The Role of Games in Security Design

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We contend that security design should be approached as a problem of game design. That is, contracts should specify the procedures that govern the behavior of contract participants in determining outcomes as well as the allocations resulting from those outcomes. We characterize optimal contracts in two nested classes: all contracts (including those that depend on the state) andstate-independent contracts. We demonstrate that, in situations in which the dependence of contracts on the state is limited, contracts designed as games can improve the allocations of resources relative to nonstrategic allocation rules.

The nature of contracts between suppliers of capital and firms determines which investments are undertaken and thus has profound effects on the efficiency of the resource allocation process. The investigation of these contracts has traditionally considered

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the determinants of the relative amounts issued by firms of various *exogenously given securities,* mainly debt and equity. A deeper question, however, is what determines the form of the contract (security) under which investors supply funds to the firm. The issue of *endogenous* contract determination (i.e., security design) has recently been the subject of increasing attention in the literature. This literature has treated securities as resource allocation rules, specifying who gets what as functions of some contractible outcome. In this article we propose a new method for analyzing corporate securities. In particular, we argue that, in some environments, contracts should specify the procedures that govern the behavior of contract participants in determining outcomes as well as the allocations resulting from those outcomes. That is, we contend that security design should be approached as a problem of *game design.* This approach allows us to investigate the important features of a contract, including a negotiation game, that results in an efficient allocation of resources.

We argue that games can improve upon nonstrategic allocation rules because financial contracting situations have natural limitations on the extent to which enforceable rules can depend on aspects of the environment. For example, inability to verify income prevents contracts that are contingent on income. Given such restrictions, the allocation of resources can sometimes be improved by designing a game in which the participants' behavior determines the outcome.[1] The improvement results because equilibrium strategies, and hence outcomes, of the game can depend on aspects of the environment even though allocation rules that depend on such aspects are not enforceable. That is, games allow some circumvention of restrictions on allocation rules.

The following example should clarify this idea. Consider the famous problem of dividing a cake fairly between two claimants. The purpose, of course, is to design an allocation scheme or game that results in as equal an allocation as possible, given that one of the claimants must cut the cake. Suppose that allocation rules must be deterministic and can specify at most which claimant cuts the cake

[1] This point has been made by Kreps (1988) for the single-person case. He argues that allocations can be improved by having contracts specify *sets* from which the single agent will be allowed to choose later. The natural extension of Kreps' approach to multiperson environments is to allow contracts to specify *games* to be played among the agents. In this regard, see Hart and Moore (1988) and Hermalin and Katz (1991). Aghion, Dewatripont, and Rey (1994) show that, in situations characterized by "lock-in" problems and incomplete contracts, first-best allocations can be achieved by specifying a renegotiation game with all bargaining power given to one party. Their results does any over to our environment because of *ex post* restrictions on available

and which piece is allocated to each claimant. Further suppose that the allocation of pieces cannot depend on any characteristic of the piece other than its location (right or left). Clearly, the only equilibrium allocation resulting from such a scheme is that one claimant takes the whole cake. If, however, we allow claimants to play the well-known "divide and choose" game in which one claimant is arbitrarily chosen to cut the cake and the other is allowed to choose which piece is his, the first-best, 50-50, allocation results (assuming the cake is homogeneous). Thus, the ability to choose games improves the allocation relative to that which could be achieved by allocation rules under the given contracting restrictions, without explicitly violating these restrictions.

Before turning to the details of our model, it is useful to review briefly the literature on security design. This literature can be categorized according to the main issue to be resolved by the security design.[2] The first models view financial contracts as resolving an agency problem between firm insiders and outside investors [e.g., Townsend (1979), Diamond (1984), Gale and Hellwig (1985), and Hart and Moore (1989)]. In these models, insiders are assumed to be able to appropriate all of the firm's income. Securities are designed to induce sufficient payout to finance the project, Inducements include income verification, penalties on managers, and liquidation of assets. A second set of models focuses on adverse selection problems [e.g., Nachman and Noe (1994), Diamond (1991, 1993), and Allen and Gale (1992)]. Here securities are designed to signal borrowers' private information to lenders. A third group considers the role of securities in the allocation of control rights [e.g., Grossman and Hart (1988), Harris and Raviv (1988), Zender (1991), and Aghion and Bolton (1992)]. In the fourth type of model, securities are designed to optimize ownership structure, including the limitation of liability [e.g., Fama and Jensen (1983) and Winton (1993)]. The final set of models focuses on the allocation of risk [e.g., Allen and Gale (1988)].

In this literature, securities are generally allocation rules specifying payments to investors or penalties on managers as functions of various outcomes. As mentioned above, we argue that, in some situations, designing contracts as games can improve the resource allocation.[3]

The environment we consider is similar to the security design models based on agency considerations, in particular that of Hart and

[2] This taxonomy follows Allen and Winton (1992). For a more detailed review, see their article along with Allen (1990) and Harris and Raviv (1992).

[3] Some papers (e.g., those on corporate control and voting rights) specify a game as part of the contract. In these models, the game structure is exogenous; only its parameters are part of the design problem.

Moore (1989). An entrepreneur needs outside financing for a positive net-present-value project. Any cash generated by the project can be appropriated by the entrepreneur. As in Hart and Moore (1989), the entrepreneur cannot appropriate proceeds from asset liquidation. The only way to induce the entrepreneur to pay out cash to investors is to threaten to seize and liquidate the firm's assets, thus eliminating future cash flows. While the liquidation values and future cash flows are uncertain as of the contracting date, it is known that there are two possible outcomes, called *states*. It is further known that, in both states, liquidation is strictly suboptimal. The state is revealed to all parties after the contract is signed.

We analyze the contract design problem, namely the choice of a game that minimizes the extent of (suboptimal) liquidation while providing sufficient payouts in the form of cash and liquidation proceeds to induce investors to contribute the required capital. Contract choice is limited in two dimensions. First, as mentioned, the entrepreneur cannot be forced to make payouts. Second, contracts may be limited in the extent to which they depend on the specifics of the firm's situation (i.e., on realized cash flows or liquidation values). Such limitations on state dependence could result from enforcement problems arising from the inability to verify the state of the firm. Consequently, we solve the contract design problem under various assumptions regarding the contractibility of the state.

In one regime, contracts are allowed to depend explicitly on the state. We refer to the best allocation achievable in this regime as the *second-best* (we reserve the term *first-best* for the best outcome achievable in the absence of contracting limitations, namely, the outcome with no liquidation). In this case a simple, nonstrategic allocation rule suffices to achieve second-best (i.e., no game is required). An optimal allocation rule resembles a debt contract with state-dependent face value. The sense in which this contract resembles debt is that it promises certain payments (albeit different payments in different states) and gives the investors rights to liquidate assets and keep the proceeds if the promised payments are not made. This second-best contract provides a benchmark against which we measure the efficiency of state-independent contracts.[4]

We also consider a second regime in which contracts cannot exhibit any state dependence. Although states are very simple abstractions in

[4] One interpretation of the second-best contract is as a "standard" debt contract augmented by a bankruptcy procedure. That is, the contract is equivalent to the following: a single face value is specified that is equal to the largest of the face values in the original contract. In the states in which the entrepreneur cannot pay this face value, a bankruptcy court forces forgiveness of the debt to the lower face value from the original contract.

our model, the assumption that contracts cannot be state-dependent is meant to capture the idea that, in reality, it may be impossible for contract-enforcing institutions (courts) to determine the true situation. The best allocation in this regime is referred to as the *third-best.* In this case, one might assume that a state-independent counterpart of the above debt contract (i.e., a debt contract with a single face value) would be optimal. We consider two such debt contracts, one in which the creditors may offer to forgive some of the debt and one in which the entrepreneur makes a take-it-or-leave-it offer to creditors. We call these contracts *creditor-favored debt* and *debtor-favored debt*, respectively.[5] We also characterize an optimal contract among state-independent contracts (including those that specify games) (i.e., one that achieves the third-best). In general, the third-best contract is strictly better than the two debt contracts just mentioned. In the third-best contract, which we call a *universal contract,* the entrepreneur, alter observing the state, deposits in an escrow account the maximum amount of first-period cash on hand he is willing to pay out. Then the entrepreneur and investors make simultaneous announcements that can be interpreted as reported states. Cash payments and liquidation amounts are then determined as functions of the amount in escrow and the two reports (but do not depend directly on the true state).[6] We characterize the optimal universal contract for any particular environment. We also show by examples that this contract may result in a less efficient allocation than second-best but a more efficient allocation than any nonstrategic allocation rule.

The remainder of the article is organized as follows. In Section 1, we discuss the modeled environment in detail and present a numerical example used throughout the article. Some of the general ideas of the article are presented by means of specific examples of contracts in Section 2. In Section 3, we carefully define the set of feasible contracts. Equilibrium of contracts and the concept of an optimal contract are defined in Section 4. In Section 5, we derive optimal contracts under various assumptions about what contracts are feasible and argue that, when the state is not contractible, games can improve the allocation of resources. Section 6 concludes.

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[5] Creditor-favored debt is essentially the same contract considered by Hart and Moore (1989), who take the form of the contract (including, the renegotiation game) as given and derive optimal values of the parameters.

[6] In fact, we show that a version of the Revelation Principle holds (namely, that the payments and liquidation amounts can be designed to implement the expected outcome of any state-independent contract); hence, the name *universal.*

1. Environment

As mentioned, we adopt an environment that is very similar to that of Hart and Moore (1989). There is a risk-neutral entrepreneur who requires K dollars of outside financing to buy assets at date 0. These assets last two periods and have zero value at the end of the second period, at date 2; they can be partially or fully liquidated at the end of the first period, at date 1. For simplicity we assume that the discount rate is zero. All uncertainty about the project is summarized by the state of nature, $s \in S = \{1, 2\}$. All participants agree that the states are equally likely. The state is revealed to all participants at date 1, although it may not be contractible (see below). The assets return a state-dependent cash flow R_{1s} at date 1, and if there is no liquidation at date 1, they also return a cash flow at date 2, R_{2s}. Any fraction $l \in [0, 1]$ of the assets can be liquidated at date 1. The technology is linear in the sense that, if the liquidation value of the assets in state s is L_s, then the proceeds from liquidating the fraction l of the assets are lL_s and accrue immediately. Moreover, in this case, returns at date 2 are reduced to $(1 - l)R_{2s}$. Following Hart and Moore (1989), we assume that liquidation is always an inferior strategy, i.e.,

$$R_{2s} > L_s, \qquad \text{for all } s. \tag{1}$$

The opportunity cost of liquidation is measured by the foregone date 2 return per dollar of liquidated assets (i.e., the cost-benefit ratio $\alpha_s \equiv R_{2s}/L_s$). This trade-off between continuation and liquidation could differ across the two states.[7] The structure of the model is shown in the following time line:

$t = 0$	$t = 1$	$t = 2$
Sign contract $K + T$ transferred Invest K	State s is revealed Cash flow R_{1s} Liquidation value L_s Play game	Cash flow R_{2s}

Outside investors are risk-neutral, and competition among them ensures that capital will be supplied, provided the expected rate of

[7] The assumptions that there are only two states and that the states have equal probabilities are for ease of exposition only. The results would be unaffected by assuming any finite number of states with any positive probabilities. Our basic intuition that games can improve on nonstrategic allocation rules by circumventing restrictions on state dependence is unaffected by increasing the number of dates or by allowing more complicated evolution of information revelation. Generalizing in these directions, however, complicates the model considerably and would certainly affect the specific nature of optimal contracts [see Hart and Moore (1989) for a three-period version of the model].

return is at least the risk-free interest rate (zero as assumed above). Thus, expected total payments to investors (cash payments plus liquidation proceeds) must be at least as great as their contribution. In what follows, we assume that investors are homogeneous and play in any game as if they were a single player; we henceforth refer to them collectively as *the investor*.

The problem for the entrepreneur is to design a least-cost contract that induces investors to contribute at least K dollars. As in Hart and Moore (1989), we allow for the possibility that an optimal contract raises more than the minimum outside financing required. We denote this *excess transfer* by T and refer to $K + T$ as the *market value* of the contract. The expected cost to the entrepreneur consists of two components. First, cost includes the actual cash payment to the investor, say C_s in state s, net of the excess transfer, T. Second, cost includes the foregone second-period cash flow resulting from asset liquidation. If the liquidation value of the assets actually liquidated in state s is A_s, then the fraction of assets liquidated is A_s/L_s. The foregone second-period cash flow is then $R_{2s}A_s/L_s = \alpha_s A_s$, and the entrepreneur's total cost in that state is $C_s - T + \alpha_s A_s$.

What makes the design problem interesting is the assumption that the entrepreneur cannot commit to making future payments to investors, nor can he be forced to pay. In particular, the entrepreneur can appropriate all *cash flows* both from the assets and from the excess transfer. This is the extreme agency cost assumption employed in the agency-driven security design literature [see, e.g., Townsend (1979), Diamond (1984), Gale and Hellwig (1985), and Han and Moore (1989)]. The idea is that cash flows are not verifiable (i.e., that managers can "hide" cash flows from institutions, such as courts, empowered to enforce contracts, even if they cannot hide these cash flows from the parties to the contract).[8] The investor, on the other hand, can liquidate assets and appropriate the proceeds from liquidation within limits specified by the contract. The only reason for the entrepreneur actually to make any payments to the investor is to avoid liquidation of assets and the resulting loss of second-period income. Since the threat of liquidation provides the only inducement for the entrepreneur to compensate investors, it is clear that no payments can

[8] Certainly, in reality, audited financial disclosures make it difficult for managers to appropriate all cash flows not paid out. Nevertheless, managers and other insiders can capture some cash flows in ways that cannot be proven in court (e.g., by overpaying suppliers that they control). If some portion of the cash flow is verifiable, our model can be viewed as addressing the part of the investment that cannot be financed by promising verifiable cash flows. The fact that actual debt contracts give creditors the power to force liquidation d assets suggests that courts cannot simply force debtors to make promised payments. Note that we are implicitly assuming the existence and necessity of an institution that enforces contracts to the extent that their contingencies are verifiable.

Table 1
Income and liquidation data[a]

s	R_{1s}	R_{2s}	L_s	α_s
1	100	200	100	2
2	40	200	50	4

[a] The table shows, for each state, $s \in \{1, 2\}$, income at dates 1 and 2, R_{1s} and R_{2s}, respectively, liquidation value, L_s, and the liquidation cost-benefit ratio, a_s.

be made from second-period income. Sometimes liquidation threats must actually be carried out for investors to recover their investment. Since liquidation is assumed to be suboptimal, the contract will be designed so as to minimize deadweight losses associated with actual liquidation.

In what follows we will make extensive use of a particular example of the type of environment assumed here. For this example:

- The investment cost is $K = 75$.
- Income at date 1 in state 1 is $R_{11} = 100$ and in state 2 is $R_{12} = 40$. Income at date 2 is 200 in either state (i.e., $R_{21} = R_{22} = 200$). Thus, expected income is $0.5(100 + 200 + 40 + 200) = 270$.
- Liquidation value of the assets in state 1 is $L_1 = 100$ and in state 2 is $L_2 = 50$.

The income and liquidation data for this example are summarized in Table 1.

As a benchmark, it is useful to calculate the lowest cost allocation for this example consistent with the inability to force the entrepreneur to pay out cash. We refer to this outcome as the second-best. Since liquidation is inefficient in both states, investors should be paid in cash to the fullest extent possible to avoid liquidation. Also, any required liquidation should occur in the state in which it is least costly (i.e., in which a_s is smallest). For this example, the maximum cash payouts are 100 in state 1 and 40 in state 2. This yields an expected payout of $0.5(100 + 40) = 70$. Since investors must receive an expected payout of at least 75 to finance the investment cost, at least 5 must be raised on average through liquidation. In state 1, each dollar of assets liquidated costs $\alpha_1 = 2$ dollars in foregone date 2 returns. In state, 2, this cost is $\alpha_2 = 4$ dollars (i.e., liquidation is twice as costly in state 2 as in state 1). Therefore, liquidation should be shifted to state 1. Consequently, the second-best outcome in this example is as follows. In state 1, the entrepreneur pays 100 in cash and liquidates assets with liquidation value 10. In state 2, he pays 40 in cash with no liquidation. The resulting, second-best expected cost to the entrepreneur is $0.5(100 + 2 \times 10 + 40) = 80$.

Before turning to general definitions and results, it is helpful to consider some examples of feasible contracts.

2. Some Special Feasible Contracts

The purpose of this section is to illustrate our concepts of feasible contracts and comparison of contracts. This will help in understanding the general definitions of feasibility and equilibrium and our results on *optimal* contract design discussed in Sections 3, 4, and 5. First, we consider a state-dependent debt contract of the type mentioned in the introduction.. We exhibit face values for which this contract produces second-best outcomes for the example of Section 1.[9] The state-dependent debt contract does not involve a true game (this is made more precise later). Next, we turn to state-independent contracts and examine the universal contract and two variations of the state-dependent debt contract, creditor-favored and debtor-favored debt. These contracts do involve games. We show that for the environment specified numerically in the previous section, debtor-favored debt dominates creditor favored debt, but both are inferior to the universal contract

2.1 State-dependent debt

State-dependent debt specifies an amount (a market value) to be contributed by investors, $K + T$, and face values, F_1 and F_2, for each state. The entrepreneur can avoid liquidation only by paying the investor the face value for the realized state, F_s. If he does not pay this amount, enough assets are liquidated to provide a total payment to the investor equal to this face value, if possible: If the assets do not have sufficient liquidation value to meet this requirement, then all assets are liquidated, the proceeds being paid to the investor. The fact that the contract allows forced liquidation if promised payments are not made is the feature that makes this contract resemble debt.

This contract does not involve a game but does require state dependence. There is no game because only the entrepreneur has any choices. The entrepreneur need not consider what the investor will choose in deciding his own optimal move (i.e., there is no strategic interaction between the entrepreneur and the investor). The contract, is state-dependent, however, because the payoffs depend on the face value, which is state-dependent.

[9] fitter, when we search for optimal contracts, we show that, in tact, state-dependent debt an always generate a second-best outcome.

We now construct a state-dependent debt contract that achieves second-best for the example environment from Section 1. This contract has market value of 75 $(T = 0)$[10] and face values of $F_1 = 110$ and $F_2 = 40$.

Next we calculate how much the investor will receive in cash and liquidation proceeds. Since each dollar the entrepreneur pays out in cash saves him more than one dollar in foregone date 2 income, in each state it is advantageous for the entrepreneur to pay cash to avoid liquidation. He is willing to pay out cash only if he can reduce his total cost to 200 or less, however, because, by paying zero, the most he can lose is the full 200 dollars of date 2 income. In state 1, therefore, the entrepreneur pays out all of his available cash, 100 dollars. Since $F_1 = 110$, assets with liquidation value of 10 are liquidated at a cost to the entrepreneur of 20. The entrepreneur's total cost in state 1 is 120, while the investor receives 110. In state 2, the entrepreneur pays out all his cash, namely 40, resulting in no liquidation (recall $F_2 = 40$). The entrepreneur's total cost and the investor's receipts are 40 in state 2.

This state-dependent debt contract results in an expected payment to investors of 0.5(110 + 40) = 75, the market value of the debt. The expected cost to the entrepreneur is 0.5(120 + 40) = SO, which is exactly the second-best outcome. Thus, this state-dependent debt contract results in the lowest possible expected cost for the entrepreneur and dead-weight liquidation costs. Indeed, we show below that this is a general result, implying that a state-dependent debt contract is the best feasible contract.

2.2 Creditor-favored debt

The *creditor-favored debt* contract that was considered by Hart and Moore (1989) is one natural adaptation of state-dependent debt to a situation in which state dependence is not feasible.[11] This contract specifies a market value, $K + T$, and a single face value, F, but allows the investor to reduce the face value voluntarily (i.e., forgive some of the debt) after observing the state. The entrepreneur can avoid liquidation only by paying the investor the (possibly reduced) face value. If he does not pay this amount, enough assets are liquidated to provide a total payment to the investor equal to the (reduced) face value, if possible. If the assets do not have sufficient liquidation value

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[10] In fact, my value of $T \leq 80$ will do equally well in this example.

[11] Actually, the game analyzed by Hart and Moore (1989) includes an extra stage at the beginning in which the entrepreneur proposes a reorganization. This stage has no effect on the outcome in the two-period environment. Consequently, we omit it.

to meet this requirement, then all assets are liquidated, the proceeds being paid to the investor.[12]

This contract involves a game since each player has a nontrivial choice. The contract is state-independent, since the game structure and payoffs as functions of the players' moves are independent of the state. Specifically, the investor's set of allowed forgiveness levels is the same for both states. Also, the set of payments the entrepreneur is allowed to make does not depend on the state. Finally, the payoffs depend only on the moves of the players and not directly on the state.

Consider the following creditor-favored debt contract for the numerical example of the previous section. The market value of this contract is 179 (financing the investment of 75 plus an excess transfer of T = 104)[13] and its face value is 200. We calculate the equilibrium of this contract by backwards induction for each state. In state 1, the entrepreneur has cash available of 204 (=100+104) dollars. Suppose the investor refuses to forgive any of the debt. Since each dollar the entrepreneur pays out in cash saves him 2 dollars in foregone date 2 income, the entrepreneur will pay the entire face value in cash in this state. If the investor forgives any of the debt, this simply reduces the cash payout but does not increase liquidation. Consequently, in state 1, the investor refuses to forgive, the entrepreneur pays the full face value of 200, and there is no liquidation.

In state 2, the entrepreneur has cash available of 144(= 40 + 104) dollars. As in state 1, he is willing to pay out as much cash as is necessary to avoid liquidation, provided that his total cost (including liquidation costs) does not exceed 200. Therefore, the most the investor can extract in this state is 144 dollars of cash plus 14 dollars of liquidation proceeds. The cost of this to the entrepreneur is $144+14 \times 4 = 200$. If the investor tried to extract more than 144 + 14 = 158 dollars by further liquidation, the entrepreneur would pay zero, and the investor would receive only the liquidation value of the assets, 50 in this state. Thus, the investor will forgive the debt to 158 in this state. The en-

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[12] Independent and related work by Bolton and Scharfstein (1992) has recently come to our attention. They consider a special case of the Hart and Moore (1989) environment and analyze a debt contract with an overall limit on liquidation but no excess transfer. They call this a debt contract with *partial security interest.* It turns out that this contract is equivalent to Hart and Moore's (1989) debt contract. Bolton and Scharfstein (1992) also consider our debtor-favored debt contract (see below) but introduce asymmetric information and multiple creditors. Like Hart and Moore (1989), Bolton and Scharfstein (1992) take the contract form as given and derive optimal parameter values.

[13] As pointed out below, this contract is optimal in the class of creditor-favored debt contracts. Surprisingly, perhaps, the additional transfer of 104 from investor to entrepreneur over and above the $75 Investment cost is essential (i.e., any smaller transfer would result in an inferior outcome, both from the point of view of the entrepreneur and socially). This point was stressed by Hart and Moore (1989).

trepreneur will pay 144 in cash, and liquidation proceeds will amount to 14.

The expected total receipts of the investor are 0.5(200 + 158) = 179. Since 179 is the market value of the bond, the loan has zero net present value to the investor. The expected cost of the bond to the entrepreneur is 0.5(200- 104 + 144 - 104 + 56) = 96,[14] which is below his expected income of 270. The expected cost of this debt contract is larger than the second-best cost. First, not all cash is paid out in state 1. Second, all liquidation occurs in state 2 instead of state 1.

The reason that “too much” liquidation occurs and that it occurs in the Wrong” state is that, in this contract, the investor has all the bargaining power when the entrepreneur defaults (hence the name of the contract). This power results from the investor’s right to set the menu from which the entrepreneur chooses [i.e., the entrepreneur must choose a combination of cash payment and liquidation proceeds that adds up to the (reduced) face value offered by the investor]. The only constraint on the investor is the entrepreneur’s right to pay nothing.[15] Thus, the investor will make a reduced-face-value offer that makes the entrepreneur’s cash and liquidation costs equal the cost of paying nothing. The investor’s ability to extract in state 2 results in “too much” liquidation in that state and “too little” cash payment and liquidation in state 1. Nevertheless, as will be shown in the Appendix, no other creditor-favored debt contract can improve on this outcome. The analysis suggests, however, that improvements are possible if a contract that weakens the investor’s bargaining power can be designed. In the next two subsections, we present two contracts that accomplish this.

2.3 Debtor-favored debt

Debtor-famed debt is another adaptation of state-dependent debt that eliminates the state dependence. It specifies market and face values as in the previous debt contract, except that now the renegotiation game confers more bargaining power on the debtor. This game consists of two stages (for each state). First, the entrepreneur offers a cash payment and offers to liquidate assets with a given liquidation value. The investor is allowed only the two choices of taking the offer *as is* or not. If he accepts the entrepreneur’s offer, then the specified cash payment is made and the specified liquidation occurs. If the investor declines, then sufficient assets are liquidated to provide the investor

[14] The solution presented here Is the one given by lemma 1 in Hart and Moore (1989).

[15] Although face value can act as a restraint on the investor, since it is not state-dependent, face value is generally binding only in one state. In this example, it is binding only in state 1, while the “excess” liquidation occurs only in state 2.

a payment equal to the face value of the contract. If this is impossible, then all assets are liquidated with the proceeds accruing to the investor. The investor is not allowed to make any counter-proposals.

This contract involves a game, since each player has a nontrivial choice. The contract is state-independent, since the game structure and payoffs as functions of the players' moves are independent of the state. Specifically, the set of allowed cash and liquidation offers of the entrepreneur is the same for both states. Also, the set of responses the investor is allowed to make (accept or reject) does not depend on the state. Finally, the payoffs depend only on the moves of the players and not directly on the state.

Now consider a debtor-favored debt contract in which $T = 0$ and $F = 100$. In each state, the investor will accept no offer in which he receives less than full liquidation value in that state. Consequently, the problem for the entrepreneur is to find the cheapest way of providing the investor with L_s in state s, for each state. Obviously, it is cheaper for the entrepreneur to pay cash as opposed to liquidating assets. In state 1, therefore, the entrepreneur offers a cash payment of 100 with zero liquidation, and the investor accepts, since $L_1 = .100$. The cost to the entrepreneur is 100. In state 2, the entrepreneur offers a cash payment of *40* and liquidation proceeds of 10. Again, since $L_2 = 50$, the investor accepts. The cost to the entrepreneur is $40 + 4 \times 10 = 80$ in this state. The investor's total expected receipts are $0.5\,(100 + 40 + 10) = 75$. Again, 75 is the market value of the contract. The entrepreneur's expected cost is $0.5(100 + 80) = 90$.

The expected cost to the entrepreneur for this debtor-favored debt contract is smaller than for the above creditor-favored debt contract (but larger than the second-best outcome). Thus the entrepreneur prefers the debtor-favored debt contract. Since the investor's transaction has zero net present value for both contracts, he is indifferent between them. Consequently, the debtor-favored debt contract is strictly Pareto-superior.[16] The reason for this is simply that the debtor-favored debt contract results in smaller dead-weight liquidation costs on average. As mentioned previously, the creditor-favored debt contract is inferior here because it gives too much bargaining power to the investor ex post. By allowing the entrepreneur to set the menu, the debtor-favored debt contract reduces the investor's bargaining power. The contract confers too much bargaining power on the entrepreneur, however, resulting in a state 1 payment that is too low. This problem can be mitigated by the next contract we study.

[16] This not true in general. For example, if liquidation values are sufficiently low, this contract may result in too little payoff to investors to raise the required funds even though the creditor-favored debt contract could raise them.

2.4 Universal contract

State-dependent debt achieves a second-best outcome but requires state dependence. We now consider a state-independent contract that, *for this environment,* also achieves the second-best. As is the case with the other contracts discussed in this section, the *universal contract* specifies an amount contributed by investors, $K + T$. After both the entrepreneur and investor observe the state, the entrepreneur deposits in an escrow account the maximum amount of cash he is willing to pay out (any cash remaining in the escrow at the end of the game reverts to the entrepreneur). He may not deposit more than his cash on hand at that time, namely $R_{1s} + T$. The deposit is observed by the investor. The parties then simultaneously announce either 1 or 2, interpreted as a claim about the state. The contract specifies the cash payment and the assets liquidated as functions of the amount in escrow and the two announcements, subject to several constraints. The first is that the cash payment cannot exceed the amount in escrow. This constraint reflects our assumption that the entrepreneur cannot be forced to make payments. The second constraint is that the cash payment and the assets liquidated cannot depend directly on the actual state. The third is that, if the liquidation that results from the parties' moves in the game exceeds the liquidation value of the assets, L_s, in some state, s, then all assets are liquidated in that state.

Like the previous two contracts, this contract involves a game, since each player has a nontrivial choice. The investor and entrepreneur can each announce either one or two, and these choice sets are the same for both states. As before, the payoffs depend only on the moves of the players and not directly on the state. Consequently, the contract is a state-independent game.[17]

Consider the following universal contract for the numerical example of the previous section. The market value of the contract is 85 $(T = 10)$. The cash payment and liquidation as functions of the escrow deposit and the announcements are given in Table 2.

An equilibrium of this contract involves an escrow deposit equal to the cash available to the entrepreneur and both parties reporting the actual state in the second stage, for both state-games. To see this, we analyze the equilibrium by backwards induction, starting with the announcements of 1 or 2 for any given feasible escrow deposit. First, suppose the true state is state 1. If the investor announces 1, the entrepreneur's cost $(c + \alpha_1 \min\{a, L_1\})$ if he announces 1 or 2 is d +

[17] The entrepreneur also must choose an amount to deposit in escrow. The choice set for this move exhibits state dependence in that the entrepreneur cannot deposit more cash than he has. This minimal state dependence is unavoidable if we require outcomes always to be feasible. This point will be discussed in more detail in the next section.

Table 2
Cash payments and liquidation amounts for the universal contract example[a]

Entrepreneur announces	Investor announces	
	1	2
1	$c = d$; $a = 120 - c$	$c = 0$; $a = 50$
2	$c = 0$; $a = 100$	$c = \min\{50, d\}$; $a = 50 - c$

[a] For any amount d deposited in escrow, this table shows, for each pair of announcements, the cash payment to the investor, c, and the proposed liquidation, a. The amount actually liquidated in state s is $\min\{a, L_s\}$.

2 min(120- d, 100) or 200, respectively. It is easy to check that 2 is the entrepreneur's best response for any $d \leq 40$, and 1 is his best response for any $d > 40$. For the investor, announcing 1 is a dominant move for any d and any announcement by the entrepreneur. If the entrepreneur announces 1 in this state, the investor's payoff $(c + \min\{a, L_1\})$ if he announces 1 or 2 is $d + \min\{120 - d, 100\}$ or 50, respectively. It is easy to check that 1 is his best response for any $d \in [0, 110]$. If the entrepreneur announces 2, the investor's payoff if he announces 1 or 2 is 100 or 50, respectively. Consequently, his best response is again 1. Thus, in the second stage, in state 1, the entrepreneur will say 1 if $d > 40$ and 2 if $d \leq 40$, while the investor will always say 1. Given this behavior in the second stage, in the initial, deposit stage, the entrepreneur's cost is

$$\begin{cases} 200 & \text{for } d \leq 40, \\ 240 - d & \text{for } d > 40. \end{cases}$$

This cost is minimized by choosing the largest. possible deposit in state 1, namely $d = R_{11} + T = 110$. This results in cost to the entrepreneur of $130 = 110 + 2 \times 10$. The investor receives 120 = 110 + 10.

Now suppose that the true state is state 2. If the investor announces 2, the entrepreneur's cost if he announces 1 or 2 is 200 or 200 - $3d$ for $d \leq 50 = R_{12} + T$, respectively. Therefore, 2 is his best response. Conversely, if the entrepreneur announces 2 and the investor announces 1, he obtains no cash and 50 from liquidation, since $L_2 = 50$. If the investor announces 2, he obtains a total of 50 in cash and liquidation (for $d \leq 50$). Consequently, announcing 2 is a best response for the investor (when a player is indifferent, we assume he announces the true state). Given that, for any escrow deposit, d, both parties will announce the true state in the second stage, it is easy to see that it is optimal for the entrepreneur to deposit $d = R_{12} + T = 50$ *in* state 2.

The contract thus results in a cash payment of 110 and liquidation of 10 in state 1 and a cash payment of 50 with no liquidation in state 2.

The expected payment to the investor is, therefore, 0.5 (110 + 10 + 50) = 85, which is the market value of the contract. The expected cost to the entrepreneur is 0.5 (130 + 50) - 10 = 80. This is exactly the second-best solution, as argued above.[18]

Even though the contract is state-independent, its expected cost to the entrepreneur is the same as that of state-dependent debt. This is not, however, a general result. In Section 5, we will show that, although no state-independent contract is superior to the best universal contract for a given environment, even the best universal contract is sometimes inferior to state-dependent debt.

3. Feasible Contracts

Having considered several examples of financial contracts, we now consider the general problem of finding an optimal contract for our environment. To solve this problem, we must first specify what contracts are feasible. We consider two concepts of feasibility depending on whether state dependence is allowed. The least restrictive concept allows fully statedependent contracts, requiring only that contracts respect the assumption that the entrepreneur cannot be forced to make payments. The more restrictive definition requires, in addition, that the contract not depend on the state. In this section, we define the two concepts of feasibility and argue that statedependent debt is a feasible, state-dependent contract, while the other contracts discussed in the previous section are feasible and state-independent.

3.1 General (possibly state-dependent) contracts

Recall that we interpret "contract" in the broad sense (i.e., as a specification of the procedures to be followed or *game to be played* in determining payoffs to the entrepreneur and the investor). Every contract starts with a transfer, $K + T$, from the investor to the entrepreneur. This transfer is then followed by a game. In what follows we refer to a game in extensive form (i.e., a specification of a tree with two initial nodes, one for each state).[19] The tree starting at the initial node for state s is a subgame called *state-game s*, denoted G_s $(s \in S)$. Each state-game tree also includes, for each node, a specification of who moves at that node and what moves are available to that player. We assume that the nodes of state-game s are labeled, and we let E_s and I_s be the sets of nodes at which the entrepreneur and investor move, re-

[18] The ash payments in this case exceed those of the optimal statedependent debt contract by 10 in both states, but this is exactly the excess transfer.

[19] For a nice discussion of games in extensive form, see Kreps and Wilson (1982).

spectively. In fact, the tree specifies *information sets* at which players move, rather than nodes. An information set is a collection of nodes among which the player cannot distinguish. For any node n in a state-game s, we denote by $\iota_s(n)$ the information set that contains n. This construction is used to represent situations in which a player is unsure of his exact situation when he chooses his move. For any node n in state-game s, let $M_s(n)$ be the set of possible moves at n, and, for any node n and move $m \in M_s(n)$, let $\sigma_s(n, m)$ be the successor node to n if m is chosen at n. Naturally, the set of moves available to a player at any node in an information set must be the same as at any other node in that set.

To define the outcome of the game, we need functions that associate final nodes with cash payments and liquidation amounts. The final node at which the game actually ends, and hence the payoffs, depends, of course, on the sequence of moves taken by both players. Let Z_s be the set of final nodes for state-game s. The cash paid and the amount liquidated in state-game s at final node z are given by the *outcome functions* $c_s(z)$ and $\min\{a_s(z), L_s\}$, respectively, where $c_s : Z_s \to \Re_+$ and $a_s : Z_s \to \Re_+$ for each state s. To summarize, the state-game $G_s = (E_s, I_s, Z_s, \iota_s, M_s, \sigma_s, c_s, a_s)$.

We are now ready to define what we mean by feasibility of a contract. A *feasible contract* $Q = (T, (G_s)_{s \in S})$ consists of an excess transfer, $T \geq 0$ (in addition to the investment cost, K), and a state-game G_s for each state, s. The state-game G_s must satisfy the following restrictions:

a) There is one initial node, denoted node o, with $o \in E_s$ and $\iota_s(o) = \{o\}$.
b) $M_s(o) \subset \Re_+$ and for any $m, m' \in M_s(o)$ with $m \neq m'$, if a node n is on a path from $\sigma_s(o, m)$ and n' is on a path from $\sigma_s(o, m')$, then $\iota_s(n) \neq \iota_s(n')$.
c) $0 \in M_s(o)$.
d) For any $m \in M_s(o)$ and for any $z \in Z_s$ that is on a path from $\sigma_s(o, m)$, $c_s(z) \leq m$.

Condition (a) says that each state game starts from a single node at which the entrepreneur moves. This rules out situations in which the investor moves first or the entrepreneur moves first but is unsure of his situation. Condition (b) requires that the entrepreneurs first move be a nonnegative number which will be interpreted as a deposit of cash into an escrow account. The rest of this condition ensures that the amount deposited is public information. Condition (c) says that the entrepreneur can always choose not to deposit any cash. Condition (d) ensures that the entrepreneur is never required to pay out more cash than he deposited in the escrow account. Conditions (c) and (d) together embody the assumption that the entrepreneur cannot be forced to pay. We refer to these two conditions as the *zero-payment guarantee.*

The reason that we require that each state-game start with a deposit by the entrepreneur into an escrow account is to ensure that, even when contracts are state-independent, the cash payments do not exceed the available cash.[20] In addition to this structure of the game, to guarantee feasible outcomes we restrict the entrepreneur's strategy so that he may not deposit more cash than is available in a given state. Formally, we define a *feasible strategy for state-game s for the* entrepreneur to be a mapping e_s, which associates with each node at which the entrepreneur moves, $n \in E_s$, a random variable $e_s(n)$, taking values in the set of possible moves $M_s(n)$ and satisfying

a) $e_s(n)$ is the same for all nodes in r,(n), and
b) $e_s(o) \leq R_{1s} + T$ with probability 1.

Condition (a) simply reflects the fact that the entrepreneur cannot distinguish among nodes in one of his information sets. Condition (b) is the formal statement that the entrepreneur cannot deposit more cash than he has.

A *feasible strategy for state-game s for the investor* is a mapping i_s, which associates with each node at which the investor moves, $n \in I_s$, a random variable, $i_s(n)$, taking values in the set of possible moves $M_s(n)$, which is the same for all nodes in $i_s(n)$.

3.2 State-independent contracts

We argued in the introduction that fully state-dependent financial contracts may not be feasible. We now propose some additional restrictions on the tree structure of the contract game to reflect a notion of state independence. What we have in mind is that the *tree structure* and payoff *rules* (as opposed to actual payoffs) of state-independent contracts not depend on the state. This does *not* mean that the moves actually made by a player at a given information set cannot depend on the state, nor does it imply that equilibrium outcomes do not depend on the state. On the contrary, it is the ability of equilibrium outcomes to depend on the state (through the equilibrium strategies) that allows contracts with games to improve on those without games when state dependence is limited. Formally, a contract is *state-independent* if state-game 1 is identical to state-game 2. This means that, for state-independent contracts, we can drop the subscript s from the sets Z_s,

[20] This is similar to the procedure adoped in the literature on implementation of competitive equilibrium [see Postlewaite (1979)]. We are grateful to Ehud Kalai and Matt Jackson for pointing us in this direction.

E_s, I_S, from the correspondences i_s and M_S, and from the functions σ_s, c_s and a_s.[21]

3.3 Contracts with and without games

One of our main points in this article is to argue that, in some situations, including games in the design of contracts can improve outcomes relative to contracts specified as nonstrategic allocation rules. To make this argument precise, we will need a definition of a contract without a game (i.e., a nonstrategic allocation rule). Intuitively, a true game situation with two players requires at least one move by each player. Thus, a contract that specifies state-games in which only one player moves in each state-game is a nonstrategic allocation rule. Formally, a *nonstrategic allocation rule* is a feasible contract in which, for each state s, either $E_s = \emptyset$ (i.e., there are no information sets at which the entrepreneur moves) or $I_s = \emptyset$ (i.e., there are no information sets at which the investor moves), or both. It is clear that the most general nonstrategic allocation rule in our environment is a contract in which, in each state, the entrepreneur chooses an amount of cash to pay, and the contract specifies how much of the assets are liquidated as a function of the state and the amount paid. We refer to this type of nonstrategic allocation rule as a *nonstrategic liquidation policy.* Of course, in the state-independent case, the liquidation outcome *rule* must be independent of the state.

3.4 State-dependent, creditor-favored, and debtor-favored debt are feasible contracts

These contracts were described verbally in Section 2. They are depicted in Figures 1, 2, and 3, respectively, and described formally in the Appendix. It is clear that these contracts satisfy our definition of a feasible contract. It is also clear that state-dependent debt is a (statedependent) nonstrategic liquidation rule (i.e., the investor has no moves, and the amount liquidated in each state is determined by the face value for that state and the amount the entrepreneur chooses to pay).[22] The other two contracts are state-independent and involve games as explained in the previous section.

[21] To ensure feasibility of the outcomes, we must restrict the entrepreneur's choice of an escrow deposit not to exceed cash available. Since the cash available depends on the state, this limited amount of state dependence is unavoidable. Nevertheless, we refer to contracts with no other state dependence as state-independent contracts.

[22] Enforcement of liquidation as called for by the liquidation rule is presumed to be carried out by some third party such as a sheriff or court.

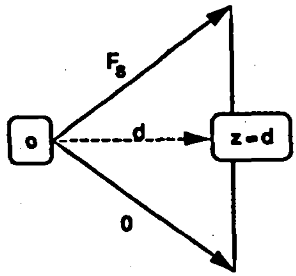

Figure 1
State-game tree for a stat-dependent debt contract *(F_1, F_2, T)*, State *s*
The entrepreneur has the only move at the initial node marked *o*. He may choose any deposit $d \in [0, F_s]$. This results in the final node marked $z = d$. The cash payment at *z* is simply *d*. Liquidation in state *s* at node *z* is $\min\{F_s - d, L_s\}$.

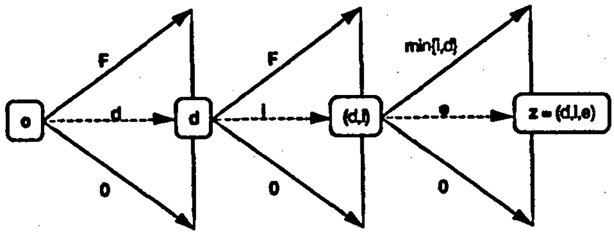

Figure 2
Game tree for a creditor-favored debt contract *(F, T)*
The entrepreneur moves at the initial node marked *o*. He may choose any deposit $d \in [0, F]$. The investor then moves at the resulting node, labeled *d*. He may choose any new face value $i \in [0, F]$. This results in the node marked *(d, i)* at which the entrepreneur moves. At node *(d, i)*, the entrepreneur can choose to pay any amount $e \in [0, \min\{i, d\}]$. This results in a final node $z = (d, i, e)$. The cash payment at *z* is simply *e*. Liquidation at *z* in state *s* is $\{i - e, L_s\}$.

3.5 Universal contract is a feasible, state-independent contract

As with all contracts, after realization of the state at date 1, first the entrepreneur makes a deposit in an escrow account. The contract can specify the set of feasible deposits, M(o), provided, of course, that *M(o)* satisfies the requirements of all feasible contracts (i.e., $0 \in M(o) \subset \Re_+$). The universal contract then requires the entrepreneur and investor to make simultaneous announcements of either 1 or 2. The cash payment and liquidation are determined as functions of the deposit and announcements. These can be any functions that satisfy the requirements of all feasible contracts. More carefully, a *universal*

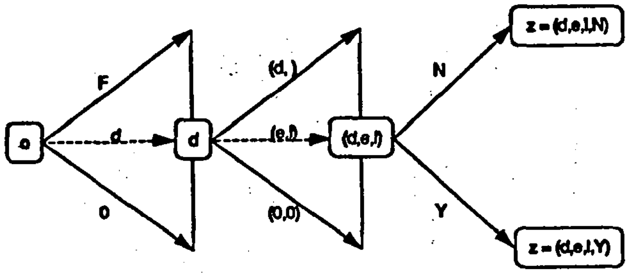

Figure 3
Game tree for a debtor-favored debt contract *(F, T)*
The entrepreneur moves at the initial node marked *o*. He may choose any deposit $d \in [0, F]$. The entrepreneur then moves again at the resulting node, labeled *d*, where he may choose any offer of cash and liquidation $l \leq 0$. This results in the node mark *(d, e, l)* at which the investor moves. He may choose a response $r \in \{Y, N\}$, resulting in final node $z = (d, e, l, r)$. If $r=Y$, the cash payment at *z* is *e*, and liquidation is $\min\{l, L_s\}$. If $r=N$, the cash payment at *z* is zero, and liquidation is $\min\{F, L_s\}$.

contract with excess transfer *T*, feasible deposits *M(o)*, and outcome functions *(c, a)* is defined in Figure 4 and formally in the Appendix.

Having defined feasible contracts and shown that our special contracts are indeed feasible, we now define equilibrium and optimality.

4. Equilibrium Strategies and Optimal Financial Contracts

Given a financial contract, the investor seeks to maximize his expected total receipts including liquidation proceeds, and the entrepreneur attempts to minimize his expected costs. Consequently, it will be convenient to define notation for expected cash payments and liquidation amounts as functions of the state and the underlying strategies that determine their distributions. For any state *s* and pair of strategies *(e, i)* for state-game *s*, let

$$\begin{aligned} C_s(e, i) &= E[c_s(\tilde{z}) \mid s, e, i], \\ A_s(e, i) &= E[\min\{a_s(\tilde{z}), L_s\} \mid s, e, i], \end{aligned} \tag{2}$$

where the expectation over final nodes, $\tilde{z}$, is with respect to the distribution induced by *(e, i)* in state-game *s*. Expected payments to the investor in state-game *s*, as a function of the strategies *(e, i)* for that state-game are then given by $C_s(e, i) + A_s(e, i)$. The entrepreneur's expected cost in state-game s, as a function of the strategies *(e, i)* for that state-game are given by $C_s(e, i) - T + \alpha_s A_s(e, i)$.

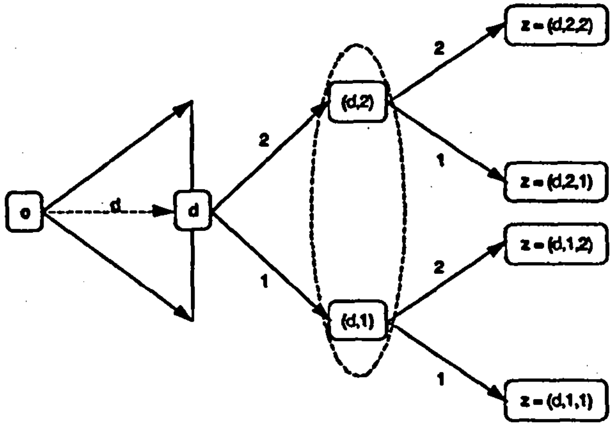

Figure 4
Game tree for a universal game ***(T,M(o),c,a)***
The entrepreneur moves at the initial node marked *o*. He may choose any deposit $d \in M(o)$. The entrepreneur moves again at the resulting node, labeled d. He may choose any $e \in S$, resulting in a node marked *(d,* 1) or *(d,* 2). These two nodes constitute an information set (shown by the broken line) at which the investor moves. At this information set. the investor can choose any $i \in S$. This results in a final node $z = (d, e, i)$ The cash payment at *(d,e,i)* is *e(d,e,i)*. Liquidation in state s at *(d,e,i)* is $\min\{a(d,e,i), L_s\}$.

Our definition of equilibrium is simply Nash equilibrium. Formally, an *equilibrium* for a feasible contract is a strategy profile (e_1, e_2, i_1, i_2) consisting of feasible strategies e_s and i_s for state-game s for the entrepreneur and investor, respectively, for each state s, such that

(a) i_s maximizes $C_s(e_s, i) + A_s(e_s, i)$ over all feasible strategies i for the investor, and
(b) e_s minimizes $C_s(e, i_s) - T + \alpha_s A_s(e, i_s)$ over all feasible strategies e for the entrepreneur.

We will generally abbreviate the expected outcome given *equilibrium* strategies as $C = (C_1, C_2)$ and $A = (A_1, A_2)$ and refer to this outcome as the *equilibrium expected outcome.*[23]

Since the entrepreneur designs the financing contract, he will choose a contract that results in minimal equilibrium expected cost to him among all feasible contracts that induce the investor to participate. Formally, the latter constraint is *investor participation:* the investor will invest $K + T$ under a contract with equilibrium expected

[23] If a contract has multiple equilibria, we assume that the one with the smallest expected cost to the entrepreneur obtains. If there is more than one such equilibrium, the particular selection is irrevelant. In this case, *(C, A)* can be taken to he any one of these equilibrium-expected outcomes.

outcome (C, A), if and only if

$$\sum_{s \in \{1,2\}} \frac{1}{2}[C_s + A_s] \geq K + T.$$

The investor participation condition guarantees that the expected payout is at least the market value of the contract. This condition reflects our assumptions that investors are risk-neutral and that competition among them ensures that capital will be supplied, provided the expected rate of return is at least the risk-free interest rate (zero in this case).

5. General Results

In this section we characterize optimal contracts within three nested classes of feasible contracts: all contracts, state-independent contracts, and creditor-favored debt contracts. First, we show that state-dependent debt is optimal in the class of all contracts. Second, we demonstrate that the universal contract is optimal in the class of state-independent contracts. Third, we show by example that, in the state-independent context, nonstrategic allocation rules (i.e., contracts without games) can be strictly inferior to the optimal contract. Fourth, we show that creditor-favored debt is, in general, suboptimal in the class of state-independent contracts.

Our approach in this section is to show that the problem of searching for an optimal *contract* can be transformed into one of searching for an optimal allocution. In particular, we first argue that any equilibrium expected outcome of any feasible financial contract in a given class must satisfy certain constraints. We then find the-allocation, among those that satisfy these constraints, that minimizes the entrepreneur's expected cost. Finally, we present a contract that implements this allocation.[24,25] This contract is then optimal in the given class of contracts.[26]

[24] An allocation (C, A) can be implemented by a contract with excess transfer T if, for some equilibrium of the contract, the equilibrium expected outcome is (C, A).

[25] These results are related to the revelation principle and to the literature on implementation in general; see Dasgupta, Hammond, and Maskin (1979), Myerson (1979), Harris and Townsend (1981), and Moore and Repullo (1988). Unfortunately, the general results of this literature cannot be used here because of the explicit restrictions we place on the set of feasible mechanisms.

[26] We make no claim that this contract is he only optimal contract.

5.1 Optimal (possibly state-dependent) contract: the second-best

In this subsection, we demonstrate that state-dependent debt is an optimal contract. First we establish some constraints that any equilibrium expected -outcome must satisfy.

Theorem 1 (Feasible outcomes). *Consider a feasible contract with excess transfer T and with equilibrium expected outcome (C, A). If the contract is acceptable to the investor, then (C, A) satisfies*

$$C_s + \alpha_s A_s \leq \alpha_s L_s = R_{2s} \text{ for } s \in \{1, 2\}, \qquad \text{(Zero-payment)}$$
$$\textstyle\sum_{s\in\{1,2\}} \frac{1}{2}[C_s + A_s] \geq K + T, \qquad \text{(Investor participation)}$$
$$C_s \in [0, R_{1s} + T], \;\; A_s \in [0, L_s], \;\; T \geq 0. \qquad \text{(Technological feasibility)}$$

Proof. That the equilibrium outcome satisfies the technological feasibility and investor participation constraints is obvious from the definition of feasible financial contracts, the definition of equilibrium, and the investor participation condition. The zero-payment constraint follows from the fact that, for either state s, the entrepreneur can choose a strategy, say ϕ_s, in which he deposits zero in the escrow account (in which case his cash payment is zero). For such a strategy, the' entrepreneur's expected cost in state s would be $A_s(\phi_s, i_s)\alpha_s = A_s(\phi_s, i_s)R_{2s}/L_s \leq R_{2s}$, since $A_s(\phi_s, i_s) \leq L_s$. Thus, the entrepreneur's cost in state s for his equilibrium strategy cannot exceed R_{2s} (i.e., the zero-payment constraint must hold). ■

The above characterization of feasible outcomes allows us to search for optimal allocations. A *feasible allocation* consists of an excess transfer, $T \geq 0$, cash payments to the investor, C_s, and liquidation proceeds, A_s, for $s \in S$, that satisfy the conditions of Theorem 1.[27] An *optimal allocation* is one that minimizes the entrepreneur's expected cost among all feasible allocations. Thus, an optimal allocation solves the *second-best problem,*

$$\min_{T,(C,A)} \sum_{s\in\{1,2\}} \frac{1}{2}[C_s - T + A_s\alpha_s],$$

subject to zero-payment, investor participation, and technological feasibility. Since the solution of this problem yields the smallest expected

27 We are abusing our notation slightly here by allowing C and A to stand for arbitrary allocations as well as equilibrium-expected outcomes. Which concept is appropriate will be clear from the context.

liquidation costs consistent with the assumed contracting restrictions, this solution is the second-best allocation.[28]

We know from Theorem 1 that it is impossible to find a financial contract whose equilibrium expected cost is below that of a second-best allocation. The only remaining question is whether such an allocation can be implemented by a feasible financial contract. We show in the next theorem that state-dependent debt can, in fact, implement any second-best allocation. This establishes that such a contract is an optimal contract.

Theorem 2 (Optimal contract). *Suppose that [T, (C, A)] is an optimal solution of the second-best problem. The state-dependent debt contract having excess transfer T and face values $F_s = C_s + A_s$ implements (C, A).*

The driving force for this result is that, because liquidation is a more costly form of payment than cash, the entrepreneur is willing to pay out in cash as much as is necessary to avoid liquidation unless this results in a cost to him that exceeds the cost under a zero-payment strategy. Consequently, the entrepreneur either pays out nothing or pays out all the cash available in that state, $R_{1s} + T$, or the face value, whichever is smaller. Since the face values are based on the second-best solution, it is always optimal for the entrepreneur to pay cash. The details of the proof are given in the Appendix.

Theorems 1 and 2 establish a method for finding an optimal contract. One simply solves the second-best problem (which is a linear program) and uses Theorem 2 to design an optimal contract. As shown in Theorem 2, this optimal contract is state-dependent debt.

5.2 Optimal state-independent contract: The third-best

In this subsection, we show that the universal contract is optimal in the class of state-independent contracts. As before, we first characterize the constraints that equilibrium expected outcomes of state-independent contracts must satisfy. This enables us to compute third-best allocations and design the universal contract needed to implement such allocations.

Theorem 3 (Feasible outcomes of state-independent contracts). *Suppose (C, A) is an equilibrium expected outcome of a feasible, state-independent contract $Q = (T, E, I, Z, \iota, M, \sigma, c^*, a^*)$ Then there exist $d_s \in M(o)$ for $s \in S$ and functions $c : M(o) \times S^2 \rightarrow \Re$ and*

[28] It can be shown that if there is an optimal solution to this problem, then there is one with $T = 0$. This implies that, when state dependence is allowed, raising funds in excess of those required for the invesment does not help reduce dead-weight liquidation costs.

$a : M(o) \times S^2 \to \Re$ *such that, for all* $s \in S$,

$$c(d_s, s, s) = C_s, \qquad a(d_s, s, s) = A_s, \tag{3}$$

$$0 \leq c(d, s, s') \leq d,\ 0 \leq a(d, s, s'),\ \textit{for all } d \in M(o) \textit{ and } s' \in S, \tag{4}$$

$$C_s + \alpha_s A_s \leq \min_{d \in M(o) \cap [0, R_{1s}+T], s' \in S} c(d, s', s) + \alpha_s \min\{a(d, s', s), L_s\} \tag{5}$$

$$C_s + A_s \geq \max_{s' \in S} c(d_s, s, s') + \min\{a(d_s, s, s'), L_s\}. \tag{6}$$

Proof. See the Appendix. ■

This result implies that, to find an optimal expected outcome of state-independent contracts that satisfies investor participation, we must solve the following *third-best problem*

$$\min_{T,(C,A),c(\cdot),a(\cdot),M(o),(d)} \frac{1}{2} \sum_{s \in S} (C_s - T + \alpha_s A_s) \tag{7}$$

subject to investor participation, technological feasibility, and, for all $s \in S$, Constraints (3) through (6), $0 \in M(o) \subset \Re_+$, $d_s \in M(o)$, and $d_s \leq R_{1s} + T$.

We now show that allocations that are feasible for the third-best problem can be implemented by a universal contract. In particular, then, a third-best allocation can be implemented by a universal contract. Thus, such contracts are optimal, state-independent contracts.

Theorem 4 (Optimal state-independent contract). *Suppose* $\{T, (C, A), c(\cdot), a(\cdot), M(o), (d)\}$ *solves the third-best problem. Then (C, A) can be implemented by a universal contract. Since its equilibrium expected outcome is third-best, this contract is an optimal state-independent contract.*[29]

Proof. Define the universal contract using the values of T, $c(\cdot)$, $a(\cdot)$, $M(o)$, and d_s from the solution of the third-best problem. Condition (5) guarantees that the entrepreneur prefers to deposit d_s in the escrow account and announce s in state s, given that the investor will announce s. Condition (6) guarantees that the investor prefers to announce s in state 5, given that the entrepreneur deposits d_s and announces s. This shows that there is an equilibrium of the universal contract in which, in state s, the entrepreneur deposits d_s and both he

[29] We actually show that any allocation that is feasible for the third-best problem can be implemented by it universal contract. Since any equilibrium expected outcome of any state-independent contract is feasible for the third-best problem (by Theorem 3), we have actually shown a version of the revelation principle for our environment. This justifies the term *universal.*

Table 3
An environment in which third-best is strictly inferior to second-best[a]

s	R_{1s}	R_{2s}	L_s	α_s
1	100	200	100	2
2	40	180	60	3

[a] The table shows, for each state, $s \in \{1, 2\}$, income at dates 1 and 2, R_{1s} and R_{2s}, respectively, liquidation value, L_s, and the liquidation cost-benefit ratio, α_s.

and the investor announce s. Obviously, in this equilibrium, condition (3) guarantees that (C, A) is the equilibrium outcome. The rest of the constraints of the third-best problem guarantee that the contract satisfies all the requirements of a universal contract. ■

Obviously, the third-best cannot be strictly superior to the second-best. We now show by example that requiring state-independence of contracts results in *strictly* lower efficiency in some environments. To show this, we first simplify the problem. It is shown in the Appendix (Lemma 2) that this problem can be rewritten as follows (assuming, without loss of generality, that $R_{11} \geq R_{12}$):

$$\min_{T,(C,A),c,a} \frac{1}{2} \sum_{s\in\{1,2\}} (C_s - T + \alpha_s A_s) \tag{8}$$

subject to zero payment, investor participation, technological feasibility, and

$$C_1 + \alpha_1 A_1 \leq c + \alpha_1 \min\{a, L_1\}, \tag{9}$$

$$C_2 + A_2 \geq c + \min\{a, L_2\}, \tag{10}$$

$$c \in [0, R_{12} + T], \quad a \geq 0.$$

The following example environment is now used to demonstrate that third-best can be strictly inferior to second-best. Let $K = 100$ and R and L be given by Table 3. The second-best solution is $C_1 = 100$, $A_1 = 50$, $C_2 = 40$, $A_2 = 10$, with minimum expected cost for the entrepreneur of 135.

For this particular example, since $L_2 < L_1$, the problem can be solved by solving two linear programs (one in which the constraint $a \leq L_2$ is added and one in which the constraint $L_2 \leq a \leq L_1$ is added) and then choosing the solution with the smallest expected cost to the entrepreneur. The result is $T = 6.67$, $C_1 = 106.67$, $A_1 = 46.67$, $C_2 = 46.67$, $A_2 = 13.33$, with minimum expected cost for the entrepreneur of 136.67 (as compared with 135 for the second-best).[30]

5.3 Games versus nonstrategic allocation rules

We turn now to the issue of whether, in fact, contracts that involve games can improve on the allocations achievable from nonstrategic allocation rules. Our main result is that games can improve allocations if and only if the state is not contractible. This result makes precise the intuition presented in the introduction that games can make equilibrium outcomes more state-dependent than can nonstrategic allocation rules.

When the state is contractible, the state-dependent debt contract is optimal. Since, as already noted, the state-dependent debt contract is a nonstrategic liquidation policy, we see that, when state dependence is allowed, contracts involving games are not required. In fact, an optimal contract involves a particularly simple nonstrategic liquidation rule in which liquidation is a fixed (state-dependent) amount minus cash paid.

When the state is not contractible, the universal contract is optimal. Clearly, this contract involves games (i.e., both entrepreneur and investor have at least one move). We now demonstrate that the universal contract (i.e., a contract that involves a game) can be strictly better than any nonstrategic allocation rule. Recall from Section 2 that, in the example environment of Section 1, the optimal state-independent contract achieves second-best. Therefore, any nonstrategic liquidation policy for this environment that is as good must achieve second-best. For this example, second-best involves liquidation of 10 in state 1 and 0 in state 2. Consequently, we need only show that there is no transfer $T \geq 0$, liquidation rule $l(c)$ as a function of the cash paid, c, and payments c_1 and c^2 such that

$$\min\{l(c_2), 50\} = 0, \tag{11}$$

$$c_1 + 20 \leq x + 2\min\{l(x), 100\}, \text{ for all } x \in [0, 100 + T], \tag{12}$$

$$\frac{1}{2}(c_1 + 10 + c_2) \geq 75 + T, \tag{13}$$

$$c_1 \in [0, 100 + T], \ c_2 \in [0, 40 + T]. \tag{14}$$

Equation (11) states that, when the entrepreneur pays C_2, liquidation must be the second-best amount for state 2 (i.e., 0) (recall $L_2 = 50$).

[30] Note that no debtor-favored debt contract satisfies investor participation for this environment. The best creditor-favored&b contract is given by $F = 172$, $T = 44$ (this will be shown below). The solutim is $C_1 = 144$, $A_1 = 28$, $C_2 = 84$, $A_2 = 32$ (the investor forgives to 116 in state 2), with expected cost of 146.

Equation (12) guarantees that the entrepreneur pays c_1 in state 1. Equations (13) and (14) are the investor participation and technological feasibility constraints in this case. Equation (11) implies that $l(c_2) = 0$. Then Equation (12) evaluated at $x = c_2$ implies that $c_1 + 20 \leq c_2$. This, together with Equation (13) implies $c_2 \geq 80 + T$, which contradicts Equation (14). This shows that there is no transfer and liquidation rule that does as well as the optimal contract for this example.

5.4 Creditor-favored debt is suboptimal

Recall that in Sections 2 and 5.2 we presented examples in which creditor-favored debt is inferior to debtor-favored debt and the universal contract. To claim that these examples demonstrate that creditor-favored debt is strictly dominated, we need only show that the particular creditor-favored debt contracts presented there are, in fact, the optimal creditor-favored debt contracts for those environments. This is established in the Appendix.

6. Conclusion

This article addresses the issue of the use of games in security design. In particular, we argue that, in situations in which the dependence of contracts on ex ante unpredictable outcomes is limited, contracts' designed as games can improve the allocation of resources relative to those specified as nonstrategic allocation rules.

We focus on the Hart-Moore (1989) environment in which only the, threat of liquidation can induce entrepreneurs to pay out income to investors. In this environment, we characterize optimal contracts in two nested classes: all contracts (including those that depend on the state of nature) and state-independent contracts. In the class of all contracts, we show that a nonstrategic allocation rule that can be described as debt with state-dependent face value is optimal. The universal contract is shown to be optimal in the class of state-independent contracts. This contract specifies cash payments and liquidation amounts that depend on the amount the entrepreneur deposits in an escrow account and the states declared by both the entrepreneur and investor. The best state-independent contract can be strictly inferior to the best state-dependent contract. Moreover, the best state-independent contract involves a game in which both players act and can be strictly superior to nonstrategic allocation rules that do not involve games. Finally, the best creditor-favored debt contract is shown to be strictly inferior to other state-independent contracts in some cases.

The model of this article addressed the issue of security design in a situation in which the only problem is one of excessive liquidation in a two-period setting. This is an important first step in understanding the role of games in the design of securities. Our contribution must be viewed as methodological as opposed to normative or positive, however, since one cannot claim to have a complete theory without addressing several other important issues. These issues include the problem of longer-term relationships between entrepreneurs and investors, distorted investment incentives, and multiple investors.

The possibility of multiperiod relationships between entrepreneurs and investors introduces the possibility that optimal financing arrangements will involve contracts with various maturities. If this is, in fact, the case, then our approach could result in a characterization of optimal maturity and seniority provisions.[31] This will lead to an evaluation of the desirability of acceleration of debt maturity, crossdefault features, and seniority in bankruptcy. Thus, one might address such questions as whether payments originally scheduled for future periods should be made before payments immediately due, how this depends on the original seniority structure, whether the courts should intervene to modify contractual seniority provisions, and whether new investments should be given seniority.

Since our model takes the investment decision as exogenous, optimal contracts derived from this model do not address the problem of distorted investment incentives caused by the financial contract. An extension of our approach of searching for optimal mechanisms to include an investment decision would allow one to design contracts that induce correct investment incentives.

When many investors contribute capital to the firm, procedures to be followed in default must address problems not present when there is a single outside investor. In particular, the procedures must specify how the investors participate in decisions regarding reorganization. For example, current procedures require unanimity of debtholders to change the provisions of a debt contract outside of bankruptcy. In Chapter 11 bankruptcy proceedings, however, smaller majorities (by number of bondholders and by value of their holdings) are required. There are two problems that such rules must address, free-rider problems and debt-holder heterogeneity. Heterogeneity could result from differences in beliefs about the value of the assets if the firm continues to operate or if it is liquidated as well as from differences in risk preferences. This heterogeneity could also lead investors to hold different securities. In this case, the problems associated with multiple investors overlap those associated with maturity and seniority

[31] Several papers have addressed the issue of maturity of debt contracts. One of the first was Myers (1977). More recent contributions include Diamond (1991, 1993) and Han and Moore (1991).

mentioned above. Again, these issues can be addressed using our approach of designing optimal financial contracts including negotiation games.

Appendix

Formal description of state-dependent debt (see Figure 1)

$$\begin{aligned}
E_s &= \{o\},\ I_s = \emptyset, \iota_s(o) = \{o\},\ M_s(o) = [0, F_s],\\
\sigma_s(o, d) &= d \in Z = [0, F_s] \qquad (\text{for } d \in M_s(o)),\\
c_s(d) &= d,\ a_s(d) = F_s - d \qquad (\text{for } d \in Z).
\end{aligned}$$

Formal description of creditor-favored debt (see Figure 2)

$$\begin{aligned}
E &= \{o\} \cup [0, F]^2,\ \iota(o) = \{o\},\ M(o) = [0, F],\\
\sigma(o, d) &= d \in I = [0, F] \text{ (for } d \in M(o)),\ \iota(d) = \{d\},\ M(d) = [0, F]\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (\text{for } d \in I),\\
\sigma(d, i) &= (d, i) \in [0, F]^2 \text{ (for } d \in M(o),\ i \in M(d)),\\
\iota(d, i) &= \{(d, i)\},\ M(d, i) = [0, \min\{d, i\}]\\
&\quad (\text{for } (d, i) \in [0, F]^2),\\
\sigma(d, i, e) &= (d, i, e) \in Z, \text{ where}\\
Z &= \{(x, y, z) \mid x, y \in [0, F]^2, z \in [0, \min\{x, y\}]\}\\
&\qquad\qquad\qquad (\text{for } d \in M(o),\ i \in M(d),\ e \in M(d, i)),\\
c(d, i, e) &= e,\ a(d, i, e) = i - d \text{ (for } (d, i, e) \in Z).
\end{aligned}$$

Since the contract is state-independent, we have omitted the *s* subscripts in the notation.

We now argue that the addition of the initial, deposit stage is innocuous. That is, it is always an equilibrium for the entrepreneur to deposit all his cash, R_{1s} + T, in the escrow account. Since the entrepreneur need not pay all the cash in the escrow account at the last stage, the only reason for him not to deposit his entire date 1 cash would be to induce greater forgiveness by the investor. In cases in which depositing less cash results in greater forgiveness, the investor chooses his forgiveness so that the cost to the entrepreneur is exactly the same if he pays all cash on deposit or if he pays zero. Thus, the entrepreneur's cost will be the same in all such cases, independent of his deposit. Consequently, the entrepreneur is at least as well off (and the investor is better off) if he deposits all available cash.[32]

[32] A formal proof is not presented here because the result is not central to the article.

Formal description of debtor-favored debt (see Figure 3)

$$\begin{aligned}
E &= \{o\} \cup [0, F], \ \iota(o) = \{o\}, \ M(o) = [0, F], \\
\sigma(o, d) &= d, \ \iota(d) = \{d\}, \ M(d) = [0, d] \times \Re_+ \ (\text{for } d \in M(o)), \\
\sigma(d, e, l) &= (d, e, l) \in I, \ \text{where} \\
I &= \{(x, y, z) \mid x \in [0, F], \ y \in [0, x], z \geq 0\}, \\
\iota(d, e, l) &= \{(d, e, l)\}, \\
M(d, e, l) &= \{Y, N\} \ (\text{for } (d, e, l) \in I), \\
\sigma(d, e, l, r) &= (d, e, l, r) \in Z = I \times \{Y, N\}, \\
c(d, e, l, Y) &= e, \ a(d, e, l, Y) = l, \ c(d, e, l, N) = 0, \\
a(d, e, l, N) &= \infty \ (\text{for } (d, e, l, r) \in Z).
\end{aligned}$$

Since the contract is state-independent, we have omitted the s subscripts in the notation.

Formal description of universal contract (see Figure 4)

$$\begin{aligned}
E &= \{o\} \cup M(o), \ \iota(o) = \{o\}, \\
\sigma(o, d) &= d, \ \iota(d) = \{d\}, M(d) = S \qquad (\text{for } d \in M(o)), \\
\sigma(d, e) &= (d, e) \in I = M(o) \times S, \ \iota(d, e) = \{d\} \times S, \ M(d, e) = S \\
& \qquad (\text{for } (d, e) \in M(o) \times S), \\
\sigma(d, e, i) &= (d, e, i) \in Z = M(o) \times S^2, \\
c(d, e, i) &\in [0, d], \ a(d, e, i) \geq 0.
\end{aligned}$$

Note that the contract is feasible, state-independent (consequently we omitted the s subscripts in defining the state-game); and involves a game.

Lemma 1. *For any optimal solution* $\{T, (C, A)\}$ *of the second-best problem and for any state s, if liquidation in state s is positive, then all cash available in state s is paid out (i.e., if* $A_s > 0$*, then* $C_s = R_{1s} + T$*).*

Proof. Suppose (T, C_s, A_s) is an optimal solution with $C_s < R_{1s} + T$ and $A_s > 0$ for some state s. Then we can choose $\delta > 0$ such that $C_s + \delta < R_{1s} + T$ and $A_s - \delta > 0$. Using Equation (1), it is clear that the solution $(T, C_s + \delta, A_s - \delta)$ is feasible and has lower expected cost for the entrepreneur. ■

Theorem 2 (Optimal contract). *Suppose* $\{T, (C, A)\}$ *is an optimal solution of the second-best problem. The state-dependent debt contract having excess transfer T and face values* $F_s = C_s + A_s$ *implements (C, A).*

Proof. First note that, because of inequality (1), the entrepreneur is willing to pay out In cash as much as is necessary to avoid liquidation unless this results in a cost to him that exceeds the cost of paying zero. That is, if an extra dollar of cash payout does not reduce liquidation (because the payout. is still so small that, even with full liquidation, the investor does not receive the full face value in the given state), then the entrepreneur will pay zero. If an extra dollar of cash payout does reduce liquidation by one dollar, thus saving the entrepreneur $a_s > 1$ dollars, he will pay the dollar. Consequently, the entrepreneur either pays out nothing or pays out all the cash available in that state, $R_{1s} + T$, or the face value, whichever is smaller.

Consider state s and recall that $F_s = C_s + A_s$. Also, since $\{T, (C, A)\}$ solves the second-best problem, (C_s, A_s) satisfies the zero-payment constraint. We argue that the entrepreneur will choose to pay C_s for each s. First, suppose that $A_s = 0$. Then $F_s = C_s$, *so* by technological feasibility $F_s \leq R_{1s} + T$, and, by the zero-payment constraint, $C_s \leq R_{2s}$. As argued above, the entrepreneur will either pay out $F_s = C_s$ (since F_s does not exceed the available cash in this state) or pay nothing. If he pays C_s, then his cost is C_s. If he pays zero, either all assets are liquidated (if $F_s > L_s$) or assets with liquidation value of F_s are liquidated (if $F_s \leq L_s$). In the former case, the entrepreneur's cost is R_{2s}, while in the latter it is $F_s\alpha_s = C_s\alpha_s > C_s$. In either case, cost is greater if the entrepreneur pays zero than if he pays C_s. Therefore, if $A_s = 0$, the entrepreneur will pay C_s. Since this is full face value in this case, there is no liquidation. Thus, we have shown that if $A_s = 0$, the given debt contract implements the solution in state s. Now suppose $A_s > 0$. Lemma 1 then implies that $C_s = R_{1s} + T$. Thus $F_s = C_s + A_s > R_{1s} + T$. Again using the argument above, the entrepreneur will pay either $R_{1s} + T = C_s$ or zero. If he pays $R_{1s} + T$, liquidation is $F_s - C_s = A_s$, and the entrepreneur's cost is $C_s + \alpha_s A_s \leq R_{2s}$, by the zero-payment constraint. If the entrepreneur pays zero, either all assets are liquidated (if $F_s > L_s$) or assets with liquidation value of F_S are liquidated (if $F_s \leq L_s$). In the former case, the entrepreneur's cost is R_{2s}, while in the latter, it is $F_s\alpha_s = (C_s + A_s)\alpha_s > C_s + \alpha_s A_s$. In either case, cost is greater if the entrepreneur pays zero than if he pays C_s. Therefore, the entrepreneur will pay C_s and assets with liquidation value $F_s - C_s = A_s$ will be liquidated. This proves that the given debt contract implements the solution in state s. ∎

Theorem 3 (Feasible outcomes of state-independent contracts). *Suppose (C, A) is an equilibrium expected outcome of a feasible, state-independent contract* $Q = (T, E, I, Z, \iota, M, \sigma, c^*, a^*)$. *Then there exist* $d_s \in M(o)$ *for* $s \in S$ *and functions* $c : M(o) \times S^2 \to \Re$ *and* $a : M(o) \times S^2 \to \Re$ *such that, for all* $s \in S$, *constraints (3) through (6) hold.*

Proof. Let $(e, i) = (e_s, i_s)_{s\in S}$ be an equilibrium strategy profile of Q. For any $d \in M(o)$ and $s \in S$, define a strategy e_{sd} for the entrepreneur in which he deposits d at node o (i.e., $e_{sd}(o) = d$ for sure), then follows e_s in the subgame starting at $\sigma(o, d)$. Note that $e_{s'd}$ is feasible in state s if and only if $d \in M(o)$ and $d \le R_{1s} + T$. Let $\tilde{z}(d, s, s')$, for $d \in M(o)$ and $s, s' \in S$, represent the distribution over final nodes induced by the strategy profile $(e_{sd}, i_{s'})$. Finally, let $\tilde{z}(s, s')$, for $s, s' \in S$, represent the distribution induced on Z by the profile $(e_s, i_{s'})$, and let $\tilde{d}(s)$ represent the distribution induced on M(o) by e_s.

It follows that, for all $s \in S$,

$$Ec^*(\tilde{z}(s, s)) = C_s \text{ and } E[\min\{a^*(\tilde{z}(s, s)), L_s\}] = A_s, \tag{A1}$$

$$\begin{aligned} C_s + \alpha_s A_s \le \min_{d\in M(o)\cap[0, R_{1s}+T], s'\in S} & E[c^*(\tilde{z}(d, s', s)) \\ & + \alpha_s \min\{a^*(\tilde{z}(d, s', s)), L_s\}], \end{aligned} \tag{A2}$$

$$C_s + \alpha_s A_s \le \min_{s'\in S} E[c^*(\tilde{z}(s', s)) + \alpha_s \min\{a^*(\tilde{z}(s', s)), L_s\}], \tag{A3}$$

if $e_{s'}$ is feasible in state s, and

$$C_s + A_s \ge \max_{s'\in S} E[c^*(\tilde{z}(s, s') + \min\{a^*(\tilde{z}(s, s'), L_s\}]. \tag{A4}$$

Now let d_s be the largest element in the support of $\tilde{d}(s)$, and for $s, s' \in S$ and $d \in M(o)$, let

$$c(d, s, s') = \begin{cases} E[c^*(\tilde{z}(s, s'))] & \text{for } d = d_s, \\ E[c^*(\tilde{z}(d, s, s'))] & \text{otherwise;} \end{cases} \tag{A5}$$

$$a(d, s, s') = \begin{cases} E[a^*(\tilde{z}(s, s'))] & \text{for } d = d_s, \\ E[a^*(\tilde{z}(d, s, s'))] & \text{otherwise.} \end{cases} \tag{A6}$$

It is obvious that c and a satisfy condition (3). Also c satisfies condition (4) since $c^*(\tilde{z}(d, s, s')) \le d$ almost surely for all $d \in M(o)$ and $E[c^*(\tilde{z}(s, s'))]$ is a weighted average of $E[c^*(\tilde{z}(d, s, s'))]$ over d in the support of $\tilde{d}(s)$, so $E[c^*(\tilde{z}(s, s'))] \le \max\{E[c^*(\tilde{z}(d, s, s'))] \mid d \in$ support of $\tilde{d}(s)\} \le d_s$.

Next, we show that condition (5) holds if $d \neq d_{s'}$. In this case, condition (5) follows from Equations (A5), (A6), and (A2). For $d = d_{s'} \le R_{1s} + T$, $e(s')$ is feasible in state s, so condition (5) follows from Equations (A5), (A6) and (A3) Finally, condition (6) follows from Equations (A5), (A6), and (A4). ■

Lemma 2. The third-best problem can be simplified as indicated in the text,

Proof. First, from condition (5), we see that removing elements from the set $M(o)$ eliminates constraints from the problem. Therefore, any solution involves including in $M(o)$ only the three required elements 0, d_1, and d_2. Second, from conditions (4) and (5), it is clear that increasing d_s relaxes constraints, so any solution involves $d_s = R_{1s} + T$. Therefore, we may assume $M(o) = \{0, R_{11} + T, R_{12} + T\}$. Third, by increasing $a(0, s', s)$ we can relax condition (5) for $d = 0$. Of course, increasing $a(0, s', s)$ above L_s does not further relax the constraint, so we may assume $a(0, s', s) = L_s$. Fourth, $c(0, s', s) = 0$ by (4).

Using these results, writing c_{rst} and a_{rst} for $c(d_r, s, t)$ and $a(d_r, s, t)$, respectively, expanding the constraints, and assuming, without loss of generality, that $R_{11} \geq R_{12}$, we can rewrite the problem as follows:

$$\min_{T,(C,A),(c_{rst}, a_{rst})_{(r,s,t)\in \bar{S}}} \frac{1}{2} \sum_{s \in S} (C_s - T + \alpha_s A_s) \tag{A7}$$

subject to investor participation, technological feasibility, and

$$c_{rst} \in [0, R_{1r} + T], \quad a_{rst} \geq 0, \quad \text{for all } (r, s, t) \in \bar{S}, \tag{A8}$$

$$C_s + \alpha_s A_s \leq \alpha_s L_s, \tag{A9}$$

$$C_1 + \alpha_1 A_1 \leq c_{121} + \alpha_1 \min\{a_{121}, L_1\}, \tag{A10}$$

$$C_1 + \alpha_1 A_1 \leq c_{221} + \alpha_1 \min\{a_{221}, L_1\}, \tag{A11}$$

$$C_1 + \alpha_1 A_1 \leq c_{211} + \alpha_1 \min\{a_{211}, L_1\}, \tag{A12}$$

$$C_2 + \alpha_2 A_2 \leq c_{212} + \alpha_2 \min\{a_{212}, L_2\}, \tag{A13}$$

$$C_1 + A_1 \geq c_{112} + \min\{a_{112}, L_1\}, \tag{A14}$$

$$C_2 + A_2 \geq c_{221} + \min\{a_{221}, L_2\}, \tag{A15}$$

where $\bar{S} = \{(r, s, t) \in S^3 \mid r, s, \text{ and } t \text{ are not all the same}\}$.

It is clear that we can relax constraints by setting $c_{121} = R_{11} + T$, $c_{211} = R_{12} + T$, $c_{212} = R_{12} + T$, $c_{112} = 0$, $a_{121} = L_1$, $a_{211} = L_1$, $a_{212} = L_2$, and $a_{112} = 0$. Substituting these values into the constraints, we find that Equation (A9) implies (A10); that Equations (A13), (A11), and (A8) imply (A12); and that Equation (A8) implies (A14). Eliminating these redundant constraints and letting $c = c_{221}$ and $a = a_{221}$ results in the problem in the text. ■

Optimal creditor-favored debt

We now characterize the optimal creditor-favored debt contract. This characterization supports our claim in Sections 2 and 5.2 that the creditor-favored debt examples considered there are the best such contracts for their respective environments.

The first step in finding an optimal creditor-favored debt contract is to characterize the set of allocations that can be implemented by such contracts. Obviously, these allocations must satisfy all the constraints of the second-best problem. In addition, the investor will extract as much as he can from the entrepreneur. When face value does not constrain the investor, then he will force the entrepreneur to his reservation cost R_{2s}. That is, the investor will force the zero-payment constraint to be binding in any state in which the face value is not binding, namely, in the default state. We refer to the second-best problem with the additional requirement that the zero-payment constraint be binding in any default state as the *fourth-best problem.*

Theorem* 5. *(Feasible outcomes of creditor-favored debt contracts). *Consider a feasible creditor-favored debt contract with excess transfer T. Suppose (C, A) is an equilibrium outcomefor this contract. Then (T, (C, A)] satisfies all the constraints of the fourth-best problem.*

Proof: Since the creditor-favored debt contract is feasible, by Theorem 1, *{T, (C, A)}* must satisfy all the constraints of the second-best problem. Our strategy is to use Lemma 1 of Hart and Moore (1989) to characterize the equilibrium allocation of the creditor-favored debt contract. Define

$$P_s(T) = \min\left\{R_{2s}, R_{1s} + T + \left[1 - \frac{R_{1s} + T}{R_{2s}}\right] L_s\right\}.$$

If $F \leq P_s(T)$, then, by Lemma 1 of Hart and Moore (1989), either $C_s = F$ and $A_s = 0$ or $C_s = R_{1s} + T$ and $A_s = F - C_s$. In either case, $C_s + A_s = F$. Therefore, if s is a default state, then $F > P_s(T)$. Consequently, by Lemma 1 of Hart and Moore (1989),

$$\frac{A_s}{L_s} = 1 - \frac{C_s}{R_{2s}}, \text{ or } C_s + \alpha_s A_s = R_{2s},$$

(i.e., the zero-payment constraint is binding). ■

Before proving a counterpart of Theorem 2, we must extend Lemma 1 to apply to the fourth-best problem.

Lemma 1'. *For any optimal solution $\{T, (C, A)\}$ of the fourth-best problem and for any state s, if liquidation in states is positive, then all cash available in states is paid out (i.e., if $A_s > 0$, then $C_s = R_{1s} + T$).*

Proof. Suppose $\{T, (C, A)\}$ is an optimal solution of the fourth-best problem. If $C_1 + A_1 = C_2 + A_2$, then the solution of the restricted problem also solves the unrestricted problem. In this case, Lemma 1 applies. Consequently, assume, without loss of generality, that $C_1 + A_1 > C_2 + A_2$, so the zero-payment constraint is binding in state 2. Therefore, the proof of Lemma 1 applies to state 1, so we consider only state 2 here. Suppose $C_2 < R_{12} + T$ and $A_2 > 0$. Since $C_1 + A_1 > C_2 + A_2 \geq 0$, at least one of C_1 and A_1 is positive.

Case 1: $A_1 > 0$. For any δ, let

$$C_1^\delta = C_1, \ C_2^\delta = C_2 + \delta, \ A_1^\delta = A_1 - \delta\left(1 - \frac{L_2}{R_{22}}\right), \ A_2^\delta = A_2 - \delta\frac{L_2}{R_{22}}.$$

Since $C_2 < R_{12} + T$, $A_1 > 0$, $A_2 > 0$, and $L_2 < R_{22}$, we can choose $\delta > 0$ such that

$$C_2 \leq C_2^\delta \leq R_{12} + T, \ A_1 > A_1^\delta \geq 0, \ A_2 > A_2^\delta \geq 0, \text{ and}$$

$$2\delta\left(1 - \frac{L_2}{R_{22}}\right) < C_1 + A_1 - (C_2 + A_2).$$

Then

$$\begin{aligned} C_1^\delta + A_1^\delta + C_2^\delta + A_2^\delta &= C_1 + A_1 + C_2 + A_2, \\ C_1^\delta + \alpha_1 A_1^\delta &< C_1 + \alpha_1 A_1, \\ C_2^\delta + \alpha_2 A_2^\delta &= C_2 + \alpha_2 A_2, \\ C_1^\delta + A_1^\delta &> C_2^\delta + A_2^\delta. \end{aligned}$$

Thus $\{T, (C^\delta, A^\delta)\}$ is feasible for the restricted problem and has a lower expected cost to the entrepreneur, contradicting the optimality of $\{T, (C, A)\}$.

Case 2: $A_1 = 0$. In this case $C_1 > 0$. Let C_2^δ and A_2^δ be the same as in Case 1, let $A_1^\delta = A_1 = 0$, and let

$$C_1^\delta = C_1 - \delta\left(1 - \frac{L_2}{R_{22}}\right).$$

It is easy to check that $\delta > 0$ can be chosen so that $\{T, (C^\delta, A^\delta)\}$ is feasible and has a lower expected cost to the entrepreneur as in Case 1. Again this contradicts the optimality of $\{T, (C, A)\}$. ■

Theorem 6 (Optimal creditor-favored debt contract). *Suppose $\{T, (C, A)\}$ is an optimal solution of the fourth-best problem. Without loss of generality, we can assume that $C_1 + A_1 \geq C_2 + A_2$ (otherwise, simply rename the states). Consider a creditor-favored debt contract with excess transfer T and face value $F = C_1 + A_1$. Then this contract implements $\{T, (C, A)\}$.*

Proof. Let $P_s(T)$ be as defined in the proof of Theorem 5. Let $(\hat{C}, \hat{A})$ be an equilibrium outcome of the debt contract defined in the statement of the theorem. Consider a given state *s*. There are two cases:

Case 1: $F > P_s(T)$. We claim that, in this case $C_s + A_s < F$, for suppose not (i.e., $C_s + A_s = F$). Then, since $F > P_s(T) \geq R_{2s}$, $C_s + A_s > R_{2s}$. This violates the zero-payment constraint for state s, which contradicts feasibility of (T, C_s, A_s) for the restricted problem. This establishes the claim that $C_s + A_s < F$. Consequently, we may conclude the zero-payment constraint is binding in state *s*.

For $F > P_s(T)$, Lemma 1 of Hart and Moore (1989) implies that

$$\hat{C}_s = \min\{R_{2s}, R_{1s} + T\}$$

and

$$\frac{\hat{A}_s}{L_s} = 1 - \min\left\{1, \frac{R_{1s} + T}{R_{2s}}\right\}.$$

First, suppose $R_{2s} < R_{1s} + T$ so that $\hat{C}_s = R_{2s}$ and $\hat{A}_s = 0$ From the zero-payment constraint, $C_s \leq R_{2s}$, so by Lemma 1'. $A_s = 0 = \hat{A}_s$. Since the zero-payment constraint is binding, this implies that $C_s = R_{2s} = \hat{C}_s$. Now suppose $R_{2s} \geq R_{1s} + T$ so that $\hat{C}_s = R_{1s} + T$ and $\hat{A}_s / L_s = 1 - \hat{C}_s / R_{2s}$. Again, since the zero-payment constraint is binding, $C_s = R_{2s}(1 - A_s/L_s)$. If $C_s < R_{1s} + T$, then by Lemma 1', $A_s = 0$, so $C_s = R_{2s} \geq R_{1s} + T$ which is a contradiction. Thus $C_s = R_{1s} + T = \hat{C}_s$ It follows from $C_s = R_{2s}(1 - A_s/L_s)$ that $A_s = \hat{A}_s$. This concludes the proof for Case 1.

Case 2: $F \leq P_s(T)$. First, suppose that $F \leq R_{1s} + T$. Then, by Lemma 1 of Hart and Moore (1989), $\hat{C}_s = F$ and $\hat{A}_s = 0$. Suppose $A_s > 0$. Then by Lemma 1', $C_s = R_{1s} + T \geq F$ which implies that $C_s + A_s > F$. But, this contradicts the definition of *F*. Therefore $A_s = 0 = \hat{A}_s$. If $C_s < F$, then the zero-payment constraint must be binding, so $C_s = R_{2s}$ (since $A_s = 0$). Consequently $R_{2s} < F \leq P_s(T) \leq R_{2s}$, which is a contradiction. Thus $C_s = F = \hat{C}_s$.

Now suppose that $F > R_{1s} + T$. By Han and Moore (1989, Lemma 1). $\hat{C}_s = R_{1s} + T$ and $\hat{A}_s = F - R_{1s} - T$. If $C_s + A_s = F$, then $A_s > 0$ (since

$C_s \leq R_{1s} + T$). Therefore, by Lemma 1', $C_s = R_{1s} + T = \hat{C}_s$, hence $A_s = F - C_s = \hat{A}_s$. If $C_s + A_s < F$, then the zero-payment constraint is biding, so $C_s + \alpha_s A_s = R_{2s} \geq P_s(T) \geq F > R_{1s} + T$. This implies that $A_s > 0$ and, by Lemma 1', that $C_s = R_{1s} + T$. Again using the fact that the zero-payment constraint is binding,

$$A_s = \left[1 - \frac{R_{1s} + T}{R_{2s}}\right] L_s.$$

Also, from the zero-payment constraint,

$$R_{2s} \geq C_s + A_s = R_{1s} + T + \left[1 - \frac{R_{1s} + T}{R_{2s}}\right] L_s.$$

Therefore, $P_s(T) = C_s + A_s < F$ contradicting the premise of this case. Consequently, we cannot have $C_s + A_s < F$. ■

Theorems 5 and 6 establish a method for finding an optimal creditor-favored debt contract (i.e., solve the fourth-best problem and use the contract described in Theorem 6). Indeed, this is how the debt contracts for the examples given above were constructed.

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