Unconditional and Conditional Takeover Offers: Experimental Evidence

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This article compares the predictions of finite-shareholder models of conditional and unconditional takeover offers with the outcomes of laboratory experiments. In addition to differentiating between types of offers, the experimental designs span small and large firms as well as different levels of offer premiums. It is found that in unconditional offers to large groups of subjects (28–40), the symmetric Nash equilibrium predicts observed tendering frequencies quite accurately. For other experimental designs, the results are mixed. The analysis of shareholder tendering strategies from the experiment yields insights into (i) the effects of takeover offer designs, (ii) the appropriateness of finite-shareholder models for research, and (iii) the costs of free riding when shareholders are nonatomistic.

Consider the problem of a firm with dispersed ownership that has received a takeover offer from a bidder who can improve the operating efficiency of the

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firm. Suppose the bidder makes an unconditional offer at a price that is between the current value of the firm and its value under the bidder's management. A shareholder's tendering decision in this situation will depend both on the value that the bidder adds to the firm and the shareholder's conjecture regarding the tendering decision of other shareholders. If he conjectures that others are not going to tender, implying that the offer will fail, then he is better off tendering his shares. On the other hand, if he conjectures that others are going to tender, he is better off not tendering and, thus, receiving the value under the bidder's management. In this scenario, it is easy to see that neither tendering nor not tendering (henceforth referred to as free riding) is a dominant strategy.

Grossman and Hart (1980) provided an important insight into this problem. They demonstrated that, given atomistic shareholders, no equilibrium exists in which the takeover succeeds. The intuition behind their result is simply that atomistic shareholders recognize that their behavior will have no effect on the outcome of the offer. If an equilibrium existed in which the tender offer succeeds, shareholders would have to be willing to tender given that they conjecture that the offer will succeed. However, since the posttakeover value exceeds the price offered by the bidder, this willingness to tender is not rational. More recently, Bagnoli and Lipman (1988), Bebchuk (1989), and Holmström and Nalebuff (1992), have relaxed the atomistic shareholder assumption of Grossman and Hart. They have shown that when the number of shareholders is finite, equilibria exist in which takeovers succeed and bidders earn a positive surplus.[1,2] The first two articles also show that, in a symmetric equilibrium, the Grossman and Hart result is approached if the number of shareholders is increased while the total number of shares remains fixed. If, on the other hand, the number of shares is also allowed to increase at a sufficiently fast rate, the bidder surplus does not fall to zero.[3]

The contribution of this article is in investigating the issues surrounding the free-rider problem in takeovers in an experimental setting.[4] The takeover setting is particularly amenable to experimental

[1] Several authors have enriched the basic Grossman and Hart model and shown that takeovers can succeed even in the presence of the free-rider problem. See Shleifer and Vishny (1986), Fishman (1988), Harris and Raviv (1988), Hirshleifer and Titman (1990), and Scharfstein (1988).

[2] The empirical study by Bradley, Desai, and Kim (1983) found positive takeover gains. See also Walkling (1985) for an empirical investigation into the determinants of takeover premiums.

[3] Determining the effective number of shareholders is a fairly complex issue. Bagnoli and Lipman (1988) point out that there may be a large number of shareholders in the firm who act atomistically. The shareholders may attempt to free ride in a manner consistent with Grossman and Hart (1980). If, however, there are sufficient but finite numbers of large block shareholders, then the outcome will be determined by the actions of these shareholders.

[4] Palfrey and Rosenthal (1990) conduct experiments on provision of public goods and the free-rider problem therein.

tests because the structure of the game is simple and, therefore, does not require a high level of sophistication on the part of subjects. Furthermore, since even in real financial markets it is unlikely that a shareholder will make a tendering decision in a takeover scenario very often, the experience differential between experimental subjects and real market participants may not be significant.

The experiment spans four scenarios: (i) unconditional any-and-all offers to a firm with a small number (five) of shareholders; (ii) unconditional any-and-all offers to a firm with a large number (28 or more) of shareholders; (iii) conditional offers to a firm with a small number (five) of shareholders; and (iv) conditional offers to a firm with a large number (25 or more) of shareholders. In addition, the takeover premium in each scenario has three possible values: low, medium, and high.

The evidence suggests that despite the presence of some free riding, takeovers do succeed at premiums that are less than the posttakeover value. The other findings are that (i) the tendering probability is not as sensitive to the offer premium as theory would suggest, (ii) bidder profit is strictly positive in all the experiments except one, (iii) free riding does not always increase when the number of shareholders is increased, (iv) for larger groups, the theory predicts outcomes in any and all offers better than in conditional offers, and (v) the theory predicts shareholder behavior better for offers that are small relative to takeover gains.

In any-and-all offers to small groups, there is a noteworthy deviation from the theoretical prediction. It is observed that shareholders demonstrate an excessive tendency to play a high-bid contrarian strategy of tendering more at the medium than at the high offer price. The theory would expect them to tender more as the offer price is raised. A possible explanation for this deviation is that, at the higher offer price, shareholders were underestimating free riding by others.[5] For large groups, the assumption that shareholders play symmetric tendering strategies also appears to be violated. The predictions of the symmetric Nash equilibrium regarding aggregate tendering probabilities of the symmetric equilibrium are, however, robust to this deviation. In contrast, for conditional offers, the theory was unable to explain most of the experimental findings.

The findings in this article have a number of implications for future research as well as for corporate and public policy. First, the fact that the symmetric Nash equilibrium predicts takeover outcomes with a

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[5] Palfrey and Rosenthal (1990) argue that deviations from equilibrium can be explained with a model that allows for a systematic underestimation of free riding.

fairly high degree of regularity for any-and-all offers should improve researchers' confidence in finite-shareholder models. Second, many models of shareholder behavior in the corporate control literature have been developed in the Grossman and Hart (1980) framework of atomistic shareholders. The success of the finite-shareholder model in predicting takeover outcomes implies that extensions of such models using the finite-shareholder paradigm would be useful in increasing the understanding of shareholder behavior in corporate control contests.

From the perspective of corporate policy, the most important implication from the experiment is that the outcome in an unconditional any-and-all takeover offer is more predictable and is characterized by larger takeover gains to the bidder, particularly when the number of shareholders is large.

Given the assumption that the posttakeover value is higher than the pretakeover value, social gains are maximized if takeovers always succeed. The basic insight provided for public policy is that while takeovers can succeed at prices below the posttakeover value without exclusionary provisions, they do not always succeed. In contrast to the Grossman and Hart (1980) result, our result implies that, even in the absence of private benefits to the bidder, bidders may undertake socially advantageous takeovers at prices that yield them positive profits. However, the nonsymmetric equilibria that would result in the socially optimum outcome is for exactly the majority of shareholders to tender every time, implying that the takeover always succeeds. This outcome is never observed in the experiments. Thus, while the free-riding problem is not as serious as indicated in Grossman and Hart (1980), exclusionary rules may still increase social welfare.

The article is organized as follows. The next section presents the derivation of equilibrium shareholder tendering strategies in any-and-all and conditional offers when the number of shareholders is finite. Section 2 describes the experimental setting, and the results from the experiments are presented in Section 3. Section 4 concludes the article.

1. Equilibrium Shareholder Tendering Strategies

Consider a simple setting in which there are N shareholders, each of whom owns one share of stock in the firm. The value of each share of the firm under current management is v_0. Assume that the firm receives a tender offer from a bidder at a bid price of b. If the bidder takes control of the firm, the value of each share will equal v_1, where $v_1 > b > v_0$. Assume that K shares are required in order for the bidder to gain control of the firm. The shareholder's problem is to determine the probability with which he will tender his share to the bidder. Then, in an any-and-all tender offer, as proved in Bagnoli and Lipman [1988, Equation (1), p. 96], for a fixed b, there is a unique $\gamma_a^* \in (0, 1)$ that supports a mixed strategy equilibrium in which all shareholders tender with probability γ_a^*, and γ_a^* is implicitly defined as the solution to the following equation:

$$b = v_o \left\{ \sum_{j=0}^{K-1} \binom{N-1}{j} \gamma_a^j (1-\gamma_a)^{N-j} \right\} + v_1 \left\{ \sum_{j=K}^{N} \binom{N-1}{j} \gamma_a^* (1-\gamma_a)^{N-j} \right\}. \tag{1}$$

A conditional offer is one in which the bidder offers to buy tendered shares only if at least K, the number of shares required for the offer to succeed, shares are tendered. Therefore, if a shareholder tenders his share, he receives b if the offer succeeds and v_0 if the offer fails. If he free rides, his payoff is v_1 if the offer succeeds and v_0 otherwise. In the case of a conditional tender offer, as shown in Holmström and Nalebuff (1992), a perfect symmetric equilibrium exists in which all shareholders tender with probability γ_c^*. The determination of γ_c^* is as follows: $\gamma_c^* = \beta^*/(1-\beta^*)$, where β^* is the unique solution to the polynomial equation

$$(v_1 - b) \sum_{j=K}^{N-1} \binom{N-1}{j} \beta^{j-(K-1)} - (b - v_0) \binom{N-1}{K-1} = 0.^6 \tag{2}$$

In a conditional offer, a shareholder gains from tendering only when he is the marginal shareholder who ensures that the offer succeeds. In the symmetric equilibrium in which the takeover succeeds with positive probability, shareholders must have an incentive to tender. This implies that as the number of shareholders increases, each shareholder remains marginal. If he did not, the incentive to tender would vanish. Thus, success in conditional offers must be "knife-edged" in that deviations by even a single shareholder must have a significant impact on the outcome. On the other hand, in symmetric equilibria for unconditional offers, shareholders need not be marginal to gain from tendering. They also gain when they are submarginal, for in this case, the offer fails and they receive the bid price rather

[6] Another Nash equilibrium exists—a free-riding equilibrium where no shareholders tender. A number of authors have noted that with a conditional tender offer to a continuum of shareholders, this free-riding outcome is a dominant strategy for each shareholder [see, for example, Spatt (1989)]. In our finite shareholder case, however, this equilibrium is not normal-form perfect.

than the smaller share value in the absence of a takeover. Thus, for unconditional offers, the per capita variance of tendering strategies does not approach zero, which implies that, in the limit, deviations from the equilibrium by a single shareholder have no impact on the probability of success [see Noe (1995)].

2. Experimental Setting

The experiments were conducted on groups of graduate business students attending Georgia State University. The experiments, the number of treatments, and group sizes for each treatment were as follows: (i) any-and-all offers to small groups were made to six groups each consisting of five subjects; (ii) any-and-all offers to large groups were made to three groups consisting of 38, 28, and 40 subjects, respectively; (iii) conditional offers to small groups were made to five groups each consisting of five subjects; and (iv) conditional offers to large groups were made to three groups consisting of 41, 25, and 37 subjects, respectively. An individual subject participated in one and only one treatment and, thus, the well-known complexities associated with experimental sequencing effects were not an issue.

At the start of each session, in keeping with standard experimental practice, subjects were told that the purpose of the experiment was to test theories of economic decision making. They were also informed that they would earn actual dollar payoffs based on their performance and that this expense would not be borne by the experimenters but would be financed by a grant from the university. Next, a detailed explanation of the decision that the subjects had to make was provided. In order to mitigate any value biases, the explanation did not use any terms or words that would indicate that the experiment was about takeovers. The entire experiment usually lasted approximately two hours, of which the first 20 minutes were spent describing the game, the payoffs, and the procedures to be followed. The next 10 minutes were spent answering questions. Each group participated in no more than 30 rounds, each of which lasted approximately three minutes. To facilitate recall, the payoff matrix relevant to each round and the number of shareholders constituting a majority were displayed during the entire course of each round.[7]

In the first experiment, the subjects were told that each of them was a member of a group of five subjects (a firm with a small number of shareholders). The identity of the other group members was not

[7] Later discussion with subjects confirmed that none of the students had guessed the purpose of the experiment correctly.

Table 1
Any-and-all offer payoffs to subjects

	Majority decision	
Subject's decision	Accept	Reject
Accept	X francs	X francs
Reject	100 francs	0 francs

revealed to the subject. The subjects were informed that the composition of their groups would remain unchanged over the duration of the experiment.[8] They were told that in each round of the experiment an offer of X francs would be made and that the subject could either accept it or reject it. The subjects were informed that the value of X, the offer price, would vary between rounds. The payoffs (in keeping with an any-and-all offer) were described as follows: (i) if the subject rejected the offer and the majority in the subject's group (at least three out of the remaining four in the case of small groups) accepted the offer, the subject's payoff would be 100 francs; (ii) if the subject rejected the offer and so did the majority in the respective group, the payoff would be zero francs; (iii) if the subject accepted the offer, the payoff would be X francs regardless of the majority decision. The second experiment, conducted on different groups of subjects, investigated any-and-all offers to a firm with a large number of shareholders. Table 1 summarizes the payoff structure in any-and-all offers.

The third and fourth experiments dealt with conditional offers to firms with small and large numbers of shareholders, respectively. The only difference between these designs and the any-and-all designs was in the payoff structure. The payoffs in this design, in keeping with a conditional offer, were as follows: (i) if the subject rejected the offer and the majority accepted it, the payoff would be 100 francs; (ii) if the subject rejected the offer and so did the majority, the payoff would be zero francs; (iii) if the subject accepted the offer and the majority accepted the offer, the payoff would be X francs; and (iv) if the subject accepted the offer and the majority rejected the offer, the payoff would be zero francs. Table 2 summarizes the payoffs in any-and-all and conditional offers.

In each of the experiments, X, the offer price, was varied between 25, 50, and 75. The offer was repeated approximately 10 times at each

[8] Group composition was unchanged in each of the experiments for the entire duration. The fact that groups are fixed and that this fact is known to the subjects may appear to invite coordination among subjects even though communication was forbidden and the groups were strictly monitored during the entire period by a team of three investigators. However, the experimental findings to be presented shortly, will show that there is little reason to suspect coordination.

Table 2
Conditional offer payoffs to subjects

	Majority decision	
Subject's decision	Accept	Reject
Accept	X francs	0 francs
Reject	100 francs	0 francs

of these offer prices. Thus, each subject participated in a maximum of 30 rounds. However, the subjects were not told how many rounds of the experiment would be conducted. The following sequence of offer prices was maintained in each experiment: 25, 50, 75, 25, 50, 75, 25, 50, 75, . . . , 25, 50, 75.[9] At the completion of each round, the outcome was tabulated and communicated to the subjects. This communication informed the participants of the number of subjects who accepted the offer in the round and whether the offer succeeded or failed. The subjects were also provided with a record-keeping sheet on which they could maintain information regarding the history of bidding in the past rounds. After every few rounds, the subjects were given time to reflect on the history of the bidding.

Since the purpose of the experiment was to model a simultaneous move game, maintaining the secrecy of their responses was necessary. In order to ensure confidentiality in the communication of the subject's decision, each subject was given a private binary code, known only to the subject and the experimenter, which was used to communicate the subject's decisions. To make the code difficult to break, two procedures were followed. First, the code was randomly varied over rounds and, second, the order in which subject responses were obtained was randomly selected.

In order to control for wealth-level effects induced by risk aversion, the Berg et al. (1986) normalization procedure was employed. Each subject's dollar payoff from participation was determined by a lottery in which the subject could win either $10 or nothing.[10] The probability of winning $10 was equal to the ratio of the subject's actual payoff to

[9] Because the order is correlated with the offer price, order effects are a possibility. One possible effect is that the deterministic sequence of offers might facilitate implicit cross-round coordination between agents. Coordination might be facilitated because when agents are aware of the exact size of total payoffs in upcoming rounds, they can more easily implement cross-round strategies requiring them to alternate in providing the necessary shares required for the success of the offer. However, both because of aspects of the design of the experiment described in footnote 8, and because of the statistical evidence presented later, it does not appear that coordination among subjects is an important issue.

[10] The average payoff is smaller than that typically paid in other experiments. The problem being investigated required the participation of a large number of subjects. This together with a tight budget constraint, forced us to employ low per-subject payoffs. Given the evident interest of the students in the experiment, however, it is unlikely that this factor affected experimental outcomes in a significant manner.

the maximum possible subject payoff possible. For example, suppose a subject earned 1,800 francs from the 30 rounds in the experiment. The maximum payoff possible in each round is 100 francs and, thus, the maximum possible payoff per subject in the experiment is 3,000. Then the probability that the subject wins $10 in the lottery would be .60. In order to make the final determination of the outcome of the lottery, a random number from a uniform distribution between zero and one was drawn for each subject. If the random number was less than or equal to the subject's probability, the subject would be paid $10. This payoff scheme was explained in detail to the subjects before the start of the experiment. For details of the experimental design, see the Appendix, which contains the instructions provided to each subject.

3. Results

3.1 Experimental outcomes

In the experiment with any-and-all offers to small groups, there were six groups with five members in each group. The outcomes in the 10 rounds for the three offer prices are presented in panel A of Table 3. The panel provides the number of accept votes for each of the six groups in each of the 10 rounds for each of the three offer prices. Despite the seemingly different behavior patterns across groups, the behavior of the subjects is not necessarily inconsistent with symmetric equilibrium shareholder strategies. Recall that in the symmetric equilibrium shareholders are playing random strategies. Therefore, different realizations across groups and across rounds could be entirely due to chance. In order to investigate this conjecture, the average number of accept votes across groups for a particular offer price was compared. At the 5% significance level, for each of the offer prices, the chi-square test did not reject the hypothesis that the accepting frequency was the same across groups in this treatment.

The results for any-and-all offers to large groups are presented in panel B (Table 3). The experiment was conducted on three groups consisting of 32, 28, and 40 shareholders.[11] The number of shares required for majority in each group were, therefore, 16, 14, and 20, respectively. The interesting observation for large groups is that the any-and-all offers succeed quite frequently, in fact, generally more often than in the small group case. It is also the case, as in the small groups, that the higher the offer price, the higher the chance of success.

[11] For the third group, the experimental session had to be terminated because of time constraints.

Table 3
Results from experiments

Offer prices	Round number 1	2	3	4	5	6	7	8	9	10
Panel A: Any-and-all offer to small (5 shareholders) groups 1–6										
Group 1										
25	**4**	2	**3**	1	**3**	1	**3**	**3**	2	2
50	**4**	**3**	**4**	1	**4**	**3**	1	1	**3**	**4**
75	2	2	**4**	**3**	**4**	**4**	**5**	**4**	**4**	**3**
Group 2										
25	0	**3**	2	1	2	2	2	1	2	2
50	0	1	2	2	**3**	**3**	**3**	**3**	2	2
75	2	**3**	**3**	**4**	**3**	2	**3**	**3**	2	2
Group 3										
25	**5**	2	0	**3**	2	**3**	2	2	**3**	**3**
50	0	2	**5**	1	2	**3**	2	2	**4**	**3**
75	1	2	**3**	**4**	**3**	**3**	2	**3**	**4**	2
Group 4										
25	2	**4**	**4**	**3**	2	2	2	**4**	**3**	1
50	**3**	2	**3**	1	2	**4**	2	2	**5**	**4**
75	1	**4**	**3**	0	2	**3**	**3**	**3**	2	**3**
Group 5										
25	**4**	2	1	0	2	1	**3**	**4**	1	**3**
50	**3**	0	1	**3**	2	**4**	**3**	2	**4**	**4**
75	**4**	**4**	**5**	1	**3**	**4**	**3**	**3**	**3**	**3**
Group 6										
25	2	**3**	**5**	1	2	**3**	**3**	0	1	2
50	2	**5**	2	**3**	2	**4**	1	2	2	2
75	**3**	**3**	2	1	**3**	**3**	2	2	**3**	**3**
Panel B: Any-and-all offers to large groups 1–3 (32, 28, 40 shareholders)										
Group 1										
25	**22**	**17**	**17**	13	12	**16**	13	13	**17**	10
50	**25**	15	**21**	11	**23**	**22**	12	**19**	**20**	15
75	**20**	**18**	**16**	**23**	15	**19**	**16**	**19**	**17**	**17**
Group 2										
25	11	**17**	**16**	10	11	**14**	**17**	4	13	**14**
50	**14**	10	**15**	11	13	**18**	12	**17**	**17**	**14**
75	**16**	**15**	**14**	**16**	**14**	**14**	12	13	**21**	13
Group 3										
25	**26**	15	16	**22**	17	**21**	18			
50	**25**	19	19	16	**30**	**21**	18			
75	15	**26**	**20**	**28**	**20**	**20**	**21**			

Panel C (Table 3) presents the outcomes for conditional offers to firms with a small number of shareholders. In this experiment there were five groups each with five members. Again, the hypothesis that, for each offer price, shareholders in each group had the same tendering frequency could not be rejected at the 5% significance level. Panel D of Table 3 presents the results on firms with a large number of shareholders for conditional offers. The group sizes were 41, 25, and 37. The number of shares constituting a majority were 21, 13, and 19, respectively. Conditional offers to firms with a large number of shareholders failed almost uniformly; at the offer price of 25 all the offers failed, at 50 all the offers except one to the group of 41 failed,

Table 3
(continued)

Offer prices	Round number 1	2	3	4	5	6	7	8	9	10
Panel C: Conditional offer to small (5 shareholders) groups 1–5										
Group 1										
25	**4**	2	2	2	2	2	**3**	1	0	1
50	**5**	2	**4**	**3**	2	**3**	**3**	1	1	2
75	**4**	2	2	**4**	**3**	2	**3**	0	**3**	**3**
Group 2										
25	**5**	1	**4**	2	2	2	1	1	2	1
50	**3**	2	**3**	**3**	**3**	1	**3**	2	**3**	**3**
75	**4**	**3**	**4**	**3**	**3**	1	1	2	1	**3**
Group 3										
25	2	1	**3**	2	1	**3**	1	1	1	**3**
50	**4**	2	**3**	0	2	2	1	1	**4**	1
75	**5**	**3**	**4**	**5**	**3**	2	**3**	**3**	**3**	2
Group 4										
25	**4**	1	**3**	1	1	2	2	2	1	**3**
50	**5**	2	**4**	1	1	2	**4**	**3**	**3**	**3**
75	2	**3**	**4**	1	1	**4**	**4**	**4**	**3**	**3**
Group 5										
25	**3**	**3**	2	**3**	0	2	1	2	1	1
50	**3**	2	**3**	2	2	2	**3**	1	2	2
75	1	1	**3**	**4**	1	**3**	**3**	**3**	**3**	**3**
Panel D: Conditional offer to large groups 1–3 (41, 25, 37 shareholders)										
Group 1										
25	14	15	13	13	10	13	18	13	14	
50	19	13	18	18	20	20	**21**	15	15	
75	**21**	11	16	16	18	19	19	19	19	
Group 2										
25	9	8	8	7	6	10	11	11	8	7
50	8	12	12	10	9	10	12	10	12	10
75	6	8	11	**15**	6	10	9	11	**13**	10
Group 3										
25	12	8	13	10	9	9	8	12		
50	14	13	10	10	14	12	11	12		
75	11	12	14	14	11	9	16	13		

Panel A presents results for any-and-all offers to small (five shareholders) groups, panel B for large groups. Panels C and D present the results for conditional offers to small and large groups, respectively. In rounds in which the offer was successful, the number of shareholders accepting is presented in bold.

and at 75, except for one to the first group and two to the second group, the offers were never successful.

3.2 Experimental results versus theoretical predictions

3.2.1 Any-and-all offers. It is possible to make direct comparisons between the outcomes predicted by the symmetric Nash equilibrium and the outcomes actually observed in the takeover experiments. Specifically, the probabilities of tendering and success, the division of synergistic takeover gains, and the total takeover gain are compared for each experiment with the values predicted by the theory. The outcomes of the tender offers and the division of gains are de-

Table 4
Comparison of theoretically predicted and experimentally observed tendering probabilities

	Tendering probability					
	Offer price = 25		Offer price = 50		Offer price = 75	
	Observed	Theoretical	Observed	Theoretical	Observed	Theoretical
Panel A: Any-and-all offers—small groups (5 shareholders each)						
Group 1	.48	.4563	.56	.6142	.70	.7569
Group 2	.34	.4563	.42*	.6142	.54*	.7569
Group 3	.50	.4563	.48*	.6142	.54*	.7569
Group 4	.54	.4563	.56	.6142	.48*	.7569
Group 5	.42	.4563	.52	.6142	.66	.7569
Group 6	.44	.4563	.50	.6142	.50*	.7569
Sample	.4533	.4563	.5067*	.6142	.5700*	.7569
Panel B: Any-and-all offers—large groups (32, 28, and 40 shareholders)						
Group 1	.4688	.4401	.5719**	.5000	.5625	.5595
Group 2	.4536	.4360	.5036	.5000	.5286	.5640
Group 3	.4821	.4465	.5286	.5000	.5357	.5535
Sample	.4682		.5347		.5423	
Panel C: Conditional offers—small groups (5 shareholders each)						
Group 1	.34	.3101	.52	.5375	.50*	.7290
Group 2	.42	.3101	.52	.5375	.40*	.7290
Group 3	.36	.3101	.40*	.5375	.68	.7290
Group 4	.40	.3101	.56	.5375	.58*	.7290
Group 5	.36	.3101	.44	.5375	.50*	.7290
Sample	.3840**	.3101	.4880	.5375	.5520*	.7290
Panel D: Conditional offers—large group (41, 25, and 37 shareholders)						
Group 1	.3333**	.2129	.4309**	.3627	.4282*	.4869
Group 2	.3400**	.2210	.4200	.3797	.3960*	.5151
Group 3	.2736**	.2142	.3243	.3656	.3378*	.4915
Sample	.3156		.3917		.3873	

*Probability of this proportion or less tendering is less than or equal to 5% in exact tests using binomial distributions.
**Probability of this proportion or more tendering is less than or equal to 5% in exact tests using binomial distributions.

termined entirely by shareholders' probabilities of tendering into the offer. In this section, therefore, first the tendering behavior of shareholders in the experiments is analyzed in detail and, then, results on the division of synergistic takeover gains are presented.

In order to generate the theoretical predictions for each variable, first the theoretical probability of tendering (γ_a^*) is determined by substituting numerical values for the number of shareholders (N), the offer price (b), and the accept votes required for majority (K) relevant for each experiment into Equation (1) and solving numerically. For example, in an any-and-all offer of 25 to small firms, $N = 5$, $b = 25$, and $K = 3$ yields the probability of tendering as .4563, which is reported in the first row of column three in panel A of Table 4.

Panel A presents the comparisons for any-and-all offers to small groups (five shareholders). For each of the three offer prices, 25, 50, and 75, the tendering probability observed in each of the six groups is compared with the predicted probability.[12] At the offer price of 25, there are no significant differences observed between predicted and observed shareholder tendering behavior. The observed tendering probabilities are not different from those predicted for any of the six groups. In addition, the tendering probability of the entire sample comprising all six groups (.4533) is also not significantly different from the predicted value (.4563). Thus, it appears that the theory predicts tendering probability quite accurately for small premium in any-and-all offers to small groups.

At higher offer prices, departures from predicted behavior become apparent. At the offer price of 50, tendering in two of the six groups is significantly lower than that predicted. At the offer price of 75, four of the six groups exhibit significantly lower tendering than theory would predict. For both offer prices, the tendering probabilities for the sample are significantly lower than those predicted. Those groups that had significantly lower tendering probabilities at 50 also tendered significantly less at the offer price of 75. Finally, the prediction that tendering probabilities would increase with the offer price is true for the samples and is usually true for individual groups. However, the sensitivity of the probability of tendering to the offer price is much less in the experiments than predicted by theory.

Panel B (Table 4) presents the comparison of tendering probability for any-and-all offers to large groups. The theory predicts shareholder tendering remarkably well. At offer prices of 25 and 75, the observed tendering probability is not different from that predicted by theory for any of the three groups. At the offer price of 50, however, the observed probability of tendering is significantly greater than that predicted for one of the three groups.

Panel C (Table 4) presents the comparison of tendering probabilities for conditional offers to small groups. At the group level, for offer prices of 25 and 50, except for one group at 50, the observed tendering probabilities are not significantly different from those pre-

[12] The null hypothesis is that the distribution of tendering into the offer is a binomial distribution with success probability equal to the theoretical probability. The number of observations for each offer price is the product of the number of shareholders in a particular group and the number of experiment rounds. For example, for small groups the number of shareholders is five and the number of rounds is 10. Because the binomial distribution is not a continuous distribution, an exact test of significance level α cannot, in general, be specified without resorting to randomized designs. In order to attach statistical significance to departures from predicted tendering probabilities, the following simple heuristic approach is adopted. Those experimental tendering frequencies less than the predicted frequency are marked with an $*$ ($**$) whenever the probability of a frequency that small or smaller (large or larger) is less than 5%. All exact tests of this type were performed using probabilities generated by the software program, Mathematica.

dicted. At the offer price of 75, for groups had significantly deviant tendering behavior. At the sample level, the observed probability was not significantly different from predicted only at the offer price of 50. Shareholders tendered more than predicted at the offer price of 25, less than predicted at 75, and usually less than predicted at 50. At the sample level, at higher offer prices, shareholders tendered more.

The results for conditional offers to large groups are presented in panel D (Table 4). Significant deviation from predicted tendering behavior is observed at offer prices of 25 and 75 in all three groups. At the offer price of 25, shareholders tendered significantly more than predicted, while at 75 they tendered significantly less. In addition, contrary to the prediction, the observed tendering probability for two of the three groups was lower at the offer price of 75 than at 50. In fact, the tendering probability for the sample was higher at 50 than at the offer price of 75.

In general, it appears that any-and-all offers, particularly to large groups, follow theoretical predictions more closely than conditional offers. Even for small groups, except for the offer price of 75 to small groups, observed tendering probabilities are almost never significantly different from those predicted by the theory. Conditional offers to small groups follow theory except at the 75 offer price. The most significant deviations from theory predictions are observed for conditional offers to large groups.

Deviations from theoretically predicted values could be explained in three ways. First, it could be that shareholders were in fact playing symmetric randomized strategies and the deviations were due to abnormal draws from distributions. The large number of significant deviations detected, however, indicates that this is unlikely. Second, it is possible that, in reality, the observed strategies were equilibrium strategies but of a nonsymmetric equilibrium. Observed deviations, therefore, could be due to inappropriate comparisons with the predictions of the symmetric equilibrium. Finally, while shareholders may in fact be playing symmetric strategies, their tendering probability may be affected by systematic biases in their beliefs regarding other players' strategies.

If the observed deviations are due to initially biased conjectures on the part of subjects, some "learning," that is, convergence toward the symmetric Nash equilibrium should be observed in the later rounds of the experiments. Table 5 presents observed tendering probabilities separately for the first five and the later rounds for each of the four experiments. For any-and-all offers to small groups, the observed tendering probabilities at 50 were significantly less than predicted in the first five rounds. In the later rounds, however, the observed tendering probabilities moved closer to the predicted value were not statistically different from the predicted value for any of the six groups. At the offer price of 75, while the observed tendering probability remained statistically lower than predicted for the later five rounds, in absolute terms, it moved closer to the theoretical value. This learning is observed in any-and-all offers to large groups as well. Recall that the only deviation from the theoretical prediction occurred in the first group at the offer price of 50. For this group, the observed tendering probability in the first five rounds was .5938, which is significantly greater than the predicted value of .5. In later rounds, the observed probability decreased to .5556, which was not significantly different from that predicted. Thus, it appears that there was some learning over rounds in the any-and-all experiments.

In conditional offers to small groups, the convergence of tendering probabilities to the predicted values was observed only for the offer price of 25. For offer prices of 50 and 75, the later round tendering probabilities were further away from the theoretical predictions. In fact, at the offer price of 50, the tendering probability for the sample, which was not different from the theoretical prediction in the first five rounds, was significantly lower in the later rounds. Thus, there does not appear to be much evidence in favor of learning over rounds in conditional offers to small groups. In conditional offers to large groups, the findings were again mixed. Over rounds, group 1 moved further away from theory for offer prices of 25 and 50, but moved closer to theoretical prediction for the offer price of 75. The same pattern was observed for the second group. The observed tendering probability of the third group, on the other hand, moved closer to the predicted value at the offer price of 25. For the higher offer prices, there did not appear to be any change in tendering probability over rounds for this group. In general, therefore, the data did not support learning in the case of conditional offers.

The null hypothesis for a symmetric equilibrium is that all shareholders in the group have identical tendering probabilities. The tendering probability was estimated from the data, and the hypothesis that the tendering frequency of all shareholders in that group was generated by this probability is tested. For example, the observed tendering probability for group 1 in any-and-all offers to small groups at the 25 offer price was .48 (Table 4, panel A). This probability value implies that the expected frequency of tendering for a shareholder in group 1 should be 4.8 in the 10 rounds. The test determines whether the observed tendering frequency of the shareholder was significantly different from 4.8.

Three test statistics were employed to measure the goodness of fit: the Pearson's chi-square statistic (PCS), the likelihood ratio statistic

Table 5
Learning over rounds in individual groups and sample

	Tendering probability								
	Raider offer price = 25			Raider offer price = 50			Raider offer price = 75		
	Rounds 0–5	Later rounds	Theory prediction	Rounds 0–5	Later rounds	Theory prediction	Rounds 0–5	Later rounds	Theory prediction
Panel A: Any-and-all offers—small groups (5 shareholders in each)									
Group 1	.52	.44	.4563	.64	.48	.6142	.60	.80	.7569
Group 2	.32	.36	.4563	.32*	.52	.6142	.60	.48*	.7569
Group 3	.48	.52	.4563	.40*	.56	.6142	.52*	.56*	.7569
Group 4	.60	.48	.4563	.44	.64	.6142	.40*	.56*	.7569
Group 5	.36	.48	.4563	.36*	.68	.6142	.68	.64	.7569
Group 6	.52	.36	.4563	.56	.44	.6142	.48*	.52*	.7569
Sample	.467	.44	.4563	.4533*	.56	.6142	.5467*	.5933*	.7569
Panel B: Any-and-all offers—large groups (32, 28, and 40 shareholders)									
Group 1	.5063	.4313	.4401	.5938**	.5556	.5000	.5750	.5556	.5595
Group 2	.4643	.4429	.4360	.4500	.5571	.5000	.5357	.5214	.5640
Group 3	.4800	.4875	.4465	.4050	.4875	.5000	.5450	.5125	.5535
Panel C: Conditional offers—small groups (5 shareholders in each)									
Group 1	.48	.28	.3101	.64	.40	.5375	.56	.44*	.7290
Group 2	.56**	.28	.3101	.56	.48	.5375	.68	.32*	.7290
Group 3	.36	.36	.3101	.64	.36	.5375	.68	.32*	.7290
Group 4	.40	.40	.3101	.52	.60	.5375	.44*	.72	.7290
Group 5	.44	.28	.3101	.48	.40	.5375	.40*	.60	.7290
Sample	.448**	.320	.3101	.528	.448*	.5375	.584*	.520*	.7290
Panel D: Conditional offers—large groups (41, 25, and 37 shareholders)									
Group 1	.3171**	.3537**	.2129	.4293	.4329**	.3627	.4000*	.4634	.4869
Group 2	.3040**	.3760**	.2210	.4080	.4320	.3797	.3680*	.4240*	.5151
Group 3	.2811**	.2613	.2142	.3297	.3153	.3656	.3351*	.3423*	.4915

*Probability of this proportion or less tendering is less than or equal to 5% in exact tests using binomial distributions.

**Probability of this proportion or more tendering is less than or equal to 5% in exact tests using binomial distributions.

(LRS), and the power divergence statistic (PDS).[13] Critical values for the statistics were approximated by using $\chi^2(N-1)$ distributions. When the alternative hypothesis involves a small local deviation from the null, the PCS test is slightly more powerful than the LRS test in small samples. On the other hand, if one or two cells have very small expected frequencies under the alternate hypothesis, the LRS test is more powerful [Read and Cressie (1988), pp. 59–60]. The PDS test is a compromise between the other two statistics.

Next, the possibility of shareholders playing nonsymmetric Nash strategies is investigated. It has been shown that, in all Nash equilibria, all randomizing shareholders use the same randomizing probability. Thus, asymmetric Nash equilibria require some shareholders to play pure strategies [see Holmström and Nalebuff (1992)]. If some shareholders play pure strategies, expected frequencies in some cells will be zero. In this case, the LRS test will be more likely to reject the false null hypothesis of symmetry. On the other hand, if observed deviations are simply the result of some idiosyncratic biases on the part of shareholders, the PCS test will be preferred.

Panel A in Table 6 presents the results from these tests for small groups. The number of small groups, by type of offer and price, in which the null of symmetry was rejected at the 5% significance level are presented. The upper left corner entry in each square presents the number for all the rounds and the bottom right entry for the later five rounds. In any-and-all offers to small groups, the LRS test rarely rejects the null of symmetry. As discussed earlier, this result would indicate that the likelihood of shareholders playing strategies from an alternative equilibrium is low. The PCS and the PDS tests reject the null more often, which suggests that the correct hypothesis may be that the actual strategies played are small deviations from symmetric strategies. The inference for conditional offers to small groups is somewhat harder to draw because all three tests usually reject the null, although the LRS test is less likely to reject the null. Comparison of the findings using PCS and PDS for the first five rounds with those for the later rounds reveals that, in general, the degree of symmetry tends to increase in later rounds. LRS is not defined for the later rounds because only one shareholder played a pure strategy over these rounds. Given this, the finding of increased symmetry over rounds is even more surprising.

For large groups, in results not reported, all three tests always rejected the null. A perusal of the numbers of shareholders playing pure strategies, reported in panel B (Table 6), is instructive. Compared to

[13] See Read and Cressie (1988) for a description of these test statistics and a comparison of their properties.

Table 6
Tests for symmetry of tendering strategies

	Offer price—any-and-all						Offer price–conditional					
	25		50		75		25		50		75	
Panel A: Small groups												
PCS	2/6		4/6		1/6		3/5		3/5		5/5	
		1/6		1/6		1/6		3/5		1/5		2/5
LRS	1/6		1/6		0/6		2/5		1/5		4/5	
		nd		nd		nd		nd		nd		nd
PDS	2/6		4/6		1/6		3/5		3/5		4/5	
		1/6		1/6		1/6		3/5		1/5		2/5
No. of constant	1/30		1/30		0/30		2/25		1/25		6/25	
actors		2/30		2/30		2/30		3/25		4/25		10/25
(small												
groups)		8/30		9/30		12/30		11/25		9/25		14/25
Panel B: Large groups												
No. of constant	6/32		6/32		10/32		11/41		12/41		10/41	
actors		8/32		6/12		11/32		16/41		16/41		20/41
(large												
group 1)		11/32		12/32		17/32		26/41		24/41		28/41
No. of constant	4/28		3/28		7/28		7/25		4/25		6/25	
actors		8/28		4/28		10/28		10/25		5/25		6/25
(large												
group 2)		7/28		7/28		8/28		16/25		15/25		14/25
No. of constant	3/40		3/40		5/40		11/37		13/37		12/37	
actors		17/40		22/40		16/40		18/37		21/37		23/37
(large												
group 3)		9/40		7/40		15/40		26/37		24/37		22/37

The first panel of the table presents the results for small groups. The first three rows present the number of small groups out of the total number for which the hypothesis of symmetric tendering strategies was rejected by the Pearson's chi-square statistic (PCS), the likelihood ratio statistic (LRS), and the power divergence statistic (PDS). The fourth row in this panel presents the number of shareholders displaying constant action choices out of the total number of shareholders in the experiment for small groups. Panel B presents the number of shareholders displaying constant action choices in large groups. Entries in the upper left corner of each box are based on all the rounds of the experiment, whereas entries in the right corner are for the first (upper value) and last five (lower value) rounds.

small groups, the fraction of shareholders playing pure strategies is much larger. This fact, together with the earlier finding that overall tendering probabilities in large groups are reasonably close to predicted ones, suggests that those shareholders not playing pure strategies adjust their tendering to offset the effects of agents playing pure strategies.

Further, the proportion of observed pure strategies is almost always greater in conditional offers than in any-and-all offers. This finding is not inconsistent with the earlier inferences regarding small groups using goodness-of-fit tests that the symmetry hypothesis is more likely to be rejected for conditional offers. Additionally, for both unconditional and conditional offers, the number of pure strategies in the later rounds is usually greater than the number of subjects who played pure strategies in the earlier rounds.[14] A possible explanation for this regularity, consistent with the literature on "learning" in games, is as follows [see, for example, Young (1993)]. Shareholders play the same strategy whenever that strategy is the unique best reply to "historical" strategies, that is, the strategies played by the other players in the previous rounds. They change their strategy when another strategy would yield a strict improvement in their payoffs if played against the historical strategies. When all strategies yield the same payoff against historical strategies, the players generally stick with the strategy that they played in the previous round. In conditional offers, whenever the number of shares tendered is less by two or more than the majority required for success, all strategies for any individual shareholder yield the same payoff against the group strategy vector. Thus, after a sequence of offer failures, shareholders, basing their response on the historical tendering pattern, see no reason to change their tendering strategy. This leads to strategic inertia, which may explain the observed pure strategy adoption in conditional offers.

For unconditional tender offers, while the rate of pure strategy adoption is generally lower than that for conditional offers, it increases more from early to later rounds in small groups. This might be explained by the fact that, because of accommodating strategy adjustments by other players in their tendering probabilities in the early rounds, some shareholders may find either tendering or not tendering to be a unique best response. This leads them to stick to their strategy rather than randomize over both strategies.

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[14] Note that in the cases of large group 3 for unconditional offers and large groups 1 and 3 for conditional offers, the number of earlier rounds is 2, 4, and 3, respectively. In these cases, simple comparisons of proportions between pure strategies in earlier versus later rounds would be misleading. The reason is that even if all subjects utilized stationary tendering probabilities over all rounds, chance alone would imply that one is more likely to observe the same tendering decision over three rounds than over five rounds.

An alternative to the symmetric Nash equilibrium hypothesis is that the shareholders may be playing a purely random strategy (PRS) of tendering with probability of .5 in all experiments at all offer prices. In this case, the Nash model will do better than the PRS when the Nash-predicted tendering probability is lower (greater) than .5 and the observed probability is lower (greater) than the Nash-predicted probability. Likelihood ratio tests comparing the Nash and PRS models show that at the offer price of 25, the PRS model is always rejected. At the offer price of 75, the Nash model is rejected for unconditional offers and conditional offers to small groups. At the offer price of 50, the Nash model is rejected for unconditional and conditional offers to small groups. In the case of large groups, the predicted probabilities are identical under Nash and PRS for unconditional groups, and for conditional offers, the PRS model is always rejected.

Further, there is no evidence in the data that subjects were able to coordinate to the joint payoff maximizing symmetric strategy vectors. We computed the symmetric joint payoff maximizing vectors for all experimental designs.[15] These strategy vectors call for significantly higher tendering probabilities than the symmetric Nash strategy vectors because the tendering by any one shareholder benefits all other shareholders. In an unconditional offer, it benefits nontendering shareholders as it increases the probability of the offer succeeding. In conditional offers, even tendering shareholders benefit because it increases the likelihood that their shares will be purchased. In all cases, the symmetric payoff maximizing tendering probability was significantly higher than the observed tendering probability. This is especially true in the case of conditional offer. For example, for conditional 75 offer to large group 2, the Nash-predicted tendering probability is .5151 and the observed tendering probability is .3960. The symmetric joint payoff maximizing tendering probability is .7229. Thus, observed tendering probabilities provide no support for the hypothesis that shareholders utilized the repeated nature of the experimental design to coordinate to non-Nash joint payoff maximizing strategies in the individual rounds.

Thus far the investigation has been able to shed light on the deviations from the predicted equilibrium. Shareholder tendering probabilities in any-and-all offers are reasonably consistent with theory for both large and small groups. In conditional offers, on the other hand, the discrepancies are significant. For both types of offers, however, the tests thus far have not been able to explain the insensitivity of

[15] These computations were performed in Mathematica, and the programs used in these computations are available upon request.

tendering probabilities to increases in the offer price from 50 to 75. In fact, for large groups, the shareholders tender less at the offer price of 75 than at 50. To investigate this anomaly, observed shareholder tendering patterns across offer prices are identified and compared with the pattern distribution generated by the symmetric equilibria in Table 7.[16]

Table 7 describes the tendering strategies used by shareholders in the four experiments for the three offer prices. In order to determine an individual's strategy for a particular offer price, the number of times (T_i) a shareholder tendered at an offer price i ($i = 25, 50, 75$) was computed for each shareholder at each offer price in all four experiments. The first column of Table 7 lists the nine collectively exhaustive combinations of shareholder tendering strategies for different offer prices. For example, the first, $T_{25} > T_{50} > T_{75}$, is the strategy in which the shareholder tendered less frequently at higher offer prices. Columns titled frequency present the number of shareholders who followed a particular strategy. Numbers in the upper left-hand corner of the square are for the first five rounds and those in the bottom right-hand corner are for the later rounds. For example, only one person out of the total of 30 followed the strategy $T_{25} > T_{50} > T_{75}$ in the any-and-all offer experiment with small firms in the first five rounds. The expected frequency generated by the symmetric equilibrium in each of the four cases is presented in the column adjacent to the observed frequency column.

For any-and-all offers to small firms, the most frequent strategy is $T_{25} < T_{50} > T_{75}$, that is, one in which shareholders tendered most when the offer price was 50. The strategy that is predicted by the theory to be most likely, $T_{25} < T_{50} < T_{75}$, that is, shareholders should tender more often at higher offer prices, was followed by relatively few shareholders. The Pearson's chi-square and the power divergence tests reject the null hypothesis that the expected cell frequencies equal the observed cell frequencies only in the earlier rounds. The likelihood ratio test, however, does not reject this null in the earlier rounds and is undefined for the later rounds.

In the case of any-and-all offers to large groups, it is difficult to make general inferences because each of the three groups exhibits different patterns of tendering. In the first group, the null hypothesis, that observed cell frequencies correspond to theoretical predictions, cannot be rejected for the first five rounds. In the second group, the null is not rejected either for the first five or for the later rounds. In

[16] In some cases, the predicted frequency of a cell is less than one, which would bias all of these goodness-of-fit tests toward rejecting the null.

Table 7
Comparison of observed and predicted shareholder voting strategies

	Any-and-all								Conditional							
	Small Groups				Large Groups				Small Groups				Large Groups			
Strategy	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.
$T_{25} >$	1	.014	3	1.16	3	0.94	6	1.61	2	0.02	2	0.26	2	0.12	1	0.25
$T_{50} >$																
T_{75}	0	0.23	1	1.07	2	0.88	5	1.44	0	0.07	1	0.36	2	0.19	0	0.32
$T_{25} <$	1	12.13	4	5.85	5	5.39	7	6.90	8	13.97	8	14.25	3	9.97	2	11.96
$T_{50} <$																
T_{75}	2	7.20	4	3.79	2	3.45	6	4.49	6	8.62	6	9.00	3	5.89	2	8.22
$T_{25} =$	1	0.42	4	0.99	1	0.84	1	1.27	2	0.13	8	0.81	1	0.36	12	0.88
$T_{50} =$																
T_{75}	3	1.33	10	2.05	2	1.77	6	2.60	5	0.65	15	2.03	14	1.14	18	1.80
$T_{25} >$	6	4.75	5	7.51	5	6.51	3	9.52	2	2.41	3	5.91	0	3.40	3	5.42
$T_{50} <$																
T_{75}	6	5.18	3	6.84	2	5.95	6	8.61	2	3.20	1	6.19	1	3.73	3	5.57
$T_{25} <$	7	4.76	10	7.52	5	6.51	6	9.52	5	3.07	8	8.10	5	4.62	3	7.33
$T_{50} >$																
T_{75}	6	5.04	6	6.84	4	5.95	5	8.61	3	3.75	3	8.15	2	4.82	4	7.32
$T_{25} =$	5	3.42	2	3.20	3	2.85	5	3.89	1	2.40	4	5.25	4	2.98	3	5.02
$T_{50} <$																
T_{75}	4	4.50	3	3.83	4	3.41	3	4.67	3	3.86	2	6.79	0	4.14	3	6.13
$T_{25} =$	1	0.28	1	1.27	2	1.05	3	1.70	2	0.06	3	0.55	3	0.24	2	0.58
$T_{50} >$																
T_{75}	2	0.69	2	1.87	5	1.58	2	2.46	2	0.26	2	1.07	1	0.59	1	0.95
$T_{25} >$	2	0.28	1	1.27	0	1.05	4	1.70	1	0.06	3	0.51	2	0.22	3	0.53

Table 7
(continued)

	Any-and-all								Conditional							
	Small Groups				Large Groups				Small Groups				Large Groups			
Strategy	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.	Obs. freq.	Pred. freq.
$T_{50} =$																
T_{75}	2	0.67	0	1.87	2	1.58	3	2.46	1	0.25	3	1.03	1	0.56	2	0.91
$T_{25} <$	6	3.82	2	3.20	4	2.85	5	3.89	2	2.88	2	5.35	5	3.09	8	5.04
$T_{50} =$																
T_{75}	5	5.15	3	3.83	5	3.44	4	4.67	3	4.35	6	6.38	1	3.94	4	5.77
PCS	32*		15		8		23*		305*		105*		87*		172*	
LRS	12		5		na		8		18*		20*		na		30*	
PDS	29*		13		7		21*		133*		76*		60*		118*	
	12		35*		14		17*		45*		128*		174*		157*	
	nd		nd		5		6		nd		27*		nd		nd	
	11		29*		13		15		33*		96*		119*		111*	
Obs.	30		32		28		40		25		41		25		37	

In the table, T_i, $i = 25, 50, 75$, represents the observed probability of a shareholder tendering at the offer price i. The strategies listed in the first column of the table represent different patterns of shareholder tendering probabilities. Thus, for example, $T_{25} > T_{50} > T_{75}$ denotes the strategy of tendering more frequently as the offer price is lowered. Entries in the upper left corner of each box are based on all the rounds of the experiment, whereas entries in the bottom right corner are for the last five rounds.

*Significant at the 0.05 level using the chi-square approximation. The first test statistic (PCS) is the Pearson's chi-square, LRS is the likelihood ratio statistic, and the third statistic is the power divergence statistic.

Table 8
Comparison of shareholder conditional voting strategies by bid price for small and large groups in any-and-all and conditional offers

Any-and-all

Bid price	Small groups Prob [tender success]	Small groups Prob [tender failure]	Large group 1 Prob [tender success]	Large group 1 Prob [tender failure]	Large group 2 Prob [tender success]	Large group 2 Prob [tender failure]	Large group 3 Prob [tender success]	Large group 3 Prob [tender failure]
\$25	.4455	.4375	.4375	.4531	.3333	.6161	.6251	.3688
\$50	.4667	.5600	.4948[b]	.6563[b]	.4763	.5625	.7250	.4063
\$75	.6111	.5333	.5508	.5938	.5833	.4048	.4542	nd[c]

Conditional

Bid price	Small groups Prob [tender success]	Small groups Prob [tender failure]	Large group 1 Prob [tender success]	Large group 1 Prob [tender failure]	Large group 2 Prob [tender success]	Large group 2 Prob [tender failure]	Large group 3 Prob [tender success]	Large group 3 Prob [tender failure]
\$25	.2800	.3657	nd[a]	.3323	nd[a]	.3378	nd[a]	.2664
\$50	.4190	.4833	.3659	.4355	nd[a]	.4311	nd[a]	.3166
\$75	.5667	.5667	.2683[c]	.4390[c]	.3200	.4400	nd[a]	.3437

[a]Not defined because a conditional offer at \$25 was never successful.
[b]Significantly different at the 5% level from each other using a Pearson's chi-square test.
[c]Not defined because an offer at \$75 never failed in the round used in the computation of the conditional probability.

the third group, while both PCS and PDS reject the null in the earlier rounds, in the latter rounds the null is either not rejected or just barely rejected.

For conditional offers, irrespective of the group size, the null hypothesis is rejected by all three statistics. The results from the above analysis indicate that the model does not predict the distribution of strategies as well as it did tendering frequencies. Nevertheless, they lend further support to the conjecture that the theory's predictions of shareholder responses are more accurate for any-and-all offers than for conditional offers.

The above analysis investigated symmetry within groups and tendering distributions across offer prices. Next, tendering behavior across rounds in investigated by presenting the probabilities after a success and after a failure in the previous round. Table 8 presents these findings. Little evidence is found for conditioning tendering probabilities on previous round outcomes in either conditional or unconditional offers to small groups or to large groups. The only exception is in unconditional offers to large group 1 where, at the 50 offer price, the tendering probability following a failure was significantly higher than that following a success.[17]

[17] Since ordering of offer prices over rounds is deterministic, conditioning effects observed may be affected by order effects.

The overall inference so far is as follows. For any-and-all offers to small and to large groups, once learning is taken into account, the theory is a powerful predictor of tendering probabilities. Some deviations from theoretically predicted tendering distributions are, however, evident. In small groups, an excessive tendency to play a high-bid contrarian strategy ($T_{25} < T_{50} > T_{75}$) was observed. A possible explanation for this finding is that at the higher offer of 75, shareholders were conjecturing that all others were going to tender and hence decided to free ride themselves.

As far as conditional offers are concerned, the theory was unable to explain most of the experimental findings. While the actual explanations may be quite complex, we conjecture two important factors that may account for the discrepancies. First, if a shareholder simply looks at the payoffs from tendering and not tendering in a conditional offer, it appears that free riding is a dominant strategy. The naive argument for this conjecture is that if the offer fails the shareholder gets nothing regardless of whether he tenders. On the other hand, if the offer succeeds, he makes more if he free rides. This argument ignores the state of the world in which the shareholder, by tendering, changes the outcome from failure to success. Casual observation of shareholders during the experiment suggested that this may indeed have affected the tendering behavior of some of the shareholders. Furthermore, as shown earlier, the outcomes of a conditional tender offer are very sensitive to small changes in the tendering probability of even one shareholder. These factors together may have made the relationship between payoffs and tendering strategies too erratic for any learning to occur.

3.2.2 Takeover outcomes and division of synergistic gains. The distribution of the tendering frequencies analyzed above determines (i) the probability of success of the takeover, (ii) the expected bidder profit, (iii) the expected shareholder profit, and (iv) the expected total synergistic gain from the takeover. Using Equation (1) the value of γ_a^* for the probability of tendering in any-and-all offer can be computed numerically. The probability of success of the takeover, that is, when at least K out of N shares are tendered, is given by the binomial formula,

$$\sum_{i=K}^{N} \binom{N}{i} \gamma_a^* (1-\gamma_a^*)^{N-i}.$$

The expected profit to the bidder is given by

$$(v_0 - b) \sum_{i=0}^{K-1} \binom{N}{i} \gamma_a^* i (1-\gamma_a^*)^{N-i} + (v_1 - b) \sum_{i=K}^{N} \binom{N}{i} \gamma_a^* i (1-\gamma_a^*)^{N-i},$$

where the first term represents the bidder's loss when the offer fails, when he must pay b for shares that are worth only v_0. For example, when $i < K$ shares are tendered, his loss is $(b - v_0)i$. The second term represents the bidder's gain when the offer is successful. In this case, the profit per share purchased is $v_1 - b$. Shareholders' gain is the difference between the total takeover gain and the bidder's profit. The total takeover gain is given by

$$N(v_1 - v_0) \sum_{i=k}^{N} \binom{N}{i} \gamma_a^* i (1 - \gamma_a^*)^{N-i}.$$

Table 9 presents the comparison of theoretical predictions with experimental findings for these variables for any-and-all offers to small (panel A) and the three large groups (panels B, C, and D), respectively. In both small and large groups, the probability of success, as predicted, is increasing in the offer price. In small groups, bidder profit per share is predicted to increase when the offer price is raised from 25 to 50 and to decrease when it is raised to 75. Experimentally, however, the bidder profit declined for both increases in the offer price. The same phenomenon was observed also in large groups. Theory predicted that bidder profit would be highest for the 50 offer price. The observed bidder profit, however, declined with the offer price.

As in the case of any-and-all offers, first the theoretical probability of tendering (γ_c^*) is determined by substituting numerical values for number of shareholders (N), offer price (b), and accept votes required for majority (K) into Equation (2) and solving numerically. For example, in any-and-all 25 offers to small firms, $N = 5$, $b = 25$, and $K = 3$ yields the probability of tendering as .3101 (which is reported in Table 7, panel C). Using this value of γ_c^*, the probability of success is given by the binomial formula

$$\sum_{i=K}^{N} \binom{N}{i} \gamma_c^* (1 - \gamma_c^*)^{N-i}.$$

Next, the expected profit to the bidder is given by

$$(v_1 - b) \sum_{i=K}^{N} \binom{N}{i} \gamma_c^* i (1 - \gamma_c^*)^{N-i}.$$

In conditional offers, since the bidder is not obligated to purchase any shares if the offer fails, he purchases shares only if the offer succeeds. For example, when $i > K$ shares are tendered, his profit is $(v_1 - b)i$. Shareholders' gain is the difference between the total takeover gain

Table 9
Comparison of experimental evidence with theory predictions for unconditional any-and-all offers.

	Raider bids					
	25		50		75	
Variables	Value in experiment	Theory prediction	Value in experiment	Theory prediction	Value in experiment	Theory prediction
Panel A: Small groups (5 shareholders each)						
Prob. of success	.4000	.4185	.4833	.7069	.6833	.9037
Raider profit	16.00	16.88	9.33	20.69	3.58	15.13
Shareholder profit	24.00	25.00	39.00	50.00	64.75	75
Total takeover gain	40.00	41.85	48.33	70.69	68.33	90.13
Panel B: Large group 1 (32 shareholders)						
Prob. of success	.5000	.3055	.6000	.5700	.9000	.8055
Raider profit	16.09	5.55	12.03	7.00	9.38	5.55
Shareholder profit	33.91	25.00	47.97	50.00	80.62	75.00
Total takeover gain	50.00	30.55	60.00	57.00	90.00	80.55
Panel C: Large group 2 (28 shareholders)						
Prob. of success	.5000	.3094	.6000	.5747	.7000	.8092
Raider profit	16.52	5.94	8.75	7.47	−0.36	5.92
Shareholder profit	33.48	25.00	51.25	50.00	70.36	75.00
Total takeover gain	50.00	30.94	60.00	57.47	70.00	80.92
Panel D: Large group 3 (40 shareholders)						
Prob. of success	.4286	.2998	.4286	.5627	.8571	.7998
Raider profit	12.59	4.98	0.72	6.27	8.04	4.98
Shareholder profit	30.27	25.00	42.14	50.00	77.68	75.00
Total takeover gain	42.86	29.98	42.86	56.27	85.71	79.98

Raider, shareholder, and total profits are per share.

and the bidder's profit. The total takeover gain is given by

$$N(v_1 - v_0) \sum_{i=K}^{N} \binom{N}{i} \gamma_c^* (1 - \gamma_c^*)^{N-i}.$$

Table 10 presents the comparison of theoretical predictions with experimental findings for these variables for conditional offers to small (panel A) and large (panels B, C, and D) groups, respectively. In offers to small groups, bidder profit is predicted to be highest for the 50 offer price. As in unconditional offers to small groups, the profit of the bidder, however, was observed to decline with the offer price. For conditional offers to large groups, bidder profit was predicted to increase with the offer price. Experimental findings, however, do not exhibit any consistent pattern. In large group 1, bidder profit increases when the offer price is raised from 25 to 50 and then decreases when the offer price is raised to 75. In large group 2, bidder profit is maximized at the offer price 75. In group 3, since no offer ever succeeded, the bidder profit was zero for all offer prices. In both small and large groups, the probability of success does not increase as predicted when the offer price is raised from 50 to 75. For example, in the case of large group 1, when the offer price is 75, the tendering probability observed (.4282) is less than that predicted probability (.4869), but the difference is not substantial. The difference in the probability of success between theory and that observed is, however, considerable (.1111 observed versus .4328 compared). In other words, almost four out of the nine offers should have been successful, but only one was successful in the experiment. This finding is consistent with the knife-edge nature of the conditional offer equilibrium discussed earlier.

4. Discussion and Concluding Remarks

In this article, predictions from the extant models of the behavior of finite shareholder groups faced with a takeover offer are tested in an experimental setting. The findings from this study are that (i) observed tendering probabilities for unconditional offers are more closely aligned with the predictions of the symmetric Nash equilibrium than those for conditional offers; (ii) the observed tendering probability is not as sensitive to the offer price as theory predicts; (iii) equilibrium tendering probabilities in unconditional offers are closer to the symmetric Nash predictions for larger groups; (iv) equilibrium tendering probabilities in unconditional offers are closer to symmetric Nash predictions at lower offer prices; (v) for conditional offers, the observed tendering probabilities are significantly at variance with those predicted by theory; (vi) as predicted by theory, observed tendering probabilities are generally lower in conditional offers than in unconditional offers; and (vii) the number of shareholders playing pure strategies is considerably greater than what theory would predict, and this number is higher in later rounds of the experiments.

The findings of this study have the following implications. First, as long as the computational demands on the part of players are not too great, the symmetric Nash equilibrium predicts aggregate tendering

Table 10
Comparison of experimental evidence with theory predictions for conditional offers

	Raider bids					
	25		50		75	
Variables	Value in experiment	Theory prediction	Value in experiment	Theory prediction	Value in experiment	Theory prediction
Panel A: Small groups (5 shareholders each)						
Prob. of success	.4000	.4185	.4833	.7069	.6833	.9037
Raider profit	16.00	16.85	9.33	20.69	3.58	15.13
Shareholder profit	24.00	25.00	39.00	50.00	64.75	75.00
Total take-over gain	40.00	41.85	48.33	70.69	68.33	90.13
Panel B: Large group 1 (41 shareholders)						
Prob. of success	.0000	.0000	.1111	.0356	.1111	.4328
Raider profit	0.00	0.00	2.85	0.95	1.42	6.04
Shareholder profit	0.00	0.00	8.26	2.61	9.69	37.24
Total take-over gain	0.00	0.00	11.11	3.56	11.11	43.28
Panel C: Large group 2 (25 shareholders)						
Prob. of success	.0000	.0010	.0000	.1087	.2000	.5606
Raider profit	0.00	0.04	0.00	2.99	2.80	8.21
Shareholder profit	0.00	0.06	0.00	7.88	17.20	47.85
Total take-over gain	0.00	0.10	0.00	10.87	20.00	56.06
Panel D: Large group 3 (37 shareholders)						
Prob. of success	.0000	.0000	.0000	.0468	.0000	.4585
Raider profit	0.00	0.00	0.00	1.25	0.00	6.46
Shareholder profit	0.00	0.00	0.00	3.43	0.00	39.39
Total take-over gain	0.00	0.00	0.00	4.68	0.00	45.85

Raider, shareholder, and total profits are per share.

probabilities fairly successfully. Second, consistent with the previous observation, bidders are able to earn positive profits from takeover attempts even in the absence of any private benefits, exclusionary rights, or preexisting ownership of shares. Third, when the number of shareholders is large, conditional offers may be less attractive to both bidders and shareholders, and therefore to society, than any-and-all offers. The experimental evidence suggests that the shareholders and the bidder are both better off with any-and-all offers.

Appendix: Player Instructions for Any-and-All Offers to Small Groups[18]

You are about to participate in an experiment in economic decision making. The experiment will be repeated several times. In each repetition or "round" of the experiment, you will choose an action that, together with the actions of other students, will determine the outcome and your payoff for that round. The outcome of each round and your payoff in it will be announced at the end of each round. The sum of your payoffs in the rounds will determine your actual dollar payoff at the end of the experiment. Your winnings will be paid out of a grant we have received from the Georgia State University Foundation.

Students in your class are divided into groups of five. You and four other students in the class constitute one of these groups. You will be informed of the number of the group you belong to. The identity of the other members of your group will not be known to you. These groups were formed completely randomly, that is, the grouping was not the result of any individual or other characteristics.

The outcome and the individual payoffs in each round will be determined by the actions chosen by members in your group. The outcome and the individual payoffs in each round depend *only* on the actions chosen by group members in that round. Individual payoffs in each round will be denominated in francs. The total number of francs you earn in all the rounds will determine your real dollar payoff at the end of the experiment. The calculation of the real dollar payoff is explained later.

Here is what you will have to do in each round. In each round you will be made an offer of X francs and you have to choose between two actions: accept or reject. Your payoffs from choosing an action are given in Table 11. After the announcement of X, you will have to decide quickly (in approximately 30 seconds) to either accept or reject the offer. Because X will always be less than 100, if the Majority in your group accept the offer, your payoff is higher if your decision is reject. On the other hand, since X will also always be greater than zero, if the majority in your group reject the offer, your payoff is higher if your decision is accept.

At the end of the minute, you will be asked to announce your choice using your personal secret code, which will be provided to you on a separate sheet. This code will require you to answer either "zero" or "one" depending on your choice and your personal secret code. The codes will not be the same for all students and, therefore, you will be unable to determine the choices made by others from their

[18] Player instructions for other experiments are similar except for the offer terms and group sizes.

Table 11

If you choose	And the majority, that is, three or more out of five, in your group choose	Then, your payoff will be
Accept	Accept	X francs[a]
Accept	Reject	X francs
Reject	Accept	100 francs
Reject	Reject	0 francs

[a]The value of X will be announced at the beginning of each round.

zero or one responses. It is absolutely essential that you respond only in code; that is, *do not say accept or reject, say zero or one* according to your personal secret code.

You will be provided with a sheet that will enable to you keep track of your payoffs in each round. Your real payoff will be determined as follows. Find your total payoff by summing your payoffs. Then calculate the following ratio:

$$Q = \frac{\text{Your total payoff}}{100 \times \text{number of rounds}}.$$

For example, if your total payoff from 10 rounds is 700 francs, then your Q is 0.7. This Q is the probability of your winning a prize of \$10. At the end of the session, we will calculate the Q for each one of you. Suppose your Q is 0.7; we will draw a random number between zero and one. If the number drawn is less than or equal to 0.7, then you will get the prize of \$10. The number drawn is greater than 0.7, you will get nothing. Thus, if you win 100 francs in every round, you are sure to get \$10. Similarly, if you get zero francs in every round, you are sure to get nothing.

Once the experiment begins, remember the following:

(i) Do not talk to others in the class.
(ii) Always announce your action choice in the zero-one code.
(iii) Make sure that your personal secret code does not become known to others in any way.
(iv) Try to ensure that you will not need to leave the room until the exercise is over.

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