Cross-Holdings: Estimation Issues, Biases, and Distortions

Mark Fedenia
James E. Hodder
Alexander J. Triantis
University of Wisconsin-Madison

Cross-holding occurs when listed corporations own securities issued by other corporations. We analyze the effect of cross-holdings on market capitalization and return measures as well as implications for econometric testing of asset pricing theories. We show that cross-holdings generally distort standard market return and risk measures. The magnitudes of such distortions are calculated for simulated economies by using a variety of cross-holding patterns. In addition, cross-holdings are shown to induce nonstationarity in the covariance matrix of security returns. We examine the effect of this nonstationarity for estimating efficient frontiers and factor structures. We also discuss the implications for risk-return estimates in equilibrium asset pricing models.

In some security markets, listed corporations own a substantial fraction of the shares issued by other listed corporations. Notable examples include the

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stock markets in Japan, Germany, and several other European countries.[1] Such *cross-holding* is found, albeit to a lesser extent, in U.S. securities markets as well.[2] It has been suggested that equity cross-holding can significantly distort certain financial measures. For example, McDonald (1989) and French and Poterba (1991) argue that estimates of aggregate equity market capitalization in Japan are inflated due to the *double counting* of cross-held shares.[3] As a consequence, they argue that the weight attached to Japanese shares in world equity market indices is overstated.[4]

We confirm the above results by using a general theoretical model in which firms may hold corporate securities. We then show that equity cross-holding generally distorts the standard market return measure, and we analyze the extent of that distortion. We demonstrate how to adjust the weights in the market portfolio to eliminate the distortion, and show that the standard value-weighted equity market portfolio will generally not be on the mean-variance efficient frontier of equity returns when the true market portfolio is efficient.

Cross-holding also has disturbing econometric implications for both domestic asset pricing tests and tests of global equity market integration. We show that the presence of cross-holding introduces nonstationarity in the covariance matrix of observed returns, which leads to significantly distorted estimates of the efficient frontier. This distortion creates the erroneous perception of a higher price of risk in the cross-held economy and increases the probability of rejecting the mean-variance efficiency of the market portfolio.

In an APT framework, we find that the number of factors for the cross-held economy is generally misestimated. Furthermore, even if the correct number of factors is specified, the estimated factor risk premia are distorted. This implies a tendency toward spurious rejections of APT in the presence of cross-holding. Some of these results are related to the Shanken (1982, 1985) critique of the APT in that they involve estimation problems that arise as a result of repackaging securities into portfolios. In this article, however, there is an additional problem of nonstationarity that compounds the factor identification issues raised by Shanken.

[1]The origins of and rationale behind the cross-holding structure of ownership in Japan have been studied by McDonald (1991), Sheard (1991), and Tachibanaki and Taki (1991), among others.

[2]See Prowse (1990) and Prowse (1992) for a comparison of the extent of aggregate cross-holding in the United States and Japan.

[3]Both these articles as well as Kester and Luehrman (1989) and Ando and Auerbach (1990) also argue that share cross-holding inflates measured price/earnings (P/E) ratios in Japan.

[4]This overstatement appears very substantial. For example, French and Poterba (1991) estimated the standard measure of equity market capitalization for Japan at twice the true market value.

Throughout, we use results from simulations to analyze the severity of the distortions that we examine. These simulations employ a return generator with known properties in economies that are constructed to be identical except for the presence of cross-holdings. This allows us to isolate the effects of cross-holding. In addition, the simulations permit us to examine the effect of a variety of different cross-holding structures. While market return distortions are typically not severe, the impact of nonstationarity induced by cross-holding on tests of asset pricing models may be quite significant.

In the next section, we develop a perfect markets model with cross-held securities and demonstrate several properties of market aggregation in such a setting. Section 2 uses this model to explore the implications of cross-holding for market returns. Section 3 analyzes econometric problems induced by cross-holding. Finally, Section 4 provides concluding comments.

1. Corporate Cross-Holding and Market Aggregation

To focus on issues that arise solely because of cross-holding, we assume that markets are complete and that there are no transaction costs or taxes. For the moment, we assume that firms are financed entirely by equity. Debt and other corporate securities are incorporated into the analysis in Appendix A. The stock market consists of shares in N firms, at least some of which hold shares of other listed firms. We treat unlisted business entities as if they were individuals.[5]

Let S_i denote the total market value of all outstanding shares issued by firm i, and let V_i^* represent the (market) value of that firm's operating assets (i.e., the firm's assets excluding its holdings of other firms' securities). By value additivity, firm i's total value will equal the sum of its operating assets and its ownership of shares in other firms.[6] That is,

$$S_i = V_i^* + \sum_j h_{ij} S_j \quad \forall i, \tag{1}$$

where h_{ij} is the fractional ownership of firm j's shares by firm i. In much of what follows, it will be useful to employ matrix notation. Consequently, we define an $n \times n$ cross-holding matrix, H, with $ijth$

[5] We do not require share ownership across corporations to be symmetric. That is, firm a may own shares in firm b regardless of whether firm b owns shares in firm a.

[6] See DeMarzo (1988) and Duffie (1988) for an extension of the Modigliani-Miller theorem to a stochastic economy where firms are allowed to trade all available securities.

element equal to h_{ij}.[7] Letting V^* be the vector of V_i^* values, we can represent the vector of S_i values by S, where

$$S = V^* + HS. \tag{2}$$

Summing (2) over all firms, we obtain

$$\iota' S = \iota'(V^* + HS), \tag{3}$$

where ι denotes a conformable unit vector. The left-hand side of (3) is the standard measure of equity market capitalization. Clearly, this market capitalization overstates the aggregate value of operating assets ($\iota' V^*$) by the extent of aggregate corporate shareholdings.

It is useful to distinguish between "inside" and "outside" shareholdings. Outside shares are those held by individuals, whereas inside shares refer to shareholdings by listed corporations. Using the superscripts I and O to designate inside and outside holdings, respectively, we can rewrite (2) as

$$S^O + S^I = V^* + HS. \tag{4}$$

Also, (3) can be rewritten as

$$\iota' S^O + \iota' S^I = \iota' V^* + \iota' HS. \tag{5}$$

Since $\iota' S^I = \Sigma_i S_i^I = \Sigma_i \Sigma_j h_{ji} S_i = \iota' HS$, iit follows that the aggregate value of outside shares equals the aggregate value of operating assets; that is, $i' S^o = i' V^*$.[8] We should emphasize that this equality holds in the aggregate, but not necessarily at the individual firm level. That is, the vector S^o will not generally equal V^*, and similarly $S^I \neq HS$. For example, a holding company whose shares are held by outsiders may have no operating assets.

We will henceforth refer to the aggregate value of outside shares as the true market capitalization.[9] One might be tempted to view the distinction between the true market capitalization and the standard measure of market capitalization as being primarily philosophical. Note, however, that the standard measure can be artificially inflated

[7] Note that we have defined this matrix to include potential cross-holdings of all N firms; however, our use of outstanding shares implies $h_{ii} = 0$. If a firm does not hold any shares in other firms and $h_{ii} = 0$. its row in the H- matrix will contain only zeros. Similarly, if a firm's shares are not held by other corporations and $h_{ii} = 0$. its column in the H- matrix will contain only zeros. Hence, it is possible (even likely) that the H- matrix is singular.

[8] One can also approach the issue of aggregate market value from an earnings perspective. In the aggregate, outside shareholders receive the total operating earnings while net payments to inside shareholders sum to zero.

[9] Brioschi, Buzzacchi, and Colombo (1989, 1991) also provide a detailed analysis of market capitalization in the presence of cross-holdings, where they use the terms gross value and net value instead of standard and true market capitalization, which we employ here. They note that their analysis [which includes equations similar to our Equations (1)-(5)] is analogous to a Leontief-type input-output model.

(without limit) by firms issuing shares to each other. This could even be accomplished by simply exchanging shares (in equivalent values) without any cash or other assets changing hands. This implies that analyses based on the standard measure of market capitalization can be seriously misleading, at least with regard to the value of operating assets.

It is also interesting to consider what constitutes the *market* in the context of an equilibrium asset pricing model (e.g., the CAPM). In such models, there is a constraint that individuals in the aggregate purchase all available shares. For our situation, this clearly corresponds to purchasing all the outside shares, since by definition inside shares are not held by individuals. Consequently, the market portfolio in such asset pricing models has weights based on the value of outside (rather than total) shares.

In Appendix A, we extend the analysis of this section to include all corporate securities (i.e., not just equity shares) that may be held by individuals as well as by other corporations. The market in this more general setting is based on weights determined by the outside holdings of all classes of securities.

2. Returns with Cross-Holding

We now consider how individual security returns as well as market returns are affected by cross-holdings. Section 2.1 presents theoretical propositions related to market returns. In Section 2.2, we examine the economic significance of these results by using numerical simulations.

2.1. Analytical properties

To keep the notation as concise as possible, we will express returns [shown in (7) and (8)] using X_i and Y_i, which denote, respectively, the operating and total earnings of firm i These are economic earnings that implicitly include any capital gains on firm i's operating assets as well as on its shareholdings. Firm i's operating earnings plus its proportional share of the earnings of the firms which it holds is

$$Y_i = X_i + \sum_j b_{ij} Y_j \quad \forall i. \tag{6}$$

The return on firm i's shares can thus be represented as

$$R_i = \frac{Y_i}{S_i} = \frac{X_i + \Sigma_j b_{ij} Y_j}{V_i^* + \Sigma_j b_{ij} S_j}. \tag{7}$$

In the absence of cross-holding, the return on firm i's equity would simply be equal to the return on its operating assets; that is,

$$R_i^* = X_i / V_i^*. \tag{8}$$

Clearly, R_i will not generally equal R_i^*. This should be no surprise, since shareholders of firm i now effectively own a portfolio of V_i^* and inside shares. This is particularly evident if we use the expression for R_i^* in (8) to rewrite (7) as

$$R_i = \frac{V_i^*}{V_i^* + \Sigma_j b_{ij} S_j} R_i^* + \sum_j \left(\frac{b_{ij} S_j}{V_i^* + \Sigma_j b_{ij} S_j} \right) R_j. \tag{9}$$

Alternatively, we can view each firm as holding a portfolio of the underlying operating assets in the economy. In fact, the N- vector of observed firm share returns can be concisely expressed as $R = PR^*$, where R^* is the N- vector of returns on the operating assets while P is a weighting matrix with elements $p_{ij} = q_{ij} V_j^* / \Sigma_j q_{ij} V_j^*$ and $Q = (I - H)^{-1}$ with q_{ij} denoting its elements. To see this, reexpress (6) in vector form as $Y = X + HY$, which simplifies to $Y = QX$; similarly, $S = QV^*$ from (2). Equation (7) can thus be expressed as $R_i = \Sigma_j q_{ij} X_j / \Sigma_j q_{ij} V_j^*$. Using $X_j = R_j^* V_j^*$ from (8), we obtain the expression above for p_{ij}.

The *true* market return (denoted by R^*_m) is the aggregate return on the operating assets in the economy.[10] Hence, we divide aggregate operating earnings by the true market value and obtain $R^*_m = \iota' X / \iota' V^*$. Alternatively, we can use a weighted average of R^*_i values across all firms to obtain

$$R_m^* = w^{*\prime} R^* = \iota' X / \iota' V^*, \tag{10}$$

where w^* is a vector of weights with elements $w^*_i = V^*_i / \iota' V^*$. Similarly, we write the value-weighted *standard* equity market return based on all issued shares (with $w_i = s_i / \iota' s$) as

$$R_m = w' R = \iota' Y / \iota' S = \iota' QX / \iota' QV^*. \tag{11}$$

Examining (10) and (11), it is clear that, in general, the standard equity market return in a cross-held economy will not equal the true market return. We discuss the difference between these two return measures below, and also derive a very restrictive condition that is necessary and sufficient for these two measures to be equal. Notice first that calculating R^*_m may be difficult in practice. The values of R^*_i cannot be readily measured by using (8), because V^*_i and X_i may not be directly observable. Fortunately, if disaggregated holdings are observable, it is possible to calculate R^*_m by using R_i (instead of R^*_i) values with an adjusted weighting scheme.[11]

[10] Recall that earnings on inside shares aggregate to zero.

[11] It should be emphasized that the calculation of R^*_m using Proposition 1 requires reweighting individual firm returns. That is, fully disaggregated data are required. Consequently, estimating

Proposition 1. *$R^*_m = \Sigma_i w^o_i R_i$, where $w^o_i = S^o_i / S^o_m$ and $S^o_m = \Sigma_i S^o_i$.*

To prove this proposition for the all-equity case we can proceed as follows:

$$\sum_i w^o_i R_i = \frac{1}{S^o_m} \sum_i S^o_i \frac{Y_i}{S_i}. \tag{12}$$

But $S^o_i Y_i / S_i$ is simply Y^o_i, the aggregate earnings on firm *i's* outside shares, so

$$\sum_i w^o_i R_i = \frac{\Sigma_i Y^o_i}{\Sigma_i S^o_i}. \tag{13}$$

We have shown in the previous section that $\iota' S^o = \iota' V^*$ and mentioned that, by similar arguments, $\iota' Y^o = \iota' X$. Using vector notation, we thus have

$$w^{o\prime} R = \frac{\iota' Y^o}{\iota' S^o} = \frac{\iota' X}{\iota' V^*} = R^*_m. \tag{14}$$

A corollary to Proposition 1 that considers both equity and nonequity corporate securities, and its proof, is contained in Appendix A.

The weighting scheme specified in Proposition 1 weights each firm's return by the proportion going to outsiders. The aggregate of outsiders' returns must equal their share of total earnings divided by their total holdings. Since in the aggregate outside shareholders receive the total operating earnings and hold shares with a total value equal to that of operating assets, the specified weighting scheme effectively eliminates the distortion caused by inside shareholding. Alternatively, by our observation in Section 1, the market portfolio must have portfolio weights based on the value of outside shares. Proposition 1 demonstrates that the true market return based on the values of operating earnings and operating assets is equivalent to the return for the value-weighted portfolio of all outside shares.

Using Proposition 1, we can derive a necessary and sufficient condition to guarantee the equality of R_m and R^*_m.

Proposition 2. *The standard equity market return (R_m) is equivalent to the true market return (R^*_m) for all V^* and X if and only if each firm has the same proportion of its shares held by insiders (i.e., the sum of each column of H must be identical).*

A proof is contained in Appendix A along with a corollary that extends this result to an economy with many classes of corporate

R^*_m by using partially disaggregated data. such as returns on industry portfolio indices, will generally be inaccurate because returns on such subindices may themselves be distorted by cross-holding.

securities. In general, we would not expect each firm to have an identical fraction of its shares held by insiders, and thus deviations between R_m and R^*_m should be expected. The following proposition provides a simple relationship between R_m and R^*_m that clarifies how inside shareholdings distort standard equity market return measures. This proposition is proved in Appendix A.

Proposition 3.

$$R_m - R^*_m = \frac{1}{(1 - h^T)S_m} \sum_i (h_i - h^T)S_i(R_i - \bar{R}),$$

where $\bar{R} = (1/N) \sum_i R_i$, h_i is the proportion of firm i's shares held by insiders, and h^T is an aggregate measure of the percentage of the standard market capitalization owned by insiders; i.e., $h^T = \sum_i h_i S_i / \sum_i S_i$.

It is clear from the above that R_m is distorted because it overweights returns of firms where the insiders' proportion of shares is higher than the cross-sectional average.[12] If firms that have a large proportion of shares held by insiders also have higher returns than the cross-sectional average return, then R_m will exceed the true market return. The expression in Proposition 3 is analogous to a weighted covariance calculation. In fact, if all firms were to have equal capitalizations *(S_i's),* then this expression simplifies to

$$R_m - R^*_m = \frac{1}{(1 - h^T)} \widehat{\mathrm{Cov}}(h, R),$$

where

$$\widehat{\mathrm{Cov}}(h, R) = \frac{1}{N} \sum_i (h_i - h^T)(R_i - \bar{R})$$

is the estimated cross-sectional covariation between the proportion of a firm's shares held by insiders and that firm's return. Note that as h^T, the aggregate cross-holding measure, increases toward unity, the market return distortion could become quite sizable even for small observed covariances. We will investigate the magnitude of this distortion in greater detail in Section 2.2.

Since R_m will generally not equal R^*_m, it is worthwhile considering

[12] Note that the reciprocal of R_m is the aggregate price/earnings ratio for the total market using economic rather than accounting earnings. Similarly, the reciprocal of R^*_m represents the aggregate P/E ratio after eliminating the distortionary effects of cross-holdings. Since R_m can be either greater or less than R^*_m, it is clear that cross-holding can distort the economic P/E ratio either down or up. This contrasts with results in the articles mentioned in note 3, which found the P/E in Japan to be biased upward. The difference is that our analysis uses economic returns in contrast to accounting returns in the other articles.

the mean-variance efficiency of the standard equity market portfolios—that is, a portfolio using w as the weighting vector. In the context of the CAPM, linear combinations of the market portfolio and the riskless asset will determine the capital market line (CML). By construction, only a single portfolio composed solely of risky securities lies on the CML. As explained in Section 1, the market portfolio for the CAPM must be what we have termed the true market portfolio. All other portfolios containing only risky securities lie below the CML; in other words, they have a lower ratio of expected excess return to standard deviation of return. Thus, the standard market portfolio with cross-holding will generally be inefficient relative to the CML. Furthermore, when the true market portfolio lies on the mean-variance efficient frontier of risky securities (as required by the CAPM), the standard equity market portfolio will also lie on that frontier only under very restrictive conditions.

Proposition 4. *Assume the true market portfolio is mean-variance efficient. For a given nonsingular covariance matrix of equity returns in the cross-held economy* (Ω), *the value-weighted portfolio of all shares (standard equity market portfolio) is also mean-variance efficient if and only if* $w - w^o_i = \lambda\,(a_i - w^o_i)$ *for all i, where* λ *is a constant and* a_i *is the weight for the ith security in the minimum-variance-efficient portfolio. Further,* $a_i = \Sigma_j v_{ij} / \Sigma_i \Sigma_j v_{ij}$ *for all i, where* v_{ij} *is the ijth element of* Ω^{-1}.

The proof of this proposition is in Appendix A. In general, the condition specified in Proposition 4 is unlikely to be satisfied (an exception clearly being when w equals w^o). Consequently, the standard equity market portfolio with cross-holding will be mean-variance inefficient in the context of the CAPM as well as being below the CML. In Section 3 we will examine additional problems that arise in the presence of cross-holding when the efficient frontier must be *estimated* with historical data.

2.2 Simulation results

We now examine simulation results that allow us to infer plausible magnitudes for the discrepancy between standard and true market return measures in economies with different cross-holding structures. Simulations are used in lieu of actual data since we can more accurately compare two economies that are constructed to be identical *except* for the presence of cross-holding. This allows us to avoid a host of confounding complications, such as those introduced by economic regulations or other market imperfections, exchange rate fluctuations, and possible nonstationarity of risk premia or return distributions.[13] Furthermore, a return generator with known properties may be used. As we will see, this advantage is quite important.

The results reported below involve simulations of eight cross-holding patterns, each at two levels of aggregate cross-holding (h^T equaling 0.2 and 0.5).[14] That is, there are 16 simulated types of cross-held economies used in our analysis. For each simulation type, the reported results are based on 100 runs using N = 50 firms, T= 300 time periods (months), and a return generator described in Appendix B. For each run, we calculate a variety of statistics for an *underlying* economy and a *cross-held* economy. These two economies have identical patterns of operating assets and operating incomes; however, in the underlying economy all shares are held by outside investors, whereas in the cross-held economy some shares are held by insiders (as specified by the particular H matrix for that economy). Each run in a simulation uses a new H matrix randomly generated according to specific rules for that type of economy. The eight patterns (or classes) of H matrices are described briefly below and in more detail in Appendix B.

In a *dense H* matrix essentially all the elements are generated to be nonzero. In other words, all firms tend to hold (generally small) positions in the shares of essentially all other firms in the economy. In contrast to this rather extreme cross-sectional dispersion of interfirm shareholdings, we generate several types of H matrices where a substantial fraction of elements is zero. The sparse matrices are generated to have 40 percent of their elements equal to zero. In the *sparse-columns* case, there is a 40 percent probability for each column that its column sum is equal to zero (and thus so is each entry in the column). In the sparse-rows case there is a 40 percent probability for each row that its row sum equals zero. These last two cases allow us to observe whether economies in which many firms either are completely owned by outside investors or, respectively, hold no equity stakes in other firms lead to different results.

Block-diagonal matrices have cross-holdings only within blocks (groups) of firms in the economy. This classification is used to simulate the situation where firms within a corporate group (e.g., the Keiretsu in Japan) are closely knit through mutual ownership but ownership across groups is relatively insignificant. In these simulations, there are five blocks (groups) with 10 firms each and the within-block cross-holdings are generated using the *dense* pattern.

[13] In addition, available data on cross-holding in major economies are incomplete. For example, Japanese data (which appear to be the most comprehensive) provide disaggregated information on roughly the top 10 to 25 shareholders for each firm. However, this frequently represents half or less of the firm's cross-held shares.

[14] The h^T value of 0.5 corresponds to the aggregate cross-holding level in Japan estimated by French and Poterba (1991). whereas h^T = 0.2 is used to represent a more moderate level of cross-holding.

The *concentrated* matrices are similar to the sparse matrices, with 90 percent instead of 40 percent of the elements equal to zero, leading to an increased concentration of corporate ownership by other corporations. The final two patterns considered are designed to illustrate the result in Proposition 3. The *risk-correlated* matrices are constructed so that firms with large (small) expected returns also have a large (small) percentage of their shares held by insiders. The *risk* and *size-correlated* matrices have the additional restriction that firm size and risk are negatively correlated.

In Table 1 we report results of our simulations for the market return measures. In each run (100 per simulation type), monthly market returns are calculated for both the underlying economy and the cross-held economy. Both value-weighted and equally weighted market returns are calculated. The value-weighted returns calculated in the underlying and cross-held economies correspond to the true market return, R^*_m, shown in (10) and the standard market return, R_m, shown in (11), respectively. We also examined the extent of distortion in equal-weighted return measures because of their widespread use in investment analysis.

The mean and standard deviation of the difference between the 300 monthly market returns in the two economies were measured for each of the 100 runs per simulation. We report the average of these 100 means and standard deviations as the result for that simulation type in Table 1. Below each average mean, we report the fraction of means that have the opposite sign from the average mean. This provides a measure of the statistical significance of the average mean differing from zero since the reported fraction of means represents the probability mass in the empirical distribution's tail.

The reported mean deviations between R_m and R^*_m are generally small and not significantly different from zero. This is true for both value-weighted and equal-weighted market returns as well as for moderate (h^T = 0.2) and rather substantial (h^T = 0.5) aggregate cross-holding levels.[15] The notable exception to this general result occurs when the pattern of cross-sectional cross-holding is correlated with returns. This is seen in the risk-correlated and risk- and size-correlated columns, where the reported means are highly significant although still rather small. The largest mean difference is -0.0015 for monthly returns or -1.8 percent on an annualized basis.

Despite the small mean differences, substantial differences occur for individual observations. This is particularly true for the equal-weighted market index, where the average standard deviation reported

[15]In addition, although not reported here, we found the correlation between R_m and R^*_m for both value- and equal-weighted indices to be generally larger than 0.99.

Table 1
Cross-holding-induced distortions in market return measures

	Dense	Sparse	Sparse columns	Sparse rows	Block diagonal	Concentrated	Risk correlated	Risk-size correlated
				A. $b^T = 0.2$, value-weighted				
Mean	0.0000	0.0000	−0.0000	−0.0000	0.0000	−0.0000	0.0003	−0.0014
	(0.41)	(0.39)	(0.52)	(0.47)	(0.49)	(0.43)	(0.00)	(0.00)
Avg. st. dev.	0.0011	0.0013	0.0027	0.0011	0.0011	0.0015	0.0038	0.0170
Risk prem. bias	−0.0004	0.0002	−0.0007	−0.0001	−0.0001	−0.0003	−0.0000	0.0014
	(0.40)	(0.36)	(0.36)	(0.50)	(0.46)	(0.40)	(0.53)	(0.49)
				B. $b^T = 0.2$, equal-weighted				
Mean	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0006	−0.0005
	(0.45)	(0.40)	(0.50)	(0.41)	(0.45)	(0.50)	(0.00)	(0.01)
Avg. st. dev.	0.0069	0.0067	0.0077	0.0048	0.0061	0.0059	0.0098	0.0055
Risk prem. bias	−0.0052	−0.0037	−0.0058	−0.0028	−0.0038	−0.0039	−0.0043	−0.0035
	(0.20)	(0.33)	(0.10)	(0.18)	(0.16)	(0.22)	(0.22)	(0.13)
				C. $b^T = 0.5$, value-weighted				
Mean	−0.0000	−0.0000	0.0000	−0.0000	0.0000	0.0000	0.0007	−0.0015
	(0.48)	(0.51)	(0.46)	(0.50)	(0.44)	(0.43)	(0.00)	(0.00)
Avg. st. dev.	0.0026	0.0023	0.0042	0.0027	0.0025	0.0024	0.0068	0.0167
Risk prem. bias	−0.0004	−0.0003	−0.0006	−0.0005	0.0004	0.0003	0.0007	0.0006
	(0.46)	(0.54)	(0.48)	(0.53)	(0.44)	(0.45)	(0.30)	(0.50)
				D. $b^T = 0.5$, equal-weighted				
Mean	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0010	−0.0009
	(0.43)	(0.40)	(0.37)	(0.35)	(0.49)	(0.46)	(0.09)	(0.00)
Avg. st. dev.	0.0108	0.0106	0.0112	0.0065	0.0093	0.0100	0.0148	0.0097
Risk prem. bias	−0.0058	−0.0055	−0.0066	−0.0033	−0.0051	−0.0045	−0.0049	−0.0046
	(0.21)	(0.23)	(0.15)	(0.28)	(0.24)	(0.22)	(0.18)	(0.23)

The mean and standard deviation of the difference between 300 monthly market returns in underlying and cross-held economies ($R_m - R_m^*$) were measured for each of 100 simulation runs (with 50 firms in the economy). The average of each of these across the 100 runs is reported as ***mean*** and ***avg. st. dev.***, respectively. ***Risk prem. bias*** measures the average difference between the ratio of the mean excess market return to the standard deviation of the market return in the cross-held and underlying economies. All these results are reported for two aggregate cross-holding levels ($b^T = 0.2$ and 0.5) and eight different cross-holding patterns. Below the ***mean*** and ***risk prem. bias*** values, we give the fraction of observations with the opposite sign from the reported mean (or bias). This represents the probability mass in the empirical distribution's tail and thus provides a significance measure for the mean (or bias) differing from zero.

is much larger than for the value-weighted returns.[16] The simulated distribution of market return differences is approximately normal; hence, an average standard deviation of 0.0069 for equal-weighted monthly returns in the dense case with $h^T = 0.2$ suggests that approximately one-third of the time the measured market return would deviate from the true return by at least 2.5 percent on an annualized basis.

We also examine the effect of cross-holding on the ratios of mean excess market return to standard deviation of market return in the cross-held and underlying economies. This ratio is frequently used as an estimate of the market risk premium. In Table 1, we report the average difference between these ratios across 100 runs under the label of *risk premium bias*. The results indicate that in the value-weighted case there is little systematic difference in the two market risk-premium estimates. For the equally weighted case, there is weak evidence of a downward bias. The risk premium in the underlying economy is approximately equal to 0.1, and thus the estimated biases are only 2 to 8 percent of this value and are not highly statistically significant. Thus, we find only a weak indication that comparing risk premia (calculated in this manner) between economies with differing cross-holding structures could potentially lead to inappropriate conclusions (e.g., rejection of the equality of risk premia). However, as we shall see shortly, other approaches to estimating risk premia across markets are much more sensitive to cross-holding.

3. Problems Associated with the Covariance Matrix of Cross-Held Returns

We have noted that firms in a cross-held economy consist of operating as well as financial assets, and thus their returns can be expressed as portfolio returns. In particular, the vector of firm returns in such an economy can be expressed as $R = PR^*$. R^* is the vector of returns on operating assets, and the weights in firm *i's* portfolio are the entries in the *ith* row of *P*. In this section, we discuss implications of this result for econometric estimates that are based on observed security returns in cross-held markets. We identify problems in estimating the efficient frontier (in Section 3.1) as well as in using factor analysis and testing the APT paradigm (in Section 3.2). International empirical

[16] The larger distortions for the equally weighted index result from its placing more weight on small firms' returns. In our simulations, except for the risk- and size-correlated column, shares are equally likely to be held by any firm. If small firms in the underlying economy hold even small equity stakes in firms with large operating assets, these stakes will have a profound effect on the small firms' returns, resulting in an accentuated deviation of the index return from that for the underlying economy.

studies are particularly susceptible to the problems we outline given the extent of cross-holdings in many foreign markets.[17]

3.1 Mean-variance analysis

Recall from the previous section that the *ijth* element of P is $p_{ij} = q_{ij}V^*_j / \Sigma_j q_{ij}V^*_j$. Thus, the weights in *firm's portfolio* will change each period as the V^*_j values change over time. We will illustrate that this nonstationarity of the elements in P can induce significant distortion in econometric estimates, particularly through its effect on the covariance matrix of security returns. We will continue to assume that the elements of H, and thus of $Q = (I - H)^{-1}$, are constant through time. That is, firms do not trade the securities that they hold and do not issue new securities. If this assumption were relaxed, the nonstationarity of the covariance matrix would likely become more pronounced, further distorting econometric estimates.

The covariance matrix of security returns plays an important role in the application, as well as the testing, of asset pricing theories. For example, this matrix together with the vector of expected security returns determine an economy's investment opportunity set and the efficient portfolios that lie on the frontier of this set. A simple linear transformation of security returns by creating portfolios (which become the new securities) does not affect the ex ante efficient frontier since these portfolios can be unbundled to recover the underlying securities. However, if the portfolio weights are not constant through time, the linear relationship between mean portfolio returns and mean underlying returns will not transform the estimated covariance matrix of portfolio returns into the covariance matrix of the returns on the underlying securities. Thus, the estimated covariance matrix and mean return vector in a cross-held economy may yield a significantly different estimate of the efficient frontier than the one that would be obtained by using returns from the underlying economy. This result is formalized in the following proposition, whose proof is in Appendix A.

Proposition 5. *Let $\bar{P}$ denote the time-series average (element by element) of the P matrices over an estimation horizon T, and use the time-series average return as the estimate of E(R). Then*

$$\text{plim}\,\frac{1}{T}\sum_t R_t = \bar{P}E(R^*),$$

[17] In addition to single-country studies involving cross-held markets. a substantial number of recent articles attempt to test international financial market integration by comparing estimated prices of risk across different stock markets. Stehle (1977), Jorion and Schwartz (1986). Solnik (1974). Errunza and Losq (1985). Korajczyk and Viallet (1989), and Harvey (1991) employ a CAPM framework. Obstfeld (1966) and Wheatley (1988) employ the CCAPM. Studies by Cho. Eun, and Senbet (1986), Gultekin. Gultekin, and Penati (1989). and Korajczyk and Wallet (1989) use the APT.

where R_t denotes the vector of returns in the cross-held economy at time t. However, plim $\hat{\Omega} > \bar{P}\Omega^*\bar{P}'$, *where*

$$\text{plim}\,\hat{\Omega} = \text{plim}\,\frac{1}{T}\sum_t (R_t - \bar{R})(R_t - \bar{R})'.$$

This proposition says that the estimated covariance matrix for the cross-held economy will be larger (element by element) than that yielded by the linear transformation of the underlying returns process using the average weighting matrix $(\bar{P})$. However, $\bar{P}$ is the matrix that transforms the underlying return process to match (in the limit) the estimated mean return for the cross-held economy. Thus, the standard estimates of $E(R)$ and Ω will yield an (estimated) efficient frontier for the cross-held economy that differs from that for the underlying economy. In fact, given that plim $\hat{\Omega} > \bar{P}\Omega^*\bar{P}'$, we expect to see a particular bias in the shape of the estimated efficient frontier in the cross-held economy relative to the underlying economy.

This bias in the shape of the frontier can be analyzed by examining the effect of cross-holding on the efficient set constants that characterize the efficient frontier. In the cross-held economy these constants are

$$A = \sum_j \sum_k v_{kj}(E(R_j) - R_f),$$

$$B = \sum_j \sum_k v_{kj}(E(R_j) - R_f)(E(R_k) - R_f),$$

$$C = \sum_j \sum_k v_{kj}.$$

In these expressions, R_f denotes the risk-free rate of return and v_{ij} was previously defined as the ijth element of Ω^{-1}. Analogous constants $(A^*, B^*,$ and $C^*)$ exist for the underlying economy, using $E(R^*)$ and Ω^*, or equivalently using $\bar{P}E(R^*)$ and $\bar{P}\Omega^*\bar{P}'$. Given our result that estimated variances and covariances will be larger in the cross-held economy, we should expect estimates with $C < C^*$. The variance of the global minimum-variance portfolio in the respective economies is equal to $1/C$ or to $1/C^*$. Consequently, the global minimum-variance point will appear to be closer to the expected return axis in the underlying economy than in the cross-held economy. Intuitively, it is harder to reduce portfolio variance when the covariances between firms' equity returns are larger. However, as we move up the frontier to portfolios with higher expected returns, we should eventually obtain portfolios using the cross-held returns, which appear to have lower variance than equivalent expected return portfolios from the underlying economy. This occurs because the larger estimated covariances in the cross-held economy can now be multiplifed by negative weights for an increasing number of pairs of firms. We should therefore obtain less apparent curvature (i.e., a more open frontier) by using the returns in a cross-held economy.

To verify this predicted distortion, we use results from the simulations discussed in Section 2.2 to plot the two efficient frontiers and to obtain related statistics. In Table 2 we provide the efficient set constants A, B, and C for the 16 simulations (eight patterns times two levels of cross-holding).[18] We then report the following biases: $A - A^*$, $B - B^*$, and $C - C^*$, where these numbers are based on the average difference in the constants over 100 runs in each simulation. As predicted, the efficient set constant C is strongly (and significantly) downward biased when estimated with the cross-held returns. We also find that B is strongly upward biased (we will return to this later), and A is weakly downward biased.[19]

Figure 1 illustrates the distortion in the efficient frontiers. These plots are based on the efficient set constants in Table 2 for the dense cross-holding matrix case with $h^T = 0.5$. In addition, the figure illustrates the tangent line (emanating from the origin) for each of the two frontiers. The figure clearly illustrates that one appears to obtain a much higher maximum ratio of return to risk (as measured by standard deviation of return) in the cross-held economy than in the underlying economy. We have calculated the slope of the tangent line, often referred to as the *Sharpe ratio,* in each of the two economies. Jobson and Korkie (1982) show that this slope is equal to $\sqrt{B}$. We have calculated the average of the 100 estimates of $\sqrt{B}$ for each simulation by using the underlying and cross-held returns and adjusting for a bias in the estimate.[20] Table 2 shows the average Sharpe ratio estimate in the underlying economy (over 100 runs) as well as the average difference between the Sharpe ratio in the cross-held and underlying economies, namely the bias induced by cross-holding. The size of the bias is large and statistically significant for each cross-

[18] All reported estimates have been adjusted for the biases arising from the small sample estimation procedure. These bias corrections can be found in Jobson and Korkie (1980). As in Jobson and Korkie (1982), we have excluded from our sample cases in which either of the two global minimum mean returns, A/C or A^*/C^*, is less than zero. Merton (1972) discusses the consequences of such cases. We have also analyzed whether discarding these observations from our sample biases our estimation procedure in any significant way. Leaving these cases in the sample decreases the value of the estimates of A^*, A, B^*, B, C^*, and C somewhat, but has a negligible impact on the differences $A - A^*$, $B - B^*$, and $C - C^*$.

[19] We also find, but do not report in Table 2, that the expected excess return on the global minimum variance portfolio, A/C, is significantly upward biased when using the cross-held returns (all 100 differences were positive in each of the 16 cases). While this indicates an upward shift in the efficient frontier, the shift is less than one basis point on average.

[20] The bias $E[\sqrt{\hat{B}}] - \sqrt{B}$ can be approximated by a Taylor series expansion as $\sqrt{B + N/T} - \sqrt{B} - \frac{1}{8}[(B + N/T)^{-3/2}]\,\mathrm{Var}(\hat{B})$ where the expression for $\mathrm{Var}(\hat{B})$ is given in Equation (2.4) in Jobson and Korkie (1980). B is the true value of the constant as calculated based on the parameters of the return generator.

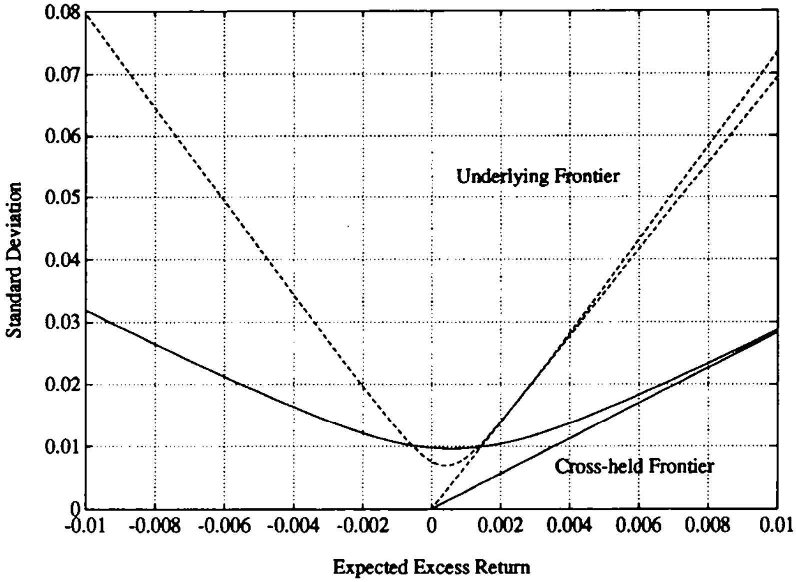

Figure 1.
Efficient frontiers in the underlying and cross-held economies
The efficient frontiers are based on efficient set constants found in Table 2 for the *dense* cross-holding case with $b^T = 0.5$. The constants are calculated from 300 monthly returns for 50 firms in each of 100 simulation runs. The tangency lines to the frontiers are also shown in the figure.

holding pattern and increases slightly with the aggregate cross-holding level.

This apparent bias in the Sharpe ratio does not imply that there is necessarily a greater price of risk in an economy that has cross-holding than in one that does not. Clearly, the result we obtain is a statistical artifact by reason of the nonstationarity of the covariance matrix in the cross-held economy coupled with an optimization procedure that attempts to exploit an apparent difference in the covariance structure. Our finding suggests, however, that one must be careful in interpreting results from market integration tests that compare the estimated market prices of risk from economies with different cross-holding structures.

Given the shapes of the two efficient frontiers illustrated in Figure 1 (and the efficient set constants in the two economies in Table 2), it is reasonable to conjecture that empirical tests of the mean-variance efficiency of the market portfolio would be more likely to reject efficiency in an economy with cross-holding than in one without cross-holding.[21] Using results from our simulations, we measure the dif-

[21]While we refer above to the market portfolio for an economy, our analysis of the efficient frontier distortion applies to portfolios generated from any given set of assets, which may or may not comprise all assets in the market.

Table 2
The effect of cross-holdings on efficient set constants, the Sharpe ratio, and mean-variance efficiency tests

	Dense	Sparse	Sparse columns	Sparse rows	Block diagonal	Concentrated	Risk correlated	Risk and size correlated
				A. $b^f = 0.2$				
A^*	9.6208	9.8439	8.7741	9.3965	9.7921	10.4556	9.5859	8.8443
B^*	0.0139	0.0155	0.0187	0.0172	0.0257	0.0168	0.0155	0.0144
C^* ($\times 10^{-4}$)	2.1254	2.1444	2.0667	2.1234	2.0695	2.0156	2.0576	2.0336
$A - A^*$	−1.8475	−2.9337	−2.2414	−0.7951	−2.9677	−3.3631	−2.4371	0.7097
	(0.37)	(0.35)	(0.38)	(0.37)	(0.25)	(0.26)	(0.42)	(0.35)
$B - B^*$	0.0962	0.1019	0.0734	0.0929	0.0718	0.0602	0.0774	0.1222
	(0.15)	(0.10)	(0.20)	(0.13)	(0.19)	(0.20)	(0.17)	(0.06)
$C - C^*$ ($\times 10^{-4}$)	−0.7663	−0.7966	−0.7150	−0.4430	−0.7063	−0.6254	−0.9149	−0.4502
	(0.00)	(0.00)	(0.00)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe ratio	0.1059	0.1079	0.1117	0.1098	0.1192	0.1094	0.1080	0.1064
Sharpe ratio bias	0.0410	0.0428	0.0303	0.0419	0.0330	0.0282	0.0309	0.0532
	(0.07)	(0.04)	(0.12)	(0.10)	(0.10)	(0.12)	(0.11)	(0.02)
Index inefficiency (underlying)	0.0105	0.0119	0.0166	0.0145	0.0234	0.0119	0.0121	0.0122
	(0.35)	(0.33)	(0.27)	(0.29)	(0.25)	(0.34)	(0.31)	(0.33)
Index inefficiency (cross-held)	0.0518	0.0545	0.0477	0.0564	0.0565	0.0405	0.0430	0.0640
	(0.08)	(0.09)	(0.06)	(0.09)	(0.12)	(0.14)	(0.07)	(0.05)

Table 2
Continued

	Dense	Sparse	Sparse columns	Sparse rows	Block diagonal	Concentrated	Risk correlated	Risk and size correlated
				B. $b^r = 0.5$				
A^*	8.6155	8.7980	8.7158	8.7361	9.6085	9.4993	8.1189	8.5444
B^*	0.0207	0.0192	0.0176	0.0210	0.0220	0.0154	0.0191	0.0168
C^* ($\times 10^{-4}$)	2.1084	2.0108	2.0511	2.0985	1.8742	2.0529	1.9983	2.0305
$A - A^*$	−2.2746	−2.7881	−1.5907	0.2711	−3.8856	−4.1141	−1.5622	−0.8620
	(0.23)	(0.27)	(0.40)	(0.52)	(0.32)	(0.24)	(0.42)	(0.40)
$B - B^*$	0.1041	0.1016	0.0900	0.1085	0.0747	0.0811	0.0980	0.1296
	(0.10)	(0.09)	(0.10)	(0.09)	(0.10)	(0.13)	(0.13)	(0.01)
$C - C^*$ ($\times 10^{-4}$)	−1.0264	−1.0217	−1.0332	−0.6255	−0.8271	−1.0122	−1.1968	−0.6423
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Sharpe ratio	0.1137	0.1118	0.1098	0.1139	0.1148	0.1072	0.1116	0.1089
Sharpe ratio bias	0.0445	0.0463	0.0404	0.0503	0.0371	0.0374	0.0463	0.0583
	(0.10)	(0.05)	(0.05)	(0.02)	(0.05)	(0.08)	(0.08)	(0.00)
Index inefficiency (underlying)	0.0168	0.0140	0.0125	0.0171	0.0183	0.0095	0.0144	0.0111
	(0.27)	(0.30)	(0.33)	(0.26)	(0.23)	(0.34)	(0.31)	(0.34)
Index inefficiency (cross-held)	0.0617	0.0607	0.0535	0.0680	0.0549	0.0466	0.0599	0.0688
	(0.00)	(0.07)	(0.05)	(0.05)	(0.05)	(0.04)	(0.05)	(0.01)

The averages of the efficient set constants, A^*, B^*, and C^* (in the underlying economy) from 100 simulation runs are reported. These constants are based on 300 monthly observations of excess returns for 50 firms. The biases in the three efficient set constants (defined as the differences between the cross-held and underlying economies' efficient set constants, $A - A^*$, $B - B^*$, $C - C^*$) are reported along with the average Sharpe ratio ($\sqrt{B^*}$) in the underlying economy and the Sharpe ratio bias ($\sqrt{B} - \sqrt{B^*}$). The index inefficiency values reported for the underlying and cross-held economies represent the average difference (over 100 runs) in the slopes of two lines: the tangency line to the efficient frontier and the line that passes through the value-weighted portfolio, both of which intersect the origin. All results are shown for 16 simulations (eight cross-holding patterns times two levels of aggregate cross-holding). The values given in parentheses below several of the reported statistics (for example, $A - A^*$) represent the fraction of observations with the opposite sign from that statistic.

ference in the slopes of the following two lines: the tangency line of the efficient frontier and the line passing through the value-weighted portfolio, both of which intersect the origin. The difference in these two slopes (measured in excess return versus standard deviation space) provides a measure of how far away the market index is from the efficient frontier.

Table 2 also reports the difference in these slopes as the *index inefficiency* for the underlying and cross-held economies. The values shown are the average differences over 100 runs in each simulation. In the underlying economy, we cannot reject that the difference in the slopes is equal to zero; that is, we cannot *reject* the ex ante efficiency of the market index. However, for the cross-held returns, the inefficiency of the market index increases markedly and appears statistically significant for a wide range of cross-holding patterns. This result implies that tests of the mean-variance efficiency for a market portfolio may be biased toward rejection of efficiency if there is even a moderate degree of cross-holding in an economy. Furthermore, this effect is even more pronounced at higher levels of aggregate cross-holding.

3.2 Factor analysis and the APT

Numerous single-market as well as market-integration studies have employed the arbitrage pricing theory (APT), where returns are assumed to be generated by a set of factors. In this section, we explore potential methodological problems induced by cross-holding when factors are prespecified and when they must be estimated from security returns. Again, we employ simulations to analyze the extent of distortions in a controlled environment.

Consider the bilinear paradigm employed by Gibbons (1982) in a market-model context and Brown and Weinstein (1983) in a multi-factor model. Suppose that the ex ante return-generating process in the underlying economy can be represented for security i in period t as[22]

$$\tilde{R}_{ti} = E_{ti} + \tilde{\delta}_{t1} b_{1i} + \cdots + \tilde{\delta}_{tk} b_{ki} + \tilde{e}_{ti}, \tag{15}$$

where δ_{tj} is the return on the *jth* stochastic factor. These factor returns each have zero expectation and are independent of returns on other factors. The weight on the *jth* factor for security i is b_{ji} and $\tilde{e}_{ti}$ has zero mean. Ross (1976) shows that under the APT assumptions, E_{ti} can be represented as

$$E_{ti} = \lambda_{t0} + \lambda_{t1} b_{1i} + \cdots + \lambda_{tk} b_{ki}, \tag{16}$$

[22] The notation and methodology in this section follows closely that of Brown and Weinstein (1983). A more detailed exposition of the model structure and some of the tests we use can be found therein.

where in equilibrium λ_{t0} is equal to the risk-free rate and the λ_{tj} 's are factor prices for period t. Let Δ_k denote the $T \times k$ matrix of factor realizations (with elements δ_{tj}), and let R now denote the $T \times N$ matrix of observed returns for the N securities. Then, from the factor process (15), we have

$$r = \Delta_k B_k + U, \tag{17}$$

where $r_{ti} = R_{ti} - E_{ti}$ and B_k is the $k \times N$ matrix of factor weightings. By assuming that $E_{ti} = E_i$ for all t, we may interpret r as differences from mean returns. From (16)) we now have

$$R = \lambda_0 \iota' + A_k B_k + U = AB + U, \tag{18}$$

where $\lambda_0 = [\lambda_{t0}]_{T\times 1}$, $A_k = [\lambda_{tj} + \delta_{tj}]_{T\times k}$, $A = [\lambda_0, A_k]_{T\times(k+1)}$, $B_k = [b_{ji}]_{k\times N}$, $B = [\iota, B_k']'_{(k+1)\times N}$, $R = [R_{ti}]_{T\times N}$, and $U = [e_{ti}]_{T\times N}$.

We first analyze whether the presence of cross-holdings in an economy distorts estimates of the factor prices *(A)* when prespecified factors are used in the bilinear model. To do so, we perform simulations, using a return generator based on a two-factor model with known properties. For simplicity, we utilize constant factor prices with $\lambda_{1t} = 0.01$ and $\lambda_{2t} = 0.02$ for all t.[23] We first generate R^*_1, a $T \times N$ matrix of return observations for the underlying economy, and then generate another $T \times N$ set of returns in the underlying economy, which are then transformed into R_2, the returns in the cross-held economy. We then examine whether factor prices are affected by cross-holding. With a two-factor model and constant factor prices, we can write the matrix containing returns from both the underlying and cross-held economies as

$$\begin{aligned}[R^*_1, R_2] = {} & [\lambda_0 \iota', (\lambda_0 + \gamma_0)\iota'] + [\lambda_1 b^*_1, (\lambda_1 + \gamma_1) b_1] \\ & + [\lambda_2 b^*_2, (\lambda_2 + \gamma_2) b_2] + [\Upsilon^*_1, \Upsilon_2],\end{aligned} \tag{19}$$

where b^*_j (b_j) is a $1 \times N$ vector of factor loadings for the *jth* factor in the underlying (cross-held) economy, and the Υ's are mean zero error matrices. Under the null hypothesis that the factor prices and the risk-free rate are unaffected by the presence of cross-holdings, we should find $\gamma_0 = \gamma_1 = \gamma_2 = 0$.

Table 3 reports the results of our regression tests using $T \times 2N = 300 \times 100 = 30{,}000$ observations for each of 100 simulation runs using a sparse H matrix with $h^T = 0.5$. We have also run these tests with several other cross-holding patterns as well as with a larger economy and find results consistent with those reported in Table 3. The iterative procedure outlined in Gibbons (1982) and Brown and Weinstein (1983) has been employed in these tests. Both WLS and

[23] Other parameters of the return generator are discussed in Appendix B.

Table 3
Tests of the arbitrate pricing model with prespecified factors

Coefficients	Weighted least squares		Generalized least squares	
	$b^T = 0.2$	$b^T = 0.5$	$b^T = 0.2$	$b^T = 0.5$
$\hat{\lambda}_0$	0.0043	0.0041	0.0038	0.0036
$\hat{\lambda}_1$	0.0079	0.0102	0.0063	0.0100
$\hat{\lambda}_2$	0.0178	0.0191	0.0166	0.0187
$\hat{\gamma}_0$	−0.0004	−0.0004	−0.0003	−0.0002
	(0.28)	(0.34)	(0.34)	(0.48)
$\hat{\gamma}_1$	0.0004	−0.0010	0.0004	−0.0012
	(0.50)	(0.50)	(0.60)	(0.50)
$\hat{\gamma}_2$	0.0009	0.0024	−0.0002	−0.0008
	(0.52)	(0.42)	(0.52)	(0.52)

Regressions based on the following equation were performed to test whether the presence of cross-holding affects the estimate of the risk-free rate of return and the factor prices:

$$[R_1^*, R_2] = [\lambda_0\iota', (\lambda_0 + \gamma_0)\iota'] + [\lambda_1 b_1^*, (\lambda_1 + \gamma_1)b_1] + [\lambda_2 b_2^*, (\lambda_2 + \gamma_2)b_2] + [\Upsilon_1^*, \Upsilon_2],$$

where R_1^* is a 300×50 matrix of security returns in the underlying economy, and R_2 is a similar matrix of security returns from a cross-held economy (using *sparse* matrices with $b^T = 0.2$ and $b^T = 0.5$). Returns are generated based on a two-factor model with $\lambda_{1t} = 0.01$ and $\lambda_{2t} = 0.02$ for all t. The reported λ and γ estimates are averages across 100 runs. Below each γ estimate, we report the fraction of observations with the opposite sign. Both WLS and GLS estimates are reported.

GLS estimations have been used, the latter to take into account the substantial cross-sectional correlation induced by cross-holding that is not captured by the factors. Examining the estimated parameters, we find that all γ 's are quite small and not significantly different from zero. Thus, with known factors, the null hypothesis that cross-holding does not distort factor price estimates cannot be rejected.[24] Not surprisingly, the estimated risk-free rate and factor prices are quite close to the parameters of the return generator.

Since the identity and even the number of factors generating returns in an economy are typically unknown, we also examine the effect of cross-holding on factor analysis and tests of the bilinear model when the number of factors and their returns must be estimated. Cross-holding-induced distortions may be anticipated for two reasons. The first reason results from recognizing that securities in a cross-held economy are essentially portfolios of the underlying economy's securities. Shanken (1982) has shown that it is possible to construct portfolios that distort estimates and may even conceal the existence of factors generating the returns of underlying securities. Although we have no a priori reason to believe that cross-holdings have the specific construction necessary to destroy factors, the increased cross-correlation between firms' security returns may well result in misleading

[24] We have also performed analogous tests using a market model and find that the null hypothesis once again is not rejected at reasonable significance levels.

estimates of the economy's underlying factor structure. The second possible reason for distortions arises from the fact that the implicit weights (P matrix) for the cross-held economy's returns are nonstationary.

We first investigate whether cross-holding affects the estimated number of factors. We generated underlying security returns, using the two-factor model described above (with T = 300 and N = 50). Using a sparse H matrix (with $h^T = 0.5$), we also obtained the corresponding returns for the cross-held economy.[25] A maximum likelihood factor analysis was performed on each set of returns to obtain B^*_k and B_k (the factor loadings in the two economies). This procedure also yields ψ^* and ψ, which at convergence of the estimation procedure contain the diagonal elements of $\Omega^* - B^*_k B^{*\prime}_k$ and $\Omega - B_k B'_k$, respectively. We then test the adequacy of a k-factor model in explaining security returns, using a maximum likelihood ratio test.[26] The standard procedure is to start with $k = 1$ and to increase the number of factors generated until the null is no longer rejected at the desired level of significance.

Using the underlying economy returns, we reject the null (for each of 100 simulation runs) when $k = 1$ but never reject (at a 1 percent significance level) when $k = 2$. Clearly, this is appropriate since the returns are in fact generated by a two-factor model. However, using returns from the cross-held portfolios of these underlying securities, we reject the null (at the 1 percent significance level) for $k = 2$ and even for $k = 3$ in every one of the 100 simulation runs. Note that the null is also rejected if we use return observations $\bar{P}R^*$—that is, a stationary transformation [$\bar{P} := (1/T)\ \Sigma\ P_t$] of the underlying returns. This indicates that even without nonstationary weights, the implicit portfolios generated by cross-holding introduce substantial cross-sectional covariance that cannot be adequately captured by two factors. Note also that it seems cross-holding has the tendency to increase the dimensionality of the apparent factor structure rather than destroying or hiding factors.

Under certain circumstances the finding of spurious factors is likely to be mitigated as the number of assets in the economy grows. Consider an economy in which each firm holds equity in at most n firms. It is possible to increase the number of firms, N, in such an economy and yet preserve a given aggregate level of cross-holding (e.g., one

[25] We also tried other cross-holding patterns as well as a larger economy and obtained results consistent with those reported below.

[26] In particular, we test the null hypothesis of $\Omega = B_k B'_k + \psi$. See page 494 of Krzanowski (1988) for a detailed description of this test. He shows that (with N securities) $W = [T - (2N + 11)/6 - 2k/3]\ (\ln |\beta\beta' + \psi| - \ln |\Omega|)$ follows a χ^2 distribution with $\frac{1}{2}[(N - k)^2 - (N + k)]$ degrees of freedom under the null.

could increase the number of blocks in our *block-diagonal* structure with each block having the same aggregate cross-holding level and value). As the number of assets in this economy grows, the fraction of firms, n/N, whose returns include a particular firm's idiosyncratic risk goes to zero. The apparent distortion in the factor structure due to cross-holding may thus eventually disappear for this economy once it has a sufficiently large number of firms.

Nevertheless, our analysis indicates that the factor structure is clearly distorted for a wide variety of cross-holding patterns, and thus it is important to examine how seriously affected are APT tests using estimated factor structures as inputs. In particular, we examine tests of the bilinear models in (17) and (18), such as those employed by Brown and Weinstein (1983), using B_k estimates obtained from the factor analysis procedure discussed above. These tests are based on partitioning a set of $2N$ securities into two disjoint groups of N securities and testing whether the risk-free rate of return and risk premia implied for the two groups of securities are identical. For example, Equation (18) can be rewritten as

$$[R_1, R_2] = A[B_1, B_2] + [U_1, U_2]. \tag{20}$$

A less restrictive form of this equation is

$$[R_1, R_2] = [A_1B_1, A_2B_2] + [U_1, U_2]. \tag{21}$$

Under the null hypothesis, we expect that $A_1 = A_2$.

We implement four tests proposed by Brown and Weinstein (1983) to study the effect of using R^*_1 and R_2, two sets of returns where the returns of the second group are based on portfolios generated by cross-holding, in lieu of R^*_1 and R^*_2. The first three tests involve F-statistics computed by using the sums of squared errors for constrained and unconstrained versions of Equation (21).[27] In the first F- statistic (F_1), the constrained regression allows only one risk-free rate to be estimated (i.e., $\lambda_{01} = \lambda_{02}$ is imposed). In the calculation of F_2, the constrained model estimates only one common set of risk premia, A_k. For F_3, both of the above constraints are imposed. We also compute a fourth F-statistic (F_4) to test whether the estimated factors are the same for both groups of returns. This F- statistic is based on the sum of squared errors of the unconstrained factor model $[r_1, r_2] = [\Delta_{k1} B_{k1}, \Delta_{k2} B_{k2}] + [U_1, U_2]$ and the constrained version $[r_1, r_2] = \Delta_k [B_{k1}, B_{k2}] + [U_1, U_2]$.

We first apply these four tests to two groups of security returns from the underlying economy (N = 50 for each group), R^*_1 and R^*_2. The

[27] These tests are described in detail in the appendix of Brown and Weinstein (1983). We did not, however, employ their approximation in (A.6), which we found to be inaccurate for our data set.

Table 4
Tests of equivalence of risk premia and the risk-free rate implied for two groups of securities

	F_1	F_2	F_3	F_4
		A. $b^T = 0.2$		
F_i^*	1.218	1.220	1.181	1.142
$F_i - F_i^*$	0.147	0.030	0.508	0.788
	(0.09)	(0.43)	(0.00)	(0.00)
$\bar{F}_i - F_i^*$	0.140	0.025	0.417	0.648
	(0.15)	(0.44)	(0.00)	(0.00)
$F_i - \bar{F}_i$	0.006	0.005	0.092	0.139
	(0.34)	(0.49)	(0.05)	(0.05)
		B. $b^T = 0.5$		
F_i^*	1.227	1.212	1.178	1.133
$F_i - F_i^*$	0.346	0.217	1.348	2.119
	(0.03)	(0.16)	(0.00)	(0.00)
$\bar{F}_i - F_i^*$	0.313	0.134	1.029	1.637
	(0.07)	(0.22)	(0.00)	(0.00)
$F_i - \bar{F}_i$	0.034	0.082	0.319	0.482
	(0.38)	(0.31)	(0.00)	(0.01)

F-statistics are presented based on constrained and unconstrained versions of the bilinear model. Each reported *F*-value is the mean across 100 runs. The first three *F*-tests are based on the arbitrage pricing model. For F_1, the constrained regression allows only one risk-free rate to be estimated. For F_2, the constrained model estimates only one common set of risk premia. For F_3, both of the above constraints are imposed. F_4 is based on constrained and unconstrained versions of the factor process model (17). There are two panels, one for each level of aggregate cross-holding. The first row of each panel provides the four *F*-statistics when both sets of security returns are from the underlying economy (R_1^*, R_2^*). The other three rows present differences in *F*-statistics, where the F_i's are based on two groups of security returns, one each from the underlying and cross-held economies, R_1^* and R_2, respectively, and the $\bar{F}_i$-statistics use the two groups, R_1^* and $\bar{P} R_2^*$. *Sparse* matrices are used to generate returns in the cross-held economy. Below the differences in *F*-values we report the fraction of observations with the opposite sign.

constrained and unconstrained regressions discussed above were performed with B_k matrices obtained from maximum likelihood factor analyses of the return data.[28] Using the errors from these regressions, we calculated F^*_1, F^*_2, F^*_3, and F^*_4, which are reported in Table 4.[29] This procedure (factor analysis and regressions) was then repeated for the two groups of returns R^*_1 and R_2, where the second group of returns now comes from a cross-held economy. This yielded F_1, F_2, F_3, and F_4. Finally, the same analysis was performed for two groups R^*_1 and $\bar{P}R^*_2$. This third set of F- statistics ($\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$, $\bar{F}_4$) allows us to explore the distortion induced by portfolios where the weights are stationary.

[28] The factor analysis used all return observations from both groups. Alternatively, the factor analysis could be performed separately on each group. We repeated our analysis in this manner, but found the results relating to distortions induced by cross-holding were quite similar.

[29] The degrees of freedom for the four F- tests are F_1: (300, 28,200), F_2: (600, 28,200), F_3: (900, 28,200), and F_4: (600, 29,600). Since the asymptotic distributions of $F_{d1,d2}$ and $\chi^2(d_1)/d_1$ approach one another as d_1 approaches infinity, we approximated all F- statistics by calculating χ^2– values.

The numbers reported in Table 4 are based on the average F- statistics for 100 simulation runs using sparse cross-holding matrices with $h^T = 0.2$ and $h^T = 0.5$ to generate the returns R_2 and $\bar{P}R_2^*$.[30] F-statistics in the first row of the table are based on the two groups of returns from the underlying economy (i.e., these test statistics represent simulations of the null hypothesis). Interestingly, the empirical distributions of our F- statistics for the null hypotheses do not conform closely to the hypothesized F- distributions. In particular, the means of our F- statistics are too far from 1, which implies that when we simulate the null hypothesis we reject it much too often. We attribute this departure from the F- distribution, in part, to our finite sample size but mainly to the error-in-variables problem associated with the factor estimates and their sentivities in the multivariate regressions.

The second row illustrates the extent to which cross-holding increases the values of the F- statistics. The average F- statistics increase when cross-held returns are included in the data. In addition, the upward biases in F_1, F_3, and F_4 are statistically significant (for example, all 100 runs yielded positive differences for F_3 and F_4).[31] Note that both F_3 and F_4 increase considerably in the presence of cross-holding. This increase is very pronounced for the high level of aggregate cross-holding, $h^T = 0.5$.

We also report the effect of a stationary portfolio transformation ($\bar{P}$) in the next two rows of Table 4. This allows us to examine the extent to which the distortion induced by cross-holding is attributable to nonstationarity of the portfolio weights. The results indicate that even a stationary $\bar{P}$ matrix transformation has a statistically significant effect on the F- statistics and explains a large fraction of the distortion. However, the last row shows that the additional effect of nonstationarity is also significant for F_3 and F_4.

In summary, cross-holding does not seem to cause significant distortions in APT-based estimates of the risk-free rate of return and factor prices as long as the appropriate factor returns are known. However, in the more realistic situation where the factor structure is not known, cross-holding appears to create significant errors. When the factors generating returns in an economy must be estimated, the factor analysis procedure does not deliver the correct underlying factors in the presence of cross-holding. Subsequent regressions using the estimated factor loadings produced distorted estimates of the

[30] Again, we have run these tests with several other cross-holding patterns as well as larger economies and obtain results consistent with those repotted in Table 4.

[31] We find that the significant biases lead to substantial increases in the rejection frequency of the null hypothesis. These rejection frequencies range from 20 percent for biases significant at the 10 percent level to 100 percent for the mast significant biases.

underlying risk-free rate and factor prices. Furthermore, the results reported in Table 4 are consistent with those for several other cross-holding patterns as well as larger economies. This suggests that these results are robust to a wide variety of cross-holding situations.

4. Concluding Comments

This article has focused on measurement distortions and a variety of estimation problems that arise when using observed returns and market capitalizations from an economy with cross-held securities. We begin by showing that equity market capitalization calculations should be based on the value of outside rather than total shares. The more general result that aggregate market value should be based only on outside securities is demonstrated in Appendix A. An implication of this result is that comparisons of aggregate leverage measures across markets should not consider inside debt or other securities held by listed firms.

Our analytic results on market returns suggest that the conditions under which cross-holding does not distort standard equity market return measures are quite restrictive. In particular, Proposition 2 requires that the fraction of inside shareholdings be equal across firms. This condition is generally not satisfied in practice. As shown in Proposition 3, the resulting distortion will depend on the cross-sectional covariation of returns and cross-holding. If more heavily cross-held firms have above-average returns, the standard market return measure will be distorted upward, and vice versa when these firms have below-average returns. Our analytical results also indicate that the standard market portfolio will generally be inefficient relative to the capital market line in a CAPM context and will also not lie on the mean-variance frontier of risky securities.

The basic problem of inappropriately measuring market returns and the resulting problem of specification error (for example, in estimating betas) is related to the Roll (1977) critique of the CAPM. However, our situation differs in that we do not have excluded assets. In other words, having returns for all assets is not a sufficient condition for a valid test of the CAPM. It is also necessary to know the cross-holding pattern (more precisely, the P matrix in our model).

Despite the above problems induced by cross-holding, our simulation results indicate that for some applications the use of cross-held returns may not lead to seriously biased results. For a wide range of cross-holding patterns, the mean bias in the cross-held market return index was small and statistically insignificant. The cross-held and underlying (true) market indices were also highly correlated. Even for those cross-holding patterns where we found statistically significant bias in the mean market return, the size of that bias was relatively small. The market risk premium measured by the ratio of mean excess market return to its standard deviation was also not significantly biased on average.

This relatively good news does not guarantee an absence of bias in all cases. It does, however, suggest that studies making relatively simple comparisons of market returns and risk premia (measured by the above ratio) across markets with widely different cross-holding patterns may still obtain unbiased estimates of these parameters. Unfortunately, our simulations indicate serious pitfalls for other types of estimation and asset pricing tests using cross-held returns.

After reflecting on our analysis, we see that the nonstationarity of the return process induced by the changing P matrix is obvious. However, the distortion induced in the estimated efficient frontier is striking. This distortion results in a large and generally significant bias in the maximum Sharpe ratio as well as a pronounced tendency to reject the mean-variance efficiency of the market portfolio in the cross-held economy. This is due to the efficient frontier optimization procedure, which implicitly magnifies the effect of distortions in the estimated variance-covariance matrix, with rather dramatic results. Notably, this effect appears robust to a wide range of cross-holding patterns.

When we shift our consideration to tests of the APT, the key issue is whether the underlying factors are known. If we can observe the underlying factor returns, our tests indicate that factor price estimates do not appear to be seriously biased by cross-holding. However, if the underlying factors are not known and must be estimated from cross-held returns, serious problems arise. First of all, the number of factors are incorrectly estimated for the cross-held economy. Even if we utilize the correct number of factors, our simulations indicate that the estimated factor risk premia are distorted. This implies a tendency toward spurious rejections of APT when some shares in the market are cross-held, or, alternatively, rejections of market integration across markets with and without cross-held shares. Again, these results appear quite robust to differing cross-holding patterns.

The problem of correctly identifying the underlying factor structure for cross-held returns is reminiscent of the debate between Shanken (1982, 1985) and Dybvig and Ross (1985) on APT testability. A key aspect of Shanken's critique was the argument that repackaging securities into portfolios could conceal the existence of factors. Perhaps the most powerful criticism of Shanken's argument was that the portfolios he constructed were artificial. While this point may have merit with regard to completely concealing factors, cross-holding represents a naturally occurring and widespread phenomenon that appears to seriously distort estimated factor structures. Consequently, a more general interpretation of Shanken's basic idea has considerable applicability in markets with cross-held securities.

In summary, cross-holding introduces a number of potential estimation difficulties. For some estimates (described above) the bias induced by using cross-held returns does not appear to be severe. For other estimates, the biases are substantial, and the only straightforward way of avoiding them is to have the time series of P matrices. That is, we need to know the weights in firms' portfolio holdings of other firms' securities. Such information is not currently available on a widespread and disaggregated basis. Until we circumvent this problem, it appears that we should treat some estimates based on cross-held returns with considerable caution.

Appendix A

Market aggregation with many classes of securities

We extend the results in Section 1 to allow for both equity and nonequity (e.g., debt) securities. Let S^k_i denote the market value of all outstanding securities of type k issued by firm i. For example, $k = 1$ may represent firms' common shares, while $k = 2$ may represent corporate bonds. Let H^k represent the cross-holding matrix for the kth type of security; that is, h^k_{ij} is the fractional ownership of firm j's type-k security by firm i. The aggregate value of firm i's securities can be expressed as follows:

$$\sum_k S^k_i = V^*_i + \sum_k \sum_j h^k_{ij} S^k_j. \tag{A1}$$

Summing over all firms, we obtain

$$\sum_k \iota' S^k = \iota' V^* + \sum_k \iota' H^k S^k. \tag{A2}$$

Splitting the security value S^k between inside and outside holdings, S^{kI} and S^{kO}, respectively, and noting that

$$\iota' S^{kI} = \sum_i S^{kI}_i = \sum_i \sum_j h_{ji} S^k_i = \iota' H^k S^k, \tag{A3}$$

we arrive at the following result, which is analogous to the all-equity case:

$$\sum_k \iota' S^{kO} = \iota' V^*. \tag{A4}$$

That is, the aggregate value of all securities held by outside investors equals the aggregate value of operating assets, and as such represents the true market capitalization for the broader securities market we have defined here. From (A4) it is also apparent that a recapitalization that involves exchanging inside securities of one type for inside securities of another type may change the standard measure of market capitalization but will not affect the true market capitalization.

Corollary to Proposition 1. *If there are many classes of corporate securities, and R^k_i is the return on firm i's type-k security, then*

$$R^*_m = \sum_i \sum_k w^O_{ik} R^k_i \qquad \text{where } w^O_{ik} = \frac{S^{kO}_i}{S^O_m} \quad \text{and} \quad S^O_m = \sum_i \sum_k S^{kO}_i. \tag{A5}$$

To prove this corollary, we proceed as in (12) and (13).

$$\sum_k \sum_i w^O_{ik} R^k_i = \frac{1}{S^O_m} \sum_k \sum_i S^{kO}_i \frac{Z^k_i}{S^k_i}, \tag{A6}$$

where Z^k_i is the portion of firm *i's* total earnings, Y_i, which accrues to holders of the *kth* class of its securities. Denoting the earnings "payable" to outside holders of firm *i's type-k* security by Z^{kO}_i, we obtain

$$\sum_k \sum_i w^O_{ik} R^k_i = \frac{1}{S^O_m} \sum_k \sum_i Z^{kO}_i. \tag{A7}$$

Using vector notation plus (A4) and analogous relation for aggregate outside earnings as discussed in the text, we obtain

$$\sum_k \sum_i w^O_{ik} R^k_i = \frac{\Sigma_k \iota' Z^{kO}}{\Sigma_k \iota' S^{kO}} = \frac{\iota' X}{\iota' V^*} = R^*_m. \qquad \blacksquare \tag{A8}$$

Proof of Proposition 2. We can express R^*_m as follows, using (13) and allowing h_i to represent the proportion of firm *i's* shares held by other corporations:

$$R^*_m = w^{O\prime} R = \frac{\Sigma_i (1 - h_i) S_i R_i}{\Sigma_i (1 - h_i) S_i} = \frac{S'R - S^{I\prime}R}{\iota' S - \iota' S^I}. \tag{A9}$$

Using (11) we can reexpress R_m as

$$R_m = S'R / \iota' S. \tag{A10}$$

Setting $R_m = R^*_m$ and cross-multiplying the scalars in (A9) and (A10), we obtain

$$\iota' S S' R - \iota' S S^{I\prime} R = \iota' S S' R - \iota' S^I S' R. \tag{A11}$$

The first term on each side of (A11) cancels. A necessary and sufficient condition for (A11) to hold for all possible return vectors, *R*, is

$$\iota' S S^{I\prime} = \iota' S^I S'. \tag{A12}$$

This requires that the column and (corresponding) row sums of $SS^{I\prime}$ must be equal; that is,

$$h_i S_i \sum S_j = S_i \sum h_j S_j. \tag{A13}$$

This will be true for all possible firm value vectors, S, if and only if $h_i = h \; \forall i$; that is, each firm must have the same proportion of its shares held by insiders. ■

Corollary to Proposition 2. *The standard market return (R_m) is equivalent to the true market return (R^*_m) for all V^* and X if and only if each firm has the same proportion of each of its types of securities held by insiders (i.e., $h^k_i = h$ for all i and k),*

The proof of this corollary is analogous to the proof in the all-equity case shown directly above.

Proof of Proposition 3. Using Proposition 1, we can express the difference between the standard and true market return measures as

$$R_m - R^*_m = (w - w^O)'R. \tag{A14}$$

Expressing the weights w, and w^O_i in terms of equity values, we get

$$R_m - R^*_m = \sum_i \left(\frac{S_i}{S_m} - \frac{(1 - h_i)S_i}{S^O_m} \right) R_i. \tag{A15}$$

Recalling that h^T is the aggregate measure of the percentage of total share value owned by insiders, we can write $S^O_m = (1 - h^T)S_m$. Substituting this into (A15) and simplifying, we obtain

$$R_m - R^*_m = \frac{1}{(1 - h^T)S_m} \sum_i (h_i - h_T)S_i R_i. \tag{A16}$$

Since $\Sigma_i \,(h_i - h^T)S_i = 0$, we can obtain the desired expression in Proposition 3. ■

Proof of Proposition 4. All portfolios on the mean-variance efficient frontier of risky assets can be represented as linear combinations of two portfolios on that frontier. Choose those two portfolios to be the true market, whose weighting vector is w^O, and the minimum-variance efficient portfolio, whose weighting vector is α. Merton (1972) has shown that elements of α are determined by the covariance matrix Ω such that $\alpha_i = \Sigma_j v_{ij} / \Sigma_i \Sigma_j v_{ij}$ for i, where v_{ij} is the *ijth* element of Ω^{-1}. Then the standard market portfolio will lie on the efficient frontier if and only if its weighting vector w satisfies

$$w = \lambda\alpha + (1 - \lambda)w^O. \tag{A17}$$

This clearly requires

$$w_i - w_i^0 = \lambda(\alpha_i - w_i^0)$$

Proof of Proposition 5. First, we demonstrate that plim $(1/T)\,\Sigma_t R_t = \bar{P}R^*$.

$$R_t = P_t R_t^*, \tag{A19}$$

$$\frac{1}{T}\sum_t R_t = \frac{1}{T}\sum_t P_t(E(R^*) + \epsilon_t^*), \tag{A20}$$

$$\operatorname{plim}\frac{1}{T}\sum_t R_t = \operatorname{plim}\frac{1}{T}\sum_t P_t E(R^*) + \operatorname{plim}\frac{1}{T}\sum_t P_t\epsilon_t^*$$

$$= \bar{P}E(R^*) + \operatorname{plim}\frac{1}{T}\sum_t P_t\epsilon_t^*, \tag{A21}$$

since $\bar{P} = (1/T)\,\Sigma_t P_t$. The elements of P_t are

$$p_{ijt} = q_{ij}V_{jt}^*/\Sigma_k\, q_{ik}V_{kt}^*. \tag{A22}$$

The V^*_{kt} values depend on R^* realizations in earlier periods; however, the error vector (e^*_t) is intertemporally independent with mean zero. Therefore,

$$\operatorname{plim}\frac{1}{T}\sum_t P_t\epsilon_t^* = 0. \tag{A23}$$

Consequently,

$$\operatorname{plim}\frac{1}{T}\sum_t R_t = \bar{P}E(R^*). \tag{A24}$$

To demonstrate that $\hat{\Omega} > \bar{P}\Omega^*\bar{P}'$, we analyze a generic element:

$$\hat{\Omega}_{ij} = \frac{1}{T}\sum_t (R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j), \tag{A25}$$

$$\operatorname{plim}\hat{\Omega}_{ij} = \operatorname{plim}\frac{1}{T}\sum_t (P_{it}R_t^*)(P_{jt}R_t^*) - (\bar{P}_iE(R^*))(\bar{P}_jE(R^*)). \tag{A26}$$

With some manipulation this reduces to

$$\operatorname{plim}\hat{\Omega}_{ij} = \bar{P}_i\Omega_{ij}^*\bar{P}_j' + \sum\sum E(R_m^*)E(R_n^*)A_{ijm} + \sum \Omega_{mm}^*A_{ijm}, \tag{A27}$$

where $A_{ijm} = \operatorname{plim}\,(1/T)\,\Sigma_t\,\theta_{imt}\theta_{jmt}$ and θ_{imt} equals the change in P_{im}, from time zero to time t. That is, $\theta_{imt} = P_{imt} - P_{im0}$.

From (A22) it is apparent that q_{imt} will be positive (negative) if V^*_m has increased more (less) than the weighted average $\Sigma_k\, q_{ik}\, V^*_k$ over the time period from 0 to t. Although the weights in q_{jmt} are slightly different, its sign will be primarily determined by the overall change in V^*_m; consequently, it will tend to have the same sign as q_{imt}. Thus, $A_{ijm} = \text{plim } (1/T) \Sigma_t\, q_{imt}\, q_{jmt} > 0$. Furthermore, the economics of pricing for risky securities imply that both $E(R^*)$ and $E(R^*_m)$ should be positive. Consequently, $\text{plim } \hat{\Omega}_{ij} > \bar{P}_i \Omega^*_{ij} \bar{P}'_j$. ■

Appendix B: Simulation Description

In this appendix, we provide details of our simulation procedures. The initial value of the operating assets (V^*_i) for each of the 50 firms is sampled from a uniform distribution between 0 and 1 (however, any positive upper bound would lead to equivalent results). Monthly equity returns in the underlying economy are generated from a market model. All distributions discussed below are normal unless otherwise indicated. The parameters of the market model are as follows: the riskless rate equals 0.0042 (i.e., 5 percent per annum); mean excess return on the market equals 0.00583 (i.e., 7 percent per annum); the variance of the monthly market return equals 0.0033 [$= (0.20)^2/12$]; the distribution of firm betas has mean equal to 1.00 and variance equal to 0.42; and, the standard deviation of firms' unsystematic risk is randomly drawn from a uniform distribution with lower and upper bounds of 0.01 and 0.07, respectively.[32] The riskless rate and the characterization of idiosyncratic risk for the two-factor model are the same as in the market model. The factor sensitivities are normally distributed with the factor means equal to 0.052 and 0.004, while the factor variances are 0.022 and 0.019. The risk premia on the two factors equal 0.01 and 0.02, respectively. The factors are independent standard normal random variables.

The restrictions used to generate the eight classes of cross-holding matrices are detailed below.

Dense: Each of the 2500 entries in H matrix is sampled from a (0, 1) uniform distribution (we assume that no firms hold short positions in other firms' equity). We also generate column sums from a uniform distribution and scale column elements so that their sums match the generated values. We then employ an iterative procedure that calculates $S = (I - H)^{-1} V^*$ and rescales H as necessary to match the specified h^T value for the economy.

Sparse: These are generated as outlined above for the dense matrices with the following exception. Individual matrix elements are

[32] The variance of the beta distribution is based on an estimate that uses the Fama and MacBeth (1973) procedure on historical U.S. stock data from the period 1926-1989.

generated based on a distribution where there is a 40 percent probability that the entry is equal to zero and a 60 percent probability that the element is drawn from the (0, 1) uniform distribution.

Sparse columns: There is a 40 percent probability that a particular column sum is equal to zero (and thus each entry in a zero column is zero).

Sparse rows; There is a 40 percent probability that a particular row sum is equal to zero (and thus each entry in a zero row is zero).

Block diagonal: These matrices are constructed with five 10 × 10 *dense* blocks (generated as described above) down the diagonal and zeros elsewhere.

Concentrated: These are generated in the same way as sparse matrices, but with a 90 percent probability of an entry being equal to zero.

Risk correlated: Prior to rescaling H to match the desired h^T, the columns of the sparse matrices are reordered such that firms with large (small) expected returns are assigned columns that have a large (small) proportion of shares held by insiders. The correlation between h_i and $E(R_i)$ across all firms for matrices generated in this manner was, on average, 0.97.

Risk and size correlated: First we construct an unscaled risk-correlated H matrix. Then we generate the vector of firm sizes based on the underlying operating assets, V^*. Next, V^* is reordered such that the expected returns are inversely related to firm size based on operating assets. Finally, H is rescaled to match the desired h^T. Thus, firm size and the proportion of the firm's shares held by insiders are also negatively correlated. The average absolute values of these correlations were greater than 0.97.

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