Costly State Verification and Multiple Investors: The Role of Seniority

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Many financial claims specify fixed maximum payments, varying seniority, and absolute priority for more senior investors. These features are motivated in a model where a firm's manager contracts with several investors and firm output can only be verified privately at a cost. Debt-like contracts of varying seniority generally dominate symmetric contracts, and, when investors are risk neutral, it is optimal to use debt-like contracts where more senior claims have absolute priority over more junior claims. In addition to motivating severalfeatures of debt and preferred stock, the model offers an explanation for structures used in leveraged buyouts, asset-backed securitizations, and reinsurance contracts.

Although firms issue many different financial claims, three features are quite common. Many of these claims have contractually fixed maximum payments, debt and preferred stock being, obvious examples. Claims often vary in their degree of seniority, which specifies who is first entitled to full payment. Moreover, the contractual order of payments typically follows

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absolute priority: senior claims are to be paid in full before more junior claims receive anything.[1]

To motivate the use of these features, I examine contracting in a model where a manager's firm requires funds from several investors. Investors can only observe and verify the firm's return privately and at a cost. This setting is described more fully in Section 1.

Section 2 briefly reviews contracting when the firm requires only one investor. Optimal incentive-compatible contracts resemble debt: the investor receives a fixed (unverified) payment when returns are good and verifies and receives a strictly lesser amount when returns are poor.[2] If the manager is risk averse, he typically receives some income in verified states; and there is a break in the payment scheme at the point where verification begins.

Section 3 turns to the situation where the manager needs funds from several investors. In general, optimal contracts have many features in common with that from the single investor case, and they have precisely the same form if contracts are required to be symmetric (all investors receive the same payments and verify in the same states of the world). More interestingly, when investors are risk neutral, debt-like contracts that specify absolute priority among investors of differing seniority are in fact optimal among all incentivecompatible contracts.

Section 4 shows that, although all investors have the same endowments and preferences, symmetric contracts are typically dominated by debt-like contracts of varying seniority. There are two reasons for this. First, symmetric contracts entail complete duplication of verification costs. Making some investors senior reduces their need to verify and increases their expected consumption; under certain conditions, their fixed payment can be reduced so as to make everyone better off.[3] Second, because symmetric contracts usually have a discontinuity when the manager is risk averse, they involve imperfect risk sharing.

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[1] Although absolute priority an be violated in Chapter 11 proceedings, it is still the rule if renegotiation fails; see White (1989).

Since the absolute priority rule describes priority for fixed payments on securities, securities with a conversion feature (warrants, convertible debt) do not fit neatly into its scheme. Although the conversion feature is in some respects junior to outstanding common stock, it only receives value if the investor converts the claim into common shares: these shares have no contractually fixed maximum payment and come last under absolute priority.

[2] The proof is similar to that in Townsend (1979), the main difference being that in this case the investor an be risk averse.

[3] This is always possible when the optimal symmetric contract is debt, as when the manager is risk neutral. Also, as discussed in Section 1, although the article focuses on the case where contracts involve deterministic verification, these results should apply when stochastic verification is possible.

Although the total cost of having all investors verify more often outweighs any benefit from improving risk sharing, having a subset of investors verify more often and receive a higher fixed payment generally allows a Pareto-improving decrease in the risk the manager bears.

Section 5 discusses empirical implications. The article's basic results are consistent with the three features mentioned at the beginning of this introduction. Although the debt-like contracts discussed here most closely resemble debt or preferred stock, the most junior class bears some likeness to common stock with a substantial and stable dividend and complete managerial expropriation of "free cash flow." To the extent that investors are better diversified and thus effectively less risk averse than the manager, absolute priority among investors may be the best rule to use as a legal standard. In addition, the result that risk-averse managers optimally receive a fixed payment when risk-neutral investors verify is consistent with U.S. bankruptcy law, which provides that employees' claims for any back pay that they are owed have higher priority than do unsecured creditors and bondholders.[5]

A closer look at the circumstances under which the model's assumptions are most likely to apply yields mote focused empirical predictions. By emphasizing the role of private information, this article implicitly uses a context most appropriate to a firm borrowing from several large investors. For large numbers of small investors, the high relative cost of private verification and the likelihood of free rider problems should make public procedures such as audits by CPAs or public bankruptcy more attractive; by contrast, relatively large investors will be more concerned about distortions caused by exclusive use of a bribeable third party or the court system. Along similar lines, the assumption that investors can commit to verify in certain states applies most clearly to institutions, since they can reap benefits from maintaining a reputation for contract enforcement. Finally, verification here is the gathering the information which *can* (but need not) be revealed in court. In practice, lenders may do workouts with defaulting borrowers rather than going to bankruptcy court; though ex post renegotiation clearly plays a role in such workouts, I focus on the simpler case where parties can commit to terms in advance.

These considerations suggest two specific applications of this model. The first is a comparison of the capital structure of reasonably large private firms and public firms of similar size and risk. Because it

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[4] See Jensen (1986). This point is discussed in more detail in Section 5.

[5] See Jackson and Scott (1989) and White (1989). Of course, this may also reflect a concern that unpaid employees will not work hard for the firm.

is harder to gather information about private firms, these firms should have more incentive to economize on monitoring costs than do public firms. Thus, the model predicts that private firms will generally have more levels of seniority. This should be most true for leveraged buyouts (LBOs), since these firms recreate most of their capital structure when the LBO occurs, rather than piecemeal over time. The model also predicts that any given large investor's holdings should be concentrated in a particular class of security, even if the securities were initially issued in "strips."

The second application is to situations where a large pool of relatively small financial claims requires verification in "bad" outcomes. Fixed costs suggest that verification should be delegated, but a single institution may be unable to take on the risk of the entire pool. If so, the model predicts a junior claim for the originator, more senior claims for other institutions, and the most senior claim for any small investors who are present. These features are found in asset-backed securitizations and excess-of-loss reinsurance contracts.

Despite a large and growing literature on optimal security design, research into optimal use of seniority among outside investors is quite recent.[6] Williams (1989), Hart and Moore (1990), and Diamond (1991, 1992, 1993) examine the use of seniority in the context of the, large publicly held firm. In contrast to this article, these authors assume that investors are small, numerous, and risk neutral, share the same information, and can collectively enforce liquidation; typically, additional institutions are imposed exogenously.[7]

This article also complements the literature on financial intermediaries as delegated monitors.[8] To the extent that perfect diversification is impossible; such intermediaries may wish to limit the amount of money they loan to any one firm. As a result, one firm may be financed by several institutions, which is the situation examined in this article.

[6] For recent reviews of the field of optimal security design, see Harris and Raviv (1992) and Allen and Winton (1994).

[7] In Williams (1989), outside investors cannot observe firm returns of managerial effort directly, but they do have a monitoring system that prevents the manager from stealing; depending on the relationship between managerial effort and firm returns, securities resembling outside equity and up to three classes of debt are optimal. In Hart and Moore (1990), the manager wishes to expand firm size without regard for investor welfare; the seniority of existing claims relative to that of subsequently issued securities can be used to control the manager's ability to raise new funding. In Diamond (1991, 1992, 1993), there is adverse selection on the firm's quality, and the manager has nontransferable benefits of control that are lost upon liquidation; the choice of debt seniority and maturity can be used to control the probability of liquidation as a function of interim signals of firm quality.

[8] See Diamond (1984) and Williamson (1986) for models of this type. More recently, Krasa and Villamil (1992a, 1992b) and Winton (1994) have addressed imperfectly diversified intermediation.

1. Basic Assumptions

Consider a one-period world with two types of agents: a manager who controls a risky production technology ("firm") and has no capital of his own and investors with one unit of capital each. The firm converts $K \geq 1$ units of capital into **X** units of consumption. **X** is a random variable with support $[0, X_m]$ continuous distribution $H(x)$, and density $h(x)$; for convenience, assume $h(x)$ is strictly positive on $(0, X_m)$. Both types of agents are expected utility maximizers; the manager's utilityof consumption $x \leq 0$ is $u(x)$, and an investor's utility of consumption x and effort e is $v(x) - e$. u and v are monotone increasing, concave, and twice continuously differentiable functions with finite values at zero.[9] Investors have reservation utility level v^*.[10]

Although the manager observes his firm's return at no cost, an investor can observe ("verify") returns only by expending effort $c > 0$ ex post.[11] Investors can commit ex ante (via contract) to verify as a deterministic, function of a message sent by the manager; as is standard in the literature, the investor's commitment to verify is assumed to be binding. Verification gives both a private observation of the return and evidence of the return that can be revealed in a court of law at the investor's discretion. If the manager and any verifying investors unanimously agree, they can alter evidence to reflect any state. Although messages are private, they too can be revealed in a court of law. If evidence and messages are revealed in court, the court has the power to enforce any contracts to which the manager and investors have agreed, imposing high nonpecuniary penalties on agents who are shown to have lied In equilibrium, the court will not be used; it serves as a backstop to make contracts credible.[12]

Since the firm's scale is fixed and each investor has one unit of capital, the manager will have to contract with K investors to get sufficient "funding." Assume that the investor's reservation return satisfies $v(0) < v^* < E[v(X/K)] - c$, which is sufficient to guarantee that an

[9] If agents did not have bounded utility at zero, it would be necessary to impose restrictions on the distribution of firm returns to prevent expected utility from being unbounded below.

[10] Although this framework implicitly rules out investment in multiple firms, allowing this would not change the results of this article so long as firm returns were independent of one another and them was no joint production in monitoring. In that case, one could define a new utility function by the expected utility of the investor's total portfolio, conditional on a given firm's "payment"; this new utility would be concave and increasing in the firm's payment, which is all that is required for my results.

[11] Although c is assumed to be fixed, most of the results of this article generalize to the case where c is nondecreasing in the true return.

[12] In related work, Lacker (1989) shows that it is possible to get sequentially rational contract enforcement in a setting where a risk-averse borrower and risk-neutral lender use the court system as a costly enforcement mechanism. He also shows that, when agents use deterministic strategies, the optimal contract is debt.

optimal contract will give neither the manager nor the investors all of the firm's returns. Contracts will specify the set of messages M_i that the manager will send to investor i, the set of messages $S_i \subseteq M_i$ for which investor i verifies, and the payment $P_i(x, m_i)$ made to the investor when the return equals x and the announcement is m_i.

These assumptions are designed to capture the role of private information in a simple way. For this reason, each investor receives a private message, and contracts are bilateral in the sense that they do not depend directly on the signals received by other investors.[13] In addition, the fact that verification is private information rules out costless public verification in court. These assumptions are the extreme form of a situation where courts are public goods and provide noisier information at higher total cost than an investor could obtain on her own. The larger the number or the smaller the size of the investors, the greater are the incentives to free ride on private verification, and the greater is the attraction of public procedures. Similarly, the larger the number of investors involved, the greater is the potential for leakage or violation of private "side bargains" between manager and investors.

Another simplifying assumption is the tacit exclusion of delegated verification ("monitoring"). Two arrangements are possible: designating an agent as auditor and relying on reputation to keep her from colluding with the manager, or investing through an intermediary and relying on portfolio diversification or the intermediary's own capital as devices to prevent cheating. In reality, neither device is perfect. As suggested by recent cases, auditors do not flawlessly assess the true state of affairs and have some incentive to either shirk in providing effort or shade reports in favor of management.[14] In any event, audited accounting information is at best a noisy and infrequent snapshot of a firm's situation. Real intermediaries are not perfectly diversifted and

[13] If public messages were used, incentive compatibility conditions would now involve total payments to groups of verifying and nonverifying investors. This would complicate the analysis of out-of-equilibrium announcements, since verifying investors might now have incentive to collude with the manager in the event of a lie. Imposing the condition that verifying investors never have such incentives would move the problem toward the constraints used in the private message case. Similarly, if contracts were allowed to depend on the private signals received by others, investors might have incentive to collude with the manager to fake messages for legal purposes.

[14] Recent examples of fraudulent accounting include the Leslie Fay Companies ("Accounting Scandal at Leslie Pay," *New York Times*, February 2, 1993), Phar-Mor Inc., College Bound, and Gottchalks ("Falsifying Corporate Data Becomes Fraud of the 90's," *New York Times*, September 21, 1992). Another article (Alison Leigh Cowan, "When Auditors Change Sides," *New York Times*, October 11, 1992) describes six instances of firms where questionable accounting that contributed to investor losses was accompanied by the hiring of auditors into senior posts: American Continental Corporation (parent of Lincoln Savings & Loan), Cannon Group, Chambers Development, Daisy World Genetics Partnerships, First Executive Corporation (parent of Executive Life), and Phar-Mor Inc. Britain's Auditing Practices Board recently reported similar practices in the United Kingdom ("Bridge-Building," *Economist*, November 28, 1992). Finally, Ernst & Young recently agreed to pay the U.S. government $400 million to settle charges that it failed to properly audit federally insured banks and S&Ls (*New York Times*, November 24, 1992).

thus have some incentives to limit exposure to any one firm. Also, for sufficiently wealthy investors, it may be cheaper to invest directly rather than through such an intermediary. Although real firms doubtless use a mixture of verification methods, I abstract from this in order to focus on the role of private information gathering.

I also assume that verification is both deterministic and sequentially rational. Although stochastic verification may be optimal [see Mookherjee and Png (1989)], it is difficult to enforce, and actual contracts generally confer fixed rights. Even if stochastic verification was possible, the general results of this model would still apply so long as different investors could not correlate their randomizations.[15] Sequential rationality may seem more problematic, since investors will have ex post incentives to shirk on verification if they know that the manager is telling the truth. Nevertheless, Lacker (1989) suggests that a costly public enforcement mechanism may overcome this problem. In addition, multiperiod reputation concerns for large investors such as institutions may make their commitments credible;

Returning to the contracting problem, the Revelation Principle [see Meyerson (1979) and Townsend (1988)] implies that the announcement set M_i can effectively be identified with the support of **X** and that the manager will report truthfully in equilibrium. Accordingly, the manager's problem is

$$\max_{\{P_i(\cdot)\},\{S_i\}} \int_0^{X_m} u\left[x - \sum_{i=1}^{K} P_i(x, x)\right] \cdot h(x)\, dx \tag{1}$$

subject to: the incentive compatibility conditions

$$P_i(x, x) \le P_i(x, y) \text{ for all } x, y \in [0, X_m] \text{ and for } i = 1, \ldots, K; \tag{2}$$

the informational constraints that unverified payments depend only

[15] From Krasa and Villamil (1994), optimal contracts with *public* stochastic verification have the following form: payments to investors increase with the borrower's return, the probability of verification decreases with the borrower's return, and the payment to investors is fixed whenever there is zero probability of verification. If verification is stochastic but private, then the same argument used in Proposition 4 of this article shows that optimal *symmetric* contracts will also have these features So long as investors are unable to correlate randomizations with one another, they will duplicate monitoring effort with positive probability.

By reducing some investors' monitoring regions, asymmetric contracts with varying seniority should improve matters. To see this, look at the case where investors are risk neutral and optimal symmetric contracts involve no verification (and thus fixed payments) in high return states. Then, as in Corollary 3 of this article, giving one investor seniority with absolute priority and reducing her maximum (unverified) claim should result in Pareto improvement: her verification costs are reduced, her expected consumption is unchanged etc.

on the message

$$P_i(x, x) = P_i(y, x) \text{ for all } x \in S_i^c, \text{ all } y \in [0, X_m], \text{ and } i = 1, \ldots, K; \tag{3}$$

and the investors' individual rationality constraints

$$\int_0^{X_m} v[P_i(x, x)] \cdot h(x)\, dx - c \cdot \int_{S_i} h(x)\, dx \geq v^* \text{ for } i = 1, \ldots, K. \tag{4}$$

This general problem is the focus of the remainder of the article.

2. Optimal Contracting When There Is Only One Investor

Although this article focuses on contracting when a firm must borrow from several investors, the analysis builds on the case where a single investor funds the entire firm. A brief description of optimal contracting in this simpler case, now follows; this extends Townsend (1979) by allowing both borrower and lender to be risk averse.

Proposition 1. *When K = 1, an optimal incentive-compatible contract will have the following features*

- *(i) The investor receives a fixed payment R in all states in which she does not verify;*
- *(ii) The investor receives payment Q(x) in all states x in which she does verify, where Q(x) < R almost everywhere and 0 £ Q(x) £ x eyerywhere;*
- *(iii) There exists g, R £ g < X_m, such that the investor verifies whenever x £ g and does not verify whenever x > g ;*
- *(iv) The investor's payment Q(x) and the manager's consumption x - Q(x) are both weakly increasing and continuous in x;*
- *(v) Either the optimal contract is the standard debt contract of Gale and Hellwig (1985) and the investor receives Q(x) = x whenever x £ R, or else the contract is discontinuous at g, and the investor receives Q(x) £ Q(g) < R whenever x £ y .*[16]

Conditions (i) and (ii) follow largely from incentive compatibility, as in Townsend (1979). Condition (iii) follows from the weak concavity of *u;* the manager prefers to make his highest payments to the investor in those states where the firm's return is highest, since his marginal utility is lowest there. Since the highest payments are unverified, it always pays to move the verified region to the left until

[16] Detailed proofs of results can be found in the Appendix.

verification occurs in a lower interval bounded above by γ ; this extends Townsend to the case of a risk-averse investor. The assumption that $v^* < E[v(X)] - c$ implies that it is never optimal for the investor to verify in all states, and so $\gamma < X_m$. Condition (iv) is a consequence of optimal risk sharing between manager and investor in the verified region; where both have complete information. Finally, condition (v) follows from the necessary first-order conditions of the problem.

Given these results, optimal contracts are *debt-like:* the investor receives an unverified fixed payment whenever firm returns exceed γ and verifies and receives a strictly lower payment whenever returns are below γ. In the verified region $[0, \gamma]$, $Q(x)$ follows an optimal risk-sharing rule and has slope between 0 and 1. If the contract does not give the investor all returns in the verified region, the contract is discontinuous at γ. The reason for the discontinuity is that, absent the limited liability constraints, the investor's payment schedule is chosen such that any marginal changes affect the expected utility of manager and investor in the same ratio. If there were no discontinuity at γ, a reduction in γ would strictly reduce the investors verification cost while having only a second order effect on either agent's consumption utility; so long as resource constraints were not binding, such an arrangement could not be optimal.[17,18] Although, ceteris paribus, a risk-averse manager would clearly prefer a continuous payment schedule, the gains from giving the manager higher consumption in verified states outweigh risk-sharing losses from the discontinuity.

I conclude this section with two useful special cases.

Risk-neutral manager. In this case, the optimal contract is standard debt (Figure 1). An easy way to see this is that, among all incentive-compatible contracts offering a given expected consumption level to each agent, the standard debt contract is least risky in the sense of Rothschild and Stiglitz (1970) and has the lowest expected verification costs.[19] Thus, the investor strictly prefers such a contract to any other contract with the same expected consumption, and the manager can maximize his own expected consumption by offering the standard debt contract that just meets the investor's reservation utility level.

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[17] This argument is formalized in necessary condition (A11) in the Appendix. If the resource constraint R £ g is binding, the discontinuity may disappear, in which case the standard debt contract of Gale and Hellwig (1985) is optimal.

[18] Note that, while optimal payments to the uninformed agent (investor) are monotone increasing, the possible discontinuity at $x = \gamma$ means that the consumption of the manager need not be so. This suggests that risk aversion rather than stochastic monitoring causes the nonmonotonicity of borrower income in Mookherjee and Png (1989). Krasa and Villamil (1994) show that monotonicity of payments to the lender holds even when stochastic verification is allowed.

[19] These results are proved in Chapter 3, Appendix B of Winton (1990).

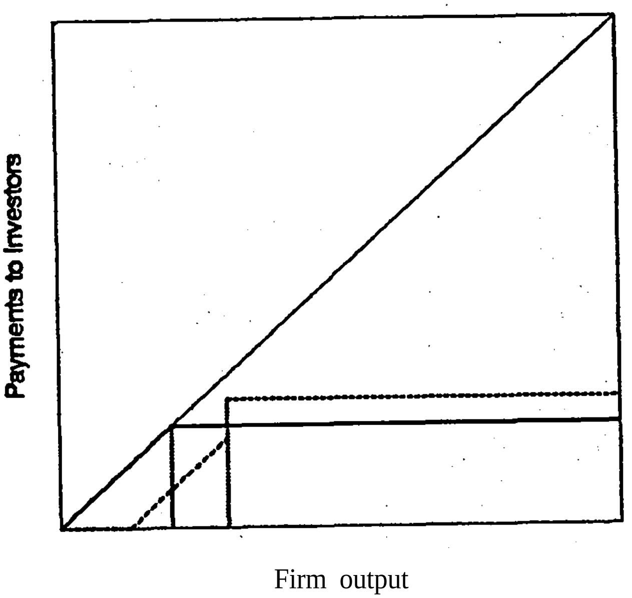

Figure 1
Optimal contracts with one investor
The solid line is the case of a risk-neutral manager, the dashed line is the case where the manager is risk averse and the investor is risk neutral. Dotted lines indicate the boundary of the verification region.

Since he is risk neutral, that he receives nothing in verified states is of no concern.

Risk-neutral investor. In this case, optimal verified payments are $Q(x) = \max[0, x - A]$, where A is a constant.[20] Thus, the manager skims all earnings above a fixed level in the best states and otherwise receives a fixed amount unless results are very low (Figure 1). Since the manager is risk averse, the investor takes on as much risk as possible within the verified region.

The following numerical example illustrates this result. Suppose **X** is uniformly distributed on [0, 8], $u(x) = 2x^{1/2}$, $v(x) = x$, $v^* = 3$, and $c = .2$. Then the optimal contract has $Q(x) = \max[0, x - .84]$, $R = 6.13$, and $g = 6.36$; note that $Q(\gamma) = 5.51$, so that the payment scheme is in fact discontinuous at γ. The manager's expected utility under this contract is 1.80. If contracts were restricted to take the form

[20] Specifially, $A = \max[0, u'^{(-1)}(\lambda_1)]$, where λ_1 is the Lagrange multiplier for the investor's reservation utility constraint. If λ_1 exceeds $u'(0)$, the optimal contract is standard debt despite the manager's risk aversion.

of standard debt, R would be 4.22 and the manager's expected utility would be 1.23—a 32% drop in expected utility.

3. Optimality of Debt-Like Contracts with Multiple Investors

From now on assume the firm's scale K exceeds 1, so that the manager must deal with several investors. This section looks at the extent to which debt-like contracts are optimal in this setting and describes conditions that optimal debt-like contracts must satisfy.

The next proposition shows that optimal incentive-compatible contracts do have certain features in common with debt-like contracts.

Proposition 2. *In an optimal incentive-compatible set of contracts with K investors, the following will be true:*

(i) Each investor i receives a fixed payment R_i in all states in which she does not verify;
(ii) Each investor i receives $Q_i(x)$ in all states x in which she does verify, where $Q_i(x) < R_i$ almost everywhere and 0 £ $Q_i(x)$ £ x everywhere;
(iii) Define N(x) Í (1,. . . K) as the set of investors whose contracts specify that they verify, state x. On any region where the set of verifying investors is constant, $Q_i(x)$ and x $\Sigma_{i \in N(x)}(Q_i(x))$ are weakly increasing in x.
(iv) Total payments to investors must be weakly increasing in x.

The reasoning behind these results is similar to that used in proving the first four results of Proposition 1. In particular, conditions (i-ii) are analogous to Proposition 1 (i-ii) and follow from incentive compatibility; condition (iii) is analogous to Proposition 1 (iv) and follows from optimal risk sharing among agents with complete information; condition (iv) follows from the manager's weakly concave utility function, which leads him to prefer contracts in which the highest total payments to investors are made in states with the highest firm returns.[21]

All that is missing from the proof that debt-like contracts are optimal is that it is optimal for each investor i to verify if and only if returns are below some $\gamma_i \in (0, X_m)$. Lemma A1 in the Appendix describes a class of incentive-compatible contracts for which optimal contracts always have this feature; this is useful in proving the optimality of

[21] Proposition 2 applies if an optimal contract exists, but it does not guarantee existence. In particular, the proof of condition (iv) shows only that a set of contracts nonmonotone total payments can always be dominated; the dominating contract that is constructed is not necessarily monotone.

debt-like contracts under the more intuitive conditions described in Propositions 3 and 4.[22]

Before proceeding to these propositions, some common features of real debt contracts need to be defined for this setting. Suppose all investors have debt-like contracts, and each investor i verifies when returns are below γ_i. If $\gamma_i < \gamma_j$, i is *senior* to j; i is "paid in full" more often than j, since she receives her fixed (unverified) payment R_i both whenever j receives her own unverified payment R_j and in some lower return states where j receives less than R_j. Conversely, j is junior to i. A set of debt-like contracts will be said to exhibit *absolute priority* if, for any i senior to j, j is paid nothing whenever i does not receive her unverified payment (i.e., when the return is below γ_i).

Proposition 3 establishes that, when investors are risk neutral, it is optimal to use debt-like contracts that specify absolute priority among investors of differing seniority.

Proposition 3 (optimality of debt-like contracts when investors are risk neutral). *Suppose investors are risk neutral. Then the optimal set of contracts exists and consists of debt-like contracts. Furthermore, this set of contracts specifies absolute priority among investors: if investors i and j verify for x below γ_i and γ_j, respectively, and $\gamma_i < \gamma_j$, then $Q_j(x) = 0$ for all $x \le \gamma_i$.*

The proof (given in the Appendix) is by construction: given any set of incentivecompatible contracts, let the investor with lowest unverified payment have "absolute priority" over the other investors (give her the smaller of her unverified payment and total payments to investors). Lower her unverified payment until other investors are as well off as before; because her verified region may be smaller, she will be at least as well off as under her initial contract. Repeating this procedure over the remaining investors leads to a dominating set of contracts that satisfies the requirement of Lemma A1; by that lemma, this new set of contracts is in turn dominated by a set of debt-like contracts.

Even if investors are not risk neutral, debt-like contracts are optimal among the class of symmetric contracts.

Proposition 4 (optimality of debt-like contracts among symmetric contracts). *Suppose contracts are required to be symmetric; that*

[22] The condition in Lemma A1 is that investors' verification "duties" are ordered: any investor who verifies more frequently than other investors must verify all the states that the others do. It is easy to show that any set of debt-like contracts satisfies this condition.

is, all investors verify in the same set of states and have the same payment schedule. Then

(i) Optimal contracts are debt-like: each investor i verifies and is paid $Q_i(x) \equiv Q(x)/K$ when $x \leq \gamma$ and does not verify and is paid $R_i \equiv R/K$ when $x > \gamma$.

(ii) Either the optimal contract is the standard debt contract of Gale and Hellwig (1985) and the investor receives $Q_i(x) = x/K$ whenever $x \leq R$, or the contract is discontinuous at γ and the investor receives payment $Q_i(\gamma) < R/K$.

Note that optimal symmetric contracts have essentially the same form as optimal contracts when there is only one investor; condition (i) is analogous to Proposition 1 (iii), and condition (ii) is analogous to Proposition 1 (v). As in the previous section, these contracts may be discontinuous at $x = \gamma$.

4. Symmetric Contracts versus Contracts with Varying Seniority

One of the results of the previous section was that, when there are several investors, the optimal set of symmetric contracts resembles debt. This section shows that this set of contracts is generally dominated by debt-like contracts where some investors are senior to others; thus symmetry is usually not optimal.

This occurs for two reasons. First, in symmetric contracts, there is total duplication of effort: whenever one investor verities, all do. Giving some investors seniority reduces their expected verification costs, and they will be willing to accept a smaller fixed payment in exchange. The cost of this change is that the position of the junior investors is riskier than before. Nonetheless, in many cases—particularly when either the manager or the investors are risk neutral—this reduction in risk sharing is more than offset by the reduction in verification costs, and introducing varying seniority is Pareto improving.

Second, if the manager is risk averse, optimal symmetric contracts generally have a discontinuity at γ, the upper bound of the verified region. This is caused by the "wedge" of verification costs and leads to inefficient risk sharing. The manager may be willing to alter the contracts of a subset of investors, giving them more junior claims in exchange for higher fixed payments; because the junior investors verify more than before, there is better risk sharing between them and the manager, and this generally offsets the increase in verification costs that the junior investors must bear.

Proposition 5 describes conditions that guarantee that giving some investors seniority allows a Pareto-improving decrease in verification costs.

Proposition 3. *Suppose the optimal symmetric set of contracts has the following properties: (i) the discontinuity in the payment to investors at the upper end of the verified region, R - Q(g), is less than kR/K, where k is between 1 and K - 1 inclusive, and (ii) the verification cost c exceeds* $\tau(k)$ *where*

$$\tau(k) = \frac{k}{K-k}\cdot\left\{\frac{K}{k}\cdot v\left[\frac{Q(\gamma)}{K}\right] - \frac{K-k}{k}\cdot v\left[\frac{R}{K}\right] - v\left[\frac{Q(\gamma)}{k} - \frac{R}{K}\cdot\frac{K-k}{k}\right]\right\}. \tag{5}$$

Then there is a set of debt-like contracts with k junior investors and K - k senior investors which strictly dominates the symmetric set of contracts.

The intuition for this result follows by construction. First, increase the fixed payment of the junior investors by a small amount and decrease the fixed payment of the senior investors accordingly (see Figure 2). Under condition (i) of the proposition, it is feasible to reward senior investors by decreasing their verified region slightly. If these variations are chosen so that the expected utility of senior investors is unchanged, condition (ii) guarantees that junior investors' expected utility increases.

A closer look at condition (ii) reveals that $\tau(k)$ is positive and depends on both the concavity of the investors' utility of consumption *v(·)* and the number of investors who receive junior positions. It is easy to show that, ceteris paribus, $\tau(k)$ is decreasing in *k;* thus, condition (ii) is most likely to hold for k equal to *K* - 1. Since (i) can be rewritten as Q(γ) > *(K - k)R/K,* it too is most likely to be satisfied for a high value of *k;* thus, the proposition's requirements are most relaxed when *K* - 1 investors take junior positions.

The next three corollaries extend Proposition 5's results to special cases.

Corollary 1. *Suppose the optimal symmetric contract is the standard debt contract; that is, g = R, and Q(x) = x for x £ y . Then the symmetric set of contracts is always dominated by debt-like contracts of varying seniority.*

The proof is immediate: in this case, *R- Q(g)* = $\tau(k)$ = 0, so both conditions from Proposition 5 hold. Note that the optimal symmetric contract will be standard debt whenever the manager is risk neutral.

Corollary 2 *(risk neutral investors). Suppose investors are risk neutral. Then so long as the optimal symmetric contract has Q(g) > R/K, the optimal set of incentive-compatible contracts will have at*

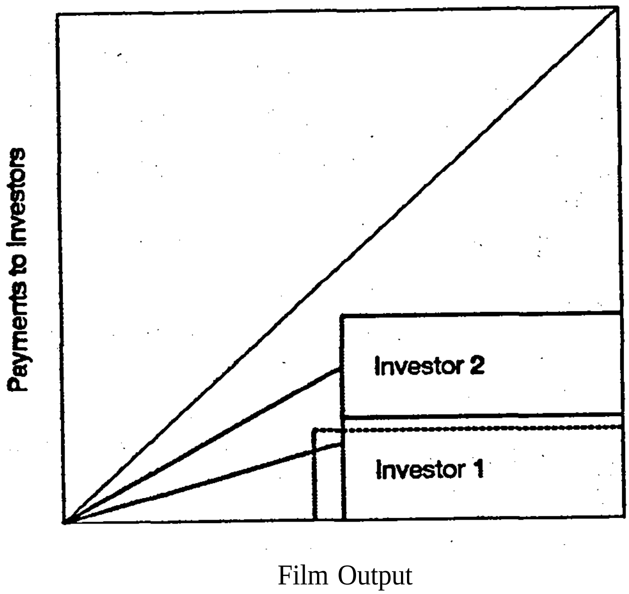

Figure 2
Proposition 5 (two investors)
The solid lines represent the initial symmetric contracts. Investor 1 is then given a senior contract (dashed line) with lower fixed payment and smaller verification region; when she verifies, she receives the same payment she would have under her original contract. Investor 2 receives the difference between the upper solid line and Investor 1's payment. Dotted lines indicate boundaries of verification regions.

least two different seniority classes of debt-like contracts, with absolute priority among investors of differing seniority.

The proof follows from two earlier results: when the corollary's condition is satisfied, Proposition 5 shows that the symmetric contract is dominated by making at least one investor senior to the others; Proposition 3 then implies that it is optimal for senior investors to have absolute priority.

As a numerical example, suppose firm returns are distributed uniformly on [0,8], there are two risk-neutral investors with reservation utility 1.5, the manager has utility $u(x) = 2x^{1/2}$, and the cost of verification is .1. The symmetric contract specifies $\gamma = 6.36$, total fixed payment $R = 6.13$ (or 3.07 to each investor), and total verified payments $Q(x) = \text{Max}[0, x - .84]$ (or Max[0, (x/2) - .42] to each investor); this gives the manager an expected utility of 1.80. Suppose the manager continues to receive $x\text{-}Q(x) = \text{Max}[x, .84]$ when $x \le \gamma$, but one investor is given seniority with fixed payment $R_s = 1.99$ and absolute priority over the other investor, while the other investor receives a fixed payment $R_j = 3.92$.[23] Then total fixed payments are reduced to 5.92 (after rounding), and the manager's expected utility increases to 1.85—a percentage increase of roughly 2.5 percent.

If more than one senior investor can be introduced, more than two layers of seniority are optimal. This follows from the next corollary, which examines optimal contracting when both manager and investors are risk neutral.

Corollary 3 (risk-neutral manager and investors). *Suppose all agents are risk neutral. Then the optimal set of contracts consists of K levels of debt, with absolute priority among investors and no compensation for the manager whenever one or more investors verify (the manager is now the most junior claimant).*

The proof is similar to that of Proposition 3. With no risk-sharing incentives, minimizing verification costs is the only concern for optimality, and there is no reason to give the manager income when one or more investors verify.[24] Furthermore, if any two investors have the same level of seniority, it is always better to give one seniority and absolute priority over the other. The senior investor's fixed payment can then be reduced until both investors have the same expected consumption; this leaves the senior investor with lower verification costs, which allows a further reduction in the payment this investor receives from the manager. Figure 3 illustrates this in the case of two investors.

The other motivation for using multiple levels of seniority is to improve risk sharing with a risk-averse manager. Proposition 6 describes conditions that guarantee' that making some of the investors junior with higher fixed payments (leaving other investors' contracts unchanged) is beneficial.

Proposition 4. *Suppose optimal symmetric contracts are discontinuous at g, and either (i) $R < g$, (ii) $v'[R/K] \cdot [1 - H(g)] > k \cdot \{v[R/K] - v[Q(g)/K] + c\} \cdot h(g)$, where k is between 1 and K - 1 inclusive, or (iii) for some $a < g$, $0 < Q(a) < a$. Then there is a set of debt-like contracts with k junior investors and K - k senior investors which strictly dominates the symmetric set of contracts.*

Again, the intuition follows by construction. Increasing the junior investors' fixed payment and increasing the set of states in which they

[23] The senior investor now verifies for $x \le (.84 + 1.99) = 2.83$, receives 0 when $x \le .84$ and Min[x - .84, 1.99] otherwise. The more junior investor verifies for $x \le \gamma = 6.36$, receives 0 when $x \le 2.83$, $x - 2.83$ when $x \in [2.83, 6.36]$, and $R_s = 3.92$ otherwise. It is easily verified that both investors receive an expected return of 1.5.

[24] Instead, the manager's share could be given to the verifying investors, in return for which their unverified payment could be reduced; this would at least weakly decrease their verification costs.

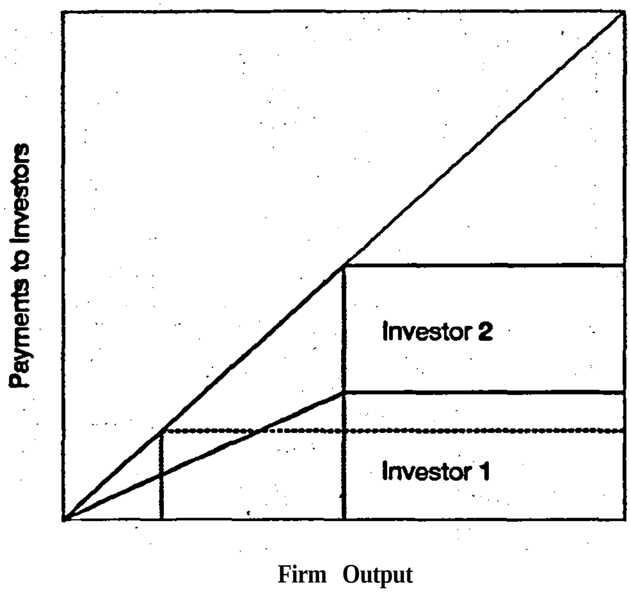

Figure 3
Corollary 3 (two investors)
The solid lines represent the initial symmetric standard debt contracts. Investor 1 is then given senior debt (dashed line) with smaller fixed payment and absolute priority. Investor 2 receives the difference between the upper solid line and Investor 1's payment. Dated lines indicate boundaries of verification regions.

verify improves risk sharing, as indicated in Figure 4. (Senior investors keep their original contracts.) Conditions (i through iii) guarantee that this can be done in a way that compensates the junior investors for increased verification costs.[25] The creation of junior contracts can improve matters even though no symmetric contract can because of the manager's risk aversion: a variation by k investors has a higher marginal effect on the manager's utility than does the same individual variation by all K investors.

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[25] When $\gamma > R$, the manager can increase γ and R in whatever ratio is necessary to achieve the desired result; when $\gamma = R$, this choice is constrained. Condition (ii) compares an investor's marginal gain from an increase in unverified payments with the marginal cost of the increase in the verified region. When the condition holds, improved risk-sharing is possible despite the constraints on the choice of γ and R; otherwise, some increase in the junior investors' verified payments is needed to compensate them for increased verification costs.

Situations where the sufficient conditions are violated are fairly limited: (i) the manager must receive nothing at the lower boundary of the unverified region, (ii) the manager must receive all firm returns within the verified region, **and** (iii) the ratio of the marginal effect on an investor's utility of changes in her fixed payment to that of changes in her verified region must be less than unity. One implication is that R must be less than $X_m/2$; otherwise, optimality conditions governing the choice of $Q(x)$ and R would be mutually contradictory.

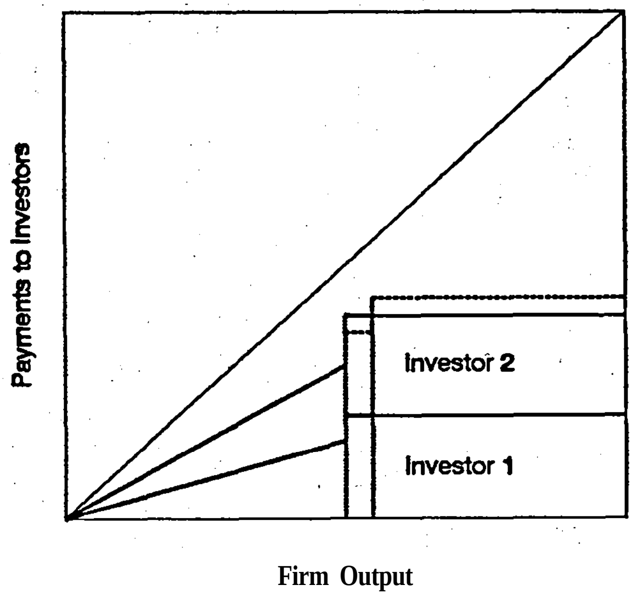

Figure 4
Proposition 6 (two investors)
The solid lines represent the initial symmetric contracts. Investor 2 is then given a junior contract (dashed line) with higher fixed payment and larger verification region; in her old verification region, Investor 2 continues to receive the payment indicated by the difference between the solid lines. Investor 1 continues to receive the payments indicated by the lower solid line. Dotted lines indicate boundaries of verification regions.

The results of this section can now be summarized. Whenever optimal symmetric contracts are standard debt or debt-like securities with a sufficiently small discontinuity, giving some of the investors senior claims improves matters by reducing total verification costs. If investors are risk neutral, senior claims will have absolute priority; if the manager is risk neutral as well, the optimal set of contracts will consist of as many classes of debt as there are investors. If the manager is risk averse and the optimal symmetric contract is discontinuous, creating junior claims with higher unverified payments allows improved risk sharing under a variety of conditions. Thus, even though all investors have identical endowments and preferences, allowing varying seniority is often preferable to using symmetric contracts.

5. Discussion

The results of the last two sections provide motivation for the contractual features discussed in the introduction: the use of "debt-like" contracts (those with fixed maximum payments), varying seniority among investors, and the absolute priority rule. This model does not require that investors have heterogeneous risk tolerance or tax treatments. Instead, if verifying firm returns is costly and private, assigning different levels of seniority to different investors reduces the duplication of verification costs in an incentive-compatible manner and also may improve risk sharing by reducing the number of investors who verify at the margin. If investors are risk neutral, absolute priority among investors of different seniority is optimal as a way of reducing verification costs; thus, this rule may provide a useful standard for contracting.[26]

Although the correspondence is far from exact, some of the features of junior outside claims in this model resemble the structure of outside equity. Jensen (1986) suggests that management tends to skim high earnings through spending on perquisites; this article produces the extreme result that the manager takes all earnings above a certain point. Models where management's ability to divert earnings or misrepresent results is more limited are capable of producing contracts that look even more like "standard" outside equity.[27]

The article also predicts that the most junior outside claimants should monitor the firm most frequently. In practice, shareholders may do this through the action of a board of directors; the board typically meets quarterly or more often through its executive committee, By contrast, loan review by banks is less frequent, and holders of public bonds seem to do little direct monitoring outside of financial distress.[28]

More specific predictions of this model can be found by examining

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[26] Using a standard saves transaction costs and improves liquidity of claims by reducing the time needed to evaluate the precise return of the contract. If investors are significantly risk averse, absolute priority may no longer be optimal. Although this suggests that ex post redistributions in court may improve matters, such redistributions open the door to opportunistic behavior by managers and junior claimants, as noted by Jackson and Scott (1989).

[27] The results of this model are due in part to the single period setting, where there is no secondary market for shares reflecting future dividend streams and proceeds are divided up once and for all. Other reasons for this result are the all-or-nothing nautres of both the verification technology and the manager's ability to lie.

In a model of costly state **falsification,** Lacker and Weinberg (1989) have shown that optimal contracts may involve paying the manager a fixed amount in the lowest states and sharing profits between manager and investor in higher states. Outside equity is also possible if management can misrepresent returns but is limited in its ability to consume them: see Williams (1989) or Chang (1993). Chang also presents conditions under which dividend smoothing occurs.

[28] Firms that receive venture capital finance provide yet another example. Such firms are typically little-known, privately held start-ups with high risk and high growth potential; thus, monitoring of such firms is both critical and costly. Venture capitalists monitor their investments quite closely, usually sitting on the firm's board of directors and meeting regularly with management. Their claims typically take the form of convertible preferred stock; this is senior to the entrepreneur's common stock but junior to claimants such as trade creditors and banks, which provide less monitoring. See Pozdena (1990).

those situations where its assumptions are most likely to hold. As noted in the introduction and again in Section 1, these are situations where the number of outside investors is relatively small (but greater than one), the investors have absolutely large stakes, and information is relatively costly to attain. In addition, because my framework is one in which investment is funded on a once-and-for-all basis, the best applications are ones in which all financial contracts are executed at the same time and last a good while.[29]

Two situations where these criteria are likely to be met come to mind. The first is that of firms that have been taken private through leveraged buyouts. Compared with firms of similar size in the same business, LBOs have a smaller number of investors, are no longer public (and thus are less subject to reporting requirements), and are largely recapitalized at a single point in time. The broad facts of LBOs are roughly consistent with this model: many classes of debt and preferred stock are typically issued, and the most junior claims (common stock) are held by management and a buyout fund that monitors management closely, as in the case of venture capital.[30]

By contrast, publicly owned firms that are otherwise similar to LBO firms should have fewer informational problems; thus, in the framework of this model, they should have less incentive to issue different classes of securities than LBO firms. The model predicts that LBOs will have more classes (by seniority) of debt and preferred stock outstanding than will similar publicly held firms and that creditor holdings in LBO firms will be more concentrated in securities of a given level of seniority than will creditor holdings in publicly held firms.[31]

If other agency problems (such as asset substitution) are present, the use of many types of securities may be detrimental. In a setting where the presence of both debt and warrants can control such behavior by equity holders, Spatt and Sterbenz (1993) show that equity

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[29] By contrast, Diamond (1991, 1992, 1993) and Hart and Moore (1990) focus on the control of *future* financing: differential seniority limits the borrower's ability to dilute existing claims by issuing claims of greater or equal priority.

[30] This model suggests that investors with lower verification costs (per dollar invested) should have more junior claims, which should cause "delegated monitors" such as banks and life insurers to hold the next most junior claims after common stock. In fact, bank loans in LBOs are often senior to other claims. This may be because of considerations beyond the scope of the model; for example, because of their relatively short-term liability structure, banks may requite shorter-term or less risky investments. As Diamond (1993) notes, shorter maturity is a partial substitute for seniority, so short-term junior claims are somewhat self-defeating.

[31] Although the use of "strip" financing may seem to contradict this, the fact that strips involve many different types of claims suggests that there must be some differentiation going on. This suggests that one should study the extent to which investors concentrate their holdings into specific securities after the initial snip financing takes place. Such ex post concentration could also be explained by tax or risk clienteles, but these considerations do not explain the form of the securities, and it is hard to see why these considerations would argue for more levels of seniority in privately held firms than in publicly held firms.

holders may have incentive to buy out more debt or warrants and then implement policies that harm the other claim. Bundling claims into a single security (such as convertible debt) forestalls such behavior. However, LBO targets are typically mature, "cash-cow" firms where opportunities for asset substitution are limited, and the greatest agency problem is managerial expropriation of free cash flow—a setting better suited to the model presented in this article.

The second application for this model is a situation where relatively small financial claims are pooled by an intermediary and then refinanced. In this case, it is relatively costly to gather information about each individual claim, so duplication of effort is to be avoided if at all possible. Although the intermediary could hold the claims itself, limited capital and the presence of systematic risk factors or other limits on diversification may cause the intermediary to seek other investors or institutions to share the financing. Examples include securitizations of pools of assets (such as automobile loans) and reinsurance by insurance companies. In both cases one finds arrangements that are consistent with the predictions of this model.

In asset-backed securitizations that do not involve government agencies, the originating institution typically agrees to take all credit losses up to a certain percentage of the assets' value; percentages vary with the expected risk of the issue. Additional "credit enhancement" is often provided by another institution (typically a bank issuing an irrevocable letter of credit), which provides coverage of additional losses up to a fixed amount.[32] Investors take losses only if the loss limits of these junior claimants (originator and credit enhancer) are exceeded; in addition, an increasing number of securitizations create two or more classes of seniority among investors.[33] Although the institutions provide contingent guarantees for a fee, the structure is analogous to debt-like claims of varying seniority with absolute priority among different classes.[34] In addition, the junior claimants are the institutions that are best placed to perform verification or monitoring at low cost, whereas the purchasers of the securities need not have such expertise.

In reinsurance, one insurer contracts with another to share potential losses on a portfolio of policies. Two general methods are used: *proportional*, where the reinsurer shares losses with the originating

[32] For details, see Rosenthal and Ocampo (1988) and Goldberg and Rogers (1988).

[33] Although the use of different "tranches" partly reflects the differing horizons of investors, one could easily imagine arrangements where these tranches had the same seniority.

[34] The various junior claimants receive a fixed amount (their fee or spread), unless the pool suffers credit losses. The originator loses money to cover losses until its loss limit is reached, at which point the "credit enhancer" loses money up to the limit of its guarantee. The contingent guarantee replaces the strict limited liability constraint of my formal model, $Q(x) > 0$, with one of the form $Q(x) > L (L > 0)$.

insurer, and excess-of-loss, where the reinsurer absorbs all losses over a set amount.[35] The latter method is analogous to the senior-junior contingent claim structure just described for asset-backed securitizations. Since a primary function of insurance firms is verifying claims by policy holders, there are potential gains to eliminating the duplication of this function by using excess-of-loss rather than proportional contracts. Costs of verification will be most sign&cant when losses on policies are least predictable and most subject to fraud, as is the case in property and casualty insurance. Furthermore, the reinsurer should feel the most need to "check up" on the originator in situations where the reinsurance market is more "arm's-length" in nature, so that repeated interactions and reputation play less of a role in enforcing behavior.

Some stylized facts about reinsurance are consistent with these predictions. Life insurers have relatively predictable claims and low risk of fraud, but the opposite is true for casualty insurers; not surprisingly, proportional is the more common type of reinsurance contract for life insurers, whereas excess-of-loss is more common for casualty insurers.[36] Similarly, in fidelity insurance (which insures against employee fraud, etc.), most policies face relatively small risks and are reinsured through "multi-line" (general) proportional arrangements, while policies for financial institutions face high risk of fraud and are typically reinsured through excess-of-loss [see Davis (1980), pp. 325-326]. Relationships also seem to play a role in determining which type of contract is used: casualty insurers do use proportional contracts when long-term relationships are likely, but there are some signs that excess-of-loss is becoming more common as the reinsurance market becomes more globalized and "transaction"-oriented.[37]

In addition to empirical investigation of this model's implications, several theoretical extensions should be pursued. The question of whether prioritized contracts are optimal when both manager and investors are risk averse remains open. Noisy verification technologies, the coexistence of public and private technologies, and sequential ra-

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[35] There are two major types of proportional reinsurance: *quota-share,* where the reinsurer gets a fixed percentage of all policies of a given type, and *surplus-share,* where only policies with indemnities above a set size are reinsured and the reinsurer takes a fixed percentage of the "surplus" above this size. See Riegel et al. (1976), pp. 131-134.

[36] For life insurers, see Riegel et al. (1976), p. 134; for casualty insurers, see Simon (1980), p. 216.

[37] On casualty insurers, see Simon (1980), p. 216: "Quota share arrangements are used in special situations involving new lines of business, special risk situations or ventures into new classes for a smaller company. In this way the primary company can rely on the professional reinsurer's expertise, or it may simply have a partner in a risky venture who is willing to rely on the specialized expertise of the primary company. . ." For comments on the increasing use of excess-of-loss contracts, see Riegel et al. (1976), p. 134, and the *Economist,* February 20, 1990, p. S8.

tionality and renegotiation also should be addressed.[38] Finally, since ex ante contracting based on exact knowledge of return distributions and individual utility functions seems unwieldy (particularly when applied to traded securities), it would be useful to examine how well combinations of simple contractual forms such as debt or equity can approximate the optimum.[39]

Appendix

Proof of Proposition 1. (i) If message m is verified, condition (2) is easily satisfied by letting $P(x, m) = P(x, x)$. From (3), unverified payments depend only on the message. Thus, $P(x, m)$ can be rewritten as $R(m)$ for $m \in S^c$ and as $Q(x)$ for $m \in S$. If the manager announces an unverified m, he will clearly choose $m \in S^c$ to minimize $R(m)$, so the contract might as well set $R(m) \equiv R$.

(ii) If $x \in S$, the manager won't reveal x if $Q(x) > R$, so $Q(x) \leq R$. If $Q(x) = R$ on a set of positive measure, the investor need not verify those states; changing the contract to reflect this reduces verification costs and permits a reduction in payments, which helps the manager. $Q(x) \in [0, x]$ because the firm's returns are the only source of consumption for both agents.

(iii) Without loss of generality, suppose the contract has unverified region $S^c = [\alpha, \beta]$, $0 \leq \alpha < \beta < X_m$, and payment scheme R, $Q(x)$. Choose γ such that $1-H(\gamma) = H(\beta)-H(\alpha)$, and define $\mu = \min[\beta, \gamma]$ and $v = \max[\beta, \gamma]$. Note that $H(\mu) - H(\alpha) = 1 - H(v)$. Now create a new contract (*) with unverified region $S^{c*} = [\gamma, X^m]$, $R^* = R$, $Q^*(x) = Q(x)$ if $x \in [0, \alpha] \cup [\mu, \gamma]$, and $Q^*(x) = Q(\phi(x))$ if $x \in [\alpha, \mu]$, where $\phi : [\alpha, \mu] \rightarrow [\nu, X_m]$ is defined by $\phi(x) = H^{-1}[H(\nu) + H(x) - H(\alpha)]$. $\phi(\alpha) = \nu$, $\phi(\mu) = X_m$, and $d\phi/dx = h(x)/h[\phi(x)] > 0$. The initial contract implies $\alpha \geq R$ and $Q(x) \leq R$ on $[v, X_m]$, so $Q^*(x)$ is feasible and incentive-compatible.

Define the expected utilities of investor and manager under the old contract by V and U, respectively, and those under the new contract by V^* and U^*. I claim that $V^* = V$ and $U^* \geq U$, so the new contract

[38] Haubrich (1991) addresses imperfect monitoring, and Seward (1990) deals with contracting in a setting where a firm can unobservably choose between technologies with observable and unobservable returns. Other extensions of costly state verification when a single investor is present include Chang (1990), who deals with multiperiod concerns, and Dionne and Viala (1992), who add unobservable managerial effort.

[39] In a recent paper, Boyd and Smith (1993) compare standard debt with optimal contracts under stochastic verification and attempt to calibrate the welfare difference between the two contracts to recent U.S. corporate data.

is Pareto improving. We have

$$V = \int_{[0,\alpha]\cup[\beta,X_m]} v[Q(x)] \cdot h(x)\,dx + v(R) \cdot [H(\beta) - H(\alpha)] - c \cdot [1 - H(\beta) + H(\alpha)] \tag{A1}$$

and

$$V^* = \int_{[0,\alpha]\cup[\mu,\gamma]} v[(Q(x)] \cdot h(x)\,dx + \int_\alpha^\mu v[Q(\phi(x))] \cdot h(x)\,dx + v(R) \cdot [1 - H(\gamma)] - c \cdot H(\gamma) \tag{A2}$$

so that

$$V^* - V = \int_\alpha^\mu v[Q(\phi(x))]h(x)\,dx - \int_\beta^{X_m} v[Q(x)]h(x)\,dx + \int_\mu^\gamma v[Q(x)]h(x)\,dx = 0, \tag{A3}$$

where the second equality in (A3) follows from substituting $y = f(x)$, $dy = [h(x)/h(y)] \cdot dx$ into the first integral term. Similarly, $U^* - U$ equals

$$\begin{aligned} &\int_\alpha^\mu u[x - Q(\phi(x))] \cdot h(x)\,dx + \int_\mu^\gamma u[x - Q(x)] \cdot h(x)\,dx \\ &+ \int_\gamma^{X_m} u[x - R] \cdot h(x)\,dx \\ &- \int_\alpha^\beta u[x - R] \cdot h(x)\,dx - \int_\beta^{X_m} u[x - Q(x)] \cdot h(x)\,dx \\ &= \int_\nu^{X_m} \big[u[\phi^{-1}(x) - Q(x)] - u[\phi^{-1}(x) - R] \\ &\quad + u[x - R] - u[x - Q(x)]\big]\, h(x)\,dx. \end{aligned} \tag{A4}$$

Since $f(x) > x$ for all $x \in [\alpha, \mu]$, $\phi^{-1}(x) < x$ for $x \in [\nu, X_m]$. Also, $R \geq Q(x)$ for all $x \in [\nu, X_m]$. The concavity of $u(\cdot)$ implies that the last integrand in (A4) is nonnegative, so $U^* - U \geq 0$ as claimed.

Assume now that the original contract had a nonverifred region consisting of a number of disjoint intervals. Then the same result can be established by induction, starting with the rightmost interval.

That $\gamma < X_m$ follows by contradiction: suppose $\gamma = X_m$ and $Q(X_m) < X_m$. Then the choice of R would be irrelevant within the interval $[Q(X_m), X_m]$. Let $R = Q(X_m)$. A differential decrease in γ leaves the manager unaffected and the investor better off. Thus $\gamma < X_m$ unless $Q(X_m) = X_m$; since $x - Q(x)$ is nondecreasing, the latter implies $Q(x) \equiv x$, but then the investor receives everything, and thus more than v^*. (v) below shows that this cannot be optimal.

(iv) If this is not true, risk sharing between the two agents in verified states is inefficient and can be improved upon.

(v) The contracting problem can now be restated as follows:

$$\max_{Q(\cdot),R,\gamma} \int_0^{\gamma} u[x - Q(x)] \cdot b(x)\,dx + \int_{\gamma}^{X_m} u[x - R] \cdot b(x)\,dx, \quad \text{(A5)}$$

such that

$$\int_0^{\gamma} v[Q(x)] \cdot b(x)\,dx + v(R) \cdot [1 - H(\gamma)] - c \cdot H(\gamma) - v^* \geq 0, \quad \text{(A6)}$$

$$\text{(a)} \quad \gamma - R \geq 0, \qquad \text{(b)} \quad X_m - \gamma \geq 0 \quad \text{(A7)}$$

For all $x \leq \gamma$,

(A8)

Following suitable changes in variable names, the problem given by (A5) through (A8) can be restated in the form given on page 657 of Takayama (1985): there is one state variable $Q(x)$, γ and R are control parameters that do not depend on x, and there are no differential constraints. It is easy to show that constraint qualification [Takayama (1985), p. 658] holds for all values of x in the interval $(0, \gamma)$; thus the conditions required for the Hestenes Theorem hold whenever an interior optimum ($\gamma > 0$) exists, which is assumed throughout this article. In addition, (A6) is binding; if this were not true, a sufficiently small decrease in R or $Q(\cdot)$ would increase the manager's expected utility without violating the constraint. Unless the investor's reservation level v^* equals $v(0)$, the constraint binds. Together with the earlier results that $Q(x) < R$ a.e., $\gamma < X_m$, and (A6) is binding, the Hestenes Theorem immediately implies that there exist nonnegative continuous functions $q_1(x)$ and $q_2(x)$ and constants $\lambda_1 > 0$, $\lambda_2 \geq 0$, such that the following conditions hold:

$$\lambda_1 \cdot \left\{ \int_0^{\gamma} v[Q(x)] \cdot b(x)\,dx + v(R) \cdot [1 - H(\gamma)] - c \cdot H(\gamma) - v^* \right\} = 0. \quad \text{(A9)}$$

$$u'[x - Q(x)] \cdot b(x) = \lambda_1 \cdot v'[Q(x)] \cdot b(x) + q_1(x) - q_2(x) \quad \text{(A10)}$$

$$\left\{ u[\gamma - R] - u[\gamma - Q(\gamma)] + \lambda_1 \cdot \left[v[R] - v[Q(\gamma)] + c \right] \right\} \cdot b(\gamma) = \lambda_2, \quad \text{(A11)}$$

$$\int_{\gamma}^{X_m} u'[x-R]\cdot b(x)\,dx - \lambda_1 \cdot v'(R)\cdot[1-H(\gamma)] = -\lambda_2, \qquad \text{(A12)}$$

$$q_1(x)\cdot Q(x) = q_2(x)\cdot[x - Q(x)] = 0 \text{ for all } x \leq \gamma, \qquad \text{(A13)}$$

$$\lambda_2 \cdot (\gamma - R) = 0.^{40} \qquad \text{(A14)}$$

If $R < \gamma$, then (A14) implies $\lambda_2 = 0$. Substituting into (A11) yields the result that $Q(\gamma) < R$. If $R = \gamma$, $\lambda_2 > 0$, and it is possible that $Q(\gamma) = R$. Since $Q(x)$ and $x - Q(x)$ are nondecreasing, it follows that $Q(x) = x$ for all $x \leq \gamma$. ∎

Proof of Proposition 2. (i-ii) follow exactly the same argument as (i-ii) in Proposition 1. (iii) follows from optimal risk sharing. If (iv) were not true, one could find two intervals such that total payments to investors on the lower interval were higher than those on the up per interval. A measure-preserving mapping of all contracts could be found which moved the higher total payments to the upper interval. As in the proof of Proposition 1 (iii), this would leave investors unaffected and increase the manager's utility. ∎

Lemma A1. *Suppose the set of contracts is required to have the following feature: for any x, the set N(x) (investors who verify state x) is one of the sets $N_1, N_2, ..., N_n$, where $N_1 \subset N_2 \subset \cdots \subset N_n \subseteq \{1, \ldots K\}$. optimal set of contracts that meets this restriction is a set of debt-like contracts.*

Proof. Take any two intervals I and I' such that $I > I'$ and $N(I) \subseteq N(I')$. (In other words, the set of verifying investors is constant on each interval, all investors who verify on interval I' verify on I, and the converse is not true.) Define $Q(x)$ as the total payment to all investors at x, whether or not they verify. Proposition 2 implies $Q_i(x)$ weakly increases on I (I') for $i \in N(I)(N(I'))$, as does $x - Q(x)$. If there were $x \in I$, $y \in I'$ such that $Q(x) < Q(y)$, then either (i) for some $i \in N(I')$, $Q_i(\cdot)$ is smaller on a neighborhood of x than on some neighborhood of y, or else (ii) $Q_i(x) \geq Q_i(y)$ for all $i \in N(I')$. If (i) were true, increasing i's payment near x and decreasing it near y would improve the expected utility of both i and the manager. If (ii) were true, $Q_i(x) < R_i$ for at least one $i \in N(I) \setminus N(I')$, and a measure-preserving mapping that moved the unverified payments of

40 q_1 and q_2 are the multipliers for the feasibility constraints $0 \leq Q(x) \leq x$, λ_1 is the multiplier for the investor's individual rationality constraint, and λ_2 is the multiplier for the feasibility constraint $R \leq \gamma$. (A10) is the optimality condition for verified payments $Q(x)$, and (A11) and (A12) are transversality for the optimal choice of γ and R, respectively.

i to I and her verified payment to I' would increase the manager's expected utility without hurting the investor's.

Thus, it must be that $Q(x) \geq Q(y)$ for all $x \in I$ and $y \in I'$. From Proposition 2, $Q_i(x) \leq R_i$ for all $i \in N(I) \backslash N(I')$, and so $Q_j(x) > Q_j(y)$ for some $j \in N(I')$. Suppose I and I' have the same probability mass. (if not, take an appropriate subset of one or the other). For all i that do not verify in the interval I', do a measure-preserving mapping that switches payments on the two regions. If the total payment to investors is left unchanged, the new total payment on I to members of $N(I')$ for whom $Q_j(x) > Q_j(y)$ will be weakly lower than before but higher than their old payments on I', and the new payments on I' will be higher than before but lower than old payments on I. This is the reverse of a mean-preserving spread, which can be apportioned so as to increase all of these investors' utilities. Thus, the original situation is always dominated by shifting verified regions to the left. ■

Proof of Proposition 3. Let investor i have the lowest unverified payment R_i (i need not be unique). Now, fixing total payments to investors, give i absolute priority: that is, extend her unverified region into all states where the total payment to investors exceeds R_i, and give her all of the investors' compensation in all other states. In each state, reduce one of the other investors' payments as much as possible; if this is not enough to make up the increased payment to i, reduce another investor's payment, and so forth. If the state was one in which one or more of the other investors did not verify under the original contract, reduce the payment of one of these nonverifying investors (j); since $R_i \leq R_j$, only j loses compensation.

Once this is done, i has (weakly) smaller expected verification costs and larger consumption. Other investors' total consumption is reduced by the amount of i's consumption gain. Also, in any state, at most one other investor has to verify where she did not before, so total expected verification costs of other investors have increased by no more than i's have decreased. Finally, i's verified region is now contained within that of any other investor.

Now reduce R_i, reducing i's verified region as necessary to maintain incentive compatibility. Share the surplus consumption among the other investors, increasing their payments in proportion to their utility loss from the first change. Eventually, they will be as well off as under the original contract, while i will be weakly better off to the extent that she does not verify in states in which ail investors originally verified. If she is better off, reduce her unverified payment further and give the surplus to the manager.

Now repeat the process among the other investors, fixing the payments of i and the manager. By induction, one ends up with a contract that dominates the original; in it, investors' verified regions are completely ordered, which implies a complete ordering of the sets $N(x)$, and investors with smaller verified regions will have absolute priority when they do verify over investors with larger verified regions. Application of Lemma A1 leads to a set of debt-like contracts with similar characteristics. ∎

Proof of Proposition 4. As noted in the text, (i) is trivial. (ii) follows from the necessary conditions for an optimum, just as in Proposition 1 (v). To see this, note that the symmetric optimization program is obtained by substituting $K\,v(y/K)$ for $v(y)$, $K\,c$ for c, and $K\,v^*$ for v^* into the program given by (A5) to (A8). This gives the following necessary conditions.

There exist nonnegative continuous functions $q_1(x)$ and $q_2(x)$, and constants $\lambda_1 > 0$, $\lambda_2 \geq 0$, such that the following conditions hold:

$$\lambda_1 \cdot \left\{ \int_0^{\gamma} v[Q(x)/K] \cdot h(x)\,dx + v(R/K) \right.$$
$$\left. \cdot [1 - H(\gamma)] - c \cdot H(\gamma) - v^* \right\} = 0. \tag{A15}$$

$$u'[x - Q(x)] \cdot h(x) = \lambda_1 \cdot v'[Q(x)/K] \cdot h(x) + q_1(x) - q_2(x), \tag{A16}$$

$$\{u[\gamma - R] - u[\gamma - Q(\gamma)] + K\lambda_1$$
$$\cdot \left[v[R/K] - v[Q(\gamma)/K] + c\right]\} \cdot h(\gamma) = \lambda_2, \tag{A17}$$

$$\int_{\gamma}^{X_m} u'[x - R] \cdot h(x)\,dx - \lambda_1 \cdot v'(R/K) \cdot [1 - H(\gamma)] = -\lambda_2, \tag{A18}$$

$$q_1(x) \cdot Q(x) = q_2(x) \cdot [x - Q(x)] = 0 \text{ for all } x \leq \gamma, \tag{A19}$$

$$\lambda_2 \cdot (\gamma - R) = 0. \tag{A20}$$

As in the proof of Proposition 1, the investors' individual rationality constraint is binding, and the rest of the proposition follows. ∎

Proof of Proposition 5. Let δ_R and δ_γ be small positive amounts, and consider the following variations in the investors' contracts. Senior investors verify and receive $Q(x)/K$ when $x < \gamma - \delta_\gamma$ and receive $R/K - \delta_R$ otherwise. Junior investors verify when $x < \gamma$ and receive $R/K + (K - k)\delta_R/k$ when $x \geq \gamma$, $\{Q(x) - (K - k) \cdot [(R/K) - \delta_R]\}/k$ when $\gamma - \delta_\gamma \leq x < \gamma$, and $Q(x)/K$ when $x < \gamma - \delta_\gamma$. These changes are feasible given condition (i), incentive compatible by Proposition 2, and do not change the compensation schedule of the manager.

Define Δ_J and Δ_S as the changes in expected utility of a junior and senior investor, respectively. For sufficiently small δ's, first-order approximations can be used:

$$\begin{aligned}\Delta_S &= -v'\left[\frac{R}{K}\right]\cdot[1-H(\gamma)]\delta_R \\ &\quad + \left\{v\left[\frac{R}{K}\right]-v\left[\frac{Q(\gamma)}{K}+c\right]\right\}\cdot b(\gamma)\delta_\gamma \qquad \text{(A21)}\\ \Delta_J &= \left[\frac{K-k}{K}\right]\cdot v'\left[\frac{R}{K}\right]\cdot[1-H(\gamma)]\delta_R \\ &\quad - \left\{v\left[\frac{Q(\gamma)}{K}\right]-v\left[\frac{Q(\gamma)-(K-k)R/K}{k}\right]\right\}\cdot b(\gamma)\delta_\gamma. \qquad \text{(A22)}\end{aligned}$$

Substituting (A22) into (A21) yields

$$\Delta_S = -\left[\frac{k}{K-k}\right]\cdot\Delta_J + [c-\tau(k)]\cdot b(\gamma)\delta_\gamma. \qquad \text{(A23)}$$

From (A21), it is clearly possible to choose δ_R and δ_γ so that Δ_J equals zero; then (A23) implies that Δ_S is positive as long as $c > \tau(k)$, which is condition (ii) of the proposition. Since a small increase in δ_R will leave both junior and senior investors better off than before, the manager can further decrease the fixed payments of all investors so as to make himself better off while leaving investors with their reservation returns. ■

Proof of Corollary 3. As shown in the text, the manager should receive no consumption in verified states. Suppose an optimal contract involves two or more investors having the same seniority: their verified regions are identical. Now give investor i priority over the others (j) in her "class," keeping the payments of all other investors and the manager fixed. Using the same argument as the proof of Proposition 3, it is easy to show that this allows a strict Pareto improvement: the more senior investor receives a lower payment and has strictly lower verification costs, and these savings can be passed through to the manager. Thus the original contract was not optimal. ■

Proof of Proposition 6. Let δ_R, δ_γ, and $\delta_Q(x)$ be small and positive, where $\delta_Q(x)$ is defined on an interval of the form (α, γ). The k junior investors verify when $x < \gamma+\delta_\gamma$ and receive $(R/K)+\delta_R$ when $x \geq \gamma + \delta_\gamma$, $Q(\gamma)/K$ when $\gamma \leq x < \gamma+\delta_\gamma$, and $(Q(x)/K)+\delta_Q(x)$ when $x < \gamma$. The other investors receive the same contract as in the symmetric case, while the manager receives the residual. These variations are feasible for appropriate choices of the δ 's and α. [Note that $Q(\gamma) < \gamma$ because of the discontinuity, so there is an interval on which $\delta_Q(x)$ can be positive.]

Define Δ_J and Δ_M as the changes in expected utility of a junior investor and of the manager. First-order approximations are as follows:

$$\begin{aligned}\Delta_J = &\int_\alpha^\gamma v'\left[\frac{Q(x)}{K}\right]\cdot\delta_Q(x)b(x)\,dx + v'\left[\frac{R}{K}\right]\cdot[1-H(\gamma)]\delta_R\\ &+\left\{v\left[\frac{Q(\gamma)}{K}\right]-v\left[\frac{R}{K}\right]-c\right\}\cdot b(\gamma)\delta_\gamma \end{aligned}\tag{A24}$$

$$\begin{aligned}\Delta_M = &-\int_\alpha^\gamma ku'[x-Q(x)]\cdot\delta_Q(x)b(x)\,d(x)\\ &-\int_\gamma^{X_m} ku'[x-R]\cdot\delta_R b(x)\,dx\\ &+\left\{u\left[\gamma-\frac{(K-k)R-kQ(\gamma)}{K}\right]-u[\gamma-R]\right\}\cdot b(\gamma)\delta_\gamma.\end{aligned}\tag{A25}$$

Substituting from (A16) through (A18) and (A24), (A25) becomes

$$\begin{aligned}\Delta_M = &-k\cdot\int_\alpha^\gamma[\lambda_1\cdot v'[Q(x)/K]\cdot b(x)+q_1(x)-q_2(x)]\cdot\delta_Q(x)\,dx\\ &-k\lambda_1\cdot v'\left[\frac{R}{K}\right]\cdot[1-H(\gamma]\cdot\delta_R+k\lambda_2\cdot\delta_R+\sigma(k)b(\gamma)\delta_\gamma\\ &+k\lambda_1\cdot\left\{v\left[\frac{R}{K}\right]-v\left[\frac{Q(\gamma)}{K}\right]+c\right\}\cdot b(\gamma)\delta_\gamma-\frac{k}{K}\cdot\lambda_2\cdot\delta_\gamma\\ = &-k\lambda_1\cdot\Delta_J-k\cdot\int_\alpha^\gamma q_1(x)\cdot\delta_Q(x)b(x)\,dx\\ &+k\lambda_2\cdot[\delta_R-\delta_\gamma/K]+\sigma(k)b(\gamma)\delta_\gamma,\end{aligned}\tag{A26}$$

where

$$\begin{aligned}\sigma(k) = &\ u\left[\gamma-\frac{(K-k)R-kQ(\gamma)}{K}\right]-\frac{K-k}{K}\cdot u[\gamma-R]\\ &-\frac{k}{K}\cdot u[\gamma-Q(\gamma)]\end{aligned}\tag{A27}$$

and I have used the fact that $q_2(x)\equiv 0$ since $Q(x) < x$. Since $u(\cdot)$ is concave, $\sigma(k)$ is positive. I now examine the three cases described in the proposition.

(i) Note that λ_2 equals zero. Set $\alpha = \gamma$, so that the terms involving $\delta_Q(x)$ disappear. For small δ_R and δ_Y, there are no relative constraints on their values, and it is feasible to choose them so that Δ_J equals zero. (A26) implies that Δ_M equals $\sigma(k)b(\gamma)\delta_Y$, which is positive.

(ii) If $R = \gamma$, δ_R and δ_Y must satisfy $k\delta_R \leq \delta_Y$. Set $\alpha = \gamma$ and choose δ_R and δ_Y so that $\Delta_J = 0$; then $\delta_Y = \delta_R \cdot v'[R/K] \cdot [1 - H(\gamma)] \cdot [b(\gamma)]^{-1} \cdot \{v[R/K] - v[Q(\gamma)/K] + c\}^{-1}$. This will be feasible when condition (ii) in the text is satisfied.

(iii) Suppose neither (i) nor (ii) holds. Setting $k\delta_R = \delta_Y$ leaves Δ_J negative. Now choose $\delta_Q(x) > 0$ on (α, γ) so that Δ_J equals zero. Because $0 < Q(x)$ for $x \in (\alpha, \gamma)$, $q_1(x) = 0$ and Δ_M equals $\sigma(k)b(\gamma)\delta_Y + k\lambda_2 \cdot [\delta_R - \delta_Y/K]$. Since $\delta_Y/K < \delta_Y/k$, both terms are positive. ■

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