Heterogeneous Beliefs and the Effect of Replicatable Options on Asset Prices

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We present two ways in which trading in a replicatable option can affect the price process of the underlying asset. In the first situation, trading an option that each investor views as payoff redundant breaks a non-fully revealing equilibrium that exists when the option market is absent. The second situation involves a market that is dynamically complete without options, but in which introducing an option market allows self-confirming conjectures of additional uncertainty about the future price of the underlying asset. Heterogeneous beliefs play important though different roles in both situations.

The 20 years that have passed since the opening of the Chicago Board Options Exchange have witnessed a dramatic growth in trading in options, futures, and other derivative securities, a proliferation in the variety of derivative security contracts, and the growth of a major academic subfield of finance concerned with such securities. All of these developments are, to differing extents, related to the success of Black and Scholes (1973) and Merton (1973) in deriving

We thank Jonathan Berk, Robert Heinkel, Burton Hollifield, Eric Hughson, and seminar participants at Stanford and UC Berkeley for helpful discussions and suggestions. We are grateful to Franklin Allen (the editor) and two anonymous referees for insightful critiques and constructive comments that substantially improved the article. Support from the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged. Address correspondence to Alan Kraus, Faculty of Commerce, University of British Columbia, 2053 Main Mall, Vancouver, B.C., Canada V6T 1Z2.

The Review of Financial Studies Fall 1996 Vol. 9, No. 3, pp. 723–756

option pricing formulas and to the subsequent research their work inspired. The Black–Scholes–Merton option pricing model is based on an assumption that trading in the option contract has no effect on the price process of the underlying asset on which the option contract is written. This article deals with two situations in which that assumption does not hold.

There is, of course, a considerable practical interest in the possibility that trading in derivative securities can affect the price process of the underlying assets. It is now common for institutional investors to take large positions in derivative securities, especially in those relating to fixed income securities. If there is a potential for these actions to affect the volatility of the underlying assets' prices, then the markets for both the underlying assets and for contingent claims written on them must be considered together by regulators and policymakers. The effects we explore in this article take place in a highly simplified world and we cannot point to direct evidence that they occur in real life. Nevertheless, our model shows a way in which the existence of a derivative securities market might alter the price path of the underlying asset.

Other researchers have considered alternative ways in which the assumption of no effect by contingent claims on underlying asset prices may be violated. One kind of departure from the Black–Scholes–Merton assumption, different from the approach here, is a situation in which the traded nonoption securities fail to span the distribution of final payoffs. Ross (1976) and Hakansson (1978) show how the introduction of options that are contingent on final payoffs of nonoption securities can increase the welfare of investors.

The payoff on commonly traded options and futures contracts is contingent not on the final payoffs of an underlying asset, however, but on the market price of that asset at a date before its final payoffs. That is a crucial difference, since it is one thing to assume that the existence of options contracts has no effect on the dividend stream of an asset, while the assumption that the existence of options has no effect on the market price of the asset is much stronger.

In the context of commodity contracts, Svensson (1981) analyzes equilibrium in a model in which there are contracts whose payoffs are contingent on the prices of other contracts. Henrotte (1992) has recently extended this analysis to financial contracts and derives restrictions on the set of admissible traded price-contingent contracts that still permit the existence of equilibrium. Danthine (1978) argues that adding a futures market can increase the price stability in the underlying spot market by bringing into the market traders with superior information about the future payoffs in the spot market. Stein (1987), on the other hand, points out that adding more traders to the market through opening trading in a contingent claim can have offsetting effects on the stability of the price of the underlying asset. On the one hand, more traders allows more risk sharing. On the other hand, if the new traders come into the market with private information, their trading may decrease the informativeness of prices to the other traders, in the sense of Grossman (1976, 1977), Grossman and Stiglitz (1980) and Hellwig (1980).

A series of articles by Chichilnisky, Dutta, and Heal (1992), Chichilnisky, Hahn, and Heal (1992), Chichilnisky et al. (1992), and Hahn (1992) show how the existence of price-contingent financial claims in the current market can affect the existence of multiple equilibria in the subsequent state-dependent goods market and, as a consequence, can affect prices today. This has similarities to some of the results in the current article, but we focus on equilibrium in financial markets alone; the possibility of multiple equilibria in goods markets plays no role in our model.

Another recent, related work is by Back (1993). He shows that introducing an option into a model of continuous trading by informed investors can cause the volatility of the price process of the underlying asset to be stochastic. This happens because adding the option changes how information is revealed in prices and trading volume. A similarity between Back's model and the first case analyzed in the present article is that both concern situations in which the equilibrium without the option appears to make the option redundant in the view of every investor, but this equilibrium is altered when the option is introduced. However, Back's results are related to the correlation between liquidity trading in the option and the underlying asset, while there are no liquidity traders in our model.

As mentioned earlier, this article deals with two situations in which introducing an option significantly affects the price process of the underlying asset despite the fact that, in both cases, the available assets without the option appear to span the payoff space. In both situations, we consider a model with two rounds of trading. In the first case, there are two types of investors, distinguished by their private information about final payoffs. At the initial trading date, the equilibrium without the option is such that the two available securities appear to each investor to offer a dynamically complete market. This is because each investor believes that the risky security price at the next trading date will be one of two values. The two types of investors, however, partly disagree on which two values the future price may take on, but neither type knows the other's beliefs exactly. Thus, each investor sees the stock price next period as having a binomial distribution, so the option can be replicated by a portfolio of stock and the riskless asset, but the different groups believe in different binomial distributions.

In these circumstances, the introduction of an option whose payoff is contingent on next period's stock price profoundly affects the current equilibrium. Since the option is a bet on the future stock price, the price of the option conveys information to each investor type about the beliefs of the other type, and thus about the other type's private information that is not conveyed by the stock price alone. This is analogous to Grossman's (1988) observation that a traded option conveys information on investors' beliefs about future price volatility that is not conveyed by the use of dynamic trading strategies as "synthetic" options. The option can be said to "complete the market for information" with the result that all private information is revealed. Ironically, having completed the market for information, the option then really is redundant. In addition, by preventing investors from hedging future endowment risk, introducing the option lowers investors' expected utility.

The second situation we analyze is, in some respects, a polar opposite to the first case. In the first situation, introducing an option decreases stock price volatility between the two trading periods since it causes the first period price to be the same as the second period price. In the second situation, introducing an option increases stock price volatility in the sense that it increases the uncertainty of next period's price. This happens because existence of the option permits certain self-consistent beliefs about future stock price uncertainty to form that are not consistent with equilibrium without the option.

What we show is that the existence of an option can allow investors to conjecture that the stock price under a given final payoff distribution can take on more than one value, depending on some extraneous event or simply an announcement that the price is to be high or low. When investors hedge against this extra uncertainty in forming their current portfolios, they take positions that make either of the conjectured future stock prices self-confirming. This happens because investors differ in their beliefs about the probabilities of the conjectured prices and so have incentives to trade option contracts with each other to bet on the conjectured future stock prices. The positions they take are such that whichever stock price is announced, the transfer of wealth that accompanies option settlement at that stock price leads to a distribution of wealth that indeed supports that stock price. Thus the existence of options can create "extra" volatility in future stock prices.

The notion that the future stock price can take on more than one value in a single state of nature depending on a purely extraneous announcement is clearly related to the "sunspot" equilibrium phenomenon of Cass and Shell (1983) as well as the work of Chichilnisky et al. noted above. That literature, however, deals with situations in which the future goods market has multiple equilibria in the absence of price-contingent contracts. Here we consider a pure securities market that is dynamically complete and is not characterized by multiple equilibria at the future date in the absence of options. One point of similarity between their results and ours is that they require some divergence of investor beliefs. Specifically, we assume investors have common beliefs over the states of nature that govern final payoff distributions and make common conjectures about the set of alternative stock prices under a given payoff distribution, but differ in the probabilities they assign to these alternative stock prices. This difference of beliefs is the reason investors will choose to bet on their conjectures when the existence of an options market allows them to do this without impinging on their ability to hedge across different final payoff distributions. In brief, investors agree on everything that has to do with final security payoffs but disagree on an aspect of future prices that in fact only exists because of their conjectures.

1. The Basic Model

To aid in understanding our main results, we choose the simplest setting in which they can occur. There are three dates in the model. In chronological order, at dates $t = 0$ and $t = 1$ securities markets are open but no consumption takes place. At date $t = 2$ security payoffs occur and are consumed. Thus the $t = 1$ payoffs to an option expiring at that date are reinvested in the securities market. "Cash" is the numéraire and has a riskless payoff of one per unit. "Stock" is a risky security whose payoff per share at $t = 2$ is A with probability π and zero[1] with probability $1 - \pi$. Some or all investors may be uncertain at $t = 0$ about the values of A and/or π and there may be a public signal about these values at $t = 1$. We discuss this in more detail later.

Investors are atomistic price takers and have von Neumann-Morgenstern utility functions of $t = 2$ consumption characterized by constant relative risk aversion. Investors having the same coefficient of relative risk aversion and the same beliefs can be combined into a single aggregate investor who holds their combined endowments and continues to act as a price taker. To simplify, we assume there are two aggregate investors, having coefficients of relative risk aversion g_1 and g_2. We specify the aggregate investors' beliefs in the two versions of the model we analyze later.

[1] This is for simplicity. At the cost of extra notation, all the main results can be obtained when the stock's payoff is positive in both states.

Z_{ci} and Z_{si} denote the $t = 0$ endowments of cash and stock, respectively, of aggregate investor $i(= 1, 2)$. At $t = 1$, aggregate investor i's holdings of cash and stock before trading are denoted by Y_{ci} and Y_{si}. The final holdings of cash and stock chosen by investor i at $t = 1$ are denoted by X_{ci} and X_{si}.

Much of our analysis considers the introduction at $t = 0$ of trading in a contingent claim (option) which expires at $t = 1$ and whose payoffs are a function of the $t = 1$ stock price. Since options are in zero net supply, we assume that $t = 0$ endowments of the option are zero for each investor. When option trading exists, we let D denote the $t = 1$ payoff per option (which depends on the $t = 1$ stock price) and let Y_{oi} denote the number of options bought ($Y_{oi} > 0$) or written ($Y_{oi} < 0$) at $t = 0$ by aggregate investor i.

To complete our basic notation, we let P_t denote the price of one share of stock in units of cash at date t, W_{it} denote the wealth of aggregate investor i at date t, and C and $\mathcal{S}$ denote the aggregate supplies of cash and stock, respectively, in the economy. Aggregate investor i's wealth is given by

$$W_{i0}(P_0) \equiv Z_{ci} + P_0 Z_{si}, \tag{1.1}$$

$$W_{i1}(P_1) \equiv Y_{ci} + P_1 Y_{si} + D(P_1) Y_{oi}, \tag{1.2}$$

assuming option trading is available. If no option market exists, then $Y_{oi} = 0$. Market clearing conditions are

$$X_{c1} + X_{c2} = Y_{c1} + Y_{c2} = Z_{c1} + Z_{c2} \equiv C, \tag{2.1}$$

$$X_{s1} + X_{s2} = Y_{s1} + Y_{s2} = Z_{s1} + Z_{s2} \equiv \mathcal{S}, \tag{2.2}$$

$$Y_{o1} + Y_{o2} = 0. \tag{2.3}$$

At $t = 1$ investors choose their final portfolios of cash and stock. Assuming investors know A and π at that date, the portfolio optimization problem of aggregate investor i is

$$\max \pi \left(\frac{1}{1-g_i}\right)(X_{ci} + AX_{si})^{1-g_i} + (1-\pi)\left(\frac{1}{1-g_i}\right) X_{ci}^{1-g_i}, \tag{3.1}$$

$$\text{subject to } X_{ci} + P_1 X_{si} = W_{i1}(P_1). \tag{3.2}$$

The optimal $t = 1$ stock holding of aggregate investor i from Equation (3) is

$$X_{si}(W_{i1}; A, P_1) = \left[\frac{-\pi^{-\frac{1}{g_i}}(A - P_1)^{-\frac{1}{g_i}} + (1-\pi)^{-\frac{1}{g_i}} P_1^{-\frac{1}{g_i}}}{\pi^{-\frac{1}{g_i}}(A - P_1)^{1-\frac{1}{g_i}} + (1-\pi)^{-\frac{1}{g_i}} P_1^{1-\frac{1}{g_i}}}\right] W_{i1}(P_1), \tag{4}$$

where we have explicitly indicated that the stock payoff parameter and the $t = 1$ stock price are parameters in the optimal stock holding. The optimal $t = 1$ cash holding is given by substituting Equation (4) in the investor's budget constraint.

At $t = 0$ investors maximize the expected value of the derived utility of $t = 1$ wealth. Substituting Equation (4) and the investor's budget constraint [Equation (3.2)] into the objective function [Equation (3.1)], aggregate investor i's derived utility of $t = 1$ wealth is

$$
\begin{aligned}
V_i(W_{i1}(P_1); A, P_1) = & \left(\frac{1}{1-g_i}\right)\pi(1-\pi)A^{1-g_i} \\
& \times \left[\pi^{-\frac{1}{g_i}}(A-P_1)^{1-\frac{1}{g_i}} + (1-\pi)^{-\frac{1}{g_i}} P_1^{1-\frac{1}{g_i}}\right]^{g_i} \\
& \times W_{i1}(P_1)^{1-g_i},
\end{aligned} \tag{5}
$$

where we have explicitly indicated that the stock payoff parameter and the $t = 1$ stock price are parameters in the derived utility function.

Having set up the basic structure above, our results come from making specific assumptions about the differences between the two aggregate investors and about trading opportunities at $t = 0$. We deal with two different situations in the sections that follow. The first of these examines a particular case of asymmetric information, while the second deals with a situation involving heterogeneous preferences. In both cases, the existence of an option market at $t = 0$ plays a pivotal role. To make the effects as transparent as possible, we assume homogeneous preferences in the first situation and symmetric information about security payoffs in the second.

2. The Effect of an Option Market on a Nonrevealing Equilibrium

We first consider a situation in which investors have identical preferences, $g_1 = g_2 \equiv g$, but there are two types of investors distinguished by their private beliefs about the final stock payoff. We construct a very simple nonrevealing equilibrium with no option market in which an option is a redundant security to every investor with respect to attainable $t = 2$ consumption patterns.[2] Specifically, we assume it is common knowledge that $\pi = \frac{1}{2}$ and that $A \in \{A_a, A_b, A_c, A_d\}$, where the true value of A will be revealed publicly after trading at $t = 0$ but before trading at $t = 1$.

[2] Our interest is in the effect of options on a nonrevealing equilibrium rather than the nature of the nonrevealing equilibrium itself. We employ the particular model described here as a convenient vehicle for demonstrating this effect, but the role of options in revealing more information is robust to the nature of the nonrevelation. We are grateful to an anonymous referee for emphasizing this point.

Table 1

		Prior beliefs about A			
True state	Investor type	A_a	A_b	A_c	A_d
A_a	1	q_{1ac}	0	$1 - q_{1ac}$	0
	2	q_{2ad}	0	0	$1 - q_{2ad}$
A_b	1	0	q_{1bd}	0	$1 - q_{1bd}$
	2	0	q_{2bc}	$1 - q_{2bc}$	0
A_c	1	q_{1ac}	0	$1 - q_{1ac}$	0
	2	0	q_{2bc}	$1 - q_{2bc}$	0
A_d	1	0	q_{1bd}	0	$1 - q_{1bd}$
	2	q_{2ad}	0	0	$1 - q_{2ad}$

Investors are endowed with prior beliefs about the four values of A such that each investor believes, correctly, that the true state is one of two possibilities. All investors of the same type have the same beliefs but different types have different beliefs. Specifically, the two possible states on which investors of type 1 place positive probabilities are not identical to the two to which type 2 investors assign positive probabilities. However, investors' beliefs are correlated with each other and with the true state, in that the true state is one to which both investor types assign positive probabilities. The specific common knowledge belief structure is shown in Table 1.

Both investor types have diffuse priors (before observing the $t = 0$ stock price) about the numerical values of the prior beliefs of the other type. After observing the $t = 0$ stock price, investors form posterior beliefs about A from inferences about the prior beliefs of investors of the other type, since the prior beliefs of investors contain information about the true state. Thus the situation is that everyone knows at $t = 0$ that other investors have different beliefs that contain information about the true value of A but do not know what those beliefs are. In addition, everyone knows that the information asymmetry will be resolved at $t = 1$.

After the true value of A is revealed, the equilibrium stock price at $t = 1$ is obtained by summing Equation (4) for $i = 1$ and $i = 2$, setting the sum equal to $\mathcal{S}$ to satisfy Equation (2.2) and solving for P_1. In general, the value of P_1 depends on the revealed value of A and also on the positions that investors took to maximize the expected value of Equation (5) in trading at $t = 0$. The latter positions depend, in turn, on the beliefs of investors at $t = 0$ about possible prices at $t = 1$. Thus, rational expectations equilibria at $t = 0$ and $t = 1$ must, in general, be obtained simultaneously.

However, since we are assuming homogenous preferences ($g_1 = g_2 \equiv g$) in this section, the conditions for aggregation prevail at $t = 1$ and the $t = 1$ stock price as a function of the revealed value of A is given by

$$P_1(A) = \frac{AC^g}{C^g + (C + AS)^g}. \tag{6}$$

At $t = 0$ all investors can calculate from Equation (6) the four possible values of the stock price at $t = 1$: $P_1(A_a)$, $P_1(A_b)$, $P_1(A_c)$, $P_1(A_d)$. In making their portfolio decisions at $t = 0$, investors of a given type will use their posterior beliefs about the possible values of A. As noted, their posterior beliefs are informed by information, if any, contained in the $t = 0$ stock price. Suppose, for the moment that the $t = 0$ stock price is uninformative about the priors of investors of the other type, and thus about the true state. In these circumstances, investors must rely on their prior beliefs about A to make their portfolio decisions so as to maximize the expected value of indirect utility given in Equation (5). When market clearing is imposed, this implies that the $t = 0$ stock price will depend on the true state and on the prior beliefs of the two investor types in that state. We write the $t = 0$ price as $P_0(A, q_1, q_2)$, where q_i is the q value in the beliefs of type i investors. Thus the four values of P_0 are $P_0(A_a, q_{1ac}, q_{2ad})$, $P_0(A_b, q_{1bd}, q_{2bc})$, $P_0(A_c, q_{1ac}, q_{2bc})$, and $P_0(A_d, q_{1bd}, q_{2ad})$.

In general, the four values of P_0 will all be different. It is critical to understand that this does *not* imply that a nonrevealing equilibrium does not exist at $t = 0$. The reason is that investors do not know the four values of P_0 because they do not know the prior beliefs of investors of the other type. Investors of type i cannot calculate $P_0(A, q_1, q_2)$ without knowing q_h for $h \neq i$. They can, of course, calculate a range of possible P_0 values corresponding to the range $0 < q_h < 1$. But since type i investors have diffuse priors about q_h, they also have diffuse priors about P_0 values in this range. Investors observe only the value of P_0 that clears the market at $t = 0$.

Investors of a given type, say type 1, have prior beliefs that the state is one of two possibilities, say A_j and A_k. They know that type 2 investors either have prior beliefs that the state is A_j or A_l (if the true state is A_j) or have prior beliefs that the state is A_k or A_m (if the true state is A_k). Suppose they observe the price P_0. Using Equation (5) to solve the optimization problem for type 2, they can solve for the possible type 2 beliefs that satisfy

$$P_0(A_j, q_{1jk}, \hat{q}_{2jl}) = P_0 = P_0(A_k, q_{1jk}, \hat{q}_{2km}).$$

If the true state is A_j and $P_0(\cdot, \cdot, q)$ is monotonic in q, the type 1 investors will obtain $\hat{q}_{2jl} = q_{2jl}$ (the true type 2 beliefs) from solving the relations above. That is, they will infer that one possibility (the correct one) is that $A = A_j$ and the prior beliefs of type 2 investors are q_{2jl}. However, if they also obtain a value $\hat{q}_{2km}$ in (0,1) from solving the above relations, they will infer that a second possibility is that $A = A_k$ and the beliefs of type 2 investors are $\hat{q}_{2km}$. If the true state is $A = A_j$, the beliefs of type 2 investors are not in fact $\hat{q}_{2km}$, but the type 1 investors don't know that because they don't know the true beliefs of the other investors. Therefore, type 1 investors learn nothing from observing P_0 as to whether the true state is A_j or A_k, and so their posterior probabilities are just their prior beliefs.

To see in general how the inferences described above are obtained, consider investors of type h who observe that the $t = 0$ stock price is P_0 and who are analyzing the possibility that the observed stock demands $Y_{si} = S - Y_{sh}$ of type i investors were produced by type i investors believing that $A = A_j$ with some probability $\hat{q}_{ijk}$ and that $A = A_k$ with probability $1 - \hat{q}_{ijk}$. Maximizing the indirect utility [Equation (5)] of the aggregate type i investor, this possibility requires that $\hat{q}_{ijk}$ lie in (0, 1) and satisfy

$$\hat{q}_{ijk} = \frac{F_k G_j}{F_j G_k + F_k G_j}, \tag{7}$$

$$\text{where } F_n \equiv [P_1(A_n) - P_0]A_n^{1-g}\{[A_n - P_1(A_n)]^{1-\frac{1}{g}} + P_1(A_n)^{1-\frac{1}{g}}\}^g,$$

$$G_n \equiv \{Z_{ci} + P_0 Z_{si} + [P_1(A_n) - P_0]Y_{si}\}^g, \qquad n = j, k,$$

and $P_1(A_n)$ is given by Equation (6).

For example, consider the situation in which type 1 investors have prior beliefs that $A = A_a$ with probability q_{1ac} and $A = A_c$ with probability $1 - q_{1ac}$ and have used these beliefs to choose $t = 0$ stock demand Y_{s1} when the $t = 0$ stock price is P_0. From the common knowledge information structure, the type 1 investors know that either (i) the true state is A_a and type 2 investors have prior beliefs that the state is A_a or A_d or (ii) the true state is A_c and the type 2 investors have prior beliefs that the state is A_b or A_c. They can use Equation (7) to calculate that the first alternative requires that the beliefs of the type 2 investors are $\{\text{prob}(A = A_a) = \hat{q}_{2ad}, \text{prob}(A = A_d) = 1 - \hat{q}_{2ad}\}$, while the second alternative requires that the type 2 investors' beliefs are $\{\text{prob}(A = A_b) = \hat{q}_{2bc}, \text{prob}(A = A_c) = 1 - \hat{q}_{2bc}\}$. Provided both $\hat{q}_{2ad}$ and $\hat{q}_{2bc}$ as calculated from Equation (7) lie in (0,1), the type 1 investors cannot distinguish between these two possibilities.

For the equilibrium to be nonrevealing, it is necessary as well that both sets of beliefs calculated for the type 2 investors, $\hat{q}_{2ad}$ and $\hat{q}_{2bc}$, be rational posterior beliefs when the $t = 0$ price is P_0. This requires, in turn, that there exist beliefs $\hat{q}_{1ac}$ and $\hat{q}_{1bd}$ for the type 1 investors

Table 2

True state	Required for nonrevealing equilibrium
A_a	$\hat{q}_{1bd} \in (0, 1)$ and $\hat{q}_{2bc} \in (0, 1)$ satisfy Equation (7) for $P_0 = P_0(A_a, q_{1ac}, q_{2ad})$.
A_b	$\hat{q}_{1ac} \in (0, 1)$ and $\hat{q}_{2ad} \in (0, 1)$ satisfy Equation (7) for $P_0 = P_0(A_b, q_{1bd}, q_{2bc})$.
A_c	$\hat{q}_{1bd} \in (0, 1)$ and $\hat{q}_{2ad} \in (0, 1)$ satisfy Equation (7) for $P_0 = P_0(A_c, q_{1ac}, q_{2bc})$.
A_d	$\hat{q}_{1ac} \in (0, 1)$ and $\hat{q}_{2bc} \in (0, 1)$ satisfy Equation (7) for $P_0 = P_0(A_d, q_{1bd}, q_{2ad})$.

that satisfy Equation (7) for the observed price P_0 and that lie in (0,1). Using the fact that the actual beliefs necessarily satisfy Equation (7) in the true state, the requirements in each possible true state for a nonrevealing equilibrium are summarized in Table 2.

It must be reemphasized that the requirements for a nonrevealing rational expectations equilibrium are not that $P_0(A, q_1, q_1)$ be the same for every state, because the $P_0(A, q_1, q_2)$ values are unknown to investors since the priors of investors of the other type are unknown. What does have to hold for a nonrevealing equilibrium, as expressed by the requirements in Table 2, is that the observed value of P_0 be consistent with $P_0(A, q_1, q_2)$ values for each state for some priors that other investors *may* hold. This implies that $\hat{q}_{ijk}$ satisfying Equation (7) when the true state is A_m need not be equal to $\hat{q}_{ijk}$ satisfying Equation (7) when the true state is A_n.

A fully revealing equilibrium at $t = 0$ always exists (except on a parameter set of measure zero). Whether a nonrevealing equilibrium also exists depends on parameter values; we present such an equilibrium in a numerical example below.

Our primary interest in a nonrevealing equilibrium at $t = 0$ is in its implications for the role of options. Note first that an option market at $t = 1$ would be superfluous. At $t = 1$ everyone knows the two possible states of nature at $t = 2$, and the $t = 1$ securities market contains two securities with linearly independent payoff vectors. There is no $t = 2$ consumption pattern that an investor could achieve by trading in cash, stock, and an option at $t = 1$ that cannot be achieved by trading only in cash and stock.

In a nonrevealing equilibrium, the source of investors' uncertainty about $t = 1$ wealth is uncertainty about the $t = 1$ stock price. Type 1 and type 2 investors know that the $t = 1$ price will be one of two values (though not the same two). Thus, in the view of each investor, the $t = 1$ price has a binomial probability distribution. But this implies that each investor sees cash and stock as sufficient to achieve any $t = 1$ wealth pattern that could be achieved if any additional security or option were also available. Hence, any role for options in the nonrevealing equilibrium must be different from Ross's (1976) treatment of the role of options in making available payoff patterns that are unattainable with stocks and bonds. In our model, each investor views an option as a redundant asset for achieving payoff patterns when cash and stock are available.

Ironically, the role that trading in an option at $t = 0$ can play in connection with a nonrevealing equilibrium is to prevent such an equilibrium from being established. This happens because of the information conveyed by investors' actions in response to the price at which the option trades. In other words, the crucial point is not the existence of an option as an alternative means for an investor to arrange a future payoff pattern, but rather the information disclosed by the price at which investors are willing to take positions in the option.

To see this, suppose for example that the true state is A_b and there is a nonrevealing equilibrium at $t = 0$ (in the absence of option trading) and that the ordering of the $t = 0$ price and the alternative $t = 1$ prices are $P_1(A_a) < P_1(A_b) < P_0 < P_1(A_c) < P_1(A_d)$.

Suppose we introduce trading in a call option on the stock expiring at $t = 1$ with exercise price Q, where $P_1(A_c) < Q < P_1(A_d)$. The $t = 1$ payoff to the call is

$$d(P_1) = \max\{P_1 - Q, 0\}.$$

Now type 1 investors, who believe that the true state is A_b or A_d, can reason as follows:

> If the true state is A_b, so that type 2 investors believe that it is A_b or A_c, then they are sure the option's payoff will be zero. On the other hand, if the true state is A_d, so that type 2 investors believe that it is A_a or A_d, then they believe that there is a positive probability that the option will pay $P_1(A_d) - Q > 0$.

Similarly, type 2 investors, who believe that the true state is A_b or A_c, can reason:

> If the true state is A_b, so that type 1 investors believe that it is A_b or A_d, then they believe that there is a positive probability that the option will pay $P_1(A_d) - Q > 0$. On the other hand, if the true state is A_c, so that type 1 investors believe that it is A_a or A_c, then they are sure that the option's payoff will be zero.

Considering how investors in each group can reason about the beliefs of investors in the other group, it is easy to see that introducing this option would cause a nonrevealing equilibrium to break down. In the true state A_b, the fact that type 2 investors are willing to write an unlimited supply of the option at any positive price reveals that they believe the true state is A_b or A_c. This informs type 1 investors that the true state is A_b and so they will take unlimited positions in the stock at any stock price different from $P_1(A_b)$.[3]

If the true state is different and/or a different ordering of the alternative $t = 1$ stock prices is considered, the details of the argument change but the final conclusion is the same: introduction of an option breaks the nonrevealing equilibrium. The key point, as we noted earlier, is that the option is a bet on the future price of the stock. Therefore, an investor's demand for the option depends on the investor's beliefs, and this implies that the price of the option informs one or both investor types of which beliefs the other type holds.

Since introduction of trading in the option reveals both investor types' private beliefs, it results in a revealing equilibrium in which the $t = 0$ price is based on all investors knowing the true state. With this knowledge, investors trade directly to final portfolio holdings at $t = 0$ and no further trading or price change occurs at $t = 1$.

The value of P_1 for the revealing equilibrium will, in general, differ from the value of P_1 that occurs at $t = 1$ in a nonrevealing equilibrium. This is because in the revealing equilibrium investors trade to final holdings at $t = 0$ from their initial endowments knowing the true value of A, while in the nonrevealing equilibrium there is a round of trade at $t = 0$ at price P_0 before investors learn A and trade to final portfolios. However, in the special case of homogeneous preferences, that is, $g_1 = g_2$, which we employ, the price in the revealing equilibrium will equal the corresponding $t = 1$ price in the nonrevealing equilibrium. This is because when preferences are homogeneous and the true state has been revealed at $t = 1$, the conditions for aggregation are satisfied and so the $t = 1$ price is not affected by how wealth is redistributed across investors as a result of $t = 0$ trading.

3. Numerical Example 1

To show how a nonrevealing equilibrium is affected by the introduction of an option market, we turn to a numerical example. The data for the example are given in Table 3. The two types of investors are identical except for their prior beliefs about A. The stock's payoff per share at $t = 2$ is A with probability π and zero otherwise, where $A \in \{0.4, 0.8, 2.4, 3.2\}$ and $\pi = 0.5$.

[3] Our assumption of atomistic, price-taking investors is crucial here. Strategic trading by large investors could lead to a different result.

Table 3
Numerical example 1—parameter values

Stock payoffs and $t = 1$ prices

$\pi = 0.5$

$A_a = 0.4$	$P_1(A_a) = 0.185$
$A_b = 0.8$	$P_1(A_b) = 0.341$
$A_c = 2.4$	$P_1(A_c) = 0.752$
$A_d = 3.2$	$P_1(A_d) = 0.867$

Preferences, aggregate endowments, and prior beliefs

	Type 1 investors	Type 2 investors
Risk aversion coefficient	$g_1 = 2$	$g_2 = 2$
Cash	$Z_{c1} = 5$	$Z_{c2} = 5$
Stock	$Z_{s1} = 1$	$Z_{s2} = 1$
Prior beliefs	$q_{1ac} = 0.5$	$q_{2ad} = 0.5$
	$q_{1bd} = 0.5$	$q_{2bc} = 0.5$

If the true state is $A = 0.4$ or $A = 2.4$ (i.e., A_a or A_c), then type 1 investors have prior beliefs that $A = 0.4$ with probability q_{1ac} and $A = 2.4$ with probability $1 - q_{1ac}$. If the true state is $A = 0.8$ or $A = 3.2$ (i.e., A_b or A_d), then type 1 investors have prior beliefs that $A = 0.8$ with probability q_{1bd} and $A = 3.2$ with probability $1 - q_{2bd}$. Type 2 investors' prior beliefs are such that if the true state is $A = 0.4$ or $A = 3.2$ (i.e., A_a or A_d), then they believe that $A = 0.4$ with probability q_{2ad} and $A = 3.2$ with probability $1 - q_{2ad}$. If the true state is $A = 0.8$ or $A = 2.4$ (*i.e.*, A_b or A_c), then type 2 investors hold prior beliefs that $A = 0.8$ with probability q_{2bc} and $A = 2.4$ with probability $1 - q_{2bc}$. For simplicity, we assume that $q_{1ac} = q_{1bd} = q_{2ad} = q_{2bc} = 0.5$. It is important to recall, however, that type 1 investors have completely diffuse priors about q_{2ad} and q_{2bc} and type 2 investors have completely diffuse priors about q_{1ac} and q_{1bd}.

The solution values for nonrevealing equilibria for each possible state are given in Table 4. Knowing the true beliefs in Table 3, we can calculate that one of four sets of numerical values of P_0 and demands Y_{c1}, Y_{c2}, Y_{s1}, Y_{s2} will occur, depending on whether the true state is A_a, A_b, A_c or A_d. Since investors do not know the prior beliefs of investors of the other type, investors cannot calculate these four sets of numerical values, however. Instead, they observe *one* set of numerical values and form their posterior beliefs from inferences using those observed values. To see how the nonrevealing equilibria work, suppose the true state is A_a (i.e., $A = 0.4$). At $t = 0$ investors observe a stock price of $P_0 = 0.451$ and each type can observe the other type's cash and stock demands. Type 1 investors, who believe that the true state is A_a or A_c, can calculate that the observed type 2 demands at $P_0 = 0.451$ are consistent with either A_a being the true state and type 2 investors having prior beliefs $\hat{q}_{2ad} = 0.5$ or A_c being the true state and type 2 investors having prior beliefs $\hat{q}_{2bc} = 0.677$. Similarly, type 2 investors, who believe that the true state is A_a or A_d, can calculate that the observed type 1 demands at $P_0 = 0.451$ are consistent with either the true state being A_a and type 1 investors having prior beliefs $\hat{q}_{1ac} = 0.5$ or A_d being the true state and type 1 investors having prior beliefs $\hat{q}_{1bd} = 0.768$. Since each type had diffuse priors about the other type's priors, neither type can exclude either of the two possibilities it calculates and so the nonrevealing equilibrium is supported by both types' Bayesian posterior beliefs, which are the same as their prior beliefs.

If we now open a market for a call option on the stock with an exercise price between $P_1(A_c)$ and $P_1(A_d)$ expiring at $t = 1$, the reaction of investors to the possibility of trading this option will undo the nonrevealing equilibrium in the manner described earlier. If the true state is A_a, type 1 investors will want to write an unlimited supply of calls at any positive price. If the true state is A_b, type 2 investors will want to write an unlimited supply of calls at any positive price. If the true state is A_c, both types will want to write calls at any positive price. If the true state is A_d, both types will want to buy calls if the price is low enough. These differences in behavior in the four states will reveal the identity of the true state.

In the nonrevealing equilibrium, every investor believes the $t = 1$ stock price has a binomial distribution. So every investor views an option's payoffs as spanned by a combination of stock and cash. However, exactly what combination of stock and cash replicates the $t = 1$ payoffs to a given option is viewed differently by the two investor types. This implies that the two types value the option differently from each other and differently from how they would value it if they had received the other signals consistent with the nonrevealing equilibrium. This is why opening trading in the option leads to the private signals being revealed.

4. Options and Self-supporting Price Uncertainty

We now consider a situation in which investors have common information about final payoffs but differ in risk aversion. Again, the securities available for trading at $t = 0$ will play a crucial role in the analysis. Before dealing with portfolio choice, however, we specify the information about final payoffs that investors possess at $t = 0$. We assume that all investors know that the $t = 2$ stock payoff will come

Table 4
Numerical example 1—nonrevealing equilibria

a) Observed price:	$P_0 = 0.451$	
Observed demands:	$Y_{c1} = 5.26$	$Y_{c2} = 4.74$
	$Y_{s1} = 0.42$	$Y_{s2} = 1.58$
Consistent alternatives:		
$A = A_a$	$\hat{q}_{1ac} = 0.500$	$\hat{q}_{2ad} = 0.500$
$A = A_b$	$\hat{q}_{1bd} = 0.768$	$\hat{q}_{2bc} = 0.677$
$A = A_c$	$\hat{q}_{1ac} = 0.500$	$\hat{q}_{2bc} = 0.677$
$A = A_d$	$\hat{q}_{1bd} = 0.768$	$\hat{q}_{2ad} = 0.500$
b) Observed price:	$P_0 = 0.546$	
Observed demands:	$Y_{c1} = 4.37$	$Y_{c2} = 5.63$
	$Y_{s1} = 2.15$	$Y_{s2} = -0.15$
Consistent alternatives:		
$A = A_a$	$\hat{q}_{1ac} = 0.259$	$\hat{q}_{2ad} = 0.466$
$A = A_b$	$\hat{q}_{1bd} = 0.500$	$\hat{q}_{2bc} = 0.500$
$A = A_c$	$\hat{q}_{1ac} = 0.259$	$\hat{q}_{2bc} = 0.500$
$A = A_d$	$\hat{q}_{1bd} = 0.500$	$\hat{q}_{2ad} = 0.466$
c) Observed price:	$P_0 = 0.497$	
Observed demands:	$Y_{c1} = 6.07$	$Y_{c2} = 3.93$
	$Y_{s1} = -1.16$	$Y_{s2} = 3.16$
Consistent alternatives:		
$A = A_a$	$\hat{q}_{1ac} = 0.500$	$\hat{q}_{2ad} = 0.339$
$A = A_b$	$\hat{q}_{1bd} = 0.739$	$\hat{q}_{2bc} = 0.500$
$A = A_c$	$\hat{q}_{1ac} = 0.500$	$\hat{q}_{2bc} = 0.500$
$A = A_d$	$\hat{q}_{1bd} = 0.739$	$\hat{q}_{2ad} = 0.339$
d) Observed price:	$P_0 = 0.538$	
Observed demands:	$Y_{c1} = 4.19$	$Y_{c2} = 5.81$
	$Y_{s1} = 2.51$	$Y_{s2} = -0.51$
Consistent alternatives:		
$A = A_a$	$\hat{q}_{1ac} = 0.256$	$\hat{q}_{2ad} = 0.500$
$A = A_b$	$\hat{q}_{1bd} = 0.500$	$\hat{q}_{2bc} = 0.533$
$A = A_c$	$\hat{q}_{1ac} = 0.256$	$\hat{q}_{2bc} = 0.533$
$A = A_d$	$\hat{q}_{1bd} = 0.500$	$\hat{q}_{2ad} = 0.500$

from one of two probability distributions. Under payoff distribution a, the stock's payoff is A_a with probability π_a and zero otherwise; under payoff distribution b, the payoff is A_b with probability π_b and zero otherwise. Investors know that the probabilities of payoff distributions a and b occurring are q and $1-q$, respectively. Investors also know that the true payoff distribution will be learned publicly after trading at $t = 0$ but before trading at $t = 1$.

Assuming equilibrium is unique, the market we have described is dynamically complete.[4] At $t = 0$ investors believe that one of two

[4] Under certain circumstances, multiple equilibria are possible. We return to this point later.

situations will prevail at $t = 1$ and there are two securities available at $t = 0$ to hedge this uncertainty. Similarly, at $t = 1$ investors believe that one of two states of nature will occur at $t = 2$ and there are two securities to hedge this risk. The addition of another trading instrument, such as an option, cannot have an effect on this market that can be ascribed to the market being incomplete without that instrument.

As before, we first consider the equilibrium in the absence of an option market. After the true payoff distribution is revealed, the equilibrium stock price at $t = 1$ is obtained by summing Equation (4) over i, equating to S, and solving for P_1. Doing this for each of the possible payoff distributions produces expressions for $t = 1$ prices $P_1(A_a)$ and $P_1(A_b)$, each of which depends on $t = 0$ portfolio choices and the $t = 0$ price P_0. The latter is given by maximizing the expected value of Equation (5) for each i and equating the sum of the optimal $t = 0$ stock demands to S. Thus the conditions for equilibrium at $t = 0$ and $t = 1$ that must be satisfied simultaneously are

$$W_{i1}(P_n) \equiv W_{i0}(P_0) + (P_n - P_0)Y_{si}, \quad i = 1, 2, \quad n = a, b, \tag{8.1}$$

$$\begin{aligned} &X_{s1}(W_{11}(P_1(A_a)); A_a, \pi_a, P_1(A_a)) \\ &\qquad + X_{s2}(W_{21}(P_1(A_a)); A_a, \pi_a, P_1(A_a)) = S, \end{aligned} \tag{8.2}$$

$$\begin{aligned} &X_{s1}(W_{11}(P_1(A_b)); A_b, \pi_b, P_1(A_b)) \\ &\qquad + X_{s2}(W_{21}(P_1(A_b)); A_b, \pi_b, P_1(A_b)) = S, \end{aligned} \tag{8.3}$$

$$\begin{aligned} &\tfrac{\partial}{\partial Y_{si}}\big[qV_i(W_{i1}(P_1(A_a)); A_a, \pi_a, P_1(A_a)) \\ &\qquad + (1-q)V_i(W_{i1}(P_1(A_b)); A_b, \pi_b, P_1(A_b))\big] \\ &\qquad = 0, \quad i = 1, 2, \end{aligned} \tag{8.4}$$

$$Y_{s1} + Y_{s2} = S, \tag{8.5}$$

where we have shown π_n as an explicit parameter since it differs in the two payoff distributions. In the general case, the equilibrium in Equation (8) exists and is unique.

Now we consider the effect of changing two aspects of the market situation we have described. First, we assume that trading is possible at $t = 0$ in a stock option (in zero net supply) exercisable at $t = 1$. Second, we assume that investors perceive uncertainty in forecasting one or both of the $t = 1$ stock prices; for simplicity, we will assume their uncertainty applies to $P_1(A_a)$ but not to $P_1(A_b)$. In other words, investors believe they know what the $t = 1$ price will be if payoff distribution b prevails but are uncertain about the $t = 1$ price if payoff distribution a occurs (even though they know all the parameters of each case). To be specific, we assume investors conjecture that under payoff distribution a the $t = 1$ price may be $P_1^l(A_a)$ or it may be $P_1^h(A_a)$, depending on some extraneous event or simply an announcement that the price is to be low (l) or high (h).

We assume that investors have common beliefs about the probability of occurrence of the payoff distributions a and b but differ in the probabilities of their conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$ conditional on payoff distribution a occurring. In other words, investors agree about the probabilities of "real" events that affect final payoffs but disagree about the likelihood of conjectured events that are not connected to final payoffs, and which, in fact, occur only because of the conjectures about them.

Without, for the moment, discussing the basis for investors' conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$, we note that holding such conjectures gives investors potentially two reasons to be interested in an option excercisable at $t = 1$. For one thing, investors want to hedge against the endowment risk associated with the $t = 1$ stock price. If they conjecture that this price can take three values [$P_1^l(A_a)$, $P_1^h(A_a)$, and $P_1(A_b)$], then the market becomes incomplete: cash and stock are insufficient to span the price uncertainty. Adding an option gives three securities for hedging the price uncertainty. Furthermore, if investors have different conjectures about the conditional probabilities of $P_1^l(A_a)$ and $P_1^h(A_a)$, then they would like to have a vehicle for making side bets about which price will prevail. Thus, investors' conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$ can lead them to hold nonzero option positions at $t = 0$.

What is more interesting is that the second of the motives above can lead to option positions taken by the investors at $t = 0$ that support the multiple $t = 1$ prices $P_1^l(A_a)$ and $P_1^h(A_a)$ that caused investors to take those option positions. To see why this can occur, assume for concreteness that the option is a put with exercise price between $P_1^l(A_a)$ and $P_1^h(A_a)$. Assume also that there are two types of investors distinguished by different degrees of risk aversion and that the more risk-averse investors conjecture a greater likelihood of $P_1^l(A_a)$ relative to $P_1^h(A_a)$ than the less risk-averse type. The more risk-averse investors have a greater demand for the put at any price than do the less risk-averse investors, so there will be a price of the put that produces equal, offsetting positions. The more risk-averse type will hold long positions in the put and will have paid cash to the less risk-averse to write the put.

The payoff to the put depends on the $t = 1$ stock price. Suppose stock payoff distribution a prevails and it is announced that the stock price is below the exercise price of the put. Then the put expires in the money and the less risk-averse investors (who wrote the put) transfer cash to the more risk-averse investors (who own the put). Thus, relative to the situation without options, the more risk-averse investors have more wealth and the less risk-averse investors have less wealth. This implies that the $t = 1$ stock price will indeed be lower than it would have been without the put [i.e., lower then $P_1(A_a)$].

On the other hand, if it is announced that the stock price at $t = 1$ is above the exercise price of the put, then the put expires worthless. But since the more risk-averse investors paid cash to the less risk-averse investors at $t = 0$ for writing the put, the less risk-averse investors have more wealth and the more risk-averse investors have less wealth relative to the situation without options. So the $t = 1$ stock price will indeed be higher than $P_1(A_a)$.

In other words, the opposite put positions held by the two investor types imply that if the $t = 1$ stock price is declared to be below the exercise price on the put, then settlement of the put implies a stock price below $P_1(A_a)$. If the $t = 1$ stock price is declared to be above the exercise price, on the other hand, then settlement of the put (i.e., that it is worthless) implies a stock price above $P_1(A_a)$. Provided the low and high $t = 1$ stock prices turn out to equal the $P_1^l(A_a)$ and $P_1^h(A_a)$ prices investors conjectured, the $t = 1$ equilibrium confirms the conjectures.

We assumed, for simplicity, that investors predict a single stock price at $t = 1$ under payoff distribution b. In general, that price will be different from the price that would prevail under payoff distribution b when options are absent. When the put option is available and investors take positions in it at $t = 0$, this alters the relative wealth of the less risk-averse investors compared to the more risk-averse investors at $t = 1$ under either stock payoff distribution. The difference in what happens under the two payoff distributions is that different outcomes for the option occur under payoff distribution a, depending on whether a low or high stock price is announced, while under payoff distribution b the outcome for the option is always the same.

To make these ideas more concrete, assume that investors' conjectures about the $t = 1$ stock price satisfy $P_1^l(A_a) < P_1^h(A_a) < P_1(A_b)$ and there is a market at $t = 0$ for a put option with price P_p that expires at $t = 1$ and has exercise price Q, where $P_1^l(A_a) < Q < P_1^h(A_a)$. The $t = 1$ payoff to the put is

$$D(P_1) = \max\{Q - P_1, 0\}.$$

All investors believe that stock payoff distributions a and b will occur with probabilities q and $1 - q$, respectively, and they all conjecture that under payoff distribution a the stock price may be either $P_1^l(A_a)$ or $P_1^h(A_a)$. However, type 1 investors believe the conditional probability of $P_1^l(A_a)$ given a is r_1, while type 2 investors believe the conditional probability of $P_1^l(A_a)$ is r_2. As we have noted, the asymmetry in beliefs concerns only conjectures about the likelihood of different stock prices conditional on the occurrence of a given payoff distribution. With respect to uncertainty about final stock payoffs, investors have common beliefs. Given these beliefs, the conditions for equilibrium at $t = 0$ and $t = 1$ are

$$W_{i1}(P_n) \equiv W_{i0}(P_0) + (P_n - P_0)\,Y_{si} + [D(P_n) - P_p]\,Y_{oi}, \quad i = 1, 2, \tag{9.1}$$

$$\begin{aligned} &X_{s1}(W_{11}(P_1^l(A_a)); A_a, \pi_a, P_1^l(A_a)) \\ &\quad + X_{s2}(W_{21}(P_1^l(A_a)); A_a, \pi_a, P_1^l(A_a)) = S, \end{aligned} \tag{9.2}$$

$$\begin{aligned} &X_{s1}(W_{11}(P_1^b(A_a)); A_a, \pi_a, P_1^b(A_a)) \\ &\quad + X_{s2}(W_{21}(P_1^b(A_a)); A_a, \pi_a, P_1^b(A_a)) = S, \end{aligned} \tag{9.3}$$

$$\begin{aligned} &X_{s1}(W_{11}(P_1(A_b)); A_b, \pi_b, P_1(A_b)) \\ &\quad + X_{s2}(W_{21}(P_1(A_b)); A_b, \pi_b, P_1(A_b)) = S, \end{aligned} \tag{9.4}$$

$$\begin{aligned} &\frac{\partial}{\partial Y_{si}} \big[q r_i V_i(W_{i1}(P_1^l(A_a)); A_a, \pi_a, P_1^l(A_a)) \\ &\quad + q(1 - r_i)\,V_i(W_{i1}(P_1^b(A_a)); A_a, \pi_a, P_1^b(A_a)) \\ &\quad + (1 - q)\,V_i(W_{i1}(P_1(A_b)); A_b, \pi_b, P_1(A_b))\big] = 0,\ i = 1, 2, \end{aligned} \tag{9.5}$$

$$\begin{aligned} &\frac{\partial}{\partial Y_{oi}} \big[q r_i V_i(W_{i1}(P_1^l(A_a)); A_a, \pi_a, P_1^l(A_a)) \\ &\quad + q(1 - r_i)\,V_i(W_{i1}(P_1^b(A_a)); A_a, \pi_b, P_1^b(A_a))\big] \\ &\quad + (1 - q)\,V_i(W_{i1}(P_1(A_b)); A_b, \pi_b, P_1(A_b))\big] = 0,\ i = 1, 2, \end{aligned} \tag{9.6}$$

$$Y_{s1} + Y_{s2} = S, \tag{9.7}$$

$$Y_{o1} + Y_{o2} = 0, \tag{9.8}$$

The equilibrium in Equation (8) is one solution to Equation (9), with $P_1^l(A_a) = P_1^b(A_a) = P_1(A_a)$ and $Y_{o1} = Y_{o2} = 0$, but not the solution we are interested in. Whether an equilibrium with $P_1^l(A_a) \neq P_1^b(A_a)$ also exists depends on parameter values. We will demonstrate the existence of this equilibrium in a numerical example in the next section. For now, we assume such a solution to Equation (9) exists.

As we have noted, the existence of the option market and the option positions taken by investors based on their conjectures about the $t = 1$ stock price alter all of the prices in the equilibrum compared to the situation without options. In particular, the values of P_0 and $P_1(A_b)$ that satisfy the equilibrium without options in Equation (8) differ from the values of those prices in the equilibrium with the put option in Equation (9).

Because all prices are changed by the existence of the option market, it is not obvious how investors' welfare is affected. Since $P_1^l(A_a) < P_1(A_a)$, the smallest potential $t = 1$ stock price is lower when the put is available than when it is absent. However, the maximum potential $t = 1$ stock price, $P_1(A_b)$, is also different in the two situations, so the effect of the put option on $t = 1$ stock price volatility is ambiguous. Furthermore, in both situations the market is dynamically complete, so investors are not constrained by market incompleteness in hedging future price risk. In particular, the existence of the option allows investors to hedge the extra stock price uncertainty under payoff distribution a that results from their conjectures.

Because investors make different conjectures about the probabilities of prices $P_1^l(A_a)$ and $P_1^h(A_a)$, and these conjectures affect their option positions, the calculation of welfare effects is ambiguous. If each investor's own conjectures are used in calculating expected utilities, the existence of an option market along with extra stock price uncertainty makes every investor better off relative to the equilibrium without an option market. However, this reflects the general result that if two individuals hold different beliefs about a future event and are allowed to make a bet between them about the event, both individuals will feel themselves to be better off because each will believe she has the odds in her favor.

As an alternative, one might assume that the two investor groups' beliefs are equally likely to be "correct" in some "objective" sense and employ the average of their beliefs to calculate expected utilities. (Obviously, these are not the utilities expected by the investors themselves). We show the results of both approaches in the following example.

Assuming an equilibrium in Equation (9) with $P_1^l(A_a) \neq P_1^h(A_a)$ exists, two aspects of this equilibrium are of particular interest. One is the effect of the difference in conditional conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$, given a, by the two investor types. The second is the role played by the option market in producing a different equilibrium than when only cash and stock are traded. Interestingly, the answers to these questions are related. We will address the economic intuition of these matters below and return to them in the context of the numerical example in the next section.

When investors take positions at $t = 0$ that support two prices, $P_1^l(A_a)$ and $P_1^h(A_a)$, conditional on distribution a being revealed at $t = 1$, they obviously add risk relative to an equilibrium which supports only a single price, $P_1(A_a)$, for distribution a. Taking on this extra risk will be Pareto dominated by avoiding it unless the ability to add expected utility through a sidebet on the extra risk offsets the negative effect. But the availability of such a sidebet can only increase expected utility for both investor types if they hold different beliefs about the outcome of the side bet, that is, about the conditional probabilities of $P_1^l(A_a)$ and $P_1^h(A_a)$.

With homogeneous conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$ the situation is equivalent to the investors agreeing at $t = 0$ to have their $t = 1$ wealth under distribution a depend on the outcome of an actuarially fair risk. However, with homogeneous conjectures we can supply Theorem 1 of Breeden and Litzenberger (1978) to show that because aggregate consumption across states of nature is the same whether $P_1 = P_1^l(A_a)$ or $P_1 = P_1^h(A_a)$, individual allocations must also be the same. Hence, a risk-averse investor would not agree to add such a risk, but might agree to add a risk she regarded as actuarially biased in her favor. Having different conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$ allows the latter situation to prevail from each investor type's viewpoint. Therefore, we would expect that the magnitude of the sidebet, and thus the divergence from the positions taken when options are not available, will depend on the degree of divergence in conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$. This implies that the difference between $P_1^l(A_a)$ and $P_1^h(A_a)$, which depends on the divergence of the investors' positions from the no-option equilibrium, will also depend on the divergence in conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$. We will examine this behavior in the numerical example.

As mentioned, the role played by the existence of an option market is related to the effect of different investor conjectures about $P_1^l(A_a)$ and $P_1^h(A_a)$. Suppose no option market exists and, for the moment, suppose everyone knows at $t = 0$ that payoff probability distribution a will prevail at $t = 1$. With two rounds of trading, at $t = 0$ and $t = 1$, it is possible for investors to take optimal portfolio positions in cash and stock at $t = 0$ that support two prices, $P_1^l(A_a)$ and $P_1^h(A_a)$, at $t = 1$.[5] This occurs when one investor type is short cash and the other is short stock at $t = 0$ such that the aggregate excess demand curve for stock at $t = 1$ is an inverted U-shape with two roots and

[5] We are indebted to an anonymous referee for pointing this out.

these positions are optimal when investors conjecture at $t = 0$ that the $t = 1$ price can be $P_1^l(A_a)$ or $P_1^h(A_a)$. This equilibrium is described by the conditions of Equations (9.1)–(9.3) and (9.5)–(9.7) with $q = 1$ and $Y_{o1} = Y_{o2} = 0$.

With only payoff distribution a being possible at $t = 1$, then even with two $t = 1$ prices, $P_1^l(A_a)$ and $P_1^h(A_a)$, the $t = 0$ market is dynamically complete with two securities—cash and stock—and no option. Suppose now that the payoff distribution to be revealed at $t = 1$ is no longer guaranteed to be distribution a but may also be distribution b. Now the $t = 0$ portfolio positions that support two prices under payoff distribution a are no longer optimal when there is also a probability of payoff distribution b occurring.

With investors conjecturing three $t = 1$ prices—$P_1^l(A_a)$, $P_1^h(A_a)$, and $P_1(A_b)$—and only two securities—cash and stock—there is a trade-off for one investor type between the gain of having a sidebet on the distribution a price and the loss of having a portfolio position that implies very low wealth under payoff distribution b. How the optimal $t = 0$ portfolio position balances this trade-off depends on the probability of distribution b occurring. If this probability is sufficiently low, the optimal $t = 0$ portfolio positions may still support two prices under distribution a. But as the probability of distribution b is increased, at some point it is no longer optimal for the $t = 0$ positions to be sufficiently extreme to support two prices under distribution a and the only equilibrium is the "usual" one described in Equation (8).

The inability of investors to have a portfolio that accommodates the distribution risk of a and b and still permits a sidebet about the price under distribution a is a result of the market being dynamically incomplete at $t = 0$ when just cash and stock are available and $P_1^l(A_a)$, $P_1^h(A_a)$, and $P_1(A_b)$ are possible $t = 1$ prices. Adding an option makes the market with two conjectured prices under distribution a dynamically complete and leads to $t = 0$ portfolio positions that support the conjectured prices. To sum up, the market with no option is dynamically complete when only the two $t = 1$ prices, $P_1(A_a)$ and $P_1(A_b)$, are contemplated at $t = 0$. In this market, an option is a redundant security. But introducing the option allows conjectures about two distribution a prices, $P_1^l(A_a)$ and $P_1^h(A_a)$, that induce $t = 0$ portfolio positions that support the conjectured $t = 1$ prices. In this three-future-price equilibrium the option is no longer redundant.

Our efforts to prove the points discussed above in the context of a general model have had limited success. What complicates the equilibria in our model, and the proving of theorems about them, is that by design the conditions for aggregation are not satisfied. The distribution of wealth across investor types affects prices, and therefore the equilibrium prices and portfolios at $t = 0$ and $t = 1$ must be de-

Table 5
Numerical example 2—parameter values

Stock payoff		
Distribution a : $A_a = 5, \pi_a = 0.5$		
Distribution b : $A_b = 10, \pi_b = 0.8$		
Preferences, aggregate endowments, and beliefs	Type 1 investors	Type 2 investors
Risk aversion coefficient	$g_1 = 1$	$g_2 = 3$
Cash	$Z_{c1} = 5$	$Z_{c2} = 5$
Stock	$Z_{s1} = 5$	$Z_{s2} = 5$
Belief about conditional probability of P_{al}	$r_1 = 0.2$	$r_2 = 0.8$

termined jointly in one system of nonlinear equations. Although we have not been able to prove the desired propositions in the context of the general case, we can demonstrate their operation in a numerical example. We turn to this exercise in the following section.

5. Numerical Example 2

The data for the example are in Table 5. The stock's payoff per share obeys one of two probability distributions. Under distribution a the payoff is 5 with probability 0.5 and zero otherwise; under distribution b the payoff is 10 with probability 0.8 and zero otherwise.

The two types of investors have identical endowments but differ in two respects: risk aversion and beliefs about the conditional likelihood of $P_1^l(A_a)$ relative to $P_1^h(A_a)$ given payoff distribution a. Both types conjecture that under payoff distribution a the stock may have either a low price, $P_1^l(A_a)$, or a high price, $P_1^h(A_a)$. The difference between the types is that type 1 investors believe the conditional probability of $P_1^l(A_a)$ is $r_1 = 0.2$, while type 2 investors believe it is $r_2 = 0.8$. They agree, however, that the probability of payoff distribution a, that is, the sum of the unconditional probabilities of $P_1^l(A_a)$ and $P_1^h(A_a)$, is q, which is a parameter we shall vary in the example.

Consider first the situation in which $q = 0.5$, so payoff distributions a and b are equally likely, and no option market exists. The market is dynamically complete and the two $t = 1$ stock prices—$P_1(A_a)$ and $P_a(A_b)$—that correspond to payoff distributions a and b, respectively, are given by Equation (8). The solution to Equation (8) for the parameter values in Table 5 and $q = 0.5$ is given in Table 6. For this equilibrium, the expected utilities of the two aggregate investors are type 1 = 3.1223 and type 2 = −0.00447.

Table 6
Numerical example 2—no option market, q = 0.5

	$t=0$	$t=1$	
Stock price	$P_0 = 0.590$	$P_1(A_a) = 0.328$	$P_1(A_b) = 1.204$
Type 1 investors' holdings			
Cash	2.79	3.03	3.03
Stock	8.75	8.02	8.55
Type 2 investors' holdings			
Cash	7.21	6.97	6.97
Stock	1.25	1.98	1.45

Now consider the situation when investors conjecture that the stock price given payoff distribution a may be $P_1^l(A_a)$ or $P_1^b(A_a)$ and a market is opened for a put option on the stock expiring at $t = 1$ with exercise price 0.400. If the conjectures are confirmed, equilibrium must now satisfy Equation (9). The solution to Equation (9) for the parameter values of the example is in Table 7.

To see why the equilibrium with the put option supports two prices under payoff distribution a, Figure 1 shows the aggregate excess demand curves for stock under payoff distributions a and b for the $t = 0$ portfolios Y_{c1}, Y_{s1} and Y_{c2}, Y_{s2} given in Table 7. As shown in Figure 1, the $t = 0$ portfolios chosen under the assumed beliefs about $t = 1$ prices imply an aggregate excess demand curve when payoff distribution a occurs that has roots at $P_1^l(A_a)$ and $P_1^b(A_a)$. The demand curve has a kink at a price of 0.400 because below that price the put pays off, while above it the put is out of the money.

The expected utilities of the two aggregate investors for the equilibrium with the put option, based on their own beliefs about the probabilities of $P_1^l(A_a)$ and $P_1^b(A_a)$, are type 1 = 3.1690 and type 2 = −0.00392. Using the average of the probability beliefs of the two aggregate investors to calculate expected utilities gives type 1 = 3.0461 and type 2 = −0.00499.

A comparison of the two equilibria shows that opening the option market leads to a greater spread in possible $t = 1$ stock prices (0.183 to 1.154 versus 0.328 to 1.204) and a lower $t = 0$ stock price (0.582 versus 0.590). The existence of the put option also causes the more risk-averse (type 2) investors to increase their $t = 0$ stock holdings

Table 7
Numerical example 2—put option with exercise price Q = 0.400, q = 0.5

	$t = 0$	$t = 1$		
Stock price	$P_0 = 0.582$	$P_1^l(A_a) = 0.183$	$P_1^h(A_a) = 0.442$	$P_1(A_b) = 1.154$
Option price	$P_{opt} = 0.069$			
Type 1 investors' holdings				
Cash	4.57	1.29	4.69	2.86
Stock	5.74	6.56	8.75	8.48
Option	−21.09			
Type 2 investors' holdings				
Cash	5.43	8.71	5.31	7.14
Stock	4.26	3.44	1.25	1.52
Option	21.09			

(4.26 in aggregate versus 1.25), which they hedge with a positive position (21.09) in the put option. The less risk-averse (type 1) investors write the puts held by the more risk-averse investors. The put option

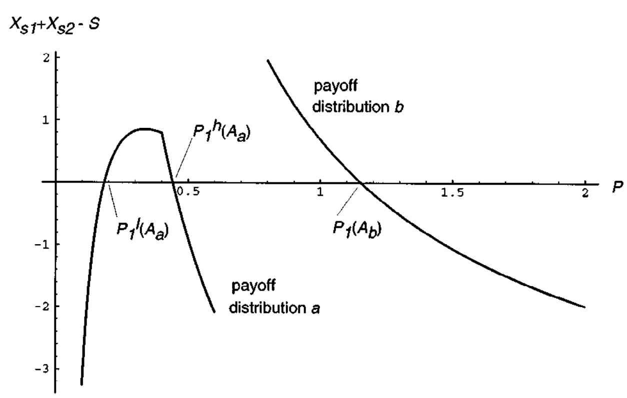

Figure 1
Aggregate excess demand for stock in market with put option
The market contains a put option with exercise price $Q = 0.400$. The aggregate excess demand curves for stock at $t = 1$ based on the parameters in Table 5 and $q = 0.5$. The two excess demand curves are distinguished by which payoff distribution, a or b, revealed at $t = 1$. The figure indicates the market-clearing prices at $t = 1$ under each payoff distribution.

pays off 0.217 (0.400–0.183) if the $t = 1$ stock price is $P_1^l(A_a)$ and is worthless if the stock price is $P_1^h(A_a)$ or $P_1(A_b)$. The $t = 0$ price of the put is 0.069.

Perhaps the most interesting point of comparison of the two equilibria is with respect to investor welfare. The opening of the option market and the investors' conjectures about price uncertainty given payoff distribution a lead to an equilibrium in which expected utility based on investors' own beliefs is higher for both the type 1 aggregate investor (3.1690 versus 3.1223) and the type 2 aggregate investor (−0.00392 versus −0.00477). The self-induced price uncertainty under payoff distribution a along with an instrument for hedging this price risk (i.e., the put) increases the welfare of all investors because each believes her put position has a positive expected gain. Using the average of beliefs, however, yields expected utilities that are lower after the introduction of option trading.

We discussed above why a difference between the two investor types in their beliefs about the conditional probabilities of $P_1^l(A_a)$ and $P_1^h(A_a)$, that is, that $r_1 \neq r_2$, is necessary for the equilibrium to exhibit $P_1^l(A_a) \neq P_1^h(A_a)$. To illustrate this point in the context of the example, we let $r_2 = 1 - r_1$ and vary r_1 from 0.2 to 0.5. Thus (r_1, r_2) go from the values in Table 5, (0.2, 0.8) to (0.5, 0.5). The resulting equilibrium prices are shown in Figure 2. We see there that $P_1(A_b)$ increases slightly and P_0 hardly changes as r_1 and r_2 jointly approach 0.5. More importantly, we see that the prices $P_1^l(A_a)$ and $P_1^h(A_a)$ supported by the investors' conjectures smoothly approach each other and become a single price, $P_1(A_a)$, as the two investor types' beliefs become identical at $r_1 = r_2 = 0.5$.

We need a somewhat more complicated analysis to illustrate how the probability of payoff distribution b affects the possibility of an equilibrium having two prices under distribution a when only cash and stock are traded at $t = 0$ (i.e., no option market). First, we assume that distribution a is certain to prevail (i.e., $q = 1$) and solve Equations (9.1)–(9.3) and (9.5)–(9.7) for $P_1^l(A_a) \neq P_1^h(A_a)$ when $Y_{o1} = Y_{o2} = 0$ (i.e., no option). That solution is given in Table 8. As we explained earlier, this equilibrium is characterized by one investor type choosing to be short cash and the other type choosing to be short stock so as to achieve a sidebet about the two possible $t = 1$ prices. These $t = 0$ portfolio choices yield the aggregate excess demand curve at $t = 1$ shown in Figure 3 which has roots $P_1^l(A_a)$ and $P_1^h(A_a)$. As we noted, in this equilibrium the market is dynamically complete without the option.

For positive probability of payoff distribution b (i.e., $q < 1$), we employ a two-step analysis. In the first step, we assume a value for $P_1(A_b)$ and solve Equations (9.1)–(9.3), and (9.5)–(9.7) for $P_1^l(A_a)$, $P_1^h(A_a)$, P_0,

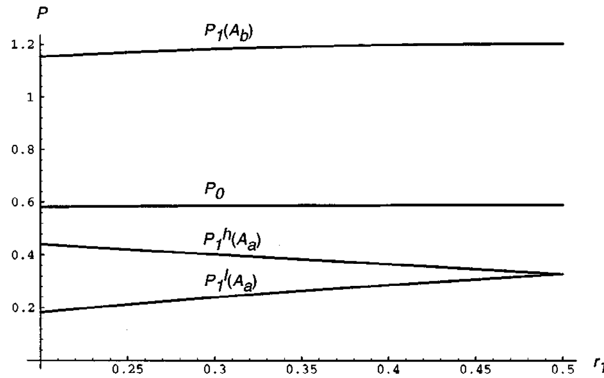

Figure 2
Relations between stock prices and investor beliefs about alternative prices
The market contains a put option with exercise price $Q = 0.400$. The prices graphed are based on the parameters in Table 5 and $q = 0.5$, but with r_1 [type 1 investors' conditional probability of $P_1^l(A_a)$ given a] varying from 0.2 to 0.5 and with r_2 [type 2 investors' conditional probability of $P_1^l(A_a)$ given a] equal to $1 - r_1$.

Table 8
Numerical example 2—no option market, q = 1

	$t = 0$	$t = 1$	
Stock price	$P_0 = 0.367$	$P_1^l(A_a) = 0.220$	$P_1^h(A_a) = 0.473$
Type 1 investors' holdings			
Cash	-2.14	1.70	5.21
Stock	24.46	7.03	8.93
Type 2 investors' holdings			
Cash	12.14	8.30	4.79
Stock	-14.46	2.97	1.07

and portfolio holdings for that value of $P_1(A_b)$, but without requiring that the resulting portfolios support the assumed value of $P_1(A_b)$ as the equilibrium under payoff distribution b, that is, we don't require Equation (9.4) to be satisfied. In the second step, we solve Equation (9.4) for the equilibrium value of $P_1(A_b)$ for the portfolios chosen by the investors in step 1. Obviously, a full equilibrium would be a fixed point in which the two values of $P_1(A_b)$ coincide.

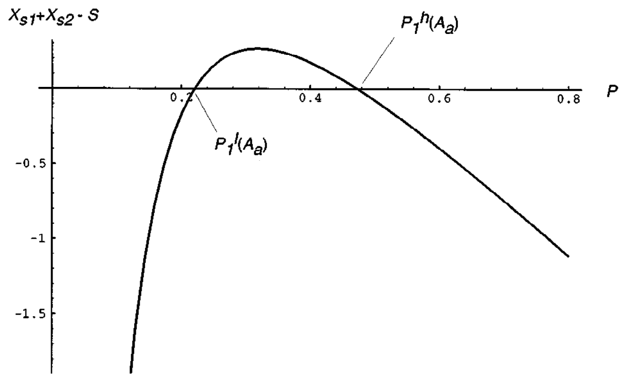

Figure 3
Aggregate excess demand for stock in market without options when payoff distribution *a* is certain
The aggregate excess demand cuves for stock at $t = 1$ are based on the parameters in Table 5 and $q = 1$ (i.e., probability 1 that payoff distribution a will prevail). The figure indicates the two market-clearing prices at $t = 1$.

Recall that our base case, as reflected in Tables 6 and 7, is $q = 0.5$, which implies that payoff distributions a and b are equally likely to prevail at $t = 1$. For values of the probability of distribution a between the base case and the extreme case in Table 8 of distribution a occurring for certain, that is, for $0.5 < q < 1$, we located by numerical search the lowest and highest values of $P_1(A_b)$, denoted P_b^l and P_b^h, respectively, that can be assumed in step 1 and still yield equilibria under payoff distribution a with distinct $P_1^l(A_a)$ and $P_1^h(A_a)$ values with no option market. Those extreme values of $P_1(A_b)$ are plotted in Figure 4. Any value of $P_1(A_b)$ assumed in step 1 that lies between P_b^l and P_b^h in Figure 4 will support an equilibrium under payoff distribution a with $P_1^l(A_a) \neq P_1^h(A_a)$. The question with respect to an overall equilibrium is whether the value of $P_1(A_b)$ assumed in step 1 is the same as the value of $P_1(A_b)$ that solves Equation (9.4) for the portfolios determined in step 1. As we shall see, such a fixed point is impossible to attain except for a very low chance of payoff distribution b (i.e., q very close to 1).

Also plotted in Figure 4 are the minimum and maximum step 2 values of $P_1(A_b)$ given by solving Equation (9.4) for the step 1 portfolios corresponding to values of $P_1(A_b)$ between P_b^l and P_b^h. For q less than approximately 0.985, the step 2 prices lie completely outside the (P_b^l, P_b^h) bounds, which implies that there is no equilibrium having $P_1^l(A_a) \neq P_1^h(A_a)$ under distribution a when there is no option market. The only equilibrium in this case is the $(P_1(A_a), P_1(A_b))$ equilibrium in Equation (8). When the probability of distribution b is less than

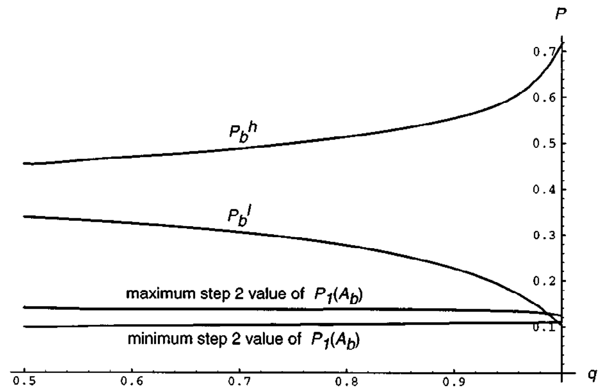

Figure 4
Relations between stock prices and investor beliefs about alternative payoff distributions in market without options
The prices graphed are based on the parameters in Table 5 and q varying from 0.5 to 1.0. P_b^b and P_b^l represent, respectively, upper and lower bounds on values of $P_1(A_b)$ that yield distinct values of $P_1^l(A_a)$ and $P_1^b(A_a)$ in the solution of Equations (9.1)–(9.3) and (9.5)–(9.7) (the step 1 solution). Also graphed are the maximum and minimum step 2 values of $P_1(A_b)$ that result from solving Equation (9.4) for the portfolios obtained in step 1. An equilibrium value of $P_1(A_b)$ must lie within both sets of bounds.

approximately 0.005 (i.e., $q > 0.995$), the step 2 prices lie inside the (P_b^l, P_b^b) bounds, so the procedure is a contraction mapping and must have a fixed point. In the region of partial overlap, $0.985 < q < 0.995$, it is also possible that a fixed point may exist.

To illustrate, Table 9 gives an equilibrium with two prices under distribution a for probability 0.01 of distribution b (i.e., $q = 0.99$). From comparison of Tables 8 and 9 we see that adding a small chance of distribution b yields relatively small changes from the case of distribution a being certain to prevail. To repeat, an equilibrium such as the one in Table 9 can only occur with no option market when the chance of distribution b is very small. When an option market is available, an equilibrium with two prices under distribution a can exist for a much wider range of the probability of distribution b. Thus the existence of an option market substantially increases the chance of the $(P_1^l(A_a), P_1^b(A_a), P_1(A_b))$ equilibrium in Equation (9).

6. Conclusions

This article has illustrated two ways in which the existence of a price-contingent contract such as an option can fundamentally alter the equilibrium in the market for the underlying security on which the contingent claim is based. What distinguishes these illustrations from the traditional views of the role of contingent claims is that both occur

Table 9
Numerical example 2—no option market, q = 0.99

	$t = 0$		$t = 1$	
Stock price	$P_0 = 0.371$	$P_1^l(A_a) = 0.270$	$P_1^h(A_a) = 0.469$	$P_1(A_b) = 0.125$
Type 1 investors' holdings				
Cash	−2.47	2.29	5.13	0.14
Stock	25.12	7.54	8.90	4.27
Type 2 investors' holdings				
Cash	12.47	7.71	4.87	9.86
Stock	−15.12	2.46	1.10	5.73

in situations where the securities market without the contingent claim appears dynamically complete to each investor. The special characteristic of a price-contingent claim that produces the effects analyzed in this article is that the payoff to such a security is based on a quantity endogenous to the market, namely, the market price of another security.

The first situation developed in this article involved an equilibrium without contingent claims in which investors have asymmetric information, every investor views the future price of the risky security as binomially distributed, but different investors see different binomial distributions and cannot determine the beliefs of others. In this case, each investor views a call option as redundant in a payoff sense since the options payoffs can be replicated by a portfolio of cash and stock. However, since heterogeneous beliefs about the future stock price imply that different investors would form different portfolios to replicate a given option, introducing trading in the option has the effect of breaking the non-fully revealing equilibrium that can exist when the option market is absent.

In the second situation studied, the effect of introducing a contingent claim market is in some respects the opposite of the first case. Here, instead of resolving uncertainty about the future stock price with asymmetrically informed agents, introducing trading in an option can allow investors to conjecture additional uncertainty respecting the future stock price, take current positions to hedge this uncertainty, and have the effect of these positions confirm the uncertainty that was conjectured. This happens because of heterogeneous risk aversion and conditional beliefs among investors and the fact that the underlying security price on which an option's payoff depends is influenced by the distribution of wealth across investors, and that distribution is itself affected by the option's payoff.

We have not modeled any particular mechanism for deciding whether the $t = 1$ stock price under payoff distribution a will be $P_1^l(A_a)$ or $P_1^h(A_a)$. It could be taken to depend on some otherwise trivial event such as whether the first order to be entered is a buy or a sell. Another possibility, slightly departing from our assumptions about the possible payoff distributions, is that nature chooses from three payoff distributions: al, ah, and b; $\pi_{al} = \pi_{ah}$ and A_{al} differs from A_{ah} by some extremely small amount. Under this interpretation, investors have homogeneous beliefs about the main $t = 1$ outcome, the occurrence of distributions al or ah versus the occurrence of distribution b, but have different beliefs about distribution al relative to distribution ah. Whatever the particular assumption made about what determines whether $P_1^l(A_a)$ or $P_1^h(A_a)$ prevails when payoff distribution a occurs, our main point is that the existence of an option market can increase the possibility of a situation in which an otherwise quite negligible difference can lead to a quite substantial difference in stock prices.

How economically significant in actual markets are the theoretical phenomena this article presents? In the case of a traded option resolving information asymmetries in a non-fully revealing equilibrium, it seems possible that the use of dynamic trading strategies (such as portfolio insurance) in place of traded option contracts may reduce investors' information about other investors' beliefs and lead to larger surprises when beliefs are revealed. Various observers have expressed opinions that something of this sort was one factor that aggravated the magnitude of the 1987 stock market crash.

It is less clear where one would look for evidence regarding the possibility that an option market may give rise to self-induced uncertainty in the price of the underlying asset. Since the phenomenon depends on the transfer of wealth from settling the option, having an effect on the price of the underlying asset, it would seem to be more relevant to trading in index options and futures than with respect to an option on an individual stock. That investors of sufficiently different risk aversion actually take opposite index option positions of sufficient magnitude to lead to the effect is unlikely to be a common occurrence. In addition, self-induced price uncertainty requires that investor's conjectures about the likelihoods of the extra price outcomes be heterogeneous and be related to their risk aversion. Clearly it would be heroic to assume that this is the usual state of affairs.

In summary, it may be the case that the effects analyzed in this article are rare events or even that they have not occurred to date in actual markets. Nonetheless, it can be argued that these effects are worth understanding, because they *may* occur in real markets and because they illuminate some important differences between securities such as common stocks that are claims to payoff streams exogenous to the securities markets and price contingent claims, such as options, whose payoffs both depend on and affect the prices of other market securities.

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