Bank Liquidity and Stability in an Overlapping Generations Model

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In an infinitely repeated version of the Diamond and Dybvig (1983) model, intergenerational transfers enable a bank to achieve interest rate smoothing and to provide depositors with liquidity insurance without Diamond and Dybvig's assumption of no side trades. The bank is subject to runs that may result from either excessive withdrawals or the hack of new deposits. The latter cause, which cannot occur in Diamond and Dybvig's one-genera&on model, implies that suspension of convertibility may n&prevent bank runs. Government intervention may be necessary to maintain bank stability.

A literature pioneered by Bryant (1980) and Diamond and Dybvig **(1983)** emerged in the 1980s examining bank liquidity services and stability. Diamond and Dybvig model a one-generation economy in which agents face private preference shocks and technologies exhibit productive illiquidity; they show that by forming a bank, agents can achieve allocations that are superior to those of exchange markets. Essentially, the bank provides agents with insurance against private preference shocks (hereafter, liquidity insur-

This article originates from a chapter of my doctoral dissertation at Washington University in St. Louis. I am indebted to Phil Dybvig, my principal advisor, for a number of very useful discussions and suggestions. I am also grateful to Franklin Allen (the executive editor), Kerry Back, John Bryant, Richard Green (the editor), Chris Lamoureux, John Nachbar, Sunil Panikkath, Steve Ross, Bruce Smith, and an anonymous referee for helpful comments Address correspondence to Jianping Qi, Department of Finance, College of Business Administration, University of South Florida, 4202 East Fowler Avenue, BSN 3403, Tampa, FL 33620.5500.

The Review of Financial Studies Summer 1994 Vol. 7, No. 2, pp. 389–417

ance), which is not directly available through traditional Arrow-Debreu contracts. Although the bank in the Diamond and Dybvig model is subject to runs induced by excessive withdrawals, suspension of deposit convertibility, government deposit insurance, or a government-sponsored lender of last resort can prevent bank runs.

In an attempt to understand the implications of Diamond and Dybvig's work for a dynamic economy, I study an infinitely repeated version of their model. The central questions are the following: (1) Are the results of the Diamond and Dybvig model generally strengthened or weakened in the dynamic economy? (2) What effects do intergenerational transfers have on the role of banks in liquidity provision? (3) What implications do intergenerational banking have on bank stability and regulatory policy?

My results show that intergenerational transfers play an important role in bank liquidity services. In particular, these transfers enable an intergenerational bank to achieve interest rate smoothing and provide depositors with liquidity insurance without Diamond and Dybvig's assumption of no side trades. As in Diamond and Dybvig (1983), the bank is subject to runs caused by excessive withdrawals. Unlike in their one-generation model, however, runs on the intergenerational bank can also result from the lack of new deposits. The latter cause implies that suspension of convertibility may not prevent bank runs. Government intervention may be necessary to maintain bank stability.

The dynamic setting of my model is the product of integrating the Diamond and Dybvig model with the overlapping generations model of Samuelson (1958). In the overlapping generations framework, intergenerational transfers enable a bank to invest a greater portion of deposits in less liquid but more productive technologies. The higher returns in turn allow the bank to achieve interest rate smoothing and provide depositors with liquidity insurance, even though the bank's deposit payoffs are made incentive compatible with respect to side trades. Imposing a similar constraint that eliminates side trades in the Diamond and Dybvig model would render it impossible for their one-generation bank to provide liquidity insurance.

As long as agents maintain confidence in the bank, intergenerational banking is very attractive. If such confidence is lost, however, the bank is vulnerable to runs which may result from either excessive withdrawals or the lack of new deposits. The latter case is possible because in the overlapping generations framework, the bank may rely on new deposits to pay off some of its deposit withdrawals. This intergenerational reliance is also the reason that suspension of convertibility may not prevent bank runs because suspension cannot force new agents to use the bank.

Government deposit insurance is one way to eliminate depositors' incentive to withdraw early, and guaranteed deposit safety also makes the bank attractive to new agents even though deposit insurance itself may not force new agents to use the bank. Alternatively, the government can sponsor a lender of last resort. During bank runs, the lender of last resort can play the role of a liquid fund recycler who, by selling government-backed securities, can attract funds from agents holding liquid assets and then inject these funds into the bank, preventing a costly liquidation of bank assets.

Besides Diamond and Dybvig's article, Chari and Jagannathan (1988), Cooper and Ross (1991), Haubrich and King (1990), Jacklin (1987), and Postlewaite and Vives (1987) also model bank liquidity services and stability in a one-generation framework.[1] Although these articles provide interesting insights, assuming a single generation prevents these models from studying the issues addressed here. Relatedly, Green (1987) studies optimal risk sharing in an economy in which agents are subject to private income shocks; he shows that the optimal allocations can be supported by borrowing and lending from a bank. Since there is only one generation of infinitely lived agents in his model, Green (1987) does not examine intergenerational transfers.

Bernanke and Gertler (1987) model a banking system in an overlapping generations framework similar to mine; however, the focus of their model is very different. Their article studies the macroeconomic implications of the aggregate liquidity demand and supply of the banking system with exogenously given banks. Freeman (1988)) Loewy (1991)) and Smith (1987) also use an overlapping generations framework to study bank liquidity services and stability, but the settings of these three models preclude intergenerational transfers. In contrast, Bryant (1981) incorporates intergenerational transfers into his banking model; although his treatment is similar to mine, his model does not address the role of banks in liquidity provision.

The article is organized as follows. Section 1 sets up the model to study the planning problem of an intergenerational bank, taking its existence as exogenously given. In Section 2, the existence assumption is dropped and the bank is formed at an arbitrary initial date. As it turns out, the major results derived in Section 1 remain qualitatively unchanged. Section 3 addresses the issues of bank stability and policy implications. Section 4 discusses topics for future study. All proofs of propositions are contained in the Appendix.

[1] See Jacklin (1989) for a survey of this literature.

1. The Model

The basic assumptions of my model are virtually identical to those of the Diamond and Dybvig model except that there are infinitely many overlapping generations of agents. Specifically, consider a sequential economy with a single consumption good. At each time $t \in Z$, the set of integers, a new generation of agents is born. Each of the identical generations is identified with the time it is born and each lives for three periods, including a planning period. The population size of each generation is constant over time, with a continuum of agents of measure 1.

Agents are identical ex ante and individually observe private preference shocks at the beginning of the middle period of life. Generation t agents' preferences are represented by

$$V(c_t, c_{t+1}, c_{t+2}) = \begin{cases} U(c_{t+1}) & \text{if Type I, with probability } \alpha, \\ U(c_{t+2}) & \text{if Type II, with probability } 1-\alpha, \end{cases}$$

where $\alpha \in (0, 1)$, c_{t+j} denotes a generation t agent's consumption at time $t + j$, and $U(\cdot)$ is an increasing, strictly concave, and twice-continuously differentiable von Neumann-Morgenstern utility function satisfying Inada conditions: $U'(0) = \infty$ and $U'(\infty) = 0$. An agent's type is only privately revealed to him in the middle period of life, and the generation to which he belongs is also only privately known.[2] There exists no technology for verifying or rejecting an agent's claim. Consequently, traditional Arrow-Debreu insurance contracts are not available in this economy because if such contracts were offered, agents would have an incentive to make nonverifiable claims of being of a particular type or generation in order to receive insurance payments.

Each agent is endowed with 1 unit of the consumption good in the first period of life and none at any other time. Two infinitely divisible investment technologies are publicly available. The first is a one-period storage technology whose per-unit payoff is 1 in one period. The second is a two-period production technology whose per-unit payoff is $R > 1$ in two periods. If the two-period technology is interrupted in one period, its per-unit payoff is only $Q \in [0, 1)$. That is, the two-period technology is illiquid.

1.1 Intergenerational banking

Suppose for now that there is an infinitely lived bank operating in this economy, taking deposits and making investments. To avoid add-

[2] The assumption that an agent's generation is unobservable to others reflects a realistic scenario. In reality, a depositor's actual lifetime is uncertain and depositors of all ages can have liquidity shocks. With many generations and nonhomogeneous depositors, the problem of generational misrepresentation would be worse.

ing other self-interested agents to the analysis, the bank is assumed to function as a social planner whose objective is to improve the welfare of depositors. Similar to the approaches taken by Diamond and Dybvig (1983)) Bhattacharya and Gale (1987)) and Jacklin (1987)) this welfare analysis of depositors replaces a potentially complex analysis of bank profit maximization and wealth distribution. More fundamentally, this assumption of the bank's objective is consistent with the characteristics of a competitive banking industry in which banks that attract deposits should be the ones that maximize the welfare of depositors.

Let D_t, denote the aggregate amount of the good that agents of generation t deposit in the bank. At each time $t \in Z$, the bank invests S_t in the storage technology and I_t in the illiquid technology. For every unit of time t deposit, depositors are offered a payoff of $r_{1t} \geq 0$ if the unit is withdrawn at time $t + 1$ or $r_{2t} \geq 0$ if withdrawn at time $t + 2$. The withdrawal made one period after deposit is called an early withdrawal; that made two periods after, a late withdrawal. In anticipation of the equilibrium, r_{1t} and r_{2t} are also referred to, respectively, as Type-I and Type-II payoffs. The ex ante, per-unit deposit payoff is denoted by $(r_{1t}\ r_{2t})$.

Consider the bank's planning problem at time $t = 0$. Agents relevant to this planning are the remaining Type-II agents of generation -1 and those who are born at and after time $t = 0$. Then, for the bank to be able to honor the promised payoffs to these agents, the following sequential budget constraints must be satisfied:

$$S_0 + I_0 \leq D_0, \tag{1}$$

$$(1 - \alpha) r_{2,-1} D_{-1} + \alpha r_{10} D_0 + S_1 + I_1 \leq RI_{-1} + S_0 + D_1, \tag{2}$$

$$(1 - \alpha) r_{2,t-2} D_{t-2} + \alpha r_{1,t-1} D_{t-1} + S_t + I_t \leq RI_{t-2} + S_{t-1} + D_t, \tag{3}$$

where (1) corresponds to time $t = 0$, (2) corresponds to time $t = 1$, and (3) holds for all $t \geq 2$. The left-hand sides of (1), (2), and (3) are the bank's outflows of the good, while the right-hand sides are its inflows. Note that in the above specification, the bank's deposits from a new generation are part of its inflows of the good; thus, intergenerational transfers are not precluded. Also, for generality, the bank is allowed to contain some initial capital and deposits. That is, I_{-1} and D_{-1} need not be 0.

Given the above setup, one formation of the bank's problem would be for the bank to maximize an aggregate welfare function of all relevant agents subject to the sequential budget constraints (1), (2), and (3) and incentive compatibility constraints to be described. As discussed in Section 4, however, there are major conceptual and technical difficulties associated with this approach. To simplify matters, I choose an alternative approach. The study will assume that all relevant agents, regardless of their generation, are treated equally. In particular, the analysis will be limited to steady-state payoffs that offer all current and future depositors an identical ex ante payoff under incentive compatibility, initially without regard to runs.[3]

Thus (r_1, r_2) is used to denote the ex ante steady-state payoff. Also, for the remaining Type-II agents of generation -1, only r_2 is relevant. Assume for now that all agents use the bank: $D_t = 1$ for $t \geq 0$. Constraints (1), (2), and (3) then become

$$S_0 + I_0 \leq 1, \tag{4}$$

$$\alpha r_1 + (1 - \alpha) r_2 D_{-1} + S_1 + I_1 \leq R I_{-1} + S_0 + 1, \tag{5}$$

$$\alpha r_1 + (1 - \alpha) r_2 + S_t + I_t \leq R I_{t-2} + S_{t-1} + 1, \tag{6}$$

where (6) also holds for all $t \geq 2$. Let $\{S_t\}$ and $\{I_t\}$ denote sequences $\{S_0, S_1, S_2, \ldots\}$ and $\{I_0, I_1, I_2, \ldots\}$, respectively. A budget-feasible steady-state payoff is now defined.

Definition 1. *A steady-state payoff (r_1, r_2) is budget feasible if there exist some nonnegative $\{S_t\}$ and $\{I_t\}$ such that (4), (5), and (6) hold.*

It should be noted that, strictly speaking, the bank's net investments in the illiquid technology can be negative since the illiquid investments can be liquidated prematurely. Given that the aggregate deposit withdrawals are deterministic, however, the bank will never plan to do this, because liquidating the illiquid investments prematurely is strictly dominated by storing the amount of the liquidation value directly.

It is further specified in this section that $D_{-1} = I_{-1} = 1$. As will be seen, this parameter restriction is actually more for convenience than for substance. In Section 2, the intergenerational bank is assumed to be formed at an arbitrary initial date, without prior bank assets and liabilities; that is, $D_{-1} = I_{-1} = 0$. The major results derived in this section will remain qualitatively invariant to different choices of the

[3] In some models, it makes sense to look at best steady-state payoffs without worrying about the conditions at an initial date and without requiring equal payoffs, Under these circumstances in my model, what were attainable in steady state would be unbounded due to scale independence of the illiquid technology, and there would be no best steady-state payoff. In particular, without a starting date and the equal-payoff condition, the bank's steadystate budget constraint, following from (3), would be *scale* independent of the form

$$\alpha r_1 + (1 - \alpha) r_2 \leq (R - 1)\frac{I}{D} + 1.$$

initial condition. With $D_{-1} = I_{-1} = 1$, the following proposition demonstrates that the sequential constraints for a steady-state payoff to be budget feasible are virtually equivalent to a simple budget constraint.

Proposition 1. *Fix $D_{-1} = I_{-1} = 1$. A steady-state payoff (r_1, r_2) is budget feasible if and only if r_1 and r_2 satisfy*

$$\alpha r_1 + (1 - \alpha) r_2 \leq R. \tag{7}$$

Proof The formal proof, given in the Appendix, is based on aggregating the resource constraints (4), (5), and (6) over time. It is shown that if (7) is false, then there exist no nonnegative $\{S_t\}$ and $\{I_t\}$ that make (4), (5), and (6) true. On the other hand, if (7) is true, then given $\{S_t\} = \{0\}$ and $\{I_t\} = \{1\}$, (4), (5), and (6) hold. ∎

Proposition 1 significantly reduces the complexity of the sequential budget constraints. In fact, given (7) holds with equality, the choices of $\{S_t\}$ and $\{I_t\}$ that make a steady-state payoff budget feasible are unique.

Proposition 2. *Fix $D_{-1} = I_{-1} = 1$ and suppose that r_1 and r_2 satisfy (7) with equality. Then (4), (5), and (6) are uniquely satisfied by $(S_t\} = \{0\}$ and $\{I_t] = \{1)$.*

Proof. See the Appendix. ∎

To construct the bank's steady-state planning problem, it remains to formulate incentive compatibility constraints. Recall that an agent's revealed type and the generation to which he belongs are both private information. To induce proper self-selection, the first incentive compatibility constraint is to require $r_1 \leq r_2$, so that Type-II depositors do not withdraw early. This condition, however, may not be strong enough in the repeated version of the Diamond and Dybvig model to prevent Type-II depositors from misrepresenting the true generation to which they belong. The bank must also impose $r_1^2 \leq r_2$ so that Type-II depositors have no incentive to withdraw early in the middle period of life and then reinvest in the bank by pretending to be agents of a new generation.

In the Diamond and Dybvig model, to eliminate the kinds of side trades illustrated in Jacklin (1987),[4] agents are required to invest only

[4]To illustrate side trades. suppose that agents need not deposit their endowments in the bank in the Diamond and Dybvig model. Let r_1 and r_2 denote the bank's deposit payoffs at time $t = 1$ and time $t = 2$. respectively. Consider a side trade game. Instead of depositing his endowment in the bank, an agent chooses to self-invest it in the illiquid technology. If the agent turns out to be a Type II, he will wait until his investment matures at $t = 2$ and obtain a return of $R > 1$. If the agent

in their bank. In my model, instead of assuming a priori that agents invest only in the bank, the problem of side trades is eliminated by imposing an incentive compatibility constraint $r_2 \geq R$.[5] Imposing this constraint in the Diamond and Dybvig model would render it impossible for their bank to provide liquidity insurance. In contrast, the restriction $r^2 \geq R$ does not prevent the intergenerational bank from providing liquidity insurance, as will be shown later in this section.

Given the assumption that agents of different generations are treated equally, the bank's problem is reduced to solving the following problem of a representative generation subject to the budget constraint (7) and the incentive compatibility constraints described above. In particular, the bank's problem is as follows:

Problem A[6]

$$\max_{\{r_1, r_2\}} \alpha U(r_1) + (1 - \alpha) U(r_2),$$

$$\text{subject to} \qquad \alpha r_1 + (1 - \alpha) r_2 \leq R, \tag{8}$$

$$r_1 \leq r_2, \tag{9}$$

$$r_1^2 \leq r_2, \tag{10}$$

$$r_2 \geq R, \tag{11}$$

$$r_1, r_2 \geq 0. \tag{12}$$

The existence of a unique solution to Problem A, denoted by (r_1^a, r_2^a) follows immediately from strict concavity of the objective function and convexity and compactness of the constraint set. More interesting, without any further restriction on agents' preferences, it will be shown that the solution to Problem A has (8) and (10) binding. It can thus be characterized precisely.

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is a Type I, he will trade the claim on the illiquid investment with a Type-II depositor who will then withdraw early at t = I. The corresponding payoff from this game for the self-investor and the early withdrawer is (r_1, R). In the Diamond and Dybvig model, if the bank provides depositors with liquidity insurance, $R > r_2 > r_1 > 1$. Both agents strictly prefer the payoff (r_1, R) from the cooperative side trade to (r_1, r_2) from the bank. The bank is therefore not stable with respect to these kinds of side trades.

[5] Wallace (1988) avoids the side trade problem by assuming that agents are in "isolation" in the middle period of life and therefore cannot trade among themselves. My model does not need this assumption either.

[6] Formulating Problem A (and later Problem B) as shown ignores a potential problem of nonmeasurability of a continuum of i.i.d. random variables. The problem is, however, technical in nature. Interested readers are referred to Anderson (1990), Feldman and Gilles (1985), and Judd (1985) for the analysis of the nonmeasurability problem and solutions.

Proposition 3. *The unique solution to Problem A has*

$$r_1^a = \frac{\sqrt{\alpha^2 + 4(1-\alpha)R} - \alpha}{2(1-\alpha)},$$

$$r_2^a = (r_1^a)^2,$$

$$r_2^a > R > r_1^a > 1.$$

Proof: See the Appendix. ■

The steady-state payoff (r_1^a, r_2^a) is implementable through demand deposits at the bank. At each time $t \geq 0$, agents of a new generation deposit their endowments at the bank. Each agent either withdraws early in one period or late in two periods, corresponding to being a Type I or a Type II. No agent wants to misrepresent his true type or true generation because he cannot obtain a better payoff through misrepresentation. Interestingly, if (10) were discarded, the first-best solution to Problem A would be attainable and would have $r_1 = r_2 = R$; this is the situation in which agents are fully insured against their preference shocks. The problem of generational misrepresentation prevents the first-best liquidity insurance.

The intergenerational bank facilitates interest rate smoothing and provides depositors with liquidity insurance. Recall the term structure of the investment technologies: The storage technology returns 1 in one period, but the illiquid technology returns $R > 1$ in two periods. With intergenerational banking, the term structure of the deposit payoffs has

$$\frac{r_1^a}{r_2^a} = \frac{1}{r_1^a} > \frac{1}{R}.$$

That is, the relative payoff to one-period deposits with respect to the payoff to two-period deposits is greater than the corresponding relative return of the one-period technology with respect to the return of the two-period technology. Moreover, $r_2^a = (r_1^a)^2$ describes a flat term structure of the deposit payoffs. Since the bank converts an upward technological yield curve into a flat interest rate yield curve, early deposit withdrawers can share with late withdrawers the higher return of the illiquid technology. The bank, therefore, provides depositors with a certain degree of liquidity insurance.

1.2 Properties of the bank payoff

One of the interesting properties of the steady-state payoff (r_1^a, r_2^a) is that this payoff is not a competitive equilibrium of simple decentralized exchange, with or without money. To illustrate this point, consider individual agents' maximization problems in simple decentralized exchange, possibly with money. Let $H_{-1} \geq 0$ denote the endowed money holdings of the remaining agents of generation –1 at time $t = -1$ and $H_t \geq 0$ the monetary endowments of agents of generation t at time $t \geq 0$. In the competitive market, agents can buy and sell the good and money in the spot or forward markets at perfectly foreseen prices, denoted p_t and m_t, respectively. Let C^i_{t+1} and C^i_{t+2} be the demands for the time $t + 1$ and the time $t + 2$ goods from agents of generation i. To be consistent with the economy described in Section 1.1, the remaining agents of generation -1 are each endowed with $I_{-1} = 1$, and the rest are each endowed with 1 unit of the good. Let S_t and I_t be the amounts agents of generation $t \geq 0$ choose to invest in the storage technology and the illiquid technology, respectively.

Individual agents' problems are described in the following. Each of the remaining agents of generation -1 solves

$$\max_{\{C_1^{-1}\}} U(C_1^{-1}), \qquad \text{subject to } p_1 C_1^{-1} \leq p_1 R + m_{-1} H_{-1},$$

$$C_1^{-1} \geq 0.$$

In contrast, each agent of generation $t \geq 0$ solves

$$\max_{\{C^t_{t+1}, C^t_{t+2}\}} \alpha U(C^t_{t+1}) + (1 - \alpha) U\left(\frac{p_{t+1}}{p_{t+2}} C^t_{t+1} + C^t_{t+2}\right),$$

$$\text{subject to} \quad p_{t+1} C^t_{t+1} + p_{t+2} C^t_{t+2} \leq p_t(1 - S_t - I_t) + p_{t+1} S_t + p_{t+2} I_t + m_t H_t,$$

$$C^t_{t+1}, C^t_{t+2} \geq 0.$$

Note that in setting up the above 2 problems, it is implicitly assumed that $p_{t+1} \geq P_{t+2}$ for all $t \geq 0$. This follows from the argument of no arbitrage. If for some $t \geq 0$, $p_{t+1} < P_{t+2}$, then agents of generation t can buy the time $t + 1$ good and store it for one period, and at the same time sell forward the time $t + 2$ good. This strategy, allowing these agents to pocket the price difference without incurring any cost, represents an arbitrage opportunity and therefore must be impossible in equilibrium.

Definition 2. *A competitive equilibrium is a collection of* $\{p_t : p_t > 0\}$, $\{m_t : m_t \geq 0\}$, $\{C^t_{t+1}\}$, *and* $\{C^t_{t+2}\}$ *such that*

1. C_1^{-1} *solves the problem of the remaining agents of generation*

2. C^t_{t+1} ***and*** C^t_{t+2} *solve the problem of agents of generation t for all*

3. $C^{t-1}_{t+1} + C^t_{t+1} = RI_{t-1} + S_t + 1 - S_{t+1} - I_t$ *for all* $t \geq 0$.

In particular, a competitive equilibrium without money is the one in which $m_t = 0$ for all $t \geq -1$, while a competitive equilibrium with money is the one in which $m_t \geq 0$ for all $t \geq -1$ and $m_t > 0$ for some $t \geq -1$.

Proposition 4. *The steady-state payoff* (r_1^a, r_2^a) *is not a competitive equilibrium, with or without money.*

Proof. The formal proof, given in the Appendix, applies the uniqueness result of Proposition 2 and shows that even if agents individually choose to pursue the investments of $\{S_t\} = \{0\}$ and $\{I_t\} = \{1\}$, there are still no positive $\{p_t\}$ and nonnegative {m,} that achieve the steady-state payoff (r_1^a, r_2^a). ■

Intuitively, the failure of decentralized exchange is due to agents' having a finite life and the fact that they do not value the good after "death." In Samuelson (1958), to achieve the superior payoff, wealth must be transferred from agents of the young generation to agents of the old generation. Simple decentralized exchange cannot accomplish these transfers because the old have nothing to give back to the young. This problem disappears when money is introduced. The old can now use money to buy the good from the young, and the young are willing to exchange because money representing claims on the future good is valuable to them. In my model, although wealth transfers are also necessary for achieving the payoff (r_1^a, r_2^a), the direction of transfers is different from Samuelson's. Here, wealth must be transferred from Type-II agents of the old generation to Type-I agents of the young generation. Decentralized exchange fails because the old do not value the future good, when they will be "dead."

Unlike in Samuelson (1958), this reversal of transfer direction also invalidates the role of money as a medium to facilitate intergenerational transfers. Money, which would give the old claims on the future good, is no longer attractive precisely because the old do not value the future good. In view of this, a centralized mechanism is probably essential to implementing the payoff (r_1^a, r_2^a). It is argued that the intergenerational bank accomplishes this goal. The result that the bank functions as a social compact to facilitate intergenerational transfers is quite general and robust to the weakening of the assumptions made in this article.

While focusing on steady-state payoffs greatly simplifies the analysis, this approach is not without conceptual difficulties. As the following example will illustrate, the payoff (r_1^a, r_2^a), although efficient within the class of steady-state payoffs, is not necessarily Pareto efficient in general. To see this, suppose $U(c) = c^{0.9}$, $a = 0.5$, and $R =$

2. Applying Proposition 3, we find $r_1^a = 1.562$ and $r_2^a = 2.440$; then the expected utilities of agents of generation $t \geq 0$ are 1.863 each. Consider now a non-steady-state payoff: $r_{2,-1} = r_2^a = 2.440$, but $r_{1t} = 0$ and $r_{2t} = 4.520$ for all $t \geq 0$; then the expected utilities of agents of generation $t \geq 0$ are 1.944 > 1.863. The non-steady-state payoff is budget feasible because with $D_{-1} = I_{-1} = 1$ and taking g $\{S_0, S_1, S_2, S_3, \ldots\} = \{0, 0.520, 0, 0, \ldots\}$ and $\{I_0, I_1, I_2, I_3, \ldots\} = \{1, 1.260, 1.260, 1.260, \ldots\}$, the sequential constraints (1), (2), and (3) hold. The non-steady-state payoff is also incentive compatible because $r_{1t} \leq r_{2t}$, $r_{1t}r_{1,t+1} \leq r_{2t}$, and $r_{2t} \geq R$. Therefore, the non-steady-state payoff Pareto-dominates the steady-state payoff (r_1^a, r_2^a).

As disturbing as it may seem, the fact that the optimal steady-state payoff may not be Pareto efficient should not invalidate the analysis of the intergenerational bank. One would expect that the bank, as a centralized mechanism, should be able to implement non-steady-state payoffs that are Pareto efficient. The difficulty is to derive Pareto-efficient payoffs in the infinitely repeated version of the Diamond and Dybvig model. This challenging task is left for future research.

2. Forming an Intergenerational Bank

Throughout Section 1, it has been assumed that an intergenerational bank exists at $t = 0$ with $D_{-1} = I_{-1} = 1$. One drawback is that under this assumption the bank appears to be exogenous. In this section, this assumption is dropped and an intergenerational bank is formed at an arbitrary initial date, without prior bank assets and liabilities. As it turns out, the major results derived in Section 1 are qualitatively invariant to different choices of the initial condition. More interesting, without an initial investment in progress, the intergenerational bank is now subject to tighter budget constraints and has to rely on new deposits to pay off some of its deposit withdrawals. Intergenerational liquidity sharing adds interesting features to the banking model.

To form a bank at an arbitrary date, let us give the economy a starting time $t \in Z$, when agents of the very first generation are born and they decide to form a bank. Unless stated otherwise, the notation used in this section is identical to that in Section 1. Without loss of generality, take the initial date to be $t = 0$. Since the bank commences operations from scratch, there are no investments in progress and no existing depositors. The only resources this bank has at the beginning are the deposits from agents of the first generation. If the first depositors of the bank expect that agents of future generations will use the bank, then for planning purposes they take $D_t = 1$ for all $t \geq 0$. As shown later, the intergenerational bank can attract agents of future generations because it can offer better payoffs than can be offered by banks with a finite life or by autarky. Forming the intergenerational bank therefore has the flavor of a rational expectations equilibrium.

Technically, the bank described in this section faces the sequential budget constraints (4), (5), and (6) with $D_{-1} = I_{-1} = 0$. Analogous to Proposition 1, the next proposition shows that these constraints can also be reduced to a simple budget constraint.

Proposition 5. *Fix $D_{-1} = I_{-1} = 0$. A steady-state payoff (r_1, r_2) is budget feasible if and only if r_1 and r_2 satisfy*

$$\alpha \frac{R+1}{2} r_1 + (1-\alpha) r_2 \leq R. \tag{13}$$

In particular, given that (13) holds with equality, (4), (5), and (6) are satisfied by the following $\{S_t\}$ and $\{I_t\}$: for $\alpha r_1 \leq 1$,

$$S_t = \begin{cases} R - \alpha r_1 - (1-\alpha) r_2 & \textit{for } t = 2k, \quad k \in Z_{++}, \\ 0 & \textit{for } t = 0 \textit{ and } t = 2k+1, \quad k \in Z_{+}, \end{cases}$$

$$I_t = \begin{cases} 1 & \textit{for } t = 2k, \quad k \in Z_{+}, \\ 1 - \alpha r_1 & \textit{for } t = 0 \textit{ and } t = 2k+1, \quad k \in Z_{+}; \end{cases}$$

for $\alpha r_1 > 1$,

$$S_t = \begin{cases} \alpha r_1 - 1 & \textit{for } t = 0, \\ \alpha r_1 + (1-\alpha) r_2 - 1 & \textit{for } t = 2k, \quad k \in Z_{++}, \\ 0 & \textit{for } t = 0 \textit{ and } t = 2k+1, \quad k \in Z_{+}, \end{cases}$$

$$I_t = \begin{cases} 2 - \alpha r_1 & \textit{for } t = 2k, \quad k \in Z_{+}, \\ 0 & \textit{for } t = 0 \textit{ and } t = 2k+1, \quad k \in Z_{+}, \end{cases}$$

where Z_+ and Z_{++} denote the sets of nonnegative andpositive integers, respectively.

Proof. The proof, given in the Appendix, is similar to that of Proposition 1. ■

From the setup of Problem A, the bank is now subject to the budget constraint (13) and the incentive compatibility constraints (9), (10), and (11). The bank's problem in this case is stated as follows.

Problem B

$$\max_{\{r_1, r_2\}} \alpha U(r_1) + (1-\alpha) U(r_2),$$

$$\text{subject to} \quad \alpha \frac{R+1}{2} r_1 + (1-\alpha) r_2 \leq R, \tag{14}$$

$$r_1 \leq r_2, \tag{15}$$

$$r_1^2 \leq r_2, \tag{16}$$

$$r_2 \geq R, \tag{17}$$

$$r_1, r_2 \geq 0. \tag{18}$$

As compared with Problem A, Problem B entails a stronger budget constraint (14). Since $0 \leq r_1 \leq 2R/(R+1) < R \leq r_2$ is implied by (14), (17), and (18) and since $(2R/(R+1))^2 < R$, (15) and (16) are thus redundant in Problem B. The existence of a unique solution to Problem B follows from strict concavity of the objective function and convexity and compactness of the constraint set. More interesting, with a mild restriction on agents' preferences, it will be shown that the solution to Problem B, denoted by (r_1^b, r_2^b) has (14) and (17) binding. It can also be characterized precisely.

Proposition 6. *Given that agents' relative risk aversion coefficient is greater than 1, the unique solution to Problem B has* $r_1^b = 2R/(R+1)$ *and* $r_2^b = R$.

Proof: See the Appendix. ■

It is easily seen that the steady-state payoff (r_1^b, r_2^b) can also be implemented through bank demand deposits. In fact, the important properties of the bank in Section 1 can be shown to apply to the bank in this section; this demonstration is omitted to avoid redundancy. Some distinctive results are, however, noteworthy. The first one is that the bank in this section can be formed at any starting point in time, since the initial date $t = 0$ is arbitrary and agents of the founding generation are not worse off initiating the intergenerational banking process, conditioned on the rational expectations that agents of future generations will use the bank. An implication of this property is that the intergenerational bank can commence operations after any out-of-equilibrium disruption. Another interesting feature is that, unlike in Section 1, the bank here uses new deposits to pay off some of its deposit withdrawals. Intergenerational liquidity sharing has implications for bank stability and regulatory policy.

Before turning to the topics of bank stability and policy implications, it is shown in the following that the intergenerational bank can achieve better payoffs than those that can be achieved by either finitely lived banks with the restriction of no side trades or by autarky. Since the results of Diamond and Dybvig (1983) imply that a one-generation bank can do no worse than autarky, what remains is to compare the payoff of the intergenerational bank with the payoffs of finitely lived banks. Specifically, consider first the constraints a finitely lived bank faces. Following from the Diamond and Dybvig model, the budget constraint of a one-generation bank can be written as

$$\alpha R r_1 + (1 - \alpha) r_2 \leq R. \tag{19}$$

By backward induction, (19) is also the budget constraint of a finitely lived bank. Then, a finitely lived bank solves the problem of maximizing the same objective function as Problem B subject to at least the following constraints: (17), (18), and (19), with (17) needed to eliminate side trades. Since (15) and (16) are redundant in Problem B and (14) is strictly weaker than (19), agents must prefer the payoff derived from Problem B to that derived from the problem of a finitely lived bank.

Intuitively, by relying on new deposits to pay off some of its deposit withdrawals, the intergenerational bank is able to improve productivity by investing a greater portion of its deposits in the illiquid but more productive technology. Since it is impossible for a finitely lived bank to achieve this productivity improvement, agents are better off investing in the intergenerational bank. At this point, note that it is possible for agents of a new generation to form a new intergenerational bank. This possibility is ignored for now because the new bank can, at best, achieve only the same payoff as can be achieved by the existing bank. The question of forming a new intergenerational bank will return when bank stability is studied in the next section.

3. Stability of the Bank

While facilitating intergenerational transfers, achieving interest rate smoothing, and providing liquidity insurance are important functions of the intergenerational bank, they are also sources of bank instability. Self-selection associated with deposits withdrawn on demand renders the bank vulnerable to runs. As long as both current depositors and new agents maintain confidence in the bank, intergenerational banking is very attractive. However, if such confidence is lost, intergenerational dependence may worsen the bank's ability to weather runs. As in the Diamond and Dybvig model, mechanisms that cause agents' beliefs to change are not modeled explicitly, but if agents believe that there is a run on the bank it is rational for them to withdraw. Thus, a bank run can occur as a Nash equilibrium.

The bank analyzed in either Section 1 or Section 2 is subject to runs. Specifically, suppose that at some time $t > 0$, a fraction $f > \alpha$ of generation $t - 1$ depositors decides to withdraw early. In the limit, total withdrawals are

$$\lim_{f \to 1} fr_1 + (1 - \alpha)r_2 = r_1 + (1 - \alpha)r_2.$$

Consider, for example, the bank payoff (r_1^a, r_2^a) of Problem A. The maximal quantity of the good available at t is $R + Q$, of which R is the return of I_{t-2} and Q is the liquidating value of $I_{t-1} = 1$. The bank is vulnerable if $r_1^a + (1 - \alpha)r_2^a > R + Q$. Given r_1^a and r_2^a from Proposition 3, the bank is subject to runs whenever

$$Q < \frac{\sqrt{\alpha^2 + 4(1 - \alpha)R} - \alpha}{2}.$$

A similar result applies to the bank payoff (r_1^b, r_2^b) of Problem B as well.

Moreover, if intergenerational liquidity sharing is important, as in Section 2, the lack of new deposits can jeopardize the bank's regular deposit withdrawals. For example, suppose that $\alpha r_1^b \leq$ At each time $t = 2k, + 1$ for $k \in Z_{++}$, following from the investment strategy of Proposition 5, the bank has the investment output of only $RI_{2k-1} + S_{2k} = (1 - \alpha)r_2^b$, which is strictly less than the aggregate payoff required for its regular withdrawals of $\alpha r_1^b + (1 - \alpha)r_2^b$. The bank relies on αr_1^b units of the new deposits to meet its deposit withdrawals.

The lack of new deposits can trigger bank runs. If agents of a new generation choose not to use the bank at some time $t = 2k + 1$, $k \in Z_+$, the bank is αr_1^b units short of resources. In order to meet its regular deposit withdrawals of $\alpha r_1^b + (1 - \alpha)r_2^b$, the bank has to liquidate $\alpha r_1^b/Q$ units of the investment in the illiquid technology prematurely. Those generation $2k$ depositors who do not withdraw r_1^b at $t = 2k + 1$ will get w at $t = 2k + 2$ when the bank liquidates, with w determined by

$$(1 - \alpha)w = \left(1 - \frac{\alpha r_1^b}{Q}\right)R \Rightarrow w = \frac{(Q - \alpha r_1^b)R}{(1 - \alpha)Q}. \tag{20}$$

From Proposition 6, $r_1^b = 2R/(R + 1)$ and $r_2^b = R$. Then

$$Q < \frac{\alpha r_1^b R}{R - (1 - \alpha)r_2^b} = \frac{2R}{R + 1} \Rightarrow w < r_2^b; \tag{21}$$

$$Q < \frac{\alpha r_1^b R}{R - (1 - \alpha)r_1^b} = \frac{2\alpha R}{2\alpha + R - 1} \Rightarrow w < r_1^b.$$

If there are no new deposits, (21) shows that Type-II depositors of generation $2k$ will not obtain the promised return r_2^b; worse yet, (22) shows that if Q is sufficiently small, they can be strictly worse off than those who withdraw early. The latter case creates an incentive for

Type-II depositors to withdraw early and hence for a run on the bank to occur.

In contrast to the result of the Diamond and Dybvig model, the threat of suspension of deposit convertibility may not prevent bank runs in the presence of intergenerational liquidity sharing. This is because although suspension of convertibility can stop excessive deposit withdrawals, it cannot force new agents to use the bank. Consider, for example, the payoff $(r_1^b, r_2^b$ in Section 2. Suppose that massive withdrawals occur at some time $t = 2k + 1$, $k \in Z_+$ During a bank run, current depositors rush to withdraw while new agents stay away from the bank. After an $f = a$ fraction of generation $2k$ depositors has withdrawn r_1^b units, the bank suspends convertibility of deposits into the good. Without new deposits, the bank is is αr_1^b units short of resources and has to liquidate the investment in the illiquid technology prematurely. Those generation $2k$ depositors who are unable to withdraw r_1^b at $t = 2k + 1$, due to suspension, will get w as determined by (20). If condition (22) holds, what they will get is strictly smaller than r_1^b. Consequently, regardless of their types, depositors will attempt to withdraw early; hence, the threat of suspension of convertibility may not prevent bank runs in this environment.

An interesting question is what new agents will choose to do with their endowments during a run on the existing bank. They can certainly take an autarkic approach, investing the good individually. An alternative is for new agents to form a new infinitely lived bank, as demonstrated in Section 2, to achieve the same payoff as the existing bank was supposed to do. If this new bank is indeed formed, presumably it can lend funds to the existing bank. In that case, suspension of convertibility together with a commitment from the new bank to lend to the existing bank can help resolve the banking crisis. This analysis is, however, not very realistic. The first problem is that the new bank's commitment to lend may not be credible. After all, its own survival depends on the confidence of current and future generations. The second problem is that although it is possible for new agents to form an intergenerational bank in the middle of a banking crisis, this scenario is unlikely, particularly given that the new bank must operate in the same way as the troubled bank has been operating.

The lack of a viable mechanism for the bank to prevent runs suggests that government intervention may be necessary to maintain bank stability. Unlike intervention by the private sector, government intervention is credible because the government can back up its commitment with the power to tax agents of the current and future generations, if necessary. One such approach is for the government to provide deposit insurance. As a result of government deposit insurance, depositors have no incentive to run the bank. Guaranteed deposit safety also makes the bank attractive to new agents even though as with suspension of convertibility, deposit insurance itself cannot force new agents to use the bank.

Another mechanism is for the government to sponsor a lender of last resort. When the bank experiences runs, it borrows emergency funds from the lender of last resort. The lender of last resort, on the other hand, attracts liquid funds by selling government-backed securities to those Type-II depositors who withdraw early and also to those new agents who stay away from the bank. Both groups are willing to invest in government securities because of the credibility afforded by the government. The lender of last resort basically functions as a liquid-fund recycler who can attract liquid funds and inject them into the banking system, preventing disruptive and costly asset liquidation.[7]

4. Conclusion

In an infinitely repeated version of the Diamond and Dybvig model, I examine bank liquidity services and stability. A bank is shown to facilitate intergenerational transfers. As a result, the bank can achieve interest rate smoothing and provide depositors with liquidity insurance without Diamond and Dybvig's assumption of no side trades. The bank is subject to runs that may result from either excessive withdrawals or the lack of new deposits. The latter case implies that suspension of convertibility may not prevent bank runs. Government intervention through deposit insurance or the lender of last resort may be necessary to maintain bank stability.

A number of issues remain. Of particular concern is the model's restriction to steady-state payoffs. An interesting but challenging extension would be to explore intergenerational banking in a more general setting in which an aggregate welfare function can be defined and the restriction to steady state can be dropped. The problem is that defining a proper aggregate welfare function is conceptually difficult in the overlapping generations framework. There is no good way to weight the expected utilities of agents of different generations. Is one generation more important than another? This problem is not unlike the difficulty encountered in trying to define an aggregate welfare function in an economy in which agents have heterogeneous preferences and endowments.

[7] In reality, the central bank can supply emergency liquidity by printing flat money, effectively creating an inflation tax. This article does not consider this approach.

In fact, in the framework of this article, even if one were to ignore the conceptual difficulty and formulated the bank's objective function as a discounted sum of the expected utilities of all relevant agents, there would be additional problems. With the sequential budget constraints (1), (2), and (3), unless the social discount rate is at least as large as $\sqrt{R} - 1$ each period, the optimal payoffs to agents of different generations would be increasing over time, and these payoffs would not converge to a steady state. This is because reducing the payoffs to agents of early generations increases the investments in the illiquid technology. In this way, the output of the good increases exponentially over time since the illiquid technology is scale independent, returning $R > 1$ every two periods. Taking the social discount rate to be $\sqrt{R} - 1$ would not make the problem of maximizing the discounted sum much easier. If the incentive compatibility constraints are taken into account, the problem becomes quite complicated and it is difficult to find a solution that can be meaningfully interpreted. The difficulty is due in part to the intertemporal dependence of consumption and investment choices that is implicit in the intertemporal budget and incentive compatibility constraints.

Another unanswered question is how important intergenerational liquidity sharing is in bank liquidity services. I examine two cases. In the first, the bank's investment returns are sufficient to cover its regular deposit withdrawals and intergenerational liquidity sharing plays a minimal role. In the second, the bank relies on new deposits to meet its regular deposit withdrawals and intergenerational liquidity sharing has a significant impact on the bank's liquidity provision and stability. The theory has nothing to say on which case is more indicative of banking in practice; this issue is better addressed by empirical investigation. A satisfactory resolution of this issue is important to our understanding of banking in a dynamic economy.

Finally, several more immediate extensions can also be made. By relaxing the assumption that the aggregate preference shocks are deterministic, one may be able to examine intergenerational pooling of the aggregate shocks. Along a related line of inquiry, Bhattacharya and Gale (1987) and Qi (1993a) use interbank liquidity sharing to resolve liquidity shocks faced by individual banks. Another extension is to study what happens if the technologies are risky and there is asymmetric information regarding asset returns. In a one-generation model, Jacklin and Bhattacharya (1988) show that such a framework may entail information-based bank runs, and Qi (1993b) demonstrates that a bank, offering informed agents bank stock and uninformed agents bank demand deposits, can improve economic efficiency.

Appendix

A.1 Proof of Proposition 1

Before proceeding to the proof, I lay some groundwork. Rewrite the sequential constraints (4), (5), and (6) in terms of intertemporal restrictions on I,:

$$I_0 \leq 1 - S_0, \tag{A1}$$

$$I_1 \leq RI_{-1} + S_0 + 1 - \alpha r_1 - (1-\alpha) r_2 D_{-1} - S_1, \tag{A2}$$

$$I_t \leq RI_{t-2} + S_{t-1} + 1 - \alpha r_1 - (1-\alpha) r_2 - S_t. \tag{A3}$$

Notice that (A1), (A2), and (A3) are interrelated, and by recursively substituting the bound on I_{t-2} into the bound on I_t, they imply a sequence of investment restrictions:

$$\begin{aligned}
I_0 &\leq 1 - S_0,\\
I_1 &\leq RI_{-1} + S_0 + 1 - \alpha r_1 - (1-\alpha) r_2 D_{-1} - S_1\\
&\leq 1 + RI_{-1} - \alpha r_1 - (1-\alpha) r_2 D_{-1} + S_0 - S_1,\\
I_2 &\leq RI_0 + S_1 + 1 - \alpha r_1 - (1-\alpha) r_2 - S_2\\
&\leq 1 - RS_0 + R - \alpha r_1 - (1-\alpha) r_2 + S_1 - S_2,\\
I_3 &\leq RI_1 + S_2 + 1 - \alpha r_1 - (1-\alpha) r_2 - S_3\\
&\leq 1 + R(RI_{-1} - \alpha r_1 - (1-\alpha) r_2 D_{-1} + S_0 - S_1)\\
&\quad + R - \alpha r_1 - (1-\alpha) r_2 + S_2 - S_3,\\
I_4 &\leq RI_2 + S_3 + 1 - \alpha r_1 - (1-\alpha) r_2 - S_4\\
&\leq 1 - R^2 S_0 + (R+1)(R - \alpha r_1 - (1-\alpha) r_2)\\
&\quad + R(S_1 - S_2) + S_3 - S_4,\\
I_5 &\leq RI_3 + S_4 + 1 - \alpha r_1 - (1-\alpha) r_2 - S_5\\
&\leq 1 + R^2(RI_{-1} - \alpha r_1 - (1-\alpha) r_2 D_{-1} + S_0 - S_1)\\
&\quad + (R+1)(R - \alpha r_1 - (1-\alpha) r_2) + R(S_2 - S_3) + S_4 - S_5,\\
&\vdots
\end{aligned}$$

The above constraints can be described in two subsequences, namely, for $k \in \mathbb{Z}_{++}$, the set of positive integers,

$$\begin{aligned}
I_{2k} &\leq RI_{2k-2} + S_{2k-1} + 1 - \alpha r_1 - (1-\alpha) r_2 - S_{2k}\\
&\leq 1 - R^k S_0 + \sum_{i=1}^{k} R^{k-i}(R - \alpha r_1 - (1-\alpha) r_2 + S_{2i-1} - S_{2i}),
\end{aligned} \tag{A4}$$

$$I_{2k+1} \leq RI_{2k-1} + S_{2k} + 1 - \alpha r_1 - (1-\alpha)r_2 - S_{2k+1}$$
$$\leq 1 + R^k(RI_{-1} - \alpha r_1 - (1-\alpha)r_2 D_{-1} + S_0 - S_1)$$
$$+ \sum_{i=1}^{k} R^{k-i}(R - \alpha r_1 - (1-\alpha)r_2 + S_{2i} - S_{2i+1}). \tag{A5}$$

Lemma 1. *A necessary condition for a steady-state payoff (r_1, r_2) to be budget feasible is that r_1 and r_2 satisfy*

$$\alpha \frac{R+1}{(R-1)I_{-1}+2} r_1 + (1-\alpha)\frac{(R-1)D_{-1}+2}{(R-1)I_{-1}+2} r_2 \leq R. \tag{A6}$$

Proof: Suppose the payoff (r_1, r_2) is budget feasible. From Definition 1, there exist some nonnegative $\{S_t\}$ and $\{I_t\}$ such that (4), (5), and (6) hold; equivalently, (A1) to (A5) hold. Take S_t ³ 0. From (A4) and (A5),

$$I_{2k} + I_{2k+1} \leq 2 + R^k(RI_{-1} - \alpha r_1 - (1-\alpha)r_2 D_{-1})$$
$$+ 2(R - \alpha r_1 - (1-\alpha)r_2) \sum_{i=1}^{k} R^{k-i}$$
$$- (R-1)\sum_{i=1}^{k} R^{k-i} S_{2i-1} - S_{2k+1}$$
$$\leq 2 + R^k\Bigg(RI_{-1} - \alpha r_1 - (1-\alpha)r_2 D_{-1}$$
$$+ 2(R - \alpha r_1 - (1-\alpha)r_2)\sum_{i=1}^{k} R^{-i}\Bigg),$$

where the last inequality obtains because $S_t \geq 0$ is taken as given.

If (A6) is false, that is, if

$$\alpha \frac{R+1}{(R-1)I_{-1}+2} r_1 + (1-\alpha)\frac{(R-1)D_{-1}+2}{(R-1)I_{-1}+2} r_2 > R,$$

$$\lim_{k\to\infty} 2 + R^k\Bigg(RI_{-1} - \alpha r_1 - (1-\alpha)r_2 D_{-1}$$
$$+ 2(R - \alpha r_1 - (1-\alpha)r_2)\sum_{i=1}^{k} R^{-i}\Bigg)$$
$$= 2 + \lim_{k\to\infty} \frac{R^k((R-1)I_{-1}+2)}{R-1}$$

$$\times\left(R-\alpha\frac{R+1}{(R-1)I_{-1}+2}r_1 - (1-\alpha)\frac{(R-1)D_{-1}+2}{(R-1)I_{-1}+2}r_2\right)$$

$$= -\infty.$$

Thus, if (A6) is false, , $I_{2k} + I_{2k+1}$ is negative for k sufficiently large, implying at least one of I_{2k} and I_{2k+1} is negative for k large. That is, there is no nonnegative $\{I_t\}$ that makes constraints (A1) to (A5) true or, equivalently, (4), (5), and (6) true. This, however, is contradictory to the assumption of budget feasibility. Therefore, a budget-feasible steady-state payoff (r_1, r_2) must satisfy (A6). ∎

Proof** of **Proposition 1. To prove necessity, suppose that the payoff (r_1, r_2) is budget feasible. Applying $D_{-1} = I_{-1} = 1$ in Lemma 1 makes (A6) become (7). Therefore, (7) is a necessary condition. To prove the sufficiency, suppose that r_1 and r_2 satisfy (7). It remains to show that there exist nonnegative $\{S_t\}$ and $\{I_t\}$ such that (4), (5), and (6) hold. Take $\{S_t\} = \{0\}$ and $\{I_t\} = \{1\}$. By substituting $D_{-1} = I_{-1} = 1$, $\{S_t\} = \{0\}$, and $\{I_t\} = \{1\}$, we see that (4), (5), and (6) hold. Therefore, (7) is also a sufficient condition. ∎

A.2 Proof of Proposition 2

To show the uniqueness result, suppose that r_1 and r_2 satisfy (7) with equality. From Proposition 1, there exist nonnegative (S_t) and $\{I_t\}$ such that (4), (5), and (6) hold; equivalently, constraints (A1) to (A5) hold. By substituting (7) and $D_{-1} = I_{-1} = 1$, constraints (A1) to (A5) become

$$I_0 \le 1 - S_0,$$

$$I_1 \le 1 + S_0 - S_1,$$

$$\vdots$$

$$I_{2k} \le 1 + \sum_{i=1}^{k} R^{k-i}S_{2i-1} - \sum_{i=0}^{k} R^{k-i}S_{2i},$$

$$I_{2k+1} \le 1 + \sum_{i=0}^{k} R^{k-i}S_{2i} - \sum_{i=0}^{k} R^{k-i}S_{2i+1},$$

where the last two inequalities hold for all $k \in \mathbf{Z}_{++}$ Since $\{I_t\}$ must be nonnegative, the two imply that

$$\sum_{i=0}^{k} R^{k-i}S_{2i+1} - 1 \leq \sum_{i=0}^{k} R^{k-i}S_{2i} \leq \sum_{i=1}^{k} R^{k-i}S_{2i-1} + 1,$$

which in turn implies that for all $k \in Z_{++}$,

$$(R - 1)\sum_{i=1}^{k} R^{k-i}S_{2i-1} + S_{2k+1} \leq 2.$$

If there exists some $j \in Z_{++}$ such that $S_{2j-1} > 0$, then

$$\lim_{k \to \infty} (R - 1)R^{k-j}S_{2j-1} = \infty > 2.$$

This is a contradiction; hence, $S_{2k+1} = 0$ for all $k \in Z_+$. To show $S_{2k} = 0$ for all $k \in Z_+$, the set of nonnegative integers, note that $S_{2k+1}, = 0$ implies that

$$-1 \leq \sum_{i=0}^{k} R^{k-i}S_{2i} \leq 1.$$

If there exists some $j \in Z_+$ such that $S_{2j} > 0$, then

$$\lim_{k \to \infty} R^{k-j}S_{2j} = \infty > 1.$$

This is again a contradiction; hence, $S_{2k} = 0$ for all $k \in Z_+$. Therefore, it must be true that $S_t = 0$ for all t ³ 0.

To show that $I_t = 1$ for all $t \geq 0$, apply (7) with equality and $S_t = 0$ to (A1), (A2), and (A3). It follows that

$$I_0 \leq 1,$$

$$I_1 \leq R(I_{-1} - 1) + 1,$$

$$I_t \leq R(I_{t-2} - 1) + 1,$$

where the last condition holds for all $t \geq 2$. Then $I_{-1} = 1$ and $I_0 \leq 1$ imply that $I_t \leq 1$ for all t ≥ 1. If there exists some $j \in Z_+$ such that I, < 1, by recursively substituting the bound, for $k \in Z_{++}$,

$$I_{j+2k} \leq R^k(I_j - 1) + 1,$$

$$\lim_{k \to \infty} I_{j+2k} \leq R^k(I_j - 1) + 1 = -\infty.$$

This is contradictory to the requirement that $I_t \geq 0$ for all t ≥ 0. Therefore, it must also be true that $I_t = 1$ for all $t \in Z_+$. n

A.3 Proof of Proposition 3

To solve Problem A, first form the Lagrangian:

$$L = \alpha U(r_1) + (1-\alpha)U(r_2) + \lambda_1(R - \alpha r_1 - (1-\alpha)r_2)$$
$$+ \lambda_2(r_2 - r_1) + \lambda_3(r_2 - r_1^2) + \lambda_4(r_2 - R) + \lambda_5 r_1 + \lambda_6 r_2,$$

where λ_i's are the multipliers for constraints (8) to (12). The first-order conditions imply the following system of equations:

$$\alpha U'(r_1) - \lambda_1\alpha - \lambda_2 - 2\lambda_3 r_1 + \lambda_5 = 0, \quad \text{(A7)}$$
$$(1-\alpha)U'(r_2) - \lambda_1(1-\alpha) + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 = 0, \quad \text{(A8)}$$
$$\lambda_1(R - \alpha r_1 - (1-\alpha)r_2) = 0,$$
$$\lambda_2(r_2 - r_1) = 0,$$
$$\lambda_3(r_2 - r_1^2) = 0,$$
$$\lambda_4(r_2 - R) = 0,$$
$$\lambda_5 r_1 = 0,$$
$$\lambda_6 r_2 = 0.$$

To find a solution, suppose first that only (8) and (10) are binding; that is, $\lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = 0$, $\lambda_1 \neq 0$, and $\lambda_3 \neq 0$. Solving (8) and (10) simultaneously gives

$$r_1 = \frac{\sqrt{\alpha^2 + 4(1-\alpha)R} - \alpha}{2(1-\alpha)},$$
$$r_2 = (r_1)^2 = \left(\frac{\sqrt{\alpha^2 + 4(1-\alpha)R} - \alpha}{2(1-\alpha)}\right)^2.$$

Note that r_1 and r_2 as characterized above have $r_2 > R > r_1 > 1$ since

$$r_1 - 1 = \frac{\sqrt{(2-\alpha)^2 + 4(1-\alpha)(R-1)} - (2-\alpha)}{2(1-\alpha)} > 0,$$
$$R - r_1 = \frac{\alpha + 2(1-\alpha)R - \sqrt{(\alpha + 2(1-\alpha)R)^2 - 4(1-\alpha)^2(R-1)R}}{2(1-\alpha)}$$
$$> 0,$$
$$r_2 = \frac{R - \alpha r_1}{1-\alpha} > R.$$

To find λ_1 and λ_3, take $\lambda_2 = \lambda_4 = \lambda_5 = \lambda_6 = 0$ and solve (A7) and (A8) simultaneously:

$$\lambda_1 = \frac{\alpha U'(r_1) + 2(1-\alpha)r_1 U'(r_2)}{\alpha + 2(1-\alpha)r_1} > 0,$$

$$\lambda_3 = \frac{\alpha(1-\alpha)}{\alpha + 2(1-\alpha)r_1}(U'(r_1) - U'(r_2)) > 0,$$

where the last inequality follows from U(·) being strictly concave and $r_1 < r_2$.

The above derivation shows that there exist nonnegative λ_i's and feasible r_1 and r_2, as characterized, which solve the system of equations. Since the objective function is strictly concave in r_1 and r_2 and since in (8) and (10) the functions $\alpha r_1 + (1 - \alpha) r_2 - R$ and $r_1^2 - r_2$ are convex in r_1 and r_2, the Kuhn-Tucker theorem [Ovarian (1984, Fact A.9, p. 322)] tells us that (r_1, r_2) as characterized is indeed a solution to Problem A. The solution is unique because the objective function is strictly concave. Finally, the inequality result follows from the above derivation. ■

A.4 Proof of Proposition 4

To show that the steady-state payoff (r_1^a, r_2^a) is not a competitive equilibrium, recall first from Proposition 2 that $\{S_t\} = 0$ and $\{I_t\} = 1$ are the unique choices that together with (r_1^a, r_2^a) make (4), (5), and (6) true. Given the investment choices, individual agents' maximization problems are restated. Each of the remaining agents of generation -1 solves

$$\max_{\{C_1^{-1}\}} U(C_1^{-1}), \qquad \text{subject to } p_1 C_1^{-1} \le p_1 R + m_{-1}H_{-1},$$
$$C_1^{-1} \ge 0.$$

For agents of generation $t \ge 0$, they each solve

$$\max_{\{C_{t+1}^t, C_{t+2}^t\}} W(C_{t+1}^t, C_{t+2}^t) = \alpha U(C_{t+1}^t) + (1-\alpha) U\left(\frac{p_{t+1}}{p_{t+2}} C_{t+1}^t + C_{t+2}^t\right),$$
$$\text{subject to } p_{t+1}C_{t+1}^t + p_{t+2}C_{t+2}^t \le p_{t+2}R + m_t H_t,$$
$$C_{t+1}^t, C_{t+2}^t \ge 0.$$

In the first problem, $U(C_1^{-1})$ increases in C_1^{-1}, implying

$$C_1^{-1} = \frac{p_1 R + m_{-1}H_{-1}}{p_1}.$$

For the second problem, the first-order conditions imply that

$$\frac{\partial W}{\partial C_{t+2}^t} = (1-\alpha) U'\left(\frac{p_{t+1}}{p_{t+2}} C_{t+1}^t + C_{t+2}^t\right) - \lambda p_{t+2} = 0,$$

$$\frac{\partial W}{\partial C_{t+1}^t} = \alpha U'(C_{t+1}^t) + (1-\alpha)U'\left(\frac{p_{t+1}}{p_{t+2}}C_{t+1}^t + C_{t+2}^t\right)\frac{p_{t+1}}{p_{t+2}} - \lambda p_{t+1}$$
$$= \alpha U'(C_{t+1}^t) > 0,$$

where $\lambda > 0$ is the multiplier for the budget constraint. The above two equations imply that $W(C_{t+1}^t, C_{t+2}^t)$ increases C_{t+1}^t; thus,

$$C_{t+1}^t = \frac{p_{t+2}R + m_t H_t}{p_{t+1}},$$
$$C_{t+2}^t = 0.$$

By Definition 2, if the payoff (r_1^a, r_2^a) is a competitive equilibrium, there must exist some $p_t > 0$ and $m_t \geq 0$ such that for all $t \geq 0$,

$$C_{t+1}^{t-1} + C_{t+1}^t = R.$$

Consider $t = 0$,

$$C_1^0 = \frac{(p_1 + p_2)R + m_{-1}H_{-1} + m_0 H_0}{p_1} = R$$
$$\Rightarrow p_2 R + m_{-1}H_{-1} + m_0 H_0 = 0.$$

But there are no positive p_2 and nonnegative m_{-1} and m_0 that would make the market clearing condition hold. Therefore, there does not exist a competitive equilibrium, with or without money, that achieves the steady-state payoff (r_1^a, r_2^a). ■

A.5 Proof of Proposition 5

For necessity, suppose that (r_1, r_2) is budget feasible. Applying $D_{-1} = I_{-1} = 0$ in Lemma 1 lets (A6) become (13). For sufficiency, suppose that r_1 and r_2 satisfy (13). Choose $\{S_t\}$ and $\{I_t\}$ as in Proposition 5. The choices are feasible because whenever r_1 and r_2 are consistent with (13), $S_t \geq 0$ and $I_t \geq 0$ for all $t \in Z_+$. By taking $D_{-1} = I_{-1} = 0$ and substituting $\{S_t\}$ and $\{I_t\}$ as chosen into constraints (A1) to (A5), the sequential budget constraints are indeed the same as imposing (13). Therefore, (13) is also a sufficient condition and $\{S_t\}$ and $\{I_t\}$ can be as chosen. ■

A.6 Proof of Proposition 6

To solve Problem B, recall first that (15) and (16) are redundant and therefore irrelevant. To find the solution, let us form the Lagrangian:

$$L = \alpha U(r_1) + (1-\alpha)U(r_2) + \mu_1\left(R - \alpha\frac{R+1}{2}r_1 - (1-\alpha)r_2\right)$$
$$+ \mu_2(r_2 - R) + \mu_3 r_1 + \mu_4 r_2,$$

where μ_i's are the multipliers for constraints (14), (17), and (18). The first-order conditions imply the following system of equations:

$$\alpha U'(r_1) - \mu_1 \alpha \frac{R+1}{2} + \mu_3 = 0, \tag{A9}$$

$$(1-\alpha)U'(r_2) - \mu_1(1-\alpha) + \mu_2 + \mu_4 = 0,$$

$$\mu_1\left(R - \alpha\frac{R+1}{2}r_1 - (1-\alpha)r_2\right) = 0,$$

$$\mu_2(r_2 - R) = 0,$$

$$\mu_3 r_1 = 0,$$

$$\mu_4 r_2 = 0.$$

To find the solution, suppose only (14) and (17) are binding; that is, $\mu_3 = \mu_4 = 0$, $\mu_1 \neq 0$, and $\mu_2 \neq 0$. By solving (14) and (17) simultaneously, $r_1 = 2R/(R+1)$, $r_2 = R$, and $r_2 > r_1 >$:. To find μ_1 and μ_2, take $\mu_3, = \mu_4, = 0$ and solve (A9) and (Al0) simultaneously,

$$\mu_1 = \frac{2}{R+1}U'(r_1) > 0,$$

$$\mu_2 = \frac{2(1-\alpha)}{R+1}U'(r_2)\left(\frac{U'(r_1)}{U'(r_2)} - \frac{R+1}{2}\right) > 0,$$

where the last inequality follows from the assumption that agents' relative-risk-aversion coefficient is greater than 1, which implies that

$$\frac{U'(r_1)}{U'(r_2)} > \frac{r_2}{r_1} = \frac{R+1}{2}.$$

Thus there exist nonnegative μ_i'such that r_1 and r_2 as characterized solve the system of equations. Since the objective function is strictly concave in r_1 and r_2, and since in (14) and (17) the functions $\alpha(R+1)/2r_1 + (1-\alpha)r_2 - R$ and $R - r_2$ are convex in r_1 and r_2 the Kuhn-Tucker theorem [Varian (1984, Fact A.9, p. 322)] again tells us that (r_1, r_2) as characterized is indeed a solution to Problem B. The solution is unique because the objective function is strictly concave. ∎

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