Mortgage Valuation Under Optimal Prepayment

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Mortgage originators offer borrowers various combinations of "points" — loan fees — and coupon: high points and low coupon or low points and high coupon. In this article points are interpreted as a device serving to separate borrowers with high prepayment probabilities from those with low prepayment probabilities. Borrowers and lenders are treated symmetrically: both are risk neutral and both have complete and frictionless access to credit markets (implying that borrowers can finance points if they wish), except that borrowers' prepayment speeds are private knowledge. Equilibria are derived, both when borrowers cannot prepay voluntarily and when they can.

Mortgage originators offer borrowers various combinations of "points" — loan fees — and coupon: high points and low coupon or low points and high coupon. Several explanations for the existence of points have been offered in the literature, as noted in Section 1. The explanation we find most persuasive — that points serve to separate borrowers with different prepayment probabilities — has been suggested informally by several authors, but has not been implemented formally in a satisfactory way.

The intuition for the account given here of the role of points is direct. In the real estate section of the Sunday newspaper the choice recommended to readers who are prospective mortgagors between various combinations of coupon and points is based on

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mobility: borrowers who are unlikely to move in the near future are advised to pay points in order to obtain a low coupon, whereas borrowers who are likely to move shortly are advised to take a mortgage with low points even though the coupon is higher. A natural inference is that lenders offer the menu of points and coupon precisely in order to separate heterogeneous borrowers who have private knowledge of their likely prepayment behavior. The present article provides formal implementation of this intuition.

A major conclusion of this article is that the simplest model capturing the foregoing intuition (and incorporating voluntary refinancing) implies a semipooling equilibrium, not a separating equilibrium. If borrowers can refinance at will (even though subject to fees), then slow borrowers — borrowers with low probability of involuntary prepayment — can always imitate fast borrowers. This result implies that in equilibrium slow borrowers are indifferent among the various alternatives, whereas fast borrowers strictly prefer the low-points mortgage.

In generalizing the model of this article, Stanton and Wallace (1995) showed that the characteristic of the equilibrium that the slowest borrowers are indifferent among all available mortgages is a consequence of the assumption here that the social costs of mortgage origination are zero, so that points and prepayment penalties are just transfers from borrowers to lenders. Stanton–Wallace relaxed that assumption and derived equilibria with many borrower types and many available mortgages. In their model each borrower class is indifferent between two mortgages and strictly rejects all the others. Their article is discussed in Section 6.

1. Literature Review

Several explanations for the existence of points were offered in the early literature on mortgage finance: most simply, points are payment for the costs of mortgage origination. Alternatively, points can be interpreted as payment for the right granted the borrower to prepay the mortgage without penalty [Dunn and McConnell (1981)]. These accounts do not explain why lenders do not simply incorporate the origination costs or option premium in the coupon rate, nor do they explain why lenders offer borrowers a menu of points-coupon combinations rather than a single set of terms. More recently, a number of articles have seriously addressed the question of why lenders charge points. As will be seen, several proceed along lines similar to the present article.

1.1 Dunn and Spatt

Dunn and Spatt's (1988) article gives a valuable overview of mortgage finance from the point of view of the theory of efficient risk sharing. Many subsequent articles on mortgage financing, including this one, can be viewed as implementing the research program outlined informally by Dunn and Spatt. Rather than examining mortgage contract features in an ad hoc fashion, as in most of the earlier mortgage literature, Dunn and Spatt proposed interpreting mortgage contract terms as devices for dealing with the asymmetric information inherent in mortgage lending.

Dunn and Spatt began by pointing to a basic tension that any institutional setup for residential financing must deal with. Lenders and borrowers are risk averse, implying that both parties are motivated to ensure fully such idiosyncratic events as borrower default and prepayment. However, adverse selection and moral hazard prevent this outcome, for reasons that have been familiar to economists at least since the work of Knight (1921), and certainly since Rothschild and Stiglitz (1976), but have not yet been fully applied in the context of mortgage finance.[1] Financial institutions and contract features are appropriately interpreted as devices that achieve satisfactory (ideally, optimal) risk sharing subject to incentive compatibility constraints.

It follows that mortgage valuation cannot be analyzed by simply examining contract terms in isolation from the asymmetric information environment that gives rise to these contract terms; doing so, Dunn and Spatt observed, implies neglect of the clientele effects that also accompany asymmetric information. Thus, Dunn and Spatt proposed that self-selection, clientele effects, and valuation be analyzed simultaneously. Dunn and Spatt interpreted most of the major features of mortgage contracts from this vantage.

For the present purpose, the most relevant sections of Dunn and Spatt's article are the discussions of fixed-rate versus adjustable-rate loans and of points. Given the prepayment option, a fixed-rate loan can be interpreted as equivalent either to a fixed-rate nonprepayable loan plus a call option or to a floating-rate loan with an option to continue the loan at the same rate without penalty. The coupon on a fixed-rate mortgage will reflect these option features. This account coincides with that modeled formally here.

Further, Dunn and Spatt observed, the option to continue the loan at the coupon rate will be more valuable to borrowers who attach low probability to the event of prepaying the loan for nonfinancial reasons. Such borrowers will self-select toward mortgages with high

[1] This interpretation of Knight is discussed in LeRoy and Singell (1987).

points and low coupons. Dunn and Spatt concluded that in each case the option features can be evaluated accurately only if account is taken of the fact that a mortgage with a given set of options will be chosen by borrowers who value these options most highly.

1.2 Chari and Jagannathan

Chari and Jagannathan (1989) attributed the existence of points to an insurance motive on the part of borrowers: if the event of selling the mortgaged property is positively correlated with income surprises, then risk-averse borrowers will accept a fairly priced option the payoff of which is negatively correlated with the event of selling the house. Mortgage points are such an option, since the annual cost of mortgage points is higher the sooner the mortgaged property is sold and the mortgage is repaid [see Dunn and Spatt (1985) for a similar explanation for the existence of the due-on-sale clause].

The adequacy of the Chari and Jagannathan explanation for mortgage points is far from clear. First, it is not obvious that the correlation between income surprises and the event of selling a mortgaged property is in fact positive (consider the current Southern California real estate market, for example, where many real estate sales have occurred as consequences of layoffs), as required by the insurance explanation. Second, the Chari–Jagannathan model predicts that fast borrowers would have a comparative advantage with a mortgage with high points and low coupon, rather than the reverse as the empirical evidence suggests.

1.3 Kazarian

Kazarian (1993) focused on the relation between borrower mobility and the choice between fixed-rate and adjustable-rate mortgages. As with most writers, Kazarian's model implies that fast borrowers have a comparative advantage with adjustable-rate mortgages, while slow borrowers prefer fixed-rate mortgages. Kazarian emphasized the clientele effects of introduction of adjustable-rate mortgages in the early 1980s: prior to introduction of adjustable-rate mortgages, borrowers with different mobilities all borrowed using fixed-rate mortgages, implying that fast borrowers were subsidizing slow borrowers. Introduction of adjustable-rate mortgages, however, gave fast borrowers a way to avoid paying the subsidy. An implication is that introduction of adjustable-rate mortgages should have reduced the speed of fixed-rate mortgages.

Kazarian's conclusions are consistent with those derived below, but his model is very different. For example, Kazarian's model implies that fast borrowers will (for some parameter values) prefer adjustable-rate mortgages to fixed-rate mortgages even in deterministic settings. In the present model and in most of the literature, in contrast, comparative advantage arises from differences in option values; such differences cannot exist in the absence of uncertainty about future interest rates. It is not clear why in Kazarian's model there is any nontrivial difference between adjustable-rate mortgages and fixed-rate mortgages in the absence of uncertainty about future interest rates.

1.4 Brueckner

Brueckner (1994) attempted to implement the Dunn and Spatt (1988) research program — that is, to apply the modern formal theory of equilibrium under asymmetric information to mortgage finance. Brueckner assumed that borrowers have either high or low prepayment probabilities, and that these probabilities cannot be directly observed by lenders. As in Rothschild-Stiglitz (1976), lenders offer contracts that separate the two classes of borrowers subject to a zero-profit constraint. Incentive compatibility requires that these contracts incorporate quantity restrictions. In these specifications Brueckner's model coincides with that presented below, except that we will assume that borrowers as well as lenders are risk neutral, a restriction that greatly simplifies derivation of equilibrium since it eliminates the need for quantity restrictions in order to induce separation.

Brueckner's model differs from that presented below in two major respects. First, the quantity restrictions imply that borrowers cannot generally finance points. This leads to a blurring of the intuition of Brueckner's main result, which is that borrowers' prepayment probabilities alter their trade-offs between fixed contract terms (points and prepayment penalties) and variable terms (coupon), thereby creating comparative advantage. Because borrowers cannot finance points in Brueckner's model,[2] the argument of Section 2 here for the equivalence of points and penalty fails. The financing restriction induces a trade-off between front-loaded payments (points) and end-loaded payments (prepayment penalties); this trade-off is irrelevant for Brueckner's result and serves only to divert attention from the fixed cost–variable cost trade-off that is central to this article.

The second difference between Brueckner's model and that reported below is that in Brueckner's model future interest rates are deterministic. This specification has the obvious implication that Brueckner cannot analyze financially motivated mortgage prepayment, as he

[2] Brueckner argued (p. 429) that the nonequivalence of points and penalty in his model is due to its incorporation of adverse selection. This is incorrect: the model below incorporates adverse selection, yet has points equivalent to penalties. Points are not equivalent to penalties in Brueckner's model because borrowers cannot finance points, a restriction that is a consequence of the quantity restrictions borrowers impose. These quantity restrictions in turn are necessary to achieve incentive-compatible mortgage choices in Brueckner's model because of borrowers' risk aversion.

recognized. Since in Brueckner's model mortgage choice does not involve valuing future prepayment options, his model cannot situate mortgage valuation as an application of options theory. An adequate analysis of mortgage selection involves valuation of the option to prepay (or, depending on the point of reference, the option to extend the loan at an unchanged rate) by borrowers with different prepayment probabilities, and the value of this option to any borrower critically depends on interest rate uncertainty.

2. Points and Prepayment Penalties

The discussion up to now has dealt with discount points. In the formal model our concern will instead be with prepayment penalties: surcharges added to the outstanding balance of a mortgage loan upon prepayment. It would appear that the two are completely different: points are a sunk cost, presumably having no bearing on whether agents prepay, whereas a prepayment penalty applies only if the loan is in fact prepaid and therefore deters prepayment. However, this is incorrect: in equilibrium high points imply low coupon, and the low coupon deters prepayment for exactly the same reason as a prepayment penalty.

In fact, under simplifying assumptions (listed below), points and prepayment penalties are virtually identical.[3] Compare a mortgage with points z versus a mortgage with prepayment penalty z. Assume that the coupon (payment per dollar borrowed) is $c/(1+z)$ on the former and c on the latter. Suppose also that the points are financed, so that the amount borrowed is $1+z$ for the points mortgage versus 1 for the penalty mortgage. The periodic payment (the product of the amount borrowed and the coupon; here we are treating the mortgages as perpetuities) is c for both mortgages, and the cash requirement if the mortgages are prepaid is $1+z$ for both mortgages. The initial proceeds of the loans differ by z^2 [the difference between $(1+z)(1-z)$ and 1] since borrowers must pay points on the points. For points on the order of 1% or 2%, this difference is trivial, implying that points and prepayment penalties are essentially equivalent.

For the most part we will work with prepayment penalties rather than points. The former specification is more convenient because, for given prepayment behavior, the coupon turns out to be related linearly to the prepayment penalty. This linearity property implies that points are related linearly to $c/(1+z)$ (see Table 1, row 4) and therefore are related nonlinearly to c.

[3] This identity was noted by Chari and Jagannathan (1989) and Dunn and Spatt (1985).

Table 1
Equivalence of points and penalty mortgages

	Points	Penalty
Points	z	0
Penalty	0	z
Nominal amount	$1+z$	1
Coupon	$c/(1+z)$	c
Effective proceeds	$(1+z)(1-z) \cong 1$	1
Periodic payment	c	c
Repayment amount	$1+z$	$1+z$

This argument for the virtual equivalence of penalty and points is subject to qualification. First, we are assuming not only that borrowers can finance points, but also that they can do so on the same terms as they pay under the mortgage itself. This assumption is equivalent to assuming that the shadow price of the constraint imposed by the lender on the amount financed — typically no more that 80% or 90% of the appraised value of the collateral — is zero or nearly so. If, on the contrary, the buyer has difficulty raising the cash for the down payment, he might elect to finance points even when doing so implies substantially higher costs.[4] When financing points results in higher costs per dollar financed, the equivalence between points and penalty breaks down.

Second, in the foregoing demonstration it was assumed that mortgages are perpetuities. In fact, mortgages usually include provision for principal reduction, so that the outstanding balance declines with the length of time the loan is outstanding. Thus the prepayment penalty decreases over time. This consideration is not incorporated in the foregoing argument. This objection, although valid, is not important quantitatively: most mortgages are prepaid in the first 10 years of their lives, during which time the reduction of principal is minor, at least for 30-year mortgages.

Third, the discussion here abstracts from default. In the event of default on a government-insured loan, the lender is repaid the outstanding principal by the government, but cannot recover the prepayment penalty.

Fourth, we are assuming that the prepayment penalty is charged whenever the mortgage is terminated. In fact, the terms of most mortgages with prepayment penalties provide that the borrower can charge the penalty only under a refinancing; when the mortgaged property is sold the prepayment penalty is generally not charged since then the

[4] For example, lenders often require buyers financing more than 80% of the appraised value of their home to obtain mortgage insurance on the entire amount financed.

mortgage is terminated at the initiative of the lender, who is exercising the due-on-sale clause.

In view of the substantial equivalence of points and penalties, it is noteworthy that the two are viewed very differently, both in the popular press and by financial regulators. The acceptability of lenders charging points in excess of origination costs is unquestioned. Prepayment penalties, on the other hand, are often viewed as an unconscionable victimization by lenders of helpless borrowers who have no alternative but to pay the charge. The Garn–St. Germain Act prohibits prepayment penalties on all adjustable-rate mortgages, and many states prohibit them on fixed-rate mortgages as well. The view implicit in this differential treatment of points and penalties is that borrowers are not aware of the prepayment penalty when they choose their loans; if, as assumed here, borrowers are aware of the penalty, by accepting a mortgage with a penalty they take a risk for which they are adequately compensated in the form of a lower coupon. Thus, except on the view that borrowers do not understand their mortgage loans, there are no grounds for the different regulatory treatment of points and penalties.

3. The Model

In the model presented here it is assumed that borrowers and lenders have equal and frictionless access to credit markets. Mortgages are assumed to be perpetuities. There exists a due-on-sale clause, so that termination of the mortgage must accompany sale of the collateral. All borrowers are subject to an exogenous probability of selling the mortgaged property and therefore involuntarily prepaying the mortgage. The probability of this event is different for different borrowers; it is assumed that this probability is known to borrowers, but cannot be credibly communicated to lenders except through their mortgage choice and prepayment behavior.

It is assumed that borrowers and lenders are risk neutral. This assumption is not innocuous. It is made in order to avoid insurance considerations like those analyzed by Chari and Jagannathan (1989). Also, risk aversion brings with it quantity restrictions, at least in Brueckner's (1994) implementation, which greatly complicates the analysis and dilutes the intuition of points and coupon as representing fixed and variable costs, respectively, as noted above.[5]

[5] A potential problem with this specification is that if utility is linear with deterministic coefficients, then in a stationary exchange economy real interest rates are deterministic and constant. It is true that in production economies with random productivity shocks, real interest rates can be random, but then we have the unattractive implication of bang-bang solutions for consumption.

The value of mortgage loans to investors depends on borrowers' prepayment probabilities, and this is true whether or not borrowers have the option to prepay voluntarily. In principle, mortgages issued to borrowers with different prepayment probabilities can be combined in heterogeneous mortgage pools or held in separate portfolios. However, whenever heterogeneous mortgagors borrow using the same instrument, some borrowers are subsidizing others. Assuming that borrowers have a prepayment option, that option is strictly more valuable the lower the borrower's probability of involuntary prepayment (except in special cases where the option is without value), since to the extent that a borrower is likely to be required to prepay, the option to prepay voluntarily is redundant. Therefore, slow borrowers will generally pay a higher coupon than fast borrowers if their mortgages are separated.

Correspondingly, if both fast and slow borrowers are combined in a mortgage pool, the former will be subsidizing the latter. In such a setting a mortgage originator could generally offer a contract that would attract the fast borrowers, and in doing so could earn a positive profit if the terms of the contract were chosen appropriately. Thus the pooling equilibrium will break down. In this article attention is restricted to separating and semipooling equilibria (except when borrowers voluntarily prepay in all interest rate states, so that they become effectively homogeneous).

A menu of choices between various combinations of coupon and points provides lenders with a device for separating borrowers with different prepayment probabilities. Under the assumption that (contrary to the preceding two paragraphs) voluntary refinancing is prohibited, this separation is strict: one expects (and we will verify, under qualifications to be stated) that fast borrowers will choose a mortgage with low points and high coupon, while slow borrowers will choose a mortgage with high points and low coupon. Since borrowers are assumed to have access to perfect capital markets, they can finance mortgage points. Therefore, the points-coupon locus represents the trade-off between fixed and variable costs, not between front-loaded and end-loaded payments, and different prepayment probabilities imply different trade-offs between fixed and variable cost.

The model to be presented has several features that render it attractive relative to what is currently available in the mortgage literature:

- Borrower heterogeneity is modeled using differing prepayment probabilities rather than differing horizons. Using probabilities is more

One alternative is to think of utility as linear, but with stochastic coefficients, so that a zero market price of risk can coexist with stochastic interest rates.

realistic because borrowers seldom know exactly when they will be prepaying their mortgages, but generally do know whether it is more or less likely that this event will occur at any given date. Also, using prepayment probabilities rather than horizons facilitates modeling mortgage pricing using the stochastic steady-state equilibrium models that have come to prevail in the economics literature on equilibrium security pricing [Lucas (1978), Mehra and Prescott (1985), for example]. These models are easier to confront with data than the three-date models used up to now in the literature on real estate finance.

- The model assumes that short-term interest rates are a Markov chain, a specification that again points toward a stochastic steady-state equilibrium as the appropriate solution concept.
- The model avoids artificial restrictions on capital market participation.

The model to be presented bears a superficial resemblance to the insurance model of Rothschild and Stiglitz (1976). In both settings, agents have private information about their own type and are induced to self-select by the contracts offered. In fact, the assumption here of risk neutrality results in a model much simpler than that of Rothschild and Stiglitz. In Rothschild and Stiglitz's setting, separating equilibrium is achieved via contracts that impose a binding constraint on the quantity of insurance purchased. This departure from standard equilibrium notions, which involve only prices, results in problems in defining an appropriate equilibrium concept, as discussed in the literature that Rothschild and Stiglitz's article stimulated [Boyd and Prescott (1986), Cho and Kreps (1987), Riley (1979), for example]. Here, in contrast, such problems are avoided since borrowers are separated using two-part prices, not quantity restrictions. Therefore, a standard equilibrium concept can be used.

Deriving a separating equilibrium in the present context consists of postulating a pattern of voluntary prepayment and mortgage choice for each class of borrowers, pricing each mortgage based on the assumed prepayment behavior of the borrowers who choose that mortgage, and then verifying that the incentive compatibility conditions for the assumed behavior pattern are satisfied so that borrowers find it in their interest to behave as postulated.

There are three classes of agents in the model: mortgage originators, mortgage borrowers, and investors. Mortgage originators lend money to borrowers, then immediately sell the mortgages to investors. Originators have no costs of processing loans. There is free entry into the mortgage origination business, implying that originators earn zero profits. Borrowers must finance their real-estate holdings with mortgage loans, so the only decisions they make are which mortgage to use and, when prepayment without sale of the collateral is permitted, whether to refinance.

The interest rate has two states: "up," for a high interest rate, and "down" (to achieve a mnemonic notation it is useful to preserve "high" and "low" for prepayment penalties). The one-period interest rate follows a Markov chain with transition probabilities $\pi_{u|u} = \pi_{d|d} = \pi$. We will take $1/2 < \pi < 1$ throughout, so that interest rates are positively autocorrelated.

There are two borrower types, "fast" and "slow," according to whether the probability of moving, and thereby triggering involuntary prepayment because of a due-on-sale clause, is high or low. Borrowers know their own type, but lenders cannot ascertain borrowers' types except through their market behavior. The prepayment probability of each borrower type is common knowledge. There are many borrowers of each type, so that pools consisting of mortgages of each type have behavior that is a deterministic function of market interest rates [see Feldman and Gilles (1986) for one way to justify this application of the law of large numbers]. The mortgage market is competitive and without transactions costs.

4. No Voluntary Prepayment

It is assumed throughout that borrowers have the option to prepay their loans when they sell the collateral, an exogenous event. In this section it is assumed that borrowers can call the loan only when the collateral is sold. Under this specification, and assuming a due-on-sale provision so that investors can put the mortgage if the collateral is sold, mortgages are prepaid whenever the collateral is sold. This is so because, in the incentive-compatible separating equilibria under analysis here, borrower types are revealed by mortgage choices, so the decision rule of any mortgagor is common knowledge, as therefore is the value of any mortgage. That being the case, a mortgage with market value less than or equal to its nominal value will be put by the lender when the collateral is sold, while a mortgage with market value greater than or equal to its nominal value will be called by the borrower. In either case, the mortgage is prepaid. Thus, the existence of a due-on-sale clause, together with the assumption that mortgage origination is costless, means that mortgages will always be prepaid when the collateral property is sold.

As noted above, mortgages are assumed to be perpetuities — that is, the periodic payment does not include a component for principal reduction. Thus, the value of a mortgage depends on its origination value and the current interest rate state, but not on the age of the mortgage.

For concreteness, we concentrate analysis on mortgages originated in the up state. We seek equilibria in which borrowers with slow (fast) prepayment speeds use mortgages with high (low) prepayment penalties and low (high) coupon. Therefore, in valuing the high- (low-) penalty mortgage the relevant prepayment speed is that of the slow (fast) borrower.

Define p_{ejo} as the price in interest rate state e ($e = u, d$) of a j-penalty mortgage ($j = h, \ell$) originated in interest rate state o ($o = u, d$). As a scale normalization, it is assumed throughout that the periodic payment is \$1. Focusing on the high-penalty mortgage issued by the slow borrower and originated in the up interest rate state, we have that p_{uhu} is given by

$$p_{uhu} = r_u^{-1}[1 + \delta_s p_{uhu}(1 + z_h) + (1 - \delta_s)(\pi p_{uhu} + (1 - \pi) p_{dhu})]. \quad (1)$$

Here r_u is the gross one-period interest rate in the up state, δ_s is the prepayment speed of the slow borrower, z_h is the prepayment penalty on the high-penalty mortgage, expressed as a fraction of origination value, and p_{dhu} is the value of the same mortgage in the down state. Similarly, p_{dhu} is given by

$$p_{dhu} = r_d^{-1}[1 + \delta_s p_{uhu}(1 + z_h) + (1 - \delta_s)((1 - \pi) p_{uhu} + \pi p_{dhu})], \quad (2)$$

where it is assumed that $1 < r_d < r_u$. Note that if the mortgage is prepaid, this occurs by convention after the periodic payment is made, implying that the periodic payment is due regardless of whether or not the borrower prepays.

The coupon on a mortgage is its periodic payment divided by its value in the origination state. Therefore, we have

$$c_{hu} \equiv \frac{1}{p_{uhu}}. \quad (3)$$

Upon substituting Equation (2) in Equation (1), dividing by p_{uhu} and using Equation (3), it is seen that the coupon depends linearly on the prepayment penalty:

$$c_{hu} = \alpha_{hu} + \beta_{hu} z_h, \quad (4)$$

where $\alpha_{hu} > 0$ and $\beta_{hu} < 0$ depend on π, r_u, r_d, and δ_s. A similar expression exists for $c_{\ell u}$.

For reference below, we note that the coupon associated with $z = 0$ is less than $r_u - 1$, the net interest rate. To see this, observe that if $z = 0$, p_{uju} and p_{dju} satisfy

$$p_{uju} = r_u^{-1}[1 + \delta p_{uju} + (1 - \delta)\{\pi p_{uju} + (1 - \pi) p_{dju}\}], \quad (5)$$

$$p_{dju} = r_d^{-1}[1 + \delta p_{uju} + (1 - \delta)\{(1 - \pi)p_{uju} + \pi p_{dju}\}]. \tag{6}$$

The solution to Equations (5) and (6) implies

$$\frac{p_{uju}}{p_{dju}} = \frac{r_d + (1 - \delta)(1 - 2\pi)}{r_u + (1 - \delta)(1 - 2\pi)}. \tag{7}$$

Since $r_d < r_u$ and $\pi > 1/2$, we have $p_{uju} < p_{dju}$. Multiplying Equation (5) by r_u and dividing by p_{uju} results in

$$r_u = c_{hu} + \delta + (1 - \delta)\left(\pi + (1 - \pi)\frac{p_{dju}}{p_{uju}}\right), \tag{8}$$

using Equation (3). From Equation (8), $p_{uju} < p_{dju}$ implies $c_{hu} < r_u - 1$.

The reason the coupon on a mortgage originated in the up state is lower than the up interest rate is that in the up state there is some probability that the interest rate will drop, implying that the mortgage will appreciate. There is no possibility of a capital loss since the interest rate cannot rise. Given that the expected capital gain on the mortgage is then positive, the equilibrium equality between the expected rate of return on the mortgage and the interest rate can obtain only if the coupon is lower than the interest rate. This is essentially a term structure argument: with the short-term interest rate likely to drop, the long-term interest rate is lower than the short rate; a (slow) mortgage is similar to a long-term bond.

Note, also for reference later, that Equation (8) implies that when $z = 0$ the coupon is higher, the higher the borrower's prepayment speed. Most simply, the faster the prepayment speed the closer a mortgage is to a short-term loan, and therefore the closer the coupon is to $r_u - 1$. More explicitly, a high prepayment probability decreases the likelihood of the joint event that the interest rate drops and the mortgage is not prepaid, implying that the consequent capital gain (to the investor) if this event does occur has lower present value. Therefore, he or she requires a higher coupon. Observe finally that if $\delta = 0$ is substituted in Equations (5) and (6), the resulting coupon coincides with the yield on a noncallable perpetuity.

In order to determine which mortgage to choose, each borrower needs to ascertain the individual cost of each mortgage. To that end, define v_{eiju} as the individual cost of mortgage type j ($j = h, \ell$), originated in interest rate state u, to borrower type i ($i = s, f$), when the current interest rate state is e ($e = u, d$). The borrower evaluates this individual cost in exactly the same way as the investor, except that the borrower figures the cost of the due-on-sale clause using his own prepayment speed. Therefore, we have

$$v_{uiju} = r_u^{-1}[1 + \delta_i p_{uju}(1 + z_j) + (1 - \delta_i)(\pi v_{uiju} + (1 - \pi) v_{diju})], \quad (9)$$

$$v_{diju} = r_d^{-1}[1 + \delta_i p_{uju}(1 + z_j) + (1 - \delta_i)((1 - \pi) v_{uiju} + \pi v_{diju})]. \quad (10)$$

As above, we have that Equations (9) and (10) determine the individual coupon, $1/v_{uiju}$, that would be consistent with capital market equilibrium if borrower i chose mortgage j.

Borrowers will choose the mortgage that minimizes the individual cost per dollar borrowed. Therefore, in an equilibrium in which borrower s chooses mortgage h and borrower f chooses mortgage ℓ, the incentive compatibility conditions in state u for mortgages originated in state u are

$$\frac{v_{ushu}}{p_{uhu}} \leq \frac{v_{us\ell u}}{p_{u\ell u}} \quad (11)$$

for borrower s, and

$$\frac{v_{uf\ell u}}{p_{u\ell u}} \leq \frac{v_{ufhu}}{p_{uhu}} \quad (12)$$

for borrower f.

Now, from comparison of Equations (1) and (9), for example, it is evident that the individual and market values of the mortgages borrowers select are equal. Since by assumption slow (fast) borrowers choose mortgage h (ℓ), the left-hand sides of Equations (11) and (12) equal unity, and the incentive compatibility conditions become

$$1 \leq \frac{v_{us\ell u}}{p_{u\ell u}} \quad (13)$$

and

$$1 \leq \frac{v_{ufhu}}{p_{uhu}}. \quad (14)$$

Thus, incentive compatibility requires that the rejected mortgages have individual costs that are not less than market values.

A simple diagrammatic interpretation of the incentive compatibility conditions is available. Figure 1 shows the penalty and coupon for two mortgages: $(z_h, 1/p_{uhu})$ and $(z_\ell, 1/p_{u\ell u})$. By assumption the slow borrower chooses the high-penalty mortgage. The line passing through $(z_h, 1/p_{uhu})$ represents the coupons consistent with different levels of the penalty, assuming the prepayment speed of the slow borrower. Therefore, the vertical coordinate of the point on the line with horizontal coordinate z_ℓ represents the coupon the slow borrower would pay if the penalty were z_ℓ. The slow borrower would thus be indifferent between mortgages $(z_\ell, 1/v_{us\ell u})$ and $(z_h, 1/p_{uhu})$. However,

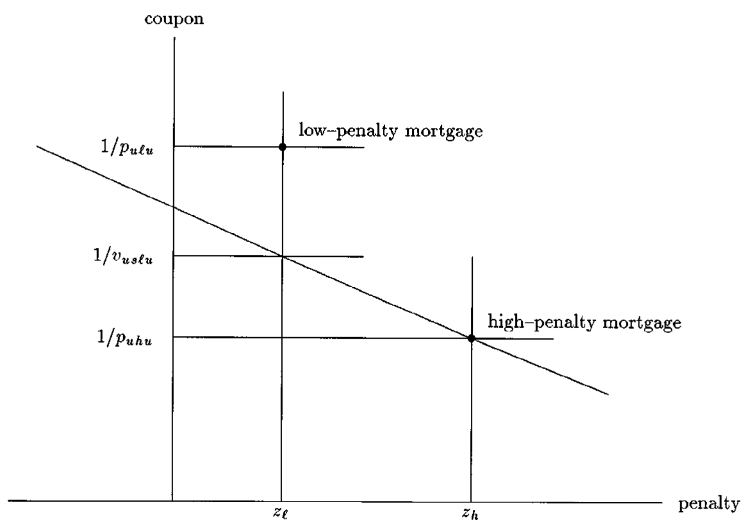

Figure 1
Incentive compatibility
The diagonal line passing through $(z_h, 1/p_{uhu})$ displays the coupon-penalty locus for the slow borrower. The slow borrower, being indifferent between contract terms $(z_h, 1/p_{uhu})$ and $(z_\ell, 1/v_{us\ell u})$, would reject the low-penalty mortgage since its coupon, $1/p_{u\ell u}$, exceeds $1/v_{us\ell u}$.

the borrower must choose between $(z_h, 1/p_{uhu})$ and $(z_\ell, 1/p_{u\ell u})$; $(z_\ell, 1/p_{us\ell u})$ is not available. The diagram is drawn so that the low-penalty mortgage has a coupon higher than $1/v_{us\ell u}$:

$$\frac{1}{v_{us\ell u}} < \frac{1}{p_{u\ell u}}, \tag{15}$$

so the slow borrower rejects the low-penalty mortgage. Equation (15) is the strict inequality associated with the incentive compatibility condition in Equation (13). Thus, the diagrammatic representation of incentive compatibility consists of the requirement that the coupon-penalty point of the rejected mortgage lie to the northeast of the coupon-penalty locus that includes the chosen mortgage.

The final step in characterizing equilibrium mortgage terms and choices consists of comparing the coupon-penalty loci for borrowers with different prepayment speeds. In Figure 2 the coupon-penalty loci intersect in the positive quadrant by virtue of the facts that (1) the coupon-penalty locus for the fast borrower is steeper than that of the slow borrower, and (2) the mortgage chosen by the fast borrower has higher coupon than that chosen by the slow borrower at $z = 0$, as noted above.

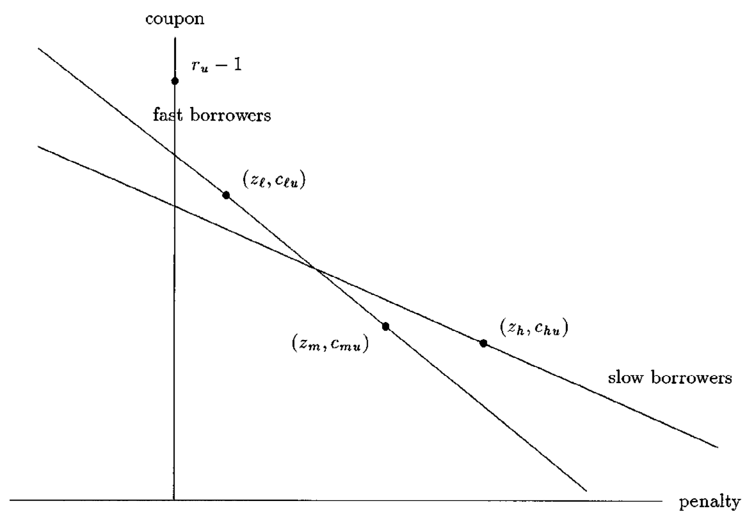

Figure 2
Mortgage choice
The pair of contract terms (z_h, c_{hu}) and (z_l, c_{lu}) generate an incentive compatible separating equilibrium: slow borrowers strictly prefer the high-penalty mortgage and fast borrowers strictly prefer the low-penalty mortgage. The pair of contract terms (z_h, c_{hu}) and (z_m, c_{mu}) would represent equilibrium mortgage pricing if the fast borrowers instead used the contract (z_m, c_{mu}). However, in the presence of the contract (z_m, c_{mu}), use of (z_h, c_{hu}) by the slow borrower is not incentive compatible.

Figure 2 also shows a pair of mortgage terms — (z_h, c_{hu}) and $(z_\ell, c_{\ell u})$ — that are associated with a separating equilibrium. Originators offer one mortgage with high penalty and low coupon, and one with low penalty and high coupon. The penalty and coupon are set so that the fast borrower is willing to pay the high coupon, $c_{\ell u}$, in order to obtain a low penalty, while the slow borrower is willing to pay a high penalty in order to obtain the low coupon, c_{hu}. Thus, the borrowers separate. Observe that for each borrower each rejected mortgage has a coupon-penalty point that lies to the northeast of the coupon-penalty locus passing through that borrower's chosen mortgage, indicating the incentive compatibility of the assumed mortgage choices. Investors, knowing that the mortgages are priced correctly with respect to prepayment speeds, are willing to hold either mortgage indifferently.[6]

Note that the fact that the coupon-penalty locus is always steeper for the fast borrower than for the slow borrower implies that it is

[6] In order to obtain separation using the same z_ℓ and z_h, it is necessary to set z_ℓ and z_h so that the value of z associated with the intersection of the coupon-penalty loci for the two borrower classes lies between z_ℓ and z_h both for mortgages originated in the up state and for those originated in the down state.

impossible to generate separating equilbria in which the borrowers choose the wrong mortgages. It is, however, possible to set the two prepayment penalties so that there does not exist a separating equilibrium. For example, suppose in Figure 2 that the penalties were set at z_m and z_h instead of z_ℓ and z_h. The incentive compatibility conditions for a separating equilibrium fail, since the slow borrowers prefer (z_m, c_{mu}) to (z_h, c_{hu}). But then the prepayment speed on the resulting pool would be an average of δ_s and δ_f,[7] so (z_m, c_{mu}) would no longer be consistent with equilibrium.

As Figure 2 makes clear, incentive compatible separating equilibria occur when mortgage originators offer two mortgages, one with a penalty less than the penalty associated with the intersection of the two coupon-penalty loci, the other with a penalty that is greater.

For any two prepayment speeds, δ_s and δ_f, there exists a value of z such that the two borrowers will receive the same coupon (indicated by the intersection of the two coupon-penalty loci in Figure 2). The interpretation is that for these two borrowers the event of involuntary prepayment in the down state, which is good for the borrower and bad for the lender, is exactly offset in expected present value terms by the event of involuntary prepayment in the up state, which is bad for the borrower and good for the lender.

This intersection of the coupon-penalty loci for borrowers with different speeds occurs at different penalty levels for different pairs of prepayment speeds.[8] As a result, when there are more than two classes of borrowers, for each borrower class there exists an interval for z such that the coupon for that borrower is maximal over borrower classes. Thus, there can be strict separation among borrower classes even when there are more than two borrower types (see Figure 3).

5. Voluntary Prepayment

In this section it is assumed that a borrower using mortgage j always has the option to prepay the mortgage at its origination value plus proportional penalty z_j. The derivation of equilibrium coupons for given penalties consists of valuing each mortgage, now assuming that prepayment as well as mortgage choice is optimal for each borrower under equilibrium prices.

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[7] The weights of the average are indeterminate since the fast borrowers are indifferent between (z_h, c_{hu}) and (z_m, z_{mu}).

[8] For example, suppose that $r_u = 1.5$, $r_d = 1$, and $\pi = 0.75$. Then for $\delta = 0$ the coupon equals 0.25 regardless of z. For $\delta = 0.2$ a coupon of 0.25 occurs with $z = 0.4165$, whereas with $\delta = 0.4$ the critical value of z is 0.3570.

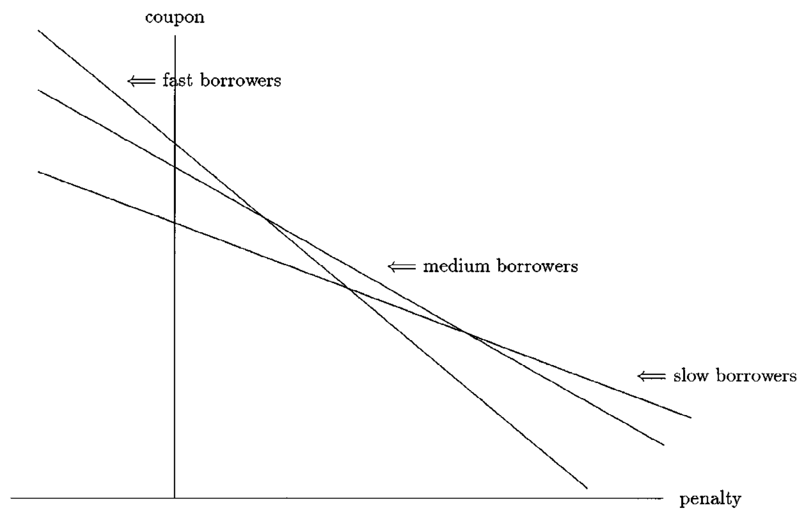

Figure 3
Mortgage choice: three borrower classes
When there are more than two borrower classes, for each borrower class there exists an interval of penalties such that the associated coupon is maximal over borrower classes. Therefore, the conclusion that there exist incentive compatible strictly separating equilibria carries over when there are many borrower classes.

There exist three undominated regimes for voluntary prepayment: (1) never prepay voluntarily, (2) prepay voluntarily in the down state but not the up state, and (3) always prepay voluntarily. The other alternative, prepay voluntarily in the up state but not the down state, is dominated (because this result is intuitive, its proof is deleted).

It is necessary first to justify Figure 4, which plots the coupon-penalty loci for a given borrower under the three prepayment regimes. A mortgage held by a borrower who elects never to prepay voluntarily is valued the same way here as in the preceding section, where it was assumed that borrowers do not have the option to prepay voluntarily.

Turn now to the regime in which borrowers prepay their mortgages in the down state but not the up state. Equation (1) is replaced by

$$p_{uju} = r_u^{-1}[1+\delta p_{uju}(1+z)+(1-\delta)(\pi p_{uju}+(1-\pi)p_{uju}(1+z))], \quad (16)$$

from which it is evident that if $z = 0$ then the coupon equals $r_u - 1$.

If voluntary prepayment occurs in both states then Equation (16) becomes

$$p_{uju} = r_u^{-1}(1 + p_{uju}(1 + z)), \quad (17)$$

so the coupon equals $r_u - 1 - z$. Thus, with $z = 0$ the coupon again equals the net interest rate. Finally, it is intuitively clear that the slope

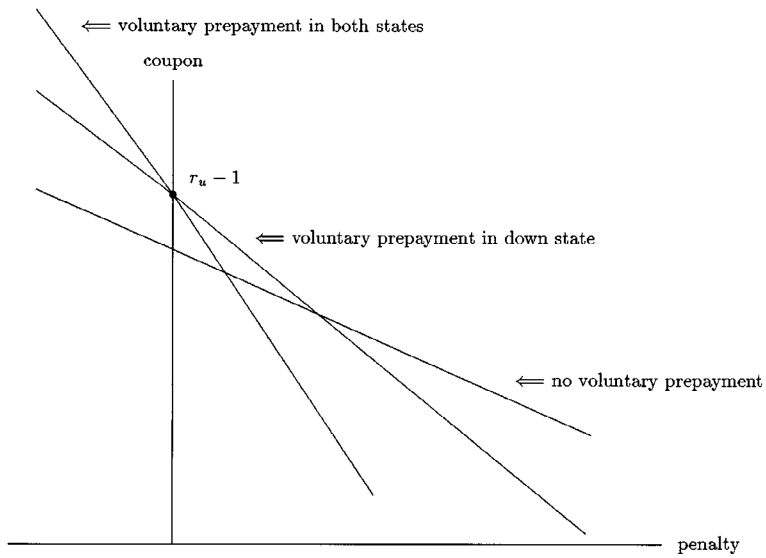

Figure 4
Prepayment regimes: mortgages originated in up state
The upper envelope of the coupon-penalty loci for different prepayment regimes determines the coupons associated with each penalty level under optimal prepayment. As expected, the higher the penalty, the less likely borrowers are to prepay volunarily.

of the coupon-penalty locus is increasingly steep in regimes 1, 2, and 3, since the more likely borrowers are to prepay, the greater the decrease in the coupon borrowers require to compensate them for an increase in the prepayment penalty.

If borrowers prepay in both states, the mortgage becomes a single-period loan. To see this, solve Equation (17) to yield

$$r_u - 1 = c_{ju} + z, \tag{18}$$

which says that the net interest rate equals the sum of the coupon and the prepayment penalty. Since the latter two variables are both measured per dollar borrowed, this makes sense. Essentially, the interpretation is that a mortgage which is priced so as to be prepaid in each period is equivalent to an adjustable-rate mortgage (without caps).

Borrowers wish to minimize the individual cost of mortgages per dollar of periodic payment. This is equivalent to choosing the prepayment regime that maximizes the coupon for each value of the prepayment penalty. Thus, the optimal mortgage choice for each value of z is the upper envelope of the coupon-penalty loci under the three prepayment regimes.

This locus, which is piecewise linear, has exactly the expected interpretation. If the prepayment penalty is nonpositive,[9] borrowers will prepay in either interest rate state a mortgage originated in u. They do this because with nonpositive z they receive a nonnegative payment if they do so (and they also lower the coupon if the current interest rate state is d). In this case a mortgage becomes essentially equivalent to a single-period loan or, alternatively, to an adjustable-rate mortgage with no caps.[10]

If z is positive but small, borrowers will prepay the high-coupon mortgage voluntarily whenever the down state occurs, since the reduction in the coupon more than offsets the prepayment penalty. However, they do not prepay if the current interest rate is u, since doing so incurs the prepayment penalty but does not reduce the coupon. Finally, if the prepayment penalty is high enough borrowers never prepay voluntarily since the coupon reduction, if any, is insufficient to offset the prepayment penalty.

Figure 5 shows the piecewise linear coupon-penalty loci for fast and slow borrowers. To justify Figure 5 it is necessary to show that the kink in the coupon-penalty locus for a slow borrower is above and to the right of that of the fast borrower. This is achieved by showing that both the value of z and the value of the coupon associated with a kink are decreasing functions of δ. At the kink, we have

$$p_{dju} = p_{uju}(1 + z), \tag{19}$$

reflecting the assumption that borrowers are indifferent between prepaying the mortgage in the down state [in which case its liability value is $p_{uju}(1 + z)$] and not prepaying (in which case its liability value is p_{dju}). Using Equation (19) to eliminate p_{uju}/p_{dju} in Equation (7) and solving for z, there results

$$z = \frac{r_u - r_d}{(1 - \delta)(1 - 2\pi) + r_d}. \tag{20}$$

Since $1 - 2\pi < 0$, z is a decreasing function of δ.

[9] It might be objected that mortgage lenders do not in fact offer mortgages with negative points. This, however, is far from clear. Adjustable rate mortgages are typically offered with below-market initial rates; these "teasers" are similar to negative points. Also, mortgage originators often charge loan fees that are lower than their costs of loan origination, planning to recover their costs with profits from servicing or selling servicing rights. In our model the costs of mortgage origination are zero, so the only way to model loan fees that are less than origination costs is to specify negative points.

[10] This generally applies only to mortgages originated in the up state. Mortgages originated in the down state will not be prepaid in the up state if the prepayment "penalty" is only slightly negative, since the borrower must pay a higher coupon on the new mortgage. Therefore, in the down state the conclusion that mortgages are always prepaid in the next period requires the assumption that the "penalty" is sufficiently negative to offset the coupon increase.

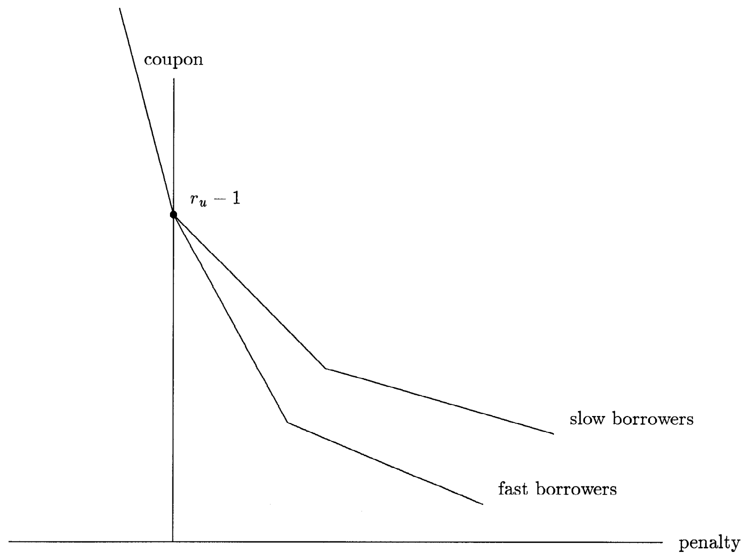

Figure 5
Coupon-penalty loci
The coupon-penalty loci for slow and fast borrowers coincide when the penalty is nonpositive. When it is positive the coupon for slow borrowers is greater than that for fast borrowers.

Multiplying both sides of Equation (16) by r_u/p_{uju}, using Equation (3), and solving for c_{bu}, there results

$$c_{ju} = r_u - 1 - z(1 - \pi(1 - \delta)). \tag{21}$$

Differentiation shows that the coupon at the kink is also a decreasing function of δ. Thus, the coupon-penalty loci are as drawn in Figure 5.

The fact that the value of z at the kink decreases with δ means that for any value of z such that fast borrowers refinance in the low interest rate state, slow borrowers also refinance. In addition, for slightly higher values of z, slow borrowers will refinance, whereas fast borrowers will not. This makes sense: slow borrowers find it worthwhile to incur the prepayment penalty in order to obtain a lower coupon, whereas fast borrowers do not refinance because the high probability of subsequent involuntary prepayment decreases the expected present value of the coupon savings.

So far the analysis has dealt with mortgages originated in the up state. Mortgages originated in the down state are less interesting since they are refinanced voluntarily only when the penalty is nonpositive (because the borrower who refinances in the down state keeps the same coupon, while the borrower who refinances in the up state raises her coupon). For completeness, we display in Figure 6 the coupon-

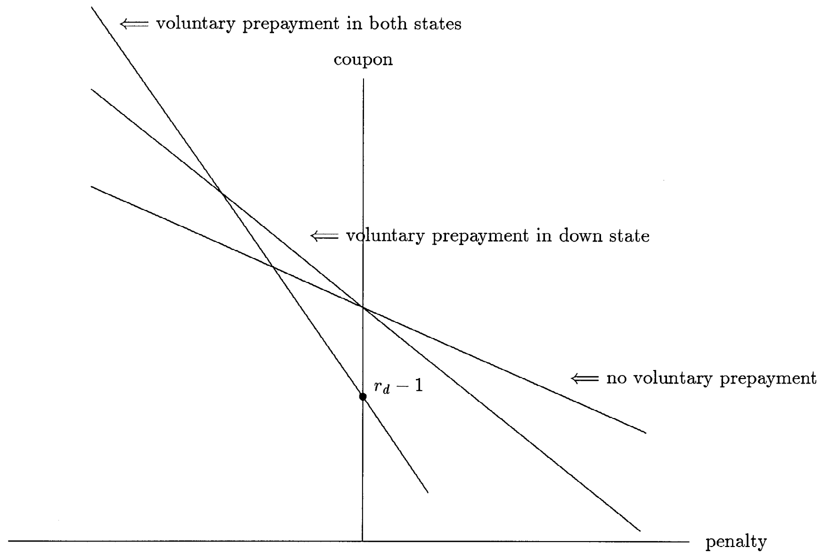

Figure 6
Prepayment regimes: mortgages originated in down state
Mortgages originated when interest rates are low are voluntarily prepaid only when the penalty is nonpositive.

penalty loci for mortgages originated in the down state. These have the expected property that borrowers optimally do not voluntarily refinance when the penalty is positive. They refinance in the down state but not in the up state when the penalty is negative and small in absolute value, and they refinance in both states when the penalty is negative and large in absolute value. The derivation of the coupon-penalty loci in Figure 6 is exactly analogous to that for mortgages originated in the up state displayed in Figure 4.

To characterize equilibria when voluntary prepayment is allowed, we have only to display diagrams of the coupon-penalty loci for different values of δ. Reverting to the case of mortgages originated in the up state, we have that for $z \leq 0$ the coupon-penalty loci coincide (obviously: with borrowers always prepaying voluntarily the probability of involuntary prepayment is irrelevant). When $z > 0$ the coupon-penalty locus for fast borrowers lies strictly below that for slow borrowers (this must be so since the kink of the fast borrower is below and to the left of that of the slow borrower, and each segment of the coupon-penalty locus is steeper for the fast borrower). Thus, to obtain a nonpooling equilibrium with two mortgages we require that the prepayment penalty on one mortgage be nonnegative and the other be nonpositive (and not both zero). The fast borrowers will choose the mortgage with nonpositive penalty, and the slow borrowers are indifferent between the two mortgages.

This is a good place to discuss an ambiguity that is sometimes encountered in informal discussions of mortgage valuation. It is stated that the prepayment rule, which is optimal in the absence of transaction costs — refinance a loan when its market value equals or exceeds the loan's nominal value, and only then — loses its validity in the presence of transactions costs, such as when points are charged for the new loan. This is held to be the case even if the selling decision is taken to be exogenous and even when the points represent a pure transfer, as in the models of this article. For example, it is suggested that a borrower might optimally not refinance a premium mortgage in order to avoid paying points on the replacement mortgage. Alternatively, it is suggested that a fast borrower might optimally not refinance a premium mortgage because of the high probability of a subsequent involuntary prepayment [Kazarian (1993, p. 12), for example]. In both cases the reasoning is incorrect or, at best, incomplete: in the model of this article mortgages are optimally refinanced when they reach par, and only then, and this is true even though the model incorporates transaction costs.

The existence or nonexistence of points on the new mortgage is immaterial (with regard to the decision to prepay an existing mortgage) because the borrower is compensated for the points in the form of a lower coupon. Points deter prepayment of existing mortgages not by invalidating the optimal prepayment rule, but by altering its terms: high points are accompanied by low coupons, implying that mortgages are more likely to trade at a discount and therefore are less likely to be prepaid voluntarily. Similarly, we saw that fast borrowers may be more reluctant than slow borrowers to prepay a mortgage with a given prepayment penalty (recall the discussion of Figure 4), but again this effect is working through the optimal prepayment rule rather than invalidating it. The interpretation is that a mortgage that would be priced at par and optimally prepaid (in a particular interest rate state) if issued by slow borrowers may be priced at a discount and optimally not prepaid if issued by fast borrowers (here ignoring incentive compatibility).[11]

A critical assumption that justifies the optimal prepayment rule is that all borrowers using a particular mortgage be homogeneous: under that assumption, if it is optimal for any borrower to prepay voluntarily,

[11] Here by a "mortgage" we mean a mortgage with a given prepayment penalty. The coupon is, of course, different for the two borrowers. This discussion is necessarily loose: we are hypothetically changing the borrower on a mortgage with given prepayment penalty, whereas in equilibrium we know that in our setting only slow borrowers will borrow using a fixed-rate mortgage.

then it is optimal for all borrowers to prepay. Obviously a mortgage that is about to prepay in its entirety will be priced at its nominal value, not above. Correspondingly, a mortgage will always be priced at a discount to its nominal value in any interest rate state in which voluntary prepayment is not in the borrowers' interest. If, contrary to this assumption, a mortgage pool contains both fast and slow borrowers, then it might be optimal for slow borrowers to refinance, despite the fact that the pool trades at a discount, because of the presence in it of fast borrowers who optimally elect not to prepay. Analysis of this case must await development of models in which heterogeneous borrowers are pooled.

An implication of the foregoing discussion is that if we are to explain the existence of premium mortgages without appealing to investor irrationality, it is not sufficient to point to the existence of transaction costs: we just saw that even in the presence of transaction costs mortgages will always trade either at par (as they are prepaid) or at a discount (when they are not). Premium mortgages will occur in equilibrium only if the environment modeled in this article is altered. If that is done, there is no difficulty in accounting for the existence of premium mortgages: a borrower who is certain he or she will sell in a year's time would not rationally refinance a mortgage valued at a slight premium now. Similarly, the apparently suboptimal decision to prepay a discount mortgage can be rationalized if the benefit of moving outweighs the loss on the mortgage. Finally, as discussed in the following section, nonzero social costs of mortgage origination generally invalidate the prepayment rule of this article, implying that it is not necessarily suboptimal to fail to prepay a premium mortgage.

6. Nonzero Origination Costs

The foregoing development was based on the assumption that the social costs of mortgage origination are zero. Under that assumption, a prepayment penalty — or, equivalently, mortgage points — is a pure transfer having no efficiency implications. It serves only to separate borrowers with different prepayment probabilities.

The picture becomes more complicated under the realistic assumption that mortgage origination involves considerable costs, so that the borrower's payments exceed the lender's receipts. In recent work Stanton and Wallace (1995) have modified the model presented here by assuming positive social costs of mortgage origination. This modification causes fundamental changes in the nature of the equilibrium.

In Stanton and Wallace's model financially motivated refinancing is inefficient, implying that borrowers and lenders are motivated ex ante to find loan contracts that minimize the probability of refinancing. Since financially motivated refinancing occurs when prevailing interest rates fall significantly below the mortgage coupon, and since in Stanton and Wallace's model interest rates can fall below any positive level [they used the Cox, Ingersoll, and Ross (1985) model of interest rates], this implies the result that the coupon on the optimal mortgage is zero. The logic of Stanton and Wallace's model implies that mortgages should coincide with non-prepayable bonds, but the only prepayable bond the payoff of which coincides with that of a nonprepayable bond is the zero bond, so the optimal mortgage is a nonprepayable zero-coupon bond.[12] Stanton and Wallace did not discuss this feature of their model.

In creating examples, Stanton and Wallace avoided the problem just discussed by imposing an ad hoc upper bound on mortgage points at a level they considered realistic. Subject to this constraint, they derived equilibria in which each borrower type is indifferent between two mortgages and strictly rejects all others.

What happens in Stanton and Wallace's model is that the existence of positive social costs of mortgage origination diminishes the incentive of the slow borrower to misrepresent himself as the fast borrower. The social costs of mortgage origination play the role of a tax on mortgage origination. It is not surprising that existence of a tax on voluntary prepayment renders the resulting equilibria similar to those that occur when voluntary prepayment is prohibited altogether; we showed in Section 4 that separating equilibria can occur if voluntary prepayment is excluded.

Introducing social costs of mortgage origination, as Stanton and Wallace did, has a further implication, one which they did not consider. We saw above that when transaction costs are pure transfers, as in this article, then the optimal prepayment rule is to prepay a mortgage when its market value equals or exceeds its nominal value, and only then. Accordingly, the decision whether to prepay the current mortgage is unaffected by whether or not the replacement mortgage carries points or a prepayment penalty, since points and penalties represent a purchased asset for which the purchaser receives (ex ante) full value. This attractive feature of the current model disappears when social costs of mortgage origination are strictly positive: in that case the optimal prepayment rule for the current mortgage will depend on features of the replacement mortgage. This is so because the transaction costs on the replacement mortgage exceed the value of the

[12] Existing zero-coupon bonds sometimes have prepayment options. These, of course, do not provide for prepayment at the nominal bond value, since then prepayment would be irrational whenever nominal interest rates are positive. Instead, they provide schedules under which prepayment occurs at discounts related to the yield to maturity.

purchased option, so borrowers have an incentive not to prepay the current mortgage even when it trades at a premium. As a consequence, it is no longer generally optimal to prepay a mortgage whenever its market value equals or exceeds its nominal value, and more seriously, the optimal prepayment rule on the current mortgage can be computed only as part of a simultaneous optimization over the subsequent mortgage choices. This exercise involves a much more complex dynamic programming problem than that analyzed here or in Stanton and Wallace's article.

The fact that there exist premium mortgage pools is customarily attributed to borrower irrationality. As noted above, the results of this model imply that if social costs of mortgage origination are zero, this conclusion is correct. Further, this is true even if borrowers pay points or prepayment penalties, as long as these costs are pure transfers. However, if social costs of mortgage origination are positive, the existence of premium mortgages is not necessarily evidence of irrationality. Put differently, if one maintains the assumption of full rationality, the existence of premium mortgages constitutes evidence that social costs of mortgage origination are not negligible.

7. Empirical Evidence

The most direct way to test the implications of this model is to seek proxies for mobility and see if these are correlated with mortgage prepayment. Brueckner (1994) estimated a probit equation relating mobility to household characteristics, computed a measure of mobility from the resulting equation, and then regressed mortgage points on this measure. He found the correlation that his model — and the model here — predicts.

Other evidence also bears out the predictions of the model. A number of authors [Brueckner and Follain (1988), Shilling, Dhillon, and Sirmans (1987), Kazarian (1993)] reported that more mobile borrowers are likely to choose adjustable-rate mortgages, whereas less mobile borrowers choose fixed-rate mortgages.

The model developed here implies the existence of clientele effects in mortgage pools. If these are present, then the prepayment rate on a given pool should be predicted not only by the contract terms on the mortgages in the pool, but also by the terms on other mortgages available at the same time, since the terms on rejected mortgages would proxy for clientele effects. Research on this line will be impeded by the fact that data on mortgage pools rarely contain information on discount points.

8. Conclusion

Existing mortgage loans give borrowers the option to prepay at book value. This option has different values for different borrowers, creating incentives to separate heterogeneous borrowers. Assuming that borrowers' characteristics are private information, some device must be found to induce self-selection. We have shown that discount points are one such device. Others are easily available: loan maturity, for example.

The option to prepay at book value gives borrowers an incentive to refinance when interest rates drop. Therefore, to the extent that mortgage origination is costly, the prepayment option is inefficient. Another function of points, distinct from their role in inducing separation, is to mitigate this inefficiency. This occurs because the existence of points reduces the coupon below what would obtain in the absence of points, therefore diminishing the incentive to refinance.

All these difficulties would vanish if mortgages were prepayable, not at their book value, but at their market value: mortgage values would not be affected by borrowers' prepayment probabilities, so there would be no need to induce separation. Further, borrowers would have no financial motive to refinance, eliminating the inefficiency. A drawback is that borrowers might perceive prepayment of premium mortgages at their market values to be unfair. This is not an argument that economists would countenance, but consumer-oriented lawyers would doubtless see opportunity here.

There are other ways to achieve the same goal that might be more acceptable. Consider a fixed-rate mortgage that refinanced itself: that is, the borrower would always have the right to benefit permanently from reductions in interest rates (perhaps upon payment of a fee), while being protected against interest rate increases. Such a "ratchet mortgage" would coincide with existing fixed-rate mortgages, except that refinancing would not involve the costly process of requalifying the borrower, reappraising the property, and paying a variety of loan fees and taxes. It is hard to understand why we do not see such instruments.

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