Insider and Liquidity Trading in Stock and Options Markets

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We analyze the introduction of a nonredundant option, which completes the markets, and the effects of this on information revelation and risk sharing. The option alters the interaction between liquidity and insider trading. We find that the option mitigates the market breakdown problem created by the combination of market incompleteness and asymmetric information. The introduction of the option has ambiguous consequences on the informational efficiency of the market. On the one hand, by avoiding market breakdown, it enables trades to occur and convey information. On the other hand, the introduction of the option enlarges the set of trading strategies the insider can follow. This can make it more difficult the market makers to interpret the information content of trades and consequently can reduce the informational efficiency of the market. The introduction of the option also has an ambiguous effect on the profitability of insider trades, which can either increase or decrease depending on parameter values.

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What are options useful for? In incomplete markets, they can be used to improve hedging opportunities for risk-averse agents [see Ross (1976)]. Financial markets are not only useful to reallocate risk across agents, however. Financial prices can reveal private information. Easley, O'Hara, and Srinivas (1993) provide empirical evidence that insiders trade in options markets. The present article analyzes how the introduction of a nonredundant option that completes the market affects information revelation and risk sharing. In this way we are able to evaluate the interaction between information asymmetry and market incompleteness.

We consider a simple one-period model where risk-neutral market makers serve market orders placed either by an insider or by a liquidity trader. The liquidity traders are rational expected utility maximizers. They trade to hedge their risk exposure to state-dependent random shocks on their final wealth. There are three final states, determining the value of the liquidity shocks and the stock. Without the option, the market is incomplete, but the introduction of the option completes the markets. The insider knows the final state of the world in advance. To exploit her private information, while avoiding full revelation, she mimics the stock and option liquidity trades that maximize her profits.

In the incomplete market, the liquidity traders can only partially hedge their risk exposure. When the option is traded, state-contingent claims (SCC) can be traded separately. Other things being equal, this additional flexibility enables the liquidity traders to better hedge their risk exposure. Without the option the insider with good news buys the stock (and sells it after bad news), but she cannot earn profits when a neutral state is realized. With the option, the full set of contingencies can be spanned. So the insider can conduct profitable trades in all states. Hence, other things being equal, the introduction of the option is beneficial to the insider because it enables her to design trades that better match her private information.

This line of reasoning can be misleading, however. The "other things being equal" argument does not hold because in equilibrium the insider trades and liquidity trades are interdependent. On the one hand, the insider mimics the liquidity trades, to obtain camouflage.[1] On the other hand, the presence of the insider creates an adverse selection cost of trading for the liquidity traders who respond to this cost by reducing their trades.[2] Consider the example of the insider with good news about the asset value. When the option is

[1] See, for example, Kyle (1985) or, in the case of options, Back (1993) and Easley, O'Hara, and Srinivas (1993).

[2] Glosten (1989), Madhavan (1992), Bhattacharya and Spiegel (1991), and Bossaerts and Hughson (1993) analyze the effects of asymmetric information on risk sharing in the stock market.

traded and the states can be spanned, a naive strategy would be to buy contingent claims associated with the good news state and to sell the other "out of the money" claims. This can be implemented by buying the call with strike K larger than the unconditional value of the stock. This makes the good news SCC expensive, however, which deters the liquidity traders from buying it. Now, if the liquidity traders do not trade this security, the strategic insider also may choose to avoid it, so there will not be excessive information revelation. This exemplifies the importance of endogenizing liquidity trading when assessing the influence of the option, one contribution of the present article.[3]

We find that the introduction of the option can reduce the profits of the insider. This is because (i) in equilibrium the insider mimics the liquidity trades to avoid being spotted and (ii) the introduction of the option can generate liquidity trading patterns that are less attractive for the insider than those prevailing in the incomplete market.

We find that the introduction of the option mitigates the market breakdown problems created by asymmetric information. Asymmetric information imposes trading costs on the liquidity traders. To determine their optimal trades, they compare these trading costs to the risk-sharing gains from trade. If the costs exceed the benefits, there is a market breakdown.[4] Market incompleteness makes the market breakdown problem more severe. In the incomplete market the liquidity traders must trade predetermined bundles of state-contingent claims, namely the stock and the risk-free asset. These may include certain claims that are not useful for hedging the liquidity traders' shocks. Hence, market incompleteness is costly for the liquidity traders because it compels them to buy unnecessary contingent claims. Asymmetric information increases the cost of purchasing these claims. As a result, the introduction of the option reduces the likelihood of market breakdowns. This is beneficial to the liquidity traders because it makes it more likely that they will be able to hedge at least part of their risk exposure.

We also find that the introduction of the option has ambiguous consequences on the informational efficiency of the market. It enables

[3] This differs from Back (1993), Easley, O'Hara, and Srinivas (1993), or John, Koticha, and Subrahmanyam (1992), who consider exogenous liquidity trades.

[4] Glosten (1989), Madhavan (1992), Bhattacharya and Spiegel (1991), and Bossaerts and Hughson (1993) analyze market breakdown problems in the stock market. Our analysis differs from theirs because we consider a situation where, even without information asymmetry, markets would be incomplete. When only the stock and the bond ate traded, the liquidity traders cannot hedge their endowment shocks contingent on the medium state. In contrast, in Glosten (1989), Madhavan (1992), Bhattacharya and Spiegel (1991), and Bossaerts and Hughson (1993) in absence of asymmetric information, agents can perfectly hedge their risk exposure, since the latter stems from stock holdings.

trades to take place in cases where there would have been a market breakdown in the incomplete market. In these cases there is some information revelation in the complete market and none in the incomplete market. The introduction of the option can also reduce the informational efficiency of the market, however. The introduction of the option increases the set of possible trades for the insider. This complexifies the trading strategies of the insider and can make it more difficult for the market makers to interpret the information content of trades. This results in a decrease in the informational efficiency of the market.

In the next section of this article, we present the model. In Section 2, we present the benchmark case where there is no asymmetric information. In Section 3 and 4, we analyze successively the equilibrium in the incomplete market and the equilibrium with the option. Sections 5 and 6 are devoted to analysis of the complete market equilibria and their comparison to the incomplete market case. Section 7 analyzes the trades in the stock and the option and the corresponding prices in the different equilibria and discusses the relation between our findings and previous empirical results. Section 8 provides concluding remarks. All proofs are in the Appendix.

1. The Model

Price formation is analyzed in a market where the risk-free asset, one stock, and one (nonredundant) option are simultaneously traded. For simplicity, the return on the risk-free asset is normalized to zero. Risk-neutral market makers face *either* one strategic, perfectly informed insider or one rational, competitive, risk-averse liquidity trader.[5] The market makers cannot determine whether the agent is informed. The only signal they observe is the order placed by the agent.

There is one period. The state of the world and the final value of the securities at the end of the period follow a three-point distribution. The stock, the bond, *and the option* are needed to complete the markets and span the three states. The strategic insider knows the true final state of the world in advance. The risk-averse liquidity traders are exposed to state-dependent liquidity shocks. They trade to hedge their risk exposure.

1.1 The final value of the securities and the market structure

The final value of the stock is u, d, or m, with equal probability. For simplicity, the distribution of the final value of the stock is symmetric

[5] The assumption that the market makers face only one agent is similar to Glosten and Milgrom (1985). It differs from Kyle (1985), where the market makers serve the aggregate net order flow from the insider *and* liquidity traders.

around m. That is, $u \equiv m + \sigma$, and $d \equiv m - \sigma$, where $\sigma > 0$. The option can be any call or put with strike price strictly between u and d, expiring at the end of the period.

For simplicity, only market orders are considered. Market makers stand ready to serve buy or sell orders in the stock and the option.[6]

1.2 The insider

With probability α, the market order stems from the insider. Before trading, the insider perfectly observes the realization of the state of the world, u, m, or d, The insider is risk neutral and strategic. She maximizes her expected trading profits. She does not have any leverage or short selling constraints.

1.3 The liquidity traders

With probability $1 - \alpha$, the market order stems from one liquidity trader. The liquidity traders are assumed to behave competitively. They do not recognize that their trades affect the prices. They do have rational expectations, however. They select their trading strategy to maximize their expected utility on the basis of the correct expectation of the price at which their trade will be executed in equilibrium.[7] For simplicity the liquidity traders are assumed to have identical constant absolute-risk aversion parameter (γ): $U(x) = -e^{-\gamma x}$. Like the insiders, they have no short selling or leverage constraints.

The liquidity traders have initial cash endowment C. They also have state-dependent endowments and place orders in the stock and the option to hedge their risk exposure. The assumption reflects the idea that the securities market is used to reallocate the risk from the real economy.[8]

There are three different types of liquidity traders, likely to enter the market with equal probability 1/3. The state-dependent endowment

[6] An alternative, equivalent assumption in the present model is that market makers in the stock and in the option markets have the same Information set; that Is. they can monitor perfectly and simultaneously orders and trades in both markets. In moat actual exchanges, the stock and the options are traded separately. For example, options on stocks listed on the NYSE are traded on the AMEX or in Chicago. Information flows rapidly across markets, however. In particular, the options market makers monitor actively the trades and orders in the stock market. For example, In the Paris Bourse and the French options markets, the traders working for the same brokerage houses actively communicate, using screens or telephones. Berkman's (1992) analysis of the European Options Exchange, in Amsterdam, suggests that the option market makers closely monitor the stock market. Note also that, in trading rooms, traders in different but related securities sit close to one another and monitor the orders, trades, and prices in the different securities.

[7] Because they are price takers, the liquidity traders consider only the equilibrium price. They do not recognize that out-of-equilibrium trades would execute at different prices, reflecting out-of-equilibrium beliefs. Assuming strategic liquidity traders would have made the analysis less tractable. In particular, it would have given rise to multiple equilibria. In the case of stock markets, Laffont and Maskin (1990) and Bhattacharya and Spiegel (1991) also consider competitive liquidity traders. For an analysis of strategic liquidity traders, see Spiegel and Subrahmanyam (1992).

[8] For a similar analysis of liquidity trading, in the case of stocks, see Dow and Gorton (1993).

of the type 1 agent is $-L$ in state u, $-l$ in state m, and 0 in state d, with $L > l$. The type 2 agent has no risk exposure. His endowment is constant (normalized to 0) in the three states. The endowment of the type 3 agent is 0 in state u, $-l$ in state m, and $-L$ in state d.

The type 1 (type 3) risk exposure can be interpreted as a "short" ("long") position. His liquidity shock is negatively (positively) correlated with the final value of the stock. Intuitively, since the type 1 liquidity trader is "short," he will engage in trades that will be attractive for the insider in state u. Symmetrically, mimicking the type 3 trades will be attractive for the insider in state d.

How would our results be affected by alterations of our simple model?

- For simplicity the endowment shocks of the type 1 and type 3 traders are assumed to be symmetric. This enables one to focus on symmetric equilibria, in which the attractiveness of the type 1 trades for the insider in state u is equal to the attractiveness of the type 3 trades for the insider in state d. Asymmetric shocks would lead to different equilibrium trades. Consider the example where the type 1 *u-state* shock would be $-L_1$ and the type 3 *d-state* shock would be $-L_3$, where $L_1 > L_3$. In this case, mimicking the type 1 trades in state u would be more attractive for the insider than mimicking the type 3 trades in state d. Consequently, the insider would trade more aggressively in state u than in state d. As a result more information would be revealed in the former than in the latter. However, the basic intuition developed in the current article (that the insider mimics the liquidity trades that best fit her information) would still hold?
- The type 2 liquidity trader is not subject to any shock and consequently does not trade. We could consider type 1 and type 3 only, without altering the qualitative nature of the results. However, because it enhances the symmetry of the model (three possible final values for the stock and three types of liquidity traders), the introduction of the type 2 liquidity trader actually simplifies the computations and enables us to obtain simpler expressions.
- Alternatively, one could have considered an intermediate liquidity shock, neither short nor long, equal to 0 in the u and d state and negative in the medium state. Agents with such an endowment shock would benefit from the introduction of the option, since when only the stock and the bond are traded they cannot obtain any risk sharing from the financial markets. Hence, this extension of the model would

[9] If we considered more than one trading period, asymmetry in the liquidity shocks (and hence in the liquidity trades) would raise the possibility of manipulation. That is, strategic agents without private information could trade in one direction to move the prices up or down and then unwind their inventory at a profit (see Allen and Gorton (1992)]. Our simple one trading period set-up precludes such behavior.

not alter our result that liquidity traders benefit from the introduction of the option.

1.4 The equilibrium

In equilibrium the following conditions are satisfied. First, the agents take optimal actions: (i) the market order chosen by each liquidity trader maximizes his expected utility, (ii) the trade of the insider maximizes her expected profits, and (iii) the prices quoted by the market makers are such that they earn zero expected profits.

Second, the agents have rational expectations: (i) when they compute their final wealth, the liquidity traders correctly anticipate the price at which their trades are executed; (ii) the insider correctly anticipates the reaction of the market makers to her trades; and (iii) in interpreting the information content of the trade, the market makers apply Bayes' rule and correctly conjecture the trading strategies of the other agents.

This is similar to the equilibrium concept in Kyle (1985), Laffont and Maskin (1990), Rochet and Vila (1994), or Vila (1989). The zero expected profit condition for the market makers stems from Bertrand competition between identical, risk-neutral agents.

2. Equilibrium Prices and Trades without Asymmetric Information

2.1 The incomplete market

The type 1 liquidity trader is exposed to the negative liquidity shocks $-L$ in state u and $-l$ in state m. To hedge the *u-state* risk exposure, he buys Q_1 shares of the stock, at the ask price A. The risk-free rate is normalized to zero. The type 1 agent must solve the following problem:

$$\max_{Q_1} [U(C - Q_1A + Q_1u - L) + U(C - Q_1A + Q_1m - l) + U(C - Q_1A + Q_1d)]/3.$$

Without asymmetric information, the ask price A is equal to the unconditional expected final value of the stock m. Noting that $u - m = \sigma$ and $m - d = -\sigma$, the objective of the type 1 liquidity trader can be rewritten:

$$\max_{Q_1} \frac{U(C + Q_1\sigma - L) + U(C - l) + U(C - Q_1\sigma)}{3}.$$

Buying the stock provides a hedge against the u-state exposure but creates new exposure to the *d-state* risk. One can easily check that the optimal compromise between these two effects is to hedge half of the *u-state* risk by trading $L/2\ \sigma$.[10] Note that the *m-state* wealth of the agent is not affected by the amount traded. The negative side of this is that the *m-state* shock cannot be hedged. The positive side is that exposure to the *m-state* shock does not prevent the liquidity trader from hedging the *u-state* shock. As a result, the optimal trade does not depend on *l*. In the next section we show that this is no longer the case with asymmetric information.

2.2 The complete market

The stock, the bond, and the option span the three states. In this case the agents can trade the *u-*, *m-*, and *d-state* -contingent claims. Because it simplifies the exposition and the proofs, we analyze trades in the complete market in terms of the SCC rather than in terms of the stock and the option. For example, the type 1 liquidity trader seeks to hedge his *w-state* and *m-state* risk exposures. To do so he purchases the *u* SCC and the *m* SCC in exchange for money (the risk-free asset).[11]

Since the market makers are risk-neutral, they are willing to trade the SCC at a price equal to the probability of the corresponding states, updated conditionally on the informational content of the trade. Let these conditional probabilities be denoted: π_u, π_m, and π_d. Note that they can also be interpreted as state prices. When there is no asymmetric information, they are simply the unconditional probabilities, equal to 1/3. In this case the final wealth of the type 1 liquidity trader who purchases $Q_{u,1}$ units of the *u* SCC and $Q_{m,1}$ units of the *m* SCC is

$$\begin{cases} C - L + Q_{u,1} - \left(\dfrac{Q_{u,1} + Q_{m,1}}{3}\right) & \text{in state } u, \\ C - l + Q_{m,1} - \left(\dfrac{Q_{u,1} + Q_{m,1}}{3}\right) & \text{in state } m, \\ C - \left(\dfrac{Q_{u,1} + Q_{m,1}}{3}\right) & \text{in state } d. \end{cases} \tag{1}$$

Because the prices are set by risk-neutral competitive market makers, who have the same information as the liquidity trader, the optimal trade is to hedge the risk exposure fully by purchasing *L* units of the *u* SCC and *l* units of the *m* SCC. This generates the same final wealth in all three states: $-[(L + l)/3]$. This contrasts with the previous, incomplete market, case.

[10] We do not prove the results presented in this section. They are particular of those presented in the next sections, in the case $\alpha = 0$.

[11] Instead of considering the *u* and *m* SCC and the risk-free asset, we could have equivalently focused on the *u, m,* and *d* SCC.

3. Equilibrium in the Incomplete Market, with Asymmetric Information

3.1 The insider trades

The market makers quote ask and bid prices *(A* and *B)* equal to the conditional expectation of the final value of the stock. Because of interdealer competition $A \leq u$ and $B \geq d$. In state u the insider buys the stock at a profit (selling would generate losses). In state d she sells. Hence, the market makers know that purchases stem either from the type 1 liquidity trader (in which case they should quote $A = m$) or from the insider in state u (in which case they should quote $A = u$). As a result, the ask price must be between u and m. Similarly, the bid is between m and d. Now consider the insider in state m. If she bought the stock, she would make losses because $A > m$. If she sold the stock, she would also make losses because $B < m$. Hence, the insider in state m does not trade. In that respect, the insider suffers from the incompleteness of the market.

3.2 The liquidity trades

As in the symmetric information case, the type 1 liquidity trader can buy the stock to hedge his u-state risk exposure. With asymmetric information, this trade is costly because the ask price is $A > m$. In this case, the final wealth of the type 1 liquidity trader if he buys the stock is

$$\begin{cases} C - L + Q_1(u - A) & \text{in state } u, \\ C - l + Q_1(m - A) & \text{in state } m, \\ C + Q_1(d - A) & \text{in state } d. \end{cases} \tag{2}$$

From now on the cash endowment of the liquidity trader *(C)* is normalized to zero. This does not affect the results, since the agents have CARA utilities, which do not exhibit wealth effects. In all states, the wealth of the liquidity trader is lower in the asymmetric information case than in the symmetric information case. This reflects the higher stock price. As a result the liquidity trade is smaller with asymmetric information than with symmetric information. Also, in contrast with the symmetric information case, the final wealth of the agent in state m us now affected by the trade. Buying the stock creates losses for the agent in the m state. Such losses are particularly damaging for the agent if his marginal utility in that state is high because his m state liquidity shock l is high. As a result the liquidity trade is decreasing in the *m-state* shock, l. The optimal demand is given in the following lemma.

The proposition states that the insider mimics the liquidity trades. This is because, if the insider traded other quantities, she would be spotted by the market makers and unable to earn profits. Note also that the bid-ask spread is $2\alpha\sigma$, consistent with the intuition that the spread increases as the volatility or the degree of asymmetric information increases. Finally, using Proposition 1 and the corollary to Lemma 1, the condition under which the quantity traded by the agent is positive is

$$e^{\gamma L} > \frac{1+\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha} e^{\gamma l}. \tag{3}$$

The right-hand side is increasing in a. If the degree of asymmetric information is high, the market breaks down.

4. Equilibrium in the Complete Market with Asymmetric Information

4.1 The liquidity trades

The liquidity trades are given in the following lemma:

Lemma 2. *The type 1 liquidity trader buys $Q_{u,l}$ units of the u SCC and $Q_{m,1}$ of the m SCC where*

$$Q_{u,1} = \text{Max}\left[0,\ L - \frac{1}{\gamma}\ln\left(\frac{\pi_{u,1}}{\pi_{d,1}}\right)\right] \quad \textit{and}$$

$$Q_{m,1} = \text{Max}\left[0,\ l - \frac{1}{\gamma}\ln\left(\frac{\pi_{m,1}}{\pi_{d,1}}\right)\right].$$

The type 2 agent does not trade. The type 3 agent buys $Q_{d,3}$ units of the d SCC and $Q_{m,3}$ of the m SCC where

$$Q_{d,3} = \text{Max}\left[0,\ L - \frac{1}{\gamma}\ln\left(\frac{\pi_{d,3}}{\pi_{u,3}}\right)\right] \quad \textit{and}$$

$$Q_{m,3} = \text{Max}\left[0,\ l - \frac{1}{\gamma}\ln\left(\frac{\pi_{m,3}}{\pi_{u,3}}\right)\right].$$

The quantities $Q_{u,1}$ and $Q_{m,1}$ are equal to the difference between the liquidity shocks in the corresponding states (*L* and *l*, respectively) and a price elasticity component that reflects the sensitivity of the liquidity trader to the prices of the contingent claims. As in the perfect information case, to hedge his risk exposure perfectly the agent can buy *L* units of the *u* SCC and *l* units of the *m* SCC.[12] When hedging

[12] The perfect hedge is the actual trade when the agent is infinitely risk-averse ($\gamma \to \infty$) so that the price elasticity component of the liquidity demand is irrelevant.

is costly because $\pi_{u,1}$ or $\pi_{m,1}$ is higher than the unconditional probabilities, the liquidity trader chooses to underhedge ($Q_{u,1} < L$ and $Q_{m,1} < l$.

4.2 The equilibrium prices and trades

On the one hand, the insider would like to buy as many claims contingent on the realized state as possible and to sell the claims contingent on the other states. Such aggressive trades are not optimal, however, because they involve too much information revelation. Rather, the insider mimics the liquidity trades to avoid being spotted by the market makers. Thus, the strategy of the insider is to mimic the liquidity trades that enable her to buy as many claims on the realized state as possible, while minimizing the number of worthless SSC purchased.[13] To the extent that the liquidity trades depend on the liquidity shocks (L and l), the insider's strategy also depends on these shocks.

Consider the trade of the type 1 liquidity trade, which involves buying the u SCC (to hedge his state-u shock, L) and the m SCC (to hedge his *state-m* shock, l). If L is large and l is small, the number of u SCCs bought is large and the number of m SCC is small. As a result this trade is attractive for the insider in state u. As L gets closer to l, the liquidity trade involves buying a larger number of m SCC and it becomes less attractive for the insider in state u and more attractive for the insider in state m. The next three propositions describe how the insider's strategy changes as the relative magnitudes of L and l vary.

When L is sufficiently larger than l, the insider in state u finds it optimal always to mimic the type 1 liquidity trade. In contrast, mimicking this trade would imply losses for the insider in state m. As a result, as in the incomplete market, the insider in state m does not trade, like the type 2 trader. This is described in the next proposition.

Proposition 2. Pure strategy equilibrium: *If*

$$L > \frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) + \frac{2+\alpha}{1+2\alpha}l, \tag{4}$$

then the insider in state u buys $L - 1/\gamma l[(1+2\alpha)/(1-\alpha)]\,\imath$ *units of the u SCC and l units of the m SCC, like the type 1 liquidity trader; the insider in state m does not trade, like the type 2 liquidity trader;*

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[13] When the insider is indifferent between two liquidity trades, her strategy involves mixing over these trades. Easley and O'Hara (1987) and Easley, O'Hara, and Srinivas (1993) also show that the Insider uses mixed strategies to limit information revelation.

Table 1
Equilibrium prices and trades in the complete market

Equ.	Q_u	Q_m	λ	μ	π_u	π_m
Pure						
	$L-\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)$	l	1	0	$\frac{1+2\alpha}{3}$	$\frac{1-\alpha}{3}$
Mixed 1						
	$L-\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)$	$l-\frac{1}{\gamma}\ln\left[\frac{1+\alpha(3\mu-1)}{1-\alpha}\right]$	1	$\mu\in]0,\frac{1}{2}]$	$\frac{\alpha+\frac{1-\alpha}{3}}{1+\alpha\mu}$	$\frac{\alpha\mu+\frac{1-\alpha}{3}}{1+\alpha\mu}$
Mixed 2						
	$L-\frac{1}{\gamma}\ln\left[\frac{\alpha\lambda+(1-\alpha)/3}{(1-\alpha)/3}\right]$	$l-\frac{1}{\gamma}\ln\left[\frac{\alpha/2+(1-\alpha)/3}{(1-\alpha)/3}\right]$	$\lambda\in[0,1]$	$\frac{1}{2}$	$\frac{\alpha\lambda+\frac{1-\alpha}{3}}{1-\alpha/2+\alpha\lambda}$	$\frac{\alpha/2+\frac{1-\alpha}{3}}{1-\alpha/2+\alpha\lambda}$

and the insider in state d buys l units of the m SCC and $L - 1/[\gamma(1 + 2\alpha)/(1-\alpha)]$ *units of the d SCC, like the type 3 liquidity trader.*[14]

The set of parameter values for which the pure strategy equilibrium prevails is region 1 in Figure 1. As l increases relative to L, the number of m SCC purchased by the type 1 or type 3 liquidity trader increases. Above a certain cutoff level for l, mimicking the type 1 or 3 trades no longer results in losses for the insider in state m. Consequently, she mixes between the type 1 and 3 trades (and the type 2 trade as long as the type 1 and 3 trades do not yield strictly positive profits). This is described more precisely in the next proposition.

Proposition 3. Partially mixed strategy equilibrium: *If*

$$\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)+\frac{2+\alpha}{1+2\alpha}l > L > \operatorname{Max}\left[\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right), \frac{1}{\gamma}\left[\ln\left(\frac{1+2\alpha}{1-\alpha}\right)-\frac{2+\alpha}{4-\alpha}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right] + \frac{2+\alpha}{4-\alpha}l\right],$$

then the insider in state u (d) buys the u (d) and m SCC, like the type 1 (3) liquidity trader. Further, there is a $\mu\in]0,\frac{1}{2}]$ *such that the insider in state m mixes between (i) buying the u and m SCC, like*

[14] In this proposition as well as in Propositions 3 and 4, the numbers of SCC bought by the agents and the corresponding state prices are in Table 1.

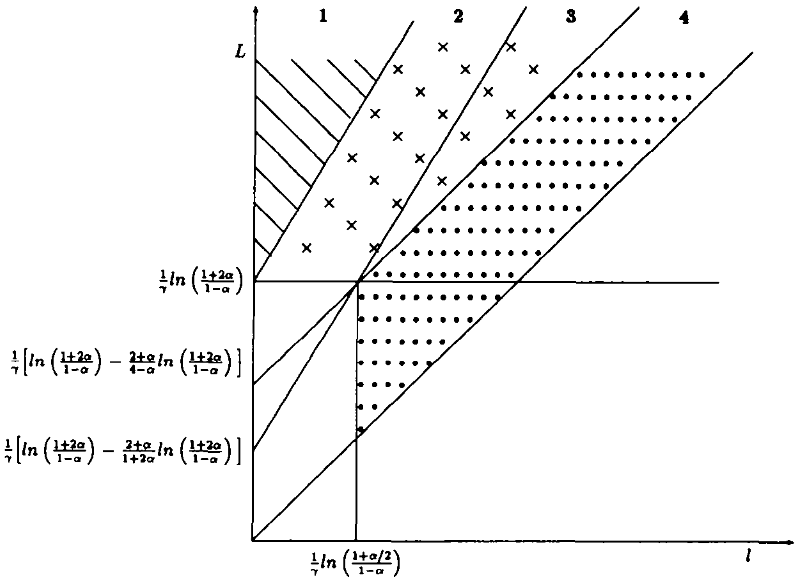

Figure 1
The equilibria in the complete market
The parameter values for which the different equilibria arise are graphically represented in Figure 1. The larger the value of the *u-state* liquidity shock *(L)* relative to the value of the *m-state* shock, the more likely it is that the pure strategy equilibrium prevails. pure strategy equilibrium; partially mixed strategy equilibrium; fully mixed strategy equilibrium. The 45° line reflects the constraint: *L > l*.

the type 1 liquidity trader, with probability $\mu \in]0, \frac{1}{2}]$, (ii) not trading, with probability $1 - 2\mu$, and (iii) buying the m and d SCC, like the type 3 liquidity trader, with probability μ.

The set of parameter values for which the partially mixed strategy equilibrium prevails is represented in regions 2 and 3 in Figure 1. Region 2 corresponds to the case where the insider in state *m* mixes over the type 1, 2, *and* 3 trades (i.e., $\mu < \frac{1}{2}$), whereas region 3 corresponds to the case where she mixes between the type 1 and 3 trades only (i.e., $\mu = \frac{1}{2}$).

As *l* keeps increasing relative to *L,* the type 1 trade becomes less attractive for the insider in state *u.* Above a certain cut-off level for *l,* mimicking the type 1 trade generates zero profits for the insider in *u.* Hence, she is indifferent (and mixes) between the type 1 and 2 trades. Symmetrically, the insider in *d* mixes between type 2 and 3 trades. In contrast, because *l* is large, the type 1 and 3 trades are very attractive for the insider in state *m,* so she mixes between these two trades. This is stated in the next proposition.

Proposition 4. Fully mixed strategy equilibrium: *If*

$$\frac{1}{\gamma}\left[\ln\left(\frac{1+2\alpha}{1-\alpha}\right)-\frac{2+\alpha}{4-\alpha}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]+\frac{2+\alpha}{4-\alpha}l > L \quad \textit{and}$$

$$l > \frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right),$$

then there is a $\lambda \in [0, 1]$ such that the insider in state u mixes between (i) buying the u and m SCC, like the type 1 liquidity trader, with probability l, and (ii) not trading, like the type 2 agent, with probability $1 - l$. Symmetrically, the insider in state d mixes between the type 2 and type 3 liquidity trades. Further, the insider in state m mixes between the type 1 and 3 liquidity trades with equal probability.

The set of parameter values for which the fully mixed strategy equilibrium prevails is region 4 in Figure 1. To conclude this section we offer some economic intuition for mixed strategies. Mixed strategies arise in our discrete model because they enable the insider to convexify the strategy space. For example, always mimicking the type 1 liquidity trade after observing that *u* is realized can be too aggressive a trading strategy because it would lead to too much information revelation. If the insider wants to pursue a less aggressive strategy (to reduce information revelation), she can mix between the type 1 and the type 2 trade. Mixing is more attractive (and arises more often) in the complete market because the introduction of options increases the set of payoffs which can be attained. For example, insiders with good news can buy either the stock or different calls. To the extent that these trades are substitutable, the insider may find it optimal to mix.

5. Analysis of the Pure Strategy Equilibrium

In this section we compare the complete and the incomplete markets in terms of pricing and welfare, for the parameter values such that the pure strategy equilibrium prevails in the incomplete market.

5.1 State prices

Proposition 5. *When the pure strategy equilibrium prevails in the complete market, the prices of the u, m, and d SCCs are the same in the complete and the incomplete markets.*

The proposition obtains because in both market structures the insider in state *u, m,* or *d,* respectively, mimics the trades of the type

1, type 2, and type 3 liquidity traders, respectively. As a result the information structure of the market makers (defined in the proof of the proposition) is the same in the complete market and the incomplete market. Consequently, the updated probabilities (and hence the state prices) of *u, m,* and *d* are the same in the two market structures.

5.2 The profits earned by the insider

The numbers of the three SCC which the liquidity traders would buy in a perfect and complete market depend on their liquidity shocks. For example, if *L* is very large with respect to *l,* the type 1 liquidity trader would like to buy many *u* SCC and relatively few *m* SCC, other things being equal. Now, in the incomplete market, the liquidity trades are constrained by the pay-off patterns of the available securities. In particular, the pay-off pattern of the stock determines the relative number of the *u, m,* and *d* SCC that the agents can buy. If *L* is very large relative to *l,* the type 1 liquidity trader is constrained by this pay-off pattern. He cannot buy as many *u* SCC as he would like to because this would involve also buying too many *m* SCC. This constraint is relaxed in the complete market. Other things being equal (and, in particular, holding prices constant), when *L* is very large relative to *l* the type 1 liquidity trader will buy more *u* SCC and less many *m* SCC in the complete than in the incomplete market.

Now, the profits of the insider are constrained by the liquidity trades because she must mimic them.[15] In state u the insider would like to buy as many *u* SCC (and as few *m* and *d* SCC) as possible. Consequently, the insider in state *u* will earn larger profits in the complete market if the type 1 liquidity trader buys more *u* SCC and less many *m* SCC in the complete than in the incomplete market.[16] As discussed above this is the case if *L* is sufficiently large with respect to *l.* The following proposition states this more precisely:

Proposition 6. *The profits of the insider in the complete market when the pure strategies equilibrium prevails are larger than in the incomplete market if and only if*

$$L > \frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) + 2l. \tag{5}$$

[15] In that sense, the market is not complete for the insider even when the option is traded. Even in that case she cannot freely choose which bundles of SCC to buy, lest she might have too strong an influence on prices.

[16] The case of the insider in state *d* is symmetric. The insider in state *m* does not trade in the incomplete market as well as in the complete market when the pure strategies equilibrium prevails.

5.3 The utility of the liquidity traders

In general, there are two differences between the complete and incomplete market cases. First, in the complete market, the liquidity trader can span the three states. Other things being equal this improves his welfare. Second, the prices at which his order is executed are, *in general,* different in the complete and the incomplete markets. Still, as stated in Proposition 5, when the pure strategy equilibrium prevails in the complete market, there is no difference in pricing between the two cases. As a result, for the liquidity trader the only difference between the complete and the incomplete market is that in the latter there is a constraint on the relative numbers of *u, m,* and *d* SCC that he can buy. The solution of the maximization problem of the liquidity trader is more efficient without the constraint than with the constraint. Consequently, the introduction of the option increases the welfare of the liquidity traders.[17] Combining this and Proposition 6, the following proposition obtains:

Proposition 7. *If*

$$L > \frac{1}{\gamma}\ln\left(\frac{1 + 2\alpha}{1 - \alpha}\right) + 2l, \tag{6}$$

then the introduction of the option leads to a Pareto improvement.

Under the condition stated in the proposition, both the liquidity trader's expected utility *and* the insider's profits are larger when the option is traded. This is because the introduction of the option creates additional surplus from trade, part of which accrues to the liquidity traders and part of which accrues to the insider.[18]

6. Comparing the Equilibria with and without the Option

6.1 Market breakdown with and without the option

As shown in Section 3.2, in the incomplete market case, the condition for no market breakdown relates to the *u-* and *m-state* shocks. In contrast, in the complete market, as shown in Lemma 2, the condition under which there is no trading in the complete market does not rely on the relative magnitude of the different liquidity shocks. Instead, it is a condition relating to each of the shocks separately. This con-

[17] Note that the expected utility of the liquidity traders is larger in this case when the option is traded, whatever their type (1, 2, or 3).

[18] Note that when the pure strategy equilibrium prevails. but condition 5 is not satisfied, the introduction of the option does not lead to a Pareto improvement, since it reduces the welfare of the insider.

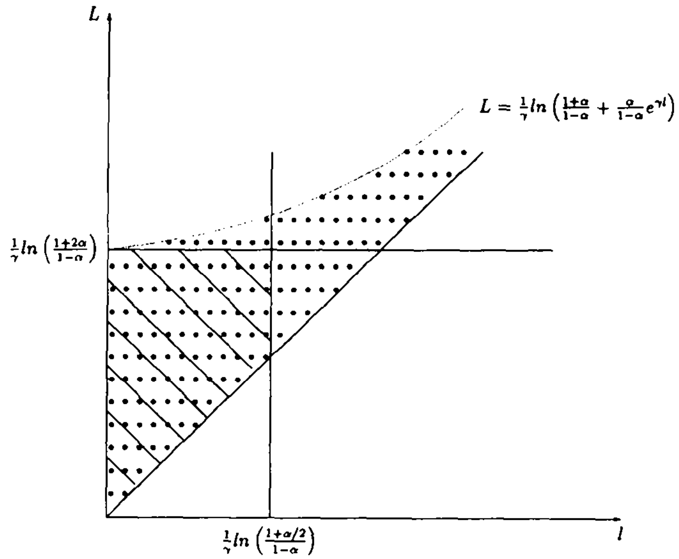

Figure 2
Market breakdown In the complete and the Incomplete markets
The sets of parameter values for which there is a marker breakdown in the complete or in the incomplete market are represented in Figure 2. The former is strictly included in the latter. That is, market breakdown is less likely to occur in the complete market than in the incomplete market. market breakdown with option; market breakdown without option.

dition is stated more precisely in the next proposition and is illustrated in Figure 2.

Proposition 8. *When the option is traded, there is no market breakdown if*

$$L > \frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) \quad or \quad l > \frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right).$$

The right-hand side of the two inequalities is increasing in a. The higher the degree of information asymmetry, the more likely the market breakdown. Also, because the two SCC can be unbundled, there is no market breakdown (i.e., some risk sharing occurs) as long as *one* of the liquidity shocks is large enough to generate trading in the corresponding SCC. Hence, the condition for no market breakdown is less demanding than in the incomplete market case. This is stated in the next corollary and illustrated in Figure 2.

Corollary to Proposition 8. *The set ofparameters for which there is a market breakdown when the option is traded is strictly included in the set of parameters for which there is a market breakdown when the option is not traded.*

This is one beneficial consequence of the option for the liquidity traders. By reducing the set of parameters for which there is a market breakdown, the introduction of the option facilitates risk sharing. In particular, if the structure of the liquidity shocks is unlike the payoff structure of the stocks (e.g., if the *u* shock *L* is close to the *m* shock *l),* risk sharing can take place only if the option is traded. This result could be examined empirically by studying the occurrence of trading halts (proxying for market breakdowns) for optioned versus nonoptioned stocks.

6.2 Information revelation

The stock price *(S)* is the expectation of the final value of the state (θ), conditional on the order flow. The difference between this conditional expectation and the realized final value of the state, $(S - \theta)$, can be interpreted as an estimation error. The lower the variance of this estimation error, the higher the informational efficiency of the market. So we define the informational efficiency of the market as $[V(S - \theta)]^{-1}$.[19] When there is a market breakdown, there is no trade on which the market makers can condition their expectations. Hence, the estimation error is simply the unconditional variance of the final state variable ($2/3\ \sigma^2$).

Proposition 9.

- *When there is no market breakdown in the incomplete market and the pure strategy equilibrium prevails in the complete market, then the informational efficiency of the market is not altered by the introduction of the option.*
- *When there is no market breakdown in the incomplete market and the (partially or fully) mixed equilibrium prevails in the complete market, then the informational efficiency of the market is lower when the option is traded.*
- *When there is a market breakdown in the incomplete but not in the complete market, then the informational efficiency of the market is higher when the option is traded.*

The introduction of the option increases the set of payoff profiles that can be traded. This has two countervailing effects on the informational efficiency of the market. On the one hand, as shown above, the introduction of the option makes it less likely that the market will break down. Consequently, it enables trades to take place and information to be revealed through trades. This tends to increase the

[19] Kyle (1985) or Grossman and Stiglitz (1980) use conditional variances to measure the informational efficiency of the market. This is because, in their normal distribution setting, conditional variances are constant. Our approach is similar to theirs, however.

informational efficiency of the market. On the other hand, the introduction of the option increases the set of profitable trades for the insider. For example, without the option the insider can only buy the stock in u and sell it in d but cannot trade profitably in m. In contrast, when the option is traded, in the two mixed equilibria, the insider in state m can mix over the type 1 and type 2 liquidity trades. This mixing blurrs the informational content of the type 1 (or type 3) trades and hence reduces the informational efficiency of the market.

7. The Informativeness of the Trades in the Stock and the Option and the Bid-Ask Spread

7.1 The informativeness of the trades in the stock and the option

We now describe the equilibrium prices and trades in terms of the actual securities, rather than in terms of the state-contingent claims. For simplicity, normalize m to σ.[20]

When several options with different strikes are written on the stock, all but one of these options are redundant. As a result there is an indeterminacy in the trades in the actual securities. Any given payoff profile can be generated by several different trades. We focus on one reasonable trading pattern by assuming that the liquidity traders first use the stock to hedge the bulk of their risk exposure and then select the simplest option trade that matches their actual trades to their optimal hedge.

For example, in the case where there is no asymmetric information, the type 1 liquidity trader wants to buy L units of the u SCC and l units of the m SCC. To do this he can buy l/σ shares of stock. This yields $2l$ in u, l in m, and 0 in d. Thus, the trader is perfectly hedged in states m and d. Given this stock purchase, the type 1 agent need only trade $(L - 2l)/(u - K)$ calls with strike K $(m < K < u)$ to achieve perfect hedging of the u-*state* shock. This call can be interpreted as an out-of-the-money call, to the extent that the strike is above the initial value of the stock, its unconditional expectation, m. Note that the trade in the out-of-the-money call is a purchase or a sale according to whether L is larger or smaller than $2l$.

Now consider the liquidity trades in the complete market with asymmetric information described in Lemma 2. To hedge his m-*state* shock, the type 1 liquidity trader can buy $[l - (1/\gamma)\ln(\pi_{m,1}/\pi_{d,1})]/\sigma$ stocks. This yields $2[l - (1/\gamma)(\pi_{m,1}/\pi_{d,1})]$ SCC in state u and $[l -$

[20] This simplifies the analysis without altering the nature of the problem. This normalization amounts to subtracting m σ to the payoff the stock in each state. This corresponds to the payoff generated by a levered position in the stock.

$(1/\gamma)\ln(\pi_{m,1}/\pi_{d,1})]$ SCC in state *m*. To fine tune his *state-u* hedge, the type 1 liquidity trader can trade

$$\frac{L - \frac{1}{\gamma}\ln\left(\frac{\pi_{u,1}}{\pi_{d,1}}\right) - 2\left[l - \frac{1}{\gamma}\ln\left(\frac{\pi_{m,1}}{\pi_{d,1}}\right)\right]}{u - K}$$

out-of-the-money calls, with strike $K \in]m, u[$. Note that, as in the symmetric information case, this can be a purchase or a sale. Consider, for example, the pure strategy equilibrium (described in Proposition 2). Substituting the values of the state prices (given in Table 1), it is easy to check that the type 1 trade in the out-of-the-money call is a purchase if and only if

$$L > \frac{1}{\gamma}\ln\left(\frac{1 + 2\alpha}{1 - \alpha}\right) + 2l.$$

Now the insider in state *u* mimics this trade; hence, if the above inequality does not hold she sells the call. Consequently, the sale of the call is not necessarily a negative signal. This is consistent with empirical results that options trades have little predictive power about the future evolution of the stock [see, for example, Vijh (1990) or Berkman (1993)].[21] Our description of the insider trades contrasts with the theoretical results of Back (1993), where the insider with good news buys the out-of-the-money call. This difference reflects the fact that Back (1993) considers exogenous liquidity trading, unaffected by the in- or out-of-the-moneyness of the call. In contrast, in our analysis, liquidity trading is endogenous. As a result, lack of liquidity trading in far out-of-the-money options can arise endogenously (consistent with casual empirical observation) and deter the insider from trading too aggressively in these options.

7.2 The bid-ask spread in the stock market

Our analysis can also be related to empirical studies of the effect of the introduction of options on the bid-ask spread in the stock market. For example, Fedenia and Grammatikos (1992) find that the reaction of the spread is ambiguous—it can either increase or decrease after the introduction of the option. To shed some light on this issue in our simple model, consider the case described in the previous subsection, where the type 1 liquidity trader buys the stock while the

[21] Our theoretical analysis points at a potential statistical problem in Vijh (1990). He regresses changes in option prices on simultaneous changes in the stock price. In contrast, our analysis shows that both the option and the stock price changes arc likely to result from the insider trades and be jointly endogenous. Easley, O'Hara, and Srinivas (1993) analyze this joint endogeneity.

type 3 liquidity trader sells it (whether the option is traded or not, as long as there is no market breakdown). In this case the ask and bid prices are

$$\begin{cases} A = \pi_{u,1}u + \pi_{m,1}m + \pi_{d,1}d = m + (\pi_{u,1} - \pi_{d,1})\sigma \quad \text{and} \\ B = \pi_{u,3}u + \pi_{m,3}m + \pi_{d,3}d = m + (\pi_{u,3} - \pi_{d,3})\sigma. \end{cases}$$

The ask price is the sum of (i) the unconditional expectation of the final value of the stock *(m)* and (ii) the update reflecting the informational content of the purchase $[(\pi_{u,1} - \pi_{d,1})\sigma]$. The latter is strong if the insider buys aggressively when she observes good news about the final value of the stock. Symmetrically, the informational content of a sale is $(\pi_{u,3} - \pi_{d,3})\sigma$. The spread *(A – B)* is the sum of the informational content of the purchase of the stock and the informational content of the sale. The spread is large when the insider responds strongly to her private information. Consequently, the spread is larger (tighter) when the option is traded when it is not if the insider responds more (less) aggressively to her private information.

The aggressiveness of the insider's strategy depends on the attractiveness of the liquidity trade she can mimic. A polar case is the fully mixed strategy equilibrium analyzed in Proposition 4. In this case the insider in state *u* is indifferent between mimicking the type 1 trade (which involves buying the stock) and not trading. This is because she earns zero profits when mimicking the type 1 trade. In this case the insider in state *u* does not react aggressively to her private information; when she observes that the final state of the world is *u*, she only buys the stock with probability λ. The bid and ask quotes in this equilibrium reflect this weak response of the insider to her private signal. The ask is $A = m + \alpha\lambda/[(1 - \alpha)/(2 + \alpha\lambda)]\sigma$, and the bid is $B = m - \alpha\lambda/[(1 - \alpha)/(2 + \alpha\lambda)]\sigma$. The spread is $2\alpha\lambda/[(1-\alpha)/(2+\alpha\lambda)]\sigma$. In contrast, in the incomplete market, the insider reacts more strongly to strong private signals: she always buys the stock on observing that the final value of the stock is *u* and sells it on observing *d*. As shown in Section 3.3, in this case the ask and bid prices are $A = m + \alpha\sigma$ and $B = m - \alpha\sigma$ and the spread is $2\alpha\sigma$. Consequently, the spread in the stock market is larger in the incomplete market than in the complete market when the fully mixed strategies equilibrium prevails.

The general point underlying this example is that the introduction of the option leads to a change in the spread because it alters the portfolios of SCC which the insider can trade and, as a result, her responsiveness to the private signal. If the introduction of the option leads to less (more) attractive trades for insiders with strong signals *(u* or *d)*, it reduces (increases) the responsiveness of the insider to these signals and consequently reduces (increases) the spread.

8. Conclusion

This article studies the effect of the introduction of an option that completes the market on risk sharing and information revelation. Thus, we study the interaction between information asymmetry and market incompleteness.

Market incompleteness reduces the *effectiveness* of hedging. In incomplete markets, the payoff structure of the securities does not perfectly match the structure of the risks faced by the liquidity traders. For example, in our model, this problem arises when the only risky security is the stock and when there is not much difference between the state-dependent endowment shocks of the liquidity trader in the good state and in the bad state. In this case the liquidity trader cannot efficiently hedge by trading the stock.

On the other hand, asymmetric information increases the cost of trading in financial markets. Thus, when the market is incomplete and the proportion of insiders is large, it is both ineffective and costly to trade. Consequently, the market can break down (i.e., there is no liquidity trading). Completing the market by introducing the option alleviates this problem and can permit some risk sharing.

To the extent that the option is necessary for trading to occur, it is also necessary for private information to be revealed. In contrast, when there is no market breakdown in the complete market, the introduction of the option can reduce the degree of information revelation. In the incomplete market the insider with very good (or very bad) information buys (or sells) the stock, while the insider with more neutral signals does not trade. In the complete market, when the endowment shocks of the liquidity traders are not too different across states, they design less extreme trades than the purchase (or the sale) of the stock. For example they can buy the stock and sell the out-of-the-money call. Such trades can be attractive both for the insider with strong signals and for the insider with more neutral signals. That insiders with different signals mimic these trades blurrs their information content. More generally, the introduction of the option complexifies the strategies of the insiders. As a result the information content of trades is more difficult to interpret for the market makers. This tends to reduce the degree of information revelation in the marketplace.

Similarly, the introduction of the option has ambiguous consequences on the profitability of insider trades. When the option is needed to avoid market breakdown, its introduction increases the insider's profit. When the liquidity trades in the complete market are less attractive for the insider, the introduction of the option reduces her profits.

Our analysis has implications for the study of endogenous market creation or security design. To the extent that the option increases the profits of the informed agent, the latter has an incentive to set up nonredundant markets. Note that, even if it is well known that the insider trades in this newly set-up market, liquidity traders may find it optimal to use it. In the pure strategy equilibrium (described in Proposition 2), the adverse selection cost is outweighed by the improved hedging opportunities. This could explain why markets created by agents who are likely to be informed attract trades. Examples of this phenomenon are the over-the-counter markets in derivatives on oil or commodities set up by nonfinancial firms that are likely to have private information, such as Elf Aquitaine (Elf Trading) or Cargill.

Another example of the interaction between securities design, asymmetric information, and market completeness is the case of corporations issuing securities. Managers designing securities are likely to subsequently obtain private information on the basis of which they can trade at the expense of the shareholders [see Diamond and Verrecchia (1991)]. To reduce the cost of capital, the managers may want to precommit not to trade on the basis of such private information. Our analysis sheds some light on this issue. If the introduction of the option reduces the profits of the insider (as is the case, for example, in the pure strategy equilibrium, when condition 5 does not hold), the managers might have an incentive, ex ante, to issue securities that improve the spanning of the markets.

Appendix

Proof of Lemma 1. The objective of the type 1 liquidity trader is

$$\max_{Q_1} = \tfrac{1}{3}U(-L + Q_1(u - A)) + \tfrac{1}{3}U(-l + Q_1(m - A)) + \tfrac{1}{3}U(Q_1(d - A)),$$

where the utility function is exponential with CARA coefficient equal to γ. The first-order condition is

$$(u - A)e^{-\gamma(-L+Q_1(u-A))} + (m - A)e^{-\gamma(-l+Q_1(m-A))} + (d - A)e^{-\gamma(Q_1(d-A))} = 0.$$

Simplifying by $e^{-\gamma Q_1(d-A)}$:

$$(u - A)e^{-\gamma(-L+2Q_1\sigma)} + (m - A)e^{-\gamma(-l+Q_1\sigma)} + (d - A) = 0.$$

Changing variables, we set $X = e^{-\gamma Q_1 \sigma}$, (or equivalently, $Q_1 = (1/$

$\gamma\sigma)\ln(1/X)$, and we get a quadratic equation:

$$(u - A)e^{-\gamma(-L)}X^2 + (m - A)e^{-\gamma(-l)}X + (d - A) = 0.$$

Since $(u - A)\, e^{-\gamma(-L)} > 0$ and $d - A < 0$, there is a unique positive root. The discriminant is

$$\Delta = (m - A)^2 e^{2\gamma l} + 4(u - A)(A - d)e^{\gamma L} > 0.$$

The positive solution is

$$\frac{(A - m)e^{\gamma l} + \sqrt{(A - m)^2 e^{\gamma l} + 4(u - A)(A - d)e^{\gamma L}}}{2(u - A)e^{\gamma L}}.$$

Imposing that Q_1 must be positive (otherwise, the price is the bid *B):*

$$Q_1 = \mathrm{Max}\left[0, \frac{1}{\gamma\sigma}\ln\left(\frac{2(u - A)e^{\gamma L}}{(A - m)e^{\gamma l} + \sqrt{(A - m)^2 e^{2\gamma l} + 4(u - A)(A - d)e^{\gamma L}}}\right)\right].$$

Q.E.D.

Proof of corollary to Lemma 1. The corollary to Lemma 1 obtains after straightforward manipulations of the expression in the lemma. Q.E.D.

Proof of Proposition 1. If the insider buys Q in state u, sells Q in state d, and does not trade in state m, then

$$\pi_{u,1} \equiv P(u \mid \mathrm{buy}Q) = \frac{P(\mathrm{buy}Q \mid u)P(u)}{P(\mathrm{buy}Q, u) + P(\mathrm{buy}Q, m) + P(\mathrm{buy}Q, d)}$$

$$= \frac{P(\mathrm{buy}Q \mid u)P(u)}{P(\mathrm{buy}Q \mid u)P(u) + P(\mathrm{buy}Q \mid m)P(m) + P(\mathrm{buy}Q \mid d)P(d)}.$$

Now $P(u) = P(m) = P(d) = 1/3$, so we can simplify the state probabilities. Also,

$$\begin{aligned} P(\text{buy } Q \mid u) &= P(\text{buy } Q \mid u, \text{insider})\, P(\text{insider}) \\ &\quad + P(\text{buy } Q \mid u, \text{liquidity})\, P(\text{liquidity}) \\ &= P(\text{buy } Q \mid u, \text{insider})\, \alpha \\ &\quad + P(\text{buy } Q \mid u, \text{liquidity})\, (1 - \alpha). \end{aligned}$$

Using the insider's strategy and given that the only liquidity trader who buys the stock is the type 1 trader, $P(\text{buy } Q \mid u) = \alpha + (1 - \alpha)/3$. Similarly, $P(\text{buy } Q \mid m) = P(\text{buy } Q \mid d = (1 - \alpha)/3$. Hence, $\pi_{u,1} = a + (1 - \alpha)/3 = (1 + 2\alpha)/3$. Similarly, $\pi_{m,1} \equiv P(m \mid \text{buy } Q) = \pi_{d,1} \equiv$

Proof of Lemma 2. The objective of the type 1 liquidity trader to choose $Q_{u,1}$ and $Q_{m,1}$ to maximize

$$\begin{aligned} &\tfrac{1}{3}U(-L + Q_{u,1}(1 - \pi_{u,1}) - Q_{m,1}\pi_{m,1}) \\ &\quad + \tfrac{1}{3}U(-l - Q_{u,1}\pi_{u,1} + Q_{m,1}(1 - \pi_{m,1})) \\ &\quad + \tfrac{1}{3}U(-Q_{u,1}\pi_{u,1} - Q_{m,1}\pi_{m,1}), \end{aligned}$$

where the utility function is exponential with CARA coefficient equal to γ. The first-order condition with respect to $Q_{u,1}$ is

$$\begin{aligned} &(1 - \pi_{u,1})e^{-\gamma(-L+Q_{u,1}(1-\pi_{u,1})-Q_{m,1}\pi_{m,1})} - \pi_{u,1}e^{-\gamma(-l-Q_{u,1}\pi_{u,1}+Q_{m,1}(1-\pi_{m,1}))} \\ &\quad - \pi_{u,1}e^{-\gamma(-Q_{u,1}\pi_{u,1}-Q_{m,1}\pi_{m,1})} = 0. \end{aligned}$$

Simplifying by $e^{-\gamma(-Q_{u,1}\pi_{u,1}-Q_{m,1}\pi_{m,1})}$,

$$(1 - \pi_{u,1})e^{\gamma(L-Q_{u,1})} - \pi_{u,1}e^{\gamma(l-Q_{m,1})} - \pi_{u,1} = 0.$$

Similarly the first-order condition with respect to $Q_{m,1}$ is

Setting $X_{u,1} \equiv e^{-\gamma Q_{u,1}}$ and $X_{m,1} \equiv e^{-\gamma Q_{m,1}}$, we have a linear system. The determinant is $D = \pi_{d,1} > 0$. The solutions are $X_{u,1} = (\pi_{u,1}/\pi_{d,1})e^{\gamma L}$ and $X_{m,1} = (\pi_{m,1}/\pi_{d,1})e^{\gamma L}$, which yields the values of $Q_{u,1}$ and $Q_{m,1}$ in the lemma. Q.E.D.

Proof of Proposition 2. First consider the liquidity trades. Suppose that, in equilibrium, the insider in state *u(d)* always mimics the trade of the type 1 (3) liquidity trader T_1 (T_3) and that the insider in state *m* never trades. As a result the updated probabilities of the market makers are the same as in the incomplete market. Substituting these probabilities in the quantities in Lemma 2, we obtain the equilibrium quantities given in Table 1. In particular, one can check that, for the parameter set where the pure strategy equilibrium prevails (condition 4).

$$L > \frac{1}{\gamma}\ln\left(\frac{1 + 2\alpha}{1 - \alpha}\right) + \frac{2 + \alpha}{1 + 2\alpha}l,$$

and, for the equilibrium prices and quantities, there is no market breakdown, $Q_{u,1}$ and $Q_{m,1}$ are strictly positive.

We now turn to the insider's trades. We show that under condition 4 the insider finds it optimal to use the pure strategy described in the proposition. We consider only the choice by the insider between the different equilibrium trades (used by the type 1, 2, and 3 liquidity traders). Out-of-equilibrium trades can be eliminated by invoking similar arguments, as in Proposition 1.

For the insider in state u, the incentive compatibility constraint is that she must be better off using the same trade as the type 1 liquidity trader than using the same trade as the type 2 liquidity trader. The condition is $Q_{u,1}(1-\pi_{u,1}) - Q_{m,1}\pi_{m,1} > 0$ or $Q_{u,1} > Q_{m,1}\pi_{m,1}/(1-\pi_{u,1})$. The incentive compatibility condition for the insider in state m is

$$-\pi_{u,1}Q_{u,1} + (1-\pi_{m,1})Q_{m,1} < 0.$$

Or

$$Q_{u,1} > Q_{m,1}\frac{1-\pi_{m,1}}{\pi_{u,1}}.$$

Only the type m constraint is relevant because $(1-\pi_{m,1})/(\pi_{u,1}) > (\pi_{m,1})/(1-\pi_{u,1})$ Substituting the values of $Q_{u,1}$ $Q_{m,1}$, $\pi_{u,1}$, $\pi_{m,1}$, the type m incentive compatibility constraint can be rewritten

$$L > \frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) + l\frac{2+\alpha}{1+2\alpha},$$

which is condition 4 in the proposition. Q.E.D.

Proof of Proposition 3. In the partially mixed strategy equilibrium described in Proposition 3, the insider in state u mimics the type 1 liquidity trade (T_1), the insider in state d mimics the type 3 liquidity trade (T_3) and the insider in state m mixes over the type 1 liquidity trade (with probability $\mu \in [0, \frac{1}{2}]$), the type 3 liquidity trade (with probability $\mu \in [0, \frac{1}{2}]$), and the type 2 liquidity trade (T_2) (with probability $1 - 2\mu$). For these trades, the probability of the state u, conditional on the type 1 trade, is

$$\pi_{u,1} \equiv P(u \mid T_1) = \frac{P(T_1 \mid u)P(u)}{P(T_1, u) + P(T_1, m) + P(T_1, d)}$$

$$= \frac{P(T_1 \mid u)P(u)}{P(T_1 \mid u)P(u) + P(T_1 \mid m)P(m) + P(\text{buy}T_1 \mid d)P(d)}.$$

Now $P(u) = P(m) = P(d) = 1/3$, so we can simplify the state probabilities. Also,

$$\begin{aligned} P(T_1 \mid u) &= P(T_1 \mid u, \text{insider})\, P(\text{insider}) \\ &\quad + P(T_1 \mid u, \text{liquidity})\, P(\text{liquidity}) \\ &= P(T_1 \mid u, \text{insider})\,\alpha + P(T_1 \mid u, \text{liquidity})\,(1-\alpha). \end{aligned}$$

Given that the liquidity trader can be of type 1 with probability 1/3, that the insider in state u always mimics T_1, and that the insider in state m mimics T_1 with probability μ,

$$\pi_{u,1} = \frac{\alpha + \frac{1-\alpha}{3}}{\left(\alpha + \frac{1-\alpha}{3}\right) + \left(\alpha\mu + \frac{1-\alpha}{3}\right) + \left(\frac{1-\alpha}{3}\right)} = \frac{\alpha + \frac{1-\alpha}{3}}{1 + \alpha\mu}.$$

Similarly, $\pi_{m,1} = [\alpha\mu + (1-\alpha)/3]/(1+\alpha\mu)$ and $\pi_{d,1} = [(1-\alpha)/3]/(1+\alpha\mu)$.

Substituting these conditional probabilities in Lemma 2, one obtains the liquidity trades described in Table 1. One can check that, when

$$\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) + \frac{2+\alpha}{1+2\alpha}l$$
$$> L > \operatorname{Max}\left\{\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right), \frac{1}{\gamma}\left[\ln\left(\frac{1+2\alpha}{1-\alpha}\right) - \frac{2+\alpha}{4-\alpha}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right] + \frac{2+\alpha}{4-\alpha}l\right\},$$

there is no market breakdown; $Q_{u,1}$ and $Q_{m,1}$ are strictly positive.

Finally, we check that for these parameter values the insider finds it optimal to mimic T_1 in u and T_3 in d and is indifferent between T_1, T_2, and T_3 in m. Consider the insider in state u. (The case of the insider in state d is symmetric and is not described.) Obviously, she will not mimic the type 3 trade (which involves buying the m and d SCC.) The incentive compatibility condition for her is that mimicking the type 1 trade be more profitable than mimicking the type 2 trade; that is,[22] $Q_{u,1}(1-\pi_{u,1}) - Q_{m,1}\pi_{m,1} > 0$. Or

$$Q_{u,1} > Q_{m,1}\frac{\pi_{m,1}}{1-\pi_{u,1}}.$$

Consider the insider in state m. Two cases must be distinguished. Either $\mu < 1/2$ and the incentive compatibility condition is

$$-\pi_{u,1}Q_{u,1} + (1-\pi_{m,1})Q_{m,1} = 0.$$

Or $\mu = 1/2$ and the incentive compatibility condition is

$$-\pi_{u,1}Q_{u,1} + (1-\pi_{m,1})Q_{m,1} > 0.$$

1. We first analyze the case where $\mu = 1/2$. In this case the i.c. con-

[22] As in the previous proposition, we focus on equilibrium trades only.

dition for the insider in state m is

$$Q_{u,1} < Q_{m,1}\frac{1-\pi_{m,1}}{\pi_{u,1}}.$$

In contrast with the pure strategy equilibrium, both the type u and type m constraints are relevant. They combine to

$$\frac{1-\pi_{m,1}}{\pi_{u,1}}Q_{m,1} > Q_{u,1} > \frac{\pi_{m,1}}{1-\pi_{u,1}}Q_{m,1}. \qquad \text{(A1)}$$

Substituting the quantities from Lemma 2 and the conditional probabilities evaluated at $\mu = 1/2$, the i.c. condition (7) is

$$\begin{aligned}&\frac{2+\alpha}{1+2\alpha}\left[l-\frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]\\ &\quad > L-\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) > \frac{\alpha+2}{4-\alpha}\left[l-\frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right].\end{aligned}$$

Or

$$\begin{aligned}&\frac{1}{\gamma}\ln\left[\left(\frac{1+2\alpha}{1-\alpha}\right)-\frac{2+\alpha}{1+2\alpha}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]+\left(\frac{2+\alpha}{1+2\alpha}\right)l\\ &\quad > L > \frac{1}{\gamma}\left[\ln\left(\frac{1+2\alpha}{1-\alpha}\right)-\left(\frac{2+\alpha}{4-\alpha}\right)\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]+\left(\frac{2+\alpha}{4-\alpha}\right)l.\end{aligned}$$

This corresponds to region 3 in Figure 1. One can check that, in this region, when the insider follows the partially mixed strategy with $\mu = 1/2$, $Q_{u,1}$ and $Q_{m,1}$ are strictly positive.

2. Second, we analyze the case where $\mu < 1/2$. In this case the i.c. condition for the insider in state m is

$$Q_{u,1} = Q_{m,1}\frac{1-\pi_{m,1}}{\pi_{u,1}}.$$

As in the pure strategy equilibrium, the type u constraint is not relevant; only the type m constraint is relevant. Substituting the quantities from Lemma 2 and the updated probabilities, for $\mu \in [0, \frac{1}{2}]$ the condition is

$$\left[L-\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)\right]=\left[l-\frac{1}{\gamma}\ln\left(\frac{1+\alpha(3\mu-1)}{1-\alpha}\right)\right]\left(\frac{2+\alpha}{1+2\alpha}\right).$$

The right-hand side of this equation is continuous, decreasing in μ, and equal to $l(2+\alpha)/(1+2\alpha)$ for $\mu = 0$ and $[(2+\alpha)/(1+2\alpha)]\{l-(1/\gamma)\ln[(1+\alpha/2)/(1-\alpha)]\}$ for $\mu = 1/2$. When

$$\frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)+l\frac{2+\alpha}{1+2\alpha}$$
$$>L>\frac{1}{\gamma}\left[\ln\left(\frac{1+2\alpha}{1-\alpha}\right)-\frac{2+\alpha}{1+2\alpha}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]+l\frac{2+\alpha}{1+2\alpha},$$

which corresponds to regions 2 in Figure 1, one can see from Figure *3A* that there exists a probability $\mu \in]0, \frac{1}{2}[$ such that the incentive compatibility condition holds. To conclude the proof, one can check that, for the parameter values in region 2, when the insider plays the partially mixed strategy with $\mu \in]0, \frac{1}{2}[$, there is no market breakdown. Q.E.D.

Proof of Proposition 4. In the fully mixed strategies equilibrium, the insider mixes between the type 1 trade (T_1), with probability λ, and the type 2 trade (T_2), with probability $1-\lambda$. The insider mixes with equal probabilities between T_1 and T_2.

First, we analyze the conditional probabilities in this equilibrium. Following the same reasoning as in the two previous propositions,

$$\pi_{u,1} \equiv P(u \mid T_1) = \frac{\alpha\lambda + (1-\alpha)/3}{1+\alpha\lambda-\alpha/2},$$

$$\pi_{m,1} \equiv P(m \mid T_1) = \frac{\alpha/2 + (1-\alpha)/3}{1+\alpha\lambda-\alpha/2},$$

and

$$\pi_{d,1} \equiv P(d \mid T_1) = \frac{(1-\alpha)/3}{1+\alpha\lambda-\alpha/2}.$$

Next, we turn to the incentive compatibility condition of the insider.[23] The incentive compatibility of the insider in state u is

$$Q_{u,1} = Q_{m,1}\frac{\pi_{m,1}}{1-\pi_{u,1}}.$$

In state m it is

$$Q_{u,1} < Q_{m,1}\frac{1-\pi_{m,1}}{\pi_{u,1}}.$$

Only the *u-state* condition is relevant. Substituting in the condition the quantities obtained in Lemma 2 and the above-computed conditional probabilities, the condition is

[23] Again we focus on the equilibrium actions.

$$\left[L - \frac{1}{\gamma}\ln\left(\frac{\alpha\lambda + (1-\alpha)/3}{(1-\alpha)/3}\right)\right]$$
$$= \left[l - \frac{1}{\gamma}\ln\left(\frac{\alpha/2 + (1-\alpha)/3}{(1-\alpha)/3}\right)\right]\left(\frac{\alpha/2 + (1-\alpha)/3}{\alpha/2 + 2/3(1-\alpha)}\right).$$

So the incentive compatibility condition is

$$\left[L - \frac{1}{\gamma}\ln\left(\frac{\alpha\lambda + (1-\alpha)/3}{(1-\alpha)/3}\right)\right] = \left[l - \frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]\left(\frac{2+\alpha}{4-\alpha}\right).$$

The left-hand side of this equation is decreasing and continuous in λ. For $\lambda = 0$ it is equal to L. For $\lambda = 1$ it is equal to $L - (1 + 2\alpha)/(1 - \alpha)$. Consider Figure *3B.* Note that

$$L > \left[l - \frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]\left(\frac{2+\alpha}{4-\alpha}\right),$$

since $L > l$. If

$$\left[l - \frac{1}{\gamma}\ln\left(\frac{1+\alpha/2}{1-\alpha}\right)\right]\left(\frac{2+\alpha/2}{4-\alpha}\right) > L - \frac{1+2\alpha}{1-\alpha}$$

(as is the case in the figure), there exists a probability λ such that the incentive compatibility condition holds. To conclude the proof, one can check that, if the above inequality holds and if $l < (1/\lambda\ (1 + \alpha/2)/(1 - \alpha)$, then there is no market breakdown (i.e., the liquidity trades are strictly positive). Q.E.D.

Proof of Proposition 5. The prices of the *u, m,* and *d* SCC are the probabilities of the respective states, conditional on the information set of the market makers. These conditional probabilities are the same in the complete market and the incomplete market because the information sets of the market makers are the same. To see this, first consider the set of all possible states of the world in the two market structures. The states of the world are differentiated by the final value of the stock *(u, m,* or *d)* and the identity of the trader: insider (denoted *i),* type 1 (denoted T_1), type 2 (denoted T_2), or type 3 (denoted T_3) liquidity trader. So there are 12 states, and the state space Ω is

$$\begin{aligned}\Omega = \{&(u, i), (u, T_1), (u, T_2), (u, T_3),\\ &(m, i), (m, T_1), (m, T_2), (m, T_3),\\ &(d, i), (d, T_1), (d, T_2), (d, T_3)\}.\end{aligned}$$

In the incomplete market the insider in state *u* always buys the stock, like the type 1 liquidity trader; the insider in state m never trades, like the type 2 liquidity trader; and the insider in state *d* always sells the stock, like the type 3 liquidity trader. So the information structure

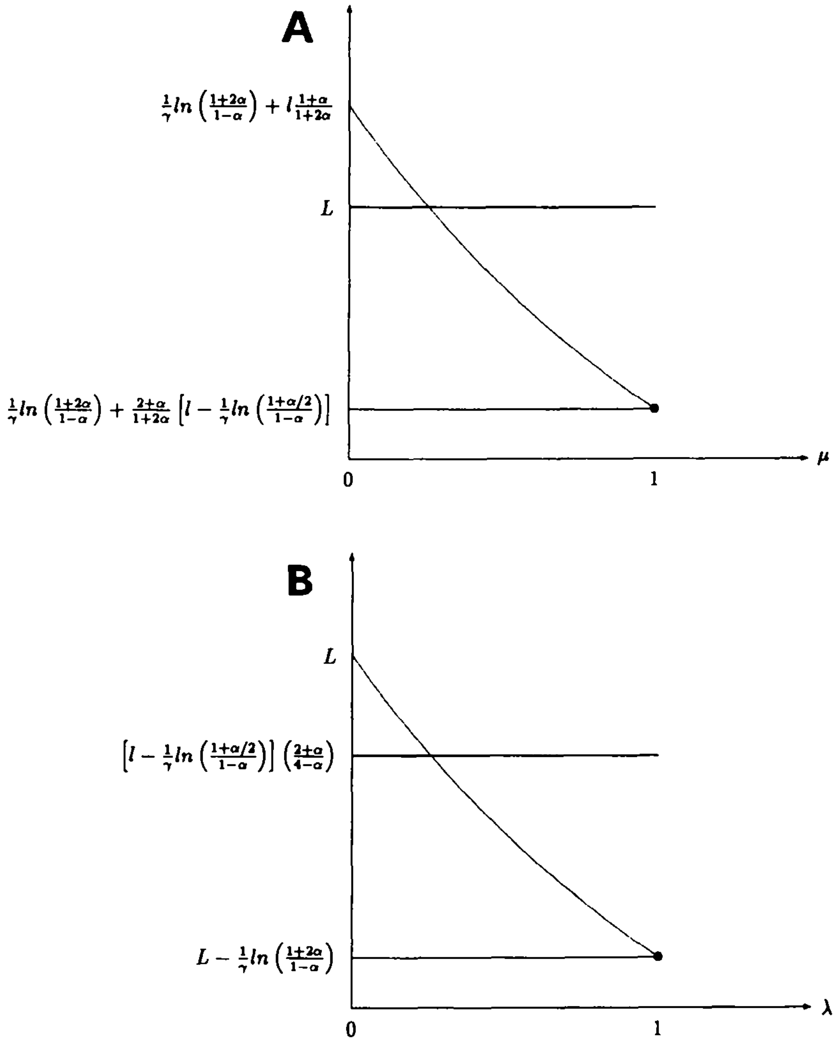

Figure 3
Graphical analysis of the existence of equilibrium mixing probabilities in the partially and fully mixed strategy equilibria
Figure 3A illustrates that, in the partially mixed strategy equilibrium, there exists a value of μ (the mixing probability) for which the equilibrium indifference condition is satisfied. This value is at the intersection of the horizontal line *(L)* and the downward sloping curve.

Figure 3B illustrates that, in the fully mixed strategy equilibrium, there exists a value of λ (the mixing probability) for which the equilibrium indifference condition is satisfied. This value is at the intersection of the horizontal line *(L)* and the downward sloping curve.

of the market makers is the following partition of the state space:

$$\{[(u, i), (u, T_1), (m, T_1), (d, T_1)], [(m, i), (u, T_2), (m, T_2), (d, T_2)], [(d, i), (u, T_3), (m, T_3), (d, T_3)]\}.$$

This reflects that, for example, the market makers cannot differentiate the purchase of the stock from the insider in state u from the purchase of the stock by the type 1 liquidity trader in state m but that they can differentiate it from the sale of the stock by the type 3 liquidity trader.

Similarly, in the complete market, in the pure strategy equilibrium, the insider always mimics the type 1 trade in state u, the type 2 trade in state m, and the type 3 trade in state d. Hence, the informational structure of the market makers is the same partition as above.

Q.E.D.

Proof of Proposition 6. Both in the incomplete market and in the complete market pure strategy equilibrium, the insider mimics the type 1 trade in u, the type 2 trade in m, and the type 3 trade in d. She earns no profit in state m, and, because of the symmetry of the game, her profits in u are equal to her profits in d. So we need only compare the insider profits in u when the option is traded and when it is not.

In state u the insider's profit when the option is not traded is

$$(u - A)Q = \frac{1-\alpha}{\gamma}\ln\left(\frac{2(1-\alpha)e^{\gamma L}}{\alpha e^{\gamma l} + \sqrt{\alpha^2 e^{2\gamma l} + 4(1-\alpha^2)e^{\gamma L}}}\right).$$

In state u the insider's profit when the option is traded is

$$(1 - \pi_{u,1})Q_{u,1} - \pi_{m,1}Q_{m,1} = \frac{1-\alpha}{3}\left\{2\left[L - \frac{1}{\gamma}\ln\left(\frac{1+2\alpha}{1-\alpha}\right)\right] - l\right\}.$$

The insider's profit is larger when the option is traded if

$$\frac{2}{3\gamma}L - \frac{2}{3}\ln\left(\frac{1+2\alpha}{1-\alpha}\right) - \frac{\gamma l}{3}$$
$$> \ln\left(\frac{2(1-\alpha)e^{\gamma L}}{\alpha e^{\gamma l} + \sqrt{\alpha^2 e^{2\gamma l} + 4(1-\alpha^2)e^{\gamma L}}}\right).$$

Taking exponentials on both sides and simplifying,

$$\sqrt{\alpha^2 e^{2\gamma l} + 4(1-\alpha^2)e^{\gamma L}}$$
$$> 2(1+2\alpha)^{2/3}(1-\alpha)^{1/3}e^{\gamma(L+l)/3} - \alpha e^{\gamma l}.$$

Squaring both sides of the inequality and simplifying,

$$(1-\alpha^2)e^{2/3\gamma L} - (1+2\alpha)^{4/3}(1-\alpha)^{2/3}e^{\gamma L/3}e^{2\gamma l/3}$$
$$+ \alpha(1+2\alpha)^{2/3}(1-\alpha)^{1/3}e^{4\gamma l/3} > 0.$$

Setting $X \equiv e^{\gamma L/3}$ and $Y = e^{2/3\gamma l}$, this is a quadratic in X and Y. The discriminant with respect to X is

$$\Delta_X = [(1-\alpha)^{2/3}(1+2\alpha)^{1/3}Y)]^2 > 0.$$

The roots are $[(1+2\alpha)/(1-\alpha)]^{1/3}Y$ and $[(1+2\alpha)/(1-\alpha)]^{1/3} \cdot Y[\alpha/(1+\alpha)]$. The quadratic is negative between the roots and positive outside them. The quadratic is positive (i.e., the profit of the insider is larger with the option) iff X is larger than the highest root.[24]

$$X > \left(\frac{1+2\alpha}{1-\alpha}\right)^{1/3} Y.$$

Substituting the values of X and Y, the above inequality hold iff $L > (1/\gamma)\ln[(1+2\alpha)/(1-\alpha)] + 2l$.

This establishes the result. Q.E.D.

Proof of Proposition 8. The proposition obtains by substituting the probabilities corresponding to the pure, partially mixed, and fully mixed strategies equilibria (described in Propositions 2, 3, 4) in the quantities given in Lemma 2. Q.E.D.

Proof of corollary to Proposition 8. The corollary obtains straightforwardly, by combining Proposition 8 and condition (3). Q.E.D.

Proof of Proposition 9. The information efficiency of the market increases as $V(S-\theta)$ decreases. $V(S-\theta) = V(S) + V(\theta) - 2\text{cov}(S, \theta)$ Note that $V(\theta) = \frac{2}{3}\sigma^2$. To analyze the informational efficiency of the market, we analyze how $V(S)$ and $\text{cov}(S, \theta)$ vary across equilibria.

The first part of the proposition stems from the identity of the trading strategies and stock prices in the incomplete market and in the pure equilibrium in the complete market. The last part of the proposition stems from the fact that, when there is trading in the complete market and not in the incomplete market, there is some information revelation in the former but none in the latter. We now establish the second part of the proposition:

1. First, consider the incomplete market when there is no market breakdown.

$$V(S) = (\alpha\sigma)^2\left[\tfrac{2}{3}\alpha + \tfrac{2}{3}(1-\alpha)\right] = \tfrac{2}{3}(\alpha\sigma)^2$$

and

$\text{cov}(S, \theta)$

$$= P(\text{insider})[P(u)(\alpha\sigma)(\sigma) + P(m)(0)(0) + P(d)(-\alpha\sigma)(-\sigma)]$$
$$+ P(\text{liquidity})\{P(u)[P(\text{type}-1)(\alpha\sigma)\sigma + P(\text{type}-2)0\sigma$$

[24] X cannot be below the lowest root when the pure strategy equilibrium prevails and there is no market breakdown in the complete market.

$$+ P(\text{type} - 3)(-\alpha\sigma)\sigma]$$
$$+ P(m)[P(\text{type} - 1)(\alpha\sigma)0 + P(\text{type} - 2)(0)(0)$$
$$+ P(\text{type } 3)(-\alpha\sigma)0]$$
$$+ P(d)[P(\text{type} - 1)(\alpha\sigma)(-\sigma)$$
$$+ P(\text{type} - 2)0(-\sigma)$$
$$+ P(\text{type} - 3)(-\alpha\sigma)(-\sigma)]\}.$$

The second term on the right-hand side [multiplied by P(liquidity)] simplifies to 0. Substituting for the values of the probabilities, the first term simplifies to cov(S, θ) = $\frac{2}{3}\alpha^2\sigma^2$. So, $V(S - \theta) = \frac{2}{3}(1 - \alpha^2)\sigma^2$.

2. Second, consider the complete market, in the partially mixed equilibrium. Taking similar steps as in the previous case,

$$V(S) = \text{cov}(\theta, S) = \frac{2}{3}\frac{(\alpha\sigma)^2}{1 + \alpha\mu}.$$

So,

$$V(S - \theta) = \frac{2}{3}\sigma^2\left(1 - \frac{\alpha^2}{1 + \alpha\mu}\right).$$

Since $(\alpha, \mu) \in [0, 1]^2$, this is higher than $V(S - \theta)$ in the incomplete market case $[\frac{2}{3}(1 - \alpha^2)\sigma^2]$.

3. Third, consider the complete market, in the fully mixed equilibrium.

$$V(S) = \text{cov}(S, \theta) = \frac{2}{3}\frac{(\alpha\lambda\sigma)^2}{1 + \alpha\lambda - \alpha/2}.$$

So,

$$V(S - \theta) = \frac{2}{3}\sigma^2\left(1 - \frac{(\alpha\lambda\sigma)^2}{1 + \alpha\lambda - \alpha/2}\right).$$

So the informational efficiency of the market is lower in this case than in the incomplete market if

$$\frac{2}{3}\left[1 - \frac{(\alpha\lambda\sigma)^2}{1 + \alpha\lambda - \alpha/2}\right]\sigma^2 > \frac{2}{3}(1 - \alpha^2)\sigma^2$$

or

$$-\lambda^2 + \alpha\lambda + (1 - \alpha/2) > 0.$$

This is a quadratic inequality in λ. The unique positive root is

$$\lambda = \frac{\alpha + \sqrt{\alpha^2 + 4 - 2\alpha}}{2},$$

which is larger than 1. Note also that for $\lambda = 0$ the inequality holds. So for all $\lambda \in [0, 1]$ the inequality holds. Consequently, the informational efficiency of the market is lower in the complete market fully mixed equilibrium than in the incomplete market (when there is no market breakdown). Q.E.D.

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