Inferring Future Volatility from the Information in Implied Volatility in Eurodollar Options: A New Approach

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We study the information content of implied volatility from several volatility specifications of the Heath–Jarrow–Morton (1992) (HJM) models relative to popular historical volatility models in the Eurodollar options market. The implied volatility from the HJM models explains much of the variation of realized interest rate volatility over both daily and monthly horizons. The implied volatility dominates the GARCH terms, the Glosten et al. (1993) type asymmetric volatility terms, and the interest rate level. However, it cannot explain that the impact of interest rate shocks on the volatility is lower when interest rates are low than when they are high.

Recent studies on the information content of implied volatility yield disappointing results for implied volatility models [e.g., Canina and Figlewski (1992), Chiras and Manaster (1974), Day and Lewis (1992), and Lamoureux and Lastrapes (1993)]. For example, Canina

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and Figlewski (1993) report that "implied volatility has virtually no correlation with future volatility, and it does not incorporate the information contained in recent observed volatility." In Day and Lewis (1992), the coefficient of the implied volatility is barely significant in the volatility forecast equation. These results contrast with a large body of academic literature that views implied volatility as the "market's volatility forecast" [see Schwert (1991)] and superior to historical data-based forecasts. Since most of our option pricing models depend only on the volatility of the underlying process, the lack of information content of implied volatility questions their validity.

In this article we study the information content of implied volatility by correcting for several biases in previous studies. First, existing studies are based on either OEX index options or individual stock options for which the transaction costs for arbitrage are nontrivial. As Canina and Figlewski (1993) argue, high transaction costs may permit market prices to deviate significantly from theoretical prices. Correspondingly, implied volatilities from OEX and individual stock options are not very informative with respect to future volatility. Therefore this issue needs to be studied in a liquid market with low transaction costs. We choose the Eurodollar futures options market. This market is dominated by sophisticated institutional investors, and the trading volume is more than twice that in any other options market (based on both the number of contracts and underlying notional value). Further, since they are futures options, Eurodollar options carry very low transaction costs for arbitrage activity.

Second, we offer some methodological improvements to the tests used in previous studies by Day and Lewis (1992) and Lamoureux and Lastrapes (1994). These studies embed the implied volatility as an exogenous term in a GARCH variance equation. However, this method implicitly includes an entire lag structure of the implied volatility through the GARCH persistence terms. This structure is apparent if we rewrite the model in a distributed lag form through recursive substitution of the lagged conditional volatility. It biases both implied volatility significance and the GARCH persistence parameter. We redesign the test equation to correct this bias. We also control for fat tails in the distribution for interest rate shocks.

Third, to avoid model misspecification, we study the implied volatility from several interest rate option models for pricing Eurodollar futures options. Previous studies have used only the Black–Scholes model with a lognormal distribution for the underlying price. We embed five different interest rate models within the Heath–Jarrow–Morton (1992) (HJM) framework to price interest rate claims. The HJM framework is based on the absence of arbitrage and requires only the initial term structure and the volatility of forward interest rates as inputs. Since these volatilities are the only unobservable parameters, we can use these models to infer the interest rate volatilities from the market prices of interest rate claims. We can then compare the forecast ability of our implied volatility to historic volatility models.

We focus on the volatility of the shortest maturity interest rate available from Eurodollar futures prices, hereafter called the *spot interest rate*. To estimate its implied volatility, we incorporate the following models in the HJM framework: Courtadon (1982), Cox, Ingersoll, and Ross (1985), (CIR), Ho and Lee (1986), Vasicek (1977), and a linear proportional volatility model studied by Amin and Morton (1994). From these models we estimate the implied volatility parameters every day. Using these implied parameters, we compute the implied volatility of the short-term Eurodollar rate from each interest rate model. We then compare the information content of the implied volatility from each model relative to historical volatility benchmarks. This approach lends insight into the validity of the HJM models and into the functional form of the volatility process.

Our historical volatility forecasts are based on both volatility persistence and interest rate levels. Since a large body of academic literature has demonstrated that economic time series exhibit volatility persistence [see the survey by Bollerslev, Chou, and Kroner (1992)], models incorporating this persistence constitute natural benchmarks to estimate volatility forecasts from time-series data. We use Bollerslev's (1986) GARCH model and its extension by Glosten, Jagannathan, and Runkle (1994), henceforth GJR, as two of our historical benchmarks. The GJR model permits an asymmetric volatility response to positive and negative shocks. This asymmetry is important in previous studies using equity data [for example, GJR and Nelson (1991)]. The GARCH and GJR models are preferable to rolling constant volatility estimates for estimating historical interest rate volatility because they optimally incorporate information decay over time through higher weighting of recent shocks.

We also include all the volatility specifications used in each of the five models used to estimate the implied volatility in our historical volatility forecasts. In addition to the GARCH and GJR forecasts, historical volatility estimates are computed separately for the CIR (1985) and Courtadon (1982) volatility specifications. In these models, the volatility depends on the interest rate level. Since we focus only on the volatility of the spot interest rate, the Ho and Lee (1986), Vasicek (1977), and the linear proportional model are special cases of the GARCH and Courtadon (1982) models for computing the historical volatility forecasts.[1]

[1] But this conclusion is not true for the implied volatility models. Since we will use options of different maturities, each of the five volatility models will yield a different implied volatility estimate for the spot rate.

Fourth, in the historical volatility forecasts, we also incorporate an interaction effect between the level and shocks to the interest rate identified by Brenner, Harjes, and Kroner (1993). Interestingly, these authors find that the volatility sensitivity to past shocks depends on interest rate levels. Koedijk et al. (1994) confirm this interaction with a different sample and a different model specification. Ignoring this effect biases the sensitivity to past shocks upwards. Furthermore, our study can offer a more powerful test of this phenomenon because we also utilize option price data.

Most of our tests are based on single-period volatility forecasts over a daily horizon. Finally, as a robustness check, we test the ability of implied volatility to explain realized volatility over monthly horizons relative to historical forecasts. We follow the regression approach of Canina and Figlewski (1993) by comparing future realized volatility over the following month with historical and implied volatility forecasts.

A brief description of the article is as follows. In the first section we describe our dataset. In Section 2, we describe the HJM (1992) approach to the valuation of Eurodollar futures options and the methodology used to estimate the term structure of forward rates and the implied volatilities. We also present various plausible forms for the volatility function. In Section 3 we estimate the historical volatility models from the time series of interest rates. In Section 4 we describe the methodology for studying the information content and forecast ability of implied volatility relative to historical volatility. We present both the standard approach used by Day and Lewis (1992) and our new approach that corrects for the potential biases under the standard approach. In Section 5 we present results comparing historical forecasts with implied volatility forecasts of single-period realized volatility. We also examine the robustness of the single period results by comparing the multiperiod forecast ability of implied volatility relative to historical volatility estimates over monthly horizons. Section 6 concludes the article.

1. Data Description: Eurodollar Futures and Options

Eurodollar futures trade with maturities up to 10 years. Almost all the activity is in contracts that expire on the second business day before the third Wednesday of March, June, September, and December. There is significant liquidity (average trading volume greater than 1000 contracts per day) in contracts with maturity up to 5 years. At maturity date T, the futures settlement price $F_T(T)$ is

$$F_T(T) = 100[1 - y(T)], \tag{1}$$

where $y(T)$ is the 3-month LIBOR[2] interest rate at date T. The notional amount for each futures contract is \$1 million. Since each contract covers a 3-month rate (0.25 years), each 0.0001 change in the interest rate corresponds to a \$25 (= 0.0001 × 0.25 × 1,000,000) change in the futures price.

Based on Equation (1), each futures contract yields information about a 3-month forward rate beginning from the maturity of the futures contract to 3 months after maturity. This 3-month period is called the accrual period of the futures contract. Since there exists a Eurodollar futures contract with maturity every 3 months, the entire term structure of forward rates can be estimated using the "strip" of Eurodollar futures prices. We assume that the instantaneous forward interest rate curve is constant to the end of the accrual period of the first futures contract and between futures maturities after that. In this study we focus on the shortest maturity Eurodollar interest rate available from the futures prices. We term this rate, which is the forward rate over the accrual period of the first futures contract, the "spot rate."

The options on this contract are American, with the same maturity date as the futures. Upon exercise the cash flow to a call option equals the difference between the current futures price and the exercise price. The owner of a put option receives the difference between the exercise price and the current futures price. Most traded options have a maturity of less than 1 year.

We use the dataset in Amin and Morton (1994). The dataset covers the period from January 1, 1988–November 10, 1992. It includes the last traded price for all Eurodollar futures and futures options contracts as of 8:30 A.M. CST (approximately an hour after trading opens). The futures and options prices are therefore sampled intraday and are based on transaction prices. Thus they are not subject to the asynchronous trading problems of prior studies using closing prices. For summary statistics on the dataset, see Amin and Morton (1994).

2. The HJM Framework

2.1 The basic model

In this section we describe the HJM class of models. Let $f(t, T)$ be the forward interest rate at date t for instantaneous and riskless borrowing or lending at date T. The spot interest rate at date t is given by $r(t) = f(t, t)$. At each trading date t, HJM specify the simultaneous

[2] London interbank offered rate. This is the offered rate on 3-month deposits in the London interbank market.

evolution of forward interest rates of every maturity T, according to the stochastic differential equation:

$$df(t,T) = a(t,T,.)\,dt + \sigma(t,T,f(t,T))\,dW(t), \tag{2}$$

where $a(t,T,.)$ and $\sigma(t,T,f(t,T))$ are the drift and dispersion coefficients for the forward interest rate of maturity T and $W(t)$ is a one-dimensional standard Brownian motion. Under complete markets, HJM show the existence of a "risk-neutral" pricing measure under which the price of every security discounted by the spot interest rate is a martingale. To illustrate, fix some final horizon date, S. Let $P(t,T)$ be the price of a discount bond at date t that matures at date T. Define

$$Z(t,T) = P(t,T)\exp\left[-\int_0^t r(u)\,du\right] \qquad \text{for } t \le T \le S. \tag{3}$$

For every discount bond of maturity $T \le S$, $Z(t,T)$ [for $0 \le t \le T$] is a martingale under the risk-neutral measure. Under this measure we can price every security as if investors are risk neutral. Therefore, the security price equals the expectation of its value at any future date discounted back to today using the spot interest rate.

Applying the martingale condition to discount bond prices yields the drift coefficient for each maturity, T:

$$\alpha(t,T,.) = \sigma(t,T,f(t,T))\int_t^T \sigma(t,u,f(t,u)\,du. \tag{4}$$

The function $\sigma(t,T,f(t,T))$, which is the instantaneous standard deviation of the forward interest rate of maturity T at date t, constitutes the parameter input(s) to the model. Specifying $\sigma(.)$ completely specifies the model since $\alpha(.)$ is then determined from Equation (4). If $\sigma(.)$ is a constant (as a function of both time and maturity), Equation (1) reduces to the Ho and Lee (1986) model. Furthermore, by appropriately specifying $\sigma(.)$, we can treat many spot rate models as special cases. For example, if the volatility of forward interest rates is an exponential function of time to maturity, the process for the spot interest rate is the same as assumed by Vasicek (1977). If $\sigma(t,T,f(t,T)) = \sigma f(t,T)^{1/2}$, then the spot interest rate process is virtually identical to that in CIR. Thus, we refer to this specification as the *CIR model*. Similarly, we refer to $\sigma(t,T,f(t,T)) = \sigma f(t,T)$ as the *Courtadon* (1982) *model*.

Consider the pricing of futures. Although futures prices do not equal forward prices if interest rates are stochastic [Jarrow and Oldfield (1981)], futures can be valued like any other contingent claim. The initial investment in a futures contract is zero. Therefore the expected change in the futures price under a risk-neutral measure must be zero.

Let $E_t[.]$ denote the expectation with respect to the risk-neutral measure conditional on the information set at date t. If the futures price at date t for maturity T is $F_T(t)$, and $F_T(T)$ is the terminal futures (and spot) price at maturity,

$$0 = E_t[F_T(T) - F_T(t)].$$

In other words,

$$F_T(t) = E_t[F_T(T)]. \tag{5}$$

Correspondingly, the futures price for a continuously marked-to-market futures contract is a martingale under the risk-neutral measure. Given the terminal spot price distribution under the risk-neutral measure, we can use Equation (5) to determine any prior futures price.

Similarly, a European option with a payoff $C(T)$ at date T can be valued using the equation

$$C(t) = E_t\left[\exp\left[-\int_t^T r(u)\,du\right] C(T)\right]. \tag{6}$$

Finally, American-style claims can be valued using a modified version of Equation (6) which accounts for early exercise. In practice, we can compute futures options prices using a path-dependent binomial-style model by discretizing Equations (2) and (4) under the risk-neutral measure. For details, see Amin and Morton (1994).

2.2 HJM volatility functions

To minimize the possibility of model misspecification, we study five volatility models. In terms of Equation (1), these can be stated as

1. Ho and Lee (1986) model: $\sigma(t, T, f(t, T)) = \sigma_0$.
2. CIR (1985): $\sigma(t, T, f(t, T)) = \sigma_0 f(t, T)^{1/2}$.
3. Courtadon (1982): $\sigma(t, T, f(t, T)) = \sigma_0 f(t, T)$.
4. Vasicek (1977): $\sigma(t, T, f(t, T)) = \sigma_0 \exp(-\lambda(T - t))$.
5. Linear Proportional (HJM): $\sigma(t, T, f(t, T)) = [\sigma_0 + \sigma_1(T - t)] f(t, T)$.

Specification (3) above is identical to the "Black model" (1976) used by practitioners for pricing interest rate caps and floors [see Brace, Gatarek, and Musiela (1995) for details]. Furthermore, since Eurodollar futures options are based on forward interest rates with maturity up to 1 year, specifications (4) and (5) permit the volatility of forward rates with maturity between 0 and 1 year to have different volatilities. However, the specific parameterizations force a unique maturity structure. By testing models that permit different maturity structures of forward rate volatilities, we can make inferences about these maturity structures.

The five separate specifications can be embedded in a unique volatility specification that nests the functions. However, since we estimate parameters each day using only the small number of available option prices, this encompassing specification yields unstable parameter estimates [see Amin and Morton (1994)]. Unlike an estimation exercise using historical interest rate changes, which uses a long time series, the implied estimation procedure uses only the small number of options traded on each date. Therefore, we study the volatility functions separately.

In this article we consider only single-factor HJM models, that is, models with a single Brownian motion (shock) in Equation (2) to determine the evolution of forward rates. We have also implemented a two-factor model. But the parameter estimates in a two-factor model are unstable because futures options do not contain sufficient information to infer the correlation structure embedded in a multiple-factor model. Even with over-the-counter caps and floors,[3] this estimation exercise is fruitless. Since we use a single-factor model to determine implied volatilities, we focus our attention on the historical volatility of a single rate, taken as the shortest maturity interest rate obtained from the Eurodollar futures rates. We term this rate the spot interest rate.

2.3 Estimating the term structure of forward rates and volatility parameters

Since forward rates do not equal futures rates with stochastic interest rates, we cannot estimate forward rates directly from the futures prices (rates). For each day in the data sample, we estimate the term structure of forward interest rates up to the maturity of the longest maturity option, using the futures prices and the previous day's parameter estimates to satisfy Equations (1) and (5). Under our assumptions, the term structure is unique and reprices all the futures exactly under each term-structure model. Since the forward-futures rate mapping is model dependent, the estimated daily term structure is different for each model. However, in practice, all models yield estimates within 1/100 of a basis point (10^{-6}) for the spot rate.

After estimating the forward rates, we use the interest rate model to estimate the parameters. Specifically, we minimize the sum of the squares of the errors between model and market prices of all available

[3] Caps and floors are essentially linear combinations of Eurodollar futures options and have much longer maturities; they are significantly less liquid than Eurodollar options.

options. Model option prices are computed using Equation (6) by discretizing Equations (2) and (4) on a path-dependent tree with 10 time steps. Repeating this process for each model, for each day in the sample period, yields a daily time series of implied parameter estimates and implied volatilities. For additional details on this estimation, see Amin and Morton (1994).

Before analyzing the implied volatility series from each model, we must interpret the different implied volatility parameters for each date, given our model assumption that volatility parameters are constant. There are two interpretations to reconcile this apparent inconsistency. First, the daily implied volatility estimation may reflect estimation errors that lead to different values. By estimating unknown parameters on several different days, we can construct reliable statistical measures. Note that even with constant parameters, the volatility need not be constant since it can depend on the spot rate level. Therefore, the forecasting exercise is still informative.

A second, more plausible interpretation is that volatility varies with both interest rate levels and time. However, the implied volatility captures the market's volatility forecast during the remaining life of the option. As Amin and Morton (1994) argue, "prices in stochastic volatility models are of similar form to those in a constant volatility model, with volatility terms in the latter replaced by their conditional expected levels in the stochastic volatility environment."

Note that the bias introduced by using a model with deterministic volatility parameters from which daily implied volatilities are computed is very small relative to a model that explicitly accounts for the stochastic nature of volatility. See Lamoureux and Lastrapes (1994) for details.

2.4 HJM implied volatility

We compute the implied volatility of the spot interest rate as follows. First, we compute implied parameter values from each model, as described in Section 2.3. Second, using the functional form of each volatility specification, we compute the daily implied volatility of the spot interest rate, $\sigma(t, t, f(t, t))$, for each model. For example, for the CIR specification, the implied volatility of the spot interest rate is

$$\sigma(t, t, f(t, t)) = \sigma_0 f(t, t)^{1/2}, \tag{7}$$

where we substitute the implied parameter estimate for σ_0 in the above equation. We then use the implied spot rate volatility from each model in our analysis.

3. Time-Series Models of Volatility

In this section we study the volatility forecasts estimated using only historical spot interest rates. In Section 4 we will compare these forecasts with implied volatility forecasts.

Figure 1 provides a model-independent overview of the data. In Figure 1 we plot the daily interest rate level, the daily interest rate change (shock), and the realized 20-day moving average of daily squared spot interest rate changes. Standardizing (for comparison), we subtract the sample mean for each series and then divide by the sample standard deviation. We then translate the standardized daily shocks series by 10 for graphic convenience. The moving average of the daily squared spot interest rate changes proxies for the realized volatility of interest rate changes.

From Figure 1 several patterns are apparent. First, volatility clustering (persistence) is more prevalent during a high interest rate period (1989). Indeed, the large shocks in 1991 and 1992, when interest rates were low, did not significantly increase volatility. Second, interest rate shocks occur at all periods and levels, even though they are more fre-

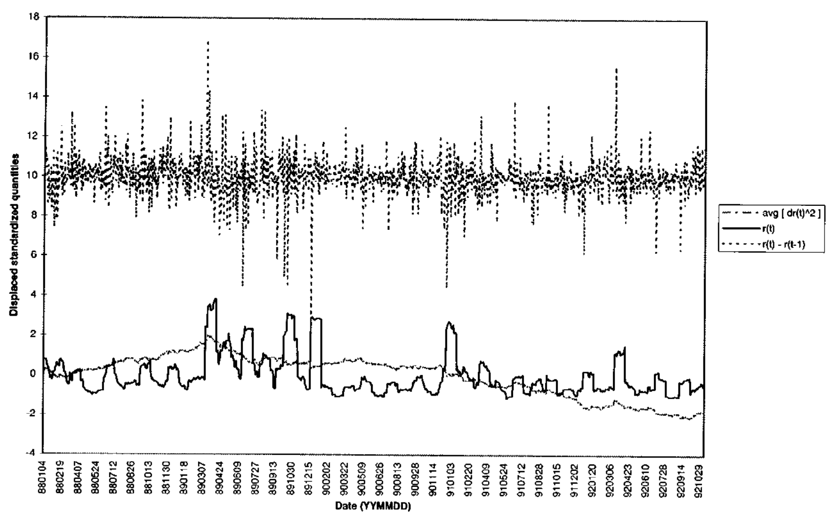

Figure 1
Daily time series of rate, rate change, and rolling standard deviation of rate changes
This figure plots the z-values for the three series represented by the daily spot Eurodollar interest rate level [$r(t)$] implied from the first Eurodollar contract, the daily change in the spot interest rate [interest rate shock $= r(t) - r(t-1)$], and the realized 20-day moving average of the daily squared spot interest rate changes (marked as avg[$dr(t)^2$] on the graph). The z-values are obtained for each series by subtracting the sample mean and dividing by the sample standard deviation. The data period is January 1, 1989–November 10, 1992.

quent during high interest rate periods. These observations affect the interest rate levels and GARCH terms in our tests.

Before applying our formal statistical tests, we must adjust spot interest rates for the effect of futures expiration dates. On the expiration date we switch from the expiring contract to the next maturity contract to estimate the spot interest rate. This contract switching leads to spurious shocks to the "spot" rate. With an upward sloping yield curve we would experience a spurious increase in rates the day after a futures expiration date.

Let $r(t)$ be the spot interest rate at date t and $r^{\text{obs}}(t)$ be the interest rate observed from the first futures contract each day. We assume these two rates are related by

$$r^{\text{obs}}(t) = r(t) + q\,Switch(t)\,YCslope(t-1), \tag{8}$$

where q is the regression coefficient, $Switch(t)$ is the indicator function for a contract switching date, and $YCslope(t-1)$ is the slope of the term structure on date $t-1$. The 1 year/1 month forward rate proxies for the slope. We then assume a simple mean reversion model for the spot rate:

$$r(t) - r(t-1) = a + br(t-1) + \varepsilon(t), \tag{9}$$

where $\varepsilon(t)$ is the shock to the interest rate at date t. Based on Equations (8) and (9), we derive the observed spot rate process:

$$\begin{aligned} r^{\text{obs}}(t) - r^{\text{obs}}(t-1) \;=\; & a + br^{\text{obs}}(t-1) + q\,Switch(t)\,YCslope(t-1) \\ & - (1+b)q\,Switch(t-1)\,YCslope(t-2) \\ & + \varepsilon(t). \end{aligned} \tag{10}$$

By estimating q in Equation (10), we can construct $r(t)$ from $r^{\text{obs}}(t)$ in Equation (8). This estimation also yields the time-series estimates for the interest rate shocks, $\varepsilon(t)$.

Table 1 provides daily volatility estimates and diagnostic statistics. A significant q parameter indicates a contract switching effect. The Box–Pierce statistic for serial correlation for $\varepsilon(t)$ does not show any evidence of misspecification in the mean equation. However, the same statistic for $\varepsilon^2(t)$ indicates significant serial correlation, which indicates time-varying volatility.

We now consider the historical volatility models. First, we consider GARCH models. These models offer a natural benchmark for assessing the information content of the implied volatility. For examples of these models, see Brenner, Harjes, and Kroner (1993), Engle, Lilian, and Robins (1987), Engle, Ng, and Rothschild (1990), Koedijk, Nissen, and Schotman (1994), and Longstaff and Schwartz (1992).

Table 1
Dynamics of the "spot rate" and adjustment for biases due to contract rollover

Mean equation representing the true interest rate process (r_t):

$$r_t - r_{t-1} = a + br_{t-1} + \varepsilon_t$$

Observed interest rate process after adjusting for the contract rollover effect[a]:

$$r_t^{\text{obs}} = r_t + q\ Switch_t\, YCslope_{t-1}$$

Model for observed interest rate (the estimation is based on this equation):

$$\begin{aligned} r_t^{\text{obs}} - r_{t-1}^{\text{obs}} &= r_t - r_{t-1} + q\ Switch_t\, YCslope_{t-1} - q\ Switch_{t-1}\, YCslope_{t-2} \\ &= a + br_{t-1}^{\text{obs}} + q\ Switch_t\, YCslope_{t-1} - (1+b)q\ Switch_{t-1}\, YCslope_{t-2} + \varepsilon_t \end{aligned}$$

Parameter estimates based on daily interest rates (in percent) from January 1, 1988, to November 10, 1992:

	a	b	q
estimate	−0.0058	0.0004	0.1164
t-statistic	(−0.72)	(0.33)	(3.18)

Summary statistics

	Mean	Std. dev.	Skewness	Kurtosis	$Q(5)$	$Q(10)$
ε_t	0.0000	0.0786	−0.3526	7.7846	12.4	19.7
ε_t^2	0.0062	0.0193	9.9664	134.5258	14.8	25.8

Since the spot interest rate is estimated to be the forward rate implicit in the first Eurodollar futures contract, we need to explicitly correct for expiration day effects when we switch from the expiring contract to the next contract. These two contracts differ in maturity by three months. On expiration dates, we subtract any effect due to the slope of the term-structure. This effect is also estimated along with the mean equation.

The interest rate is in percent and the sample period is from January 1, 1988–November 10, 1993. The t-statistics reported are White heteroskedasticity consistent t-statistics. $Q(n)$ is the nth-order Box–Pierce statistics for serial correlation.

[a] $Switch_t$ is a dummy variable that takes a value of one if a new Eurodollar futures contract is used starting at day t and $YCslope_{t-1}$ is the slope of the yield curve at time $t-1$ (before the switching date). *YCslope* is proxied by the observed spread between the 1-year forward rate and the 1-month forward rate.

Let $h(t)$ be the variance of $r(t) - r(t-1)$ conditional on all information available at time $t-1$. In a typical GARCH(1, 1) model, $h(t)$ is specified as

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1), \tag{11}$$

where ω, β, and α are parameters of the model. In this equation the shock effect, or dependence on past interest rate shocks, contrasts with the "level effect" in many interest rate models (including the CIR, Courtadon, and HJM linear proportional models) where volatility depends only on past interest rate levels.

In addition to the GARCH(1, 1) model, we also consider the GARCH model due to Glosten et al. (1993) (the GJR model), which incorporates an asymmetric volatility response due to positive and negative shocks. Several studies have found that stock return volatility increases more after negative shocks than after positive shocks. This difference may represent involuntary liquidation of positions due to margin calls, stop-loss strategies, or other market frictions following significant price drops. Volatility dependence on the direction of rate moves can also create an asymmetric effect. To incorporate this asymmetry, GJR specify the volatility equation as

$$h(t) = \omega + \beta h(t-1) + \alpha \varepsilon^2(t-1) + \alpha_- \eta^2(t-1), \tag{12}$$

where $\eta(t-1) = \max[0, -\varepsilon(t-1)]$. Here $\eta(t-1)$ represents the asymmetric negative shock to the interest rate. A positive (negative) α_- coefficient implies that the volatility increase is larger (smaller) after a negative shock than after a positive one.

To estimate the volatility equations, we take the estimated residual, $\varepsilon(t)$, from Equation (10) as a predetermined variable and then use maximum likelihood to estimate the variance equation. This procedure follows the two-step estimation approach of Pagan and Schwert (1990) and Gallant, Rossi, and Tauchen (1993).

The diagnostic statistics in Table 1 indicate that the distribution of $\varepsilon(t)$ is fat tailed. Therefore, we assume a t-distribution (versus normal) for $\varepsilon(t)$ for more precise statistical inference [Bollerslev (1987)]. To ensure convergence of the estimates, we assume that the degrees of freedom parameter of the t-distribution is given by a logistic function dependent on a parameter, θ, which is unrestricted. Specifically we assume

$$\text{df } (t\text{-distribution}) = 5 + 30\left[1 - \frac{1}{1 + \exp[-10(\theta - 0.5)]}\right]. \tag{13}$$

This transformation bounds the degrees of freedom of the t-distribution between 5 and 35. The lower bound ensures a finite fourth moment of the t-distribution, while the upper bound ensures that we do not have to compute very high-order factorials when evaluating the gamma function in the t-distribution density. The other parameter values in the logistic function ensure a smooth transition to the upper and lower bounds. These parameter values mainly affect the speed of convergence of the maximum likelihood estimation. When θ is negative, the degree of freedom of the t-distribution is very close to 35, providing a virtually normal distribution. Therefore a significantly positive θ indicates a rejection of normality. In our estimations θ is significantly positive (around 1.8). The corresponding asymptotic t-statistic is always significantly greater than 2.0. For conciseness, these values are not reported.

In Table 2 we report the estimation results for the GARCH and GJR specifications [Equations (11) and (12)] and the level effect models. As expected, the coefficient of $\varepsilon^2(t-1)$ is significant in the GARCH equation. Further, the coefficient of the asymmetric response term in the GJR specification is significantly negative. Therefore interest rate volatility increases more after an interest rate increase than after a decrease. This finding parallels the finding that return volatility increases by a larger amount after negative return shocks than after positive return shocks [Engle and Ng (1993) and Nelson (1991)]. Both results are consistent if viewed in price terms since bond prices decrease as interest rates increase. Therefore price decreases lead to higher volatility in both the stock and fixedincome markets.

To study historical volatility models based on rate levels, we estimate the Courtadon (1982) and CIR (1985) models. The Courtadon (1982) model mimics the linear proportional model if we focus on a single rate. Further, the Ho and Lee (1986) and Vasicek (1977) models reduce to a constant volatility model which is a special case of the GARCH model.[4] Specifically the estimated models are

$$h(t) = \gamma\, r(t-1), \qquad \text{(Courtadon);} \tag{14}$$

$$h(t) = \gamma \sqrt{r(t-1)}, \text{(CIR)} \tag{15}$$

Using only historical data, these models cannot capture the persistence in the volatility that the GARCH and GJR models capture. Therefore their log-likelihood is significantly lower. Including an additional constant term in Equations (14) and (15) does not significantly affect the estimates for γ or the log-likelihood.

We now incorporate interest rate levels in the GARCH and GJR models to explain volatility. Further, we also test the interaction between rate levels and the GARCH-type volatility clustering found by Brenner, Harjes, and Kroner (1993) and Koedijk et al. (1994). Specifically these authors find that the impact of shocks to volatility is lower when interest rates are low than when they are high. One explanation is as follows. At low rates, if a large shock were to cause a large and persistent [through the GARCH term $h(t)$] increase in volatility, the probability of negative interest rates would be very high. In fact, a criticism of GARCH-type models for interest rate volatility is that they do not preclude negative interest rates. The interaction term scales the squared shocks by the level of interest rates. Therefore it incorporates GARCH-type effects in the determination of interest rate volatility without permitting a very high probability of negative interest rates.

On the economic side, it is also likely that investors react differently under different interest rate regimes. For example, many investors

[4] Note that this conclusion is not true for the implied volatility estimation since we use options of all maturities to estimate the volatility of a single rate.

maintain a duration target for their asset portfolio. As rates change, these investors rebalance their bond portfolio to maintain their duration target. Since bond prices are convex functions of interest rates, an interest rate increase will decrease the duration of a bond portfolio. Therefore the investor will buy long-term bonds and sell short-term bonds. This rebalancing will counter the initial rate shocks, as he will be buying (selling) the short end following a short rate increase (decrease). Furthermore, since bond prices are more convex at low rate levels, the counteraction effect will be stronger when rates are low. Hence, following large shocks, we are less likely to see high volatility at low rates.

Ignoring the interaction of rate levels and shocks can lead to significantly mis-estimated parameters [Brenner, Harjes, and Kroner (1993)]. For example, the power parameter for the volatility sensitivity to the rate level [as estimated by Chan et al. (1992)] is almost three times as high without making allowances for the interaction.

To address this interaction, we consider four combinations of the shock and level effects by combining the GARCH and GJR models individually with the Courtadon and CIR volatility specifications. For consistency with the term incorporating the interest rate level, we define the interaction term as $J(t-1) = r(t-1)\varepsilon^2(t-1)$ for the Courtadon model and $C(t-1) = \sqrt{r(t-1)}\varepsilon^2(t-1)$ for the CIR model. The resulting equations are

1. Mixed GARCH and Courtadon model:

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi J(t-1) + \gamma r(t-1). \quad (16)$$

2. Mixed GARCH and CIR model:

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi C(t-1) + \gamma\sqrt{r(t-1)}. \quad (17)$$

3. Mixed GJR and Courtadon model:

$$\begin{aligned} h(t) \;=\; & \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi J(t-1) \\ & + \gamma r(t-1) + \alpha_- h^2(t-1). \end{aligned} \quad (18)$$

4. Mixed GJR and CIR model:

$$\begin{aligned} h(t) \;=\; & \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi C(t-1) \\ & + \gamma\sqrt{r(t-1)} + \alpha_- h^2(t-1). \end{aligned} \quad (19)$$

Table 2 presents the estimates for these hybrid volatilities. Interestingly, the interaction terms [$J(t-1)$ and $C(t-1)$] swamp both interest rate level effects and the GARCH-type terms. The GJR asymmetric

Table 2
Alternative volatility models (historical estimates)

GARCH models (based on past interest rate shocks)

GARCH(1, 1)

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}
Coefficient	1553.2	.000	.937	.035
t-statistic		2.5	59.8	4.0

Glosten–Jagannathan–Runkle (GJR)

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}	η^2_{t-1}
coefficient	1557.1	.000	.921	.071	$-.051$
t-statistic		2.7	52.9	4.0	-2.9

$[\eta_{t-1} = \max(0, -\varepsilon_{t-1})]$

Models based on the level of interest rates

Cox–Ingersoll–Ross (CIR)

	$\log L$	$\sqrt{r_{t-1}}$
Coefficient	1535.7	.002
t-statistic		21.9

Courtadon

	$\log L$	r_{t-1}
Coefficient	1539.5	.001
t-statistic		22.1

Hybrid models

Mixed GARCH(1, 1) and Courtadon

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}	J_{t-1}	r_{t-1}
Coefficient	1557.4	.001	.809	$-.118$	.025	.000
t-statistic		5.9	17.8	-8.8	5.9	-1.7

$(J_{t-1} = r_{t-1}\varepsilon^2_{t-1})$

Mixed GARCH(1, 1) and CIR

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}	C_{t-1}	$\sqrt{r_{t-1}}$
Coefficient	1560.7	.000	.890	$-.096$	.054	.000
t-statistic		1.5	30.7	-1.8	2.4	$-.3$

$(C_{t-1} = \sqrt{r_{t-1}}\varepsilon^2_{t-1})$

Mixed GJR and Courtadon

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}	J_{t-1}	r_{t-1}	η^2_{t-1}
Coefficient	1561.9	.000	.891	.009	.007	.000	$-.034$
t-statistic		1.5	26.7	.3	1.7	.8	-1.8

Mixed GJR and CIR

	$\log L$	Constant	h_{t-1}	ε^2_{t-1}	C_{t-1}	$\sqrt{r_{t-1}}$	η^2_{t-1}
Coefficient	1562.8	.000	.906	$-.065$	.047	.000	$-.037$
t-statistic		1.9	38.3	-1.3	2.5	$-.9$	-2.2

This table reports the maximum likelihood parameter estimates for several historical volatility models with volatility dependent on both interest rate levels and past shocks to the interest rate. For each model, the date t volatility, h_t, is assumed to be linear in the specified variables with a conditional t-error distribution. The degree of freedom parameter of the t distribution (not reported) is estimated jointly with the other parameters. The sample period is from January 1, 1988–November 10, 1993. r_t is the spot interest rate, ε_t is the interest rate shock, and h_t is the volatility at date t.

response term is still significant after incorporating the interaction, although it is much weaker.

To contrast the GARCH/GJR, level effect, and hybrid models, we plot the volatility $[h(t)]$ against the shock $[\varepsilon(t-1)]$ at selected interest rate levels $[r(t-1)]$. These plots are a two-dimensional extension of the news impact curve of Engle and Ng (1993). Figure 2 represents the GJR model. It shows that a positive shock (an unexpected increase in the interest rate) causes a higher volatility than a negative shock of the same magnitude. Note that the model does not permit the lagged

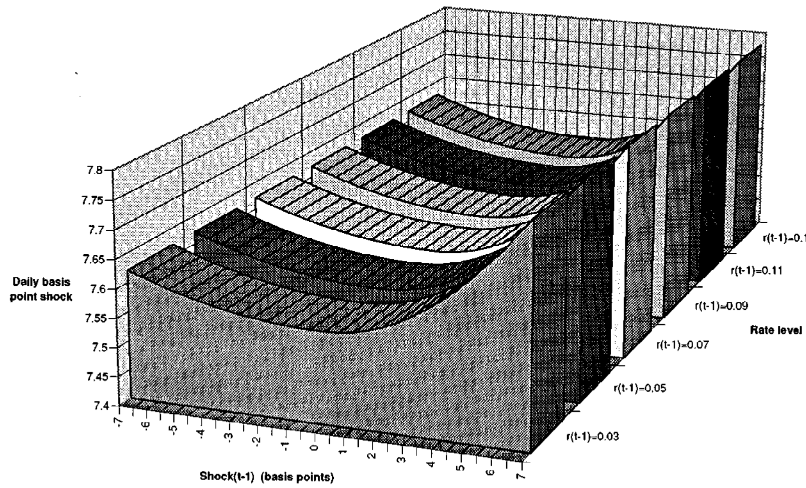

Figure 2
GJR volatility as a function of lagged shock and rate level
This figure plots the next periods forecast volatility for the spot interest rate based on the previous periods interest rate shock and rate level, using the GJR volatility model. The parameters of the GJR model are estimated using maximum likelihood with the daily time series of the spot rates (3-month Eurodollar interest rate implied from the first Eurodollar futures contract) during January 1, 1988–November 10, 1992.

interest rate level to affect the volatility. Figure 3 represents the CIR model. It shows a flat graph along the axis for $\varepsilon(t-1)$ since the model does not permit a shock (GARCH) effect. Finally, Figure 4 of the mixed GJR and CIR models shows the interaction effect. For a shock of the same magnitude, volatility is impacted less by rate shocks at low rates than at high rates.

4. Implied HJM Volatility and Historical Volatility Based on GARCH and GJR Models

In this section we compare the relative forecast ability of the implied HJM volatilities to the historical volatility models studied in Section 3.

4.1 Test framework

One way to test the individual significance of historical and implied volatility is to test the nested form:

$$h(t) = \kappa\, h_{\text{historical}}(t) + \delta \sigma_I^2(t), \tag{20}$$

where $h_{\text{historical}}(t)$ is the conditional volatility based on past returns and shocks (from a GARCH-type model) and $\sigma_I^2(t)$ is the implied (HJM) volatility from traded options. If κ is significant, but δ is not, then

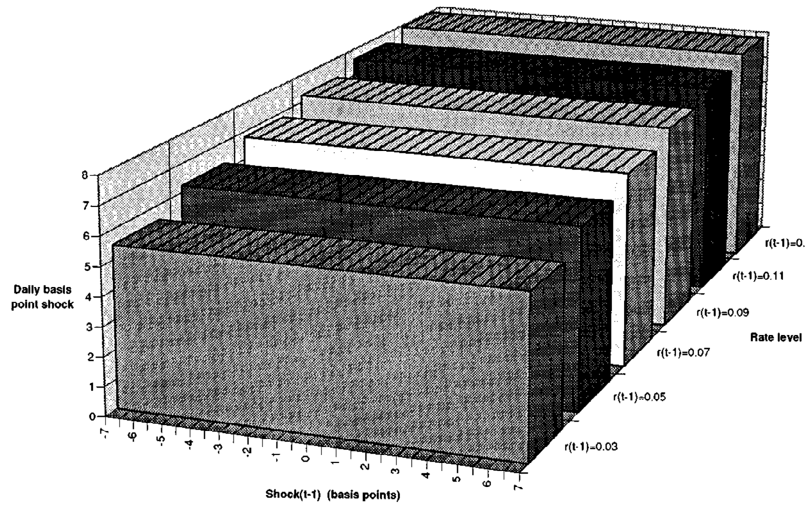

Figure 3
CIR volatility as a function of lagged shock and rate level
This figure plots the next periods forecast volatility based on the previous periods interest rate shock and rate level, based on the Cox, Ingersoll, and Ross (1985) volatility model. The spot rate volatility is proportional to the square root of the spot interest rate. The proportionality parameter is estimated using maximum likelihood with the daily time series of the spot rates (3-month Eurodollar interest rate implied from the first Eurodollar futures contract) during January 1, 1988–November 10, 1992.

implied volatility provides no additional information about the actual volatility. Similarly, if δ is significant, but κ is not, then the historical volatility contains no additional information.

In a GARCH historical volatility model, κ is multiplied by each coefficient in the GARCH variance equation. Thus it cannot be identified. In this case the significance of κ is equivalent to the significance of some ARCH/GARCH coefficients. Thus, unless we study nonlinear historical volatility models, the relevant form is

$$h(t) = h_{\text{garch}}(t) + \delta\sigma_I^2(t). \tag{21}$$

Further, the implied volatility estimation yields $\sigma_I(t)^2$ to be the expectation of the average volatility between time t and the options expiration [Amin and Morton (1994)]. However, the above equation is justifiable for two reasons. First, exchange traded options are near to-the-money options whose price is almost linear in the expected average volatility. Second, if the volatility follows a simple mean reversion model [an AR(1)] such as

$$\sigma(t)^2 = \kappa + \theta\sigma(t-1)^2 + v(t),$$

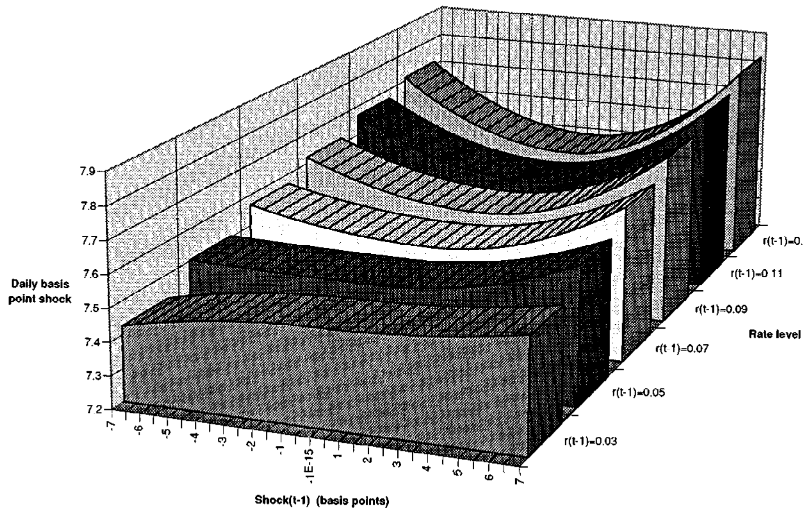

Figure 4
Hybrid GJR and CIR volatility as a function of lagged shock and rate level
This figure plots the next periods forecast volatility based on the previous periods interest rate shock and rate level, based on the hybrid GJR and CIR volatility model. The parameters are estimated using maximum likelihood with the daily time series of the spot rates (3-month Eurodollar interest rate implied from the first Eurodollar futures contract) during January 1, 1988–November 10, 1992.

where $v(t)$ is i.i.d. with a zero mean, then the time t expectation of the average volatility between time t and a later time T is just a simple linear function of $\sigma(t)^2$. Thus, if an AR(1) model approximates the volatility dynamics, $\sigma_I(t)^2$ is a reasonable substitute for $\sigma(t)^2$.

To test for the information effect of implied volatility, we first include the current implied volatility, $\sigma_I^2(t)$, in Equation (21). Since $\sigma_I^2(t)$ contains current information, this specification yields a strong test. Furthermore, the significance of $\varepsilon^2(t-1)$ in the testing equation provides a strong rejection of the informational efficiency of the implied volatility under all the maintained assumptions. Secondly, we construct a predictive test, including the lagged implied volatility, $[\sigma_I^2(t-1)]$, in Equation (21). The statistical significance of δ in the predictive test indicates the implied volatility's forecast ability relative to historical forecasts.

4.2 A new testing approach

Day and Lewis (1992) and Lamoureux and Lastrapes (1993) implement a version of Equation (21) by embedding the implied volatility in a GARCH equation:

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon(t-1)^2 + \delta\sigma_I(t)^2. \tag{22}$$

However, Equation (22) implies significantly different restrictions compared to Equation (21). Recursively substituting out the lagged conditional volatility in Equation (22), we find that

$$\begin{aligned} h(t) &= \frac{\omega}{1-(\alpha+\beta)} + \alpha\Sigma_{i=0,t-1}(\alpha+\beta)^i v(t-1-i) \\ &\quad + \delta\sigma_I(t)^2 + \delta\Sigma_{i=1,t-1}(\alpha+\beta)^i\sigma_I(t-i)^2 \end{aligned} \tag{23}$$

Notice that in a standard GARCH(1, 1) model

$$\begin{aligned} h(t) &= \omega + \beta h(t-1) + \alpha\varepsilon(t-1)^2 \\ &= \omega + (\alpha+\beta)h(t-1) + \alpha[\varepsilon(t-1)^2 - h(t-1)] \\ &= \frac{\omega}{1-(\alpha+\beta)} + \alpha\Sigma_{i=0,t-1}(\alpha+\beta)^i v(t-1-i). \end{aligned} \tag{24}$$

Comparing Equations (23) and (24), we observe that the first two terms of Equation (23) are equivalent to the GARCH conditional variance. If we term the GARCH volatility given by the righthand side of Equation (24) as $h_{\text{garch}}(t)$, we can rewrite Equation (23) as

$$h(t) = h_{\text{garch}}(t) + \delta\sigma_I(t)^2 + \delta\Sigma_{i=1,t-1}(\alpha+\beta)^i\sigma_I(t-i)^2.$$

Thus Equation (22) is equivalent to the estimation equation,

$$h(t) = h_{\text{garch}}(t) + \delta\sigma_I(t)^2 + \Sigma_{i=1,t-1}\delta_i\sigma_I(t-i)^2,$$

with the constraints

$$\delta_i = \delta(\alpha+\beta)^i \quad \text{for all } i = 1, \ldots, t-1.$$

If the lagged implied volatility is uninformative, its coefficient will be zero. However, our constraints do not allow a zero coefficient. Specifically, if α and β are significant and the current implied volatility contains some volatility information (δ is significantly different from zero), then the δ_i's cannot be zero.

Thus these constraints provide a joint test of the significance of current and multiple lagged implied volatilities with a specific lag structure. However, since this joint test does not directly address the current implied volatility, it cannot test the information content of this volatility. Futhermore, it yields a downwardly biased estimate for δ. If the ARCH and GARCH coefficients are not significantly affected, we can approximate the magnitude of this bias. Let h_m be the sample unconditional variance. At the maximum of the likelihood function, the expected conditional variance (evaluated at the estimated parameters) should be fairly close to the sample unconditional variance. Similarly, the expected value of the implied volatility should be proportional to the sample unconditional variance. Denoting this proportionality factor as g, for parameters estimated from Equation (21) (ω, α, β, and δ), we find

$$h_m = \frac{\omega}{1-(\alpha+\beta)} + \delta g h_m.$$

Note that the unconditional variance under a GARCH(1, 1) model is equal to $\frac{\omega}{1-(\alpha+\beta)}$. Conversely, using parameters estimated from Equations (22) or (23) (ω, α, β, and δ^*) provides

$$h_m = \frac{\omega}{1-(\alpha+\beta)} + \delta^* g h_m [1-(\alpha+\beta)^T] \frac{1}{1-(\alpha+\beta)},$$

where T is the number of observations.

If the number of observations is large enough and the ARCH and GARCH parameters are not significantly affected by specification choice, we can identify the relationship between δ^* and δ:

$$\delta^* = \delta(1-\alpha-\beta).$$

Thus, if $\alpha + \beta = 0.8$, a Day and Lewis (1992) δ estimate could be as small as 20% of the true δ. Of course the actual bias also depends on the effect of the specification on the estimates of α and β. In fact, the Day and Lewis (1992) approach will likely yield a downwardly biased β. In their article Day and Lewis (1992) estimate β to be negative, implying a very unusual lag structure.

To correct for this bias we eliminate the implicit lagged implied volatility terms. Consider the following equation:

$$\begin{aligned} h(t) &= h_{\text{garch}}(t) + \delta\sigma_I(t)^2 \\ &= \frac{\omega}{1-(\alpha+\beta)} + \alpha \Sigma_{i=0,t-1} (\alpha+\beta)^i v(t-1-i) + \delta\sigma_I(t)^2. \end{aligned}$$

Rewriting this equation in autoregressive form, with L as the lag operator creates

$$h(t) = \frac{\omega}{1-(\alpha+\beta)} + \frac{\alpha}{1-(\alpha+\beta)L} v(t-1) + \delta\sigma_I(t)^2.$$

Multiplying both sides by $1-(\alpha+\beta)L$ and rearranging terms yields the corrected test equation:

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon(t-1)^2 + \delta\sigma_I(t)^2 - \delta(\alpha+\beta)\sigma_I(t-1)^2. \quad (25)$$

Equation (25) yields an unbiased method for testing the information content of the current implied volatility. If the market is informationally efficient and the option pricing model is valid, $\sigma_I(t)^2$ incorporates all current and past information. Thus the ARCH terms, $h(t-1)$

and $\varepsilon(t-1)^2$, should provide no added explanatory power for $h(t)$. However, if α and β are significant, then either the market is not informationally efficient or the option pricing model is misspecified.

In addition to the Day and Lewis (1992) type tests based on Equation (22), we have also performed tests based on the bias corrected Equation (25) and the corresponding equations for the GJR and hybrid models.

To conduct our predictive test, we derive the test equation by replacing the implied volatility in Equation (20) by its one-period lagged counterpart:

$$\begin{aligned} h(t) &= h_{\text{garch}}(t) + \delta\sigma_I(t-1)^2 \\ &= \frac{\omega}{1-(\alpha+\beta)} + \alpha\Sigma_{i=0,t-1}(\alpha+\beta)^i v(t-1-i) + \delta\sigma_I(t-1)^2. \end{aligned}$$

We can rewrite this equation as

$$h(t) = \omega + \beta h(t-1) + \alpha\varepsilon(t-1)^2 + \delta\sigma_I(t-1)^2 - \delta(\alpha+\beta)\sigma_I(t-2)^2. \quad (26)$$

Equation (26) compares the one-period-ahead implied volatility forecast to the GARCH forecast. For testing the HJM model and the information content of the implied volatility, this test is cleaner than a multiperiod forecast comparison, which requires a dynamic implied volatility model. This model becomes complicated if the HJM model is not correct. Furthermore, multiperiod comparisons require computation of the multiperiod expected average volatility from the GARCH model using numerical integration. This computation introduces additional numerical errors. Nonetheless, for completeness, we perform a multiperiod forecast comparison similar to that of Canina and Figlewski (1993), which we report in Section 6.

5. Empirical Results Comparing Historical Forecasts with Implied Volatilities

In this section we analyze the results of the empirical tests developed in Section 4.

5.1 Contemporaneous implied volatility

We first estimate Equations (22) and (25) with the current implied volatility. (See Table 3 for GARCH results). From Table 3 we see that implied volatilities from all five HJM models are strongly significant. However, the Day–Lewis equation coefficient is significantly weaker. In fact, the coefficient of the implied volatility term (δ) in our persistence corrected equation is at least three times its corresponding value from the uncorrected equation. For example, in the CIR case, the co-

Table 3
Implied volatility and GARCH models

	log L	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{It} δ
Variance equation without persistence adjustment (Day–Lewis approach)					
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It}$					
Ho–Lee implied volatility					
Coefficient	1563.2	.000	.838	.050	.08
t-statistic		.0	16.3	3.2	2.4
Courtadon model implied volatility					
Coefficient	1565.2	.000	.662	.042	.234
t-statistic		.0	4.4	2.1	2.1
Linear proportional implied volatility					
Coefficient	1576.9	.000	.780	.025	.251
t-statistic		−1.5	11.2	1.8	2.9
Cox–Ingersoll–Ross model implied volatility					
Coefficient	1564.9	.000	.807	.047	.120
t-statistic		−.5	11.3	2.8	2.2
Vasicek model implied volatility					
Coefficient	1576.0	.000	.733	.029	.283
t-statistic		−.3	6.9	1.8	2.3
Variance equation with persistence adjustment					
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It} - \delta(\beta+\alpha)\sigma^2_{It-1}$					
Ho–Lee model implied volatility					
Coefficient	1563.9	.000	.860	.039	.725
t-statistic		.0	17.6	2.8	6.1
Courtadon model implied volatility					
Coefficient	1567.8	.000	.863	.036	.795
t-statistics		−.2	16.7	2.7	6.7
Linear proportional model implied volatility					
Coefficient	1566.3	.000	.788	.030	.854
t-statistic		1.4	6.9	1.9	9.9
Cox–Ingersoll–Ross model					
Coefficient	1566.4	.000	.858	.040	.805
t-statistic		−.2	16.8	2.7	6.3
Vasicek model implied volatility					
Coefficient	1569.9	.000	.766	.022	1.04
t-statistic		.7	4.5	1.5	9.3

This table reports the maximum likelihood parameter estimates for the coefficients of HJM implied volatility and GARCH-style historical volatility in the variance equation. Several HJM volatility functions are tested. The implied volatility is tested in the volatility equation in two ways. The first is based on Day and Lewis (1992), and the second corrects for an implicit persistence structure for the lagged implied volatilty in Day and Lewis. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t.

efficient increases from 0.12 to 0.805 after the correction. As expected, the Day–Lewis and Lamoureux–Lastrapes implicit lag structure creates an underestimate of the effect of the implied volatility.

When computing implied volatility with the Ho–Lee, Courtadon, or the CIR models, we find that the squared residual term, $\varepsilon^2(t-1)$ (which represents the ARCH effect), is strongly significant. However, it is only marginally significant for the HJM linear proportional model and marginally insignificant for the Vasicek model. Thus the HJM linear proportional and Vasicek models capture the spot rate volatility

Table 4
Implied volatility and GJR volatility

	$\log L$	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{It} δ	η^2_{t-1} α_-
Ho–Lee model implied volatility						
Estimate	1563.8	.000	.870	.038	.712	−.006
t-statistic		.1	19.0	2.3	5.6	−.3
Courtadon model implied volatility						
Estimate	1567.7	.000	.869	.038	.793	−.008
t-statistic		−.1	18.1	2.2	6.0	−.4
Linear proportional model implied volatility						
Estimate	1567.1	.000	.776	.016	.888	.019
t-statistic		1.1	5.3	1.0	9.3	.7
Cox–Ingersoll–Ross model implied volatility						
Estimate	1566.1	.000	.871	.036	.741	−.005
t-statistic		−.1	18.5	2.2	5.9	−.3
Vasicek model implied volatility						
Estimate	1569.9	.000	.764	.016	.950	.007
t-statistic		.9	4.0	1.0	8.5	.3

This table reports the maximum likelihood parameter estimates for the coefficients of the HJM implied volatility and GJR historical volatility terms in the variance equation. Several different HJM volatility specifications are tested. The variance equation is corrected for the implicit lag structure of implied volatility in the Day and Lewis (1992) approach. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t.

Variance equation estimated:

$$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It} - \delta(\beta+\alpha)\sigma^2_{It-1} + \alpha_-\eta^2_{t-1}$$

where, $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$

better than the GARCH models. Since the Vasicek and linear proportional HJM models permit the volatility of different maturity rates to be different, the term structure of interest rate volatility seems important in explaining interest rate volatilities.

In Table 4 we report the results of comparing the GJR specification with the implied volatility in our persistence corrected equation. Note that the ARCH terms are insignificant for both the Vasicek (1977) and the linear proportional HJM model. Interestingly, the implied volatility captures the asymmetric response effect, making the asymmetric response term insignificant even in equations with significant ARCH terms.

Although the GARCH/GJR parameter, β, is significant in Table 4, there is no GARCH effect when the α parameter is insignificant. We recall that Equation (25) is equivalent to

$$h(t) = \frac{\omega}{1-(\alpha+\beta)} + \frac{\alpha}{1-(\alpha+\beta)L}[\varepsilon^2(t-1) - h(t-1)] + \delta\sigma^2_I(t), \quad (27)$$

where L is the lag operator. When α is zero, the second term in the above equation drops out, leaving multiple optima that correspond to different values of ω and β that yield the same value for $\omega/(1-\beta)$.

In this case, since we cannot estimate β, we also cannot interpret its standard error.

In our test of implied volatility, a significant interaction effect for interest rates and previous period shocks implies an inefficient implied volatility forecast. Recall that our results with the hybrid models using historical data indicate a strong interaction term in Table 2. By testing whether the implied volatility from simple volatility models captures this interaction effect, we can provide a more powerful test of the interaction hypothesis.

To test for an interaction effect, we add $r(t-1)\varepsilon^2(t-1)$ and $r(t-1)$ into Equation (25) to obtain

$$\begin{aligned} h(t) \;=\; & \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi r(t-1)\varepsilon^2(t-1) + \gamma r(t-1) \\ & + \delta\sigma_I^2(t) - \delta(\alpha+\beta)\sigma_I^2(t-1). \end{aligned} \tag{28}$$

With Equation (28) we can evaluate the information content of implied volatility relative to the mixed GARCH and Courtadon models. Specifically, a significant ϕ coefficient suggests an interaction effect between the GARCH term and the interest rate level, while a significant γ coefficient indicates misspecification of the sensitivity of volatility to the interest rate level. The test equation using the mixed GJR and Courtadon model as the benchmark is

$$\begin{aligned} h(t) \;=\; & \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi r(t-1)\varepsilon^2(t-1) + \gamma r(t-1) \\ & + \alpha_-\eta^2(t-1) + \delta\sigma_I^2(t) - \delta(\alpha+\beta)\sigma_I^2(t-1). \end{aligned} \tag{29}$$

For conciseness, only the results with Equation (29) are reported in Table 5. The results with Equation (28) are similar.

To test our mixed GJR and CIR models, we replace $r(t-1)$ with $\sqrt{r(t-1)}$ in Equation (29):

$$\begin{aligned} h(t) \;=\; & \omega + \beta h(t-1) + \alpha\varepsilon^2(t-1) + \phi\sqrt{r(t-1)}\varepsilon^2(t-1) \\ & + \gamma\sqrt{r(t-1)} + \alpha_-\eta^2(t-1) + \delta\sigma_I^2(t) \\ & - \delta(\alpha+\beta)\sigma_I^2(t-1). \end{aligned} \tag{30}$$

Considering both $r(t-1)$ and $\sqrt{r(t-1)}$, we can also test to see if the functional form of the volatility dependence on interest rate levels is misspecified. These results are also reported in Table 5.

Notice that the coefficient for the GARCH term $\varepsilon^2(t-1)$ in all cases is negative. It is also negative in the estimates with the GARCH model in Equation (28) (results not reported). However, note that the total impact of a shock to volatility under Equation (30) is $[\alpha + \phi r(t-1)]\varepsilon^2(t-1)$. Hence, even if α is negative, the total impact is

Table 5
Implied volatility and mixed GJR/interest rate dependent models

Implied volatility vs. mixed GJR and Courtadon model
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It} - \delta(\beta+\alpha)\sigma^2_{It-1} + \phi J_{t-1} + \gamma r_{t-1} + \alpha_{-}\eta^2_{t-1} + \phi r_{t-1}\eta^2_{t-1}$ where, $J_{t-1} = r_{t-1}\varepsilon^2_{t-1}$ and $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$.

	log L	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{It} δ	J_{t-1} ϕ	r_{t-1} γ	η^2_{t-1} α_{-}
Ho–Lee model implied volatility								
Estimate	1568.5	.000	.840	−.026	.349	.012	.000	−.026
t-statistic		1.0	14.4	−.7	3.3	2.0	−.3	−1.1
Courtadon model implied volatility								
Estimate	1570.4	.000	.844	−.022	.446	.010	.000	−.022
t-statistic		1.5	13.1	−.6	3.4	1.8	−1.1	−1.0
Linear proportional model implied volatility								
Estimate	1576.9	.000	.806	−.048	.672	.012	.000	−.018
t-statistic		31.7	10.0	−1.5	7.2	2.5	−.4	−.9
Cox–Ingersoll–Ross model implied volatility								
Estimate	1569.2	.000	.848	−.029	.400	.010	.000	−.017
t-statistic		1.3	13.0	−.7	3.3	1.8	−.7	−.8
Vasicek model implied volatility								
Estimate	1575.3	.000	.823	−.037	.737	.012	.000	−.027
t-statistics		2.3	13.3	−1.3	5.5	2.9	−2.2	−1.4

Implied volatility vs. mixed GJR and CIR model
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It} - \delta(\beta+\alpha)\sigma^2_{It-1} + \phi C_{t-1} + \gamma\sqrt{r_{t-1}} + \alpha_{-}\eta^2_{t-1} + \phi\sqrt{r_{t-1}}\eta^2_{t-1}$ where, $C_{t-1} = \sqrt{r_{t-1}}\varepsilon^2_{t-1}$ and $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$.

	log L	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{It} δ	C_{t-1} ϕ	$\sqrt{r_{t-1}}$ γ	η^2_{t-1} α_{-}
Ho–Lee model implied volatility								
Estimate	1568.1	.000	.847	−.079	.269	.049	.000	−.016
t-statistic		.7	14.6	−1.1	2.8	1.8	−.3	−.7
Courtadon model implied volatility								
Estimate	1570.3	.000	.830	−.085	.365	.053	.000	−.023
t-statistic		1.5	12.2	−1.2	3.2	1.8	−1.3	−1.0
Linear proportional model implied volatility								
Estimate	1577.4	.000	.832	−.072	.619	.040	.000	−.014
t-statistic		.5	10.2	−1.3	5.0	1.7	−.4	−.7
Cox–Ingersoll–Ross model implied volatility								
Estimate	1569.3	.000	.834	−.080	.338	.049	.000	−.019
t-statistic		1.0	12.1	−1.1	3.1	1.7	−.7	−.8
Vasicek model implied volatility								
Estimate	1576.8	.000	.819	−.082	.657	.046	.000	−.021
t-statistic		1.6	10.4	−1.5	4.7	2.0	−1.5	−1.1

This table reports the maximum likelihood parameter estimates for the coefficients of the persistence corrected HJM implied volatility and historical volatility terms, which include the GJR terms, interest rate levels, and the interaction between the interest rate level and shocks to the interest rate in the variance equation. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t.

still positive as long as $[\alpha + \phi r(t-1)]$ is positive, a condition that is always satisfied in our estimation. Since the interest rate is expressed in percent, the product of $r(t-1)$ and ϕ dominates α. Further, the estimated implied volatility coefficient is significantly lower than with only the GARCH/GJR terms (see Tables 3 and 4). This decline is more significant for the Ho and Lee, Courtadon, and CIR models, which also do not perform well under previous GARCH tests.

As in Table 4, the asymmetry term in the GJR specification is insignificant. Further, the interest rate level is also insignificant. But the interaction terms $J(t-1) = r(t-1)\varepsilon^2(t-1)$ and $C(t-1) = \sqrt{r(t-1)}\varepsilon^2(t-1)$ are significant. Therefore, both the interest rate level and the GARCH term $\varepsilon^2(t-1)$ seem to affect interest rate volatility only through the interaction term. That is, when interest rates are low, shocks to the interest rate lead to smaller volatility increases than when they are high. Since this behavior is not completely captured by any of the HJM models, they are not informationally efficient.

So far we have used the current period's implied volatility to forecast the realized volatility and to test to see if historical information provides additional explanatory power. However, even though the volatility forecast from the contemporaneous implied volatility can be improved using historical information, we also want to determine the forecasting ability of lagged implied volatilities relative to the GARCH and GJR forecasts.

5.2 Predictive power of lagged implied volatility

To compare the predictive ability of GARCH forecasts to implied volatility, we consider the effect of lagged implied volatility in the persistence corrected variance equation. Since it does not contain current information, the lagged implied volatility permits more significant ARCH terms. Thus we can use this test to judge the forecast usefulness of lagged implied volatility.

In Table 6 we report results for the GARCH/GJR forecasts and HJM implied volatilities. As with the current implied volatility, the ARCH coefficient [for $\varepsilon(t-1)^2$] is strongly significant for the Ho and Lee, Courtadon, and CIR models. However, this parameter is insignificant for the HJM linear proportional and Vasicek models with the GJR benchmark and only marginally significant with the GARCH benchmark. Thus these two models capture spot rate volatility better than the GARCH/GJR models do.

To study the interaction effect in the GARCH/GJR equation in conjunction with the lagged implied volatility, we replace $\sigma_I^2(t)$ with $\sigma_I^2(t-1)$ and $\sigma_I^2(t-1)$ with $\sigma_I^2(t-2)$ in Equations (29) and (30). The results are reported in Table 7. As with the current implied volatility, parameters for the GARCH, interest rate level, and the asymmetric response term are insignificant. However, the interaction terms are still significant for both the linear proportional HJM model and the Vasicek model. Further, these terms significantly reduce the implied volatility coefficients. For example, by including the interaction terms, the Vasicek implied volatility coefficient declines from about 0.7 in Table 5 to about 0.5 in Table 7. This decline is even more significant for the single parameter models (Ho and Lee, Courtadon, and CIR).

Table 6
Predictive power of lagged implied volatility versus GARCH and GJR

Lagged implied volatility vs. GARCH
Variance equation: $h_t = \omega + \beta h_{t-1} + \alpha\varepsilon_{t-1}^2 + \delta\sigma_{It-1}^2 - \delta(\beta+\alpha)\sigma_{It-2}^2$

	$\log L$	Constant ω	h_{t-1} β	ε_{t-1}^2 α	σ_{It-1}^2 δ
Ho–Lee model implied volatility					
Coefficient	1563.8	.000	.860	.044	.772
t-statistic		$-.2$	18.3	3.0	5.5
Courtadon model implied volatility					
Coefficient	1567.9	.000	.867	.037	.859
t-statistic		$-.6$	17.1	2.7	6.4
Linear proportional model implied volatility					
Coefficient	1560.7	.000	.795	.043	.765
t-statistic		1.7	8.4	2.2	5.9
Cox–Ingersoll–Ross model implied volatility					
Coefficient	1566.2	.000	.864	.038	.808
t-statistic		$-.5$	17.3	2.8	6.0
Vasicek model implied volatility					
Coefficient	1563.5	.000	.767	.038	.915
t-statistic		1.3	6.6	2.0	6.8

Lagged implied volatility vs. GJR
Variance equation: $h_t = \omega + \beta h_{t-1} + \alpha\varepsilon_{t-1}^2 + \delta\sigma_{It-1}^2 - \delta(\beta+\alpha)\sigma_{It-2}^2 + \alpha_-\eta_{t-1}^2$,
where, $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$

	$\log L$	Constant ω	h_{t-1} β	ε_{t-1}^2 α	σ_{It-1}^2 δ	η_{t-1}^3 α_-
Ho–Lee model implied volatility						
Estimate	1563.6	.000	.874	.040	.729	$-.008$
t-statistic		.0	19.6	2.4	5.2	$-.4$
Courtadon model implied volatility						
Estimate	1568.1	.000	.867	.046	.913	$-.014$
t-statistic		$-.5$	18.1	2.4	5.5	$-.8$
Linear proportional model implied volatility						
Estimate	1561.0	.000	.849	.022	.766	.003
t-statistics		1.6	11.4	1.5	6.3	.2
Cox–Ingersoll–Ross model implied volatility						
Estimate	1566.4	.000	.864	.046	.858	$-.013$
t-statistic		$-.4$	17.9	2.5	5.3	$-.7$
Vasicek model implied volatility						
Estimate	1563.5	.000	.768	.031	.898	.011
t-statistic		1.3	6.4	1.6	6.5	.4

This table reports the maximum likelihood parameter estimates for the coefficients of the persistence corrected lagged HJM implied volatility and the GARCH and GJR historical volatility terms in the variance equation. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t.

5.3 A robustness check with multiperiod volatility forecasts

So far we have studied the forecast ability of HJM implied volatility models relative to historical volatility models for one-period-ahead forecasts. However, since the implied volatility is computed from longer maturity options, it also contains information about longer period volatility. We check the robustness of our results and assess multiperiod forecast ability with a simple test described below. To differentiate between the ARCH and interaction effects, we construct

Table 7
Predictive power of implied volatility relative to mixed GJR models

Lagged implied volatility vs. mixed GJR and Courtadon models
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It-1} - \delta(\beta+\alpha)\sigma^2_{It-2} + \phi J_{t-1} + \gamma r_{t-1} + \alpha_- \eta^2_{t-1}$ where, $J_{t-1} = r_{t-1}\varepsilon^2_{t-1}$ and $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$.

	$\log L$	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{It-1} δ	J_{t-1} ϕ	r_{t-1} γ	η^2_{t-1} α_-
Ho–Lee model implied volatility								
Estimate	1567.2	.000	.861	.002	.394	.008	.000	−.026
t-statistic		.2	17.5	.1	2.9	1.6	.4	−1.2
Courtadon model implied volatility								
Estimate	1569.7	.000	.865	−.007	.520	.007	.000	−.019
t-statistics		1.3	15.3	−.2	3.1	1.5	−1.1	−.9
Linear proportional model implied volatility								
Estimate	1571.1	.000	.855	−.042	.572	.011	.000	−.010
t-statistic		1.4	13.4	−1.2	4.1	2.0	−.8	−.5
Cox–Ingersoll–Ross model implied volatility								
Estimate	1568.6	.000	.855	−.012	.418	.009	.000	−.022
t-statistic		1.0	14.8	−.3	3.0	1.6	−.4	−1.0
Vasicek model implied volatility								
Estimate	1571.2	.000	.837	−.042	.614	.012	.000	−.020
t-statistic		2.1	13.5	−1.3	4.0	2.5	−1.7	−1.0

Lagged implied volatility vs. mixed GJR and CIR model
$h_t = \omega + \beta h_{t-1} + \alpha\varepsilon^2_{t-1} + \delta\sigma^2_{It-1} - \delta(\beta+\alpha)\sigma^2_{It-2} + \phi C_{t-1} + \gamma\sqrt{r_{t-1}} + \alpha_- \eta^2_{t-1}$ where, $C_{t-1} = \sqrt{r_{t-1}}\varepsilon^2_{t-1}$ and $\eta_{t-1} = \max[0, -\varepsilon_{t-1}]$.

	$\log L$	Constant ω	h_{t-1} β	ε^2_{t-1} α	σ^2_{t-1} δ	c_{t-1} ϕ	$\sqrt{r_{t-1}}$ γ	η^2_{t-1} α_-
Ho–Lee model implied volatility								
Estimate	1566.9	.000	.852	−.065	.275	.042	.000	−.013
t-statistic		.6	14.3	−.9	2.5	1.5	−.2	−.6
Courtadon model implied volatility								
Estimate	1569.2	.000	.843	−.044	.419	.036	.000	−.020
t-statistic		1.1	13.3	−.7	2.7	1.4	−1.0	−.9
Linear proportional model implied volatility								
Estimate	1572.8	.000	.819	−.107	.462	.054	.000	−.010
t-statistic		1.2	10.6	−1.7	3.9	2.1	−.8	−.5
Cox–Ingersoll–Ross model implied volatility								
Estimate	1568.5	.000	.843	−.064	.347	.044	.000	−.020
t-statistic		.9	13.2	−.9	2.7	1.6	−.6	−.9
Vasicek model implied volatility								
Estimate	1573.1	.001	.829	−.109	.504	.058	.000	−.015
t-statistic		1.8	12.2	−1.7	3.5	2.2	−1.7	−.7

This table reports the maximum likelihood parameter estimates for the coefficients of the persistence corrected lagged HJM implied volatility and historical volatility terms, which include the GJR terms, interest rate levels and the interaction between the interest rate level and shocks to the interest rate in the variance equation. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t.

volatility forecasts using the moving average of the realized volatility due to each of these terms over the last 20 business days. These tests are based on Canina and Figlewskis (1993) use of the realized historical volatility (estimated by the moving average of squared shocks) to forecast the future realized volatility. We also include terms to represent shock asymmetry and interaction effects.

The historical volatility forecasts are proxied by the variables:

1. ARCH term (lagged realized volatility): $V_{\varepsilon t} = \frac{1}{19}\sum_{i=1}^{20} \varepsilon^2_{t+1-i}$.
2. Asymmetric shocks: $V_{\eta t} = \frac{1}{19}\sum_{i=1}^{20} \max[0, -\varepsilon^2_{t+1-i}]$
3. Interaction terms: $V_{Ct} = \frac{1}{19}\sum_{i=1}^{20} r_{t+1-i}\ \varepsilon^2_{t+1-i}$ or $\frac{1}{19}\sum_{i=1}^{20} \sqrt{r_{t+1-i}}\ \varepsilon^2_{t+1-i}$.

The realized volatility over the next 20 business days is proxied by the mean of the squared shocks. Therefore our test equation becomes

$$h_{t,t+20} = \frac{1}{19}\sum_{i=1}^{20} \varepsilon^2_{t+1+i} = \omega + \alpha V_{\varepsilon t} + \gamma r(t) + \alpha_- V_{\eta t} + \phi V_{Ct} + \delta \sigma^2_I(t), \tag{31}$$

where $h_{t,t+20}$ is the realized volatility between time t and t+20. Table 8 presents the results of this test. In Table 9 we report results using $\sqrt{r(t)}$ as a rate level proxy and incorporating an interaction term. Note that the reported t-statistics are corrected for heteroskedasticity and serial correlation for up to the 20th order using the approach of Newey and West (1987).

The results of our multiperiod tests coincide with the single-period forecast results. With all of the HJM models, the implied volatility plays a dominant role in explaining the realized volatility. We see the most significant results for models that permit a term structure of volatility (the Vasicek and linear proportional volatility models). In fact, for these models, the coefficient of the implied volatility is insignificantly different from one. Overall, while the realized historical volatility, interest rate level, and asymmetric volatility are all insignificant, the interaction term is still quite significant.

6. Conclusions and Economic Implications

This article offers several important conclusions. First, in contrast to earlier studies on the information content of implied volatility from equity options [Canina and Figlewski (1993), Day and Lewis (1992), Lamoureux and Lastrapes (1993) and others], we find that HJM implied volatility from Eurodollar futures options explains most of the realized interest rate volatility. Our test equation corrects for some biases in Day and Lewis (1992) and Lamoureux and Lastrapes (1993). Since the

Table 8
Forecasting realized volatility over the next 20 days using current and past shocks, the interest rate level, and the implied volatility

Dependent variable: $\Sigma_{i=1,20}\varepsilon^2_{t+i}/19$ (realized volatility)

	Independent variables						
	Constant	$V_{\varepsilon t}$	$V_{\eta t}$	V_{Jt}	r_t	σ^2_{It}	R^2
Estimate	0.004	0.281	−0.128				0.050
t-statistic	6.68	4.02	−0.99				
Estimate	0.003	−0.949	0.150	0.109	0.000		0.177
t-statistic	1.20	−2.85	1.00	3.38	1.22		
Implied volatility from the Ho–Lee model							
Estimate	0.002	−0.941	0.185	0.099	0.000	0.357	0.192
t-statistic	0.94	−2.82	1.29	2.97	0.74	1.33	
Estimate	0.002					0.753	0.129
t-statistic	1.31					3.53	
Implied volatility from the Courtadon model							
Estimate	0.003	−0.926	0.176	0.095	0.000	0.530	0.207
t-statistic	1.31	−2.71	1.24	2.79	0.18	1.93	
Estimate	0.002					0.808	0.175
t-statistic	1.60					4.49	
Implied volatility from the linear proportional model							
Estimate	0.002	−0.840	0.097	0.084	0.000	0.714	0.242
t-statistics	0.91	−2.40	0.67	2.40	0.65	3.79	
Estimate	0.002					0.929	0.190
t-statistic	3.83					5.88	
Implied volatility from the Cox–Ingersoll–Ross model							
Estimate	0.003	−0.934	0.182	0.097	0.000	0.443	0.199
t-statistic	1.09	−2.77	1.28	2.88	0.48	1.62	
Estimate	0.002					0.793	0.154
t-statistic	1.36					4.04	
Implied volatility from the Vasicek model							
Estimate	0.003	−0.755	0.065	0.074	0.000	0.802	0.246
t-statistic	1.07	−2.12	0.44	2.04	0.40	3.69	
Estimate	0.002					0.971	0.211
t-statistic	3.23					5.72	

This table presents the results of regressing the 20-day moving realized volatility on the implied volatility, the interest rate level, the lagged realized volatility, the lagged asymmetric shock effect, and the lagged effect due to the interaction between the shocks and interest rate levels. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t. The t-statistics reported are the Newey–West heteroskedasticity and serial correlation consistent t-statistics (for up to the 20th order).

Variable definitions:
$V_{\varepsilon t} = \Sigma_{i=1,20}\varepsilon^2_{t+1-i}/19$ (lagged realized volatility)
$V_{\eta t} = \Sigma_{i=1,20}\max(0, -\varepsilon_{t+1-i})^2/19$ (asymmetric shock effect)
$V_{Jt} = \Sigma_{i=1,20} r_{t+1-i}\varepsilon^2_{t+1-i}/19$ (shock and interest rate interaction effect)

transaction costs for arbitrage activity are significantly lower for Eurodollar futures options than for OEX options or individual stock options, this market is more efficient. Therefore the implied volatility is more likely to reflect the market's ex ante expectation of the future volatility in the Eurodollar options market. Our findings support this expectation, using several volatility models to minimize specification errors.

Second, based on the historical time series of interest rates, we find that the GARCH persistence terms and the GJR asymmetric shock

Table 9
Forecasting realized volatility over the next 20 days using current and past shocks, the square root of the interest rate level, and the implied volatility

Dependent variable: $\Sigma_{i=1,20}\varepsilon^2_{t+i}/19$ (realized volatility)

	Independent variables						
	Constant	$V_{\varepsilon t}$	$V_{\eta t}$	V_{Jt}	$\sqrt{r_t}$	σ^2_{It}	R^2
Estimate	0.004	0.286	−0.133				0.050
t-statistic	6.700	4.059	−1.021				
Estimate	0.002	−1.777	0.113	0.615	0.002		0.167
t-statistic	0.334	−3.290	0.761	3.684	0.973		
Implied volatility from the Ho–Lee model							
Estimate	0.002	−1.684	0.158	0.555	0.001	0.397	0.187
t-statistics	0.418	−3.108	1.135	3.195	0.479	1.489	
Estimate	0.002					0.753	0.129
t-statistics	1.307					3.526	
Implied volatility from the Courtadon model							
Estimate	0.004	−1.617	0.151	0.523	0.000	0.572	0.203
t-statistic	0.889	−2.934	1.086	2.952	−0.094	2.132	
Estimate	0.002					0.808	0.175
t-statistics	1.604					4.486	
Implied volatility from the linear proportional model							
Estimate	0.002	−1.462	0.068	0.468	0.001	0.738	0.237
t-statistic	0.433	−2.566	0.479	2.577	0.423	3.911	
Estimate	0.002					0.929	0.190
t-statistic	3.831					5.880	
Implied volatility from the Cox–Ingersoll–Ross model							
Estimate	0.003	−1.652	0.156	0.539	0.000	0.485	0.194
t-statistics	0.625	−3.030	1.126	3.080	0.211	1.797	
Estimate	0.002					0.793	0.154
t-statistics	1.362					4.036	
Implied volatility from the Vasicek model							
Estimate	0.003	−1.285	0.037	0.407	0.000	0.831	0.243
t-statistics	0.629	−2.186	0.262	2.141	0.173	3.807	
Estimate	0.002					0.971	0.211
t-statistic	3.234					5.721	

This table presents the results of regressing the 20-day moving realized volatility on the implied volatility, the square root of the interest rate level, the lagged realized volatility, the lagged asymmetric shock effect and the lagged effect due to the interaction between the shocks and interest rate levels. r_t is the spot interest rate, ε_t is the interest rate shock, σ_{It} is the implied volatility, and h_t is the volatility at date t. The *t*-statistics reported are the Newey–West heteroskedasticity and serial correlation consistent *t*-statistics (for up to the 20th order).

Variable definitions:
$V_{\varepsilon t} = \Sigma_{i=1,20}\varepsilon^2_{t+1-i}/19$ (lagged realized volatility).
$V_{\eta t} = \Sigma_{i=1,20}\max(0, -\varepsilon_{t+1-i})^2/19$ (asymmetric shock effect).
$V_{ct} = \Sigma_{i=1,20}\sqrt{r_{t+1-i}}\varepsilon^2_{t+1-i}/19$ (shock and interest rate interaction effect).

terms are highly significant. Specifically, interest rate volatility increases more with shocks that increase the interest rate (decrease bond prices) than with shocks that decrease the rate (increase bond prices). Both these effects are captured by the implied volatility from the different HJM models. They are no longer significant once we incorporate the implied volatility into the volatility forecast equation.

Third, the Vasicek (1977) and linear proportional volatility models perform better than the other implied volatility models. The implied volatility from these two models explains most of the time variation in realized volatility. These two models permit the volatility of different points of the term structure to be different. Volatility of short-term rates is significantly different from that of even 1-year forward rates. Incorporating the implied volatility from these two models in the volatility equation, we find that the interest rate level, the GARCH term, and the asymmetric volatility effect are insignificant.

Fourth, using the hybrid models as historical volatility benchmarks, we find that the HJM models fail to capture the interaction between rate shocks and rate levels in determining realized volatility. The impact of an interest rate shock on volatility is significantly lower when interest rates are low than when they are high. Further, the implied volatility coefficient drops significantly when we include these interaction terms. This drop is more significant for the Ho and Lee, Courtadon, and CIR implied volatilities. Although the importance of the implied volatility in the linear proportional and Vasicek models is reduced, these models still explain a dominant portion of the time variation in volatility.

The interaction effect exists under all volatility models we examine. Since the Eurodollar futures options market provides low arbitrage costs in an efficient market setting, it is unlikely that our results reflect market frictions.

Fifth, we have also examined the predictive ability of implied volatility by using lagged implied volatility in our tests. The results are weaker but still consistent with the current implied volatility results.

Finally, we find that implied volatility has significant explanatory power even over monthly horizons. In fact, historical volatility terms representing ARCH and asymmetric shock effects provide no additional explanatory power. Nonetheless, even in monthly tests, the interaction term is still significant. Once again, models that permit a term structure of volatility yield better explanatory power.

These results have several important economic implications. First, our results are much more optimistic regarding the usefulness of option pricing models relative to those from recent studies on equity options. They also help us identify interest rate models for pricing contingent claims. Even for options based on forward rates of maturity of 1 year or less, we need to specify a term structure of volatility.

Second, we can use the interaction effect results to improve pricing and hedging strategies based on interest rate models. For example, after a large shock in a low interest rate period, we should price and hedge options with a lower volatility than we do during high interest rate periods. This differential impact is not captured by implied volatility measures/models.

Finally, our results can be used to optimally combine the implied volatility from HJM models with historical volatility terms (incorporating the interaction effect) in a volatility forecast equation. These volatility forecasts are important for determining the risk premia in equilibrium asset pricing models.

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