# Financial and Industrial Structure with Agency

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*A subgame perfect Nash equilibrium is characterized for an industry with dissipative costs of agency. In sequence, firms can enter the industry, raise capital with external debt and/or equity, invest in a capital-intensive technology or dissipate capital in perquisites, and finally produce output. For plausible values of two critical parameters, some firms forego in equilibrium investments with positive net present values. Although more managers would like their firms to invest in the capital-intensive technology, they cannot raise the required cash in the capital market. In equilibrium, the industry can have both a profitable core of large, secure, capital-intensive firms, with some debt but no unique optimal capital structure, and a competitive fringe of small, risky, labor-intensive firms. Even as the cost of entry converges to zero, capital-intensive firms can earn extraordinary profits, while all labor-intensive firms fail. With costly agency, access to capital can become a barrier to entry.*

The relationship between investment, risk, prerequisites, and capital structure has been studied extensively in corporate finance.[1] In many models, man-

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[1] The original contributions by Jensen and Meckling (1976) and Myers (1977) have been most influential. Subsequent contributions include Haugen and Senbet (1981), Green (1984), Titman (1984), Green and Talmor (1986), Williams (1987), and Kim and Maksimovic (1990). Other papers are cited in Barnea, Haugen, and Senbet (1985).

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agers maximize the market value of their stock plus their utility from perks. Depending on their firm's capital structure, these managers may forego profitable investments, pick excessively risky projects, and consume dissipative perks. Anticipating agency problems, managers then select capital structures with optimal combinations of external debt and equity. With few exceptions, these now standard results have been derived for a single firm in isolation rather than an industry in equilibrium. As illustrated by the exceptions, issues of industrial organization can be critical. In Brander and Lewis (1986), for example, managers who maximize the market value of their stock optimally sell no corporate debt to outsiders and thereby avoid all agency costs in both monopolistic and perfectly competitive industries. With imperfect competition, however, the same managers sell dissipative debt in equilibrium because it effectively precommits their firms to produce more output and thereby induces rivals to produce less output.[2] In other words, industrial structure can affect corporate conduct, including both the initial acquisition of capital and the subsequent production of output. Thereby, it can affect investment, risk, perks, and optimal capital structures.

The contrast between the now standard stories of corporate agency and the emerging literature on financial and industrial structure is most striking in Maksimovic and Zechner (1991), hereafter called MZ. In MZ each firm can invest in either a technology with a known marginal cost or another with an uncertain cost. All capital is raised with external debt, and all managers are assumed to maximize the market value of their internal equity. For a firm operating in isolation without interaction in an industry, standard results hold. Given no taxes, the firm sells no risky debt to outside investors because its manager *then* picks the risky project and thereby decreases its market value. However, when firms compete in a product market, capital structure becomes irrelevant. In their presumably perfectly competitive product market, the risky and riskless technologies are adopted by sufficient numbers of firms so that the two net present values are equal in equilibrium. Although risky debt still induces managers to pick the risky project, the choice has no affect on a firm's market value. A similar result holds with corporate taxes and tax-deductible interest expenses. Also, risk is endogenous. As more firms adopt the risky technology, the aggregate output and thereby the price of output more closely reflect the marginal cost of production. Thereby, the risky technology becomes

[2] Capital structures are also affected by the strategic behavior of firms in the agency models of Allen (1986) and Maksimovic (1988).

a better hedge against the uncertain cost of production. If the risky technology is selected by sufficiently many firms in equilibrium, then it is less risky than the riskless technology.

Despite this recent progress, theoretical corporate finance has yet to explain the limited empirical evidence on the determinants of capital structure. Empirically, capital structures vary considerably across firms, even within the same industry.[3] Also, capital structures are related to capital intensity. Specifically, capital intensity seems to be positively correlated with book values of debt but nearly uncorrelated with ratios of book debt to value. In the literature on industrial organization, the following empirical observations have been labeled "stylized facts."[4] Capital intensity is correlated positively with concentration. In turn, concentration is correlated positively with accounting profitability--at least for industrial leaders in American manufacturing, but not for firms with small market shares. Also, accounting profitability is correlated positively across industries with measures of both market share and capital requirements or economies of scale. Finally, firm size seems to be negatively related to intertemporal and cross-sectional variability of profits.

The theoretical results in this article are consistent with this empirical evidence. In this model each firm within an industry can produce a homogeneous good using one of two technologies: one labor-intensive and the other capital-intensive. The labor-intensive technology requires no initial investment but has a higher variable cost of production. To finance the investment required for capital-intensive production, a firm can sell a bond and stock in a perfectly competitive capital market. Given plausible values of two critical parameters, some firms are forced in the resulting equilibrium to forego investments with positive net present values (NPVs). More managers would like their firms to invest, but they cannot raise the required capital from outside investors. Because the foregone investments have positive NPVs, capital-intensive firms earn positive profits in equilibrium, while labor-intensive firms can have negative NPVs after the cost of entry. This foregone investment is a consequence of competition within an industry and thus is unrelated to previous explanations of

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[3] Recent empirical evidence appears in Bradley, Jarrell, and Kim (1984) and Titman and Wessels (1988). A range of optimal capital structures, rather than a unique optimal capital structure, is consistent with Maksimovic and Zechner (1991) and Fischer, Heinkel, and Zechner (1989). In the latter model, debt can be controlled continuously through time in an option pricing framework, and the ratio of long-term debt to the market value of equity has a negative, insignificant coefficient with size.

[4] See the survey by Schmalensee (1989).

optimal investments for isolated firms.[5] Capital-intensive firms are typically less risky and always much larger in size than are labor-intensive firms. Also, at least some debt is optimal for capital-intensive firms, but the optimal debt is not unique. In short, the industry has a core of large, profitable, secure, capital-intensive firms, each with at least some external debt, and a competitive fringe of small, marginally profitable or unprofitable, risky, labor-intensive firms. Even as the cost of entry converges to zero, a few capital-intensive firms can continue to earn extraordinary profits, while all labor-intensive firms can fail and thereby effectively exit the industry. In this sense, industries can be concentrated even with vanishingly small costs of entry because access to capital is effectively another barrier to entry, given costly agency.

These results differ dramatically from the conclusions in MZ. The differences reflect two major differences in assumptions. In MZ perks are precluded and the industry is assumed to be perfectly competitive. Here, dissipative perks are possible and the number of firms is determined endogenously in equilibrium. As shown subsequently, if perks are precluded, then the aggregate investment in equilibrium is Pareto-optimal, as in MZ. With dissipative perks, however, some investment may be forgone in equilibrium, depending on the values of two parameters. Also, if investment is foregone, then capital-intensive firms are profitable in equilibrium. Moreover, these profits can persist even with vanishingly small costs of entry. Consequently, the industry need not be perfectly competitive, given endogenous entry.[6]

The article is organized as follows. The model and its major results are sketched in Section 1. The analysis then begins in Section 2 with the fourth and final period and proceeds backward through time, section by section. In Section 2 the firms' optimal production and resulting endogenous risks are identified. The trade-off between perks and investment, conditional on capital structure, is presented in Section 3, and optimal capital structures are calculated in Section 4. In Section 5 the initial, optimal entry of firms is characterized, including both limiting, analytic solutions for small costs of entry and numerical solutions for larger costs. Extensions are discussed in Section 6,

[5] In Myers (1977), for example, identical firms, acting in isolation, invest identically, depending on the value of their withstanding debt. In MZ, by contrast, firms are not isolated and investment is not foregone.

[6] Two minor differences in assumptions also distinguish the models. In MZ, firms can issue only internal equity and external debt; here. firms can also sell external debt. Also, in MZ only the marginal cost of the risky technology is uncertain, whereas only aggregate demand is uncertain here. The additional security introduces into this model some indeterminancy in optimal capital structures, whereas the difference in technologies affects the source and interpretation of endogenous risk.

and the results are summarized in Section 7. All proofs appear in the Appendix.

## 1. Overview

In the model all corporate decisions are made in a subgame perfect equilibrium in pure strategies at four points in time. The sequence of events is shown in Figure 1. First, firms can enter the industry. Second, entrants can raise cash by selling corporate debt and equity to outside investors in a perfectly competitive capital market. Third, managers can consume perks and invest in a capital-intensive technology that reduces the variable costs of production. Finally, managers observe the aggregate demand in their industry, and their firms can then produce and sell their outputs. The model has four stages because each contributes significantly to the major results. For example, many of the new results in this article arise in the initial stage when firms enter endogeously. Since the model, with its four stages, is complicated, all other assumptions are designed to simplify the exposition as much as possible. Initially, all firms are identical. For each the cost of entry is the same exogenous constant. After entry, all cash for corporate investment must be raised by selling a bond or stock to outside investors. If a firm raises sufficient cash, then it can invest in a capital-intensive technology that reduces its variable cost of production. If it does not Invest; then it must produce using a second, labor-intensive technology. Subsequently, the aggregate demand for the homogeneous good can be either high or low. Both the aggregate demand curve and the cost functions are linear. Unlike Mz, the cost functions are deterministic.[7]

In the fourth and final time period, each firm has a fixed technology-either capital-intensive or labor-intensive. After all managers observe the aggregate demand for their industry's output, each firm produces in a pure-strategy, Cournot-Nash equilibrium. Because both the aggregate demand and the cost functions are linear, the unique optimal outputs can be calculated explicitly. With lower variable costs of production, capital-intensive firms always produce larger outputs than do labor-intensive firms. Also, the optimal output of each firm is decreasing in the number of capital-intensive firms and, with a restriction on the parameters, the total number of firms in the industry. In this sense, capital-intensive firms are always larger than labor-intensive firms, and each firm is smaller with more competition from other

[7] In Mz the marginal cost of production is uncertain with the risky technology and firms' cost functions are quadratic. Here, the intercept in aggregate demand is uncertain, and all cost functions are linear.

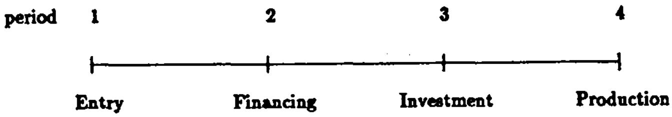


**Figure 1**
**Sequence of events**
1. Entry: Firms can enter the industry simultaneously. The fixed cost of entry is an exogenous constant. the equilibrium is subgame perfect. 2. Financing: To raise the capital required for investment, firms can sell securities simultaneously in a perfectly competitive capital market. Each firm can offer to outside investors and some fraction of its stock. The firm's insider retains all remaining stock. 3. Investment: Firms can invest simultaneously in a capital-intensive technology. Firms that do not invest can produce subsequently using a labor-intensive technology with a higher variable cost of production: A firm's insider can expend all remaining corporate cash on perks. 4. Productions After the insiders in all firms observe the aggregate demand in the market, firms an produce simultaneously using their capital-intensive or labor-intensive technology. The price is determined by the aggregate output in the industry. The aggregate demand curve and all cost curves are linear.

firms. In addition, risk is determined endogenously in equilibrium. As more firms either enter the industry or invest in the capital-intensive technology, the cash inflow from incremental investment decreases for each level of aggregate demand. Thereby, the capital-intensive technology changes from stochastically dominant, to more risky, to stochastically dominated, relative to the labor-intensive technology. Capital-intensive production, with its higher fixed cost and lower variable costs, has operating leverage relative to labor-intensive production. The risk of operating leverage from capital-intensive production is then determined in equilibrium by both the number of firms in the industry and their aggregate investment. This result holds even though all risk arises from aggregate demand.[8]

In the second and third periods, each firm can sell securities in a perfectly competitive capital market and then invest in the capital-intensive technology. By assumption, a firm can raise cash by selling to outsiders both a bond and some fraction of its stock. All feasible cap ital structures must satisfy two constraints. The investment constraint identifies all capital structures that induce a firm's insider to invest and not to consume dissipative perks. The financial constraint identifies all capital structures that induce outside investors to contribute the cash required for investment. As the number of capital-intensive firms increases, the NPV of incremental investment decreases. This tightens both the investment and financial constraints. More precisely, for

[8]In MZ the marginal cost of production is risky because either the technology or the price of an input is uncertain. There, the risky cost is a hedge against the risky price resulting from the uncertain output of all firms. Here, no such hedge is possible.

any possible promised payment on external debt, additional aggregate capital has two effects. First, it decreases the maximum fraction of external equity that just induces a firm's insider to invest and to forego perks. Second, it increases the minimum faction of external equity that just induces outsiders to contribute the required capital. If both the investment and financial constraints are satisfied at the value-maximizing aggregate investment, then the subgame perfect equilibria starting in period 2 are first-best or Pareto-optimal-conditional on the number of firms entering into the industry during the previous period. In these conditionally first-best equilibria, the insider in each firm is approximately indifferent. at the margin between investing or not, given optimal investments by all other firms. The equilibria are distinguished only by the identities of the capital-intensive firms and their optimal capital structures, and the solution is approximate only because the number of capital-intensive firms must be an integer. Given risk-neutral agents, operating leverage is then approximately a mean-preserving spread in equilibrium. Associated with each firm's investment in the capital-intensive technology is a set of optimal cap ital structures, but no unique optimal capital structure. As shown in Section 3, the equilibria are first-best only under special circumstances. Either perks must be precluded, as in MZ, or perks must be sufficiently dissipative and the fixed cost of the capital-intensive technology must be sufficiently large.

More importantly, this first-best solution may be infeasible. If, at the first-best solution, the intersection of the investment and financial constraints is empty, then the largest feasible aggregate investment is less than the first-best investment. In this case, the associated subgame perfect equilibria starting in period 2 are second-best or constrained-Pareto-optimal, again conditional on the number of firms in the industry. As before, these equilibria differ only in the identities of the capital-intensive firms and their capital structures. Each second-best equilibrium has the following properties. Some firms invest in capital-intensive production, while other, identical firms forego identical investments with positive NPVs. More managers would like their firms to invest, but they cannot credibly commit to forego their dissipative perks and thus cannot raise the required capital from outside investors. In this sense, capital is rationed even though the capital market is perfectly competitive. With rationing the NPVs of capital-intensive firms must be positive. As shown in Section 3, the NPVs of capital-intensive firms are positive in equilibrium if perks are possible and either perks are not too dissipative or the fixed cost of the capital-intensive technology is not too large.

The optimal aggregate investment in period 3 is supported by optimal capital structures in period 2. For each capital-intensive firm, at least some external debt is optimal, but the optimal debt is not unique. This lower bound on optimal debt can be explained as follows. Suppose that an insider announces his firm's investment in the capital-intensive technology and then raises the cash partly by selling a corporate bond that exceeds the market value of a labor-intensive firm. If, subsequently, he cancels the investment and expends the cash as perks, then his internal equity in his now labor-intensive firm becomes worthless. Thereby, debt diminishes the insider's incentive ex post to consume perks and to forego investment. Outsiders understand this relationship between debt and the insider's subsequent action and then contribute the required cash when otherwise they would not. Consequently, debt that is infeasible for labor-intensive firms is optimal for capital-intensive firms in a second-best equilibrium.[9]

In the initial period, firms can enter the industry. In the subgame perfect equilibrium, entry must maximize each firm's NPV, measured net of the fixed cost of entry, conditional on optimal entry by all other firms. The optimal number of entrants and the resulting number of capital-intensive firms are calculated analytically for vanishingly small costs of entry and numerically for larger costs of entry. Other properties of the equilibria are also identified. As the cost of entry decreases toward zero, the industry becomes more competitive only in the sense that the total number of entrants grows without bound. both the number and profitability of capital-intensive firms depend *on* the values of various parameters. If perks are not too dissipative and the fixed cost of capital-intensive production is sufficiently large, then competitive industries with low costs of entry consist solely of small, labor-intensive firms. In the remaining cases, even with very small costs of entry, relatively few capital-intensive firms persist and prosper, while labor-intensive firms produce no output, receive no revenues, and thereby fail, depending on the demand in the market. In the latter equilibria the capital-intensive technology is either stochastically dominant or more risky than the labor-intensive technology, but never stochastically dominated. Given larger costs of entry, the numerical solutions have the following properties. All equilibria are second-best. Capital-intensive firms have large, positive NPVs, whereas labor-intensive firms have slightly positive, zero, or slightly negative NPVs, when measured net of the cost of entry. Capital-intensive firms are less risky and much larger in size than labor-intensive firms. This lower risk occurs in equilibrium despite the operating leverage of capital-intensive firms and the assumed absence of diversification in the model.

[9] This has some similarities with the dissipation of free cash flows in Jensen (1986).

## 2. Production

All production occurs in the fourth and final time period. At this time each firm, $i \in \{1, \ldots, n\}$, has one of two technologies or capita-intensities, $k_i \in \{0,1\}$. The labor-intensive technology, $k_i = 0$, has a lower fixed cost but higher variable costs of production than the capital-intensive technology, $k_i = 1$. If firm $i$ produces the quantity of output $q_i$, then it incurs the variable cost, $(\alpha - \beta k_i)q_i$, with the constants, $\alpha > b > 0$. Also, it has the fixed cost, $g\ k_i$, with the constant, $\gamma > 0$. Under this convenient normalization, labor-intensive production, $k_i = 0$, has no fixed cost. The variable cost is incurred at the time of production when revenues are received, whereas the fixed cost of the capital-intensive technology, $k_i = 1$, is incurred at the time of investment and must be financed. The industry has k capital-intensive firms: $k \equiv \sum_{i=1}^{n} k_i$. Excluding firm $i$, it has $k_{-i}$ other capital-intensive firms: $k_{-i} \equiv \sum_{\ell \neq i} k_\ell$. All firms in the industry produce a homogeneous good with the aggregate output: $q \equiv \sum_{i=1}^{n} q_i$. Collectively, all other firms have the total output: $q_{-i} \equiv \sum_{\ell \neq i} q_\ell$.

The aggregate demand in the industry depends on the size of the market. The market is either small, $m = 0$, or large, $m = 1$. A large market, $m = 1$, occurs with probability $\pi$ satisfying $0 < \pi < 1$. This probability, together with the entire structure of the model, is common knowledge among managers Each manager sees the size of the market $m$ after selecting his firm's technology but before choosing its output. Conditional on the market $m$ and the aggregate output $q$, the price of the homogeneous good $p$ is determined by its inverse demand curve: $p = \zeta + \eta m - \theta q$. The intercept $\zeta$ satisfies $\zeta > \alpha - \beta > 0$, so that at least some firm can earn positive profits by producing in each market. Also, the slopes satisfy $\eta, \theta > 0$, so that a larger market $m$ generates a larger demand $q$. Given this linear demand and the, linear costs, the subsequent problems of production, investment, and capital structure have explicit solutions.[10] Although the restriction to two technologies is substantive, the restriction to two states is merely an expositional convenience. Extensions with more general technologies are did in Section 5.

The game has four stages with observed actions throughout and a subgame perfect equilibrium. In each subgame all agents act simultaneously in a pure-strategy Nash equilibrium. In the final period firm $i$ is endowed with the capital-intensity or technology 4. After observing the size of the market $m$, insider $i$ chooses his firm's output. The

[10]In MZ the demands are linear, the variable costs are quadratic, and the market is assumed to be perfectly competitive. Here an explicit solution is possible with linear costs because the market is generally imperfectly competitive. With quadratic costs the solution is similar; only the parameter $\theta$ must be reinterpreted.

optimal output maximizes his firm's final cash inflow, $(p-\alpha+\beta k_i)q_i$, in a Cournot-Nash equilibrium:

$$q^*_{im} = \arg\max\{[\zeta + \eta m - \theta(q_i + q^*_{-im}) - \alpha + \beta k_i]q_i : q_i \geq 0\}, \quad (1)$$

for all firms, $i = 1, \ldots n$.[11] In (1) the optimal output $q^*_{im}$ of each firm $i$ is a best response to the optimal outputs of all other firms $q^*_{-im}$. In other words, all agents move simultaneously following pure strategies that depend on their firms' technologies and the state of the market. The strategies can also depend on the numbers of capital and labor-intensive firms, k and $n$ - $k$, in the industry. The industry can be monopolistic, $n = 1$, imperfectly competitive, $0 < n < \infty$, or perfectly competitive, $n = \infty$.

The Cournot-Nash equillibrium is identified in the Appendix. The unique equilibrium in pure strategies is symmetric within technologies. All firms with the same technology produce in each market $m$ the same output: $q^*_{im} = q^*_{i'm}$, for all firms $i$ and $i'$ with $k_i = k_{i'}$. In other words, each labor-intensive firm and each capital-intensive firm have the respective optimal outputs: $Q_{0m}(k, n)$ and $Q_{1m}(k, n)$.To simplify the subsequent notation, this output is identified as $Q_{jm}(k, n)$, with the index, $j = 0, 1$. That is, a labor-intensive firm, $k_i = 0$; has the index, $j = 0$, while a capital-intensive firm, $k_i = 1$, has the index, $j = 1$. This optimal output $Q_{jm}(k, n)$ can be calculated explicitly. It depends on both the number of capital-intensive firms k and the total number of firms in the industry $n$:

$$Q_{jm}(k, n) = \frac{1}{\theta}\left[\beta j + j\min\left(0, \frac{\zeta + \eta m - \alpha - \beta k}{k+1}\right) + \max\left(0, \frac{\zeta + \eta m - \alpha - \beta k}{n+1}\right)\right], \quad (2)$$

for $j$, $m = 0.1$, $k = 0, 1, \ldots, n$ and $i = 1, \ldots, n$. Three properties of this solution are important in the subsequent analysis. First, labor-intensive firms always produce less than capital-intensive firms: $0 \leq Q_{0m} < Q_{1m}$. Second, if a firm produces in market $m$, then its output decreases in the number of capital-intensive firms $k$: $Q_{0m}(k, \cdot) > Q_{0m}(k+1, \cdot)$, whenever $Q_{0m}(k, \cdot) > 0$, and $Q_{1m}(k, \cdot) > Q_{1m}(k+1, \cdot)$. Third, if labor-intensive firms produce in market $m$, then the output of each type decreases in the total number of firms $n$: $Q_{jm}(\cdot, n) > Q_{jm}(\cdot, n+1)$, for $j = 0, 1$, whenever $Q_{0m}(\cdot, n) > 0$. In short, a labor-

[11] This maximand does not distinguish between bankrupt and solvent firms. If a firm is bankrupt, then its manager is assumed to maximze its residual value in (1). This avoids adding more types of firms.

intensive firm is always smaller than a capital-intensive firm, and each firm is smaller with more competition from other firms.

A firm's optimal output affects its final cash inflow. If a firm with technology $j$ produces in market $m$ the output $q_{jm}$, then it realizes the cash inflow: $y_{im} \equiv (p - \alpha + \beta k_i) q_{im}$. With the optimal output, $q^*_{im} = Q_{jm}(k, n)$, from (2) this cash inflow is

$$y^*_{im} = Y_{jm}(k, n) \equiv (p - \alpha - \beta f) Q_{jm}(k, n) = \theta [Q_{jm}(k, n)]^2. \quad (3)$$

The final cash inflow (3) exhibits the same monotonicity with respect to all state variables as the optimal output (2). The same monotonicity is also exhibited by the incremental cash inflow from incremental capital: $\Delta Y_m(k, n) \equiv Y_{1m}(k, n) - Y_{0m}(k-1, n)$. Specifically, the incremental cash inflow is positive and increasing in the size of the market m: $0 < \Delta Y_0 < \Delta Y_1$. Also, it is decreasing in the number of capital-intensive firms $k$: $\Delta Y_m(k, \cdot) > \Delta Y_m(k+1, \cdot)$. Finally, it is decreasing in the number of firms $n$ when labor-intensive firms are induced to produce: $\Delta Y_m(\cdot, n) > \Delta Y_m(\cdot, n+1)$, whenever $Q_{0m}(\cdot, n) > 0$. In other words, a firm has a lower return on investment in both smaller markets and more competitive markets, where competition can come from either investment by other firms or entry into the industry. These three properties of the incremental cash flow generate many of the subsequent results.

Operating risk is endogenous in MZ. There, the cost of production is uncertain because either the technology or the prices of inputs are uncertain. Here, a similar result holds even' though all uncertainty arises from aggregate demand. Consider the three, mutually exclusive and exhaustive cases:

$$\begin{aligned} &(a) \quad \gamma \le \Delta Y_0(k, n) < \Delta Y_1(k, n), \\ &(b) \quad \Delta Y_0(k, n) < \gamma < \Delta Y_1(k, n), \qquad (4) \\ &(c) \quad \Delta Y_0(k, n) < \Delta Y_1(k, n) \le \gamma. \end{aligned}$$

Measured net of the fixed costs $\gamma$, the incremental cash inflow in market $m$ from investing in capital-intensive production is $\Delta Y_m(k, n) - \gamma$. With the inequalities in (4 $a$ ), labor-intensive production, $k_i = 0$, is (first-order) stochastically dominated by capital-intensive production, $k_t = 1$; whereas with (4 $c$ ) the reverse is true. As shown in the Appendix, case (4 $a$ ) can occur with sufficiently few capital-intensive firms: $\Delta Y_0(0, n) > \gamma$ if $\beta^2/4\theta \ge \gamma$. Similarly, (4 $c$ ) holds with sufficiently many capital-intensive firms: $\Delta Y_1(n, n) \to 0$ as $n \to \infty$. In the remaining case (4 $b$ ), the capital-intensive technology, with its higher fixed cost and lower-variable costs, makes the firm more risky. This incremental risk from operating leverage is mean-preserving in the sense of Rothschild and Stiglitz (1970) if $(1-\pi)\Delta Y_0(k, n)+\pi\Delta Y_1(k, n) = \gamma$.

## 3. Investment

*Problem.* In the third period the owner-manager of each firm allocates his corporate cash between investment and perks. At this time firm $i$ has the cash $c_i$ that it raised in the previous period by selling securities to outside investors. If the corporate cash $c_i$ exceeds or equals the fixed cost $\gamma$, then firm $i$ can invest $\gamma$ dollars in capital-intensive production; otherwise, it can invest nothing and must produce using the labor-intensive technology. Simultaneously, the insider in firm $i$ can consume on the job $x_i$ dollars of perks and thereby realize the utility $U(x_i)$, for all $x_i \geq 0$. This utility from perks is measured in dollars, and the manager's total utility is additively separable in the utility from perks plus his dollars of direct compensation. The utility function $U$ is continuously differentiable, nondecreasing, and concave, with $U(0) = 0$ and $U'(\infty) = 0$. Also, perks are dissipative: $0 \leq U(x_i) < x_i$ for all $x_i > 0$. The dissipation, $x_i - U(x_i) > 0$, can be interpreted as the firm's cost of maintaining internal accounting controls and its managers' costs of responding to those controls. In any case, the total expenditure on capital and perks cannot exceed the available corporate cash: $0 \leq \gamma k_i + x_i \leq c_i$. Also, as in other agency models the manager cannot avoid dissipative perks by renegotiating his contractual payoff with outside investors.** Consequently, insider $i$ always consumes all residual cash as perks: $x_i = c_i - \gamma k_i \geq 0$.

The firm can be capitalized with at most a bond and stock.[13] The bond of firm $i$ can have any promised payment $b_i$ up to the largest possible corporate cash inflow: $0 \leq b_i \leq Y_{11}(k, n)$. For analytical simplicity, all investors are assumed to be risk-neutral. Also, for notational simplicity the riskless rate of interest is set equal to zero. Because the state of the market $m$ is not revealed until the fourth and final period, the value of the firm's bond in the third period is

$$B_j(b_i, k, n) = (1-\pi)\min\{b_i, Y_{j0}(k, n)\} + \pi\min\{b_i, Y_{j1}(k, n)\}, \quad (5)$$

for $j = 0, 1$. Again, $\pi$ is the probability of a large market, $m = 1$.

12 The manager cannot renegotiate in period 3 the contract initiated in period 2 when his firm sold securities to outsiders. If manager $i$ could renegotiate, then the would sell his perks back to his Investors for the price $x_i$ and thereby realize the gain: $x_i - U(x_i) > 0$. As in other agency models, renegotiation is precluded by unmodeled costs.

Simultaneously, the firm's stock has the market value:

$$S_j(b_i, k, n) = (1-\pi)\max\{0, Y_{j0}(k, n) - b_i\} + \pi \max\{0, Y_{j1}(k, n) - b_i\}, \tag{6}$$

for $j = 0.1$. In (5) and (6) the values of debt and equity depend on the firm's final cash inflow (2) after its optimal production (1) in the two possible markets, $m = 0, 1$. The sum of these two values, (5) and (6), is the total market value of the firm: $F_j(k, n) \equiv B_j(b_i, k, n) + S_j(b_i, k, n)$.

A manager can be compensated with both perks and inside equity. Suppose that firm $i$ has raised the cash $c_i$ by selling to outside investors some fraction $a_i$ of its stock and a bond with the promised payment $b$. Insider $i$ then retains the residual share, $1 - a_i$, of his firm's stock. Again, the analysis is restricted to pure strategies. Also, without further loss of generality, the firm is assumed always to invest in capital-intensive production when its insider is indifferent between investing or not. Conditional on the capital structure $(a_i, b_i)$ and the resulting cash $c_i$, insider $i$ then chooses for his firm the capital-intensity $k_i$ and thereby the perks, $x_i = c_i - \gamma k_i$. The optimal choice maximizes his utility of perks plus the market value of his personal equity in another Nash equilibrium:

$$k_i^* = \arg\max\{U(c_i - \gamma k_i) + (1 - a_i)S_{k_i}(b_i, k_i + k_{-i}^*, n)\}, \tag{7}$$

subject to $k_i \in \{0, 1\}$, and $\gamma k_i \leq c_i$. In Equation (7) insider $i$ chooses the capital-intensity $k_i^*$ to maximize hi personal welfare, conditional on his correct conjecture in equilibrium that the insiders in $k_{-i}^*$ other, firms choose the capital-intensive technology. Thereby, $k^*$ firms become capital-intensive in the Nash equilibrium.

*Investment constraint.* This problem can be simplified as follows. If the firm has insufficient cash $c_i$ for capital-intensive production, $0 \leq c_i < \gamma$, then it invests nothing, $k_i = 0$, and the insider expends all cash on perks, $x_i = c_i$. Alternatively, if the cash is sufficient for investment, $c_i \geq \gamma$, then two solutions are possible. Either the firm invests nothing and the insider consumes everything, $k_i = 0$ and $x_i = c_i$, or the firm invests in capital-intensive production and the insider consumes the residual, $k_i = 1$ and $x_i = c_i - \gamma$. Also, outsiders understand the insider's incentives and price the firm's securities correctly. As a result, the insider must effectively pay to outsiders $x_i$ dollars for perks with the utility $U(x_i)$, and then realize the loss: $x_i - U(x_i) > 0$ for all $x_i > 0$. For this reason, no insider ever sells securities solely to finance his perks. Instead, each insider $i$ voluntarily restricts his corporate cash to the two values: $c_i \in \{0, \gamma\}$. These two strategies, $c_i = 0$ or $c_i = \gamma$, dominate all other strategies in period 2. Henceforth; the dominated strategies are eliminated, and the analysis is restricted to the remaining possibilities: $c_i \in \{0, \gamma\}$. Also, the only possible response in period 3 to the choice in period 2 of no corporate cash, $c_i = 0$, is no investment and no perks: $k_i = 0 = x_i$. Therefore, the remainder of this subsection is restricted to the case: $c_i = \gamma$. In this case, an insider either invests nothing and consumes everything or invests everything and consumes nothing.

Under these conditions, problem (8) can be restated as follows. Insider $i$ is induced to invest all corporate cash $\gamma$ whenever his perks plus the market value of his personal stock in a labor-intensive firm, $k_i = 0$, is no greater than the market value of his personal stock in a capital-intensive firm, $k_i = 1$:

$$U(\gamma) + (1 - a_i)S_0(b_i, k^*_{-i}, n) \leq (1 - a_i)S_1(b_i, 1 + k^*_{-i}, n). \qquad (8)$$

This investment constraint identifies all capital structures $(a_i, b_i)$, consisting of the fraction of external equity $a_i$ and the promised payment on debt $b_i$, that induce insider $i$ to invest all his firm's funds and to forego all perks. When the reverse inequality holds, the insider invests nothing and consumes everything. As in other agency problems, this incentive-compatibility constraint is necessary for virtually all subsequent results. Also, it is binding only because a forcing contract is infeasible. That is, (8) constrains feasible contracts because outside investors cannot force a firm's insider to invest in capital-intensive production. Although a forcing contract may be possible with two distinctly different technologies--as in this model--the sharp distinction here between one technology with capital and another without capital is merely an expositional convenience. Any pair, any finite or infinite number, or even a continuum of technologies is sufficient for the subsequent results if two conditions are satisfied. The choice between the technologies must be restricted by an investment constraint like (8), and no technology can be dominated by another. These issues are discussed in more detail in Section 6.

The investment constraint is calculated in the Appendix. The calculation covers two mutually exclusive and exhaustive cases: $Y_{10}(k, n) < Y_{01}(k - 1, n)$ and $Y_{10}(k, n) \geq Y_{01}(k - 1, n)$. The investment constraint for the first case is shown in Figure 2. The set of feasible capital structures $(a_i, b_i)$ that induce insider **i** to invest, $k^*_i = 1$, lie on and below the curve with three kinks. With fractions of external equity $a_i$ and promised payments on debt $b_i$ in this lower region, the additional value of the insider's stock from investing the cash $\gamma$ exceeds his utility from the foregone perks. In this region the insider retains enough equity in his firm to be induced to invest all corporate cash and to forego all his dissipative perks. With larger fractions of external equity or larger promised payments on external debt such that the reverse inequality holds, the manager invests nothing and consumes everything.[14] In this model there is a sharp boundary between the two regions of investment without perks, $k_i^* = 1$, and perks without investment, $k_i^* = 0$, only because the technology is discrete; production is either labor-intensive or capital-intensive.[15] For any feasible capital structure, this constraint completely characterizes a firm's investment and its manager's perks.

*Financial constraint.* To understand the implications of the investment constraint, it is helpful to look back briefly to the previous period. At time 2 firms can sell securities to outside investors. Initially, no firm is endowed with any internal cash.[16] To raise the cash required for investment $\gamma$, firm $i$ must sell to outside investors some fraction of its stock $a_i$ and a bond with the promised payment $b_i$ that satisfy the financial constraint:

$$\gamma \leq B_1(b_i, k^*, n) + a_i S_1(b_i, k^*, n). \tag{9}$$

As indicated in (9), the market values of the firm's bond and stock depend on the optimal number of capital-intensive firms $k^*$ from period 3. This financial constraint is shown in Figure 2 as all capital structures $(a_i, b_i)$ above and to the right of the curve with one kink at $Y_{10}(k, n)$. The figure is drawn for an arbitrary number of capital-intensive firms $k$, not lust the optimal number $k^*$. Any &dominated capital structure must satisfy the financial constraint (9) with equality and thus must lie along the boundary of the financial constraint. Also, any feasible capital structure $(a_i, b_i)$ must satisfy the inequalities:

$$0 \leq a_i \leq 1, \qquad 0 \leq b_i \leq Y_{11}(k^*, n). \tag{10}$$

The upper bound $Y_{11}(k^*, n)$ is the largest possible corporate cash inflow with the optimal number of capital-intensive firms $k^*$ and the optimal output $Q_{11}(k^*, n)$ in (2).

Together, the investment and financial constraints and the above inequalities determine the feasible set of capital structures. Again, fo-

[14] The flat segments of the boundary occur because in these regions the promised payment $b_i$ has the same marginal effect on the stock of both the labor and capital-intensive firms. With the reverse inequality, $Y_{01}(k-1, n) \leq Y_{10}(k, n)$, only the initial interval, $0 \leq b_i < Y_{00}(k, n)$, is flat. See (A7) and (A8) in the Appendix.

[15] If the average variable costs were continuous and convex in the firm's capital $k_i$ on some interval around the capital, $k_i = 1$, then the optimal investment would be continuously decreasing in both the fraction of external equity $a_i$ and the promised payment on external debt $b_i$. See Section 6.

[16] Internal cash is introduced in Section 6. The extension is straightforward.

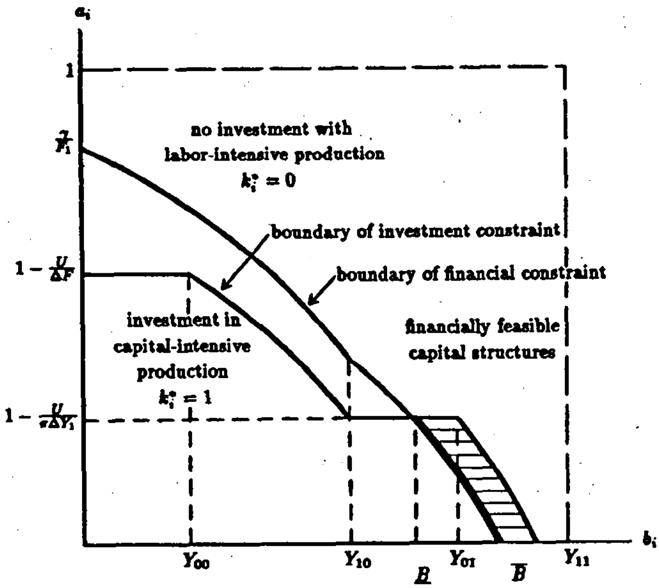


**Figure 2**
**Feasible capital structures**
Conditional on the numbers of capital-intensive and labor-intensive firms in the industry, $k$ and $n - k$, firm $i$ with technology $j$ has in market $m$ the final cash inflow $Y_{jm}(k, n)$, for $j, m \in \{0, 1\}$. The firm can sell to outside investors the fraction of its stock $a_i$ and a bond with the promised payment $b_i$. All feasible capital structures $(a_i, b_i)$ must satisfy the investment constraint (8), the financial constraint (9), and the inequalities (10). Capital structures below and to the left of the investment constraint induce the manager to invest in capital-intensive production, $k_i^* = 1$, and to forego all perks. Capital structures above and to the right of the financial constraint induce outside investors to contribute the cash required for capital-intensive production $\gamma$. All feasible capital structures are shown in the hatched area. All undominated, feasible capital structures lie along the dark line between the promised payments, $\underline{B}(n)$ and $\overline{B}(n)$. With additional capital-intensive firms $k$, the investment constraint is shifted downward and the financial constraint is shifted upward. The investment constraint is shown for the special case: $Y_{10}(k, n) < Y_{01}(k - 1, n)$.

cus on Figure 2. For a firm that invests in capital-intensive production, all feasible capital structures $(a_i, b_i)$ must satisfy the investment constraint (8), the financial constraint (9), and the two inequalities (10). Capital structures on or below the investment constraint induce the manager to forego perks; capital structures on or above the financial constraint induce outsiders to contribute the required capital. All capital structures satisfying both constraints are shown in the hatched area of Figure 2. Also, all feasible and undominated capital structures with positive. debt, $b_i > 0$, appear along the dark line. The size of the hatched area depends on the numbers of both capital-intensive and labor-intensive firms. As shown in the Appendix, the boundary of the investment constraint (8) shifts down as the number of capital-intensive firms k increases. It also shifts down as the total number of firms $n$ increases, whenever $0 \leq k < (\zeta + \eta - \alpha)/\beta$. Similarly, the boundary of the financial constraint (9) shifts up as either $k$ or $n$ increase. This property of feasible capital structures can be interpreted as follows. As competition among firms increases, either because more firms k become capital-intensive or more firms $n$ enter the industry, each firm's NPV from investment decreases. In turn, this reduces each insider's incentive to forego perks and thereby reduces the fraction of external equity $a_i$ for any given promised payment $b_i$ that firm $i$ can sell to outsiders and still induce its insider to invest. It also raises the fraction $a_i$ for any given payment $b_i$, required to finance the firm's investment. Consequently, each firm's feasible set of capital structures collapses as competition among firms increases. This property is critical to the subsequent results.

*Optimal investment.* The aggregate investment in equilibrium can now be identified. The argument in this subsection has several steps. Initially, the investment constraint (8) is ignored and the resulting aggregate investment is identified. This investment is first-best or Pareto-optimal, conditional on the number of firms in the industry. In Proposition 1A this first-best solution is shown to be the aggregate investment in all subgame-perfect Nash equilibria once perks are precluded. Next, the investment constraint is reintroduced and the resulting second-best or constrained-Pareto-optimal investment is identified. In Proposition 1B this second-best solution is shown to be the aggregate investment in equilibrium for some values of two critical parameters.

Initially ignore the investment constraint (8). In this case, by the argument below (7), each manager optimally consumes no perks and hence derives no utility from perks: $U(0) = 0$. Also, by then-below (9), all undominated capital structures must satisfy the financial constraint (9) with equality. Dropping the investment constraint (8), inserting the binding financial constraint (9) into the maximand in (7), and rearranging terms produces the problem:

$$k_i^* = \arg\max\{F_{k_i}(k_i + k_{-i}^*, n) - \gamma k_i\}, \tag{11}$$

subject to $k_i = c_i/\gamma \in \{0, 1\}$, for $i = 1, \ldots, n$. For each firm $i$ the optimal capital $k$ is a best response to all other firms' optimal capital $k_{-i}^*$. That is, firm $i$ can have one of two NPVs: $F_0(k_{-i}^*, n)$ with no investment and labor-intensive production, $k_i = 0$, or $F_1(1 + k_{-i}^*, n) - \gamma$ with investment in the capital-intensive technology, $k_i = 1$. In a Nash equilibrium no insider can increase his firm's NPV by switching unilaterally from labor-intensive production to capital-intensive production, or vice versa.

The solution to this problem can be sketched as follows. A firm with technology $j$ in an industry with the aggregate capital $k$ and the number of firms n has the total market value: $F_j(k, n)$. If this firm switches from labor-intensive to capital-intensive production and no other firms switch, then its incremental value is $\Delta F(k, n) \equiv F_1(k, n) - F_0(k-1, n)$. In a Nash equilibrium the NPV of switching unilaterally to capital-intensive production must be nonnegative at the value-maximizing aggregate capital $K_1(n)$ and negative at the aggregate capital $K_1(n)+1$:

$$\begin{aligned} &\text{either } K_1(n) = 0 \text{ or } \quad \gamma \le \Delta F[K_1(n), n] \quad \text{with} \quad K_1(n) \in \{1, \ldots, n\}, \\ &\text{either } K_1(n) = n \text{ or } \gamma > \Delta F[K_1(n) + 1, n] \text{ with } K_1(n) \in \{0, \ldots, n-1\}, \end{aligned} \tag{12}$$

for all $n = 1, 2, \ldots$. Also, the difference $\Delta F(\cdot, n)$ is decreasing in its argument $k$, so that the necessary optimality conditions (12) are sufficient for a unique optimum to (11). This solution to (11) is called the first-best number of capital-intensive firms $K_1(n)$. It is the Pareto-optimal or value-maximizing number of capital-intensive firms, conditional on the total number of firms in the industry $n$, which itself may be less than first-best. This first-best aggregate investment is chosen collectively by all insiders if the investment constraint (8) is not binding at $k^*_{-1} = K_1(n) - 1$. As shown in the Appendix, the investment constraint (8) is never binding at the first-best solution if perks are precluded. These results are summarized in the first proposition.

***Proposition 1A.*** *Without perks the number of capital-intensive firms in equilibrium (11) is always first-best: $k^* = K_1(n)$, for all $n = 1, 2, \ldots$. It is nonincreasing everywhere in both the number of firms $n$ and the fixed cost of capital-intensive production $\gamma$.*

Without perks the agency problem disappears in equilibrium. In these Nash equilibria to the subgame starting with an arbitrary number of firms $n$, the NPV from investing in the capital-intensive technology is nonnegative and nearly zero.[17] The NPV is nearly zero only because the first-best number of firms $K_1(n)$ invest in capital-intensive production. If fewer firms invest, then at least one firm switches to, capital-intensive production; if more firms invest, then at least one switches back to labor-intensive production. In equilibrium the two technologies are selected by sufficient numbers of firms so that the

The multiple equilibria differ only in the identities of the $k^*$ capital-intensive firms. Also, the NPV can be positive in (14) only because $k$ must be an integer: $k \in \{1, \ldots, n\}$. In MZ, firms are assumed to be perfectly competitive, so that discreteness can be ignored.

NPV from investment is driven nearly to zero. Once this occurs, no insider has an incentive to switch unilaterally between technologies. As in MZ, this result reflects competition between firms. For each firm the cash inflows in (3) and thereby the NPV in (11) decrease as more firms k invest in capital-intensive production. Without this competition in the product market, the NPV would always be positive or negative for every firm, depending only on the technology.

However, the first-best solution may be infeasible. If the investment constraint (8) is strictly binding at the first-best solution $K_1(n)$, then the optimal solution must be second-best. To see this, again focus on Figure 2. If $k > 0$, then the feasible set of capital structures-the hatched area in the figure-cannot be empty for any firm. If it is empty for one firm at $k > 0$, then it is empty for all firms so that no investment can be financed, which is inconsistent with $k > 0$. Also, by the argument below (9), the hatched area decreases in size as the aggregate capital $k$ increases. Let $\hat{K}_2(n)$ be the largest integer such that the hatched area is nonempty, and set $K_2(n) \equiv \max\{0, \min[\hat{K}_2(n), n]\}$, for $n = 1, 2, \ldots$. Clearly, the integer $K_2(n)$ is an upper bound on the feasible number of capital-intensive firms: $k \in \{0, \ldots, K_2(n)\}$. As shown in the Appendix, this upper bound uniquely satisfies the conditions:

$$\begin{array}{llll} \text{either} & K_2(n) = 0 & \text{or} & \gamma + U(\gamma) \leq F_1[K_2(n), n] \\ & & \text{with} & K_2(n) \in \{1, \ldots, n\}, \\ \text{either} & K_2(n) = n & \text{or} & \gamma + U(\gamma) > F_1[K_2(n) + 1, n] \\ & & \text{with} & K_2(n) \in \{0, \ldots, n-1\}, \end{array} \tag{13}$$

for any feasible number of firms $n$.

The optimal aggregate capital can now be identified. By the argument below (3), the present value of switching from labor-intensive to capital-intensive production $\Delta F(k, n)$ is decreasing in the aggregate number of capital-intensive firms $k$. With (12) the NPV of switching must then be positive at all values of $k$ below the first-best number of capital-intensive firms $K_1(n)$: $\Delta F(k, n) - \gamma > 0$ for all $0 \leq k < K_1(n)$. In this case, the optimal aggregate capital $k^*$ must be the minimum of the first-best aggregate capital from (12) and the upper bound from (13):

$$k^* = K(n) \equiv \min\{K_1(n), K_2(n)\}, \tag{14}$$

for all $n$. Given (14), the optimal aggregate capital $k^*$ is called first-best if $k^* = K_1(n) \leq K_2(n)$ and second-best if $k^* = K_2(n) < K_1(n)$. In the second-best solution, some firms are prevented from investing in the capital-intensive technology by the strictly binding upper bound, $K_2(n) < K_1(n)$. As As shown in the next proposition, the upper bound is weakly binding, $K_2(n) \leq K_1(n)$, and the NPV of investment is positive at the margin, $\Delta F[K_2(n), n] > \gamma$, whenever the fixed cost $\gamma$ is sufficiently small or perks are not too dissipative.

***Proposition 1B.*** *With dissipative perks, $0 < U(\gamma) < \gamma$, the number of capital-intensive firms in equilibrium (14) satisfies*

$$k^* = \begin{cases} K_1(n) & \gamma \geq \Gamma(n) \\ K_2(n) & \text{otherwise,} \end{cases} \tag{15}$$

*whenever $n/(n+1)^2 \leq \beta/(\zeta - \alpha + \beta)$, for all $n = 1, 2, \ldots$. The bound $\Gamma$ in (15) satisfies $0 < \Gamma(n) < \infty$ if and only if $U(\infty) < F_0(0, n)$; otherwise, it vanishes, $\Gamma(n) = \infty$. If $0 < \gamma < \Gamma(n)$, then capital-intensive production has a positive NPV: $\Delta F[K_2(n), n] > \gamma$. The number of capital-intensive firms $k^*$ is nonincreasing in both the total number of firms $n$ and the fixed cost of investment $\gamma$.*

The economically significant properties of this solution depend critically on the fixed cost of investment. If the positive cost y is sufficiently small, $0 < \gamma < \Gamma(r$, or perks are not too 'dissipative, $U(\infty) \geq F_0(0, n)$, then the upper bound on the feasible number of capital-intensive firms in (13) is weakly binding, $t^* = K_2(n) \leq K_1(n)$, whenever the number of firms in the industry *n is* sufficiently large, $n/(n+1)^2 \leq \beta/(\zeta - \alpha +$ It would be strictly binding if the aggregate capital-intensity *k* could be any real number, not just an integer. As a result, all capital-intensive firms then have positive NPVs in equilibrium. If the upper bound in (13) is strictly binding, $k^* = K_2(n) <$ *K,(n),* then some firms are forced by the capital market to forego investments with positive NPVs. In this case, more managers would like their firms to invest, but they cannot credibly commit to forego their dissipative perks and thus cannot raise the required cash y from outside investors. Moreover, the $K_2(n)$ firms that do invest in equilibrium -do not differ in period 1 from the remaining $n$- $K_2(n)$ *firms* that do not invest.'[18] Consequently, the foregone investment, $K_1(n) - K_2(n) > 0$, cannot be caused solely by such familiar factors as prior securities or prior compensation to managers. Instead, it is a consequence of competition within an industry.

## 4. Capital Structure

The' optimal aggregate investment is supported by optimal capital structures. Specifically, each insider *i* chooses between investment and perks in (11), conditional on his firm's fraction of external equity

[18] Foregone investment with heterogeneous firms is discussed in Section 6.

$a_i$ and its promised payment on external debt $b_i$. Firm $i$ then invests in capital-intensive production if and only if its capital structure $(a_i, b_i)$ satisfies the investment constraint (8), the financial constraint (9), and the inequalities (10), with $b_i > 0$. Again, all such feasible capital structures are shown in the hatched area of Figure 2 and all undominated, feasible capital structures are shown along the adjacent dark line. If the insiders in $k^*$ firms choose capital structures along this line and the insiders in all other firms choose to sell no corporate securities, then these capital structures support the optimal number of capital-intensive firms (14). That is, these capital structures induce the insiders in only $k^*$ firms to choose the capital-intensive technology. All subgame perfect equilibria starting in period 2 then have $k^*$ capital-intensive firms, $n - k^*$ labor-intensive firms, and the associated capital structures.[19] Without loss of generality, the $n$ firms can then be ordered so that the first $k^*$ firms, $i = 1, \ldots, k^*$, are capital-intensive, and the remainder, $i = k^* + 1, \ldots, n$, are labor-intensive.

All optimal capital structures can be identified as follows: Again focus on Figure 2. The undominated capital structures that support an investment in capital-intensive production lie along the dark line below the hatched area. These capital structures $(a_i, b_i)$ satisfy all constraints for all promised payments $b_i$ between two points, labeled $\underline{B}$ and $\overline{B}$ in the figure. At the minimum promised payment $\underline{B}(n)$, the investment constraint (8) and binding financial constraint (9) first meet. At the maximum promised payment $\overline{B}(n)$ the binding financial constraint (9) intersects the $b_i$-axis. To display the lower bound, define by $\Delta S(b, k, n) \equiv S_1(b, k, n) - S_0(b, k - 1, n)$ the incremental value of a firm's stock when it switches from labor-intensive to capital-intensive production, given the promised payment on its debt $b$, the number of capital-intensive firms after switching $k$, and the total number of firms $n$. As shown in the Appendix, the minimum promised payment $\underline{B}(n)$ is uniquely determined by the condition:

$$\underline{B}(n) \equiv \min\left\{ b \geq 0 : \frac{\Delta S[b, K(n), n]}{S_1[b, K(n), n]} = \frac{U(\gamma)}{F_1[K(n), n] - \gamma} \right\}, \qquad (16)$$

for all $n = 1, 2, \ldots$. Also, the maximum promised payment uniquely satisfies the second condition:

$$S_1[\overline{B}(n), K(n), n] = F_1[K(n), n] - \gamma, \qquad (17)$$

for $n = 1\ 2, \ldots$. The two bounds, $\underline{B}(n)$ and $\overline{B}(n)$, are well defined:

[19]The number of such equilibria is the number of combinations of $k^*$ capital-intensive firms in $n$ firms: $\binom{n}{k^*}$. These equilibria are distinguished only by the assignment of the identical firms to the two technologies, $j = 0, 1$, and the associated capital Structures.

$0 \leq \underline{B}(n) \leq \overline{B}(n) \leq Y_{11}[K(n), n]$. These results are summarized in the second proposition.

**Proposition 2.** *The optimal number of capital-intensive firms (14) is supported by the following capital structures. Each capital-intensive firm i has a capital structure* $(a_i^*, b_i^*)$ *satisfying*

$$a_i^* = \frac{\gamma - b_i^* + \max\{0, b_i^* - Y_{10}[K(n), n]\}}{(1-\pi)\max\{0, Y_{10}[K(n), n] - b_i^*\} + \pi\{Y_{11}[K(n), n] - b_i^*\}} \quad (18)$$

*and*

$$\underline{B}(n) \leq b_i^* \leq \overline{B}(n), \quad (19)$$

*with* $0 \leq \underline{B}(n) \leq \overline{B}(n) \leq Y_{11}[K(n), n]$, *for all* $i = 1, \ldots, K(n)$. *Each labor-intensive firm i optimally sells no external securities:* $a_i^* = 0 = b_i^*$, *for all i* $K(n) + 1, \ldots, n$.

In this model capital-intensive firms have optimal, but not uniquely optimal capital structures. The optimal promised payment $b_i^*$, and thereby the resulting market value of its debt, $B_1[b_i^*, K(n), n]$, is bounded below in (16) and above in (17). Because the financial constraint (9) is binding, the bounds on optimal equity are also determined by (16) and (17). Again, see Figure 2. As illustrated, the feasible set of capital structures is smaller or, equivalently, these bounds are tighter, other things equal, when the aggregate capital is second-best versus first-best. Also, the lower bound on debt has an interesting economic interpretation. Suppose that insider $i$ announces his plan to invest in capital-intensive production and sells corporate debt with the promised payment: $b_i \geq Y_{01}[K(n), n]$. This is the lower bound on optimal debt in Figure 2 when the interior of the hatched area is empty-that is, when the integer constraint on aggregate capital is not binding and the optimal aggregate investment is second-best. If, subsequently, the insider chooses labor-intensive production and expends the cash y on perks, then his stock becomes worthless: $S_0[b_i, K(n), n] = 0$ for all $b_i \geq Y_{10}[K(n), n]$. Clearly, this diminishes. his incentive in (7) to choose labor-intensive production and thereby to consume perks. In this sense, sufficiently high levels of corporate debt relax the binding investment constraint (8).[20]

[20] This differs from debt as a precommitment to increase operating risk.In this model risk is endogenous in equilibrium. In the equilibria itientjfied in Section 5, capital-intensive production that is financed partly by debt is never more risky than labor-intensive production without debt.

for which the NPV from entry, $V(n) - \delta$, is nonnegative. For the next larger number of firms, $n^* + 1$, the NPV is negative in (21). Thereby, only $n^*$ firms are induced to enter noncooperatively in equilibrium.

*Limiting solutions.* In general, explicit solutions are possible only for equilibria with large numbers of entrants, including a perfectly competitive equilibrium, $n = \infty$. The latter equilibrium can be constructed as the limit of a sequence of solutions to the optimality conditions, (12), (13), (14), and (21), with progressively smaller costs of entry $\delta$ and thereby progressively larger numbers of entrants in equilibrium $n^*_\delta$. Each equilibrium in this sequence can be characterized by its number of capital-intensive firms $K(n^*_\delta)$ and total number of entrants $n^*_\delta$. For some values of the parameters, explicit solutions, $K(n^*_\delta)$ and $n^*_\delta$ can be calculated for all sufficiently small but still positive costs of entry: $0 \leq \delta \leq \delta_0$, with some $\delta_0 > 0$. For other values of the parameters, only approximate solutions can be calculated explicitly. With the approximate solutions, $K(n^*_\delta)$ and $n^*_\delta$ the inequalities (21) are satisfied to order $1/(n^*_\delta)^2$: $V(n^*_\delta) - \delta = O(1/n^{*2}_\delta)$ In the former case, the equilibria are called competitive, but not perfectly competitive. In the latter case the equilibria are called approximate. In all cases, all equilibria with identical numbers of capital-intensive and labor-intensive firms are grouped together and identified henceforth as one equilibrium

To display these limiting solutions, some additional notation is necessary. Three sets of parameters, $\kappa_s$ and $\nu_s$, for s = 0, 1, 2, are defined in the Appendix in (A25) through (A30). These constants are shown next to be the number of capital-intensive firms and total number of firms in three limiting economies. The limiting economies are characterized below in three propositions.

**Proposition 3A.** *If $\gamma + U(\gamma) > \beta^2/\theta$ then a positive constant $\delta_0$ exists such that the unique equilibrium for each cost of entry, $0 \leq \delta \leq \delta_0$, has the aggregate capital and the number of entrants:*

$$K(n^*) = K_2(n^*) = \kappa_0 = 0, \qquad n^* = \nu_0, \tag{22}$$

*with $\nu_0 = 1/\sqrt{\delta})$. In these equilibria, labor-intensive firms always produce in (2): $Q_{01}(0, \nu_0) > Q_{00}(0, \nu_0) > 0$. In the limit as $\delta \to 0$, operating leverage is stochastically dominant if $\gamma \leq \beta^2/\theta$, and stochastically dominated otherwise.*

Proposition 3B. *If $\lambda\beta^2/\theta < \gamma + U(\gamma) \leq \beta^2/\theta$, then the unique perfectly competitive equilibrium can be constructed as the limit, when $\delta \to 0$, of a sequence of approximate equilibria with the aggregate* *capital and the number of entrants:*

$$K(n^*) = K_2(n^*) = \kappa_1, \qquad n^* = \nu_1, \tag{23}$$

*with* $\nu_1 = O(1/\delta)$. *In these approximate equilibria, labor-intensive firms do not produce in (2) for the small market,* $m = 0$: $Q_{01}(\kappa_1, \nu_1) > Q_{00}(\kappa_1, \nu_1) = 0$. *In the limit as* $\delta \to 0$, *operating leverage is stochastically dominant if*

$$\frac{U(\gamma)}{\gamma} \geq \pi\left(\frac{\beta^2}{\gamma\theta} - 1\right).$$

*Operating leverage is never stochastically dominated.*

*Proposition 3C. If* $\gamma + U(\gamma) \leq \lambda\beta^2/\theta$, *then the unique equilibrium for each cost of entry,* $0 \leq \delta < \infty$, *has the aggregate capital and the number of entrants:*

$$K(n^*) = K_2(n^*) = \kappa_2, \qquad n^* = \nu_2,$$

*with* $\nu_2 = O(1/\delta)$. *In these equilibria, labor-intensive firms never produce in (2):* $Q_{0m}(\kappa_2, \nu_2) = 0$, *for* $m = 0, 1$. *Also, operating leverage is stochastically dominant if*

$$\frac{U(\gamma)}{\gamma} \geq \eta\pi\frac{\eta + 2(\zeta - \alpha + \beta)}{(\zeta - \alpha + \beta)^2}.$$

*This inequality becomes necessary and sufficient as* $\theta \to 0$. *Operating leverage is never stochastically dominated.*

These equilibria have plausible properties. If the cost of entry is very small, then the number of entrants in equilibrium $n^*$ is very large, while the number of capital-intensive firms $K(n^*)$ is relatively small. In these equilibria two costs are critical: the fixed cost of capital-intensive production y and the dissipation from perks, $\gamma - U(\gamma)$. When the former cost is sufficiently large and the latter cost is sufficiently small, so that the inequality in Proposition 3A is satisfied, then all firms are labor-intensive in the competitive equilibria. With small costs of entry, many firms enter and none become capital-intensive: $\nu_0 = O(1/\sqrt{\delta})$ and $K(n^*) = 0$. As a result, all entrants earn only competitive profits: $F_0[K(n^*), n^*] \approx \delta$. Alternatively, if capital-intensive production is not too costly and perks are sufficiently dissipative, then even more firms are induced to enter: $\nu_s = O(1/\delta)$, for $s = 1, 2$. Nevertheless, a few firms always invest in the capital-intensive technology: $\kappa_s > 0$ with $\kappa_s/\nu_s = O(\delta)$, for $s = 1, 2$. Capital-intensive firms always produce large outputs profitably, while labor-intensive firms fail to produce in at least one market and, under the conditions of Proposition 3C, fail in both markets. At best, labor-intensive firms axe small and unprofitable relative to capital-intensive firms: $Q_{01}(\kappa_1, \nu_1)/Q_{11}(\kappa_1, \nu_1) = O(\sqrt{\delta})$ and $Y_{01}(\kappa_1, \nu_1)/Y_{11}(\kappa_1, \nu_1) = O(\delta)$ in Proposition 3B. In this sense, some capital-intensive firms can persist and prosper in large markets, while many small, labor-intensive firms enter and then fail. Even vanishingly small costs of entry are not sufficient for competitive profits. Costly agency access to capital effectively becomes another barrier to entry.

The equilibria have other interesting properties. If the cost of entry $\delta$ is sufficiently small, then the upper bound on feasible capital (13) is always weakly binding: $K(n^*) = K_2(n^*) \leq K_1(n^*)$ in (22) through (24). If the aggregate capital-intensity $k$ were not restricted to an integer, then this upper bound would always be strictly binding in (23) and (24). This has three implications. First, the NPV of capital-intensive production is positive in the equilibria of Propositions 3B and 3C. Because the NPV of foregone investment is positive at the margin, capital is effectively rationed in these equilibria. Second, capital rationing affects the endogenous risk of operating leverage. With rationing in Propositions 3B and 3C, the capital-intensive technology is either stochastically dominant or more risky than the labor-intensive technology, but never stochastically dominated. Finally, optimal capital structures for capital-intensive firms can be tightly constrained in the latter equilibria, (23) and (24). As illustrated in Figure 2, the feasible set of capital structures is smaller, other things equal, when the number of capital-intensive firms $k$ is closer to the upper bound $K_2(n)$. When the upper bound is binding, $k^* = K_2(n) < K_1(n)$, and the integer constraint is not, $K_2(n) = K_2^*(n)$ from (A14) in the Appendix, then the interior of the feasible set is empty and all optimal capital structures are restricted to the dark line.

*Numerical Solutions.* In general, the number of entrants in equilibrium must be calculated numerically. Numerical solutions for selected values of all parameters are shown in Table 1. Values of the parameters for the base case are shown in the upper left. At the head of each column is the value of the parameter that is perturbed; only it can be different from the base case. Just below is the number of entrants in equilibrium $n^*$, and below that is the number of capital-intensive firms $K(n^*)$. For example, in the base case the industry has in equilibrium 9 entrants and 2 capital-intensive, firms: $n^* = 9$ and $K(n^*) = 2$. In most cases the equilibrium is second-best: $K(n^*) = K_2(n^*) < K_1(n^*)$.

The next three numbers measure the marginal return from investment in equilibrium. The first such variable, $MR_0$, is the marginal return from the labor-intensive technology, calculated as the NPV per dollar invested in entry: $F_0[K(n^*), n^*]/\delta - 1$. The next variable, $MR_1$, is the marginal return from capital-intensive production; also calculated as the NPV per dollar invested in entry: $\{F_1[K(n^*), n^*] - \gamma\}/\delta - 1$. In the

**Table 1**
**Numerical solutions[a]**

| parameter values for the base case | variables in rows | $\alpha$ variable cost of production with labor-intensive technology | | |
|---|---|---|---|---|
| | | **1.0** | **2.0** | **3.0** |
| $\alpha = 2.0$ | $n^*$ | 13 | 9 | 2 |
| $\beta = 0.5$ | $K(n^*)$ | 3 | 2 | 2 |
| $\gamma = 500$ | $MR_0$ | −0.566 | −0.500 | 0.111 |
| $\delta = 100$ | $MR_1$ | 2.291 | 2.500 | 3.444 |
| $\zeta = 4.0$ | $MRS$ | 0.571 | 0.600 | 0.667 |
| $\eta = 1.0$ | $MRS_0$ | 0.429 | 0.400 | 0.000 |
| $\theta = .0005$ | $GRR$ | 0.235 | 0.255 | — |
| $\pi = 0.5$ | $NRR$ | 0.593 | 0.619 | — |
| $U(\gamma)/\gamma = 0.5$ | $RS$ | 19.12 | 17.00 | — |

| $\beta$ reduction in variable cost with capital-intensive technology | | | $\gamma$ fixed cost of investment in capital-intensive technology | | |
|---|---|---|---|---|---|
| **.25** | **.50** | **.75** | **200** | **500** | **800** |
| 10 | 9 | 14 | 6 | 9 | 10 |
| 2 | 2 | 4 | 6 | 2 | 0 |
| 0.074 | −0.500 | −1.000 | −1.000 | −0.500 | 0.074 |
| −1.403 | 2.500 | 2.650 | 0.776 | 2.500 | 1.620 |
| −0.295 | 0.600 | 0.730 | 0.888 | 0.600 | 0.193 |
| −0.386 | 0.400 | 0.210 | 0.276 | 0.400 | 0.080 |
| — | 0.255 | — | — | 0.255 | — |
| — | 0.619 | — | — | 0.619 | — |
| — | 17.00 | ∞ | — | 17.00 | — |

| $\delta$ cost of entry into industry | | | $\zeta$ intercept of inverse demand curve in weak market, $m = 0$ | | |
|---|---|---|---|---|---|
| **50** | **100** | **150** | **3.0** | **4.0** | **5.0** |
| 14 | 9 | 6 | 2 | 9 | 13 |
| 1 | 2 | 3 | 2 | 2 | 3 |
| −0.244 | −0.500 | −0.660 | 0.111 | −0.500 | −0.556 |
| 5.089 | 2.500 | 1.245 | 3.444 | 2.500 | 2.291 |
| 0.533 | 0.400 | 0.571 | 0.667 | 0.400 | 0.571 |
| 0.400 | 0.600 | 0.286 | 0.000 | 0.600 | 0.429 |
| 0.223 | 0.255 | 0.274 | — | 0.255 | 0.235 |
| 0.589 | 0.619 | 0.682 | — | 0.619 | 0.593 |
| 21.29 | 17.00 | 16.40 | — | 17.00 | 19.12 |

[a] At the head of each column is the value of the indicated parameter. All other parameters have the values in the base case. Below the head of each column are nine numbers. From top to bottom these numbers are as follows: (1) $n^*$—the total number of firms entering the industry in equilibrium;

**Table 1**
**Continued**[a]

| $\eta$ incremental inverse demand in strong market, $m = 1$ | | | $\theta$ slope of inverse demand with respect to aggregate output $q$ | | |
|---|---|---|---|---|---|
| **0.0** | **1.0** | **2.0** | **.0002** | **.0005** | **.0008** |
| 7 | 9 | 11 | 26 | 9 | 8 |
| 2 | 2 | 3 | 6 | 2 | 0 |
| −0.688 | −0.500 | −0.549 | −1.000 | −0.500 | 0.003 |
| 1.812 | 2.500 | 1.951 | 3.439 | 2.500 | 1.600 |
| 0.500 | 0.600 | 0.500 | 0.888 | 0.600 | 0.319 |
| 0.500 | 0.400 | 0.167 | 0.276 | 0.400 | 0.181 |
| — | 0.255 | 0.284 | — | 0.255 | 0.370 |
| — | 0.619 | 0.765 | — | 0.619 | 1.080 |
| 25.00 | 17.00 | 17.62 | ∞ | 17.00 | 7.57 |
| **$\pi$ probability of strong market with high demand, $m = 1$** | | | **$U(\gamma)/\gamma$ utility of perks relative to cost of perks** | | |
| **.25** | **.50** | **.75** | **.25** | **.50** | **.75** |
| 8 | 9 | 4 | 4 | 9 | 10 |
| 2 | 2 | 4 | 4 | 2 | 1 |
| −0.568 | −0.500 | −0.400 | −0.600 | −0.500 | −0.298 |
| 2.210 | 2.500 | 2.600 | 1.400 | 2.500 | 3.339 |
| 0.556 | 0.600 | 0.600 | 0.400 | 0.600 | 0.727 |
| 0.444 | 0.400 | 0.000 | 0.000 | 0.400 | 0.545 |
| 0.211 | 0.255 | — | — | 0.255 | 0.282 |
| 0.538 | 0.619 | — | — | 0.619 | 0.607 |
| 19.00 | 17.00 | — | — | 17.00 | 13.29 |

[a] (2) $K(\pi^*)$—the number of capital-intensive firms in equilibrium; (3) $MR_0$—the marginal return from labor-intensive production, calculated as the NPV per dollar of entry cost, $F_0[K^*(\pi^*), \pi^*]/\delta - 1$; (4) $MR_1$—the marginal return from capital-intensive production, calculated as the NPV per dollar of entry cost, $\{F_1[K^*(\pi^*), \pi^*] - \gamma\}/\delta - 1$; (5) $MRS$—the marginal return of switching from labor-intensive to capital-intensive production, calculated as the incremental NPV per dollar of investment, $\Delta F[K^*(\pi^*), \pi^*]/\gamma - 1$; (6) $MRS_0$—the marginal return in the small market, $m = 0$, of switching from labor-intensive to capital-intensive production, calculated as the incremental cash inflow per dollar of investment, $\Delta Y_1[K^*(\pi^*), \pi^*]/\gamma - 1$; (7) $GRR$—the gross relative risk, calculated as the coefficient of variation for the cash inflow (3) from a capital-intensive firm divided by the corresponding value for a labor-intensive firm; (8) $NRR$—the net relative risk, calculated as the same ratio, but with cash inflow from the capital-intensive firm measured net of the fixed cost of investment $\gamma$; and (9) $RS$—the relative size of a capital-intensive firm, calculated as the expected cash inflow (3) from a capital-intensive firm, divided by the corresponding cash inflow for a labor-intensive firm.

table $MR_0$ is both positive and negative, whereas $MR_1$ is almost always positive and often substantially so: These results reflect the gamble that firms face upon entry. In (20) a firm becomes capital-intensive with probability $K(n^*)/n^*$ and labor-intensive otherwise. Because capital-intensive firms can earn substantial profits in a second-best equilibrium, competitive entry in (21) guarantees that labor-intensive firms often earn negative returns. Finally, the variable, *MRS*, measures the marginal return of switching from labor-intensive to capital-intensive production, as reflected in the incremental NPV per dollar of investment: $\Delta F[K(n^*), n^*]/\gamma - 1$. These returns are also positive with one exception and often quite large. Such large marginal returns to investment suggest that second-best equilibria can have substantial dissipative costs of agency.

The next number in each column measures the endogenous risk from operating leverage. The variable, $MRS_0$, is the marginal return of switching from labor-intensive production, $j = 0$, to capital-intensive production, $j = 1$, in a small market, $m = 0$. It equals the incremental cash inflow in the small market per dollar of investment: $\Delta Y_0[K(n^*), n^*]/\gamma - 1$. If this return is nonnegative, then capital-intensive production is stochastically dominant-case (a) in (4)-otherwise it is more risky than labor-intensive production. In a second-best equilibrium, operating leverage cannot be stochastically dominated-case (c) in (4) – since $\Delta Y_1[K_2(n^*), n^*] > \Delta F[K_2(n^*), n^*] \geq \gamma$. As indicated, $MRS_0$ is nonnegative in all but one case, so that the capital-intensive technology is stochastically dominant in all but one case. This result reflects the binding investment constraint (9) in the second-best equilibria that severely restricts aggregate corporate investment.

The next two ratios also measure the risk of a capital-intensive firm relative to a labor-intensive firm. The gross relative risk, *GRR*, is the coefficient of variation for the cash inflow (3) in equilibrium for a, capital-intensive firm divided by the corresponding coefficient of variation for a labor-intensive firm. The net relative risk, NRR, is the same ratio with one modification; for a capital-intensive firm, the cash. inflow (3) is measured net of the fixed cost of investment y. In both ratios coefficients of variation are used to adjust for the substantially different sizes of the two firms. As indicated by the dashes in the table, relative risk is undefined in the 10 equilibria without labor-intensive firms: $K(n^*) = n^*$. In all remaining equilibria *GRR* is much less than one, while *NRR* is less than one in all remaining equilibria but one. By these additional measures, small labor-intensive firms are again more risky than large capital-intensive firms. Surprisingly, large firms are less risky despite the operating leverage from capital-intensive production and the absence of diversification in this model. This result reinforces the previous observation that operating leverage does not increase risk because outside investors severely ration corporate capital in the second-best equilibria.

The last number measures the size of a capital-intensive firm relative to a labor-intensive firm. The relative size, *RS*, equals the market value of a capital-intensive firm relative to the corresponding market value of a labor-intensive firm: $F_1[K(n^*), n^*]/F_0[K(n^*), n^*]$. Again, as indicated by the dashes in the table, relative size is undefined in all equilibria without labor-intensive firms. In two equilibria labor-intensive firms fail in both markets, $Q_{0m}[K(N^*), n^*] = 0$ for $m = 0, 1$, and hence have no market value. In these equilibria the relative size is infinite, again as indicated in the table. In all remaining equilibria, capital-intensive firms are much larger than labor-intensive firms. The smallest relative size is 7.57; in the base case it is 17.00.

Some comparative statics from the table are worth noting. First, the dissipative costs of agency decrease as the dissipation from perks increases. Specifically, smaller ratios, $U(\gamma)/\gamma$, are associated in equilibrium with smaller marginal returns, $MR_0$, $MR_1$, and *MRS*. This seemingly surprising result has a simple explanation. When perks are more dissipative, the investment constraint (8) is less binding, so that investments with smaller, positive NPVs are financed in equilibrium. Second, the marginal returns, $MR_0$, $MR_1$, and *MRS*, are not monotone in several parameters, including the fixed cost $\gamma$ and the entry cost $\delta$. These complex comparative statics reflect the relationship between profitability and both-entry and investment in equilibrium, the latter as measured by $n^*$ and $K(n^*)$. Given the complex comparative statics, it is not surprising that so many of the empirical results on industrial structure and performance are contradictory.[21]

## 6. Extensions

*Competition.* In this model firms compete only in the product market. All firms in a representative industry produce a homogeneous good with a decreasing aggregate demand curve, so that the output of any one firm affects the price and thereby the optimal outputs of all other firms. In turn, this affects the optimal investment and thus the optimal capital structure of each firm in equilibrium. In fact, firms compete in multiple markets. For example, a variable input used by all firms in an industry can have an upward sloping aggregate supply curve, or the cost of the capital-intensive technology to each firm can increase with the aggregate investment by all firms. In either case, if the cost curves are linear, then the equilibria can be characterized

[21] See Schmalensee (1989) and the references cited therein.

explicitly, as in this model: In particular, if the average variable cost per unit of output is linear in the aggregate demand by all firms for the variable factor, then the parameter $\boldsymbol{\theta}$ is replaced everywhere by the sum of the coefficients from the inverse demand and supply functions; otherwise, the previous solution is unchanged. Alternatively, if the cost of capital is linear in the aggregate capital, then fewer firms are rationed in the second-best equilibrium relative to the first-best equilibrium. As firms are rationed and the cost of capital is reduced, the investment and financial constraints, (8) and (9), are relaxed, and more firms are financed in equilibrium.

*Inside cash.* In the model all firms start with no inside cash and identical opportunities to invest. In this case, the subgame starting in the second period has multiple Nash equilibria, distinguished only by the identities of the $k^*$ capital-intensive firms. Fortunately, this unappealing indeterminancy is easily eliminated by introducing some realistic heterogeneity across firms. Specifically, some firms can be endowed with internal cash. In this case, a ‘pecking order” develops in which firms with more internal cash invest before firms with less initial cash. For example, some firms can start with no cash, while all others begin with the initial cash c satisfying $0 \leq c < \gamma$. In the resulting subgame perfect equilibria, all firms with inside cash invest if some firms without initial cash also invest. Also, depending on the parameters, at least some firms with internal cash invest even when no firm without initial cash invests. These results follow immediately from a minor modification of the financial constraint (9). For firms with internal cash, the required capital $\gamma$ is replaced by the difference $\gamma - c$. This relaxes the financial constraint (9) and thereby raises the critical number of capital-intensive firms, $K_2(n)$ in (12), for firms with internal cash above the corresponding value for firms without internal funds. Thereby, firms with internal funds are less severely rationed than firms without such funds.

*Other securities.* In this model each firm is restricted to external debt and both external and internal equity. In fact, firms can issue a complicated mix of securities, including secured and unsecured debt, callable and convertible debt, and preferred stock to insiders, together with salaries, bonuses, and warrants to insiders. Additional securities have no effect on the previous results, other than enlarging the feasible set of capital structures. For example, if all insiders are compensated with warrants rather than internal equity, then the lower bound on optimal debt in Proposition 2 is replaced by a lower bound on all external securities that are senior to warrants. Otherwise, the previous results are unchanged. If managers are risk-averse, then the second-best solution in Propositions 1B and 2 may be precluded by the trade-off between optimal risk-sharing and efficient investment. This tradeoff may persist even with multiple corporate securities. Alternatively, if managers can consume perks from the final cash flows in period 4, then these perks may effectively function like senior securities. As in *Williams* (1987), this result depends on the priorities between debt, equity, and perks. If perks precede principal plus interest on debt, then perks in the final period may also preclude the above second-best solution.

*Markets.* In Section 3 aggregate demand is restricted to one of two possible states: a small market, $m = 0$, and a large market, $m = 1$. With more discrete states or even continuous states, the previous results change only slightly. Again, focus on Figure 2. As illustrated, the minimum optimal debt of a capital-intensive firm $\underline{B}(n^*)$ is determined in large part by the cash flow from a labor-intensive firm in the larger market, $Y_{01}[K^*(n^*), n^*]$. If the aggregate capital-intensity $k$ could be any nonnegative real number, rather than an integer, then the interior of the hatched area would be empty at a second-best solution and the minimum promised payment would be equal to this cash flow. With many discrete markets, $m = 1, \ldots, M$, or continuous markets, $0 \leq m \leq M$, *the* cash inflow in the largest market $M$ is again critical. For example, if $k$ could be any real number, then the minimum promised payment on debt in the second-best solution would be $Y_{0M}[K_2(n^*), n^*]$. In this case, the minimum optimal debt of a capital-intensive firm would be larger, other things equal, with more states of the market $M$. Moreover, this larger minimum debt may make the previous results more realistic. For example, with two states, $M = 2$, and the largest possible minimum payment, $\underline{B}(n^*) = \max\{Y_{01}[K(n^*), n^*], Y_{10}[K(n), n^*]\}$, the resulting debt-to-value ratio, $B_1[\underline{B}(n), K(n^*), n^*]/F_1[K(n^*), n^*]$, is roughly in the range of .10 to .15 for the numerical solutions in the table.

*Technology.* For expositional ease, each firm was previously assumed to have one of two possible technologies: labor-intensive, $j = 0$, or capital-intensive, $j = 1$. With other discrete or continuous technologies, the previous results are altered either slightly or substantially, depending on the specifics. As argued below (8), the main results in this article require at least two distinct technologies satisfying two properties. The choice between the technologies' must satisfy an investment constraint like (8), and neither technology can dominate the other. For example, each firm $i$ could have an average variable cost $C(k_i)$ that decreases continuously its capital intensity, $k_i \geq 0$. The cost function $C$ could be concave on some initial interval and 'convex thereafter. This could produce two local optima: one at the comer, $k_i = 0$, and another at an interior point, conveniently normalized at $k_i = 1$. Again, the two technologies, 0 and i, must be sufficiently similar so that switching between technologies after securities have been sold to outsiders cannot be precluded contractually. In other words, the investment constraint (8) must apply to these two technologies. In this case, capital would again be rationed in equilibrium, as in Proposition 1B. With this technology, however, managers may consume dissipative perks in equilibrium, unlike the results in Section 3. If the cost function $C$ is convex around the local maximum at $k_i = 1$, then the sharp distinction is lost in the investment constraint (8), and thereby Figure 2, between investment without perks and perks without investment. Instead, manager $i$ would invest marginally less cash and consume marginally more perks with either slightly larger fractions of outside equity $a_i$ or slightly larger promised payments on debt $b_i$. In equilibrium, firms with the capital-intensive technology would then forego at the margin some investments with positive NPV. Otherwise, the previous solution would be unchanged.

With other technologies the previous solution can change radically. For example, the average variable cost of production C could be convex everywhere. In this case, one solution dominates all others, and all firms choose the same capital and produce the same output in equilibrium. The distinction then disappears between labor-intensive and capital-intensive firms. Alternatively, two technologies could be local optima on a discrete or continuous set of technologies, and some contractual constraints on perks could be legally enforceable. For example, investors and courts could observe and penalize large but not small perks, so that another investment constraint, like (8), would apply locally only to small perks. If the perks gained by switching between technologies were large, then the switch could be precluded contractually. In this second case, the investment constraint (8) and thereby the upper bound in (13) would not constrain aggregate investment in equilibrium, rationing could not occur as in Proposition 1B; and large, capital-intensive firms would not necessarily sell at least some debt as in Proposition 2. Instead, the second-best equilibrium would be characterized by some foregone investment at the margin, as in previous models of corporate agency with isolated firms. In equilibrium, investment and perks would still reflect competition among firms in the product market, but outsiders would not ration capital.

## 7. Conclusion

The literature on corporate agency has produced some now familiar predictions: if a manager maximizes his personal welfare, then, depending on his firm's capital structure, he may pick excessively risky projects, forego profitable projects, and consume dissipative perks. With few exceptions these standard results have been derived for a single firm in isolation, rather than an industry in equilibrium. As shown by the exceptions, the equilibrium may have very different properties if firms within an industry compete in a product market. This suggests that the standard predictions may have only limited empirical content. More importantly, the theoretical literature on corporate agency has yet to explain the limited empirical evidence on financial and industrial structure. This evidence includes the observed correlations between a firm's profitability, its physical capital, and the book value of its debt.

In this article, corporate agency is shown to have interesting empirical implications for financial and industrial structure. If the fixed cost of the capital-intensive technology is sufficiently small, then some firms are forced in equilibrium to forego investments with positive NPVs-but for different reasons than those discussed in previous papers. The managers of more firms would like to invest in capital-intensive production, but they cannot credibly commit to avoid dissipative perks. Thus, some firms cannot sell to outside. investors sufficient securities to raise the cash required for the capital-intensive technology. In equilibrium, capital-intensive firms are then profitable, while labor-intensive firms can be unprofitable when measured net of the cost of entry. Also, capital-intensive firms are always larger than labor-intensive firms and much larger in the numerical solutions. Despite their operating leverage with their higher fixed cost and lower variable costs, capital-intensive firms are also less risky in the numerical solutions than labor-intensive firms. In equilibrium, capital-intensive firms finance their investments with at least some outside debt but do not have unique optimal capital structures. Even as the cost of entry approaches zero, capital-intensive firms can earn extraordinary profits, while labor-intensive firms can fail; In this sense, access to capital effectively becomes a barrier to entry with costly agency.

## Appendix

*Derivation of the optimal outputs (2).* The optimal output from (1) is

$$q_{im}^* = \frac{1}{2\theta}\max\{0, \zeta + \eta m - \theta q_{-im}^* - \alpha + \beta k_i\}, \tag{A1}$$

for firms, $i = 1, \ldots, n$, and markets, $m = 0.1$. These equations have a symmetric solution within technologies: $q_{im}^* = Q_{jm}(k, n)$, with $j = k_i$ for all firms $i = 1, \ldots, n$. This is the unique solution in pure strategies by the argument in Szidarovsky and Yakowitz (1977). All other firms produce the total quantity: $q^*_{-im} = (n-k-1)Q_{0m}(k, n) + kQ_{1m}(k, n)$ if $k_i = 0$, and $q^*_{-im} = (n-k)Q_{0m}(k, n) + (k-1)Q_{1m}(k, n)$ if $k_i = 1$. If $Q_{0m}(k, n) > 0$ then (A1) has the unique solution:

$$Q_{jm}(k, n) = \frac{1}{\theta}\left(\beta j + \frac{\zeta + \eta m - \alpha - \beta k}{n+1}\right), \tag{A2}$$

for $j$ = 0.1. Alternatively, if $Q_{0m}(k, n) = 0$, then (A2) has for $j$ = 1 the unique solution:

$$Q_{1m}(k, n) = \frac{\zeta + \eta m - \alpha + \beta}{\theta(k+1)}. \tag{A3}$$

Together, (A2) and (A3) imply (2).

*Calculation of the incremental cash inflow in (4).* The incremental cash in&w, $\Delta Y_m(k, n) \equiv Y_{1m}(k, n) - Y_{0m}(k-1, n)$ is calculated directly from (3). If $0 \le k < (\zeta + \eta m - \alpha)/\beta$, then (3) implies that

$$\Delta Y_m(k, n) = \frac{\beta n}{\theta(n+1)^2}[\beta n + 2(\zeta + \eta m - \alpha + \beta) - 2\beta k]. \tag{A4}$$

Alternatively, if $(\zeta + \eta m - \alpha)/\beta \le k < 1 + (\zeta + \eta m - \alpha)/\beta$, then (3) gives

$$\begin{aligned}\Delta Y_m(k, n) &= \frac{1}{\theta}\left(\frac{\zeta + \eta m - \alpha + \beta}{k+1}\right)^2 \\ &\quad - \frac{1}{\theta}\left(\frac{\zeta + \eta m - \alpha + \beta - \beta k}{n+1}\right)^2.\end{aligned} \tag{A5}$$

Finally, if $1 + (\zeta + \eta m - \alpha)/\beta \le k \le n$, then (3) yields

$$\Delta Y_m(k, n) = \frac{1}{\theta}\left(\frac{\zeta + \eta m - \alpha + \beta}{k+1}\right)^2. \tag{A6}$$

All properties of the incremental cash inflow that are identified below (3) are derived from (A4) through (A6).

*Properties of the investment and financial constraints, (8) and (9).* When a firm switches from labor-intensive production, $j$ = 0, to capital-intensive production, $j$ = 1, it realizes the incremental value: $\Delta F(k, n) \equiv F_1(k, n) - F_0(k-1, n)$ Its stock has the concurrent increment: $\Delta S(b, k, n) \equiv S_1(b, k, n) - S_0(b, k-1, n)$. If $Y_{00}(k-1, n) \le$

$Y_{10}(k, n) < Y_{01}(k-1, n) \leq Y_{11}(k, n)$, then the latter value is

$$\Delta S(k, n) = \begin{cases} \Delta F(k, n) & 0 \leq b < Y_{00}(k-1, n) \\ \Delta F(k, n) - (1-\pi) \\ \times[b - Y_{00}(k-1, n)] & Y_{00}(k-1, n) \leq b < Y_{10}(k, n) \\ \pi \Delta Y_1(k, n) & Y_{10}(k, n) \leq b < Y_{01}(k-1, n) \\ \pi[Y_{11}(k, n) - b] & Y_{01}(k-1, n) \leq b < Y_{11}(k, n). \end{cases} \tag{A7}$$

Alternatively, if $Y_{00}(k-1, n) \leq Y_{01}(k-1, n) \leq Y_{10}(k, n) \leq Y_{11}(k, n)$, then the stock's incremental value is

$$\Delta S(k, n) = \begin{cases} \Delta F(k, n) & 0 \leq b < Y_{00}(k-1, n) \\ \Delta F(k, n) - (1-\pi) \\ \times[b - Y_{00}(k-1, n)] & Y_{00}(k-1, n) \leq b < Y_{01}(k-1, n) \\ F_1(k, n) - b & Y_{01}(k-1, n) \leq b < Y_{10}(k, n) \\ \pi[Y_{11}(k, n) - b] & Y_{10}(k, n) \leq b < Y_{11}(k, n). \end{cases} \tag{A8}$$

The two cases, (A7) and (A8), are mutually exclusive and exhaustive.

The investment and financial constraints can be rewritten as follows. Define the lower and upper bounds:

$$\underline{A}(b, k, n) \equiv 1 - \frac{F_1(k, n) - \gamma}{S_1(b, k, n)},$$

$$\overline{A}(b, k, n) \equiv 1 - \frac{U(\gamma)}{\Delta S(b, k, n)}. \tag{A9}$$

With this notation the investment constraint (8) is

$$a_i \leq \overline{A}(b_i, 1 + k^*_{-i}, n), \tag{A10}$$

for $0 \leq b_i < Y_{11}(1 + k^*_{-i}, n)$. Similarly, the financial constraint (9) is

$$a_i \geq \underline{A}(b_i, k^*, n), \tag{A11}$$

for $0 \leq b_i < Y_{11}(k^*, n)$. The upper (lower) bound in (A9) is decreasing (increasing) in $k$. Also, the upper (lower) bound is decreasing (increasing) in $n$ if $0 \leq k < (\zeta + \eta - \alpha)/\beta$. Figure 2 is drawn from (A10) and (A11) for case (A7).

*Derivation of the first-best and second-best aggregate capital in* (12) *and (13).* Temporarily ignore both the integer constraint on the number of capital-intensive firms $k$ and the remaining bounds, $0 \leq k \leq n$. Without these constraints the optimality condition (12) for the first-best aggregate capital simplifies to

for $n = 1, 2, \ldots$ The difference $\Delta F$ is decreasing everywhere in $k$. Also, it satisfies the comer conditions: $\Delta F(-\infty, n) > \gamma > \Delta F(\infty, n)$. In this case, (A12) is both necessary and sufficient for a unique, real-valued, first-best aggregate capital-intensity K;(n).

Next, consider the second-best, real-valued aggregate capital-intensity. If firm $i$ invests, $k_i = j = 1$, then its capital structure $(a_i, b_i)$ must satisfy the investment and financial constraints, (A10) and (All), and the inequalities, $0 \leq a_i \leq 1$ and $0 \leq b_i \leq Y_{11}(k^*, n)$. Again, the continuous lower (upper) bound in (A9) is everywhere increasing -(decreasing) in k. Also, the bounds in (A9) satisfy the comer conditions: $\overline{A}(b, -\infty, n) - \underline{A}(b, -\infty, n) > 0 > \overline{A}(b, \infty, n) - \underline{A}(b, \infty, n)$. Thus, there must exist a finite real number $K_2^*(n)$ such that

$$K_2^*(n) = \max\{k \geq 0: \underline{A}(b, k, n) < \overline{A}(b, k, n), \text{ for some } 0 \leq b < Y_{11}(k, n)\}, \tag{A13}$$

for $n = 1, 2, \ldots$ To simplify (A13), note that the bounds in (A9) have -the following properties: $\Delta S(b, k, n) \leq S_1(b, k, n)$ with an equality if and only if $b \geq Y_{01}(k-1, n)$; $F_1(k, n) > F_1(k+1, n)$, for all feasible $k$ and $n$; and $F_1(-\infty, n) > \gamma > F_1(\infty, n)$. With these properties, the second-best, real-valued, aggregate capital-intensity, $K(n^*)$ from (A13), is uniquely determined by the condition:

$$\gamma + U(\gamma) = F_1[K_2^*(n), n], \tag{A14}$$

with $b \geq Y_{01}(k-1, n)$ for each n = 1, 2, . . .

The first-best and second-best number of capital-intensive firms follow immediately from the unique solutions to (A12) and (A14). Represent by $[z]$ the largest integer not exceeding the real number $z$: $z - 1 < [z] \leq z$. Using this notation, define two integers:

$$K_s(n) = \begin{cases} 0 & K_s^*(n) < 0 \\ [K_s^*(n)] & 0 \leq K_s^*(n) \leq n \\ n & n < K_s^*(n), \end{cases} \tag{A15}$$

for s = 1, 2, and $n = 1, 2, \ldots$ With s = 1, the integer (A15) is the first-best number of capital-intensive firms in (11). With $s = 2$, it is the second-best number or, equivalently, the upper bound on the number of capital-intensive firms in (13).

*Proof of Proposition 1A.* If $U(\gamma) = 0$, then (A12) and (A14) imply that

$$F_1[K_2^*(n), n] = \gamma = \Delta F[K_1^*(n), n] \leq F_1[K_1^*(n), n]. \tag{A16}$$

Because $F_1$ is decreasing in $k$, then $K_1^* \leq K_2^*$, so that $K_1 \leq K_2$ from (A15). This produces the desired result: $k^* = K_1$. The monotonicity of $K_1^*$ and thereby $K_1$ follows from the monotonicity in (A12) of $\Delta F$ with respect to $k$.

*Values of firms.* If $0 \leq k < (\zeta - \alpha)/\beta$, then labor-intensive firms, $j = 0$, produce in both markets, $m = 0, 1$. Given (3), labor-intensive and capital-intensive firms then have the respective values:

$$F_0(k, n) = \frac{1}{\theta(n+1)^2}[(\zeta + \eta\pi - \alpha - \beta k)^2 + \eta^2\pi(1-\pi)] \tag{A17}$$

and

$$F_1(k, n) = \frac{1}{\theta(n+1)^2}\{[\zeta + \eta\pi - \alpha + \beta(n-k+1)]^2 + \eta^2\pi(1-\pi)\}. \tag{A18}$$

Alternatively, if $(\zeta - \alpha)/\beta \leq k < (\zeta + \eta - \alpha)/\beta$, then labor-intensive firms produce only in the large market, $m = 1$. In this second case, the two types of firms have the values:

$$F_0(k, n) = \frac{\pi}{\theta}\left(\frac{\zeta + \eta - \alpha - \beta k}{n+1}\right)^2 \tag{A19}$$

and

$$F_1(k, n) = \frac{1-\pi}{\theta}\left(\frac{\zeta - \alpha + \beta}{k+1}\right)^2 + \frac{\pi}{\theta}\left(\beta + \frac{\zeta + \eta - \alpha - \beta k}{n+1}\right)^2. \tag{A20}$$

Finally, if $k \geq (\zeta + \eta - \alpha)/\beta$, then labor-intensive firms never produce. As a result, the two types have the values: $F_0(k, n) = 0$ and

$$F_1(k, n) = \frac{1}{\theta(k+1)^2}\left[(\zeta + \eta\pi - \alpha + \beta)^2 + \eta^2\pi(1-\pi)\right], \tag{A21}$$

for $n = 1, 2, \ldots$. For labor-intensive firms, the valuation function $F_0$ is continuously differentiable in the $k$, for all $k \geq 0$. For capital-intensive firms, the valuation function $F_1$ is continuous and piecewise differentiable on $k \geq 0$, with kinks at $k = (\zeta - \alpha)/\beta$ and $k = (\zeta + \eta - \alpha)/\beta$.

*Proof of Proposition 1B.* The monotonicity of $K_2^*$ in (A14) and thereby the monotonicity of $K_2$ in (A15) follow immediately from the monotonicity of $F_1$ with respect to $k$ and n. For the remaining results it is sufficient to show that $K_1^*(n) > K_2^*(n)$ for $0 < \gamma < \Gamma(n)$ and $K_1^*(n) < K_2^*(n)$ for $\Gamma(n) < \gamma < \infty$.
tion, temporarily suppress the variable n and express $K_1^*$ and $K_2^*$ from (A12) and (A14) as functions solely of the cost y. With (A12), (A14), and the monotonicity of $\Delta F$ in γ, then $K_1^* > K_2^*$ only if $U = F_1^*(K_2^*) - \gamma = \Delta F(K_2^*) - \gamma + F_0(K_2^*) > \Delta F(K_1^*) - \gamma + F_0(K_2^*) = F_0(K_2^*) > F_0(K_1^*)$. Alternatively, if $K_1^* < K_2^*$, then the same argument yields $U < F_0(K_2^*) < F_0(K_1^*)$. Therefore, it is sufficient to show that the continuous function, $G \equiv U - F_0(K_1^*)$, is positive on an initial interval, $0 < \gamma < \Gamma$, and negative on the subsequent interval, $\Gamma < \gamma < \infty$.

The function $G$ has the following properties. It is piecewise continuously differentiable everywhere in y except possibly at the above kinks in $F_l$. These kinks can occur only at two values, $\gamma_1$ and $\gamma_2$, such that $K_1^*(\gamma_1) = (\zeta + \eta - \alpha)/\beta$ and $K_1^*(\gamma_2) = (\zeta - \alpha)/\beta$. These two costs, $0 < \gamma_1 < \gamma_2 < \infty$, correspond to the smallest values of k such that $Q_{0m}(k, n) = 0$ in (2) for $m = 1$ and $m = 0$, respectively. These properties follow from (A17) through (A21) and the monotonicity of $K_1^*$ in $\gamma$. In turn, these two possible kinks define three intervals: (a) $0 < \gamma < \gamma_1$, (b) $\gamma_1 < \gamma < \gamma_2$, and (c) $\gamma_2 < \gamma < \infty$. On each of these three intervals, G has the slope:

$$G' = U' - \frac{\partial F_0/\partial k}{\partial \Delta F/\partial k}, \tag{A22}$$

evaluated at $k = K_1^*(\gamma)$. This derivative is calculated using (A12).

Focus first on the initial interval (a). Here, the aggregate capital, $k = K_1^*(\gamma)$, satisfies $k \geq (\zeta + \eta - \alpha)/\beta$. In this case, the slope (A22) simplifies to $G' = U' > 0$. Also, with U(0) = 0 and the argument in Proposition 1A, $G$ has the initial value: G(0) = 0. Thus, G is positive on (a), so that $\gamma_1 < \Gamma(n)$.

Next, consider the intermediate interval (b). Now, the aggregate capital, $k = K_1^*(\gamma)$, satisfies $(\zeta - \alpha)/\beta \leq k < (\zeta + \eta - \alpha)/\beta$. In this second situation, the slope (A22) is

$$G' = U' + \frac{K_1^* - \frac{\zeta+\eta-\alpha}{\beta}}{n + \left(1 + \frac{\zeta-\alpha}{\beta}\right)^2 \frac{1-\pi}{\pi} \frac{(n+1)^2}{(K_1^*+1)^3}}, \tag{A23}$$

for $\gamma_1 < \gamma < \gamma_2$. In the second term of (A23), the numerator is decreasing in y, and the denominator is greater than 1. Given these properties and $U'' < 0 < G(\gamma_1)$, the function $G$ can have at most one zero on (b). Also, if $G$ has a zero on (b), then $G(\gamma_2) < 0$.

Finally, focus on the upper interval (c). Here, the aggregate capital, $k = K_1^*(\gamma)$, satisfies $0 \leq k < (\zeta - \alpha)/\beta$. In this final situation, the slope (A22) is

$$G' = U' + \frac{K_1^* - \frac{\zeta+\eta\pi-\alpha}{\beta}}{n}, \tag{A24}$$

for $\gamma_2 < \gamma < \infty$. Since $K_1^*$ is decreasing in y, then G is strictly concave on (c). Under the hypothesis in the proposition, $n/(n+1)^2 \leq \beta/(\zeta - \alpha + \beta)$, if $G'(\gamma_2-) < 0$, then $G'(\gamma_2+) < 0$. In this case, if $G$ has a zero on (b), then it can have no zero on (c), whereas if $G$ has no zero on (b), then it can have at most one zero on (c). Also, if G has a zero on (b) or (c) then and only then $G(\infty) < 0$. Therefore, the bound in (15) satisfies $0 < \Gamma(n) < \infty$ if and only if $G(\infty) < 0$; otherwise, it satisfies $\Gamma(n) = \infty$. Since $\Delta F$ is decreasing in $k$ and $\Delta F(0, \cdot) < \infty$, then $K_1(n, \gamma) \leq K_1^*(n, \gamma) = 0$ for all sufficiently large $\gamma$. Thus, $U(\infty) = U(\infty) - F_0(0, n) \geq 0$, and the desired results follow from the indicated inequalities for $U(\infty)$ and $F_0(0, n)$. This completes the proof.

*Notation for Propositions 3A-C.* First, define the constant:

$$\lambda \equiv \frac{(\zeta + \eta\pi - \alpha + \beta)^2 + \eta^2\pi(1 - \pi)}{(\zeta + \eta - \alpha + \beta)^2}. \tag{A25}$$

It satisfies $\pi < \lambda < 1$. Next, construct two constants:

$$\kappa_1^* \equiv (\zeta - \alpha + \beta)\sqrt{\frac{1 - \pi}{\theta[\gamma + U(\gamma)] - \beta^2\pi}} - 1 \tag{A26}$$

and

$$\kappa_2^* \equiv (\zeta + \eta - \alpha + \beta)\sqrt{\frac{\lambda}{\theta[\gamma + U(\gamma)]}} - 1. \tag{A27}$$

These constants have the corresponding integers: $\kappa_s \equiv [\kappa_s^*]$, for $s$ = 1, 2. In turn, each integer is associated with another constant:

$$\nu_1^* \equiv \frac{\kappa_1}{\delta} U(\gamma) \tag{A28}$$

and

$$\nu_2^* \equiv \frac{\kappa_2}{\delta}\left[\frac{(\zeta + \eta\pi - \alpha + \beta)^2 + \eta^2\pi(1 - \pi)}{\theta(\kappa_2 + 1)^2} - \gamma\right]. \tag{A29}$$

The latter constants have the corresponding integers: $\nu_s \equiv [\nu_s^*]$, for

$s = 1, 2$. Finally, construct the constant:

$$\nu_0^* \equiv \sqrt{\frac{1}{\delta\theta}[(\zeta + \eta\pi - \alpha)^2 + \eta^2\pi(1-\pi)]} - 1, \qquad \text{(A30)}$$

and the corresponding integer $\nu_0 \equiv [\nu_0^*]$. For notational symmetry, the latter integer is matched with the integer $\kappa_0 = 0$. The parameters, $\kappa_s$ and $\nu_s$, for $s = 0, 1, 2$, are shown in Propositions 3A-C to be the aggregate capital and the number of entrants in three limiting economies.

*Proof of Proposition 3C.* If $k \geq (\zeta + \eta - \alpha)/\beta$, then labor-intensive firms never produce, and the two types of firms have the values: $F_0(k, n) = 0$ and (A21). Inserting these values into (A12) and (A14) gives $K_1^*(n) = (\zeta + \eta - \alpha + \beta)\sqrt{\lambda/\gamma\theta} - 1$ and $K_2^*(n) = \kappa_2^*$. The first function is strictly larger than the second: $K_1^* > K_2^*$. From its definition in the text above Proposition 3A, the constant $\kappa_2$ satisfies $\kappa_2^* \geq (\zeta + \eta - \alpha)/\beta$ if and only if $\gamma + U(\gamma) \leq \lambda\beta^2/\theta$. Because the latter inequality holds by hypothesis, the above upper interval for $k$ applies, along with the subsequent results. In this case (A15) requires that $K_1(n) \geq K_2(n)$, for $n = 1, 2, \ldots$. Also, with $F_0(k, n) = 0$, *the* maximand (20) simplifies to

$$V(n) = \frac{\kappa_s}{n}\{F_1(\kappa_s, n) - \gamma\}, \qquad \text{(A31)}$$

with $s = 2$. Inserting (A21) and (A31) into the optimality conditions (21) yields $n^* = \nu_2$. The results on stochastic dominance follow from (4); (24), and (A6).

*Proof of Proposition 3B.* Under the hypothesis, $\gamma + U(\gamma) > \lambda\beta^2/\theta$, the above proof of Proposition 3C ensures that $K^*(n) > (\zeta + \eta - \alpha)/\beta$, for some real valued $n$, only if $K^*(n) = \kappa_2 < (\zeta + \eta - \alpha)/\beta$, a contradiction. Hence, the hypothesis requires that $K^*(n) \leq (\zeta + \eta - \alpha)/\beta$, for $n = 1, 2, \ldots$. In this case, (A17) through (A20) apply, so that switching from labor-intensive to capital-Intensive production has the value:

$$\Delta F(k, n) = \frac{\beta n}{\theta(n+1)^2}[\beta n + 2(\zeta + \eta\pi - \alpha + \beta - \beta k)], \qquad \text{(A32)}$$

for $0 \leq k < (\zeta - \alpha)/\beta$, and

$$\Delta F(k, n) = \frac{1-\pi}{\theta}\left(\frac{\zeta - \alpha + \beta}{k+1}\right)^2 + \frac{\pi}{\theta}\frac{\beta n}{(n+1)^2}[\beta n + 2(\zeta + \eta - \alpha + \beta - \beta k)], \qquad \text{(A33)}$$

for $(\zeta-\alpha)/\beta \leq k < (\zeta+\eta-\alpha)/\beta$.] Because $F_1$ and $\Delta F$ are now decreasing everywhere in both $k$ and $n$, then $K_1^*$ and $K_2^*$ from (A12) and (A14) are decreasing everywhere in n. This leaves two possibilities. The first possibility, $(\zeta-\alpha)/\beta \leq K^*(n) \leq (\zeta+\eta-\alpha)/\beta$, for all $n = 1, 2, \ldots$, is considered here. The remaining possibility, $K^*(n) < (\zeta-\alpha)/\beta$ for some $n$, is considered in the proof of Proposition 3A.

With $(\zeta-\alpha)/\beta \leq k \leq (\zeta+\eta-\alpha)/\beta$, the two types of firms have the market values, (A19) and (A20), and the incremental value (A33). both (A20) and (A33) have the same uniform approximation:

$$F_1(k,n), \Delta F(k,n) \to \frac{1}{\theta}\left[(1-\pi)\left(\frac{\zeta-\alpha+\beta}{k+1}\right)^2 + \beta^2\pi\right] + O\left(\frac{1}{n}\right), \tag{A34}$$

as $n \to \infty$. Because $\gamma + U(\gamma) > \lambda\beta^2/\theta > \pi\beta^2/\theta$, then Equations (A14) and (A34) imply that $K_2^*(n) \to \kappa_1^*$ as $n \to \infty$. Also, (A12) and (A34) imply that $K_1^*(n) > K_2^*(n)$, and thereby $K_1(n) \geq K_2(n)$ for all sufficiently large integers $n$. Inserting (A34) with $k = \kappa_1$ into (A31) with $s = 1$ yields $V(v_1^*) - \delta = O(1/v_1^{*2})$. In addition, $\kappa_1^* \geq (\zeta-\alpha)/\beta$ holds if $\gamma + U(\gamma) \leq \beta^2/\theta$. Hence, (23) holds if $\lambda\beta^2/\theta < \gamma + U(\gamma) \leq \beta^2/\theta$. The results on stochastic dominance then follow from Equations (4), (23), (A4), and (A5).

*Proof of Proposition 3A.* In the remaining case, the optimal aggregate capital satisfies $K^*(n) < (\zeta-\alpha)/\beta$, for some integer $n$. In this case, (A14) and (A18) imply that

$$K_2^*(n) = n + 1 + \frac{\zeta + \eta\pi - \alpha}{\beta} - \frac{1}{\beta}\sqrt{\theta[\gamma + U(\gamma)](n+1)^2 - \eta^2\pi(1-\pi)}. \tag{A35}$$

This function is strictly decreasing in $n$, with the limit:

$$\frac{K_2^*(n)}{n+1} \to 1 - \frac{1}{\beta}\sqrt{\theta[\gamma + U(\gamma)]} < 0, \tag{A36}$$

as $n \to \infty$. The inequality in (A36) follows from the hypothesis of the proposition: $\gamma + U(\gamma) > \beta^2/\theta$. Thus, there exists some $n_0 < \infty$ such that $K_2^*(n) = 0$ for all integers $n \geq n_0$. With $k = 0$ and (A17), the maximand (19) simplifies to

$$V(n) = F_0(0, n) = \frac{1}{\theta(n+1)^2}[(\zeta + \eta\pi - \alpha)^2 + \eta^2\pi(1-\pi)]. \quad \text{(A37)}$$

Inserting (A37) into (21) gives $n^* = v_0$. Setting $v_0^* = n_0$ in the definition of $v_0^*$ yields the critical entry cost, $\delta_0 > 0$. This is (22). The results on stochastic dominance follow from (4), (22), and (A4).

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