Dynamic Banking: A Reconsideration

Sudipto Bhattacharya
Université Catholique de Louvain

A. Jorge Padilla
CEMFI and CEPR

Financially intermediated and stock market consumption-investment allocations, with and without governmental interventions, are compared in a welfare sense in overlapping generation economies with (and without) shocks to agents' intertemporal preferences. We first show that, in economies with preference shocks, governmental interventions subject to the same informational requirements as those imposed on financial intermediaries, lead to stock market allocations that are not inferior to those attained under financial intermediation. Second, we argue that the necessary interventions are qualitatively no different from those required to implement stationary optimal allocations in OLG models without shocks to agents' intertemporal consumption preferences.

This article contributes to the long-standing theoretical debate on the relative merits of financial intermediaries and financial markets in promoting economic development [see Allan and Gale (1995), Mayer (1988), and Sussman (1995) for a broader discussion]. From a theoretical perspective, it is well-known that

This research was started while S. Bhattacharya was a visiting professor at CEMFI from February to April 1994. Discussions with Jayashree Dutta, Paolo Fulghieri, Herakles Polemarchakis, and Ricardo Rovelli have been helpful. We gratefully acknowledge the comments on an earlier version of this article by Franklin Allen, Kai-Uwe Kühn, Bruce Smith, Oren Sussman, Fabrizio Zilibotti, and two anonymous referees. We are solely and jointly responsible for all remaining errors and omissions. Address correspondence to Dr. A. Jorge Padilla, CEMFI, Casado del Alisal 5, 28014 Madrid, Spain.

The Review of Financial Studies Fall 1996 Vol. 9, No. 3, pp. 1003–1032

in a world of complete markets there is no need for financial intermediaries (other than, perhaps, in order to reduce transaction costs). The modern information-based literature on banking — pioneered by Bryant (1980) and further extended by Bhattacharya and Gale (1987), Diamond and Dybvig (1983), and Jacklin (1987), among others — has examined the role of financial intermediaries in economies with shocks to intertemporal preferences for consumption, long-lived and technologically illiquid investment opportunities, and, most importantly, incomplete insurance markets. The intermediaries analyzed in these models are static intragenerational institutions whose role is to provide goods with superior liquidity services, or de facto insurance within a generation for shocks to the intertemporal preferences for consumption of its members.

These static models of intertemporal liquidity risks, or preference shocks, have recently been extended to the realm of dynamic overlapping generation (OLG) economies. In three closely related recent articles, Dutta and Kapoor (1993), Fulghieri and Rovelli (1993), and Qi (1994) have considered the problem of designing and implementing ex ante optimal stationary allocations in such dynamic OLG economies. These articles aim to study the possibility of sharing liquidity risks across generations, taking advantage of the "life-cycle" structure of the savings decisions made by each generation. Their common setup includes (1) overlapping generations of agents who face uncertainty about their preferences regarding the timing of consumption, (2) endogenous investment choices, and (3) investment returns that may accrue over multiple periods (lags). In this setting, these authors examine the functioning of financial intermediation in helping to attain a set of (first-best or constrained) optimal allocations of investment and consumption in different periods of agents' lives.[1] They contrast these results with those attained under stock market value-maximizing choices of investment, coupled with the underlying optimal portfolio choices for financial market equilibrium.

These articles suggest that, in a stationary (steady-state) sense, that is, omitting consideration of the very first generation, stock markets attain allocations that are inferior for agents, and typically result in underinvestment relative to the allocations attained by competing, and thus expected utility maximizing, financial intermediaries serving ex ante identical agents.[2] This phenomenon occurs despite the fact that

[1] For earlier work on overlapping generations models, with or without endogenous investment choices, see the excellent survey by Geanakoplos and Polemarchakis (1991).

[2] There is, to our knowledge, no result comparing financially intermediated and stock market allocations once the first generation is explicitly considered. An attempt is made by Qi (1994), who studies the attainability of financially intermediated allocations, but not of stock market

the provision of liquidity (*interim* consumption for agents preferring to consume early in their lives) does *not* necessitate any technological investment in liquid or short-term assets in these economies, under either intermediated or stock market modes of functioning. Instead, such liquidity is provided from long-term investment made in the past, and *interim* withdrawal/liquidation rights are provided for either by design of bank deposit contracts or through the trading of shares in the long-term investments.[3]

In this article we critically examine these conclusions regarding intermediated versus stock market allocations in dynamic OLG economies. We show that, first, once all possible arbitrage opportunities arising in a financially intermediated stationary OLG economy subject to preference stocks and characterized by illiquid production technologies are properly considered, financial intermediaries can no longer improve on the stock market equilibrium.[4] Second, we show that governmental interventions, such as tax-subsidy schemes (or, alternatively, the introduction of real money balances), that respect the same informational requirements as those needed for financial intermediation to outperform the stock market can lead to stock market equilibrium allocations with endogenous investment level choices that are identical, or even superior, to those attained under financial intermediation. Finally, we argue that the necessary governmental interventions in the former model are qualitatively no different from those required to implement stationary optimal allocations in an OLG model without uncertainty regarding agents' intertemporal consumption preferences.[5] Thus, the provision of liquidity is tangential vis-à-vis stock market efficiency in an OLG economy for the set of agents' preferences analyzed by Dutta and Kapoor (1993), Fulghieri and Rovelli (1993), and Qi (1994). Issues arising with more general structures for agents' intertemporal preferences, and shocks to these, are briefly discussed in the concluding section of this article.

In Section 1, we set out and analyze a simple OLG model with preference shocks and endogenous investment choices subject to constant

allocations, in such a setting. See also Section 3 for further elaboration on this issue.

[3] This is a fundamental distinction between these models and the earlier, static (one-generation) models of Bhattacharya and Gale (1987), Diamond and Dybvig (1983), Hellwig (1994), Haubrich and King (1990), and Jacklin (1987).

[4] This result parallels those by Haubrich and King (1990) and Jacklin (1987) for an earlier static model of liquidity provision by intermediaries [Diamond and Dybvig (1983)].

[5] These interventions are also somewhat similar to those in the standard pure exchange OLG model [Gale (1973)]: (1) the introduction of tax-subsidy schemes when stationary optimal choices require the transfer of resources from a later to an earlier period of agents' lives (the Classical case), and (2) the introduction of real money holdings when resources should be transferred from the present to the future (the Samuelson case). See Sections 2 and 3 and the Appendix for further details.

returns to scale, comparing the relative performance of financially intermediated and stock market economies subject to the same informational requirements. Economies with deterministic preferences (i.e., without liquidity shocks) are the focus of Section 2. In the concluding Section 3, we discuss some extensions of our previous analysis, including issues related to the feasibility of transition to the steady-state allocations we focus on. Finally, we generalize our main results to the case of investment technologies exhibiting decreasing returns to scale in the Appendix.

1. Dynamic Economies with Liquidity Shocks

1.1 The model

We study an infinite-horizon economy with an infinite sequence of overlapping generations of agents. A new generation, consisting of a continuum of agents of measure 1, is born at each time point $t \in \{0, 1, 2, \ldots, \infty\}$. There are eventually two types of individuals in each generation, differing in their respective life spans. Individuals of the first type live over one period only and are denoted as "early diers." Individuals of the second type, instead, live over two periods and are denoted as "late diers." Thus, an early dier born at time t consumes and dies at time $t+1$, whereas a late dier lives until time $t+2$ and, by assumption, consumes only then. Each newly born agent, irrespective of his type, is endowed at his time of birth with one unit of a homogeneous good, which can be used for either consumption or productive investment. Furthermore, no agent (again irrespective of his type) derives any utility from consumption at the beginning of his life, making his initial savings decision trivial. An agent born at t gets to know its own type at $t+1$, that is, one period after birth. There is a common probability ε of being an early dier ($o < \varepsilon < 1$), an event that is independently distributed over agents in any generation. Agents' realized types are private information but the a priori probability distribution on types is common knowledge.[6] Agents' ages, however, may be either private or public information. A newborn agent of generation t has lifetime expected utility equal to

$$EU_t = \varepsilon u(C_{t1}) + (1-\varepsilon)u(C_{t2}), \tag{1}$$

where $u(.)$ is strictly increasing and concave: $u'(.) > 0, u''(.) < 0$; $C_{t1} \geq 0$ is the level of consumption if the consumer is an early dier, and

[6] There is no available technology to verify any private information on types. This is a crucial assumption since, otherwise, individuals could write state-contingent insurance contracts to obtain risk sharing. Such contracts are thus ruled out in our model.

$C_{t2} \geq 0$ is the corresponding consumption level if the consumer is a late dier. Agents' ex post preferences are assumed to have a corner nature, that is, $u(C_{t1})$ or $u(C_{t2})$ only, as in Diamond and Dybvig (1983). This is a simplifying assumption, which allows us to implement (first-best) ex ante optimal allocations subject to resource constraints only, without taking explicit account of "incentive compatibility" constraints for preferring their allocated consumption profiles by agents of differing types; see Jacklin (1987) for further discussion of this point. This modeling assumption represents a simplified case of circumstances in which background events of an uninsurable nature affect agents' utilities for withdrawals from their financial resource base.

To sum up, note that there are four groups of individuals coexisting at each time $t \geq 2$: (1) newborns, of measure 1; (2) early diers, of measure ε; (3) late diers in the first period of their lives, of measure $1-\varepsilon$; and (4) late diers in the final period of their lives, of measure $1-\varepsilon$. So, in each period there are $3 - \varepsilon$ individuals. Individuals in groups (2) and (4) consume (so that there is a measure 1 of consumers); the remainder [groups (1) and (3)] invest their wealth in the assets available in this economy.

There is a single constant returns to scale (CRS) investment/production technology with convex returns over time. It yields $R > 1$ units of output at $t + 2$ per unit invested at t if the productive assets remain employed in the production process for two periods, and zero otherwise. In this sense, the technology is completely illiquid.[7] The Appendix extends our analysis to technologies exhibiting decreasing returns to scale (DRS). Let $I_t \geq 0$ denote the aggregate investment at t in this technology. A new round of production is started at every time point t provided that $I_t > 0$. This leads to an overlapping sequence of production processes: at each time t, one production process is just starting up, one is in the (unproductive) intermediate stage, and one comes to maturity. Therefore, the total resources available for consumption at any given t are equal to

$$1 - I_t + RI_{t-2}, \tag{2}$$

which is the sum of the endowment of newborns at t, net of current investment, plus the output from production processes currently maturing.

As it is standard for the welfare analysis of OLG models, in the following sections we shall exclusively focus on steady-state alloca-

[7] None of our results are substantially modified if we assume that the investment/production technology is not completely illiquid but, nevertheless, can only be terminated at a loss, for example, if the per-unit payoff in case of termination at $t + 1$ is $Q \in [0, 1]$.

tions that yield an identical ex ante payoff to all current and future generations, so that $C_{t1} = C_1$, $C_{t2} = C_2$, and $I_t = I$ for all t, hereafter.[8] Notice that the optimal steady-state allocations may not be uniquely Pareto efficient, especially once we take consideration of the very first generation [see Gale (1973) and Zilcha (1991)]. Questions of transition to the (optimal) steady-state allocations under alternative institutional arrangements are discussed in Section 3.

1.2 The stationary optimal allocation

We characterize the optimal stationary (steady-state) allocation of consumption and investment over time when consumers face ex ante uncertainty concerning their life spans. This allocation features both intergenerational risk sharing and full investment of the initial endowments, and will hereafter constitute our benchmark for future comparisons. An optimal allocation for our model $\{C_1^*, C_2^*, I^*\}$ maximizes the lifetime expected utility of each generation $[\varepsilon u(C_1) + (1-\varepsilon) u(C_2)]$, subject to the aggregate resource constraint:

$$(1 - \varepsilon) C_2 + \varepsilon C_1 + I = 1 + RI, \tag{3}$$

and the steady-state (stationary optimality) constraint:[9]

$$0 \leq I \leq 1. \tag{4}$$

We denote I^* as the optimal steady-state level of investment, and (C_1^*, C_2^*) as the pair of optimal steady-state consumption levels for early and late diers, respectively.

Solving the previous maximization program, we obtain the following expressions for optimal investment and consumption choices:

$$I^* = 1 \qquad C_1^* = C_2^* = R. \tag{5}$$

Hence, the efficient steady-state allocation requires that the initial endowment of each new generation is fully committed to productive investment, so that the economy sustains maximum consumption over time. Furthermore, given that agents' preferences exhibit risk aversion, efficiency also requires giving an agent a constant consumption level over time, that is, complete insurance against the random liquidity shocks to preferences for timing of consumption.

---

[8] There is no obvious way of studying a model like ours without restricting the analysis to steady-state consumption-investment allocations. First, there is an important conceptual problem regarding the proper definition for an aggregate social welfare function in an OLG framework. Second, otherwise, there may be no best allocation because of the scale independence of the CRS investment technology assumed [see Qi (1994) for further discussion].

[9] The constraint $I \leq 1$ reflects the endowment constraint of the very first generation, and it also holds for private endowment constraints under decentralization of the optimal plan.

Note that the above allocation cannot be implemented by individual agents acting autarkically because of the illiquidity of the production technology and the existence of an uncertain demand for liquidity. In this case, newborns are forced to invest all their endowment in the production technology and to terminate production prematurely if they happen to be early diers. Therefore, an autarkic economy will be characterized by an optimal investment level and also by an uncertain consumption profile: $C_1 = 0$, $C_2 = R$. (We could allow $C_1 = Q \leq 1$, where Q denotes the proceeds from early liquidation.) Of course, if there is not risk of premature death ($\varepsilon = 0$), the efficient outcome will be achieved even under autarky, in spite of the illiquidity of the production technology. In this last case, there is no need for financial intermediaries or, alternatively, for a stock market to operate.

1.3 The intermediated economy

In this section, which borrows in part from Fulghieri and Rovelli (1993) and Qi (1994), we analyze the consumption-investment allocations that can be achieved in an economy with competitive, profit-maximizing financial (depository) intermediaries under our previous assumptions on preferences and technology. Suppose that all agents deposit their initial endowments. A depository intermediary that receives as a demand deposit the whole initial endowment of a newborn promises to pay a total return D_1 for withdrawals made after one period or D_2 for withdrawals made after two periods.[10]

We initially assume that there is no interbank market for loanable funds, which implies that intermediaries invest their funds in the only available productive technology (which may be run by the intermediary himself or by a third party). Furthermore, we assume that intermediaries cannot price discriminate between early diers and late diers, because age is private information and/or there is a free secondary market for deposit certificates. In this setting, as shown by Fulghieri and Rovelli (1993), any depository intermediary will be able to offer only deposit contracts (D_1, D_2) such that no late dier finds it optimal to emulate an early dier and/or there is no arbitrage in the secondary market, that is, contracts such that $D_2 \geq (D_1)^2$. If $D_2 < (D_1)^2$, late diers will emulate early diers by withdrawing D_1 in the first period of their lives and then redepositing those funds to obtain $(D_1)^2$ anyway. Given that the pair (D_1, D_2) must satisfy the zero-profit constraint

[10] In all of the intermediated allocations derived below, every newborn finds it privately optimal to deposit his initial endowment rather than directly invest it into the production technology, with the possibility in the event of premature death of trading his claim on the production technology with a late dier depositor who will then withdraw early. This is because, in any of these allocations, $D_2 \geq R$ [see Qi (1994, p. 394)].

$\varepsilon D_1 + (1-\varepsilon)D_2 = 1 + RI - I$, agents' expected utility falls with increases in the ratio D_2/D_1, since consumption risk is increasing in it. Hence, $D_2 = (D_1)^2$ at the optimum and depository intermediaries choose $D_1 = D$ to maximize expected utility [Equation (1)] subject to the zero-profit constraint:

$$\varepsilon D + (1-\varepsilon)D^2 + I = 1 + RI, \quad I \leq 1. \tag{6}$$

The solution to this problem yields $I^d = 1$, $C_1^d = D$, and $C_2^d = D^2$, where $D^2 > R > D > \sqrt{R}$, which implies that, although the intermediated economy attains the efficient level of investment, it results in a consumption pattern that is inefficient: consumption is not sufficiently smooth.[11]

This last inefficiency would, nevertheless, disappear if intermediaries were able to price discriminate according to age, that is, if there were no active market for deposit certificates and intermediaries could commit to allow access to deposit contracts only to newborns (that is, if they could impose a *no-redepositing condition*[12]), so that the constraint $D_2 \geq (D_1)^2$ would drop from the depository problem. In fact, Fulghieri and Rovelli (1993) and Qi (1994) show that, under the latter assumptions, an intermediated economy achieves the optimal steady-state allocation of consumption and investment: $I^d = 1$, $C_1^d = C_2^d = R$.

Suppose, however, that the depository intermediaries cannot distinguish among depositors by age, and also that intermediaries can freely deposit in other intermediaries, that is, individuals and other banks cannot be distinguished by a given bank. Then, any given intermediary can either invest his funds in the only production technology, which yields R units of the consumption good per unit invested after two periods or, alternatively, deposit them into another intermediary, thus obtaining D^2 units of the consumption good per unit invested after two periods. If $D^2 > R$, as in our previous results, then no intermediary chooses to invest in the production technology so that, eventually, the whole system of intermediation collapses.[13] Therefore,

[11] Notice that the depository intermediaries considered here are intergenerational banks who allow risk sharing across generations and, therefore, differ from the intragenerational banks, who just allow risk sharing within generations, studied by Diamond and Dybvig (1983) and others.

[12] Intermediaries need to observe the ages (or consumption levels) of *all* depositors in order to impose such a condition effectively.

[13] If a bank offers $D_2 > R$, then he will not be able to finance the uncertain liquidity needs of his own individual depositors *and* reward the funds obtained from other banks operating in the interbank market using only the proceeds of the production technology unless D_1 is set to be sufficiently low. In particular, $D_1 < \sqrt{D_2}$. But then all his individual depositors will prefer to place their savings in those banks investing in the interbank market to sustain a consumption profile of $(\sqrt{D_2}, D_2)$. Hence, $D_2 \leq R$. Finally, in equilibrium, D_2 cannot be smaller than R because the banking industry is modeled as perfectly contestable.

D^2 must be equal to R, which leads to a consumption allocation given by $C_1^d = \sqrt{R}$ and $C_2^d = R$, and an aggregate investment level equal to $I^d = 1 - \varepsilon\frac{R-\sqrt{R}}{R-1} < 1$, using Equation (6). Hence, the intermediated economy with both a secondary market for deposits and an interbank market will be characterized by a consumption profile that is not smooth and also by underinvestment. We show below that a competitive stock market allocation, without governmental (tax subsidy) interventions, results in exactly the same degree of underinvestment and the same consumption pattern for early and late diers.

1.4 Equilibrium with a competitive stock market

What consumption-investment allocation can be sustained by a competitive stock market without governmental interventions? How does it compare to the steady-state optimal allocation and to the allocation achieved by an intermediated economy? Provided that the competitive equilibrium allocation is suboptimal, are there any interventions that can implement the efficient outcome? How rich is the information required to implement those interventions? The following subsections deal with these questions in order.

1.4.1 Steady-state equilibrium. In any given period t, each individual within the newborn generation starts up a new round of production by investing an amount of I. Simultaneously, he issues one share per unit of investment. Therefore, at any period t, there are three different kinds of firms: "new" firms (denoted by super index 0), which have just started production; "intermediate" firms (denoted by super index 1), which are ongoing technologies not yet matured; and "mature" firms, which are all liquidated at t paying dividends R per unit of initial investment. Let θ_i^0 and θ_i^1 be the number of shares of a new firm and an intermediate firm held by an individual of age i, respectively, $i = 0, 1$, where $i = 0$ denotes newborns, and age $i = 1$ "late dier" individuals after the first period of their lives. Individuals of age $i = 2$ never find it optimal to invest in ongoing technologies and limit themselves to consume the returns from their previous investments, as do the "early diers" of age $i = 1$.

At any point in time there are three markets open in this economy: (1) a market for goods, with a unit price normalized to one; (2) a market for shares of investments in new firms with a unit price P_0; and (3) a market for shares in intermediate firms with a unit price P_1. Any newborn individual planning his consumption and investment choices over time will have to take into account the following sequence of constraints and events. At $i = 0$, he invests an amount I in the production technology and purchases θ_0^0 and θ_0^1 shares of new and intermediate firms at prices P_0 and P_1, respectively. The resources available to him at this time are given by his own unit endowment plus the sale (or short-sale) of shares in his own (or others') firms. Hence, he faces an initial budget constraint at $i = 0$ equal to

$$1 + (P_0 - 1)I = P_0\theta_0^0 + P_1\theta_0^1. \tag{7}$$

We assume that unlimited short sales are allowed to individual agents or, equivalently, that free borrowing and lending contracts at a market rate of interest are feasible between agents who are going to be both alive at borrowing and repayment points. At age $i = 1$, an agent comes to know whether he is an early dier or a late dier. If he happens to be an early dier (which occurs with probability ε), he liquidates the dividends in his portfolio of maturing firms, θ_0^1, and sells his holdings of intermediate firm shares, θ_0^0, in order to finance consumption C_1. Buyers of intermediate firm shares are thus newborns from the generation that follows and survivors (late diers) of the current generation. Hence, the second budget constraint, for early diers, is that

$$P_1\theta_0^0 + R\theta_0^1 = C_1. \tag{8}$$

If, on the other hand, an individual realizes that he is a late dier (which occurs with probability $1-\varepsilon$), his only action at age $i = 1$ is to rebalance his portfolio. This leads to the following budget constraint:

$$P_1\theta_0^0 + R\theta_0^1 = P_0\theta_1^0 + P_1\theta_1^1. \tag{9}$$

Finally, the late dier ($i = 2$) consumes his accumulated wealth. This implies that C_2 satisfies a final budget constraint:

$$P_1\theta_1^0 + R\theta_1^1 = C_2. \tag{10}$$

We now characterize the steady-state competitive equilibrium. Consider first the asset markets; the no arbitrage conditions between these markets require that the rate of returns on the two assets (new-firm shares and intermediate-firm shares) is equalized, that is,

$$(1 + r) = P_1/P_0 = R/P_1 = \sqrt{R/P_0}. \tag{11}$$

Hence, the common one-period net rate of return on the two kinds of shares, r, equals the implicit one-period rate of return from investing in new-firm shares and holding them up to maturity. Similarly, no arbitrage between the goods market and the assets market requires $P_0 = 1$, which, from Equation (11), implies that $P_1 = \sqrt{R}$. Otherwise, from Equation (7) and given CRS, if $P_0 > 1$, newborns would find it profitable to demand and invest an infinite amount of the good. This demand could be financed by short selling intermediate firm shares[14] or, equivalently, by borrowing at an interest rate r. On the other hand, if $P_0 < 1$, no investment in new firms would be made; but then no old firm would exist.[15]

We then require that both the market for goods and the assets markets clear, which, given asset prices, implies that

$$\varepsilon C_1 + (1-\varepsilon) C_2 = 1 + (R-1)I, \tag{12}$$

$$\theta_0^0 + (1-\varepsilon)\theta_1^0 = 1, \tag{13}$$

$$\theta_0^1 + (1-\varepsilon)\theta_1^1 = 1. \tag{14}$$

The first equation constitutes the "material balance" constraint and simply restates the aggregate resource constraint in Equation (3). The other two equations ensure that the market for new firm shares [Equation (13)] and the market for intermediate shares [Equation (14)] clear. Note that only a measure $(1-\varepsilon)$ of individuals of age $i = 1$ hold shares. Given our previous (no arbitrage) assumptions, we can set $\theta_0^0 = 1$, without loss of generality. We can now state the following result.

Proposition 1 [Fulghieri and Rovelli, (1993)]. *The steady-state competitive equilibrium with a stock market is characterized by*

$$P_0^m = 1,\, P_1^m = \sqrt{R},\, C_1^m = \sqrt{R},\, C_2^m = R,$$

and

$$0 < I^m = 1 - \varepsilon \frac{R - \sqrt{R}}{R-1} < 1.$$

The consumption-investment allocation sustained in a stock market competitive equilibrium is therefore suboptimal. First, there is a problem of underinvestment: each newborn *only* invests $I^m < 1$ in a new production process. The remainder of his endowment $(1 - I^m)$ is diverted from the production technology into the stock market in order to smooth his consumption profile (that is, in order to obtain insurance against potential liquidity shocks). Second, the consumption pattern in the competitive equilibrium also differs from

[14] A newborn would short sell $1/P_1$ units of the intermediate firms shares to raise one unit for incremental investment, sell shares to it to raise $P_0 > 1$, and buy P_0/P_1 units of intermediate firm shares, making an arbitrage profit of $(\frac{P_0-1}{P_1})R$ in the next period, for consumption (if he is an early dier) or further investment (if he is a late dier) until age $i = 2$.

[15] If $P_0 < 1$, but $(P_1/P_0) = (R/P_1)$, then any newborn agent would prefer buying a share in another new firm of another newborn to investing on his own, and thus obtain a two-period return of $(R/P_0) > R$, or a one-period return of $(P_1/P_0) > P_1$.

the optimal consumption pattern. The aggregate consumption level $\varepsilon C_1^m + (1-\varepsilon)C_2^m = \varepsilon\sqrt{R} + (1-\varepsilon)R$ is smaller than the optimal consumption level, which equals $\varepsilon C_1^* + (1-\varepsilon)C_2^* = R$. This is precisely because investment in the production technology is too low from an ex ante perspective. Furthermore, the competitive equilibrium does not achieve the first-best allocation of consumption risk (over an agent's life) since consumption is not smooth over time.

In conclusion, the suboptimality of the competitive equilibrium outcome with a stock market arises from the fact that individuals can reallocate their consumption intertemporally (and across generations) only via the stock market at a positive net rate of return, which distorts the investment decision and makes consumption too variable. The steady-state optimality of the intergenerational allocation of consumption would require that the one-period gross rate of return on new firms' investments be equal to R, and hence that the gross market rate of return on intermediate firms' shares equal unity. However, at such a configuration of returns, there would be an excess supply of intermediate firms' shares.[16] Therefore, the stock market cannot decentralize the optimal allocation. [See also Dutta and Kapoor (1993) for further explanation of closely related results.]

1.4.2 Government interventions. Age-dependent lump-sum taxes and transfers. Suppose that the government subsidizes newborns and middle-aged individuals by transferring to them lump-sum subsidies equal to s and z, respectively. These subsidies are financed by levying taxes on old survivors, t, so that the government budget is balanced:

$$s + z = (1-\varepsilon)t. \tag{15}$$

The purpose of these interventions is, of course, to solve the problems of underinvestment and lack of smoothness in consumption that characterize the competitive equilibrium allocation. This is done by transferring some of the resources from the liquidation of maturing firms ($t > 0$) in order to finance consumption in the event of premature death ($z > 0$) and to subsidize investment of the initial endowment ($s > 0$).[17]

[16] In other words, if $P_1 = R$ and $P_0 < R$, then late diers would sell intermediate firm shares to buy new firm shares; if $P_0 > 1$, then the newborns would short sell intermediate firm shares to invest $I > 1$ with unbounded demand.

[17] Of course, the required interventions may be different (for instance, the government may have to introduce real money balances) in economies with more general production technologies. The reader is referred to the Appendix for an analysis of the optimal governmental interventions in economies with decreasing returns to scale technologies.

No arbitrage between shares in new firms and shares in intermediate firms, and also between the stock market and the market for goods, implies that $P_0 = 1$ and $P_1 = \sqrt{R}$. Hence, the set of budget constraints faced by individuals of a given generation becomes

$$1 + s = \theta_0^0 + \sqrt{R}\theta_0^1, \tag{16}$$

$$\sqrt{R}\theta_0^0 + R\theta_0^1 + z = C_1, \tag{17}$$

$$\sqrt{R}\theta_0^0 + R\theta_0^1 + z = \theta_1^0 + \sqrt{R}\theta_1^1, \tag{18}$$

$$\sqrt{R}\theta_1^0 + R\theta_1^1 - t = C_2. \tag{19}$$

In addition, we assume that Equations (12), (13), and (14) hold, implying that both the asset markets and the markets for goods clear. To implement the first-best allocation we calculate (s, z, t) so that $I^m = 1$ and $C_1^m = C_2^m = R$.

Proposition 2. *The stock market equilibrium implements the first-best allocation* $\{C_1^*, C_2^*, I^*\}$ *if and only if the government subsidizes newborn and middle-aged individuals with lump-sum transfers equal to* $s = \sqrt{R} - (1 - \varepsilon)R$ *and* $z = \sqrt{R}((1 - \varepsilon)R - 1)$*, respectively, and taxes old survivors with a lump-sum tax equal to* $t = R(\sqrt{R} - 1)$.

Proof. Set $\theta_0^0 = 1$, without loss of generality, then from the budget constraints of middle aged-individuals we have that (i) $(1+s)\sqrt{R}+z = \theta_1^1\sqrt{R}$ and (ii) $(1 + s)\sqrt{R} + z = C_1$. Thus, $C_1 = C_1^* = R$ implies that $\theta_1^1 = \sqrt{R}$ and $z = R - \sqrt{R}(1+s)$. Similarly, from the budget constraint for old survivors, we have that $C_2 = C_2^* = R$ yields $t = R(\sqrt{R} - 1)$. Now, financial market clearing requires that $\theta_1^0 = 0$ and $\theta_0^1 + (1 - \varepsilon)\theta_1^1 = 1$, that is, $\theta_0^1 = 1 - (1 - \varepsilon)\sqrt{R}$. Substituting θ_0^1 into the budget constraint applying to newborns, we have that $s = \sqrt{R} - (1 - \varepsilon)R$, and $z = \sqrt{R}((1-\varepsilon)R-1)$. It follows immediately that for such values of s, t, and z, the government's budget balance constraint and the aggregate resource constraint are both trivially satisfied. ■

Subsidies to investment and taxes on dividends. We have just shown that there is a set of governmental interventions that ensures the optimality of the decentralized equilibrium. However, these interventions are not simple to implement in practice since they require the government to single out individuals according to age. In this subsection we consider interventions that require less information to be implemented: subsidies to investment and taxes on dividends.[18] More precisely, we will distinguish between subsidies conditional on total investment of the initial endowment (or, simply, subsidies per agent), and subsidies that are proportional to the actual level of investment. As in the previous subsection, these interventions facilitate the intergenerational transfers required to achieve efficiency but, contrary to the previous case, they can only improve on the intervention-free stock market allocation without attaining the first-best outcome.

Let s denote the subsidy paid conditional on full investment of the initial endowment, and t the tax on dividends. Government budget balance requires, in equilibrium, that $s\gamma = t$, where $\gamma = 1$ if $I = 1$ and zero otherwise. Given these subsidies and taxes, the set of budget constraints faced by any given individual becomes

$$(1 + s\gamma) + (P_0 - 1)I = P_0\theta_0^0 + P_1\theta_0^1, \tag{20}$$

$$P_1\theta_0^0 + (R - s\gamma)\theta_0^1 = C_1, \tag{21}$$

$$P_1\theta_0^0 + (R - s\gamma)\theta_0^1 = P_0\theta_1^0 + P_1\theta_1^1, \tag{22}$$

$$P_1\theta_1^0 + (R - s\gamma)\theta_1^1 = C_2. \tag{23}$$

Following a line of argument that parallels those in previous sections, we can easily show that no arbitrage conditions yield

$$P_0 = 1 \text{ and } P_1 = \sqrt{R - s\gamma}.$$

Proposition 3. *The steady-state optimal stock market equilibrium with fixed conditional subsidies to (unit) investment and proportional taxes on dividends is characterized by*

$$C_1^{mf} = (1 + s)\sqrt{R - s},\ C_2^{mf} = (1 + s)(R - s),\ I^{mf} = 1,$$

where $s = t$ *solves* $\varepsilon\sqrt{R - s} = s(1 + (1 - \varepsilon)\sqrt{R - s})$.

Proof. Since $P_0 = 1$ and $s > 0$, it is clear from Equation (20) and the arbitrage conditions above that $I^{mf} = 1$ and, hence, $\gamma = 1$. Let $(1 + r) = \sqrt{R - s}$ and set $\theta_0^0 = 1$ (which immediately yields $\theta_1^0 = 0$), then from the budget constraints in Equations (20)–(23) we have that (i) $(1 + s) = \theta_1^1 = 1 + (1 + r)\theta_0^1$; (ii) $(1 + s)(1 + r) = C_1^{mf}$, and (iii) $C_2^{mf} = (1 + r)C_1^{mf} = (1 + s)(1 + r)^2$. Now, financial market clearing requires that $\theta_0^1 + (1 - \varepsilon)\theta_1^1 = 1$, that is, $\theta_0^1 = 1 - (1 - \varepsilon)(1 + s)$. Solving

[18] Sussman (1992) considers a similar set of interventions for the earlier static Diamond and Dybvig model. However, in his model, these interventions successfully implement the first-best in contrast with our results below.

for θ_0^1 from the budget constraint applying to newborns, we have that $\theta_0^1 = s/(1+r)$. Hence, s solves implicitly:

$$\frac{\varepsilon(1+r)}{1+(1-\varepsilon)(1+r)} = (R-(1+r)^2).$$

It is immediate that for such values of s and t the government's budget balance constraint and the aggregate resource constraint are both trivially satisfied. To complete the proof we just need to ensure that the previous implicit equation has a real solution. For that purpose, let $A((1+r)) = \frac{\varepsilon(1+r)}{1+(1-\varepsilon)(1+r)}$ and $B((1+r)) = (R-(1+r)^2)$. Notice that $B(\cdot)$ is a strictly decreasing and concave function of $(1+r)$, and that $B((1+r)) = 0$ at $(1+r) = \sqrt{R}$. On the other hand, $A(\cdot)$ is a strictly increasing and concave function of $(1+r)$, with a slope that is strictly smaller than 1 for all $(1+r)$. Therefore, we can ensure that there exists a unique value of $(1+r) < \sqrt{R}$, and thus a unique value of $s > 0$, that solves the implicit equation above. ■

Corollary 1. *(i)* $C_1^{mf} > \sqrt{R}$ *and (ii)* $(C_1^{mf})^2 > C_2^{mf} > R$.

Proof. From the previous Proposition, we have that $C_1^{mf} = (1+r)(1+s)$. Then, $C_1^{mf} - \sqrt{R} = (1+r)(1+s) - \sqrt{(1+r)+s}$. Substituting $s = \varepsilon(1+r)/(1+(1+r)(1-\varepsilon))$, we obtain $C_1^{mf} - \sqrt{R} = (1+r)^2 + (1+r)(1-(1-\varepsilon)^2) - (1-\varepsilon) > 0$ as $(1+r)$ exceeds 1. Similarly, $C_2^{mf} = (1+r)C_1^{mf} = (1+r)^2(1+s)$ exceeds $R = (1+r)^2 + s$ since $(1+r) > 1$. Finally, $C_2^{mf} < (C_1^{mf})^2$. ■

One might criticize the previous result by arguing that it assumes some of the features of a *command* economy, namely, the fact that subsidies are granted if and only if a prespecified level of investment is made. Consequently, we now consider the performance of a decentralized economy where subsidies are proportional to the amount being invested. This new assumption just alters the budget constraint for the government, which here becomes $sI = t$, and the set of budget constraints faced by individuals of different age which here turn out to be given by

$$1 + sI + (P_0 - 1)I = P_0\theta_0^0 + P_1\theta_0^1, \tag{24}$$

$$P_1\theta_0^0 + (R - sI)\theta_0^1 = C_1, \tag{25}$$

$$P_1\theta_0^0 + (R - sI)\theta_0^1 = P_0\theta_1^0 + P_1\theta_1^1, \tag{26}$$

$$P_1\theta_1^0 + (R - sI)\theta_1^1 = C_2. \tag{27}$$

Proposition 4. *The steady-state optimal stock market equilibrium with propositional subsidies to investment and taxes on dividends is characterized by*

$$C_1^{mp} = \sqrt{\frac{R-s}{1-s}},\ C_2^{mp} = (C_1^{mp})^2 = \frac{R-s}{1-s},\ I^{mp} = 1, \tag{28}$$

where $s = t$ solves

$$\varepsilon\sqrt{\frac{R-s}{1-s}} = s\left(1 + \sqrt{\frac{R-s}{1-s}}\right). \tag{29}$$

Proof. No arbitrage conditions imply that $P_0 = 1 - s$ and $P_1 = \sqrt{(R - sI)(1 - s)}$. Setting $I^{mp} = 1$ is optimal because increasing I both raises total consumption and makes consumption smoother over time. Let $(1+r) = \sqrt{\frac{R-s}{1-s}}$ and set $\theta_0^0 = 1$ (which immediately yields $\theta_1^0 = 0$), then from the budget constraints in Equations (28), (29), and (24) we have that (i) $1/(1-s) = \theta_1^1 = 1 + (1+r)\theta_0^1$; (ii) $C_1^{mp} = (1+r)$, and (iii) $C_2^{mp} = (C_1^{mp})^2 = (1+r)C_1^{mp} = (1+r)^2$. Now, financial market clearing requires that $\theta_0^1 + (1-\varepsilon)\theta_1^1 = 1$, so that $\theta_0^1 = 1-(1-\varepsilon)/(1-s)$. Solving out θ_0^1 from the budget constraint applying to newborns, we have that $\theta_0^1 = s/(1+r)(1-s)$. Hence, s solves implicitly:

$$\varepsilon\sqrt{\frac{R-s}{1-s}} = s\left(1 + \sqrt{\frac{R-s}{1-s}}\right),$$

or, equivalently,

$$s^2(R + 2\varepsilon - 1) - s(2\varepsilon R + \varepsilon^2) + \varepsilon^2 R = 0,$$

which is a quadratic equation with a unique real solution with $1 > \varepsilon > s > 0$. Note that $\varepsilon > s$ is necessary to be consistent with financial market clearing (since $s > \varepsilon$ implies that $\theta_0^1 > 1$). Finally, it is immediate that for such values of s and t the government's budget balance constraint and the aggregate resource constraint with $I^{mp} = 1$ are both trivially satisfied. ∎

Corollary 2. *(i) $C_1^{mf} > C_1^{mp} > \sqrt{R}$ and (ii) $(C_1^{mp})^2 = C_2^{mp} > C_2^{mf} > R$.*

Proof. First, note that $C_1^{mp} > \sqrt{R}$ and $C_2^{mp} > R$ follow directly from their definitions in the previous Proposition. The inequalities $C_1^{mf} > C_1^{mp}$ and $C_2^{mf} < C_2^{mp}$ stem directly from the fact that $C_2^{mf} < (C_1^{mf})^2$ and $C_2^{mf} = (C_2^{mp})^2$, and the material balance constraint. ∎

Propositions 3 and 4 show that the simultaneous introduction of subsidies to investment (whether lump-sum or proportional) and taxes on dividends in an otherwise decentralized economy yields the optimal level of investment but fails to fully insure agents against consumption risk over their stochastic life spans.

Equilibrium investment decisions are optimal because the two kinds of subsidies are tailored so as to provide newborns with exactly the right incentives for full investment of their initial endowments. However, consumption is not completely smooth across ages (of death) because the equilibrium net market rate of return in any of the two scenarios is still larger than zero despite the introduction of taxes on dividends. Finally, note that in the decentralized economy with proportional subsidies to investment, agents face a greater consumption risk than in the otherwise identical economy with lump-sum subsidies. In the former economy, the gross market rate of return is equal to $\sqrt{\frac{R-s}{1-s}}$, whereas in the latter, it equals $\sqrt{R-s}$. But, for any $0 < s < 1$, which is the case in a stock market equilibrium with proportional subsidies, $\sqrt{\frac{R-s}{1-s}} > \sqrt{R-s}$.

1.5 Welfare comparisons of stock market and intermediation

Table 1 summarizes all relevant information concerning consumption and investment allocations under different institutional settings. First of all, notice that both an unrestricted intermediated economy (that is, with active secondary markets for deposits and interbank markets for loanable funds) and a stock market economy with no interventions yield the very same consumption-investment allocation, thus failing to implement a steady-state (first-best) optimum. Consumption is not sufficiently smooth and investment too low with respect to their corresponding efficient levels.

Steady-state optimality can be obtained in *both* cases if we intervene using age-dependent instruments: no-redepositing clauses and no interbank depositing possibilities in an intermediated economy,[19] and age-dependent taxes and subsidies in a stock market economy. If, instead, we restrict the set of potential interventions to be age-independent, we may still be able to achieve the steady-state optimal level of investment of the initial endowment, but at the cost of excessive consumption risk. Furthermore, Table 1 shows that the same consumption allocation that can be achieved in an intermediated economy where the interbank market is restricted not to operate can also be achieved by a stock market economy where dividends

[19] Together with the additional prohibition on trading deposit certificates in a secondary market.

Table 1
Consumption-investment allocations

1. **First-best**:

$$I^* = 1, \quad C_1^* = C_2^* = R.$$

2. **Intermediated economy**:

a) with secondary market + interbank market:

$$I^d = 1 - \varepsilon \frac{R - \sqrt{R}}{R - 1} < 1,\ C_1^d = \sqrt{R},\ C_2^d = (C_1^d)^2 = R.$$

b) with secondary market only:

$$I^d = 1,\ C_1^d = D > \sqrt{R},\ C_2^d = (C_1^d)^2 = D^2 > R.$$

c) with age-dependent depository clauses:

$$I^d = 1,\ C_1^d = C_2^d = R.$$

3. **Stock market economy**:

a) without interventions:

$$I^m = 1 - \varepsilon \frac{R - \sqrt{R}}{R - 1} < 1,\ C_1^m = \sqrt{R},\ C_2^m = (C_1^m)^2 = R.$$

b) with subsidies to investment and taxes on dividends:

b.1) fixed subsidies:

$$I^{mf} = 1,\ C_1^{mf} > \sqrt{R},\ (C_1^{mf})^2 > C_2^{mf} > R.$$

b.2) proportional subsidies:

$$I^{mp} = 1,\ C_1^{mf} > C_1^{mp} > \sqrt{R},\ C_2^{mp} = (C_1^{mp})^2 > C_2^{mf} > R.$$

c) with age-dependent subsidies and taxes:

$$I^{ma} = 1,\ C_1^{ma} = C_2^{ma} = R.$$

are taxed and investment is subsidized proportionally. In this latter case, if subsidies to investment are not proportional to the actual investment level but are lump-sum investment-contingent subsidies per agent, overall efficiency is further improved since agents' consumption patterns become smoother over their stochastic life spans.

2. Dynamic Economies Without Liquidity Shocks

How essential are liquidity (early versus late dier) shocks for the results discussed above? To consider this issue, in this section we briefly analyze an OLG economy in which the agents of a generation are all identical ex ante and ex post: they have identical consumption preferences (as well as endowments), and each agent wishes to consume in all periods after being born. That is, we assume that all individuals live for two periods (three-time points) and that each of them is endowed at his time of birth with one unit of a homogeneous good, which can be used for either consumption or productive investment. As in the previous section, there is a single production technology with constant returns to scale: it yields $R > 1$ units of output per unit invested in period t if the productive assets remain employed in the production process for two periods, and zero otherwise. Let I_t again denote the aggregate investment in the production technology at t. No individual derives any utility from consumption at the beginning of his life, thus making his initial savings decision trivial. However, each individual derives utility from consumption in the first and second periods of his life: any newborn at t will thus consume at $t+1$ and $t+2$ and will die immediately after. His lifetime utility will be then given by

$$U_t = u(C_{t1}) + u(C_{t2}), \tag{30}$$

where $u(\cdot)$ is a strictly increasing and concave function ($u'(.) > 0, u''(.) < 0$); $C_{t1} \geq 0$ is the level of consumption in the first period of his life; and $C_{t2} \geq 0$ is the corresponding consumption level in the second period of his life. The additive separability (and lack of discounting) assumption is made solely to keep comparison with the previous scenario, which incorporated shocks to agents' preferences for timing of consumption, tractable.

Therefore, there are three different groups of individuals in each period, each of size 1: (1) the newborn; (2) consumers in the first period of their lives; and (3) consumers in the second period of their lives. In total, there is a measure 3 of individuals per period, of whom a measure 2 [groups (2) and (3)] are consumers and the remainder [group (1)] invest their wealth in the assets available in the economy. As in the previous section, we shall focus on steady-state allocations exclusively, so that $C_{t1} = C_1$, $C_{t2} = C_2$, and $I_t = I$, for all t, hereafter.

In this setting, the optimal stationary allocation $\{C_1^*, C_2^* I^*\}$ solves

$$\max u(C_1) + u(C_2) \tag{31}$$

subject to

$$C_1 + C_2 = 1 - I + RI, \tag{32}$$

and

$$I \leq 1, \tag{33}$$

where Equation (32) simply represents the material balance constraint. Solving out the previous maximization program, we obtain the following expressions for the optimal consumption-investment allocation:

$$C_1^* = C_2^* = C^* = \frac{R}{2}, \text{ and } I^* = 1. \tag{34}$$

Hence, the efficient allocation requires full investment of the initial endowments and constant consumption over agents' life spans.[20] Can the stock market (with value-maximizing investment choices) attain this allocation, in equilibrium? The answer is no, as we proceed to show below.

Let θ_i^0 and θ_i^1 be the number of shares of new firms and intermediate firms, respectively, held by an individual of age i, $i = 0$, 1, and P_0 and P_1 be their corresponding stock market price levels. No arbitrage between these assets markets requires that the one-period rates of return of these two assets is equalized to the implicit rate of return from investing in new shares and holding them up to maturity. That is,

$$(1 + r) = P_1/P_0 = R/P_1 = \sqrt{R/P_0},$$

where r denotes the one-period stock market net rate of return.

Agents then choose:

$$\{C_1^m, C_2^m, I^m, (\theta_i^0, \theta_i^1) i = 0, 1\} = \arg\max\{u(C_1) + u(C_2)\} \tag{35}$$

subject to

$$1 + (P_0 - 1)I = P_0\theta_0^0 + P_1\theta_0^1, \tag{36}$$

$$(1 + r)(P_0\theta_0^0 + P_1\theta_0^1) = C_1 + P_0\theta_1^0 + P_1\theta_1^1, \tag{37}$$

$$(1 + r)(P_0\theta_1^0 + P_1\theta_1^1) = C_2, \tag{38}$$

where Equations (36)–(38) denote the per-period budget constraints faced by any (representative) agent in this stationary world. Any newborn deciding upon his consumption and investment choices over time will have to take into account the following sequence of constraints and events. Initially, he may commit an amount I of his total endowment and/or invest in shares θ_0^0 and θ_0^1 at their corresponding prices P_0 and P_1. Resources for investment are given by the unit endowment plus the sale of shares in his own new firm at price P_0. Hence, he faces an initial budget constraint at $i = 0$ given by Equation (36). At $i = 1$, he uses the dividends on his portfolio to finance consumption C_1 and/or rebalance his portfolio [Equation (37)]. Finally, at $i = 2$, the individual just consumes his accumulated wealth [Equation (38)].

[20] All our observations in this section are easily generalized to the case in which future utility $u(C_2)$ is discounted.

Financial markets clear if and only if

$$\theta_0^0 + \theta_1^0 = 1, \tag{39}$$

$$\theta_0^1 + \theta_1^1 = 1. \tag{40}$$

We also need the material balance (goods market clearing) condition in Equation (32) which, however, is redundant by Walras Law. Notice that, as in the previous section, we can set $\theta_0^0 = 1$ and thus $\theta_1^0 = 0$ without loss of generality. This is because an agent just needs one financial instrument to transfer income intertemporally given the lack of arbitrage.

Proposition 5. *The stock market equilibrium, defined by Equations (35)–(40), cannot attain the stationary optimal allocation.*

Proof. First, optimality requires $r = 0$, since otherwise (for all $r > 0$) agents maximizing Equation (35) subject to Equations (36)–(40) choose $C_1 \neq C_2$. Then, the stock market equilibrium is never optimal since, otherwise, in order to have $I^* = 1$, we must have that $P_0 = 1$, which in turn implies that $(1 + r) = \sqrt{R}$, or $r > 0$, contradicting optimality.[21] ■

The suboptimality of the stock market equilibrium outcome results from the fact that the only way for agents to smooth consumption is via the stock market, at a strictly positive net market rate of return r, while optimality requires a net interest rate equal to zero. Note that $r > 0$ implies underinvestment in equilibrium and also a consumption profile that is not smooth over time. An optimal consumption-investment allocation (which features $r = 0$ and $I = 1$) could be decentralized in equilibrium only if the market for intermediate firm shares clears. But, setting $r = 0$ and $I = 1$ implies that $\theta_0^1 + \theta_1^1 = 1/2 < 1$, so that there is an excess supply of intermediate firms shares. This is because in this case newborns find it unattractive to divert resources from the production technology, which yields an implicit rate of return per period strictly larger than zero.

What, if any, governmental intervention can result in stock markets reaching the stationary optimal allocation? We have the following result:

Proposition 6. *Consider a stock market economy where the government sets taxes of $(R - 1)$ at rate $t = (\frac{R-1}{R})$ on investment returns,*

[21] No arbitrage between the goods market and the assets markets requires $P_0 = 1$, which implies that $P_1 = \sqrt{R}$. Otherwise, from Equation (36) and given CRS, if $P_0 > 1$, newborns would find it profitable to demand and invest an infinite amount of the good; this demand is financed through short selling intermediate firm shares. If $P_0 < 1$, investment would be zero.

distributed as subsidies of $s = (1 - \frac{R}{2})$ *to the young, and* $z = (\frac{3R}{2} - 2)$ *to the middle-aged agents; then there exists a stock market equilibrium with interest rate* $r = 0$ *which attains the stationary optimal allocation.*[22]

Proof. This tax-subsidy scheme results in $r = 0, I^* = 1, P_0 = P_1 = 1$, and holdings of intermediate firm shares of $\theta_0^1 = s = 1 - \frac{R}{2}$ by the young and $\theta_1^1 = s + z + P_0 - C_1^* = \frac{R}{2}$ by the middle-aged, so Equation (40) is satisfied, with $C_1^* = C_2^* = \frac{R}{2}$ chosen by maximizing in Equation (35) subject to Equations (36) and (39) at $r = 0$. ∎

Hence, decentralizing the steady-state optimal consumption-investment allocation requires the government to intervene by introducing subsidies to young and middle-aged individuals and taxes on the investment returns of the old. This intervention helps individuals to transfer income from the last period of their lives to earlier periods and, at the same time, reduces the implied one-period rate of return associated with the productive investment, which leads to smoother consumption and optimal investment. Let us remark that, as in the previous section where we studied the impact of liquidity shocks, we need age-dependent interventions to attain efficient outcomes in equilibrium.

Note that the results derived for optimal government tax-subsidy schemes in this section can be viewed as being analogous to their corresponding results in Section 1, when $\varepsilon = \frac{1}{2}$, so that the ex ante expected utility of an agent at the beginning of her life is $(u(c_1) + u(c_2))/2$. Thus, we conclude that the issue of provision of liquidity is tangential to that of stock market optimality, vis-à-vis investment in OLG economies analyzed across steady states. All that matters for our results is the impossibility, in a stock market economy without governmental taxes and subsidies, of transferring resources across generations without compensating shareholders for the opportunity cost of not investing in the production technology.

3. Extensions and Concluding Remarks

In this article we have compared financially intermediated and stock market consumption-investment allocations, with and without governmental interventions, in overlapping generations economies with and without shocks to agents' intertemporal preferences. We have shown that, in economies with preference shocks, governmental in-

[22] As a result, budget constraints are altered to $(1+(P_0-1)I+s)$ in the right-hand side of Equation (36), $(1+r)(1+(P_0-1)I+s)+z$ in the right-hand side of Equation (37), and $\theta_1^1 = C_2$ in Equation (38).

terventions subject to the same informational requirements as those imposed on financial intermediaries lead to stock market allocations that are not inferior in a welfare sense to those attained under financial intermediation. Furthermore, we have demonstrated that the necessary interventions are qualitatively no different from those required to implement stationary optimal allocations in OLG models without shocks to agents' intertemporal consumption preferences.

These results have been obtained under the assumption that agents' ex post preferences have a corner nature. A straightforward extension of our work would be to incorporate preference shocks that leave agents with ex post interior preferences (satisfying Inada Conditions), as in Jacklin (1987). With such a setup, the comparison of stock market and bank-intermediated allocations may not produce equivalence results. Financial intermediation may lead to welfare superior consumption-investment allocations even with government taxes and subsidies, and even without no redepositing (but with restricted interbank depositing) clauses. The reason is, as Jacklin noted, that traded stock market contracts have to satisfy stronger coalitional incentive-compatibility constraints across early and late consumption-preferring agents, compared with nontraded deposit contracts, and tax-subsidy interventions may not be sufficient to overcome this added constraint.

Our work could also be extended to compare alternative equilibria with banks versus stock markets in growth models of the OLG type. Bencivenga and Smith (1991) as well as Greenwood and Smith (1993) have contrasted allocations resulting with stock markets and with competing representative banks in an overlapping generations model [based on the model by Bryant (1980)] which does not allow for intergenerational banking or transfers. They have elaborated on the implications of the fact that the [Bhattacharya and Gale (1987)] interim value-maximization criterion implies that stock market economies will grow faster (slower) that intermediated [Diamond and Dybvig (1983)] economies, owing to greater investment in higher-yielding illiquid capital stock, as the agents' coefficient of relative risk aversion is larger (smaller) than one. We are less than comfortable with the general validity of this type of inference. First, simple contractual restrictions (such as prohibiting interim trade of shares) yield equivalence results in their setting, which are similar in spirit to those obtained in Propositions 3 and 4, and in the Appendix. Moreover, since dynamic stationary economies require no investment in short-term assets to provide liquidity, the issues raised by Bhattacharya and Gale (1987) do not seem to be a priori relevant in the setting of OLG models without further extensions. For example, the riskiness of longer-term returns, and the resulting illiquidity (through greater transactions costs arising from asymmetric information) in their secondary markets, could modify our equivalence results above.

Furthermore, it should be clear that the steady-state optimal allocations we have examined are not uniquely Pareto efficient across generations. In particular, once the welfare of the very first generation is considered, full investment of its initial endowment by the first and all subsequent generations does not lead to the consumption allocation of Equation (5) for the first generation, since there is no prior investment. This is the case with both the intermediated allocations which involve $I = 1$, or with the tax-subsidy based stock outcomes of Propositions 2, 3, and 4. Thus, it is entirely possible that alternative allocational mechanisms (such as a stock market without government intervention[23]) will provide the first generation with higher utility than in our steady-state optimal allocation. Indeed, the same observation would be valid without either (1) liquidity shocks to agents intertemporal preferences, or (2) time lags in obtaining the return from investment if consumption in the period of investment is valued. In fact, very little about the qualitative results developed and analyzed above depend essentially on the specific structure of preferences and technologies assumed, other than the fact that the marginal return at investment of the full endowment exceeds the population growth rate.[24]

There is, however, a sense in which OLG models of investment and consumption with long-lived technologies, and inelastic initial savings decisions, are different from the usual exchange or production models analyzed in the OLG literature, surveyed in Geanakoplos and Polemarchakis (1991). In the former models, any interim consumption in the first period of life for the very first generation must be provided through either (1) underinvestment by the second (and possibly subsequent) generations, or (2) premature liquidation of the long-lived physical investment. In a steady-state sense, alternative (1), if repeated by all generations may still be preferable to alternative (2) — see Section 1.4. However, there may also be a transition path that performs "better" than either alternative while at the same time leading the dynamic OLG economy to the steady-state optimal intergenerational allocation of Equation (5), or some constrained version thereof, as in Propositions 3 and 4.

In Bhattacharya, Fulghieri, and Rovelli (1995), the authors attempt

[23] Indeed, the tax-subsidy schemes of our Propositions 2, 3, and 4 hurt the welfare of members of the first generation, since they do not get any investment subsidies but only pay the first dividend taxes.

[24] We are in the classical case [Gale (1973)] of the OLG model of Samuelson (1958). Hence, both the steady-state optimal and the stock market (without intervention) equilibrium allocations satisfy the Pareto efficiency criteria developed in Gale (1973) and Zilcha (1991).

to provide an answer to the following related question. Are there intertemporal (intergenerational) transition paths that (1) lead to a (constrained) steady-state optimal intergenerational allocation in finite time, and (2) which cannot be improved on by any generation proposing a different (and feasible) alternative allocational mechanism? Note that the set of alternative mechanisms (e.g., intergenerational stock markets, intragenerational banking) must be restricted by prior property rights and subsequent sustainability in order to obtain a nontrivial answer to this question. It appears from their work that there exist circumstances (preference and technology parameters) under which such a transition path, which is superior to the intergenerational stock market or intragenerational banking allocations for the first generation, indeed exist.[25] This "turnpike" path, which leads to an intergenerational steady-state (constrained) optimal allocation in finite time, is also (expected) utility improving for the countable infinity of generations that follow the transition, relative to the stationary stock market or intergenerational banking allocations. However, earlier generations, other than the first, which live during the transition, may not be all made better off than with other alternative stationarity equilibrium allocational mechanisms.[26] Hence, while the stationary (constrained) optimal intergenerational allocations we have examined above (in Sections 1 and 2) may not be uniquely Pareto efficient, there is a case to be made for studying these allocations and their attainability through decentralized institutions, such as financial intermediaries or stock markets, subject to contractual (governmental) restrictions (interventions).[27]

Appendix: Decreasing Returns to Scale

Consider our basic OLG model with liquidity shocks (Section 1) where now the return on the production technology, $F(I)/I$, exhibits decreasing returns to scale (DRS), that is, $F(.)$ is such that $F'(0) > 1 > F'(1)$ and $F''(.) < 0$. (Under this assumption, contrary to the CRS

[25] Qi (1994), in his "Problem B' formulation, also attempts a characterization of optimal intermediated allocations that do not rely on the returns from investments made in the past. However, he confines his attention to steady-state intermediated allocations, rather than (fastest) transition paths, subject to such a restriction on resources available for consumptions cum investment.

[26] Nevertheless, generations subsequent to the first are not able to propose initiating an alternative allocational mechanism (within a restricted class) that is (1) consistent with the property rights of the prior generation, and (2) utility improving at the outset for the proposing generation.

[27] Intergenerationally stationary optimal allocations may also not be in the core of an intergenerational (cooperative) game. The methodology of Esteban and Sákovics (1994), and also Bhattacharya, Fulghieri, and Rovelli (1995), "surmounts" this problem by analyzing an explicit (noncooperative) intergenerational proposal game with a strategy space that is constrained "appropriately."

case, investment and production yields pure profits, which we take to accrue to shareholders.) In this setting we show that a decentralized stock market economy may yield either *too little* or *too much* investment, but always excessive consumption variability, from a social standpoint. In the former case, a welfare-maximizing government can implement the first-best by introducing proportional subsidies to investment and taxes on investment returns in an otherwise decentralized stock market economy (as was the case under CRS). Otherwise, if the problem is one of overinvestment in equilibrium, the government intervenes by introducing real money balances. This contrasts with Qi (1994), who argues that in this class of economies (with preference shocks and illiquid technologies) money is irrelevant, but this is just the case in his model because he restricts himself to technologies exhibiting CRS.

A.1 The efficient outcome

The first-best allocation $\{C_1^*, C_2^*, I^*\}$ solves

$$\max \varepsilon u(C_1) + (1-\varepsilon)u(C_2)$$

subject to

$$\varepsilon C_1 + (1-\varepsilon)C_2 = 1 + F(I) - I,$$

which yields

$$C_1^* = C_2^* = 1F(I^*) - I^*,$$

and

$$I^* \text{ solving } F'(I^*) = 1.$$

A.2 Stock market equilibrium

$\{C_1^m, C_2^m, I^m\}$ maximize $[\varepsilon u(C_1) + (1-\varepsilon)u(C_2)]$ subject to a material balance constraint $\varepsilon C_1 + (1-\varepsilon)C_2 = 1 + F(I) - I$ as before and a set of budget constraints (where we shall fix $\theta_0^0 = 1$ and $\theta_1^0 = 0$, without any loss of generality):

$$1 - I + P_0(I) = P_0(I) + P_1(I)\theta_0^1,$$

$$(1+r)(1 - I + P_0(I)) = C_1,$$

$$(1+r)(1 - I + P_0(I)) = P_1(I)\theta_1^1,$$

$$F(I)\theta_1^1 = C_2,$$

where r is the net real interest rate; $P_0(I) = F(I)/(1+r)^2$; $P_1(I) = (1+r)P_0(I)$; and $\theta_0^1 + (1-\varepsilon)\theta_1^1 = 1$, so that the stock market clears.

Steady-state ("golden rule") optimality requires that $r = 0$ and $I = I^*$. An interest rate $r = 0$ ensures that consumption is optimally allocated in equilibrium, that is, $C_1^m = C_2^m = 1 + F(I^*) - I^*$. This allocation requires that the agents' holdings of intermediate firms' shares equal $\theta_0^1 = (1 - I^*)/F(I^*)$ and $\theta_1^1 = (1 - I^* + F(I^*))/F(I^*)$. But then,

$$\theta_0^1 + (1-\varepsilon)\theta_1^1 = \frac{(2-\varepsilon)(1-I^*) + (1-\varepsilon)F(I^*)}{F(I^*)} = S > (<)1.$$

When $S < 1$ (i.e., $(1 - I^*) < \frac{\varepsilon}{2-\varepsilon}F(I^*)$), there is excess supply of shares in equilibrium. The optimal intervention in this case is to introduce a combination of subsidies and taxes: it is easy to show that there are age-dependent subsidies to investment and taxes on returns such that full efficiency is attained. If, instead, we restrict ourselves to non-age-dependent proportional subsidies to investment and taxes on returns, we will still be able to implement first-best investment levels but we will fail to achieve the optimal intertemporal allocation of consumption. On the other hand, when $S > 1$ (i.e., $(1 - I^*) > \frac{\varepsilon}{2-\varepsilon}F(I^*)$), so that there is excess demand for shares, the government will introduce money achieving both efficient investment levels and optimal consumption patterns.

Case A. $1 - I^* < \frac{\varepsilon}{2-\varepsilon}F(I^*)$. The government introduces a subsidy s proportional to investment and taxes dividends at rate τ so that $\tau F(I) = t = sI$ (i.e., the government budget is balanced). The optimal subsidy is chosen so that, in equilibrium, investment is optimal, that is, s is such that $(1 - s) = P_0'(I^*)$, which given that $F'(I^*) = 1$, implies that $P_0(I) = (1 - s)F(I)$. On the other hand, no arbitrage conditions yield

$$P_0(I) = \frac{(1-\tau)F(I)}{(1+r)^2} = \frac{F(I) - sI}{(1+r)^2}.$$

Thus, in equilibrium we must have that

$$P_0(I) = \frac{F(I) - sI}{(1+r)^2} = (1-s)F(I),$$

which implies that the equilibrium interest rate r is larger than zero so that the consumption pattern exhibits excessive volatility. Indeed, $\{C_1^{mp}, C_2^{mp}\} = \arg\max[\varepsilon u(C_1) + (1-\varepsilon)u(C_2)]$ subject to

$$1 + (s-1)I^* + P_0(I^*) = P_0(I^*) + \theta_0^1 P_1(I^*),$$

$$(1+r)(1 + (s-1)I^* + P_0(I^*)) = C_1 = \theta_1^1 P_1(I^*),$$

$$\theta_1^1(F(I^*) - sI^*) = C_2,$$

which requires that

$$\theta_0^1 = \frac{1 + (s-1)I^*}{P_1(I^*)} \text{ and } \theta_1^1 = 1 + (1+r)\theta_0^1.$$

Therefore, s has to be such that the market for intermediate shares clears, that is, such that $\theta_0^1 + (1-\varepsilon)\theta_1^1 = 1$ or, equivalently, s must solve

$$\varepsilon = A(s) + (1-\varepsilon)B(s),$$

where

$$A(s) = \frac{1 + (s-1)I^*}{\sqrt{(1-s)F(I^*)}\sqrt{F(I^*) - sI^*}},$$

and

$$B(s) = \frac{1 + (s-1)I^*}{(1-s)F(I^*)}.$$

Notice that $A'(s)$ and $B'(s)$ are both positive, which means that the previous equation has a unique solution. Moreover, note that if $s = 0$, $A + (1-\varepsilon)B = \frac{1-I^*}{F(I^*)}(2-\varepsilon)$, which is smaller than one in this case. On the other hand, if $s = 1$, both A and B tend to infinity, so that $A + (1-\varepsilon)B > 1$. In conclusion, there is a unique $s \in (0, 1)$ that implements the constrained stationary optimum.

Case B. $1 - I^* > \frac{\varepsilon}{2-\varepsilon}F(I^*)$. Money is then given to the newborn who holds real balances of m which can only be exchanged for consumption. Is there an $r = 0$ equilibrium with money? If $r = 0$, then $P_0 = P_1 = F(I^*)$, and the set of budget constraints faced by any given individual becomes

$$1 - I^* - m = \theta_0^1 F(I^*),$$

$$(1 + \theta_0^1)F(I^*) + m = C_1,$$

$$(1 + \theta_0^1) = \theta_1^1,$$

$$\theta_1^1 F(I^*) + m = C_2.$$

The equilibrium consumption pattern thus coincides with the optimal one, that is, $C_1 = C_1^* = C_2^* = C_2$, and investment is set at its first-best level $I = I^*$, provided that the financial markets clear. That is, we require shareholdings equal to

$$\theta_0^1 = \frac{\varepsilon}{2-\varepsilon} \text{ and } \theta_1^1 = \frac{2}{2-\varepsilon},$$

and real money balances equal to

$$m = 1 - I^* - \frac{\varepsilon}{2-\varepsilon} F(I^*).$$

These results represent an example of the role for the public debt in the context of investment and growth studied, for example, in the important article of Diamond (1965).

References

Allen, F., and D. Gale, 1995, "A Welfare Comparison of Intermediaries and Financial Markets in Germany and the US," *European Economic Review*, 39, 179–211.

Bencivenga, V., and B. Smith, 1991, "Financial Intermediation and Endogenous Growth," *Review of Economic Studies*, 58, 195–210.

Bhattacharya, S. P., P. Fulghieri, and R. Rovelli, 1995, "Turnpike Banking," working paper in progress, Universitá Bocconi, Milan.

Bhattacharya, S., and D. Gale, 1987, "Preference Shocks, Liquidity, and Central Bank Policy," in W. A. Barnett and K. J. Singleton (eds.), *New Approaches to Monetary Economics*, Cambridge University Press, Cambridge.

Bryant, J., 1980, "A Model of Reserves, Bank Runs, and Deposit Insurance," *Journal of Banking and Finance*, 4, 335–344.

Diamond, D., and P. Dybvig, 1983, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

Diamond, P., 1965, "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150.

Dutta, J., and S. Kapoor, 1993, "Liquidity and Financial Intermediation," Working Paper 188, University of Cambridge, Cambridge.

Esteban, J. M., and J. Sákovics, 1994, "Intertemporal Transfer Institutions," *Journal of Economic Theory*, 61, 189–205.

Fulghieri, P., and R. Rovelli, 1993, "Capital Markets, Financial Intermediaries, and the Supply of Liquidity in a Dynamic Economy," working paper, IGIER, Milan.

Gale, D., 1973, "Pure Exchange Equilibrium of Dynamic Economic Models," *Journal of Economic Theory*, 6, 12–36.

Geanakoplos, J., and H. Polemarchakis, 1991, "Overlapping Generations," in W. Hildebrand and H. Sonnenschein (eds.), *Handbook of Mathematical Economics*, Elsevier Science Publishers, Netherlands.

Greenwood, J., and B. D. Smith, 1993, "Financial Markets in Development, and the Development of Financial Markets," working paper, Cornell University, Ithaca, New York.

Haubrich, J. G., and R. G. King, 1990, "Banking and Insurance," *Journal of Monetary Economics*, 26, 361–386.

Hellwig, M., 1994, "Liquidity Provision, Banking and the Allocation of Interest Rate Risk," *European Economic Review*, 38, 1363–1390.

Jacklin, C., 1987, "Demand Deposits, Trading Restrictions, and Risk Sharing," in E. C. Presscott and N. Wallace (eds.), *Contractual Arrangements for Intertemporal Trade*, University of Minnesota Press, Minneapolis.

Mayer, C., 1988, "New Issues in Corporate Finance," *European Economic Review*, 32, 1167–1188.

Qi, J., 1994, "Bank Liquidity and Stability in an Overlapping Generations Model," *Review of Financial Studies*, 7, 389–417.

Samuelson, P. A., 1958, "An Exact Consumption-Loan Model of Interest, With or Without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467–482.

Sussman, O., 1992, "A Welfare Analysis of the Diamond-Dybvig Model," *Economics Letters*, 38, 217–222.

Sussman, O., 1995, "Investment and Banking: Some International Comparisons," *Oxford Review of Economic Policy*, 10, 79–93.

Zilcha, I., 1991, "Characterizing Efficiency in Stochastic Overlapping Generations Models," *Journal of Economic Theory*, 55, 1–16.