The Rationality of Early Exercise Decisions: Evidence from the S&P 100 Index Options Market

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This study provides a comprehensive empirical analysis of the early exercise history of S&P 100 put and call options. Even though the S&P 100 index options market is generally considered to be the most efficient options market in the world, we show that many exercise decisions are inefficient because they occur when recorded bids are greater than exercise values. Due to market imperfections, some of the cases of inefficient exercise are still rational, but we show that a substantial number of these decisions are clearly irrational, since it would have been possible to realize a larger riskless net cash flow by selling. Unlike previous studies of early exercise, our tests of efficiency and the rational decisions that presumably lead to efficient markets are model independent. We also provide evidence concerning the relative significance of dividends and the wild card to index option pricing models, and introduce and document the importance of the option to exercise and avoid the indirect costs of the spread. We also find evidence of a significant day-of-the-week exercise effect, and present some likely explanations for that effect.

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Efficiency and the rational investment decisions that presumably lead to efficient markets are notoriously difficult to test for. Tests based on contingent claim boundary conditions tend to involve fewer joint hypotheses than other tests of efficiency and rationality, and several recent studies have considered whether actions prescribed by options exercise boundary conditions are consistent with observed actions. Since these tests still rely upon pricing models to determine the boundary conditions, previous studies of options exercise, including those conducted by Ingersoll (1977), Bodurtha and Courtadon (1986), Overdahl(1988)) Dunn and Eades (1989), Gay, Kolb, and Yung (1989), Zivney (1991), and French and Maberly (1992), suffer from the problems that afflict most tests of market efficiency: They are joint tests of rationality and pricing models derived from sets of simplifying assumptions. While many of these studies document violations of boundary conditions, the results cannot be interpreted as unambiguous evidence of inefficiency because of the joint hypothesis problem.

By way of contrast, the efficiency of S&P 100 index (OEX) option early exercise can be unambiguously assessed by comparing market maker bids to exercise values. If early exercise, defined as any exercise prior to the maturity of the contract, occurs when the highest registered bid is below the exercise value of the contract, the decision to exercise is efficient with respect to the quoted bid, since it maximizes cash flow and fully reflects all relevant information. Exercise that occurs when bids exceed the exercise value are inefficient, and may be irrational. If there are no imperfections that would have prevented selling the option at the higher bid, in addition to being inefficient, the decision to exercise is also irrational.

Since S&P 100 index options are cash settled at the closing index level, and trading in the options continues after the closing index value is public information, it is possible to differentiate between efficient exercise decisions and decisions that are irrational or attributable to market imperfections without using pricing models, estimates of volatilities, discount rates, or other unobserved variables. All that is required is an exercise decision, the closing index level, the strike price, and a registered bid price recorded after the closing index level is known.

This study exploits the nature of the exercise procedures for S&P 100 index options and directly tests the hypothesis that the market for S&P 100 index options is efficient and investors and traders exercise index options in a rational manner. Using a sample of daily exercise activity for S&P 100 index options that spans the period from April 1983 through December 1988, we examine historical exercise activity by comparing cash flows realized by exercise to the cash flows that would have been realized by selling the option. Evidence is presented that a substantial proportion of the early exercise activity for S&P 100 index options is inefficient and that some of the inefficient cases of exercise are due to irrational decisions.

This study also documents the empirical significance of the determinants of rational early exercise. Several American option pricing models for index options have appeared in the recent literature, and each is based on the assumption that early exercise will occur if and only if conditions for rational early exercise are met. While these models have been used in empirical studies of valuation, option market efficiency, and volatility, little consideration has been given to the question of whether the options are exercised in accordance with the conditions on which the pricing models are based. Although there is some disagreement in the literature concerning their importance, two features of index options have been identified as factors that may induce optimal early exercise: the payment of cash dividends on the underlying stocks in the index, and the wild card option that allows the holder of the option to exercise as late as 3:15 P.M. CST at the closing index level.[1] In addition to dividends and the wild card, this study considers the possibility that transactions costs may also trigger rational early exercise. We show that the holder of an American index option has the option of exercising and avoiding the indirect costs of the index option bid-ask spread, and that the spread can be large enough to justify rational early exercise, even in the absence of dividends and the wild card.

While we test the efficiency and rationality of exercise decisions without the use of pricing models, our empirical assessment of the factors that trigger rational early exercise is generally model dependent. A simple comparison of bids and exercise values unambiguously determines whether an exercise decision is efficient, but by using pricing models to classify the cases of efficient exercise as dividend or wild card induced, we are able to provide a more thorough understanding of early exercise decisions.

In summary, this article makes several contributions. Even though the S&P 100 index options market is generally considered to be the most efficient options market in the world, we provide evidence of inefficiency and irrational investment decisions. We also provide evidence concerning the relative importance of dividends and the wild card to index option pricing models. We also document and explain a day-of-the-week effect for early exercise. Finally, we describe and document the importance of the option to exercise and avoid the indirect costs of the spread.

[1] All times used in this article are Central Standard Time. Throughout our sample period, the NYSE closed at 3:00 P.M. CST, while the CBOE closed at 3:15 P.M. CST. The 3:15 P.M. exercise cutoff imposed by the CBOE was in effect throughout out sample period, but it has since been changed to 3:20 P.M.

The remainder of this article is organized as follows. Section 1 clarifies our definitions of efficient and rational exercise decisions, Section 2 discusses the theoretical factors that may trigger early exercise, and the data used in the study are described in Section 3. Section 4 contains the results of tests designed to assess the empirical relevance of the various factors, Section 5 documents the presence of inefficiencies and irrational exercise decisions, and Section 6 examines the day-of-the-week exercise effect. Section 7 discusses the pricing implications of the option to exercise and avoid the spread, and conclusions are presented in the final section.

1. Efficiency, Rationality, and Imperfections

Before proceeding with our empirical tests, it is helpful to be more explicit concerning our definitions of rational and efficient exercise decisions. The information set that we use to define an efficient exercise decision consists of the prices that are available on the floor of the options exchange after the closing price of the index is known. More specifically, we assume that the exercise value of the option (F_x), the last bid registered after F_x is known (F^j_b), and the true value of the contract (F^i) are part of trader *i's* information set when the exercise decision is made. Note that F^i may be trader specific.

We define an efficient exercise decision as one that is consistent with F_x, F^j_b, and F^i:[2]

$$F^j_b \leq F^i \leq F_x. \tag{1}$$

Since the exercise value is at least as great as the value to investor *i*, and the cash flows from exercising are at least as great as the cash flows from selling at the bid, in addition to being consistent with the quoted bid, exercise is rational. If, instead, early exercise occurs when

$$F^j_b > F_x, \tag{2}$$

exercise is inefficient with respect to the quoted bid and may be irrational. If the quantity exercised could have been sold at the bid, the decision is irrational. If, however, market imperfections prevent selling the desired number of contracts at the quoted bid, the decision to exercise may still be rational, even though it is not efficient with respect to the quoted bid. In Section 5 we present the results of model-independent tests of condition (2). In Section 4, we use estimated American model prices to approximate F^i and gain some insights

[2] If condition (1) is satisfied. the decision to exercise is efficient with respect to the quoted bid, but the bidding process is also a potential source of inefficiency. If $F^j_b \neq F^j - c^j$, where c^j represents trader *j's* minimum average cost of buying the specified number of contracts, the bid is inefficient.

into the relative empirical importance of dividends, the wild card, and transactions costs.

2. Theoretical Factors Influencing the Occurrence of Early Exercise

2.1 Dividend payments

Merton (1973) provides a set of assumptions under which the only factors that can possibly trigger the early exercise of American calls are dividend payments and exercise price changes. Merton also provides the foundations for the Roll (1977), Geske (1979), Whaley (1981) American call option model by proving that early exercise will only occur just prior to dividend payments or exercise price changes. Since index options have constant exercise prices, we ignore the possibility of changing exercise prices and concentrate on dividends.

There is some disagreement in the literature concerning the relevance of dividend payments to the early exercise of American index call options. Several authors [e.g., Day and Lewis (1988) and French and Maberly (1992)] use European pricing models in studies involving S&P 100 index options, arguing that the nature of the dividend stream on the S&P 100 component stocks is such that the likelihood of dividend-induced early exercise is negligible. Others argue that the dividends on the S&P 100 stocks can be large enough to induce early exercise. Harvey and Whaley (1992a) estimate the historical dividend series for the S&P 100 stocks and use their series to show that pricing models that do not explicitly consider discrete cash dividends are likely to be misspecified. In a related article, Harvey and Whaley (1992b) present a model for pricing American index options with discrete cash dividends that explicitly incorporates early exercise.

It is possible to derive a minimum dividend payment ($\bar{D}_t$) that will trigger early call exercise at any point in time $t \in [0, T]$:

$$C(I_t - \bar{D}_t, K, r_f, T, \sigma, D) = I_t - K, \tag{3}$$

where $I_t - K$ is the value of the call if exercised early at time t, and $C(\cdot)$ is the value of the call if not exercised, as a function of the index level at time t (I_t), the strike price (K), the riskless rate (r_f), the time to maturity (T), the volatility of the index (a), and the vector of dividends on the index stocks over the remaining life of the option (D).

While condition (3) is commonly used in the literature, and there is a growing literature on early exercise, no previous tests consider whether investors and traders make exercise decisions in accordance with these commonly accepted conditions. This question is addressed empirically in Section 4 by examining the relationship between observed cases of index call option early exercise and dividends paid on the S&P 100 index.

For the tests that follow, it is convenient to modify condition (3) slightly. In practice, index stocks effectively go ex dividend when the markets are closed, and dividend-induced early call exercise should only occur just before the close of trading on the trading day prior to the ex dividend date in question. Defining I_t as the closing (cum dividend) index level, and $D_{\tau+\Delta\tau}$ as the dividend that will be paid after time τ and before the market opens on the following trading day, the dividend is large enough to induce early exercise if

$$C_d \equiv C(I_\tau - D_{\tau+\Delta\tau}, K, r_f, T, \sigma, D) \leq I_\tau - K \equiv C_x, \tag{4}$$

where C_d is the dividend-adjusted value of the call option if it is not exercised at time τ, and C_x is the time τ exercise value of the call.

Although dividend payments will not induce early put exercise and optimal early put exercise is more likely to occur after a dividend is paid, American index puts may still be rationally exercised prior to maturity. If we define $P(\cdot)$ as the American put value, a put should rationally be exercised if

$$P_d \equiv P(I_\tau - D_{\tau+\Delta\tau}, K, r_f, T, \sigma, D) \leq K - I_\tau \equiv P_x, \tag{5}$$

where P_d is the dividend adjusted value of the put option if it is not exercised at time τ, and P_x is the time-r exercise value of the put. In the absence of the wild card, the value of the American put will never be less than its current exercise value, and (5) can be written as an equality.

2.2 The wild card option

A second factor that may induce early exercise in the case of index options that are near maturity is the wild card feature. The wild card option allows holders of S&P 100 index options to exercise at any time prior to 3:15 P.M. at the closing index level. On days when the market drops substantially after the close, call options may be exercised early, while early put option exercise may be triggered by substantial postclosing increases.

As with dividends, there is some disagreement in the literature concerning the importance of the wild card option. Estimates of the wild card option made by Harvey and Whaley (1992b), Valerio (1991), and French and Maberly (1992) tend to be rather modest. However, the comparisons of dividend and wild card early exercise premiums provided by Fleming and Whaley (1992) indicate that at least for calls, the wild card premium can be larger than the dividend premium. Empirically, French and Maberly conclude that although the value of the wild card option is small relative to the value of the index option, it is still a major cause of early exercise.

Assuming the next trading day is not an ex dividend day for any of the stocks in the index $(D_{\tau+\Delta\tau} = 0)$, the conditions for the wild card option may be incorporated by modifying the closing index value to reflect postclosing changes. We define R_τ as the postclosing index return, so the condition for early call exercise becomes

$$C_w = C(I_\tau e^{R_\tau}, K, r_f, T, \sigma, D) \leq I_\tau - K, \tag{6}$$

and the condition for rational put exercise may be expressed as

$$P_w \equiv P(I_\tau e^{R_\tau}, K, r_f, T, \sigma, D) \leq K - I_\tau, \tag{7}$$

where C_w and P_w are the respective wild card adjusted values of the call and put options if they are not exercised.

Recognizing that dividends and the wild card may interact, we may combine conditions (3) and (6) to provide a general condition for early call exercise:

$$C_{dw} \equiv C(I_\tau e^{R_\tau} - D_{\tau+\Delta\tau}, K, r_f, T, \sigma, D) \leq I_\tau - K, \tag{8}$$

and an analogous condition for rational put exercise may be expressed as:[3]

$$P_{dw} \equiv P(I_\tau e^{R_\tau} - D_{\tau+\Delta\tau}, K, r_f, T, \sigma, D) \leq K - I_\tau, \tag{9}$$

where C_{dw} and P_{dw} are the values of the options if they are not exercised, adjusted for both dividend and wild card effects.

2.3 Transactions costs

If the transactions costs incurred by exercising an option are lower than the direct and indirect costs of selling an option, a rational trader may optimally choose to exercise an option, even if the dividend and wild card conditions for early exercise are not satisfied. By selling an option, the investor incurs both the indirect cost of the bid-ask spread and any direct costs associated with the sale. While there are no indirect costs associated with the exercise of an index option, even though they are cash settled, investors and traders still incur direct costs.

[3] The pricing functions $C(\cdot)$ and $P(\cdot)$ in (6) and (7) should price both dividend- and wild-card. related exercise. Valuing American index options with discrete dividends and the wild card option is, however, a complex problem, that has only recently been addressed by Fleming and Whaley (1992). In the tests of the importance of the wild card option that follow. we do not explicitly price the wild card option, which will tend to bias the results toward accepting the wild card as a relevant factor in early exercise decisions. We discuss this problem more fully in Section 4.

Somewhat surprisingly, the presence of differential direct transactions costs is not an issue for individual investors and other nonmember traders. Nonmember traders are charged the same commissions for exercises as sales. For Chicago Board Options Exchange (CBOE) market makers there is a difference in, the direct costs of exercising and selling an index option. At the clearing level, there is a charge of $1.00 per exercise, regardless of the number of contracts exercised. Market makers who clear their trades through a clearing member are charged $1.00 per contract exercised, up to a maximum of $20.00 per exercise block. For comparative purposes, we follow Phillips and Smith (1980) and assume a relatively high trading and clearing cost for a market maker executing a sale of $1.00 per contract. Based upon these estimates, the largest difference in costs would exist for market makers who clear their own trades. For exercises of a very large number of contracts by *a* clearing member, the difference in direct costs between exercising and selling could approach $1.00 per contract, or $0.01 for each point of the quoted option premium. Since premium and exercise settlement both occur on the following business day, timing differences need not be considered. Similarly, while the tax treatment of call options on individual issues of common stock may create incentives to exercise and postpone the realization of capital gains, since index options are cash settled, there are no differences in the tax consequences of exercising and selling.

Beyond these direct trading costs, nonmember traders and market makers may also incur the indirect cost of the bid-ask spread when trading with other market makers or the order book official. While the difference in direct costs appears to be trivial, the indirect cost of the spread associated with selling an option may induce optimal early exercise. The holder of a long American index call or put has the option of selling the contract at the bid or exercising the contract and avoiding the cost of the spread. This option is in the money if, when the position is closed, the bid is below the exercise value of the option, and the incremental cash flow (X_τ) at the time the position is closed is

$$X_\tau = \begin{cases} F_x - F_b, & \text{if } F_x > F_b \text{ and } \tau < T, \\ 0, & \text{otherwise,} \end{cases} \tag{10}$$

where F_x is the exercise value of the put or call and F_b is the bid price of the put or call.

The cash flows defined in (10) are the cash flows for a put option. The underlying asset is the bid price, and the exercise price of the put is equal to the original exercise price of the contract. The maturity of this put option to avoid the cost of the spread matches the holder's investment horizon. If the investor's horizon matches the stated maturity of the contract, this option has no value since the spread is not a cost for positions closed at maturity. In Section 7 we show that for a trader with a fixed horizon that is less than maturity, or a random horizon, this option may have value.

3. The Data

3.1 Data sources

The data are compiled from a variety of sources. The number of put and call contracts exercised from 4/13/83 to 12/31/88 are obtained from the Options Clearing Corporation's (OCC) Historical Exercise Report for OEX Options. The exercises are separated by option type, strike price, and maturity, but only aggregate daily totals by contract are contained in the report; information concerning the size of separate exercises is not available. While exercise notices can be filed at any time during the day, the actual mechanics of exercise always occur after the end of the trading day, and the exchange does not record the time at which the exercise notices are filed. We therefore define early exercise as any exercise that occurs prior to the last day of trading for the contract. Even though an exercise decision made prior to the close during the last day of trading would technically be a case of early exercise, since we do not know the time of day when the exercise notices are filed, all other cases of exercise are assumed to occur at maturity. In a number of instances, the OCC's Exercise Report contains strike prices and maturities that do not correspond to any listed options. These records and those containing other obvious data entry errors are eliminated from the sample.

The absence of information concerning the timing of exercise decisions introduces a few complications. When an index option is exercised, exercise should occur as late in the day as possible. In theory, both dividend and wild card induced exercise should only occur just prior to the close of options trading. More generally, regardless of the time of day at which the exercise notice is filed, the cash flows from exercise are realized on the following business day, and since the realized cash flows depend only on the closing index and the strike price, it is simple to show that waiting to exercise until after the closing index is known dominates any strategies involving exercise prior to that time. While these considerations might be used to justify the assumption that all exercise decisions are made just prior to the close of the CBOE, imperfections faced by individual investors may cause some options to be exercised earlier in the day.

In practice, individual investors trading through brokers are often required to notify their brokers of their exercise decisions before the closing index value is known. While we found both discount and full service brokers that would accept exercise notices until 3:15 P.M., many impose earlier deadlines, with some as early as 1:00 P.M. Since individual investors may face timing constraints that are not present for traders, the exercise decision can be more complex for individuals. By exercising before the closing index value is known, the individual faces the risk that a higher net cash flow could have been realized by selling the option. Exercising before the closing index is known can even result in out-of-the-money exercises with negative net cash flows. Depending on the preferences of the constrained investor, the gain from avoiding the cost of the spread may justify the risk of exercising before the close of the NYSE. If it turns out that the option could have been sold at a higher price, the decision to exercise will be inefficient, but because of the presence of the timing constraint the decision may still be rational.

Since it is possible for individuals to find brokers that will accept late exercise notices, an argument can be made that a rational investor that plans to exercise options should only trade through brokers that do not impose *early exercise* deadlines, and any exercises that occur prior to 3:15 P.M. are irrational. Although we do not know the times at which the exercises in our sample occur, we have been told that some contracts are exercised before 3:15 P.M. and that it is not unheard of for investors to exercise early in the morning, long before any broker-imposed deadlines. While we tend to favor the argument that any exercise that occurs prior to the 3:15 P.M. cutoff imposed by the CBOE is irrational, we accept the possibility that some individuals may be constrained, and we consider the implications of timing constraints for our tests of efficiency and rationality in Section 5.

Volume, index levels, and bid-ask quotes are obtained from the CBOE's Market Data Report (MDR) tapes. Since the index stocks do not trade after 3:00 P.M., the post-NYSE closing S&P 100 index return is estimated from transactions data obtained from the CME for the near-maturity S&P 500 index futures contract. Daily dividends on the S&P 100 and closing index levels are obtained from a series constructed by Whaley and Harvey, and the riskless rates required for estimating model prices are calculated from maturity-matched Treasury-bill quotes. Daily open interest for each contract is obtained from the CBOE, although the necessary data are unavailable prior to October 1985.

3.2 Historical exercise activity

Options on the S&P 100 index are the most heavily traded options in the world, and early exercise is common for both puts and calls. Table 1 summarizes the exercise activity for each type of option from 4/13/83 to 12/31/88, listing the total number of contracts exercised,

Table 1
Early exercise activity for S&P 100 index options: 4/13/83–12/31/88

Year	Trading days	Call contracts exercised	Calls exercised early	Put contracts exercised	Puts exercised early
1983	205	66,362	12,883	101,103	34,792
		(n.a.)	(n.a.)	(n.a.)	(n.a.)
1984	253	517,736	82,393	928,725	526,194
		(n.a.)	(n.a.)	(n.a.)	(n.a.)
1985	252	1,252,517	440,753	810,234	350,257
		(1.24)	(0.44)	(2.16)	(0.93)
1986	253	1,585,139	657,440	580,605	128,667
		(2.09)	(0.86)	(1.37)	(0.30)
1987	253	1,528,633	343,836	700,171	194,067
		(2.49)	(0.56)	(2.69)	(0.75)
1988	253	957,701	253,126	369,511	108,338
		(2.71)	(0.72)	(1.58)	(0.46)
Totals	1469	5,908,088	1,790,431	3,490,349	1,342,315

Early exercise is defined as exercise prior to maturity. Average daily exercise as a percentage of average daily open interest is listed in parentheses. Open interest data is not available prior to 1985.

the number of contracts exercised early, and exercise rates, calculated as the average daily exercise as a percentage of average daily open interest, by year and type of option. During the sample period 30 percent of the exercised calls and 38 percent of the exercised puts are exercised prior to maturity.

While the level of exercise activity appears small as a percentage of open interest and is on average less than 1 percent of average daily volume, these percentages are misleading. The mean ratios for all traded contracts do not reflect the fact that the percentage of open contracts exercised early is significantly higher for shorter-maturity in-the-money options. In addition, hedgers and other traders are often concerned with their ability to maintain an open long position for extended periods of time. While the percentage of open contracts exercised on any one day may be small, over longer periods total early exercise can be substantial.[4]

The greatest level of early exercise activity for S&P 100 calls occurs on the Thursday prior to expiration, and the exercise activity declines fairly steadily as the number of days to maturity increases. The earliest instance of early exercise for calls occurs 108 days before maturity, when 555 contracts are exercised. The instances of early exercise for

[4] Using historical open interest and exercise data, we estimate probabilities that an open contract will be exercised prior to maturity for different maturities and exercise values. For deep in-the-money calls and puts, our probability estimates run as high as .75 for the longer maturity options, and never fall below .50 for options with *at* least a week to maturity. Furthermore, the incidence of early exercise is. in fact, high enough to concern traders. Near the close, traders monitor the box on the Boor of the exchange where exercise notices are deposited, and our conversations with traders suggest that they incorporate this information in their trading strategies.

Table 2
Exercise activity categorized by the day of the week: 4/13/83–12/31/88

	Exercised during maturity week		Exercised during all other weeks	
Day	Number of cases	Contracts exercised	Number of cases	Contracts exercised
Calls				
Friday	434	4,117,657	400	371,859
Thursday	254	342,218	242	208,927
Wednesday	179	173,390	192	157,920
Tuesday	118	138,073	214	167,815
Monday	115	121,030	158	109,209
	1100	4,892,358	1206	1,015,730
Puts				
Friday	288	2,148,034	426	223,454
Thursday	156	126,848	356	179,347
Wednesday	136	131,259	347	150,660
Tuesday	135	153,897	334	96,757
Monday	129	129,815	365	150,278
	844	2,689,853	1828	800,496

A case is defined as the exercise of a particular contract on a particular date. Exercises occurring at expiration are listed as occurring on Friday of the maturity week. All of the other entries in the table are cases of early exercise.

S&P 100 put options are distributed more uniformly than the call option exercises during the week of expiration. The level of early put exercise activity generally decreases with the number of days to maturity, but there are more cases of extreme early exercise for puts. The earliest case of put exercise occurs 133 days prior to maturity, and while the number of contracts exercised tends to be relatively small, early put exercise is consistently observed up to 110 days before maturity.

It is interesting to note that exercise activity tends to be unusually high on nonexpiration Fridays for both puts and calls. Table 2 contains the total number of call and put contracts exercised for each day of the week, together with the number of times early exercise occurred on each day of the week for each type of option. Since the majority of index options are exercised at maturity or during the maturityweek, exercises are separated into those occurring during the maturity week and all other weeks. Early exercise is more commonly observed on Fridays than on any other day of the week for puts and calls, and the total number of contracts exercised also tends to be the greatest on Fridays for both types of options. Some possible explanations for this apparent day-of-the-week effect are considered in Section 6.

It is clear that early exercise is common for both put and call contracts on the S&P 100 index, and a substantial number of contracts are exercised more than one month prior to maturity. While dividend payments on the underlying index stocks and the presence of the wild card option may at least partially explain the cases of exercise that occur near maturity, the instances of exercise that occur long before maturity suggest that the existing theory of early exercise may not adequately describe the exercise decisions of traders and investors.

4. Empirical Relevance of Dividends, the Wild Card, and Transactions Costs

We evaluate the relative empirical importance of the three factors that are expected to induce rational early exercise by classifying the cases of early call and put exercise on the basis of whether each case is consistent with exercise induced by dividends and the standard American boundary conditions, the wild card option, or by transactions costs. To conduct these tests, it is necessary to limit our sample to observations where we have estimates of all of the necessary option pricing model parameters as well as a quoted bid price for the option that is recorded after the exercise value of the contract is part of the trading crowd's information set.

Due to reporting delays and day end return effects [see, for example, Wood, McInish, and Ord (1985) or Harris (1986)], the index level reported on the MDR tapes (and the floor of the CBOE at the 3:00 P.M. CST close of the NYSE) differs systematically from the actual closing index level. Figure 1 illustrates this problem. Average index levels are calculated from the MDR tapes at two-minute intervals for the entire sample period, from 8:32 A.M. until 3:14 P.M., one minute before the close of the CBOE. Since the index level continues to be posted and updated even after the 3:00 P.M. CST stock market close, we can use the index levels reported after the NYSE close to estimate the magnitude of reporting delays for the index stocks. The first solid vertical bar represents the 3:00 P.M. CST NYSE close, and the second solid bar is drawn at 3:16 P.M., one minute after the close of the CBOE. To check for errors in the index values reported on the MDR tapes, we use the actual closing index level reported by Standard and Poors as the 3:16 P.M. index value.

Consistent with previous studies, most of the daily return on the index stocks occurs near the open and the close. Because of reporting delays, prior to 3:06 P.M., the mean reported index level is significantly different from the mean closing index level. The dashed bar at 3:06 P.M. represents the point at which the reported index is not significantly different from the closing index level. At this point we can safely assume that the closing index level is part of the trader's information set, and for the tests that follow we use the last bid registered after 3:06 P.M. While reported delays do not preclude the possibility

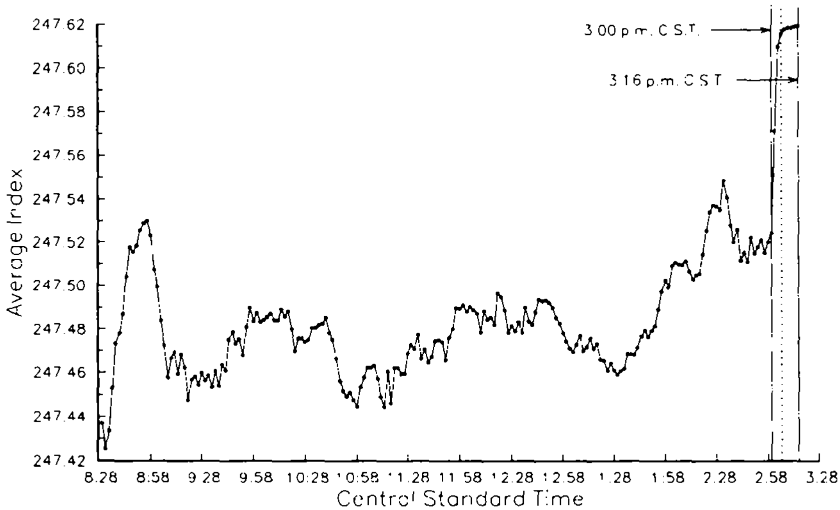

Figure 1
Two-minute average S&P 100 index levels: June 1, 1985-December 31, 1988
Average index levels from the CBOE's MDR tapes are plotted at two-minute intervals from 8:32 A.M. CST to 3:14 P.M. CST for the sample period. The vertical bar drawn at 3:00 represents the closing time of the NYSE, and the dotted line drawn at 3:06 represents the cutoff time used for the bids in our sample. The last index level (plotted at 3:16) is the average official closing index level reported by Standard and Poors for the sample period.

that the closing index is known prior to 3:06 P.M., we adopt a conservative cutoff time, and if no bids are recorded after 3:06 P.M. the observation is discarded. Since all index stocks do not trade near the close, even the reported closing index level may be a stale estimate of the true closing index level, but because exercise value is determined by the reported closing value, this is not a problem for our study.

After eliminating any observations where the option is exercised out-of-the-money, the recorded bid and contemporaneous quotes on other contracts imply arbitrage violations, and any observations where the reported bid is inconsistent with previous bid and trade prices, we obtain 1366 cases of early call exercises, and 1550 cases of early put exercise. The average number of calls exercised for each case is 1158 contracts, and the average number of puts exercised is 754 contracts.

4.1 Test procedures

For each observed case of early call exercise, the exercise value of the call (C_x) and the last bid recorded after 3:06 P.M. (C_b) are compared to model prices that reflect the impact of dividends and the wild card. If we assume that price formation in the options markets is consistent with the assumptions of the pricing models, these model prices may be interpreted as estimates of F^i in condition (2), the true value to the trader that is exercising the option. C_x is calculated as the difference between the closing index value reported by Standard and Poors and the strike price. The value of C_d, the dividend adjusted value of the call if not exercised, ignoring any wild card effects, is estimated using the specification in (5) and a binomial American call pricing model, adjusted for discrete cash dividends throughout the life of the option. Volatility is estimated by calculating binomial dividend-adjusted American call implied volatilities at 15-minute intervals using the mean of the bid and ask prices recorded closest to the close of the 15-minute interval for the shortest maturity, closest-to-the-money call with at least seven days remaining prior to maturity. The intraday volatilities are averaged on a daily basis to obtain the estimate of volatility.[5]

The estimation of C_w, the value of the call with the wild card effect only, is more difficult because the required postclosing index return is unobservable. Following Harvey and Whaley (1992b), Valerio (1991), and French and Maberly (1992), we use the 3:00 P.M.–3:15 P.M. return on the near maturity S&P 500 futures contract as a proxy for the post-NYSE closing S&P 100 index return. To separate any dividend induced effects from the wild card effects, the ex dividend amount is set to zero, and C_w is estimated by adjusting the recorded index level for the observed postclosing futures return as specified in condition (6), using the American binomial model adjusted for any subsequent dividend payments.[6] C_{dw} is then estimated by using (8) with the actual ex dividend amount together with the wild card adjustment.

Analogous prices, P_x, P_b, P_d, P_w, and P_{dw} are estimated for the sample

[5] To approximate "true" prices, we use the mean of the bid-ask spread and eliminate indirect transactions costs from the estimated model prices. The nonsimultaneous price problem discussed by Harvey and Whaley (1991) can be reduced by averaging put and call implied volatilities. Using averages calculated in this manner does not materially affect the results of the tests presented in this section. More generally, since the classification tests conducted depend on the change in value induced by dividends and the wild card rather than on estimates of absolute prices, and since the options can be exercised after the exercise value is known, the tests are relatively insensitive *to* volatility estimates.

[6] More specifically, since completely ignoring the impact of the ex dividend amount on the index would provide biased estimates of call prices, to isolate the wild card effects, we simply move the dividend to the second binomial period and set the dividend in the first binomial period to 0. This guarantees that as long as the futures return is nonnegative, the model price of the call will be strictly greater than the exercise value. Since we have more than one binomial period per trading day, this procedure does not affect our modeling of the subsequent dividends. For consistency, we use the same procedure for puts even though shifting the dividend by one period does not have a material effect on our estimates of put prices. Ignoring the remaining value of the wild card may bias our estimates of C_u downward. Since we compare C_w to the exercise value, our tests may inflate the importance of the wild card. This problem is partially mitigated by calculating implied volatilities without pricing the wild card. For our sample of 1366 observations, the mean model prices (C_w) we estimate, ignoring any future wild card effects, are \$0.017 lower than the mean midpoint of the quoted spread.

of early put exercise, using (5), (7), and (9). The implied volatilities used for pricing the puts are average intraday implied volatilities calculated for short-maturity at-the-money puts with maturities of at least seven days.

The relative importance of each factor is assessed by classifying each case of early call or put exercise into one of six mutually exclusive categories. For F as a generic symbol for the call *(C)* or put *(P)* prices, the categories used are

1. Dividends only ($F_d \leq F_X \cap F_u, > F_x$): Ignoring any wild card effects, dividends alone (or, in the case of puts, the standard American boundary conditions) are consistent with the observed case.
2. Wild card only ($F_d > F_x \cap F_w \leq F_x$): Ignoring the overnight dividend payment, the wild card alone is consistent with the case.
3. Dividends plus wild card ($F_d > F_x \cap F_w > F_x \cap F_{dw} \leq F_x$): The combined effects of dividends and the wild card are sufficient, while neither factor alone is sufficient.
4. Either dividends or wild card ($F_d \leq F_x \cap F_w \leq F_x$): The case is consistent with both dividends alone and the wild card alone.
5. Transactions costs ($F_d > F_x \cap F_w > F_x \cap F_{dw} > F_x \cap F_b < F_x$) : None of the dividend or wild card classes are consistent with the case, but the cash flow from exercising exceeds the cash flow from selling.
6. Unexplained ($F_d > F_x \cap F_w > F_x \cap F_{dw} > F_x \cap F_b > F_x$): The case is not consistent with any of the explanations listed above.

The first four classes consist of exercises that are consistent with the factors that have previously been proposed in the literature. The fifth class is composed of exercises that may be explained by transactions cost differentials and the option to avoid the spread. The last class, consisting of contracts that are exercised even though a bid that exceeds the exercise value has been registered after the closing index level is known, cannot be explained by any of the other factors, unless they simply represent data errors. If these cases are not the result of measurement errors or market imperfections, they are the result of irrational exercise decisions.

4.2 Classification results for calls

Table 3 contains the number of call contracts and percentages of contracts exercised in each category, together with some additional descriptive information concerning the observations in each class. The mean maturity, percentage of open interest, and median percentage of open interest are weighted by the number of contracts, so a case involving 100 contracts is weighted 10 times as heavily as a case involving 10 contracts.

Table 3
Early call exercise explained by dividends, the wild card, and transactions costs

Exercise condition	Number of cases	Number of contracts exercised	Percentage of total exercise	Mean daily exercise	Mean post-closing volume	Mean days to maturity	Mean (median) % of open interest
$C_d \le C_x \cap C_w > C_x$ (div only)	63	103,813	6.56	1648	268	10	25.15 (22.01)
$C_d > C_x \cap C_w \le C_x$ (w.c. only)	322	352,141	22.26	1093	127	7	21.15 (9.16)
$C_d > C_x \cap C_w > C_x \cap C_{dw} \le C_x$ (div plus w.c.)	16	17,800	1.13	1112	12	16	17.37 (7.60)
$C_d \le C_x \cap C_w \le C_x$ (either div or w.c.)	41	40,067	2.53	977	108	8	18.20 (9.90)
$C_d > C_x \cap C_w > C_x \cap C_{dw} > C_x \cap C_b < C_x$ (transactions costs)	461	762,994	48.24	1655	195	9	24.03 (15.12)
$C_d > C_x \cap C_w > C_x \cap C_{dw} > C_x \cap C_b > C_x$ (unexplained)	463	304,974	19.28	656	254	12	12.40 (2.32)
Totals	1366	1,581,789	100.00				

Early exercise is defined as exercise prior to maturity. C_x is the exercise value of the call. C_d is the dividend adjusted American binomial model estimate of the midpoint of the spread, C_w is the wild-card-adjusted American binomial model price, C_{dw} is the dividend- and wild-card-adjusted American model price, and C_b is the last bid recorded after 3:06. Mean postclosing volume is the average number of contracts traded from 3:00-3:15. Open interest reflects the number of contracts exercised. and the percentage of open interest is calculated as contracts exercised/(open interest + contracts exercised). Mean percentages of open interest are calculated from exercises occurring between 10/1/85 and 12/31/88. The sample period is 4/13/83-12/31/88.

For our sample of 1366 cases of early call exercise, less than 7 percent of the contracts exercised can be explained by dividends alone, while 22 percent can be explained by the wild card alone. The combined effects of dividends and the wild card only account for an additional 1 percent that cannot be explained by either factor alone. At more than 48 percent, the indirect transactions costs of the spread appear to be more important than dividends and/or the wild card. While we can explain the majority of early call exercise, 19 percent of the contracts are exercised when the bid and all of the estimated model prices are higher than the exercise value of the contract.

The last column in Table 3 lists the mean and median percentages of open interest exercised for each of the classes. At 25 percent, the mean ratio is highest for dividend related exercise, but it is clear that early exercise is not an all-or-none phenomenon. Assuming that traders agree upon model specification and the values of the relevant variables, option pricing theory implies that if dividend or wild card boundary conditions for early exercise are met, all open contracts should be exercised. Unless traders are indifferent to exercising, which would occur in the unlikely case that the exercised value is exactly equal to the value of the open contract, either all of the open contracts or none of the open contracts should be exercised on any given day. The fact that the observed exercises represent a relatively small percentage of open interest suggests that option values are trader specific.

The unexplained exercises occur more frequently than any of the other types, but the average number of contracts exercised is smaller. These cases also tend to occur earlier than the others, with the exception of the contracts that can be explained by the combined effects of dividends and the wild card. Since the combined effects of dividends and the wild card can potentially overcome more remaining time value than either factor alone, this exception is not surprising. It is also interesting to note that the volume during the last 15 minutes of trading for these contracts does not appear to be unusually low, but is actually higher than average. The average number of contracts exercised in the unexplained category is 656, which is less than half the average for the other categories. The percentage of open interest exercised under these conditions is also the lowest of all of the classes. On average, these exercises account for only 12.4 percent of open interest, and half of the contracts exercised in the unexplained category make up less than 2.3 percent of open interest. This suggests that these cases are qualitatively different from the explained cases and are not the result of classification errors. The differences in frequency, size, timing, and percentage of open interest for the unexplained exercises suggest that these cases are systematically different from the ones we are able to explain.

4.3 Classification results for puts

Table 4 contains the classification results for the put option sample. To facilitate comparisons with Table 3, we refer to the case where the standard American boundary alone is sufficient to induce exercise as dividend induced. There are no cases in the third category of Table 4 because while the combination of a large dividend and a negative postclosing return may cause put exercise to be optimally postponed, synergy effects between dividends and the wild card should not induce early put exercise. Compared to the call results, the distribution across the remaining categories is more uniform for puts. The wild card appears to be marginally more important than dividends, and the two factors explain 62.3 percent of the puts exercised prior to maturity. Transactions costs explain about 25 percent of the puts exercised early, and only 12 percent of the contracts remain unexplained. The unexplained instances tend to involve a relatively small number of contracts (467 on average), a very small proportion of open interest, and the mean postclosing volume for the unexplained exercises is higher than any of the other classes. Once again, the unexplained exercises differ systematically from the cases that we are able to explain.

Table 4
Early put exercise explained by dividends, the wild card, and transactions costs

Exercise condition	Number of cases	Number of contracts exercised	Percentage of total exercise	Mean daily exercise	Mean post-closing volume	Mean days to maturity	Mean (median) % of open interest
$P_d \le P_x \cap P_w > P_x$ (div only)	215	221,498	18.95	1030	171	7	18.12 (8.43)
$P_d > P_x \cap P_w \le P_x$ (w.c. only)	467	305,071	26.10	653	122	14	25.74 (12.95)
$P_d > P_x \cap P_w > P_x \cap P_{dw} \le P_x$ (div plus w.c.)	0	0	0.00	0	n.a.	n.a.	n.a. (n.a.)
$P_d \le P_x \cap P_w \le P_x$ (either div or w.c.)	256	201,209	17.21	786	85	8	24.51 (14.40)
$P_d > P_x \cap P_w > P_x \cap P_{dw} > P_x \cap P_b < P_x$ (transactions costs)	289	290,242	24.83	1004	146	20	23.43 (11.30)
$P_d > P_x \cap P_w > P_x \cap P_{dw} > P_x \cap P_b > P_x$ (unexplained)	323	150,983	12.91	467	277	15	6.25 (0.81)
Totals	1550	1,169,003	100.00				

Early exercise is defined as exercise prior to maturity. P_x is the exercise value of the put, P_d is the dividend-adjusted American binomial model estimate of the midpoint of the spread, P_w is the wild-card-adjusted American binomial model price, P_{dw} is the dividend- and wild-card-adjusted American model price, and P_b is the last bid recorded after 3:06. Mean postclosing volume is the average number ofcontracts traded from 3:00-3:15. Open interest reflects the number ofcontracts exercised, and the percentage of open interest is calculated as contracts exercised/(open interest + contracts exercised). Mean percentages of open interest are calculated from exercises occurring between 10/1/85 and 12/31/88. The sample period is 4/13/83-12/31/88.

To summarize our findings concerning the relative importance of the factors that are expected to induce rational early exercise, for the period from 4/13/83 to 12/31/88 we find that dividends account for only a small proportion of early call exercise, while the wild card is relatively more important. Transactions costs account for more cases of early call exercise than the other two factors combined, but a substantial number of early call exercise decisions remain unexplained. For puts, the majority of the cases can be explained by dividends and the wild card, but transactions costs are also a major determinant. A smaller proportion of put exercise is unexplained. The relatively high incidence of unexplained exercise suggests that some of the observed exercise decisions are inefficient and may be irrational. In the following section, we present the results of simple tests of efficiency that are independent of any pricing models.

5. Tests of Efficiency and Rationality

Our definition of efficiency with respect to the quoted bid requires that the exercise value of the contract be at least as great as the last

Table 5
Early exercise activity for S&P 100 index call options classified by the bid price minus the exercise value

Range of E_t	Number of cases	Contracts exercised early	Percentage of early exercise	Percentage explained	Mean daily exercise	Mean postclosing volume	Mean % of open interest	Median % of open interest
$E_t >$ 4.00	1	4	0.00	0.00	4	12	n.a.	n.a.
$3.50 < E_t \leq$ 4.00	0	0	0.00	n.a.	0	n.a.	n.a.	n.a.
$3.00 < E_t \leq$ 3.50	0	0	0.00	n.a.	0	n.a.	n.a.	n.a.
$2.50 < E_t \leq$ 3.00	4	151	0.01	0.00	38	185	0.05	0.03
$2.00 < E_t \leq$ 2.50	4	36	0.00	0.00	9	115	0.94	0.05
$1.50 < E_t \leq$ 2.00	9	4,143	0.26	95.99	460	316	19.68	6.03
$1.00 < E_t \leq$ 1.50	31	11,135	0.70	74.64	359	133	3.27	0.24
$0.50 < E_t \leq$ 1.00	70	26,546	1.68	37.32	379	137	11.63	1.44
$0.25 < E_t \leq$ 0.50	121	34,619	2.19	54.00	286	229	7.64	1.00
$0.00 < E_t \leq$ 0.25	431	381,500	24.12	36.46	855	216	14.67	5.15
$-0.25 < E_t \leq$ 0.00	630	1,080,992	68.34	100.00	1716	208	25.49	16.81
$-0.50 < E_t \leq -0.25$	45	36,802	2.33	100.00	818	21	33.70	25.51
$-1.00 < E_t \leq -0.50$	17	3,947	0.25	100.00	232	8	22.62	15.29
$-1.50 < E_t \leq >1.00$	3	1,914	0.12	100.00	638	0	25.89	16.67
Totals	1366	1,581,789						

Early exercise is defined as exercise prior to maturity. E_t is defined as the bid price recorded for the call after 3:06, minus the exercise value of the call, adjusted for differential direct transactions costs. The percentage explained is the percentage of contracts exercised in each class that is consistent with dividend, wild card, or transactions costs explanations. Mean postclosing volume is the average number of contracts traded from 3:01-3:15. Open interest reflects the number of contracts exercised, and the percentage of open interest is calculated as contracts exercised/(open interest + contracts exercised). Mean and median percentages of open interest are calculated from exercises occurring between 10/1/85 and 12/31/88. The sample period is 4/13/83-12/31/88.

bid quoted after the exercise value of the contract is known. Since all of the bids used in the empirical tests are recorded after the closing index level is known, the hypothesis of efficiency can be tested directly. Modifying condition (2) to reflect the fact that there may be as much as a l-cent difference in the direct costs of selling and exercising, exercise is inefficient if

$$E_t \equiv F_b^j - F_x - 0.01 > 0, \tag{11}$$

where F_b^j is the bid for the put or call, and F_x is the exercise value.

To examine the efficiency of early call exercise, values of E_t are calculated for each case of early exercise using the sample that is used to construct Table 3. Table 5 contains the number of contracts exercised, together with additional descriptive information, classified by values of E_t. The number of contracts in each class as a percentage of total early call exercise, and the percentage of contracts in each class explained by dividends, the wild card, or transactions costs, are also included.

The majority of the exercise is efficient, and most occurs in the –0.25-0.00 class. Inefficient exercise does, however, occur frequently; 29 percent of the exercise occurs when the bid is higher than the exercise value, and while the more extreme cases only represent a small number of contracts, a few contracts are exercised when the bid is more than $4.00 above the exercise value. The prevalence of inefficient exercise suggests that some of this exercise may also be irrational.

To classify exercise as irrational, we also must be sure that in addition to satisfying (11), the contracts could actually have been sold at the quoted bid. In some cases, lack of depth at the quoted bid might prevent a trader from selling at the quoted price, and although the exercise would be inefficient it is still rational. Although direct evidence concerning OEX options market depth is not available, Vijh (1990) finds that for CBOE common stock options, liquidity is excellent. The consistently high volume for S&P 100 options suggests that the same conclusions are likely to apply to the CBOE index options. In practice, lack of depth does not appear to be a major problem: the large OEX options traders we spoke with did not feel that depth was a significant constraint, and while we are unable to measure market depth directly we generally find substantial trading volume for exercised options after the NYSE close.

While depth may not be a major factor, the trader's ability to locate the bid in a trading crowd of 400 may explain some of the instances of inefficient, and seemingly irrational, exercise decisions. When a trading strategy involves closing a position near the end of trading, even though the trader can see the posted bid, it may not be possible to locate the person that made the bid. Failure to locate the bid may prevent the trader from closing a position. This appears to be a real concern for OEX options traders, and in such cases, traders may rationally decide that the potential costs of trying to locate the bid outweigh the loss incurred by exercising at a lower net price. Similar considerations also apply to individual investors. Even if an individual trades through a broker that allows exercise decisions to be made after the closing index value is known, the time required for order placement and execution may make it impossible to get a sell order filled during the last few minutes of trading on heavy trading days.

Although we do not have the data to directly measure depth or the trader's search costs, Table 5 contains strong evidence that at least some of the observed exercise activity is irrational. For the extreme cases where $E_t > 2.00$, none of the cases are consistent with dividends or the wild card, the mean postclosing volume is greater than the mean number of contracts exercised (suggesting that depth is not a problem), and a careful check of index levels, surrounding quotes, and trade prices reveals nothing unusual. Furthermore, although it is not shown in the table, in every one of these instances trades occurred at prices that were at least as high as the bid during the last five minutes of trading, and for the contracts where open interest is available, the exercises are uniformly less than one percent of open interest.

Since it is possible that individual investors facing early broker-imposed exercise cutoff times may be responsible for these exercises, we also compare the bids registered between 1:00 P.M. and 3:00 P.M. to the difference between the concurrent index value and the strike price. Empirically, we find no support for this explanation. In all cases, these bids are consistently greater than the index level recorded at the time of the bid, minus the strike price. These extreme cases of inefficient exercise clearly appear to be irrational.

For the intermediate cases where $1.00 < E_t \leq 2.00$, the fact that most of the exercised contracts can be explained by dividends or the wild card implies that many of these cases may be rational. All of the explained cases in these classes are, in fact, consistent with wild-card-induced early exercise and occur when there is a substantial post-NYSE close drop in the S&P 500 futures price. Since each of these cases also involves exercise of at least 500 contracts, these may be attributable to lack of depth at the quoted bid. Even though we use bids quoted after the closing index is known, traders would be unlikely to buy more than 10 contracts at that bid once a substantial drop in the futures price has occurred.[7] The unexplained exercises in this intermediate class are qualitatively similar to the exercises that occur when $E_t > 2.0$, and for the same reasons, it may be argued that these too are irrational. Once again, a close record-by-record examination of the cases that are not explained by dividends or the wild card fails to reveal any other rational explanation for why they occurred.

For the remaining cases of inefficient exercise that occur when $0.00 < E_t \leq 1.00$, some may be due to traders that are unable to locate the bid, and some may be attributable to individuals that face timing constraints for exercises and sales. Since less than half of these cases are consistent with dividends or the wild card, and since they tend to represent a smaller fraction of open interest than the efficient exercises, it seems likely that in addition to being inefficient with respect to the quoted bid, at least some are irrational. When the bid is more than $0.50 above the exercise value, none of the model based conditions for early exercise are satisfied, the exercise is a very small percentage of open interest, and there is substantial volume relative

[7] Currently, under the CBOE's rule 8.51b (effective July 1989), traders are required to buy a minimum of 10 contracts at the displayed bid when a sell order reaches the trading Boor. During our sample period, a five-contract minimum was in effect after January 1987. Prior to that time there was no minimum requirement.

to the number of contracts exercised, a strong case can be made that these decisions are irrational.

In the more marginal cases where the bid is less than \$0.25 above the exercise value, the case for irrationality is not as strong, for it is likely that due to the presence of market frictions many of the inefficient exercise decisions are rational. Unfortunately, since we cannot observe investor's utility functions or trader's demand curves, and since we do not know who is responsible for the exercise decisions or when they occur, it is not possible to unambiguously distinguish between rational and irrational decisions for the more marginally inefficient cases of early exercise.

An examination of the cases of exercise that are efficient with respect to the quoted bid ($E_t < 0$) in Table 5 suggests another potential source of inefficiency. Since these bids are made when the exercise value is known, we would expect the bids to be closer to the exercise value. Buying one contract at a bid that is \$0.50 below the exercise value and exercising represents a riskless cash flow of \$50.00, and it is hard to imagine that the trader's costs of buying and exercising could be this large. When the bids are this low, exercise is efficient with respect to the quoted bid, but the bid itself may be inefficient. While a thorough examination of this question is beyond the scope of this article, this may be a promising direction for future research into the efficiency of the options markets[8]

Table 6 contains the results for the sample of puts classified by E_t. Exercise is inefficient for about 35 percent of the put contracts, but most of these are in the marginally inefficient 0.00-0.25 class. There are more extreme positive values of E_t for puts than for calls, but all of the exercises for $E_t > 3.00$ occur on October 19 and 20, 1987. Because of the unusual market conditions on those days, we cannot determine whether these extreme cases are rational, but clearly they are inefficient. More generally, since we are able to explain a larger percentage of the put exercise with the wild card and the standard American boundary conditions, the case for irrationality is weaker for index put options than it is for index calls. When we examine the put exercise on a case-by-case basis, there are, however, definite instances of unexplained exercise involving a small number of contracts that are clearly irrational.

The lower incidence of irrational put exercise suggests an interesting possibility: the markets for index calls and puts may be char-

[8] Vijh (1990) considers some of the determinants of the spread for CBOE common stock options, but different factors may be relevant for index options. For example, while rule 8.07.02 of the CBOE requires that in the absence of unusual market conditions, market makers must ordinarily refrain from purchasing common stock options at more than \$0.25 below parity. this rule does not apply to index options.

Table 6
Early exercise activity for S&P 100 index put options classified by the bid price minus the exercise value

Range of E_t	Number of cases	Contracts exercised early	Percentage of early exercise	Percentage explained	Mean daily exercise	Mean postclosing volume	Mean % of open interest	Median % of open interest
$E_t >$ 4.00	8	47	0.00	89.36	6	48	2.12	0.02
$3.50 < E_t \leq$ 4.00	1	3	0.00	100.00	3	0	1.36	1.36
$3.00 < E_t \leq$ 3.50	1	10	0.00	0.00	10	921	0.02	0.02
$2.50 < E_t \leq$ 3.00	3	13	0.00	61.53	4	79	0.52	0.52
$2.00 < E_t \leq$ 2.50	4	69	0.01	85.51	17	0	0.61	0.61
$1.50 < E_t \leq$ 2.00	14	158	0.01	43.04	11	146	3.52	0.31
$1.00 < E_t \leq$ 1.50	29	2,035	0.17	88.94	70	256	12.88	0.74
$0.50 < E_t \leq$ 1.00	88	20,059	1.72	63.58	228	161	12.87	1.20
$0.25 < E_t \leq$ 0.50	113	71,374	6.11	58.59	632	132	12.79	3.36
$0.00 < E_t \leq$ 0.25	435	314,648	26.92	67.44	723	243	14.88	5.34
$-0.25 < E_t \leq$ 0.00	733	736,418	63.00	100.00	1005	137	29.51	19.57
$-0.50 < E_t \leq -0.25$	81	12,077	1.03	100.00	149	9	16.80	8.33
$-1.00 < E_t \leq -0.50$	28	11,321	0.97	100.00	404	2	51.28	54.03
$-1.50 < E_t \leq -1.00$	8	721	0.06	100.00	90	1	31.07	18.20
$-2.00 < E_t \leq -1.50$	1	20	0.00	100.00	20	0	12.50	12.50
$-2.50 < E_t \leq -2.00$	2	10	0.00	100.00	5	0	0.67	0.67
$-3.00 < E_t \leq -2.50$	0	0	0.00	n.a.	0	n.a.	n.a.	n.a.
$-3.50 < E_t \leq -3.00$	1	20	0.00	100.00	20	14	1.31	1.31
Total	1550	1,169,003						

Early exercise is defined as exercise prior to maturity. E_t is defined as the bid price recorded for the put after 3:06, minus the exercise value of the put, adjusted for differential direct transactions costs. The percentage explained is the percentage of contracts exercised in each class that is consistent with dividend, wild card, or transactions costs explanations. Mean postclosing volume is the average number of contracts traded from 3:00-3:15. Open interest reflects the number of contracts exercised, and the percentage of open interest is calculated as contracts exercised/(open interest + contracts exercised). Mean and median percentages of open interest are calculated from exercises occurring between 10/1/85 and 12/31/88. The sample period is 4/13/83-12/31/88.

acterized by different clienteles of investors. If it is assumed that unsophisticated investors are responsible for the observed cases of irrational early exercise, the lower incidence of irrational put exercise suggests that the market for index calls tends to appeal to more unsophisticated investors, while the market for index puts may attract relatively more sophisticated investors. The fact that puts are more commonly used for portfolio insurance strategies would be consistent with the existence of put and call clienteles.[9] Unfortunately, since we do not have access to the necessary data, we can only speculate, but the comparatively small number of contracts exercised in the cases that appear to be irrational does suggest that small traders are making these exercise decisions.

To summarize, for both calls and puts, a substantial proportion of

[9] Using a similar argument, Harvey and Whaley (1992b) suggest that the use of puts for portfolio insurance may explain differences in index put and call implied volatilities.

the observed exercise is inefficient, and although our inability to measure market depth and search costs does not always allow us to identify irrational exercise decisions, at least some of the inefficient exercise is also irrational.

6. Day-of-the-Week Patterns

The total contracts exercised early for each day of the week listed in Table 2 suggest a concentration on Fridays. For exercise that is unexplained, inefficient, and possibly irrational, there is no reason to expect a day-of-the-week pattern, but there are several reasons why rational early exercise may occur more frequently on particular days of the week. To the extent that early exercise is dividend induced, day-of-the-week patterns in dividend payments on the index stocks might influence the likelihood of early exercise on particular days. Harvey and Whaley (1992a) find a U-shaped day-of-the-week pattern for dividends, with a tendency for dividends to be highest on Mondays. Higher call exercise on Fridays (before the Monday dividend) and higher put exercise on Tuesdays would be consistent with this finding.

In a similar fashion, systematic patterns in post-Friday closing (weekend) returns might affect wild card exercise patterns. The well-documented significant negative returns over weekends would tend to make wild-card-induced call exercise higher on Fridays; since larger negative postclosing returns are more commonly observed for Fridays than for other days of the week, it is more likely that wild card boundary conditions for call options will be satisfied on Fridays. The opposite effect would be expected for puts. Negative returns after Friday's close would prevent the wild card boundary from being reached for put options.

Early exercise that can be explained by transactions costs may also exhibit a day-of-the-week pattern. For the portion of our sample where open interest is available, open interest declines significantly on nonexpiration Fridays. Traders may tend to close positions at the end of the week because of margin requirements, an inability to hedge over the weekend, or other exogenous factors. Excess selling pressure would also cause bids to be lower, and the indirect cost of the spread would cause traders to exercise if the bids are below the exercise value of the contract.[10] Exercise induced by transactions costs would therefore tend to be higher on Fridays for both puts and calls.

Since Table 2 only contains daily exercise aggregates, and different reasons for exercise may induce different day-of-the-week patterns, a

[10] See Admati and Pfleiderer (1989) for an example of a model that can explain the presence of concentrated selling at particular times.

closer examination of these patterns is warranted. We examine these patterns by regressing the number of contracts exercised for each of our observations on a set of day-of-the-week dummy variables while controlling for maturity and moneyness. The analysis is conducted for both puts and calls, for the full samples and each of the subsamples used for Tables 3 and 4. The estimated model is

$$N = \hat{\alpha} + \hat{\beta}_1 T + \hat{\beta}_2 M + \hat{\beta}_3 \mathrm{MON} + \hat{\beta}_4 \mathrm{TUE} + \hat{\beta}_5 \mathrm{THU} + \hat{\beta}_6 \mathrm{FRI} + \hat{\epsilon}, \tag{12}$$

where N is the number of contract exercised, and $M = I_t/K$ for calls and K/I_t for puts. We use OLS as a convenient method of estimating (incremental) conditional mean exercise for each day of the week. To ensure that our conclusions are not attributable to specification errors, we also conduct nonparametric tests of the day-of-the-week effects, with similar results. The OLS estimates are presented in Tables 7 and 8.

For the full samples, the Friday effect is not as strong as Table 2 might suggest: the FRI coefficient is only marginally significant for calls and insignificant for puts. While the FRI coefficient for the dividend-only call subsample is not significant, the TUE coefficient for the dividend-only put sample is positive. Although the coefficient is only significant at the 10 percent level, this supports the hypothesis that traders may tend to wait to exercise puts until after the relatively high Monday dividends are paid.

Our previous results indicate that the wild card is stronger than the dividend effect for calls, and for the wild-card-only call sample exercise is significantly higher on Fridays. This is consistent with the hypothesis that expectations of negative returns over weekends can induce early call exercise. For the transactions cost subsamples, there are significant Friday effects for both puts and calls. Our previous results indicate that a higher percentage of call exercise is attributable to transactions costs, and the effect is stronger for calls than for puts.

Consistent with our hypothesis that there are no systematic patterns for the unexplained cases of early exercise, none of the day-of-the-week dummies are significant for this group. For the calls, the coefficients on T and I/K are also insignificant, providing further support for the notion that the unexplained cases are not the result of classification errors.

7. The Price Impact of the Option to Avoid the Spread

Several previous studies have used numerical approximations of dividend and wild card premiums to judge their importance, but none have considered the potential value of the option to avoid the bid-

Table 7
Day-of-the-week regression results for call early exercise

Subsample	Number of observations	Intercept	Coefficient of T	I/K	MON	TUE	THU	FRI
Full	1366	13,246	−19.555	−10,962	71.458	37.578	214.881	433.661
		(0.000)	(0.000)	(0.000)	(0.771)	(0.874)	(0.312)	(0.067)
Dividends only	63	11,283	−40.014	−29,000	667.693	428.932	−24.662	−869.566
		(0.004)	(0.476)	(0.006)	(0.632)	(0.738)	(0.985)	(0.724)
Wild card only	322	9,869	−1.318	−8,146	−349.475	1.485	479.843	958.847
		(0.000)	(0.935)	(0.001)	(0.441)	(0.997)	(0.182)	(0.028)
Div plus w.c.	16	19,685	−62.454	−15,595	−1312.823	−20.986	−135.552	186.201
		(0.351)	(0.515)	(0.428)	(0.507)	(0.996)	(0.957)	(0.921)
Either div or w.c.	41	20,579	−13.284	−16,804	−1365.118	−786.003	−933.711	n.a.
		(0.002)	(0.874)	(0.006)	(0.168)	(0.451)	(0.342)	(n.a.)
Transactions costs	461	27,653	−35.232	−23,354	240.555	−132.020	−35.304	831.771
		(0.000)	(0.002)	(0.000)	(0.652)	(0.796)	(0.941)	(0.095)
Unexplained	463	1,395	−6.406	−576	−146.598	−36.308	236.564	−271.166
		(0.453)	(0.209)	(0.738)	(0.600)	(0.897)	(0.318)	(0.288)

The estimated model is

$$N = \hat{\alpha} + \hat{\beta}_1 T + \hat{\beta}_2 (I/K) + \hat{\beta}_3 \mathrm{MON} + \hat{\beta}_4 \mathrm{TUE} + \hat{\beta}_5 \mathrm{THU} + \hat{\beta}_6 \mathrm{FRI} + \hat{\epsilon},$$

where N is the number of contracts exercised, T is days to maturity, I is the closing index, K is the exercise price, and MON, TUE, THU, and FRI are day-of-the-week dummy variables. OLS is used for estimation, and significance levels are reported in parentheses. For the dividend or wild card subgroup, no contracts are exercised on Fridays, and the FRI dummy is omitted from the regression. The sample period is 4/13/83-12/31/88.

Table 8
Day-of-the-week regression results for put early exercise

			Coefficient of					
Subsample	Number of observations	Intercept	*T*	*K/I*	MON	TUE	THU	FRI
Full	1550	6,602	−15.126	−5,175	115.427	8.894	25.819	246.594
		(0.000)	(0.000)	(0.000)	(0.439)	(0.952)	(0.860)	(0.120)
Dividends only	215	11,640	−20.994	−9,795	181.625	736.199	247.566	−27.282
		(0.000)	(0.008)	(0.001)	(0.756)	(0.093)	(0.577)	(0.972)
Wild card only	467	4,718	−13.105	−3,561	154.689	64.157	−156.411	242.448
		(0.000)	(0.001)	(0.000)	(0.502)	(0.800)	(0.518)	(0.324)
Either div or w.c.	256	7,977	−18.112	−6,089	−10.871	−713.509	−306.997	−118.990
		(0.000)	(0.007)	(0.000)	(0.975)	(0.032)	(0.353)	(0.831)
Transactions costs	289	15,058	−26.082	−12,735	510.436	100.422	399.264	1097.151
		(0.000)	(0.000)	(0.000)	(0.210)	(0.804)	(0.342)	(0.008)
Unexplained	323	3,996	−9.212	−3,221	−43.484	131.411	332.859	195.317
		(0.012)	(0.005)	(0.030)	(0.875)	(0.622)	(0.225)	(0.455)

The estimated model is

$$N = \hat{\alpha} + \hat{\beta}_1 T + \hat{\beta}_2 (K/I) + \hat{\beta}_3 \mathrm{MON} + \hat{\beta}_4 \mathrm{TUE} + \hat{\beta}_5 \mathrm{THU} + \hat{\beta}_6 \mathrm{FRI} + \hat{\epsilon},$$

where N is the number of contracts exercised. *T* is days to maturity, *J* is the closing index, *K* is the exercise price, and MON, TUE, THU, and FRI are day-of-the-week dummy variables. OLS is used for estimation, and significance levels are repotted in parentheses. The sample period is 4/13/83-12/31/88.

ask spread. Since our results indicate that this option is at least as important as the others in explaining observed early exercise, we provide some simulated values of this option that are estimated using parameter values that are representative of the options in our sample. The simulations are conducted for index calls only, but the same procedures can be used for valuing the option to avoid the spread for index puts.

Defining 0 as the current time, T as the maturity of the call, and t_x, as the predetermined point at which the position will be closed ($0 < t_x \leq T$), we use Boyle's (1977) Monte Carlo technique to simulate the time t_x index level. To simplify the problem, we assume that there are no dividends and no wild card effects, so that we can use the Black-Scholes model to estimate the true time t_x value of the call for the simulated index level.[11] For our sample, the quoted spreads range from \$0.0625 to \$1.00, and the mean spread is about 2.5 percent of the average of the bid and ask prices. Assuming that spreads are quoted symmetrically around the Black-Scholes value, the quoted bid at time t_x is approximated as

$$C_b = C_e - 0.5\mathrm{MIN}[1.00, \mathrm{MAX}[0.0625, 0.025C_e]], \tag{13}$$

where C_e is the Black-Scholes price for a European call calculated using the simulated time t_x index level and a maturity of $T - t_x$. For each of the simulated index levels at time t_x, the maximum of the exercise value and the estimated bid from (13) is discounted at the riskless rate to time 0, and the mean discounted value is used as the estimate of the time 0 value of the American index call option. A European index call option value is also estimated by assuming the call is always sold at the bid at time t_x. The difference in these two values is the estimate of the premium associated with the option to exercise and avoid the spread.

The simulations are conducted using 90 time periods and 500 repetitions with control and antithetic variates. Table 9 contains the results of the simulations for a maturity of 10 days, a riskless rate of 6 percent, a volatility of 16 percent, and a strike price of 200. The simulations are conducted for values of the holding period (t_x) ranging from 1 to 10 days and current index values of 200, 220, and 240.

The simulated American and European call prices for $t_x = T = 10$ are both equal to the Black-Scholes price because neither option is

[11] Simulating the value of the option for puts requires setting the exercise value of the index put option equal to the strike price minus the index value and using an American put pricing model instead of the European Black-Scholes call model. The binomial model can also be modified to incorporate the option to avoid the spread for both puts and calls. The modification involves using the option values at the time t_x nodes to estimate the spread, subtracting half the spread from these values, and setting the prices at the nodes to the maximum of the adjusted value and the exercise value.

Table 9
Simulated American premiums for the option to avoid the spread

Number of days to exercise	American price (index = 200)	American premium (index = 200)	American price (index = 220)	American premium (index = 220)	American price (index = 240)	American premium (index = 240)
1	2.2450	0.0000	20.0751	0.0005	40.0329	0.2085
2	2.2431	0.0000	20.0834	0.0089	40.0657	0.2413
3	2.2416	0.0000	20.1054	0.0309	40.0986	0.2742
4	2.2405	0.0000	20.1341	0.0595	40.1314	0.3070
5	2.2396	0.0000	20.1656	0.0910	40.1643	0.3399
6	2.2392	0.0003	20.1978	0.1233	40.1972	0.3728
7	2.2402	0.0015	20.2304	0.1558	40.2300	0.4056
8	2.2447	0.0048	20.2630	0.1884	40.2629	0.4385
9	2.2540	0.0127	20.2959	0.2213	40.2957	0.4713
10	2.2796	0.0000	20.3287	0.0000	40.3285	0.0000
Mean premium		0.0019		0.0880		0.3059

The parameter values used in the simulations are time to maturity = 10 days, riskless rate = 6%, strike price = 200, and volatility = 16%. The bid-ask spread is assumed to be MIN[1.00, MAX[0.0625. $0.025C_e$]], where C_e is the Black-Scholes value. The simulations use 90 periods with 500 repetitions, and all of the estimated standard errors of the simulated prices are less than 0.003.

exercised prior to maturity, and the Black-&holes model is used as a control variate in the simulations. Since closing a position prior to maturity involves incurring the cost of the spread or exercising an option with remaining time value, the simulated American and European prices are always less than the Black-Scholes price when the positions are closed prior to maturity. This illustrates the fact that there must be some exogenous reason for closing the position; otherwise the position should be held to maturity.

Table 9 shows that conditional upon a fixed holding period, the premium for the option to avoid the spread can be substantial, at least for relatively short maturity deep in-the-money options. For the at-the-money call ($I = K = 200$) the value of the option is close to zero regardless of the holding period, but for the calls that are in-the-money the value of the option to avoid the spread increases as the holding period approaches maturity. The mean premiums listed in Table 9 provide rough estimates of the value of the option to avoid the spread for random (uniformly distributed) holding periods. While the mean premium for the deep in-the-money options of $0.31 is only a small fraction of the average total call value, it does represent $30.59 on a contract basis.

The option to avoid the spread will be in-the-money when bids are below exercise values, which is more likely to occur when the value of the underlying call option is close to the exercise value. This will be true for short-maturity, deep in-the-money options and when the trader's horizon is close to, but less than, the maturity of the option. The question of whether this option is priced is beyond the scope of this article, but it is interesting to note that Sheikh (1991) finds that short-maturity deep in-the-money S&P 100 index calls are systematically underpriced by a dividend-adjusted version of the Black-Scholes model. While there are other factors that may also explain this underpricing, it is consistent with what we would expect if market prices reflect the value of the option to avoid the spread.

While we only conduct the simulations for index calls, the option to avoid the spread is also valuable for short-maturity index puts that are deep in the money. As the index level decreases and the put moves deeper in the money, the probability that the option will be in the money at the end of the investor's holding period increases. At the same time, the difference between the value of the put and its exercise value decreases, and it becomes more likely that the bid will be below the exercise value at the end of the holding period.

Since the value of the option to avoid the spread increases with index level for calls and decreases with the index level for the puts, it is possible that the differences in implied index put and call volatilities documented by Harvey and Whaley (1992b) might be at least partially explained by the option to avoid the spread. This explanation, however, seems doubtful, since Harvey and Whaley use near-the-money options, and the value of the option to avoid the spread is likely to be small unless the options are deep in the money.

Although ignoring the option to avoid the spread when pricing at-the-money options is not likely to introduce pricing errors, biases may be introduced if the cost of the spread is not considered for in-the-money American options. Our empirical results show that the cost of the spread is at least as important as dividends and the wild card in explaining the observed cases of early exercise, and to accurately price American index options it may be necessary to allow for all three factors. Furthermore, in addition to the biases reported by Sheikh (1991), a second piece of evidence supports the idea that the option to avoid the spread may be priced by the markets. The fact that the ratio of volume to open interest is much higher for the S&P 100 American options than for the S&P 500 European options suggests that traders with shorter horizons prefer to trade American options that can be exercised without incurring the cost of the spread, while traders with longer horizons prefer the European options. These preferences are consistent with a positive price for the option to avoid the spread. If it is unlikely that the position will be closed prior to maturity, European options would be the preferred choice; otherwise, the trader would be paying for an option that would not be used.

Unlike the values of dividend- and wild-card-induced early exercise, the value of the option to avoid the spread depends on the investor's holding period, and standard arbitrage arguments may not provide a feasible method of determining an equilibrium price for this option unless the holding period is known. Pricing this option for an unknown holding period appears to be an interesting area for future research that may improve the accuracy of existing pricing models.

8. Summary and Conclusions

This article provides a comprehensive empirical analysis of early exercise for S&P 100 index calls and puts. By comparing bid quotes to exercise values we are able to unambiguously show that these markets do suffer from some inefficiencies, even though they are generally considered to be the most efficient options markets in the world. These tests are independent of any option pricing models. We are also able to show that at least some of the inefficient exercise is also irrational.

In addition to our tests of efficiency, we use dividend- and wild-card-adjusted American boundary conditions to examine the relative impact of these two factors on early exercise. For calls, dividends explain a relatively small proportion of the observed cases of early exercise, and the wild card appears to be more important. The most important factor, however, appears to be the indirect cost of the index option bid-ask spread. Approximately half of the call exercise occurs when dividends and the wild card effect are insufficient for early exercise, but bids are below exercise values. The distribution of put exercise across the three factors is more uniform.

The results of these classification tests suggest that index option pricing models that do not incorporate dividends and the wild card are likely to be misspecified for American index options. Furthermore, since the cost of the spread is responsible for a large proportion of the observed early exercise, our results support the notion that ignoring transactions costs may also lead to specification errors.

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