

Repurchase Premia as a Reason for Dividends: A Dynamic Model of Corporate Payout Policies

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We propose that it is precisely because firms' repurchases of their own stock through tender offers are associated with large stock-price increases that repurchases are unattractive as a means of distributing cash. As a result, firms distribute some cash in the form of dividends – despite the tax disadvantage-and carry the rest to future periods. However, when their stock is sufficiently undervalued, firms distribute all accumulated cash through stock repurchases. We show that dividends are smoothed and are positively related both to earnings innovations and to previous period's dividends. Also, the stock-price reaction to a repurchase announcement, of a given size, is increasing in the previous period's dividends.

The puzzle about why firms pay dividends despite the substantial tax disadvantage associated with dividend payments has been debated extensively by financial

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economists.¹ Stock repurchases, as an alternative means of cash distribution to equity holders, are well known to be significantly less tax disadvantaged than dividends. Despite that fact, stock repurchases are used relatively infrequently. Firms can repurchase shares either through an intrafirm tender offer or by engaging in an open market repurchase program. Open market repurchases can be costly, however, on account of their effect on the liquidity of corporations' securities, as shown in Barclay and Smith (1988).² We argue that repurchases through intrafirm tender offers may also involve substantial costs.

A stylized fact about repurchases of stock by firms is that investors tend to react positively to the announcement of a firm's tender offer for its own stock, resulting in a substantial increase in the firm's stock price [the magnitude of the increase is about 20 percent; see Dann (1981) and Vermaelen (1981)]. We propose that it is precisely the fact that tender offer repurchases are associated with large stock-price increases that makes them unattractive as a means of distributing cash. The occasions when repurchases would not be costly to the firm, despite the large premium it must pay to acquire its own shares, are when the firm's stock is indeed undervalued by the market compared to its true value which is observed by its managers. The intuition here is analogous to that in Myers and Majluf (1984). In their model, firms may decide to forgo positive net present-value projects if external equity financing is required and the price at which new equity can be sold is substantially below the true value observed by the firm managers. In our model, the firm may choose not to repurchase shares, despite the tax advantage, if the price at which the shares can be repurchased is substantially higher than the true value. We analyze the trade-offs firms face in choosing between cash dividends and tender offers for their own stock in an intertemporal setting.

Our argument suggests why firms repurchase their stock relatively infrequently through tender offers. Firms carry cash through time so as to be able to distribute it in the form of repurchases at appropriate times. However, it may not be as efficient for a firm as it would be for investors themselves to carry cash through time. Firms, therefore, trade off the marginal benefit associated with distributing cash in the form of dividends to the marginal benefit associated with carrying

¹See Miller and Modigliani (1961), Black (1976), Miller and Scholes (1978), Bhattacharya (1979), DeAngelo and Masulis (1980), Easterbrook (1984), John and Williams (1985), Miller and Rock (1985), Miller (1986), Barclay and Smith (1988), Brennan and Thakor (1990), among others.

²They argue: "Open-market repurchases increase the fraction of informed traders in the market. This widens the bid-ask spread, raises the required rate of return (opportunity cost of capital) on the claims of the corporation, reduces corporate investment and lowers firm value..." They find that open market repurchases increase the bid-ask spread, and they estimate the implied loss in firm value to be 7.9 percent.

cash forward to future periods in which the cash could be distributed in the form of a repurchase. The optimal payout policy, in general, is to distribute some cash in the form of dividends and carry the rest to future periods; whenever the firm's stock is sufficiently undervalued, the firm distributes all its accumulated cash by engaging in a stock repurchase program. This argument explains not only why firms pay dividends but also why firms repurchase stocks relatively infrequently. The amount of cash disbursed through a stock repurchase is also likely to be large compared to the dividend payments in any period.

We develop a model that formalizes the arguments presented above. The basic idea of the model is as follows. Firms generate a random quantity of surplus cash each period. Managers of a firm can distribute part or all of this and any cash held over from earlier periods in the form of dividends or stock repurchases and carry the rest of the cash forward. Shareholders prefer a tax-free distribution of excess cash to having the firm invest this cash for them in marketable securities. There are several reasons why this may be true. First, there may be significant costs to having the firm retain large quantities of "free cash flow" [see Jensen (1986) and Stulz (1990)]. Surplus cash may lead to waste through organizational inefficiencies, for example, through excessive perquisite consumption. In addition, large surplus cash holdings may also necessitate high monitoring costs to prevent the possibility of embezzlement and fraud. The "deep pockets" of a cash-rich firm may also make the firm more likely to be a target of lawsuits by other stakeholders. Finally, the corporate income tax rate may exceed the personal income tax rates. To capture these ideas we assume that any surplus cash retained in the firm grows at a rate (which could be different each period) less than the rate on marketable securities that the investors could earn were they to invest the cash on their own.

Managers of a firm have more timely information about the evolution in the value of the firm's assets in place than outside investors. There is no information asymmetry about the total cash held by the firm, nor do managers have any incentive to engage in costly signaling to reveal information about firm value. Managers are assumed to choose payout policies that maximize the present value of the after-tax cash flows for the average investor left in the firm at the end of the period. This objective can be justified on several grounds. First, managers themselves usually own shares in their firm, which they do not tender in a repurchase plan.³ Second, it is the long-term investors in the firm

³A plausible explanation for this institutional practice is as follows: If managers or informed insiders were expected to tender their shares for repurchase, this would result in other shareholders tendering their shares as well. The reason is that given that managers and insiders have superior information about the true value of the firm, tendering shares would strictly dominate not tendering

who are likely to be instrumental in determining managerial compensation and tenure.

Since repurchases result in long-term shareholders acquiring a larger proportion of the equity of the firm, the decision about whether to repurchase stock depends on whether managers believe that the firm is being undervalued by the market. Firms that are being undervalued by the market in a particular period may engage in a stock repurchase, whereas firms that are not being undervalued by the market may find it simply too expensive to repurchase equity that period. They may prefer to pay out some dividends and carry the rest of the cash forward in the expectation that a share repurchase could be conducted at a later period. Given that repurchases are tax advantaged, when firms find it optimal to repurchase they will use all their accumulated cash in the process. In a period when it is not optimal to repurchase, a firm's decision about the quantity of cash to pay out in dividends will be determined by factors such as the effective marginal cost of paying dividends compared to the cost of carrying cash through time and the expected marginal cost of paying out dividends and repurchasing in future periods.

The model generates a number of time-series and cross-sectional predictions about firm dividend and repurchase policies.

In our model, dividends are increasing in the level of accumulated cash and so is the quantity of cash retained for future distributions. Since firms retain more cash as the accumulated level of cash increases, the expected level of dividends, or the size of the stock repurchase next period is positively related to the level of dividends in the current period and to the level of unexpected earnings next period. Dividends exhibit a smoothing behavior: an increase (decrease) of a dollar in unexpected earnings is, on average, associated with less than a dollar increase (decrease) in dividends.⁴

The model implies that firms that are less likely to be able to repurchase shares pay out larger dividends and carry forward a smaller fraction of their cash in any period in which they are not repurchasing.

shares. The net result of this would be to leave the proportional stake of shareholders in the firm largely unchanged. The distribution would, in this case, be treated as being essentially equivalent to a dividend and would not qualify for preferential tax treatment under Section 302 of the Internal Revenue Code. If managers do not to tender their shares, then, in equilibrium, tendering shares may no longer be the dominant strategy for other shareholders. Given shareholder heterogeneity, the likelihood of a repurchase resulting in a proportional redemption is small. This may be one reason why managers commit *not* to submit their shares for repurchase and often make this commitment known in the offering circular itself [see Vermaelen (1981)].

An alternative reason that has been suggested in the literature for this institutional practice may be managerial concern about the possibility of shareholder lawsuits and the risk of being charged with insider trading violations.

⁴Evidence on dividend smoothing is provided in the classic study by Lintner (1956).

Hence, the model predicts that firms that repurchase less frequently will have higher dividend yields. These firms will also have a higher stock-price reaction associated with a repurchase announcement of a given size.

Another prediction is that stock-price return associated with the announcement of a repurchase of a particular size will be increasing in the size of the previous period's dividend. The rationale is that firms that had a smaller likelihood of repurchasing would have paid out a larger dividend in the earlier period for a given repurchase size.

The model also suggests a possible explanation for some of the aggregate patterns in repurchase activity by firms. It has been observed in many instances that the number of stock repurchases tends to increase following a stock market decline.⁵ We explore the possibility that the arrival of negative public information about the evolution of a firm's asset value may lead to a switch from a separating to a pooling equilibrium in which the firm distributes cash through a repurchase, irrespective of its asset value. In such a pooling equilibrium the repurchase premium would have to be small enough to induce even a firm being overvalued by investors to engage in a repurchase. If the evolution in the asset values of firms are contemporaneously correlated on account of publicly observed common economic factors, this leads to the prediction that repurchases may increase following downturns in the stock market.

Little of the work on dividends or repurchases has focused on treating both forms of cash distribution together. Ofer and Thakor (1987) provide a signaling model that allows for both taxable dividends as well as stock repurchases. Despite the differences between their model and ours, the common feature that emerges is that less valuable (overvalued) firms will be less likely to engage in significant repurchases. Brennan and Thakor (1990) take a different approach and assume firm managers to be uninformed and investors to be heterogeneously informed. Since uninformed investors are put at a disadvantage when the firm repurchases shares, under some circumstances these investors may require the firm to make taxable dividend payments rather than distribute cash through a stock repurchase. The differences between these models and ours are substantial, however. Managers in our model have no incentive to signal higher firm value and are essentially attempting to optimally distribute cash through time in a dynamic setting. As far as we are aware, there are no existing models that have treated both dividends and repurchases in a dynamic context.

⁵ There was, for instance, heightened repurchase activity following the October 1987 market crash [Netter and Mitchell (1989)] Also, see Choe (1992).

The article is organized as follows. Section 1 presents the model. The empirical implications are discussed in Section 2. Section 3 discusses a possible explanation for the aggregate patterns in stock repurchase activity. Section 4 concludes.

1. The Model

Consider an all-equity firm with investments in place that generate a cash flow each period until the firm liquidates. The value of the firm derives both from the present value of its after-tax cash flows and from the value of its properties and assets upon liquidation. Investors are assumed to be risk-neutral and to discount after-tax cash flows at one plus the risk-free rate. The cash flow in any period t denoted $\tilde{c}_t$, is bounded, nonnegative, and independently drawn from the same distribution each period. With probability $1 - \pi$ the firm's existing technology becomes obsolete in any period, upon which the cash flow ceases and the firm liquidates. Hence, if the firm liquidates in period t , the cash flow for the period is given by $c_t = 0$. The salvage value of the firm's assets in place, in the event of liquidation at time t , is denoted by A_t . The evolution in the salvage value A_t is stochastic and independent of the cash flow in any period. The expected rate of growth in A_t is assumed to equal the risk-free rate r .

The cash flow c_t is publicly observable at the beginning of the period. There is, however, information asymmetry about A_t : managers learn the true value of A_t at the beginning of period t , and this information becomes public only at the end of the period. With probability π_b , the salvage value A_t is higher than the last period value A_{t-1} and, with complementary probability, it is equal to A_{t-1} .

Denote the value of A_t in the high state by hA_{t-1} . Since A_{t-1} is expected to grow at the risk-free rate, we have

$$1 + r = \pi_b h + (1 - \pi_b)$$

or

$$h = r/\pi_b + 1. \quad (1)$$

We denote the firm's accumulated cash holdings at time t by Z_t , where Z_t includes $\tilde{c}_t$, the period t cash flow. The final liquidating distribution received by shareholders is assumed to be tax free. Hence, if the firm liquidates at time t , shareholders receive a total payment of $A_t + Z_t$. If the firm does not liquidate, it can distribute cash to stockholders either with a cash dividend, which is taxed, or with a stock repurchase, which is not taxed. Cash that is not distributed can be carried forward to the next period. For reasons suggested in the introduction, we assume that the firm is inefficient in carrying this

excess cash forward in that the accumulated cash grows at a rate that is less than the rate on competitive riskless marketable securities, $1 + r$. Specifically, we assume that \$1 of surplus cash at date t in the firm grows to Y_{t+1} , where Y_{t+1} each period is independently and identically distributed over $[0, (1 + r)]$ with an expected value equal to $(1 + r)(1 - \delta)$. The parameter $\delta \in (0, 1)$ could be interpreted as the expected cost per period of carrying accumulated cash forward in the firm.

As discussed in the introduction, managers choose payout policies that maximize the present value of the after-tax cash flows for the average investor left in the firm at the end of the period. The cost of distributing cash through dividends is that dividends are taxed. The cost of carrying cash forward is that it earns a rate of return that is less than the return on competitive risk-free securities. Repurchases are not taxed but may involve potential costs to long-term shareholders, depending on the true value of the firm's assets. The cost to the shareholders who remain after the repurchase is the cash paid out of the firm, which is the same regardless of the true value of the firm. The benefit to these remaining shareholders is that they now own the entire firm. Obviously this benefit is greater if the firm has more valuable assets. Therefore, managers are more likely to accept a given trade-off of some cash paid out in return for increased ownership of the firm by long-term shareholders, when the true value of the firm's assets is high than when it is low.

If managers observe the value of A_t to be hA_{t-1} , which happens with probability π_h , they will choose to distribute all the accumulated cash Z_t through a repurchase. This is because they would not be overpaying even if the market believed the value of A_t to be hA_{t-1} . But if managers discover that the value of A_t has remained at A_{t-1} , they may not engage in a repurchase program because investors may suspect the value of A_t to be hA_{t-1} and demand a high price accordingly. We will make an assumption that ensures that this penalty is so high that the managers will choose not to engage in a repurchase program unless they discover that A_t has indeed gone up to hA_{t-1} . Thus, in equilibrium, managers' choice of whether they distribute cash through a repurchase program will reveal their private information about the value of A_t . The sequence of events within a period has been summarized in Figure 1.

If the firm has no accumulated cash (i.e., $Z_t = 0$), let $V(0)$ denote the present value of all future after-tax cash flows of the firm, excluding the salvage value of assets in the event of liquidation. Also, let S_t denote the total value of the firm at time t . With probability π_h , managers discover the value of A_t has gone up to hA_{t-1} and they distribute the accumulated cash Z_t through a stock repurchase. Using the def-

inition of $V(0)$, the total value of the firm in this case can be expressed as

$$S_t^b = V(0) + Z_t + hA_{t-1}.$$

With probability $1 - \pi_b$, managers discover that the value of A_t has stayed at A_{t-1} and, consequently, do not engage in a stock repurchase. In this case let the function $V(Z_t)$ denote the present value of all current and future after-tax cash flows of the firm, excluding the salvage value of assets in the event of liquidation. Here $V(Z_t) - V(0)$ represents the incremental value at date t of having Z_t in accumulated cash inside the firm. The total value of the firm can be expressed as

$$S_t^i = V(Z_t) + A_{t-1}.$$

Hence, prior to a dividend or repurchase announcement, the value of the firm's stock is $\pi_b S_t^b + (1 - \pi_b) S_t^i$.⁶

Let ϕ_t denote the number of outstanding shares (normalized to 1) the firm repurchases to distribute the accumulated cash Z_t . Equating the payoff per share of those who tender during a repurchase to those who retain their shares, we get

$$\frac{Z_t}{\phi_t} = \frac{1}{1 - \phi_t} [V(0) + hA_{t-1}].$$

Rearranging the above equation, we get

$$\phi_t = \frac{Z_t}{V(0) + Z_t + hA_{t-1}}. \quad (2)$$

To ensure that managers would choose not to repurchase shares unless they discover the value of A_t to be hA_{t-1} , the following incentive compatibility condition must be satisfied:

$$[V(Z_t) + A_{t-1}] \geq \frac{1}{1 - \phi_t} [V(0) + hA_{t-1}]. \quad (3)$$

If the salvage value of the assets A , is large enough compared with the value of the expected cash flow each period, then the incentive compatibility condition in Equation (3) is satisfied as the following lemma formally demonstrates:

⁶Since the firm pays dividends only when it is being overvalued by the market, a dividend announcement in our model would be associated with a drop in firm stock price. This counterfactual implication arises from the simplifying assumption in the model that dividends contain no information about the firm's present or future earnings. We later show that the level of dividends is positively related to the level of unexpected earnings in the current period. Therefore, if the information about current earnings was conveyed through dividend announcements or if earnings were observed concurrent with dividend announcements, an unexpected increase in dividends may lead to an increase in firm stock price.

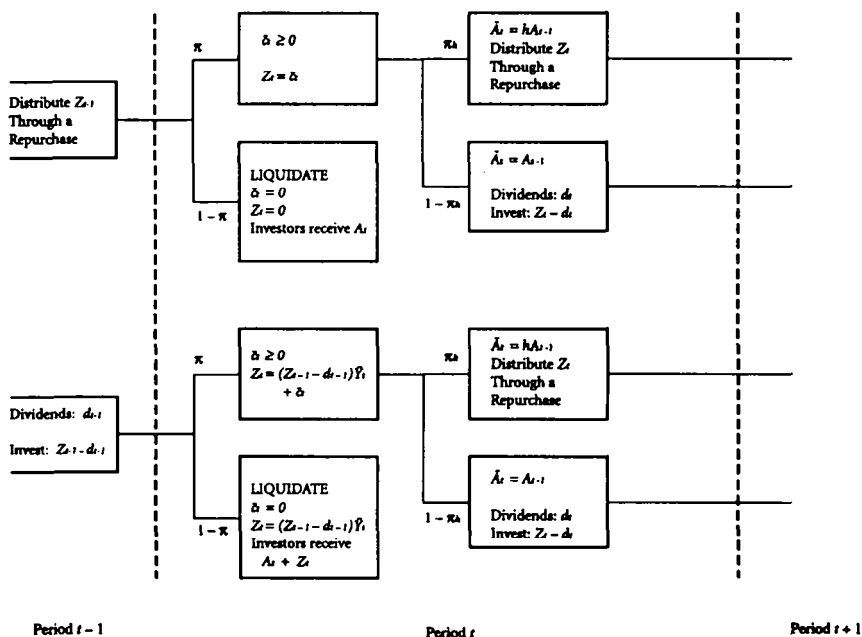


Figure 1
Sequence of events

Z_{t-1} , Z_t denote the firm's accumulated cash holdings at times $t-1$, t . Z_t is the cash flow at t . d_{t-1} , d_t represent the dividends paid in periods t , $t-1$. Y_t is the $(1 + \text{rate of growth})$ of the accumulated cash retained by the firm. π denotes the probability with which the firm does not liquidate in any period. The salvage value of the firm's assets in place at $t-1$ is denoted by A_{t-1} . With probability πh , the salvage value grows to hA_{t-1} in period t .

Lemma 1. *The incentive compatibility condition (3) is always satisfied if*

$$\left[\frac{\pi}{r + (1 - \pi)} \right] \left[\frac{1}{(r/\delta)((1 - \pi)/\pi_b + \pi)(1 - \delta) - 1} \right],$$

where A_{t-1} denotes the value of assets in place at $t = 0$ and $\tilde{c}$ denotes the expected value of the cash flow $\tilde{c}$.

Proof: See the Appendix ■

The intuitive interpretation of the sufficient condition (4) is as follows. The value of the firm consists of two components: the present value of the cash flows and the value of the assets in place, A_t . The magnitude of the repurchase premium depends on the asymmetry in information about the total value of the firm. The asymmetry in information about firm value comes only through A_t . Therefore, if A_t were

small compared with the value of the cash flows, the repurchase premium would be small and the firm may choose to repurchase shares even if it is not undervalued. The smallest value of the assets is given by A_0 , since assets in place are nondecreasing through time. Hence, ensuring that A_0 is large enough compared with $\bar{c}$ guarantees that $A_t \geq A_0$ would always be large enough to induce the firm to repurchase shares only when undervalued. Later we consider the case when firms may sometimes repurchase shares whether or not they are undervalued.

Let us now analyze how managers determine optimal payout policy. If managers discover that the value of A_t is high, as discussed above, they pay out all accumulated cash through a stock repurchase. If, however, the value of A_t is not high, the managers must decide how much cash to distribute in the form of taxable dividends and how much cash to carry forward. Let d_t denote the level of dividends in period t . Let $g(d_t)$ denote the value of the dividends to the shareholders where $0 < g(d_t) < d_t$ for $d_t > 0$ and $g(d_t) = 0$ for $d_t = 0$. We assume that $g(d_t)$ is increasing and strictly concave in d_t for all values of d_t less than a certain level d^{max} . This is justified on the grounds that the marginal benefit to the shareholders may be decreasing in the level of dividends. For instance, a graduated income tax rate schedule for personal income taxes may reduce the marginal benefit of receiving high levels of dividends. This would be particularly important for those shareholders who own a significant stake in the firm, as is likely for the managers and insiders. In addition, the ability of these investors to shield dividends from income taxation is likely to be limited and decreasing in the quantity of dividends.⁷

The policy of carrying cash until a period in which the firm can distribute the cash tax free through a repurchase is always available to the firm. In the proof of Lemma 1, we show that the present value of \$1 under such a policy is

$$m \equiv \frac{(1 - \pi + \pi\pi_b)(1 - \delta)}{\delta + (1 - \pi + \pi\pi_b)(1 - \delta)}. \quad (5)$$

This provides an upper bound to the maximum level of dividends, d^{max} , the firm would be willing to pay in any period since the marginal benefit is decreasing in the level of dividends. The value of d^{max} then is given by the following equation

$$g'(d^{max}) = m. \quad (6)$$

To ensure that $d^{max} > 0$ exists, we impose additional restrictions as

⁷Miller and Scholes (1978) discuss some of the ways in which investors may be able to shield dividend income from income taxes.

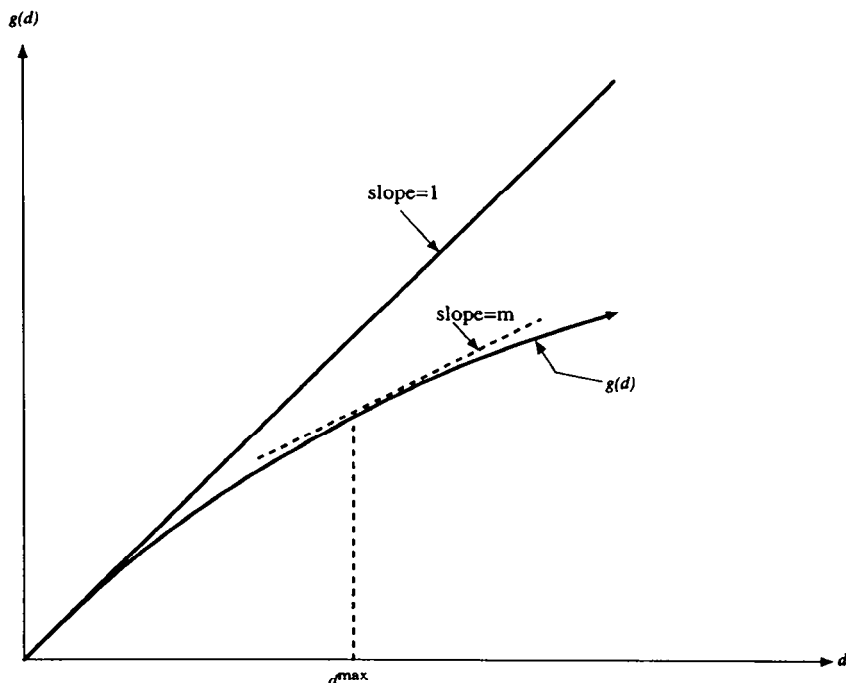


Figure 2
Value of dividends $g(d)$ to shareholders as a function of the dividend payout d . $d^{\max}$ is the maximum level of dividends the firm will pay out in any period.

follows:

$$\lim_{d_i \rightarrow \infty} g'(d_i) < m < g'(0).$$

An illustration of the function $g(d)$ is given in Figure 2.

Proposition 1. *If the probability distribution of $\tilde{c}_t$ is such that c_t will always be at least as large as $d^{\max}$, then the optimal dividend policy is to pay out a constant level of dividends equal to $d^{\max}$ in any period in which the firm does not repurchase shares.*

Proof: We know that the firm would never want to pay more than $d^{\max}$ in dividends. If the cash every period is going to be at least as high as $d^{\max}$, there is no benefit to paying a cash dividend less than $d^{\max}$ since the firm would never be able to distribute its excess cash at a lower marginal cost in any period in which it was not repurchasing. ■

In what follows we will assume that $\tilde{c}_t$ could, with positive probability, take on values that are less than $d^{\max}$. We also assume that

the marginal benefits to shareholders from cash dividends are not so low that managers would never want to pay out any dividends.

Proposition 2. *If the marginal benefits from dividends are sufficiently high, there exists a level of Z_t such that the optimal dividend policy is to pay out all the available cash in dividends.*

Proof. If the firm decides to carry some cash to the next period, the best possible scenario next period is that the firm is able to distribute this cash tax free to the investors. The maximum possible present value of a dollar carried forward is $EY_{t+1}/(1+r)$, which equals $1 - \delta$. If the marginal benefit for low levels of dividends is such that $g'(d_t) > 1 - \delta$, then for low enough levels of accumulated cash it is optimal for the firm to pay out all the cash in dividends. ■

In any period in which managers decide not to repurchase shares, they must choose the level of dividend payments, d_t . Cash not distributed through dividends in that period, $Z_t - d_t$, is carried to the next period to yield $(Z_t - d_t)\tilde{Y}_{t+1}$. With probability $1 - \pi$, the firm is liquidated next period, and the accumulated cash is distributed tax free. Since the cash flow $\tilde{c}_{t+1} = 0$ in the event of liquidation, the accumulated cash flow is given by $(Z_t - d_t)Y_{t+1}$. With probability π , the firm continues to generate income, in which case with probability π_b it will disburse accumulated cash through a repurchase. Otherwise, the firm faces a similar decision problem about choosing the level of dividend payment d_{t+1} .

The value of the firm derives from two separable components: value from future liquidation of firm assets in place and value from the payout of cash flows. Since managers cannot affect the first source of value, their decision problem is one of choosing an optimal cash disbursement policy. The managers' decision problem can be written as the following dynamic optimization problem:

$$V(Z_t) = \text{Max}_{d_t} \left\{ g(d_t) + \frac{1}{1+r} E_t[(1-\pi)(Z_t - d_t)\tilde{Y}_{t+1} + \pi\pi_b\{\tilde{Z}_{t+1} + V(0)\} + \pi(1-\pi_b)V(\tilde{Z}_{t+1})] \right\}$$

subject to the constraint $d_t \leq Z_t$
 where $\tilde{Z}_{t+1} = (Z_t - d_t)\tilde{Y}_{t+1} + \tilde{c}_{t+1}$.

Notice that the maximization problem maximizes the present value of the disbursed cash flow. It is easy to see that this is equivalent to the problem that maximizes the present value of the after-tax cash

flows for the average investor left in the firm at the end of the period. The reason is that even though these shareholders do not tender their shares in a stock repurchase plan, the stock repurchase is always done at fair market value.

Lemma 2. *There exists a unique continuous value function $V(Z_t)$ satisfying the above dynamic optimization problem. This value function is strictly increasing, strictly concave, and differentiable in Z_t .*

Proof. See the Appendix. ■

The fact that the value function is increasing in the level of accumulated cash is obvious. The strict concavity of the value function implies that the marginal benefit of carrying cash forward is decreasing in the quantity of cash retained. This is because the cash that is retained will be paid out in the future either in the form of a repurchase or in the form of future dividends, and the marginal benefit of dividends is strictly decreasing in the level of dividends.

Proposition 3. *The optimal level of dividends will always be strictly less than d^{max} .*

Proof: See the Appendix. ■

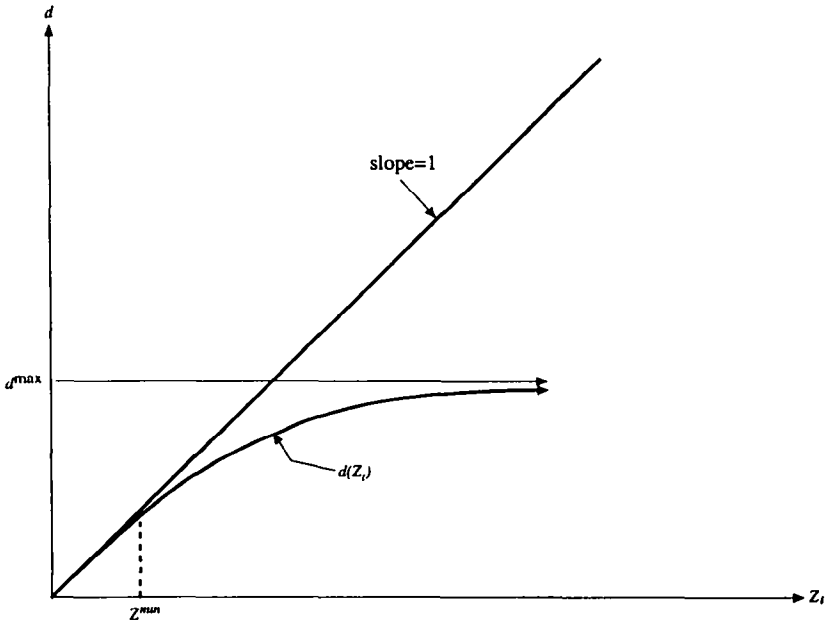
The marginal benefit to the shareholders of receiving d^{max} in dividends equals the marginal benefit of the policy in which the firm carries cash forward until it is able to distribute the cash flow tax free to the investors. Since the accumulated cash in any period could, with positive probability, take on values that are less than d^{max} , then, by saving the last dollar paid out in current dividends, there is some possibility that the firm could distribute this incremental cash in the form of future dividends whose marginal benefit would be higher. Therefore, the optimal policy is to decrease the level of dividends from d^{max} to a lower level.

Proposition 4. *The optimal dividend policy function, denoted $d(Z_t)$, has the following properties:*

$$\begin{aligned} d(Z_t) &= Z_t, & Z_t &\leq Z^{\min}, \\ d(Z_t) &< Z_t, & Z_t &> Z^{\min}, \\ 0 &< d'(Z_t) &< 1, & Z_t > Z^{\min}, \end{aligned}$$

where $Z^{\min}$ denotes a certain minimum level of cash Z_t .

Proof. See the Appendix. ■



Dividend payout $d(Z_t)$ as a function of accumulated cash Z_t

The firm will pay dividends strictly less than $d^{\max}$. For $Z_t \leq Z^{\min}$, all the accumulated cash Z_t is paid out in dividends.

We know from Proposition 2 that for low levels of dividends, the marginal benefit of dividends is high enough that if the accumulated cash is low enough it is better to pay out all of it in the form of dividends. $Z^{\min}$ denotes that level of accumulated cash at which the marginal benefit of current dividends just equals the marginal benefit of carrying any cash forward. If the accumulated cash is higher than $Z^{\min}$, then it could not be optimal to either pay out all the cash in excess of $Z^{\min}$ in the form of current dividends, since the marginal benefit of dividends is decreasing, nor to carry all the cash in excess of $Z^{\min}$ to the next period, since the marginal benefit of doing that is also decreasing. An illustration of the optimal dividend policy is provided in Figure 3.

Proposition 5. For firms with a smaller probability, π_b , of being undervalued:

1. The optimal level of dividends, for a given level of accumulated cash Z_b , is higher.
2. The percentage increase in the stock price, for a given size of a repurchase Z_b , is higher.

Proof. See the Appendix. n

Firms with a smaller probability of being undervalued are less likely to be able to distribute cash through a repurchase. Thus, the marginal benefit of carrying cash forward, for these firms, is smaller. As a result, payout in the form of current dividends is higher. For these firms, since a repurchase is less likely, the surprise when a repurchase does occur is larger, leading to a larger price increase. Moreover, the price before the announcement is also smaller since these firms are less able to distribute cash tax free in the form of repurchases.

Proposition 6. *For firms that have larger costs, d , of carrying cash forward to future periods:*

1. *The optimal level of dividends, for a given level of accumulated cash, is higher.*
2. *The percentage increase in the stock price, for a given size of a repurchase Z_v , is higher.*

Proof. See the Appendix. ■

The marginal benefit of carrying cash forward is smaller for firms that have larger costs of carrying cash forward. As a result, optimal dividend payout is higher for these firms for a given level of accumulated cash. Also for these firms, the relative benefit of being able to distribute cash with a repurchase is larger.

Proposition 7. *Right after a repurchase, the level of dividends is expected to increase over time until the period in which the next repurchase occurs.*

Proof. See the Appendix. ■

The firm disburses all its cash when it repurchases shares. Right after a repurchase, since the firm is expected to distribute only a part of its cash flow in the form of dividends and carry forward the rest, the accumulated cash level is expected to increase over time. Therefore, the dividend payout is also expected to increase over time.

2. Empirical Implications

The propositions in the previous section imply a number of empirical predictions that are discussed as follows.

Implication 1. *For a firm that engages in repurchases less frequently:*

1. In periods in which the firm decides not to distribute cash through a repurchase:
 - (a) The level of dividends tends to be higher.
 - (b) The dividend yields tend to be higher.
2. The stock price reaction to the announcement of a repurchase is larger (controlling for repurchase size).

This implication follows from Proposition 5. Firms that engage in repurchases less frequently are firms that are less likely to be undervalued (low π_b). For these firms, since the marginal benefit of retaining cash is smaller, the dividend payout is higher. The stock price of these firms is also smaller, since they are not able to distribute cash in tax-free repurchases as often. Therefore, the dividend yields for these firms is higher. For these firms, when a repurchase does occur, since the surprise is greater, the stock-price reaction is also larger.

Implication 2. For a firm that pays dividends in a period and then engages in a repurchase the following period:

1. The expected size of the repurchase is positively related to the last period's dividend.
2. The stock-price reaction to the announcement of a repurchase is positively related to the level of dividends (controlling for the level of last period's accumulated cash).

For any given firm, a higher level of dividends implies a higher level of accumulated cash [since $d'(Z_t) > 0$]. Since the amount carried forward [$Z_t - d(Z_t)$] is also greater [since $d'(Z_t) < 1$], the expected amount available for a potential distribution through a repurchase is also higher.

Firms that have larger dividend payouts, for a given level of accumulated cash, are likely to be those that are able to repurchase less frequently or those that have larger costs of carrying cash forward (from Propositions 5 and 6). For these firms, the stock price is lower, and when a repurchase does occur it is more unexpected (from Propositions 5 and 6). Therefore, the stock-price reaction is larger for these firms.

Implication 3. Dividends exhibit the following patterns through time:

1. If the level of dividends in the current period is sufficiently high ($d_t > Z^{\min}$), then the expected level of dividends next period ($E_t d_{t+1}$) is positively related to the level of dividends in the current period (d_t) and to the level of unexpected earnings next period ($\tilde{e}_{t+1} - E_t \tilde{e}_{t+1}$).

An increase (decrease) of \$1 in unexpected earnings is, on average, associated with less than \$1 increase (decrease) in dividends.

2. However, if the level of dividends in the current period is low ($d_t \notin Z^{\min}$), then the level of dividends nextperiod (d_{t+1}) is unrelated to the level of dividends thisperiod (d_t) and is highly positively related to the level of earnings next period (c_{t+1}).

3. The expectedgrowth in dividends between two consecutive dividend payments is smaller if there is also a stock repurchase in the intervening period.

4. For firms with a stable pattern of earnings (i.e., earnings in all periods are within a region; $c_L \leq c_t \leq c_U \forall t$; where c_L and c_U are constants):

- If the earnings are sufficiently low ($c_U \leq Z^{\min}$), the firm distributes its earnings entirely each period,
- If the earnings are sufficiently high ($d^{\max} \leq c_L$), the firm pays a constant amount of dividends each period and accumulates the surplus through time for periods in which it repurchases.
- If the earnings are in the intermediate region ($Z^{\min} \leq c_L$, $c_U \notin d^{\max}$ and $c_L < c_t \leq c_U$ for some t), the expected dividend payments increase between repurchases.

The expected accumulated cash available for distribution next period is increasing in the cash retained in the current period, which is higher if the current period's dividends are higher. Since the accumulated cash is also increasing in the unexpected earnings next period, the expected level of dividends next period is increasing in both the current period's dividends as well as in the size of unexpected earnings next period. The smoothing result follows from the fact that when the accumulated cash is relatively high, it could not be optimal to pay out all the incremental cash in the form of dividends since the marginal benefit of dividends is decreasing, nor could it be optimal to carry all the incremental cash to the next period since the marginal benefit of doing so is also decreasing.

For low levels of cash, however, since the marginal benefit of dividends is high, the firm pays all the cash flow from earnings in the form of dividends. Moreover, since the cash flow from earnings each period is independent of the cash flow from earnings in the following period, the dividends next period are independent of the current period's dividends.

Dividends are increasing in the level of accumulated cash, which decreases drastically when the firm repurchases shares. Between two repurchases, however, the accumulated cash levels, and therefore the dividend payments, are expected to increase.

For a firm with relatively stable earnings, if the earnings are low enough, the firm would distribute all its earnings either in a repurchase or through dividend payments since the marginal benefit of dividend payments is high enough to induce the firm not to carry cash forward. If the earnings are sufficiently high, the firm would pay dividends up to a level at which the marginal benefit of dividends was equal to the marginal benefit from carrying cash forward until it was able to distribute this cash tax free. If the earnings are in the intermediate region, some cash is retained so that the accumulated cash is expected to grow over time until it is paid out completely in a *stock* purchase.

3. Stock Market Behavior and Repurchase Activity

We have so far analyzed the case in which the repurchase premiums are high enough that a firm chooses not to repurchase unless it is undervalued. We now explore extensions to our analysis. Since we do not provide a formal analysis of these extensions, the following discussion should be viewed as being suggestive.

Consider the possibility that the repurchase premiums may be small enough so that the firm may decide to repurchase shares, whether or not it is undervalued. This may happen, for instance, if the probability π_b of the asset value A_t being high is sufficiently small so as to allow for the existence of such a pooling equilibrium. In this type of equilibrium, the cost to the overvalued firm would be relatively small, given investor beliefs that the firm would be engaging in a repurchase, irrespective of its asset value.

The fact that under the appropriate conditions the equilibrium might switch from being a separating to a pooling equilibrium, may conceivably account for some of the observed patterns in repurchase activity through time. The arrival of some public information, for example, may alter investor beliefs about π_b and, consequently, the nature of the equilibrium. Consider the case in which π_b drops because of some public information revelation about the firm's assets in place. In response to the decrease in π_b , the stock price S_t will fall. At the same time, this decrease in π_b may also lead to a situation in which the pooling equilibrium exists, whereas it may not have existed for higher values of π_b . Thus, if the information about the value of the firm is "bad," in the sense that π_b is low, then the likelihood of a share repurchase may go up, since the equilibrium may switch from a separating to a pooling equilibrium. This suggests that in some periods the likelihood of a firm repurchasing its shares may increase following a substantial drop in its stock price. Hence, it may well be the case that as π_b varies, for some region (the separating region), a

decrease in the value of π_b may be associated with a decrease in the likelihood of a repurchase. If the value of π_b is low enough, however, the likelihood of a repurchase may increase dramatically (in the pooling region).⁸

This argument is easily extended to provide an explanation regarding the observed aggregate patterns in the volume of repurchases as a function of the overall stock market performance. Suppose that there are economy-wide factors that result in positive correlations in the innovations in asset values across firms. Then, based on the above arguments, there may well be a concentration of repurchases in periods following stock market declines. Such aggregate increases in share repurchase activity have been observed, for example, following the October 1987 stock market crash [see Netter and Mitchell (1989)]. Choe (1992) provides evidence of such a relation between stock market performance and repurchase activity over the 1970-1989 period.

An episode that has received considerable attention in the literature on repurchases and dividends is the Nixon price-and-dividend control period. The increase in repurchase activity during the years 1973 and 1974 has often been attributed to the 4 percent annual growth rate limit on annual dividends in effect from August 1971 through 1974. According to our arguments, however, firms affected by the dividend controls would not necessarily increase the frequency of repurchases; instead, they would carry cash to future periods. Choe (1992) suggests that this may not be too different from what happened,⁹ and a plausible alternative explanation is that this repurchase activity was the result of the fall in the stock market price levels over this period. As discussed, a period of "bad" information about firm value may well be associated with an increase in stock repurchase activity.

4. Conclusion

A stylized fact about stock repurchases through intrafirm tender offers is that investors tend to react positively to the announcement of a stock repurchase by a firm, resulting in a substantial increase in its stock price. We propose that it is precisely the fact that repurchases through intrafirm tender offers are associated with large stock-price increases that makes them unattractive as a means of distributing cash.

⁸Since our formal model did not allow for variation over time in the values of parameters such as π_b , one cannot be sure that the suggested results would survive in a model where it was common knowledge that the parameters could change in this way.

⁹First, despite the fact that dividend controls were into effect in 1971, it was only in 1973 that there was a dramatic increase in repurchase activity, by which time the dividend controls had been substantially relaxed. Moreover, Choe finds that firms that were being affected by the dividend controls were no more likely to engage in repurchases than firms that were not so affected.

Despite the tax disadvantage of dividends, the optimal payout policy is to distribute some cash in the form of dividends and carry the rest to future periods; however, when the firm's stock is sufficiently under-valued, the firm would distribute all its accumulated cash by engaging in a stock repurchase program. This explains not only why firms pay dividends but also why firms repurchase stocks relatively infrequently. The amount of cash disbursed through a stock repurchase is also likely to be large compared to the dividend payments in any period.

Dividends as well as the quantity of cash retained are shown to be increasing in the quantity of accumulated cash. The level of dividends or the size of the stock repurchase is positively related to the level of dividends in the previous period and to the level of unexpected earnings in that period. Dividends are smoothed in the sense that an increase (decrease) of \$1 in unexpected earnings is, on average, associated with less than \$1 increase (decrease) in dividends.

The model predicts that firms that repurchase less frequently will have higher dividend yields and a larger stock-price reaction associated with a repurchase announcement of a given size. The stock-price return associated with a repurchase of a particular size will be increasing in the size of the last period's dividend. The model also suggests that periods associated with negative information about firm values may be those in which firms are more likely to repurchase. If firm asset values are affected by common factors, this leads to the prediction that repurchases may increase following downturns in the stock market.

Appendix

A.1 Proof of Lemma 1

Substituting for ϕ_t from Equation (2) in Equation (3) and rearranging, we write the incentive compatibility condition as

$$\frac{V(Z_t) - V(0)}{Z_t} \geq \frac{1 + A_{t-1}/V(0)}{1 + bA_{t-1}/V(0)}. \quad (\text{A1})$$

To obtain a lower bound on $V(Z_t) - V(0)$, we argue as follows. Suppose the firm sets the current accumulated cash Z_t aside and carries it forward to future periods until it liquidates or is able to distribute it in a stock repurchase. With respect to all other future cash flows, it follows the optimal policy as if its accumulated cash in period t were zero. Hence, $V(Z_t)$ must be at least as large as the sum of $V(0)$ and the present value of undistributed cash Z_t under this suboptimal policy. The present value of the undistributed cash Z_t under the suboptimal policy is equal to

$$Z_t(1 - \pi + \pi\pi_b)(1 - \delta)[1 + \pi(1 - \pi_b)(1 - \delta) + \{\pi(1 - \pi_b)(1 - \delta)\}^2 + \dots],$$

which simplifies to

$$Z_t \frac{(1 - \pi + \pi\pi_b)(1 - \delta)}{\delta + (1 - \pi + \pi\pi_b)(1 - \delta)}.$$

Therefore, it follows that

$$V(Z_t) \geq V(0) + Z_t \frac{(1 - \pi + \pi\pi_b)(1 - \delta)}{\delta + (1 - \pi + \pi\pi_b)(1 - \delta)}.$$

Rearranging, we get

$$\frac{V(Z_t) - V(0)}{Z_t} \geq \frac{(1 - \pi + \pi\pi_b)(1 - \delta)}{\delta + (1 - \pi + \pi\pi_b)(1 - \delta)}. \quad (\text{A2})$$

So, if we can ensure that

$$\frac{(1 - \pi + \pi\pi_b)(1 - \delta)}{\delta + (1 - \pi + \pi\pi_b)(1 - \delta)} \geq \frac{1 + A_{t-1}/V(0)}{1 + b(A_{t-1}/V(0))}, \quad (\text{A3})$$

we see that the incentive compatibility condition (A1) will be satisfied. Rearranging condition (A3) and substituting from Equation (1), we write condition (A3) as

$$\frac{A_{t-1}}{V(0)} \geq \frac{1}{(r/\delta)((1 - \pi)/\pi_b + \pi)(1 - \delta) - 1}.$$

Since the above condition must hold for all periods, it is sufficient to assume that

$$A_0 \geq V(0) \left[\frac{1}{(r/\delta)((1 - \pi)/\pi_b + \pi)(1 - \delta) - 1} \right] \quad (\text{A4})$$

since the value of A_t , by assumption, never falls. The present value of the cash flow stream in a situation in which the firm is able to distribute its cash flow tax free each period provides an upper bound on the value of $V(0)$. Therefore, we get

$$\begin{aligned} V(0) &\leq \frac{\pi}{1 + r} E_t[\tilde{c}_{t+1}] + \left(\frac{\pi}{1 + r} \right)^2 E_t[\tilde{c}_{t+2}] \\ &\quad + \left(\frac{\pi}{1 + r} \right)^3 E_t[\tilde{c}_{t+3}] + \dots \end{aligned} \quad (\text{A5})$$

Since the cash flows $\tilde{c}_t$ are independently drawn from the same distribution, their expected values must be identical. If we let $\bar{c}$ denote $E_t[\tilde{c}_{t+i}] \forall i$, the above condition can be simplified to

$$V(0) \leq \frac{\bar{c}\pi}{r + (1 - \pi)}.$$

Therefore, in order to ensure that condition (A4) is satisfied it is sufficient to assume the following restriction on the exogenous parameters:

$$A_0 \geq \left[\frac{\bar{c}\pi}{r + (1 - \pi)} \right] \left[\frac{1}{(r/\delta)((1 - \pi)/\pi_b + \pi)(1 - \delta) - 1} \right]. \quad (\text{A6})$$

A.2 Proof of Lemma 2

The dynamic optimization problem can be rewritten as

$$V(Z_t) = \text{Max}_{d_t} [f(d_t, Z_t) + \beta E_t V(\tilde{Z}_{t+1})]$$

subject to the constraint $d_t \leq Z_t$

where $\tilde{Z}_{t+1} = (Z_t - d_t) \tilde{Y}_{t+1} + \tilde{c}_{t+1}$.

$$f(d_t, Z_t) \equiv g(d_t) + (1 - \pi + \pi\pi_b)(Z_t - d_t)(1 - \delta)$$

$$+ \frac{\pi\pi_b}{1 + r} \{\bar{c} + V(0)\},$$

$$\beta \equiv \frac{\pi(1 - \pi_b)}{1 + r} < 1.$$

The proof is based on standard dynamic programming techniques and is sketched below.

Existence. The contraction mapping theorem can be used to show that there exists a unique value function that is the solution to the dynamic optimization problem above and is continuous and strictly increasing in Z , [see Stokey and Lucas (1989, pp. 263-264, Theorems 9.6, 9.7)]. Since the value function $V(Z_t)$ is monotonically increasing in Z_t (therefore, potentially unbounded), in applying the contraction mapping theorem $V(Z_t)$ is first appropriately transformed to a bounded function $B(Z_t) = V(Z_t)/(Z_t + C/r)$, where C represents the upper bound on $\bar{c}$ the cash flow realization each period. It is easy to verify that this transformed function, $B(Z_t)$, is bounded for any Z_t and that all the conditions for contraction are satisfied. From this we have a proof for the existence of a unique continuous value function $V(Z_t) = (Z_t + C/r)B(Z_t)$.

Strict concavity and differentiability. The value function can be rewritten as

$$V(Z_t) = \text{Max}_{d_t} [g(d_t) + K_1(Z_t - d_t) + K_2 + \beta E_t V(\tilde{Z}_{t+1})],$$

where

$$K_1 \equiv (1 - \pi + \pi\pi_b)(1 - \delta),$$

$$K_2 \equiv \frac{\pi\pi_b}{1+r} \{\bar{c} + V(0)\}.$$

To show concavity, consider $Z_t^1, Z_t^2, Z_t^1 \neq Z_t^2$. Define

$$\tilde{Z}_t \equiv \theta Z_t^1 + (1 - \theta) Z_t^2, \quad 0 < \theta < 1.$$

Let d_t^1, d_t^2 be the optimal choices of d_t that correspond to Z_t^1, Z_t^2 . Let $\{d_{t+i}^1\}_{i=1}^\infty$ and $\{d_{t+i}^2\}_{i=1}^\infty$ denote the optimal sequences of dividend payments corresponding to realizations of the sequences $\{Z_{t+i}^1\}_{i=1}^\infty$ and $\{Z_{t+i}^2\}_{i=1}^\infty$ respectively. Also define

$$\hat{d}_{t+n} \equiv \theta d_{t+n}^1 + (1 - \theta) d_{t+n}^2$$

and

$$\hat{Z}_{t+n} \equiv \theta Z_{t+n}^1 + (1 - \theta) Z_{t+n}^2.$$

By definition of $V(Z_t)$ we have, for $j = 1, 2$,

$$V(Z_t^j) = \sum_{i=0}^T \beta^i E_t [g(d_{t+i}^j) + K_1(Z_{t+i}^j - d_{t+i}^j) + K_2] + \beta^{T+1} E_t V(Z_{T+1}^j).$$

Note that since Z_{t+i+1} is linear in Z_{t+i} and d_{t+i} , if $Z_{t+i} = \hat{Z}_{t+i}$ and $d_{t+i} = \hat{d}_{t+i}$, then

$$\begin{aligned} \tilde{Z}_{t+i+1} &= [\theta(Z_{t+i}^1 - d_{t+i}^1) + (1 - \theta)(Z_{t+i}^2 - d_{t+i}^2)] \tilde{Y}_{t+i+1} + \tilde{c}_{t+i+1} \\ &= \hat{Z}_{t+i+1}. \end{aligned}$$

Hence we have

$$\begin{aligned} V(\tilde{Z}_t) &\geq \sum_{i=0}^T \beta^i E_t [g(\hat{d}_{t+i}) + K_1(\hat{Z}_{t+i} - \hat{d}_{t+i}) + K_2] + \beta^{T+1} E_t V(\hat{Z}_{T+1}) \\ &> \sum_{i=0}^T \beta^i E_t [\theta g(d_{t+i}^1) + (1 - \theta) g(d_{t+i}^2) \\ &\quad + K_1(\hat{Z}_{t+i} - \hat{d}_{t+i}) + K_2] + \beta^{T+1} E_t V(\hat{Z}_{T+1}). \end{aligned}$$

The first weak inequality follows since $\hat{d}_{t+i}$ is not necessarily the optimal choice of d_t for Z_{t+i} . The next strict inequality follows from the fact that $g(\cdot)$ is strictly concave, and we are assured of the fact that all d_{t+i}^1 and d_{t+i}^2 cannot be identical from the fact that there

is always a positive probability of the Z_{t+1}^1, Z_{t+1}^2 being in a region (as in Proposition 2) in which the Z_t is entirely paid out in dividends.

Therefore, using the expressions for $V(Z_t^1)$ and $V(Z_t^2)$, we obtain

$$V(\hat{Z}_t) > \theta V(Z_t^1) + (1 - \theta) V(Z_t^2) \\ + \beta^{T+1} E_t[V(\hat{Z}_{T+1}) - \{\theta V(Z_{T+1}^1) + (1 - \theta) V(Z_{T+1}^2)\}]$$

First notice that the term $E_t[V(\hat{Z}_{T+1}) - \{\theta V(Z_{T+1}^1) + (1 - \theta) V(Z_{T+1}^2)\}]$ is bounded since the probability of Z_{T+1} being unbounded, as $T \rightarrow \infty$, is arbitrarily small (since there is a finite probability each period that all accumulated cash will be paid out in repurchases or that the firm will be liquidated). Now, since $\beta^{T+1} \rightarrow 0$ as $T \rightarrow \infty$, the term

$$\beta^{T+1} E_t[V(\hat{Z}_{T+1}) - \{\theta V(Z_{T+1}^1) + (1 - \theta) V(Z_{T+1}^2)\}]$$

can be made arbitrarily small. It follows, therefore, that $V(Z_t)$ is strictly concave since

$$V(\hat{Z}_t) > \theta V(Z_t^1) + (1 - \theta) V(Z_t^2).$$

The differentiability of the value function $V(Z_t)$ follows from the fact that the value function is strictly concave and that $g(d_t) + K_1(Z_t - d_t) + K_2$ is concave and differentiable [see Stokey and Lucas (1989, p. 266, Theorem 9.10)]. ■

A.3 Proof of Proposition 3

The optimization problem is

$$\text{Max}_{d_t} f(d_t, Z_t) + \beta E_t V(\tilde{Z}_{t+1}) - \lambda(d_t - Z_t).$$

The first-order condition for the above maximization problem is

$$g'(d_t^*) - (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(\tilde{Z}_{t+1})] - \lambda = 0. \quad (\text{A7})$$

The second-order condition for the maximization problem is also satisfied since

$$g''(d_t^*) + \pi\pi_b(1 + r)(1 - \delta)^2 E_t V''(\tilde{Z}_{t+1}) < 0. \quad (\text{A8})$$

Since the policy of carrying any cash forward until the firm's technology ceases to be viable or until the high state is realized is always possible, it must be that [from reasoning similar to the one leading to condition (A2)] $V(Z_{T+1}) \geq m$. Since the accumulated cash flow next period, $\tilde{Z}_{t+1}$, could, with positive probability, take on values that are less than $d_t^{\max}$, by distributing cash in the form of dividends next period the firm could do strictly better. Therefore, it follows that

$$E_t V'(\tilde{Z}_{t+1}) > m. \quad (\text{A9})$$

Rearranging the first-order condition (A7), we get

$$\begin{aligned} g'(\mathbf{d}_t^*) &= (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(\tilde{Z}_{t+1})] + \lambda \\ &\geq (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(\tilde{Z}_{t+1})] \\ &> (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)m] \quad [\text{from (A9)}] \\ &= m \quad [\text{from (5)}] \\ &= g'(\mathbf{d}^{\max}) \quad [\text{from (6)}]. \end{aligned}$$

Since $g'(d_t)$ is decreasing in d_t , we get $\mathbf{d}_t^* < \mathbf{d}^{\max}$.

A.4 Proof of Proposition 4

Rearranging the first-order condition (A7), we get

$$\lambda = g'(\mathbf{d}_t^*) - (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(\tilde{Z}_{t+1})]. \quad (\text{A10})$$

Consider the case when $\lambda > 0$, in which case $\mathbf{d}_t^* = Z_t$, which implies that $Z_{t+1} = c_{t+1}$ and

$$g'(Z_t) > (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(c_{t+1})].$$

Let $Z^{\min}$ denote that value of Z , such that

$$g'(Z^{\min}) = (1 - \delta)[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(c_{t+1})]. \quad (\text{A11})$$

Clearly then $Z^{\min}$ denotes the largest value of Z , below which the constraint $d_t \leq Z_t$ is satisfied as an equality.

Consider the case when $Z_t > Z^{\min}$. In this case $\mathbf{d}_t^*$ cannot be equal to Z_t . To see this let us suppose $\mathbf{d}_t^* = Z_t$. The first-order condition, in this case, is

$$\delta[(1 - \pi + \pi\pi_b) + \pi(1 - \pi_b)E_t V'(c_{t+1})] - \lambda = 0.$$

It is easy to see that the first-order condition (A10) cannot be satisfied since $g'(Z_t) < g'(Z^{\min})$, $\lambda \geq 0$, and, from condition (A11), the right-hand side of the first-order condition is negative. It follows then that the only way the first-order condition will be satisfied is if $\mathbf{d}_t^* < Z_t$.

To determine the properties of the optimal dividend policy function $d(Z_t)$, we differentiate the first-order condition for the case when $Z_t > Z^{\min}$ and obtain

$$d'(Z_t) = \frac{\pi(1 - \pi_b)(1 + r)(1 - \delta)^2 E_t V''(\tilde{Z}_{t+1})}{g''(\mathbf{d}_t^*) + \pi(1 - \pi_b)(1 + r)(1 - \delta)^2 E_t V''(\tilde{Z}_{t+1})}.$$

Since, both $g(\cdot)$ and $V(\cdot)$ are strictly concave, we get $0 < d'(Z_t) < 1$. ■

A.5 Proof of Proposition 5

1. The value function can be rewritten as

$$V(Z_t; \pi_b) = g(d(Z_t; \pi_b)) + K_1(Z_t - d(Z_t; \pi_b)) + K_2 + \beta E_t V(\tilde{Z}_{t+1}; \pi_b), \quad (\text{A12})$$

where

$$\begin{aligned} K_1 &\equiv (1 - \pi + \pi\pi_b)(1 - \delta), \\ K_2 &\equiv \frac{\pi\pi_b}{1 + r} \{\tilde{c} + V(0; \pi_b)\}, \\ \tilde{Z}_{t+1} &= [Z_t - d(Z_t; \pi_b)]\hat{Y}_{t+1} + \tilde{c}_{t+1}, \\ \beta &= \frac{\pi(1 - \pi_b)}{1 + r}. \end{aligned}$$

Differentiating the value function with respect to π and invoking the envelope theorem, we get

$$\begin{aligned} \frac{\partial}{\partial \pi_b} V(Z_t; \pi_b) &= \frac{\pi}{1 + r} [E_t \tilde{Z}_{t+1} + V(0; \pi_b) - E_t V(\tilde{Z}_{t+1}; \pi_b)] \\ &\quad + \frac{\pi\pi_b}{1 + r} \frac{\partial}{\partial \pi_b} V(0; \pi_b) + \beta E_t \frac{\partial}{\partial \pi_b} V(\tilde{Z}_{t+1}; \pi_b). \end{aligned}$$

Now

$$\begin{aligned} &\frac{\partial}{\partial Z_t} [E_t \tilde{Z}_{t+1} + V(0; \pi_b) - E_t V(\tilde{Z}_{t+1}; \pi_b)] \\ &= [1 - d'(Z_t; \pi_b)](1 + r)(1 - \delta)[1 - E_t V'(\tilde{Z}_{t+1}; \pi_b)] > 0. \quad (\text{A14}) \end{aligned}$$

We follow the convention that a prime on a function denotes the derivative with respect to the first argument.

From (A14), (A13), and the contraction mapping theorem, we get

$$\frac{\partial}{\partial Z_t} \frac{\partial}{\partial \pi_b} V(Z_t; \pi_b) = \frac{\partial}{\partial \pi_b} V'(Z_t; \pi_b) > 0. \quad (\text{A15})$$

Suppressing the dependence on the parameters, we write the first-order condition as

$$g'(d(Z_t)) = K_1 + \pi\pi_b(1 - \delta)E_t V'(\tilde{Z}_{t+1}). \quad (\text{A16})$$

Also differentiating (A12) with respect to Z_t and invoking the envelope theorem, we get

$$V'(Z_t) = K_1 + \pi\pi_b(1 - \delta)E_t V'(\tilde{Z}_{t+1}). \quad (\text{A17})$$

From (A16) and (A17), we get

$$V'(Z_i) = g'(d(Z_i)). \quad (\text{A18})$$

Rewriting (A18) with explicit dependence on π_b , we get

$$V'(Z_i; \pi_b) = g'(d(Z_i; \pi_b)). \quad (\text{A19})$$

Differentiating (A19) with respect to π_b , we get

$$\frac{\partial}{\partial \pi_b} V'(Z_i; \pi_b) = g''(d(Z_i; \pi_b)) \frac{\partial}{\partial \pi_b} d(Z_i; \pi_b).$$

Substituting from (A15), we get $g''(d(Z_i; \pi_b))(\partial/\partial \pi_b)d(Z_i; \pi_b) > 0$. Since $g''(\cdot) < 0$, we get $(\partial/\partial \pi_b)d(Z_i; \pi_b) < 0$.

2. The price reaction to the announcement of a stock repurchase is

$$\Delta P = S_i^b - [\pi_b S_i^b + (1 - \pi_b) S_i^s].$$

We first show that ΔP is decreasing in π_b :

$$\begin{aligned} \Delta P &= (1 - \pi_b)[S_i^s - S_i^b] \\ &= (1 - \pi_b)[(b - 1)A_{i-1} + V(0; \pi_b) + Z_i - V(Z_i; \pi_b)] \\ &= (1 - \pi_b) \left[\frac{r}{\pi_b} A_{i-1} + V(0; \pi_b) + Z_i - V(Z_i; \pi_b) \right]. \end{aligned}$$

We know that $1 - \pi_b$, and $1/\pi_b$ are decreasing in π_b . From (A15) we also know that $(\partial/\partial \pi_b)V(Z_i; \pi_b)$ is higher for higher values of Z_i ; therefore, $V(0; \pi_b) - V(Z_i; \pi_b)$ is also decreasing in π_b . Therefore, ΔP is decreasing in π_b .

We now show that P is increasing in π_b :

$$\begin{aligned} P &= \pi_b[V(0; \pi_b) + Z_i + bA_{i-1}] + (1 - \pi_b)[V(Z_i; \pi_b) + A_{i-1}] \\ &= [\pi_b b + (1 - \pi_b)] A_{i-1} + \pi_b[V(0; \pi_b) + Z_i] \\ &\quad + (1 - \pi_b)[V(Z_i; \pi_b)] \\ &= (1 + r)A_{i-1} + \pi_b[V(0; \pi_b) + Z_i] + (1 - \pi_b)[V(Z_i; \pi_b)]. \end{aligned}$$

Therefore, differentiating P with respect to π_b , we get

$$\begin{aligned} \frac{\partial}{\partial \pi_b} P &= \pi_b \frac{\partial}{\partial \pi_b} V(0; \pi_b) + (1 - \pi_b) \frac{\partial}{\partial \pi_b} V(Z_i; \pi_b) \\ &\quad + [V(0; \pi_b) + Z_i - V(Z_i; \pi_b)] > 0. \end{aligned}$$

Since ΔP is decreasing in π_b and P is increasing in π_b , the percentage price reaction $\Delta P/P$ is decreasing in π_b . ■

A.6 Proof of Proposition 6

The arguments for this proof are similar to those in the proof of Proposition 5.

1. The value function can be rewritten as

$$V(Z_i; \delta) = g(d(Z_i; \delta)) + K_1(Z_i - d(Z_i; \delta)) + K_2 + \beta E_t V(\tilde{Z}_{i+1}; \delta),$$

where

$$K_1 \equiv (1 - \pi + \pi\pi_b)(1 - \delta),$$

$$K_2 \equiv \frac{\pi\pi_b}{1+r} \{\bar{c} + V(0; \delta)\},$$

$$\tilde{Z}_{i+1} = [Z_i - d(Z_i; \delta)]\tilde{Y}_{i+1} + \tilde{c}_{i+1}.$$

Differentiating the value function with respect to δ and invoking the envelope theorem, we get

$$\begin{aligned} \frac{\partial}{\partial \delta} V(Z_i; \delta) &= -(1 - \pi + \pi\pi_b)[Z_i - d(Z_i)] \\ &\quad + \frac{\pi\pi_b}{1+r} \frac{\partial}{\partial \delta} V(0; \delta) + \beta E_t \frac{\partial}{\partial \delta} V(\tilde{Z}_{i+1}; \delta). \end{aligned} \quad (\text{A21})$$

Since $Z_i - d(Z_i)$ is increasing in Z_i , from the contraction mapping theorem we get

$$\frac{\partial}{\partial Z_i} \frac{\partial}{\partial \delta} V(Z_i; \delta) = \frac{\partial}{\partial \delta} V'(Z_i; \delta) < 0. \quad (\text{A22})$$

Rewriting (A18) with explicit dependence on δ , we get

$$V'(Z_i; \delta) = g'(d(Z_i; \delta)). \quad (\text{A23})$$

Differentiating (A23) with respect to δ , we get

$$\frac{\partial}{\partial \delta} V'(Z_i; \delta) = g''(d(Z_i; \delta)) \frac{\partial}{\partial \delta} d(Z_i; \delta). \quad (\text{A24})$$

Substituting from (A22), we get $g''(d(Z_i; \delta))(\partial/\partial \delta)d(Z_i; \delta) < 0$. Since $g''(\cdot) < 0$, we get $(\partial/\partial \delta)d(Z_i; \delta) > 0$.

$$\begin{aligned} \frac{\Delta P}{P} &= \frac{(1 - \pi_b)[S_i^b - S_i^l]}{\pi_b S_i^b + (1 - \pi_b)S_i^l} = \frac{(1 - \pi_b)[S_i^b - S_i^l]}{S_i^b - (1 - \pi_b)[S_i^b - S_i^l]} \\ &= \frac{1 - \pi_b}{S_i^b/[S_i^b - S_i^l] - (1 - \pi_b)} = \frac{1 - \pi_b}{1/[1 - S_i^l/S_i^b] - (1 - \pi_b)}. \end{aligned}$$

Therefore, $\Delta P/P$ is positively related to S_i^b/S_i^l .

$$\frac{\partial S'_i}{\partial \delta} = \frac{S'_i(\partial/\partial \delta)S'_i - S'_i(\partial/\partial \delta)S'_i}{(S'_i)^2} = \frac{S'_i(\partial/\partial \delta)V(0; \delta) - S'_i(\partial/\partial \delta)V(Z_i; \delta)}{(S'_i)^2}.$$

Since $S'_i < S^p_i$ and $\partial/\partial \delta V(Z_i; \delta) < \partial/\partial \delta V(0; \delta) < 0$, we get $(\partial/\partial \delta)(S'_i/S^p_i) > 0$, which implies that $(\partial/\partial \delta)(\Delta P/P) > 0$. ■

A.7 Proof of Proposition 7

Without loss of generality, let us assume that the cash flow in any period c , is drawn from a discrete distribution and could take one of N possible values: $c_t \in (0 = c^1 < c^2 < \dots < c^N)$. The period in which the firm does a repurchase, say period t , it distributes all its accumulated cash Z_t . The accumulated cash next period then is simply $c_t + 1$. It is easy to see that, in that case,

$$E_t[d_{t+k}] = E_t[d_{t+k+1} | c_{t+1} = c^1 = 0] \quad (\text{A25})$$

since the distribution of the stream of cash flows from period $t + 1$ to $t + k$ is identical to the distribution of the stream of cash flows period $t + 2$ to $t + k + 1$ if the cash flow in period $t + 1$ was equal to $c^1 = 0$. Now since

$$E_t[d_{t+k+1} | c_{t+1} = c^1] < E_t[d_{t+k+1} | c_{t+1} = c^i] \quad \forall i \geq 2,$$

from (A25), we get

$$E_t[d_{t+k+1}] > E_t[d_{t+k+1} | c_{t+1} = c^1] = E_t[d_{t+k}]. \quad \blacksquare$$

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