'_m |x_{ijn} - \bar{x}| + \beta''_m(m\bar{n} - 1). \end{aligned} \tag{A2}$$

The inequality in Equation (A2) follows from the upper bounds, β'_m and β''_m , in Equation (A1). Given these uniform bounds and the upper bound, $|x_{ijn} - \bar{x}| \leq 1$, the right side of Equation (A2) converges to 0 as $m \rightarrow \infty$. This completes the argument for the asymptotic approximation $F(x_{ijn}, \bar{x})$.

Consider next the arrival of new clients for broker i . All brokers other than i allocate to their search for new clients the time, $y_{-in} = (y_{1n}, \dots, y_{i-1n}, y_{i+1n}, \dots, y_{mn})$. In general, the average arrival rate of new clients for each broker has the form, $\bar{\Phi}(y_{-in}) + y_{in}\bar{\Psi}(y_{-in})$. The nonnegative, real-valued functions, $\bar{\Phi}$ and $\bar{\Psi}$, are everywhere continuously differentiable, nondecreasing, and symmetric in their $m - 1$ arguments. Their partial derivatives have the same asymptotic properties as the partial derivatives of the function $\bar{F}$ with respect to its last $m\bar{n} - 1$ arguments. Define the two functions: $\Phi(\bar{y}) \equiv \bar{\Phi}(\bar{y}, \dots, \bar{y})$ and $\Psi(\bar{y}) \equiv \bar{\Psi}(\bar{y}, \dots, \bar{y})$. By the above argument, the differences, $\bar{\Phi}(y_{-in}, \dots, y_{-in}) - \Phi(\bar{y})$ and $\bar{\Phi}(y_{-in}, \dots, y_{-in}) - \Phi(\bar{y})$, are both uniformly bounded above and below by a bound of order $m o(\frac{1}{m})$. As $m \rightarrow \infty$, this bound also converges to 0.

Proof of Proposition 1. This lengthy proof has five parts. (i) Any solution to Equations (2) through (4) is shown to be the same for all brokers and sellers: $r_{ijn}^* = r_n^*$, $x_{ijn}^* = x_n^*$, and $w_{in}^* = w_n^*$ for all ijn . (ii) The problem is shown to have a constant solution: $r_n^* = r^*$, $x_n^* = x^*$, $w_n^* = w^*$, and $\Delta v_n^* = \Delta v^*$, with $v_n^* \equiv V(n)$ for all n . (iii) For this constant solution the critical value θ^* is then calculated. (iv) The constant solution is shown to be unique. (v) The steady-state distribution in Equation (13) is derived.

(i) Introduce the new notation: $M(r) \equiv \int_r^\infty [1 - G(p)] dp$. Integrate the maximand in Equation (2) by parts; divide all terms by Δt ; let $\Delta t \rightarrow 0$; and reorganize terms. This produces the problem:

$$0 = \max_{r_i, x_i, y_i} \left\{ b^* \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) M(r_{ijn}) - \theta H \left(\sum_{j=1}^n x_{ijn} + y_{in} \right) \right. \\ \left. + \alpha \bar{n} (y_{in} / \bar{y}^*) \Delta v_n^* + \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) [1 - G(r_{ijn})] (b^* r_{ijn} - \Delta v_{n-1}^*) \right. \\ \left. - v_n^* \right\}, \quad (\text{A3})$$

subject to $r_{in}, x_{in}, y_{in} \geq 0$ and $\sum_{j=1}^n x_{ijn} + y_{in} \leq 1$ for $j = 1, \dots, n$ and $i, n = 0, 1, \dots$. Given the corner conditions of F and H , any solution to Equation (2) must satisfy for r_{ijn} , x_{ijn} , and y_{in} the respective necessary optimality conditions:

$$r_{ijn}^* = b^* \Delta v_{n-1}^*, \quad (\text{A4})$$

$$\theta H'(w_{in}^*) = b^* F_1(x_{ijn}^*, \bar{x}^*) M(r_{ijn}^*), \quad (\text{A5})$$

and

$$\theta H'(w_{in}^*) \geq \frac{\alpha \bar{n} \Delta v_n^*}{\bar{y}^*}, \quad (\text{A6})$$

with an equality if $y_{in}^* > 0$ for all ijn .

Next, it is shown that $r_{ijn}^* = r_n^*$, $x_{ijn}^* = x_n^*$, and $w_{in}^* = w_n^*$ for all ijn . The first equality follows immediately from Equation (A4). Also, H' is increasing in its argument, whereas F_1 is decreasing in its first argument. If $y_{in}^* > 0$, then from (A5) and (A6) any solution must satisfy $x_{ijn}^* = x_n^*$, $w_{in}^* = w_n^*$, and thereby $y_{in}^* = y_n^*$ for all ijn . Alternatively, if $y_{in}^* = 0$, then $x_{ijn}^* = \frac{1}{n} w_{in}^*$ for all ijn . With the above monotonicity of H' and F_1 , then Equation (A5) requires that $x_{ijn}^* = x_n^*$ and $w_{in}^* = w_n^*$ for all ijn . Given this solution, the maximand in Equation (2) simplifies

to

$$\iota v_n^* = b^* n F(x_n^*, \bar{x}^*) M(r_n^*) + \alpha (y_n^* / \bar{y}^*) \Delta v_n^* - \theta H(w_n^*). \quad (\text{A7})$$

The maximand in Equation (2) is strictly concave in some neighborhood around this solution, so that the solution must be a local maximum.

(ii) Now try the constant solution: $r_n^* = r^*$, $x_n^* = x^*$, $w_n^* = w^*$, and $\Delta v_n^* = \Delta v^*$ for $n = 0, 1, \dots, n^*$. Here $n^* \equiv \left\lfloor \frac{w^*}{x^*} \right\rfloor$ is the largest integer not exceeding the real value $\frac{w^*}{x^*}$: $n^* \leq \frac{w^*}{x^*} < n^* + 1$. With this solution, the residual time spent searching for new sellers, $y_n^* = w^* - nx^*$, satisfies $y_n^* > 0$ for $n = 0, 1, \dots, n^* - 1$; $y_{n^*}^* \geq 0$ with an equality iff $n^* = \frac{w^*}{x^*}$; and $y_{n^*+1}^* = 0$. In this case, Equations (A2) through (A5) simplify to

$$r_{ijn}^* = r^*, \quad \Delta v = b^* r^*, \quad (\text{A8})$$

$$x_{ijn}^* = x^*, \quad \theta^* H'(w^*) = b^* F_1(x^*, x^*) M(r^*), \quad (\text{A9})$$

$$y_{in}^* = w^* - nx^*, \quad \theta^* H'(w^*) = \frac{\alpha \bar{n}^* \Delta v^*}{w^* - \bar{n}^* x^*}, \quad (\text{A10})$$

for $n = 0, 1, \dots, n^*$. Also, Equation (A7) holds for all $n = 0, 1, \dots, n^*$ if and only if

$$\iota_0^* = \gamma, \quad \gamma \iota = \frac{\alpha \bar{n}^* w^* \Delta v^*}{w^* - \bar{n}^* x^*} - \theta^* H(w^*), \quad (\text{A11})$$

$$\Delta v_n^* = \Delta v^*, \quad \iota \Delta v^* = b^* F(x^*, x^*) M(r^*) - \frac{\alpha \bar{n}^* x^* \Delta v^*}{w^* - \bar{n}^* x^*}. \quad (\text{A12})$$

In steady state the expected number of assets per broker $E(\tilde{n})$ is constant: $E(\Delta \tilde{n}) = 0$ in Equation (1). With Equations (A8) through (A10) this holds if and only if

$$\alpha = F(x^*, x^*) [1 - G(r^*)]. \quad (\text{A13})$$

Equations (6) through (12) follow from Equations (4) and (A8) through (A13). Specifically, Equation (7) follows from Equations (A9), (A10), and (A12); Equation (6) follows from Equations (7), (A8), and (A10); Equation (8) follows from Equations (7), (A8) through (A10), and (A13); Equation (9) follows from Equations (A9) and (A13); Equation (10) follows from Equations (A10) and (A11); and Equation (12) follows from Equations (A8), (A10), and (A11). To derive Equation (11), divide Equation (3) by Δt , let $\Delta t \rightarrow 0$, and insert Equations (8) and (A13) into the result. The resulting equation has the unique solution in Equation (11) whenever $U(n) = U(n+1)$. To show that the

latter equality must hold, set $b = 1$, drop Equation (4), and replicate the derivation of Equations (A8) through (A13). In this case, these conditions again hold with one deletion. In Equation (A11) the left side is dropped and γ on the right side is replaced by v_0^* . Most importantly, the solution with $b = 1$ satisfies $\Delta v_n^*|_{b=1} = r^*$ and $v_n^*|_{b=1} = v_n^*|_{b^*} + nU(n)|_{b^*}$. This requires that $U(n) = U(n+1)$.

Next, the right side of Equation (8) is shown to have a unique solution. To this end, define $\lambda \equiv \log \frac{\alpha}{\iota}(1 - \eta)$ and $L(r) \equiv \log \frac{r[1-G(r)]}{M(r)}$ so that Equation (8) simplifies to $L(r^*) = \lambda$. The continuous function L satisfies $L(0) = -\infty$ and $L(\infty) = \infty$. The latter condition is guaranteed by the nondecreasing hazard function $\frac{g}{1-G}$, assumed in Section 1. Thus Equation (8) must have at least one solution, $0 < r^* < \infty$. Suppose that it has at least two solutions, $0 < r_1^* < r_2^* < \infty$. With the derivative, $L'(r_k^*) = [1 + \frac{\alpha}{\iota}(1 - \eta)]\frac{1}{r_k^*} - \frac{g(r_k^*)}{1-G(r_k^*)}$, for $k = 1, 2$ and the nonincreasing hazard function, the two solutions must satisfy $L'(r_1^*) > L'(r_2^*)$. Given this inequality and the above boundary conditions, Equation (8) can have at most one solution r^* . Hence it has a unique solution.

(iii) The integer constraint is eliminated by carefully choosing a value of the parameter θ in a neighborhood of $\theta = 1$ so that $n^* = \frac{w^*}{x^*}$. From Equations (8) and (9) both r^* and x^* are independent of the parameter θ , whereas w^* and thereby n^* are not: $w^* = W(\theta)$ and $n^* = N(\theta) \equiv \left\lceil \frac{W(\theta)}{x^*} \right\rceil$. With Equation (10) and the strict monotonicity and convexity of H , the function W is decreasing everywhere: $W' = -\gamma\iota/\theta^2 WH'' < 0$ for $0 < \theta < \infty$. Also, it satisfies the initial and limiting conditions: $W(0) = 1$ and $W(\infty) = 0$. Write $w_1 = W(1)$ and $n_1 = N(1)$. Since $0 < w < 1$, there exists some value, $0 < \theta^* < \infty$, satisfying $W(\theta^*) = x^* n_1$. To calculate this value, set $w^* = W(\theta^*)$ and $n^* = n_1$ so that $n^* \leq \frac{w_1}{x^*} < n^* + 1$ and thereby $0 \leq w_1 - w^* < x^*$. Next, note from Equation (10) that

$$\begin{aligned} 1 \leq \theta^* &= \frac{w_1 H'(w_1) - H(w_1)}{w^* H'(w^*) - H(w^*)} \\ &= 1 + \frac{1}{\gamma\iota} w^* H''(w^*)(w_1 - w^*) + O[(w_1 - w^*)^2]. \quad (\text{A14}) \end{aligned}$$

This generates the bounds below Equation (12).

(iv) By the above argument, a constant, symmetric solution analogous to Equations (10) through (12) holds, with the parameter θ^* replaced by an arbitrary parameter $0 < \theta < \infty$. It is the unique solution to (2) through (5). This is shown as follows. The maximum in Equation (2) is well defined because the maximand is continuous and the feasible set of controls is compact. Also, with (5) the function V has the asymptotic bound: $\lim_{n \rightarrow \infty} V(n)/n \leq \delta$. In this case, the trans-

formation defined by the right side of Equation (2) is a contraction map with the modulus $e^{-\iota\Delta t}$. This follows from the modified Blackwell sufficiency condition of Stokey and Lucas (1989), Theorem 4.12. Thus the transformation on the right side of Equation (2) has a unique fixed point $V(\cdot, \Delta t)$ for all $\Delta t > 0$. By the derivation of Equation (A3), the function $V(n, \cdot)$ is continuous in some neighborhood of $\Delta t = 0$ with the limit: $V(n) \equiv V(n, 0) = \lim_{\Delta t \rightarrow 0} V(n, \Delta t)$.

(v) To complete the proof, the distribution in Equation (13) must be derived. For notational convenience construct the constant: $\zeta \equiv \frac{\eta\iota}{1-\eta}$. Given Equations (8) through (10), the generalized birth-death process in Equation (1) simplifies to

$$\Delta n = \begin{cases} +1 & \zeta(n^* - n)\Delta t + o(\Delta t) \\ 0 & 1 - \zeta(n^* - n)\Delta t - \alpha n\Delta t + o(\Delta t) \\ -1 & \alpha n\Delta t + o(\Delta t), \end{cases} \quad (\text{A15})$$

for $n = 0, 1, \dots, n^*$. With the evolution of states in Equation (A15), the steady-state distribution of $\tilde{n}$ must satisfy the second-order difference equation:

$$[\zeta(n^* - n) + \alpha n] \Pr\{\tilde{n} = n\} = \zeta(n^* - n + 1) \Pr\{\tilde{n} = n - 1\} + \alpha(n + 1) \Pr\{\tilde{n} = n + 1\}, \quad (\text{A16})$$

for $n = 0, 1, \dots, n^*$, with $\Pr\{\tilde{n} = -1\} = 0 = \Pr\{\tilde{n} = n^* + 1\}$ and

$$1 = \sum_{n=0}^{n^*} \Pr\{\tilde{n} = n\}. \quad (\text{A17})$$

Together Equations (A16) and (A17) have the unique solution: $\Pr\{\tilde{n} = n\} = v_n / \sum_{n=0}^{n^*} v_n$, with

$$v_n = \frac{\zeta(n^* - n + 1)}{\alpha n} \frac{\zeta(n^* - n + 2)}{\alpha(n - 1)} \dots \frac{\zeta n^*}{\alpha} = \binom{n^*}{n} \left(\frac{\zeta}{\alpha}\right)^n \quad (\text{A18})$$

for $n = 0, 1, \dots, n^*$. Next note that

$$\frac{\zeta}{\alpha + \zeta} = \frac{\eta\iota}{\eta\iota + (1 - \eta)\iota} = \pi, \quad (\text{A19})$$

from the definitions of the constants, ζ and π , above Equations (A15)

and (6), respectively. Now, rewrite Equation (A18) as follows:

$$v_n = (1 - \pi)^{-n^*} \binom{n^*}{n} \pi^n (1 - \pi)^{n^* - n}, \quad (\text{A20})$$

for $n = 0, 1, \dots, n^*$. The desired result in Equation (13) follows from Equation (A20) since $\sum_{n=0}^{n^*} v_n = (1 - \pi)^{-n^*}$. This completes the proof of Proposition 1.

Characterization of partial equilibrium in steady state

Drop the competitive entry condition for brokers in Equation (4) and replicate the derivation of Equations (6) through (10) and (12). In this case, the endogenous commission rate, b^* in Equation (6), is replaced by the exogenous constant b , while the average number of assets per broker, $\bar{n}^*$ in Equation (7), and the total time spent working, w^* in Equation (10), are replaced by $N(b)$ and $W(b)$, respectively. In this partial equilibrium the necessary optimality conditions are Equations (8), (9), and

$$N(b) \equiv E(\tilde{n}) = \pi \frac{W(b)}{x^*}, \quad (\text{A21})$$

$$j_{in}^* = W(b) - nx^*, \quad \theta^* H'[W(b)] = \frac{\eta b}{1 - \eta} \frac{r^*}{x^*}, \quad (\text{A22})$$

$$V(b, n) = \frac{1}{l} \{ W(b) \theta^* H'[W(b)] - \theta^* H[W(b)] \} + n b r^*, \quad (\text{A23})$$

for $i = 1, \dots, m$; $j = 1, \dots, n$; and $n = 0, 1, \dots, [\frac{W(b)}{x^*}]$. The unique solution to Equations (8) and (9) is independent of the exogenous commission rate b in partial equilibrium for a large market in steady state. Measured per asset, brokers spend searching for new sellers the average time: $\frac{W(b)}{N(b)} - x^* = \frac{1-\pi}{\pi} x^*$. The latter allocation is also independent of b . In equilibrium the commission rate, b^* in Equation (6), is the unique solution to the competitive entry condition in Equation (4): $V(b^*, 0) = \gamma$.

Proof of Proposition 2. By the argument in Equation (A3), the modified problem I of seller ijn can be rewritten as follows:

$$0 = \max_{r_{ijn} \geq 0} \left\{ (1 - b^{**}) F(x_{ijn}, \bar{x}) M(r_{ijn}) + F(x_{ijn}, \bar{x}) [1 - G(r_{ijn})] [(1 - b^{**}) r_{ijn} - u_n^*] - u_n^* \right\}. \quad (\text{A24})$$

Similarly the modified problem I of broker i can be rewritten as follows:

$$0 = \max_{x_{in}, y_{in}} \left\{ b^{**} \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) M(r_{ijn}^*) - \theta^* H(w_{in}) + \alpha \bar{n}^{**} (y_{in}/\bar{y}^*) \Delta v_n^* + \sum_{j=1}^n F(x_{ijn}, \bar{x}^*) [1 - G(r_{ijn}^*)] (b^{**} r_{ijn}^* - \Delta v_{n-1}^* - v_n^*) \right\}, \quad (\text{A25})$$

subject to $x_{in}, y_{in} \geq 0$.

In steady state the unique solution to Equations (A24) and (A25) uniquely satisfies the necessary optimality conditions:

$$b^{**} = \left(\frac{1-\eta}{\eta\iota} + \frac{1}{\alpha+\iota} \right) \frac{x^{**}}{r^{**}} \theta^* H'(w^*), \quad (\text{A26})$$

$$\lim_{m \rightarrow \infty} \bar{n}^{**} = E(\tilde{n}) = \pi \frac{w^*}{x^{**}}, \quad (\text{A27})$$

$$r_{ijn}^* = r^{**}, \quad \frac{\alpha}{\iota} = \frac{r^{**}[1 - G(r^{**})]}{\int_{r^{**}}^{\infty} [1 - G(p)] dp}, \quad (\text{A28})$$

$$x_{ijn}^* = x^{**}, \quad \alpha = F(x^{**}, x^{**})[1 - G(r^{**})], \quad (\text{A29})$$

$$y_{in}^* = w^* - nx^{**}, \quad \gamma\iota = w^* \theta^* H'(w^*) - \theta^* H(w^*), \quad (\text{A30})$$

$$U(n) = (1 - b^{**})r^{**}, \quad (\text{A31})$$

$$V(n) = \gamma + \left[1 - \frac{\eta\iota}{\alpha(1-\eta) + \iota} \right] nb^{**}r^{**}, \quad (\text{A32})$$

for $i = 1, \dots; j = 1, \dots, n$; and $n = 0, 1, \dots, [\frac{w^*}{x^*}]$. The uniqueness of this solution, Equations (A26) through (A32), is shown by an argument that is almost identical to the argument below Equation (A14) in the proof of Proposition 1.

The inequalities in Equation (16) can be verified as follows. Compare Equations (8) and (A28). Replace $\lambda = \log \frac{\alpha}{\iota} (1 - \eta)$ by $\lambda' = \log \frac{\alpha}{\iota}$ in the argument in Equation (A13). By this argument it follows that $r^* < r^{**}$. In this case, the steady-state conditions in Equations (9) and (A29) require that $x^* > x^{**}$. This completes the proof.

Proof of Proposition 3. By the argument above Equation (A3), the modified problem II of seller ijn with the contractual asking price,

$a_{ijn} > 0$, can be rewritten as follows:

$$\begin{aligned} 0 = & \max_{r_{ijn} \geq a_{ijn}} \{F(x_{ijn}, \bar{x}^*)[M(r_{ijn}) - a_{ijn}M(a_{ijn})] \\ & + F(x_{ijn}, \bar{x}^*)[1 - G(r_{ijn})](r_{ijn} - u_{ijn}^*) \\ & - F(x_{ijn}, \bar{x}^*)[1 - G(a_{ijn})]b^*a_{ijn} - \iota u_{ijn}^*\}. \end{aligned} \quad (A33)$$

Similarly, the modified problem II of broker i can be rewritten as follows:

$$\begin{aligned} 0 = & \max_{x_{in}, y_{in}} \left\{ b^* \sum_{j=1}^n F(x_{ijn}, \bar{x}^*)M(a_{ijn}) - \theta^* H(w_{in}) + \alpha \bar{n}^* (y_{in}/y^*) \Delta v_n^* \right. \\ & + \sum_{j=1}^n \{F(x_{ijn}, \bar{x}^*)[1 - G(a_{ijn})]b^*a_{ijn} - [1 - G(r_{ijn}^*)]\Delta v_n^*\} \\ & \left. - \iota v_n^* \right\}, \end{aligned} \quad (A34)$$

subject to $x_{in}, y_{in} \geq 0$.

If $r_{ijn}^* = a_{ijn} = r^*$ from Equation (8), then the broker's problem in Equation (A34) has the unique solution, Equations (9) and (10). This follows from a minor modification of the proof of Proposition 1. For the desired result, it is then sufficient to show that the asking price, $a_{ijn} = r^*$, induces from the seller's problem in Equation (A33) the unique solution, $r_{ijn}^* = r^*$. For this result it is necessary and sufficient that the maximand in Equation (A33) be decreasing in r_{ijn} for all $r_{ijn} > a_{ijn} = r^*$. From Equation (A33) this holds if and only if $r_{ijn} > u_{ijn}^*$ for all $r_{ijn} > a_{ijn} = r^*$. Equivalently, this holds if and only if $u_{ijn}^* \leq r_{ijn}^* = a_{ijn} = r^*$. Solve Equation (A33) for u_{ijn}^* and apply the right sides of Equations (8) and (9) to the result. This yields $u_{ijn}^* = U(n)$ from Equation (11). Clearly $U(n) \leq r^*$ if and only if the desired inequality in (16) is satisfied.

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