Collusion in Uniform-Price Auctions: Experimental Evidence and Implications for Treasury Auctions

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We provide experimental evidence that nonbinding preplay communication between bidders in auctions of shares facilitates the adoption of equilibrium strategies: collusive strategies in uniform-price auctions, and the unique equilibrium in undominated strategies in discriminatory auctions. When communication between bidders is introduced, clearing prices and auctioneer profits in uniform-price auctions fall below those observed in discriminatory auctions. This evidence suggests that uniform-price auctions of Treasury securities may result in lower revenues than the currently employed discriminatory procedure.

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Historically, the U.S. Treasury has marketed securities employing a sealed-bid discriminatory mechanism. Individual bidders submit bids at multiple prices and vary the number of units bid for at each price. Each bidder pays the price he bids for each unit received. Until recently, economists have advocated the use of sealed-bid uniform-price auctions for Treasury bills [see, for example, Milgrom (1989)]. In these auctions, too, individual bidders can submit a schedule of bids at multiple prices. However, in uniform-price auctions, unlike discriminatory auctions, all units are sold at the same price, the market clearing price.

Economists' advocacy of uniform-price auctions is based on the theory of auctions of units—auctions in which each participant can bid only for a single unit of the good being auctioned. However, as demonstrated by Back and Zender (1993) and Wilson (1979), the theory of single unit auctions is not readily extended to "auctions of shares," multiple-unit auctions in which bidders can submit bids at multiple prices.[1] In fact, in uniform-price auctions of shares, there exist self-enforcing strategies that allow bidders to "collude." By colluding, they are able to maximize their payoffs at the expense of the auctioneer. Other self-enforcing strategies also exist in which "competitive" outcomes obtain, and the auctioneer is able to extract the entire surplus from the auction. In contrast, in discriminatory auctions of shares, there exist no collusive equilibria. In fact, if prices at which bidders can submit bids are discrete, there exists a unique equilibrium in undominated strategies in which the auction clears at the highest price that is lower than the value of the good. Thus, the optimal mechanism from the perspective of the auctioneer depends on the equilibria most likely to obtain under the uniform-price mechanism.

In this article we provide experimental evidence on the effect nonbinding preplay communication has on clearing prices, auctioneer's surplus, and demand schedules in uniform-price and discriminatory auctions of shares. The highest clearing prices, auctioneer's surplus, and aggregate demand at the competitive price are observed in uniform-price experimental treatments without preplay communication. The lowest clearing prices and auctioneer's surplus are observed in uniform-price treatments that permit communication between bidders. Our experiments indicate that the sensitivity of outcomes to preplay communication varies significantly with the auction mechanism. In uniform-price treatments, communication results in significantly lower clearing prices and auctioneer's surplus. In stark contrast, in discriminatory treatments, preplay communication has little impact on aggregate demand, clearing prices and auctioneer's surplus. Opportunities for

[1] The term "auctions of shares" was used by Wilson (1979) to describe such auctions.

communication in uniform-price auctions result in the emergence of collusive equilibria. Similarly, in the discriminatory treatments, though communication had little effect on aggregate outcomes, it resulted in the emergence of the unique equilibrium in undominated strategies that supports a clearing price one "tick" below the competitive price. These equilibria emerged because communication appeared to create a common knowledge base among bidders. When communication is not permitted, the hypothesis that bidders adopt Nash equilibrium strategies has no predictive success [see Selten (1991)]. Communication also appears to increase the symmetry of strategies in uniform-price experiments. Further, in both discriminatory and uniform-price experiments, variation across rounds in bidder strategies increases when communication is permitted.

This is not the first article to analyze the issuance of Treasury securities. A number of researchers have investigated the choice of auction mechanism by the Treasury.[2] Some of this analysis has investigated the primary market for Treasury bills from an industrial organization perspective: examining the degree of market concentration and participants' profits [see, for example, Meltzer and von der Linde (1960) and Reiber (1964)]. Friedman (1960), taking a different approach, examined the range of bids in 13 successive Treasury auctions. A third approach, adopted by Simon (1992), relied on a comparison of the markup of Treasury auction yields over when-issued yields. A fourth approach was adopted by the Treasury itself. In September 1992, the Treasury undertook a 1-year experiment using the uniform-price auction format for its 2-year and 5-year note auctions.

These studies, while providing insights into auctions of shares, do not permit the isolation of strategic bidder behavior from institutional factors. For example, if the group of bidders is small enough and they either have other linkages or they expect to participate in a number of auctions, collusive behavior may emerge even when it is not self-enforcing in any given auction [see, for example, Fudenberg and Maskin (1986)]. Similarly, institutional factors make studies such as the Treasury's experiment difficult to interpret. If dealers believe that by eschewing profits during the experimentation period they can ensure that the uniform-price auction mechanism is adopted and they can earn larger profits after the adoption, they may have an incentive to utilize self-enforcing competitive strategies during the experimentation period, switching to collusive strategies afterwards.

Our experimental data complement these empirical studies. In a

[2] In addition to empirical investigations of the U.S. Treasury auctions, researchers have also studied auction mechanisms in other contexts. For example, Umlauf (1993) examines Mexican Treasury auctions and Tenorio (1993) examines the Zambian foreign exchange markets.

controlled experimental setting such as ours, it is possible to isolate the effects of communication and alternative allocation mechanisms.[3] Our article also extends the extant experimental literature on auctions. This literature is extensive [see, for example, Cox, Smith, and Walker (1984) and Smith (1967)]. Much of this literature has also focused on comparing the uniform and discriminatory auction mechanisms. However, this strand of research on auctions has been limited to examining the outcomes of auctions of units. These auctions do not allow for self-enforcing bidding strategies that extract the auctioneer's surplus.

This article is organized as follows. In Section 1, a simple model is developed that we use to characterize the Nash equilibrium strategy vectors in the various experimental treatments. In Section 2 we describe the procedures followed in performing the experiments. In Section 3 we describe the results of the experiments in detail. Section 4 contains some concluding remarks. Appendix A presents the proofs of some of the results derived in Section 1. The instructions presented to bidders in the experiments are presented in Appendix B.

1. Uniform and Discriminatory Auctions of Shares

As a first step to examining bidder strategies in an experimental setting, we characterize equilibrium behavior in auctions. This involves the specification of bidder and auctioneer payoffs and strategies. Our parameterizations have been selected to make the auction mechanisms transparent to the bidders, ease their computational burden, and conform with some salient institutional characteristics of Treasury auctions. To simplify the computations and make the auction mechanism transparent, we specify a common unit value. To conform with the institutional characteristics of Treasury auctions, we restrict the number of units that can be bid for by a bidder and restrict bidders to placing bids at fixed and discrete price levels.[4]

1.1 Description of the experimental setting

In each auction 100 units of a good are for sale. Each unit of the good has a value of 20 to the bidders and a value of 10 to the auctioneer. There are 11 bidders in each auction.[5] They attempt to maximize their

[3] Experiments have also provided insights into other important issues in financial economics [see, for example, Cadsby, Frank, and Maksimovic (1990)].

[4] These specifications differentiate our model from those of Back and Zender (1993) and Wilson (1979), who allow for bidding strategies to range over a continuum of prices and quantities and allow for more general informational structures.

[5] Most experiments involved exactly 11 bidders. Formal analysis of bidder behavior and experimental outcomes is conducted only on this core group of experiments. In some experiments,

monetary payoffs. Bidders simultaneously specify demand schedules. Each schedule specifies the number of units the bidder is willing to purchase at each of three prices: 10, 15, and 20. Each bidder can submit only nonnegative-integer valued bids that sum to no more than 100. Let d_{ip} represent the number of units of the good demanded by bidder i at price p. Then bidder i's demand schedule can be represented by a 3-tuple $d_i \equiv (d_{i20}, d_{i15}, d_{i10})$.

A bidder's allocation of the good is determined by her demand schedule and the aggregate demand schedule. Let d represent a vector of demand schedules, where $d \equiv (d_1, d_2, \cdots, d_{11})$. Let $A_p(d)$ represent the aggregate demand of bidders at a price p, where $A_p(d) \equiv \Sigma_i d_{ip}$. Similarly, let $C_p(d)$ represent the cumulative aggregate demand at price p, where

$$c_p(d) \equiv \Sigma_{p' \leq p} A_{p'}(d). \tag{1}$$

Bidders' payoffs are determined by their allocations and the "clearing" or "stop-out" price. A clearing price exists only if at least 100 units of the good are demanded. If less than 100 units are demanded, the auction is canceled and bidders receive no payoffs. On the other hand, if the auction is successful, the clearing price for the auction is the highest price at which cumulative demand equals or exceeds 100. To complete the description of the auction, let $C^+(d)$ represent the cumulative aggregate demand at the price immediately above the clearing price, p^*, where $C^+(d) \equiv C_{p^*}(d) - A_{p^*}(d)$. Given the clearing price, each bidder receives the number of units she demanded at prices above the clearing price and a prorated share of her demand at the clearing price. No allocations are received for demand at prices lower than the clearing price. Let the proration factor at price p be represented by the function $\pi_p(d)$, where $\pi_p(d) = 1$ at prices above the clearing price and $\frac{100 - C^+(d)}{A_{p^*}(d)}$ at the clearing price. Thus, bidder i's allocation at prices greater than or equal to the clearing price is $r_{ip}(d) = d_{ip}\pi_p(d)$.

A bidder's payoff is determined by the amount she is required to pay for her allocation. This amount varies with the rules of the auction. In a uniform-price auction, the price paid for all units is the clearing price. Thus, bidder i's payoff in a successful auction can be represented by $V_i(d)$, where

$$V_i(d) \equiv (20 - p^*) \Sigma_{p \geq p^*} r_{ip}(d). \tag{2}$$

In a discriminatory auction, for each unit they receive, bidders pay

because of unavoidable circumstances, the number of bidders differed from 11. The rationale for using 11 bidders in our experiments is elucidated below.

the price at which the bid was submitted. Thus, bidder *i*'s payoff in successful auction can be represented by $V_i(d)$, where

$$V_i(d) \equiv \Sigma_{p \geq p^*} r_{ip}(d)(20 - p). \tag{3}$$

Because the auctioneer makes the same profit per unit auctioned, in a uniform-price auction, his surplus can be represented by $100(p^* - 10)$. In discriminatory auctions, on the other hand, the auctioneer's per-unit surplus varies across units. In this case, his surplus can be represented by

$$\Sigma_{p \geq p^*} A_p(d)(p - 10). \tag{4}$$

1.2 Outcomes of the auctions

In characterizing the outcomes of these auctions, we focus our attention on Nash equilibria. In a Nash equilibrium the demand schedule submitted by each bidder is a best response to the demand schedules of other bidders. In line with much of the literature on strategic decision making, we formally examine only Nash equilibria in which bidders adopt pure strategies. When multiple equilibria exist producing the same clearing price, following Back and Zender (1993), we focus on the symmetric equilibria supporting these outcomes. These equilibria are focal for two reasons: (1) all the bidders are identically endowed and it is more likely that coordination would implement outcomes that would not discriminate between bidders; (2) in our experiments, communication between bidders seemed to indicate that they expected equal treatment.[6]

Some properties of these Nash equilibria are fairly obvious. Regardless of the auction mechanism employed, submitting a demand vector of less than 100 units is a dominated strategy. The logic behind this result is simple, submitting demand at the lowest price of 10 is never worse than, and is sometimes better than, not submitting any demand at all.[7] This implies the following obvious corollary: in any Nash equilibrium in undominated strategies, total demand, d^*, is no lower than the number of units for sale and the auction is not canceled.

In a uniform-price auction, a multiplicity of equilibria exist. Some of the outcomes supported by these equilibria are "competitive" in

[6] Almost all the literature on auctions has focused on symmetric equilibria [see, for example, Vickrey (1961)]. Symmetric equilibria have also been the focus of research in related problems such as corporate takeovers [Holmstrom and Nalebuff (1992)].

[7] Using the elimination of dominated strategies as a solution concept is common in the literature [see, for example, Kohlberg and Mertens (1986)]. The support for this solution concept is based on both classical decision theory [Luce and Raiffa (1957)] and the theory of evolutionary stable strategies [Samuelson (1991)]. However, Samuelson (1992) points out that this solution concept cannot be deduced from the common knowledge of rationality. Further discussion on this subject and its relationship with our research appears below.

that they ensure that the clearing price equals the bidders' reservation price. These competitive outcomes are supported by strategy vectors in which total demand at a price of 20 is large enough to ensure that no individual bidder can lower the clearing price by changing her demand. This implies that, from the perspective of any one bidder, the sum of all other bidders' demand at a price of 20 exceeds 100, or equivalently

$$A_{20}(d) - \max_i d_{i20} \geq 100. \tag{5}$$

Thus, in symmetric equilibria supporting a competitive clearing price, each bidder must demand at least 10 units at the price of 20. Given the equilibrium strategy vector, the payoff from all possible strategies available to any agent is zero.

Proposition 1. *In any symmetric Nash equilibrium of a uniform-price auction, the clearing price equals the competitive price if and only if each bidder demands at least 10 units at a price of 20.*

In addition to the competitive outcomes characterized in Proposition 1, in uniform-price auctions there also exist "collusive" outcomes. In these outcomes, bidders are able to extract the maximum possible value from the auction. As Back and Zender (1993) and Wilson (1979) demonstrate, the cumulative aggregate demand schedule induced by equilibrium strategies is highly inelastic. Thus, any attempt by an individual bidder to increase her allocation by placing a larger demand at a higher price results in a large jump in the clearing price. Because of this large increase in the clearing price, the bidder incurs a large loss on her original allocation. Further, given the inelasticity of the cumulative demand schedule, her allocation increases by only a small amount. Thus, her loss from the increase in the clearing price more than offsets the gain from the increased allocation. Given our parameter choices, the requisite inelasticity of total demand is achieved if aggregate demand at a price of 20 is 99, or equivalently

$$A_{20}(d) = 99. \tag{6}$$

Symmetric equilibria exist because the number of bidders, 11, is a divisor of 99.

Proposition 2. *(i) In a uniform-price auction, there exists a symmetric Nash equilibrium such that bidders extract all the surplus from the auction. (ii) In this equilibrium, bidders demand 9 units at a price of 20, 0 units at a price of 15, and 91 units at a price of 10.*

It is easy to demonstrate that the equilibrium outcomes characterized in Proposition 2 are coalition-proof [see Bernheim, Peleg, and Whinston (1987)], while outcomes in which the clearing price is either

20 or 15 are not coalition-proof. Further, Bernheim, Peleg, and Whinston (1987) argue that the logical candidate equilibria that result from costless preplay communication between agents are coalition-proof. Thus, outcomes in which the auction clears at 10 seem of particular interest.

In uniform-price auctions, in addition to the competitive outcomes characterized by Equation (5) and the symmetric collusive outcomes characterized in Proposition 2, there exist other equilibria in which the clearing price is lower than the competitive clearing price. In some of these equilibria, the clearing price is 15. The enforcement mechanism that sustains these equilibria is virtually identical to that which sustains the symmetric equilibria characterized in Proposition 2—the cumulative aggregate demand schedule induced by equilibrium strategies is highly inelastic, ensuring that penalties for deviations from equilibrium strategies through the placement of bids at prices above the clearing price are sufficient to deter deviations. The requisite inelasticity of total demand is once again achieved if and only if Equation (6) is satisfied. When Equation (6) is satisfied, the only additional requirements for equilibria are (1) all bidders earn sufficient profits from colluding to make an increase in the clearing price prohibitively costly, and (2) bidders maximize their share of prorated demand at the clearing price. These conditions are satisfied by any outcome in which (1) all bidders receive at least two units and (2) each bidder's demand sums to 100 units. Thus, there exist numerous asymmetric equilibria supporting clearing prices lower than the competitive price.

As can be seen from the above discussion, there exists great variation in equilibrium clearing prices of uniform-price auctions. There is less variation for discriminatory auctions [see Back and Zender (1993)]. In these auctions, demanding any units at the competitive price locks in a zero profit on those units and thus is a dominated strategy. However, Nash equilibria exist in which the auction clears at a price of 20. In these competitive equilibria, each bidder is held to a zero profit regardless of her strategies, with the other bidders playing dominated strategies by submitting sufficiently large bids at the price of 20 to force the auction to clear at this price. The equilibrium strategies in this case are identical to the strategies that induce a clearing price of 20 in uniform-price auctions. Again Equation (5) is necessary and sufficient for the competitive outcome.

As we demonstrate in Proposition 3, the only equilibrium in undominated strategies for discriminatory auctions ensures that the clearing price is 15. In equilibrium, demand is concentrated at a price of 15, with every bidder maximizing her demand at this price. To see the uniqueness of this equilibrium, note that collusive outcomes with a clearing price of 10 are not sustainable. Because of the discriminatory nature of the auction, a bidder is able to switch some of her demand to a higher price without affecting her profits on the unchanged portion of her demand. The increased allocation resulting from this switch increases the bidder's payoff.

Proposition 3. *In a discriminatory-price auction, a unique Nash equilibrium in undominated strategies exists in which all bidders submit all demand at a price of 15, the price one tick below the competitive price.*

The above results demonstrate that uniform-price auctions may clear at price levels of 10, 15, or 20, while discriminatory auctions will tend to clear at 15. Further, if preplay communication facilitates the adoption of collusive strategies, the auctioneer's surplus in uniform-price auctions permitting preplay communication should be lower then clearing prices in either discriminatory auctions or uniform-price auctions without communication. In discriminatory auctions, however, because there is a unique equilibrium in undominated strategies, opportunities for preplay communication should have little effect on bidder behavior.

2. Experimental Methodology

Experiments were conducted on groups of graduate business students. The auction mechanism, as well as the opportunities for bidder communication, varied. Four experimental auction treatments were performed: (1) uniform price without communication (U), (2) uniform price with communication (UC), (3) discriminatory without communication (D), and (4) discriminatory with communication (DC). Henceforth, each experiment is referred to by a name. This name indicates both the treatment and a number to distinguish the experiment from other repetitions of the same treatment. For example, UC2 refers to repetition 2 of the uniform-price treatment with communication. Details of the number of bidders, rounds, and the repetitions of each treatment are presented in Table 1.

Attempts were made to ensure that 11 bidders participated in each experiment. In some instances, however, the number of bidders could not be controlled. At least three repetitions of each treatment were conducted with 11 bidders. The uniform-price treatment without communication was repeated four times for groups of 11, 11, 11, and 14. The uniform-price treatment with communication was repeated five times with groups of 11, 11, 11, 12, and 12. The discriminatory treatment without communication was repeated four times with groups of 11, 11, 11, and 10. The discriminatory treatment with bidder communication was repeated three times. Each repetition was conducted

Table 1
Descriptions of the experiments

Panel A: Uniform-price treatments

	Uniform price without communication				Uniform price with communication				
Experiment	U1	U2	U3	U4*	UC1	UC2	UC3	UC4*	UC5*
Price levels	3	3	3	3	3	3	3	3	3
Units	100	100	100	100	100	100	100	100	100
Rounds	14	14	12	15	12	12	12	16	12
Bidders	11	11	11	14	11	11	11	12	12

Panel B: Discriminatory treatments

	Discriminatory without communication				Discriminatory with communication		
Experiment	D1	D2	D3	D4*	DC1	DC2	DC3
Price levels	3	3	3	3	3	3	3
Units	100	100	100	100	100	100	100
Rounds	12	12	12	12	12	12	12
Bidders	11	11	11	10	11	11	11

This table presents a summary or the parameters for all experiments. There were four treatments investigated: uniform price without communication (U), uniform price with communication (UC), discriminatory without communication (D), and discriminatory with communication (DC).

with a group of 11. No bidder was involved in more than one experiment. An asterisk is affixed to the name of each experiment involving a number of bidders unequal to 11.

The experiments without communication lasted approximately 45 minutes, while experiments involving communication lasted 10–15 minutes longer. First, bidders were assembled in a classroom and presented an instructional handout explaining the rules of the treatment and the process used in determining payoffs.[8] They were given 5 minutes to peruse the instructions, after which, one of the experimenters read the instructions and verbally explained the auction mechanism and computer interface. Bidders were then given an opportunity to ask questions of the experimenters. The reading and answering of bidder questions took approximately 10 minutes. Bidders were then moved to a computer laboratory. The logistics involved in this change of location allowed for bidder communication and discussion after the instructions were completed but before the experiment commenced. The time available for such discussion varied across experiments. However, these opportunities for communication were similar across all experiments. In the computer laboratory, bidders were seated so as

[8] The cohorts of students participating in the experiments consisted of entire MBA-level classes. As the size of these classes typically exceeded the number terminals we could utilize in any laboratory, we were forced to break each class into groups of 11 bidders. To facilitate this division, bidders were first assembled in a classroom then moved to a computer laboratory.

to prevent others from observing their computer screens. The experiments were performed using a local area network to communicate bids, allocations, and payoffs.

An experiment commenced when bidders first entered demand schedules into their terminals. Once all bids had been entered, the results of the auction were electronically computed. Each bidder was electronically informed of the clearing price, her allocation, and her payoff from the auction. At this point, bidders were given an opportunity to record their payoffs and allocations for their own reference. Other than the clearing price, bidders were not presented any information regarding other bidders' demand schedules or the aggregate demand schedule. Once this process, or round, was complete, the auction was repeated. Each of the first four rounds took approximately 4 minutes to complete. Subsequent rounds took approximately 2 minutes to complete. All experiments were run for at least 12 rounds, with most consisting of exactly 12 rounds. Variations in the number of rounds across experiments resulted from attempts to maximize the number of rounds subject to time constraints. Bidders were not informed of the number of rounds to be played, and a perusal of the results indicates that the deletion of results from rounds after round 12 would have no qualitative impact on our conclusions.

In treatments in which communication was not permitted, bidders were not allowed to speak to each other once the experiment commenced. In treatments allowing communication, bidders were allowed to speak to each other every two rounds.[9] They were allocated 5 minutes for the first discussion and 3 minutes for subsequent discussions. However, no communication was allowed when bidders were entering their strategies or recording their payoffs. Communication was governed by the following rules: bidders were not allowed to leave their terminals or show any of their personal records or notes to other bidders. However, verbal communications were unrestricted in that bidders were allowed to propose strategies for future rounds and comment on the outcomes of previous rounds. Three experimenters enforced these rules governing communication.

Bidders' payoffs in each round were determined using the payoff functions described in the previous section. All prices and payoffs were denominated in "francs," a notional currency. Payoffs were summed across all rounds to determine bidders' payoffs in each ex-

[9] Bidder communication was restricted to alternate rounds to ensure that experiments did not last too long and thus result in reduced bidder interest. Analysis of outcomes in odd and even numbered rounds in experiments permitting bidder communication did not reveal any significant difference in clearing prices, bidder demand, or auctioneer's surplus. This would seem to indicate that bidder strategies in rounds immediately following discussions were not significantly different from their strategies in the subsequent round.

periment. These experiment payoffs were translated into monetary payoffs using the following formula:

$$\$ \text{ payoff } = \text{ sum of round by round payoffs } \times \mathcal{F},$$

where $\mathcal{F}$ represents a scaling factor. Payoffs were not revealed to other bidders and were dispensed in sealed envelopes. These payments were made from funds provided by university research funding. Bidders were informed regarding both the payment procedure and the source of the funding at the beginning of each experiment.

The scaling factor for payoffs, $\mathcal{F}$, which varied between 0.01 and 0.02, was made known to bidders before each experiment. We expected large payouts to be made in treatments involving bidder communication. In order to most efficiently utilize our limited budget, we attempted to keep the total payoff the same across treatments and experiments. This called for using lower scaling factors in experiments involving communication. The resulting payoffs to individual bidders ranged between \$0.50 and \$10.00. The average payoff was approximately \$5.00.[10] Despite the lower scaling factors, payoffs were significantly higher in treatments that allowed for bidder communication. Payoffs were also relatively sensitive to the nature of the adopted strategy. For example, in each round of the uniform-price treatments, each bidder could earn approximately \$0.90 from adopting collusive strategies, while earning \$0.00 from adopting competitive strategies. Because each round lasted approximately 2–4 minutes, the marginal reward to the individual bidder for adopting collusive strategies was between \$0.22 and \$0.45 per minute.[11] Because coordination to collusive equilibria requires significantly more effort than playing competitive strategies, any bias against effort induced by lower scaling factors for the treatments with communication would bias results against collusion.

3. Experimental Results

In this section we examine the outcomes of the experiments. We begin with a preliminary analysis of clearing prices, auctioneer surplus,

[10] We observed many obvious signs of bidder interest in the experiments. For example, bidders asked a number of questions of the experimenters during their explanations of the rules of the experiments, and they entered into animated discussions when they were permitted to communicate with one another. While we are fairly confident that our results are representative of the types of bidder behavior that would obtain in similar experiments, we realize that higher payoffs might have lead to greater frequency of collusive behavior by bidders.

[11] This contrasts with the relatively small sensitivity of payoffs to bidder strategies observed in unit sealed-bid auction experiments. Thus, the criticisms of experimental methodology by Harrison (1989) have less force in our setting.

and demand in all the experiments. Then we conduct statistical tests to evaluate the effects of communication and the choice of auction mechanism on experiment outcomes. To control for biases induced by changes in group size and learning, these tests are restricted to the first 12 rounds of experiments conducted with 11 bidders.

3.1 Preliminary findings

The round-by-round evolution of clearing prices for the first 12 rounds of each experiment is illustrated in Figure 1. Figure 2 presents the distribution of clearing prices. In the uniform-price treatment with communication, the clearing price displayed the greatest range and variance. The collusive clearing price of 10 was observed 36% of the time, while the competitive price of 20 obtained 34% of the time. With the exception of UC2 and the last round of UC1, the clearing price was never 20 in the last three rounds. Further, in UC1, UC3, and UC5*, the clearing price was usually 10 in the later rounds. In UC2, clearing prices displayed a contrasting pattern. The clearing price was 10 for the first six rounds and 20 for the last four rounds. It appeared that bidders were able to collude at the inception of the experiment, but coordination broke down as the experiment progressed. In the uniform-price treatment without communication, the clearing price was never 10. With the exception of U1, the clearing price was 20 in most rounds. In the discriminatory treatments both with and without communication, the clearing price of 10 was never observed, and 15 was the most frequent clearing price. In fact, in the last two rounds of all of these experiments, 15 was the only clearing price.

The evolution of the auctioneer's surplus in each treatment is illustrated in Figure 3. As can be expected, given the pattern of clearing prices, the auctioneer's share of the surplus was smallest in the uniform-price treatment with communication and largest in the uniform-price treatment without communication. Further, the variations in auctioneer's surplus appear to be highest in the uniform-price and discriminatory treatments permitting communication.

Figure 4 illustrates the pattern of subject demand and the importance of communication and auction mechanism in shaping the experimental outcomes. Aggregate demand always exceeded 100 units, indicating that auctions were never canceled. However, weakly dominated strategies in which individual bidders demanded less than 100 units were observed. In the uniform-price treatment without communication, bidder demand at the competitive price of 20 was higher than in any other treatment. In contrast, not only was the relative demand at the collusive price of 10 highest in the uniform-price treatment with communication, but the *majority* of demand was placed at this price. Consistent with our earlier analysis, regardless of whether

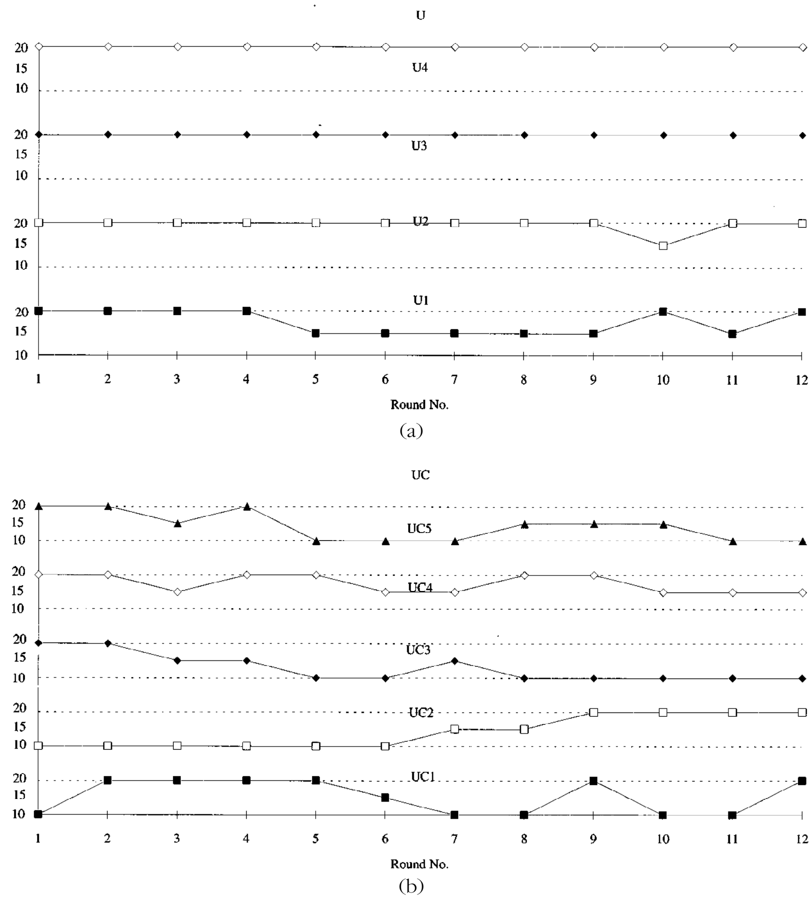
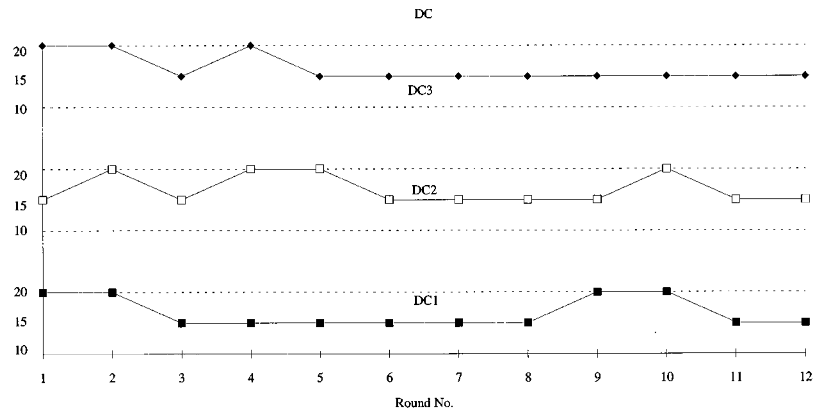


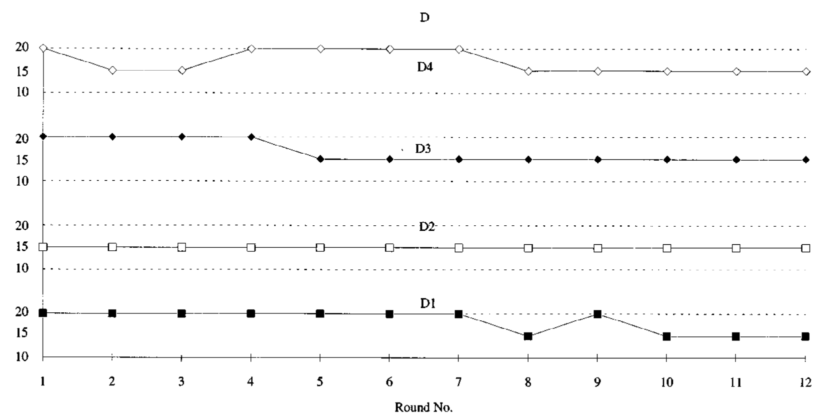
Figure 1
The evolution of clearing prices
Each panel presents the clearing price for the first 12 rounds of each repetition of a given treatment. Panel U presents this information for the uniform-price treatment without communication. Panel UC presents this information for the uniform-price treatment with communication. Panels D and DC present this information for the discriminatory counterparts of these treatments. *Continued next page.*

communication was permitted, the majority of demand was placed one tick below the competitive price at a price of 15 in the discriminatory treatments.

In addition to the general patterns of bidder behavior highlighted by the averages in Figure 4, there were some interesting behaviors unique to some of the experiments. For example, for UC1, UC3, and UC5*, the cumulative demand in the last six rounds corresponded almost exactly to that characterizing collusive outcomes. In contrast, in UC2, during the first six rounds, bidder strategies corresponded ex-



(c)



(d)

Figure 1
The evolution of clearing prices
Continued.

actly to naive collusive strategies of placing maximal demand at the lowest price of 10. In round seven, a bidder demanded 100 units at the price of 15, eliminating any gains to the other bidders. As indicated by the evolution of clearing prices, at this point, coordination between bidders appeared to break down.[12] In the uniform-price treat-

[12] From bidder communication subsequent to round seven, it was apparent that they were attempting to revert to the naive collusive strategies played in earlier rounds. On realizing that the strategies were not self-enforcing, they attempted, albeit unsuccessfully, to agree on a trigger mechanism to enforce penalties for future deviations from the naive strategies. Coordination across rounds via the adoption of a "super-game" strategy was not evident in any other experiment. A priori, such strategies are not likely to be played in an experimental setting such as ours. First, because bidders do not know the number of rounds to be played, they lack the common knowledge base

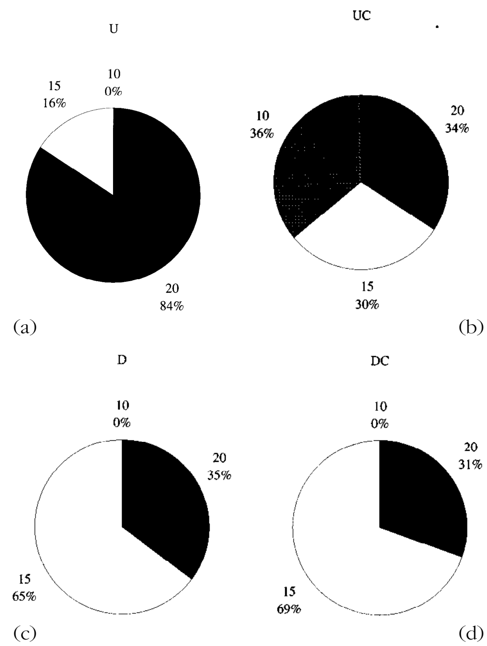


Figure 2
The frequency of clearing prices
This figure presents the frequency with which the experiments cleared at each of the three clearing prices. Panel U presents this information for the uniform-price treatment without communication. Panel UC deals with the uniform-price treatment with communication. Panels D and DC deal with the discriminatory counterparts of these two treatments.

ment without communication, aggregate demand at the price of 20 displayed a tendency to decline over time. In the discriminatory treatment without communication, aggregate demand at the price of 20 tended to exceed 100 during the first six rounds. In the later rounds, however, it was consistently below 100. Aggregate demand at the price of 20 displayed a similar pattern in the discriminatory treatment with communication.

3.2 Assessments of experimental outcomes

We assess the experimental outcomes along five dimensions: clearing price, auctioneer's surplus, aggregate demand, predictive success, and symmetry and stability of demand. Tables 2 and 3 present statistical tests for differences in clearing prices across treatments. Parametric

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necessary to agree on penalty mechanisms to enforce super-game strategies. Second, at least in the uniform-price treatments, bidders can capture all the surplus employing Nash equilibrium strategies in the single-shot games, making any attempt at coordinating to super-game strategies superfluous.

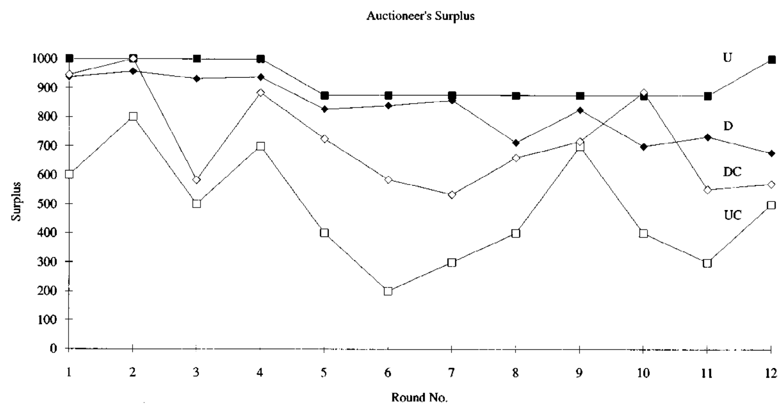

Figure 3
Auctioneer's surplus
This figure presents the round-by-round average of auctioneer's across the experiments in each of the four treatments. The average surplus series for the uniform-price treatment without communication is labeled U, while the series for the uniform-price treatment with subject communication is labeled UC. The corresponding discriminatory surplus series are labeled D and DC.

tests for differences in mean clearing prices are presented in Table 2. Person chi-square goodness-of-fit tests comparing clearing price distributions are presented in Table 3. Both tests suggest that auction mechanisms significantly influenced clearing prices, while communication between bidders exerted a significant influence on clearing prices only in uniform-price treatments, in which case communication led to significantly lower clearing prices. When communication was permitted, prices were lower in the uniform-price treatment than in the discriminatory treatment. On the other hand, when communication was prohibited, the discriminatory treatment resulted in lower clearing prices than the uniform-price treatment. Tests on auctioneer's surplus presented in Table 2 mirror these results.

Table 4 presents the results of Mann–Whitney–Wilcoxon tests for differences in aggregate demand. The results of these tests are consistent with the tests on clearing prices and auctioneer's surpluses. Communication in uniform-price treatments resulted in significantly higher demand at the price of 10 and significantly lower demand at the prices 15 and 20. In the discriminatory treatments, the hypothesis that communication had no effect cannot be rejected at conventional confidence levels. Tests for the influence of auction mechanism on demand indicate that discriminatory treatments produced significantly higher demand at 15 and significantly lower demand at 20 than uniform-price treatments. When communication was permitted, the uniform-price treatment produced significantly higher demand at 10

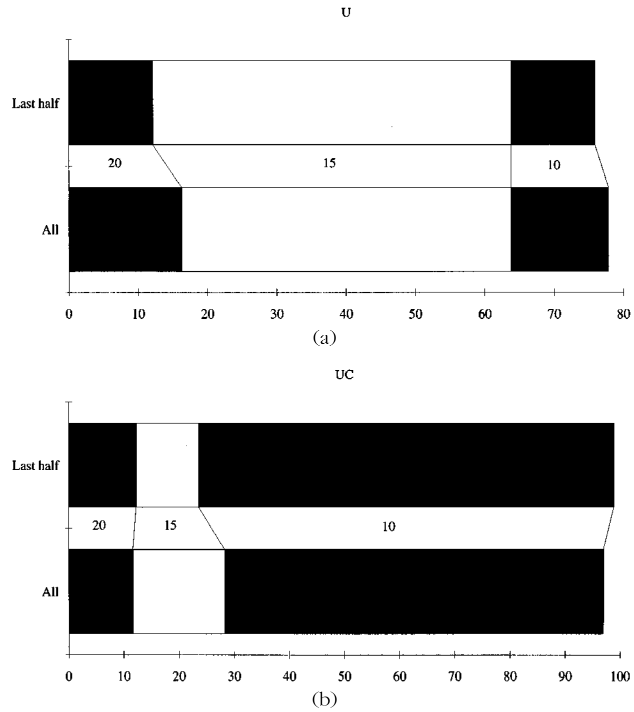

Figure 4
Aggregate demand
This figure presents bidders' aggregate demand at each of the three price levels across all rounds of the experiments and the second half of all repetitions in each of the four treatments. Each panel presents this information for one of the treatments. Panel U presents this information for the uniform-price treatment without bidder communication. Panel UC deals with the uniform-price treatment with bidder communication. Panels D and DC deal with the discriminatory counterparts of these two treatments. *Continued next page.*

than the discriminatory treatment.

Table 5 presents evidence on the predictive success of equilibria. The measure used to assess predictive success is that of Selten (1991).[13] This measure of predictive success is given by $r-a$, where r is the relative frequency, the fraction of actual outcomes that conform with the given equilibrium hypothesis, and a is the area predicted by the hypothesis, the probability that outcomes consistent with the hypothesis occur by random chance assuming a uniform probability distribution over the strategy space. Because high levels of r imply that the theory accurately describes the results of the experiment, r measures the accuracy of a theory. On the other hand, because low

[13] We wish to thank an anonymous referee for directing us toward the literature on predictive success.

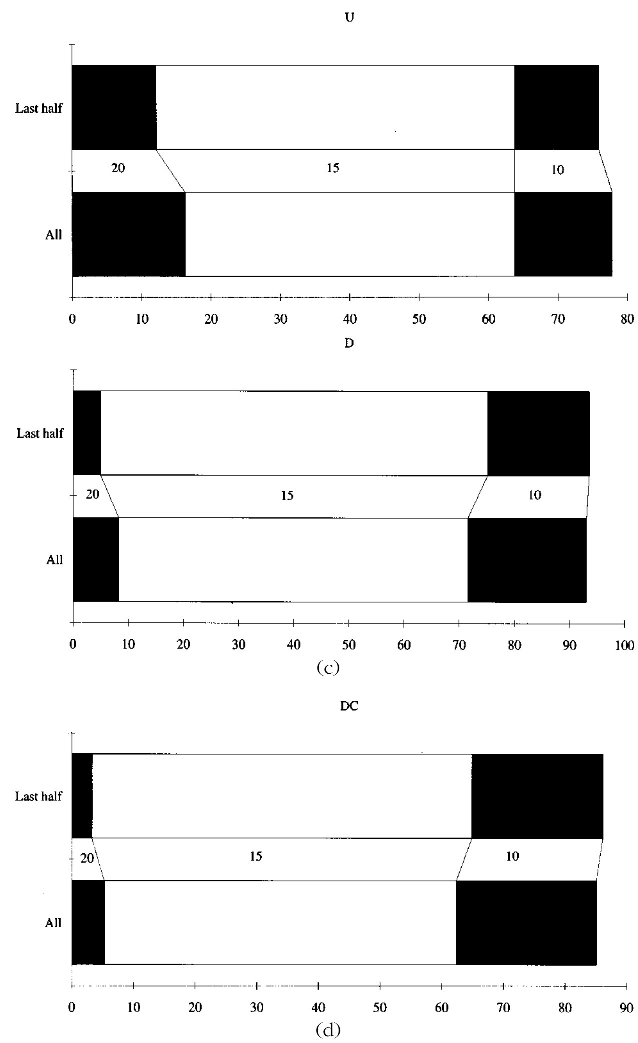

Figure 4
Aggregate demand
Continued.

levels of a indicate that the theory makes a precise prediction regarding where the observation lies in the outcome space, a is a measure of the precision of the theory.[14]

We present the predictive success of two theories for each auction

[14] Selten (1991) uses an axiomatic argument to show that the difference between r and a, rather than, say, their ratio, is the correct measure of predictive success.

Table 2
Tests for the differences in clearing price and auctioneer's surplus

	Uniform price with communication CP	Uniform price with communication W	Discriminatory with communication CP	Discriminatory with communication W	Uniform price without communication CP	Uniform price without communication W	Discriminatory without communication CP	Discriminatory without communication W
Uniform price with communication	—	—	−2.36[a]	−303[a]	−4.86[a]	−486[a]	NC	NC
Discriminatory with communication	—	—	—	—	NC	NC	−0.139	−97.8
Uniform price without communication	—	—	—	—	—	—	2.34[a]	85.0
Discriminatory without communication	—	—	—	—	—	—	—	—

[a]Significant at the 1% level.

This table presents differences in mean clearing prices (CP) and auctioneer's surplus (W). The number in each cell represents the mean for the row less the mean for the column. The null hypothesis that the mean clearing prices and auctioneer's surplus are identical across treatments is tested using *t*-tests for differences in means.

Table 3
Tests for the efffect of communication and auction mechanism on the distribution of clearing prices

	Uniform price with communication	Discriminatory with communication	Uniform price without communication	Discriminatory without communication
Uniform price with communication	0	35.61[a]	94.53[a]	NC
Discriminatory with communication	—	0	NC	0.06
Uniform price without communication	—	—	0	16.36[a]
Discriminatory without communication	—	—	—	0

[a]Significant at the 1% level.

This table presents the Pearson chi-square goodness-of-fit test statistics for clearing price distributions across auction mechanisms and communication regimes. The null hypothesis is that, for any given cell, the price distribution of the row and column treatments are the same.

mechanism. In the uniform-price case the two theories are that (1) bidders play competitive Nash equilibrium strategies, and (2) bidders play the symmetric collusive equilibrium strategies. In the discriminatory case, the two theories are that (1) competitive equilibria are played and (2) the unique Nash equilibrium in undominated strategies is played. As these are the only equilibrium outcomes for the discriminatory treatments, it follows that we present evidence on the predic-

Table 4
The effects of communication and auction mechanism on demand

Panel A: Effects of communication on demand

	Round	Uniform price without communication			Discriminatory without communication		
		T_{20}	T_{15}	T_{10}	T_{20}	T_{15}	T_{10}
Uniform price with communication	All	114[c]	78[d]	221[b]			
	First	22[d]	21[d]	56[b]			
	Second	41	21[d]	57[b]			
Discriminatory with communication	All				120[c]	142	155
	First				31	38	37
	Second				30	35	42

Panel B: Effects of changing the auction mechanism on demand

	Round	Discriminatory with communication			Discriminatory without communication		
		T_{20}	T_{15}	T_{10}	T_{20}	T_{15}	T_{10}
Uniform price with communication	All	193[b]	78[d]	221[b]			
	First	42	21[d]	56[b]			
	Second	55[b]	21[d]	57[b]			
Uniform price without communication	All				206[b]	85[d]	104[b]
	First				53[a]	22[d]	25[c]
	Second				57[b]	21[d]	25[c]

[a]Prob{Σ rank(row) $>$ observed} < 0.05
[b]Prob{Σ rank(row) $>$ observed} < 0.01
[c]Prob{Σ rank(row) $<$ observed} < 0.05
[d]Prob{Σ rank(row) $<$ observed} < 0.01

This table presents the results of Mann–Whitney–Wilcoxon tests comparing average aggregate demand at each of the three price levels across auction mechanisms and communication regimes. The null hypothesis is that, for any given cell, the sums of the ranks for the row and column treatments are the same. The numbers in each cell represent the sum of the ranks of the row treatment.

tive success of all equilibrium theories for discriminatory treatments. As discussed above, there are other possible equilibrium hypotheses for uniform-price auctions. However, evidence on the predictive success of these equilibrium theories is not presented, as the data revealed that the equilibria predicted by these theories were never played; the necessary condition that cumulative aggregate demand sum to 99 was never satisfied except when the clearing price was 10 and bidders adopted identical strategies. These tests indicate that the competitive equilibrium hypothesis had no positive predictive power. Further, its predictive power declined in later rounds. The collusive hypothesis for the uniform-price treatments and the hypothesis that bidders play strategies supporting the unique equilibrium in undominated strategies in the discriminatory treatments had positive predictive power only when communication was permitted. This positive predictive power increased over rounds.

Finally, we consider the stability and symmetry of bidder strate-

Table 5
Predictive success of equilibrium hypotheses

Round	Uniform price with communication		Discriminatory with communication		Uniform price without communication		Discriminatory without communication	
	comp	col	comp	undom	comp	col	comp	undom
1–12	−0.791	0.278	−0.791	0.056	−0.429	0.000	−0.763	0.000
1–6	−0.765	0.111	−0.652	0.000	−0.207	0.000	−0.541	0.000
7–12	−0.818	0.444	−0.930	0.111	−0.652	0.000	−0.985	0.000

This table presents Selten's (1991) measures of the predictive success. For uniform-price treatments we measure the predictive success of the competitive Nash equilibrium hypothesis (comp) and the symmetric collusive equilibrium hypothesis (col). For discriminatory treatments, we measure the predictive success of the competitive hypothesis (comp) and the hypothesis that bidders play the unique equilibrium in undominated strategies (undom). The numbers presented in the table represent $r - a$, where r represents the relative frequency of the theory, the fraction of the experimental outcomes conforming with the theory, and a represents the area of the theory, the probability of outcomes conforming to the theory, given that the strategies are produced by a uniform measure over the strategy space. The areas of the symmetric collusive theory and the equilibrium in undominated strategies are approximately 0.000. The area of the competitive theory is approximately 0.985. The computations determining the areas of these theories are available from the authors upon request.

gies. Symmetry is measured by the average Euclidean distance of individual demand vectors from the average demand vector. Stability is measured by the variance of each individual bidder's demand vector. Mann–Whitney–Wilcoxon tests for the effects of communication and the auction mechanism on these measures are presented in Table 6. The results do not indicate a significant change in the symmetry of bidder strategies over time. Stability increased over time except in the discriminatory treatments with communication. This increase in stability was significant only for uniform-price and discriminatory treatments without communication. Comparisons of the symmetry of bidder demand across treatments suggest that symmetry increased with communication in uniform-price treatments. Further, communication resulted in significantly greater symmetry in the uniform-price treatment relative to the discriminatory treatment. In contrast, the symmetry of bidder strategies was significantly lower in the uniform-price treatment without communication than in the discriminatory treatment without communication. No significant differences, across auction mechanisms, were observed in stability of bidder strategies. However, in both uniform-price and discriminatory treatments, communication resulted in significant decreases in stability.

In summary, the above results demonstrate that uniform-price auctions can result in significantly lower clearing prices and auctioneer's surplus. This is likely to occur only when bidders have opportunities to communicate. In the absence of preplay communication, uniform-price auctions generate higher clearing prices and auctioneer's surplus than discriminatory auctions. Communication appears to facilitate the

Table 6
The role of communication and learning

Panel A: Learning over rounds

	Uniform price with communication	Discriminatory with communication	Uniform price without communication	Discriminatory without communication
Symmetry	340	345	315	366
Stability	1193	1095	1415[a]	1427[a]

Panel B: Auction design, symmetry, and stability

	Uniform price with communication		Discriminatory with communication		Uniform price without communication		Distriminatory without communication	
	Stability	Symmetry	Stability	Symmetry	Stability	Symmetry	Stability	Symmetry
Uniform price with communication	—	—	994.5	1088[b]	1130[c]	995[b]	NC	NC
Discriminatory with communication	—	—	—	—	NC	NC	1467[c]	1457
Uniform price without communication	—	—	—	—	—	—	1100	1733[c]
Discriminatory without communication	—	—	—	—	—	—	—	—

[a]Prob{Σ Rank(1st half) > observed} < 0.01
[b]Prob{Σ Rank(Row) < observed} < 0.01
[c]Prob{Σ Rank(Row) > observed} < 0.01

This table presents the results of Mann–Whitney–Wilcoxon tests comparing the symmetry and stability of demand. Symmetry is measured by the variance in demand across bidders. Stability is measured by variance of each bidder's demand across rounds, averaged across all bidders. Higher levels of these measures represent lower levels of symmetry and stability. Panel A presents results comparing the first and second halves of the experiments. The numbers in each cell represent the sum of ranks from the first half of the experiments in each treatment. Panel B presents results on the effect of communication regimes and auction mechanisms. The numbers in each cell represent the sum of ranks for the row treatment.

adoption of symmetric collusive strategies in uniform-price auctions. It also appears to facilitate the adoption of equilibrium strategies in discriminatory auctions. However, unlike the case of uniform-price auctions, it has little impact on either clearing prices or auctioneer's surplus in discriminatory auctions.

4. Concluding Comments

In this article we provided experimental evidence on strategy choice in auctions of shares. Our experiments indicate that nonbinding preplay communication facilitates convergence to equilibrium outcomes. When opportunities for communication are available, in uniform-price auctions, bidders are more likely to gravitate toward symmetric self-enforcing collusive strategies. This produces a lower surplus for the auctioneer than in a discriminatory auction. In the absence of communication opportunities, convergence to Nash behavior is less evident.

These results have important implications for the design of Treasury auctions. Because participants in these auctions have ample opportunities to communicate, it would appear that uniform-price auctions will net the Treasury lower revenues. Our evidence is consistent with that of Simon (1992), but is inconsistent with the predictions of researchers such as Friedman (1960). Our results also have interesting implications for researchers. The effect of communication on the symmetry of strategies, especially when there exist collusive Nash equilibria, would seem to indicate that greater emphasis should be placed on the existence of symmetric equilibria in facilitating the attainment of Pareto optimal self-enforcing agreements. Second, our results also point to the dynamic instability of competitive equilibria in which agents' payoffs are minimized and all feasible strategies are best responses to the equilibrium strategy vector. When this is the case, agents' strategy choices tend to wander.[15] Although a change in any individual agent's strategy by itself can have no effect on the outcome, because all agents exhibit a tendency to change their strategies, divergence from the equilibrium competitive price is observed fairly frequently.

Our investigation focused primarily on bidders' strategies. The experimental design did not permit us to analyze the effects of private information regarding valuations, transparency of the auction process, and secondary markets on equilibrium auction behavior. Extensions of our design to incorporate these effects seem fairly obvious. There exist numerous examples of experimental auction designs in which bidders possess private information regarding their reservation prices [see, for example, Smith (1967)]. In fact, it is the performance of uniform-price auctions in this setting that has led to their appeal among economists. A synthesis of existing experimental designs with ours will permit the examination of the impact of private information on auction outcomes. The effect of the transparency of the auction process can also be examined by a fairly straightforward extension of our design. For example, the experiment could be performed while revealing both the clearing price as well as the aggregate demand schedule to the bidders after each round. Changing the auction design to study the effect of secondary markets on bidder behavior is

[15] See Young (1993) for an analysis of best-reply structures and the evolutionary adaptation needed for convergence.

not as simple. One alternative would be to meld the existing auction model with an experimental implementation of a double auction market, where winning auction participants can trade their allocations after the completion of each round of the auction.

Appendix A: Proofs of Propositions

Definition 1. *Let* $(d|^{-i}d'_i)$ *represent the vector obtained by replacing the ith element of the d vector with* d'_i*. That is,* $(d|^{-i}d'_i) = (d_1, d_2, \ldots d_{i-1}, d'_i, d_{i+1}, \ldots, d_{11})$*. A Nash equilibrium is a feasible demand vector* d^* *such that* $V_i(d^*) \geq V_i(d^*|^{-i}d'_i)$ *for all feasible demand schedules* d'_i *and all bidders* i*.*

Proof of Proposition 1. Clearly, if other bidders, in aggregate, demand more than a total of 100 units at a price of 20, any demand schedule is a best response. In the event that other bidders' aggregate demand at a price of 20 is lower than 100 units, a bidder will never submit a bid that would force the clearing price to 20, as this would result in a payoff of zero. Thus, if the demand vectors are symmetric, for the auction to clear at 20, each bidder must bid for at least 10 units at this price. ■

Proof of Proposition 2. First note that the clearing price under the equilibrium strategy is 10. Now consider deviations by any bidder from the equilibrium strategy. This deviation either induces the same clearing price, a clearing price of 15, or one of 20. Deviations that maintain the clearing price cannot result in a higher payoff for the bidder because they cannot increase her allocation. Deviations that raise the clearing price to 20 are clearly suboptimal. Deviations that raise the clearing price to 15 are suboptimal by our choice of the parameters. ■

Proof of Proposition 3. (i) First, note that any strategy in which $d_{i20} > 0$ is a dominated strategy as demand at this price can never produce a positive payoff. (ii) Now we show that there is no Nash equilibrium in undominated strategies where the clearing price is either 10 or 20. The latter result follows directly from (i). To see the first result, note that the assumption that $N = 11$, along with our choice of parameters, ensures that given a clearing price of 10, bidders can always increase their allocation and payoff by moving some demand from a price of 10 to place bids at a price of 15. (iii) Now, to complete the proof, we establish that, in any Nash equilibrium, bidders will concentrate all demand at a price of 15. To see this, suppose that bidders adopt another strategy. Switching all demand to a price of 15 will increase bidder payoffs. This follows because payoff from bids made at prices of 10 and 20 are zero, given that the clearing price must be 15. The proof is concluded by noting that concentrating all demand at a price of 15 is a Nash equilibrium. ■

Appendix B: Sample of Bidder Instructions

Presented below is a sample of instructions issued to bidders in the uniform-price treatment with communication. Instructions in all other treatments were virtually identical. The only differences being the discussion of communication opportunities and the description of the auction mechanism.

1. General Instructions

You have been selected to participate in an experiment on economic decision making. The experiment is being sponsored by the Department of Finance, College of Business Administration. The experiment consists of several rounds. At the end of each round your payoffs will be calculated. At the end of the experiment, the payoffs in each round will be added up. The sum of your round by round payoffs will determine your payoff from the experiment. Your payoff from the experiment could range from $0 to $120. This payoff will be made from funds provided by the university.

2. The Game

In this experiment, you will be required to bid for units of a good. There will be 100 units of the good available. The value of each unit is Fr. 20. You will be submitting a schedule of bids. This schedule you submit indicates the number of units of the good you are willing to buy at a given price level. The permissible price levels will be Fr. 10, Fr. 15, and Fr. 20. Once all schedules have been submitted, the computer will assign units of the good to players submitting the highest bids. When more units are demanded than are available at a given price, the available units will be allocated in proportion to the bids submitted at that price. The price paid for each unit (the clearing price) will be equal to the price at which the cumulative demand equals the supply. When there is no price at which cumulative demand exactly equals supply, the clearing price will be the price at which cumulative demand first exceeds supply. Your payoff from a bid will equal the difference in the value per unit of the good and its clearing price times the number of units of the good you are allocated.

To illustrate the bidding process, consider the following example. Consider a game with three bidders: A, B, and C. They are required to submit their demands at price levels of Fr. 10, Fr. 15, and Fr. 20. The value per unit of the good is Fr. 20. Suppose that A, B, and C submit the following schedules:

	A	B	C
Fr. 20	40	20	0
Fr. 15	20	10	10
Fr. 10	40	0	70

In this case the demand at a unit price of Fr. 20 is $40 + 20 + 0 = 60$. The demand at unit price Fr. 15 is $20 + 10 + 10 = 40$. Finally, the demand at unit price Fr. 10 is $40 + 0 + 70 = 110$. It follows that the cumulative demand at Fr. 20 is 60. The cumulative demand at Fr. 15 is $60 + 40 = 100$, and the cumulative demand at Fr. 10 is $60 + 40 + 110 = 210$. In this case, the clearing price is Fr. 15. Player A receives $40 + 20 = 60$ units at the clearing price. Player B receives $20 + 10 = 30$ units, and player C receives $0 + 10 = 10$ units. It follows that player A's payoff is $(20 - 15) \times 60 =$ Fr. 300, player B's payoff is $(20 - 15) \times 30 =$ Fr. 150, and player C's payoff is $(20 - 15) \times 10 =$ Fr. 50.

To illustrate the method used for allocating shares and determining payoffs if there exists no price at which demand exactly equals supply, suppose that A, B, and C submit the following schedules:

	A	B	C
Fr. 20	40	20	20
Fr. 15	20	10	10
Fr. 10	40	0	70

In this case the demand at a unit price of Fr. 20 is $40 + 20 + 20 = 80$. The demand at unit price Fr. 15 is $20 + 10 + 10 = 40$. Finally, the demand at unit price Fr. 10 is $40 + 0 + 70 = 110$. It follows that the cumulative demand at Fr. 20 is 80, the cumulative demand at Fr. 15 is $80 + 40 = 120$, and the cumulative demand at Fr. 10 is $80 + 40 + 110 = 230$. In this case too, the clearing price is Fr. 15.

All players will be allocated the number of shares they demand at Fr. 15. In addition, each player will receive $(100 - 80)/40 = 0.5$ unit for each unit they demand at Fr. 15. This implies that player A will receive $40+10 = 50$ units, player B will receive $20+5 = 25$ units, and player C will receive $20 + 5 = 25$ units. Because the clearing price is Fr. 15, player A's payoff is $(20 - 15) \times 50 =$ Fr. 250, player B's payoff is $(20 - 15) \times 25 =$ Fr. 125, and player C's payoff is $(20 - 15) \times 25 =$ Fr. 125.

The computation of payoffs marks the completion of a round of bidding. You will be required to participate in a number of rounds of bidding. After the first two rounds you will be given an opportunity to discuss strategies with the other participants in the experiment. This discussion will be limited to 5 minutes. Thereafter, after every two rounds you will be given 3 minutes to discuss strategies with the other participants. During the time allowed for discussion, you can choose either to announce the strategies that you recommend be played, or you could enter in personal discussions with the other participants. You are free to change your bidding strategies between rounds. Your actual bids in each round will be known only to the experimenter and yourself. They will not be revealed to other players. The experimenter will end the experiment after some number of rounds of bidding. This number of rounds is known only to the experimenter. After the experiment is ended, your cumulative dollar payoff will be determined using the following formula:

$$\$ \text{ payoff } = \text{ sum of round by round payoffs (in Fr.) } \times 0.01.$$

Your payoff from the experiment will only be known to you and the experimenter.

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