Production Flexibility, Stochastic Separation, Hedging, and Futures Prices

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We study a dynamic model where uncertainty about interim output adjustments causes producers to face price, cost and output uncertainty. Stochastically separable production decisions are independent of the producer's risk preferences and expectations and are based on the prevailing futures price as a certain output price. Conditions under which futures contracts achieve stochastic separation are established Optimal hedging and maturity structure of futures contracts, equilibrium futures prices, and the effects of futures trading on output are studied. The systematic risk premium depends on the product of the futures beta and the covariance of the market return with production revenues.

When a complete set of Arrow-Debreu securities is traded, all producers, regardless of their risk preferences or beliefs, agree on the appropriate production decisions. But what if markets are incomplete? Will futures contracts provide the spanning function of Arrow-Debreu securities? Townsend (1978) shows that if there are as many linearly independent spot prices as there are states of the world, then futures

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contracts have the spanning property of a complete set of Arrow-Debreu securities. This result, while of great theoretical interest, provides little insight into the effective role of futures markets in our economy because of the limited number of futures contracts compared with states of the world. Townsend's article thus leaves open the interesting question: are there reasonable economies where spanning will occur when the number of traded futures contracts is significantly lower than the number of possible states of the world?

Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) show that when producers under price uncertainty can trade futures contracts on their output, their optimal production rule is as follows: choose the input level that equates the marginal product valued at the prevailing futures price with the marginal cost. That is, all producers produce as if they lived in a world of certainty where the output price they face is the prevailing futures price. Because production decisions are independent of the producer's risk preferences and expectations, they are called stochastically separable. When a firm's production decisions are stochastically separable, its managers and shareholders agree unanimously on the optimal output levels [Baron (1976)].

These studies, which provide important insights into producers' production decisions and hedging behavior, examine a two-date world. This study extends the analysis of the behavior of producers under price uncertainty in the presence of futures markets into a dynamic world. In a dynamic model, unless the production process is instantaneous, producers can adjust their output levels at interim dates. (It is the existence of an option to adjust the output level at interim dates, at a cost, that is the critical difference between the static and dynamic models.) Therefore, even if production and costs functions are deterministic, the output level and cost of production are uncertain at the beginning of the production process because producers are uncertain of what output adjustments they may wish to make on interim dates in response to new information regarding output prices.[1]

We examine the following issues. Section 1 establishes conditions on the production and cost functions that achieve stochastic separation in a dynamic world. Section 2 examines the implications of these conditions for temporal hedging patterns in futures markets and the optimal maturity structure of futures contracts and provides empirical support for these implications. Section 3 examines the effects

[1] Anderson and Danthine (1983b) and Marcus and Modest (1984) study stochastic separation of production and optimal hedging in a multiperiod setting. This article is more general than theirs in that they do not allow for interim production decisions, and thus essentially study a two-date production setting. Their articles are more general than this in that they also study producers with stochastic production functions (that is, when input levels are known: output is deterministic in ours but stochastic in theirs).

of futures trading on optimal output, and establishes conditions under which futures trading increases output. Section 4 studies the implications of the flexibility to adjust output levels at interim dates for optimal hedging in futures and stock markets, extending the two-date models of Stoll (1979) and Hirshleifer (1988a).[2] Section 5 derives the equilibrium futures prices and examines the implications for tests for futures risk premia.

The study provides new insights into the determinants of futures risk premia. The systematic risk premium depends on the net hedging position in the dynamic model but not in the static model. The residual (nonmarketable) risk premium depends on the net hedging position in both the dynamic and static models. Moreover, all CAPM-based tests of futures risk premia [see, for example, Dusak (1973), Carter, Rausser, and Schmitz (1983), and Bessembinder (1992)] assume that the systematic risk premium is a linear function of the futures beta and is a function of beta alone. This study shows that, in a dynamic world, the premium is nonlinear with beta and is not a function of beta alone but of the product of beta and the covariance of the market return with production revenues.

1. The Stochastic Separation of Production Decisions

Consider a three-date model (t = 1, 2, 3) with a perfectly competitive representative producer with a nonstochastic production function $G(X_1, X_2)$, where X_1 and X_2 are the levels of inputs used at time 1 and time 2. The final output, $G(X_1, X_2)$, is available on date 3. The nonstochastic factor pricing function, describing the cost of inputs, is $C(X_1, X_2)$.[3] On date 2 the producer faces uncertainty about the

[2]Karp (1988), Ho (1984), and Hirshleifer (1991) also study optimal hedging in a multiperiod world with production inflexibility and stochastic production functions. Anderson and Danthine (1983a) and Hirshleifer (1988b, 1989) study optimal hedging and equilibrium futures prices in special cases of a multiperiod model with production flexibility. Anderson and Danthine (1983a) and Hirshleifer (1988b) study a two-stage production process: growers who produce an intermediate commodity choose their output level on date 1 and produce if on date 2, and processors who use the intermediate commodity as input on date 2 to produce the final output. Only futures contracts on the intermediate good from date 1 to date 2 exist. Anderson and Danthine (1983a) also study production inflexibility by examining the case where processors must choose their input level (for dare 2) on date 1 instead of on date 2. Hirshleifer (1989) studies a single-period production model, but "output" next period can be adjusted through storage. The futures contracts are single-period contracts. The treatment of production flexibility here is more general in that production is a multiperiod process and producers react to information arriving during the process. This study also allows producers to hedge in a stock market and in both single- and multiperiod futures contracts. Breeden (1984) studies optimal hedging and consumption allocation in a continuous time multigood exchange economy with instant-maturity commodity futures and options contracts.

[3]The production function obeys $G_1(X_1, X_2) > 0$, $G_{1,}(X_1, X_2) < 0$, $G_2(X_1, X_2) > 0$, and $G_{22}(X_1, X_2) < 0$, where subscripts denote partial derivatives with respect to the corresponding argument. Moreover, $G(X_1, 0) = 0$ and $G(0, X_2) = 0$. so the production process must be a three-date process. The factor pricing function obeys $C_{1,}(X_1, X_2) > 0$, $C_{11}(X_1, X_2) \geq 0$, $C_2(X_1, X_2) > 0$, and $C_{22}(X_1, X_2) \geq 0$. Turnovsky (1973) studies the behavior of producers with production flexibility without futures markets.

output price on date 3. On date 1 the producer faces, *in addition,* uncertainty about the final output level and the cost of production because X_2 is unknown on date 1.[4] The producer can also trade on date 1 futures contracts maturing on date 3. The futures contract is an obligation to deliver or accept a delivery of one unit of the output, on its maturity date, at a known price. For simplicity, we assume in this section that producers have no other sources of income.[5] The producer's profits at date 3 are

$$\pi = {}_3P_3 G(X_1, X_2) - C(X_1, X_2) + {}_2F_3({}_3P_3 - {}_2P_3) + {}_1F_3({}_2P_3 - {}_1P_3), \tag{1}$$

where ${}_tP_T$ is the price on date t of a futures contract calling for a delivery of one unit of output on date T; the futures price on the contract's expiration date, ${}_TP_T$, equals the spot price; and ${}_tF_T$ is the number of futures contracts maturing on date T held on time t, where $F > 0$ represents a purchase and $F < 0$ represents a short position.

Interest rates are nonstochastic, and all cash flows are expressed in their date 3 values. The futures positions generate gains or losses of ${}_1F_3({}_2P_3 - {}_1P_3)$ on date 2 and gains or losses of ${}_2F_3({}_3P_3 - {}_2P_3)$ on date 3. This reflects the fact that producers can adjust their futures position at any time during the life of a futures contract.

The producer maximizes the expected value of a concave utility function of her profits. Let $E_t[U(\pi)]$ denote the expected utility conditional on the information available on date t. Futures prices are unknown prior to their trading date, but the producer knows their joint distribution. On date 2, the producer chooses X_2 and ${}_2F_3$, depending on X_1 and ${}_1F_3$,. On date 1, the producer chooses (in a recursive fashion) X_1 and ${}_1F_3$, taking into account her subsequent adjustment possibilities. Let $G_i(X_1, X_2)$ and $C_i(X_1, X_2)$ denote partial derivatives of $G(X_1, X_2)$ and $C(X_1, X_2)$ with respect to X_i $(i = 1, 2)$. The first-order conditions on date 2 are[6]

$$C_2(X_1, X_2) E_2[U'(\pi)] = G_2(X_1, X_2) E_2[U'(\pi)\,{}_3P_3] \tag{2}$$

[4]One way is to think of X_1 as capital and X_2 as labor. Another is to think of X_1 as some combination of capital of labor and allow the producer to adjust output level in response to new information by adjusting labor and capacity utilization on date 2. In this latter case, the factor pricing function is not necessarily linear.

[5]The producer will also be able to trade in (i) futures contracts expiring on date 2-that is, in futures contracts expiring before final output becomes available (which can also settle in cash)-and (ii) the stock market. Explicitly studying these additional assets does not alter the results in this section but makes the derivation more cumbersome. It is therefore postponed to subsequent sections.

[6]The sign of the second partial derivative with respect to the futures position equals the sign of $U''(\pi)$. Thus, an interior solution holds for risk-averse producers.

and

$$ {}_2P_3E_2[U'(\pi)] = E_2[U'(\pi)\,{}_3P_3]. \tag{3}$$

Consequently,

$$C_2(X_1, X_2) = {}_2P_3G_2(X_1, X_2). \tag{4}$$

The optimal level of input on date 2 is the level that equates the marginal product valued at the prevailing futures price with the marginal cost [Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980)]. The futures contract allows producers to produce as if they lived in a world of certainty in which the output price they face is the prevailing futures price. One property of this result is that the producer's risk preferences and expectations do not affect her production decision. The production decision is therefore called *stochastically separable.* The futures contract completes the set of markets upon which she, *as a producer,* relies because her revenues from output are known with certainty. When the production decisions are stochastically separable, output levels and incremental revenues from output produced by X_2 are identical in all states of the world on date 3. If the production and factor pricing functions are identical, output levels are identical for all producers.

Definitions. *A production function $G(X_1, X_2)$ is said to be intertemporally separable if $G_1(X_1, X_2)$ is independent of X_2.*

A factor pricing function $C(X_1, X_2)$ is said to be intertemporally separable if $C_1(X_1, X_2)$ is independent of X_2 [for example, $C(X_1, X_2) = a(X_1) + b(X_2)$].

Proposition 1. *On date 1, if (i) the production function is intertemporally separable, (ii) the factor pricing function is intertemporally separable, and (iii) there exists a futures contract with time to maturity exactly matching the production process (i.e., with delivery on date 3), the optimalproduction rule is*

$$C_1(X_1, X_2) = {}_1P_3G_1(X_1, X_2). \tag{5}$$

Proof: All proofs are in Appendix A.

In a dynamic setting producers can adjust their output levels at the interim period. Therefore, even if the factor pricing and production functions are deterministic, producers are uncertain on date 1 of what output adjustments they may make at date 2. With intertemporally-complementary production and factor pricing functions, $G_1(X_1, X_2)$ and $C_1(X_1, X_2)$ are unknown on date 1, and the production decision on date 1 is[7]

$$E_1[U'(\pi)\cdot C_1(X_1, X_2)] = E_1[U'(\pi)\cdot G_1(X_1, X_2)\cdot {}_2P_3]. \tag{6}$$

Consequently, with intertemporally-complementary production and factor pricing functions, the output decision on date 1 depends on the producer's risk preferences and expectations,

The first condition in Proposition 1 ensures that interim decisions do not affect the marginal productivity of the initial input, so that $G_1(X_1, X_2)$ is deterministic on date 1. The second condition ensures that interim decisions do not affect the marginal cost of the initial input, so that $C1\ (X_1, X_2)$ is deterministic on date 1. The third condition ensures that the producer can fully hedge the revenues from the output produced by X_1 so that the value of the marginal productivity of X_1 is certain on date 1. Together these conditions allow the production decisions to be stochastically separable.[8,9] The futures contract ensures that whenever a producer with intertemporally separable production and factor pricing functions decides on an input, the value of marginal productivity of *that input* and the *incremental* revenues from the output produced by that input are known with certainty. That is, the contract eliminates the uncertainty of production revenues in a sequential manner. For a given futures price on date 1, the revenues from output produced by X_1 are certain on date 1, but the incremental revenues from output produced by X_2 remain uncertain. Then, for a given futures price on date 2, the incremental revenues from output produced by X_2 become certain on date 2. Futures trading enables the producer, on each date, to choose the input level that equates the marginal product valued at the prevailing futures price with the marginal cost [Equations (4) and (5)].

The futures contract also achieves stochastic separation for producers with production inflexibility (i.e., producers who cannot adjust their output on date 2). In this case, as in a two-date model, production revenues uncertainty is completely eliminated on date 1. Whenever needed, we distinguish between stochastic separation in the static (two-date or many dates with production inflexibility) model and in the dynamic model by defining the latter as *dynamic stochastic separation.*

[7]Using Equations (A3) and (3).

[8]The factor pricing function is likely to be intettemporally separable when X_1 and X_2 are different inputs at different points in time. But when X_1 and X_2 are different amounts of the same input and the price of X_2 is nonlinear with X_2, the factor pricing function is intertemporally complementary. Also, when X_1 and X_2 are different amounts of same input, diminishing returns imply that $G_{12}(X_1, X_2) < 0$.

[9]Suppose that $C(X_1, X_2)$ is intertemporally separable. Without stochastic separation G_{11} must be negative for the second order condition to hold, but with stochastic separation G_{11} must be non-positive. Hence, stochastic separation affects the requirements on marginal products and costs.

The requirements of Proposition 1 also hold for risk-neutral producers. With intertemporally-complementary production and factor pricing functions, their production decisions depend on their expected output price, expected marginal productivity, and expected cost of input on date 1. Hence, the ability of futures markets to achieve stochastic separation of production decisions depends on intertemporal inflexibilities and intertemporal linearities in the production and factor pricing functions and not on linearities in risk preferences.

2. Implications for Temporal Hedging Patterns

Although Section 1 assumed the existence of a futures contract expiring on date 3, the production process is sometimes only spanned by a continuous sequence of short-lived (single-period in our model) futures contracts. In this case, producers hedge on date 1 in a futures contract expiring on date 2 [with profits of ${}_1F_2({}_2P_2 - {}_1P_2)$] and then roll over their positions into a futures contract expiring on date 3 [with profits of ${}_2F_3({}_3P_3 - {}_2P_3)$]. Because the difference between ${}_2P_3$ and ${}_2P_2$ can be stochastic on date 1, the production decisions of producers with intertemporally separable production and factor pricing functions need not be stochastically separable on date 1. A sufficient condition for dynamic stochastic separation on date 1 for these producers is that the relation between ${}_2P_3$ and ${}_2P_2$ is exactly linear so that they can perfectly hedge the price spread risk on date 1.[10]

The ability of futures contracts to achieve stochastic separation thus depends on the existence of intertemporal separabilities and intertemporal inflexibilities in the production process, and, when the production process is spanned only by a continuous sequence of short-lived futures contracts, on the existence of exact linearity in the relation between the prices of these contracts. When these conditions fail, the length of the period of commitment in the production process (i.e., the period in which output cannot be adjusted), the maturity dates of futures contracts, and the degree of nonlinearity in the relation between prices of consecutive futures contracts emerge as crucial determinants. This is best illustrated by the following example: Consider a four-date world with a producer, a storer, and an exporter, and suppose that the relations between prices of different futures contracts are not linear. The producer, who has intertemporally separable production and factor pricing functions, uses inputs on dates 1 and 2 and produces on date 3. The exporter makes a binding commitment on date 1 to export given quantities on dates 2 and 4. The storer decides on each date on the optimal level of storage. The simulta-

[10] For (static) stochastic separation, exact linearity is both necessary and sufficient.

neous existence on date 1 of futures contracts for dates 2, 3, and 4 completes the set of markets on which each of them can rely. But, the exporter does not need the futures contract expiring on date 3, whereas the producer needs only the futures contract expiring on date 3 and, as we will see, needs to revise the futures position on date 2. Finally, the storer needs all three contracts, but does not need them to exist prior to a single period before expiration.

The simultaneous existence of many futures contracts will increase the cost of trading in each of the contracts. The greater the number of futures contracts that are traded simultaneously, the smaller the number of traders that are likely to trade in each contract, and the higher the liquidity cost of trading in each of them. The equilibrium maturity structure of futures contracts is, therefore, a function of (i) the periods of "real" commitments of the potential traders, (ii) the degree of linearity in the relation between the prices of these contracts, (iii) the stability of these price relations over time," and (iv) the relation between the number of futures contracts with different maturities and the cost of trading. The degree of linearity in the relation between prices of different futures contracts serves as a measure of the hedging and informational substitution of the contracts.[12] When the price relation is close to linear, the hedging effectiveness of the nearest-maturity contract is close to that of a futures contract with time to delivery exactly matching the production process; and the implicit price of the latter can be deduced accurately from the price of the former.

It is common that even when several futures contracts are traded simultaneously, hedging in futures contracts tends to concentrate in the nearest-maturity contract. The nearest-maturity futures contract is typically the most liquid and least costly to trade contract. The foregoing analysis provides an explanation for this phenomenon. Dealers and storers have a relatively short period of commitment in the cash market. The speculators' period of commitment in the futures market is also short. Dealers, storers, speculators, and producers whose period of commitment is relatively short thus tend to trade in the nearest-maturity contract, whereas producers whose period of commitment is relatively long choose among the various contracts by weighing

[11] While both a futures contract maturing on date 3 and a sequence of single-period futures contracts achieve stochastic separation with an exactly linear price relation, producers need not be indifferent between them. Adjusting for differences *in* trading costs, *any* producer, regardless of the properties of her production and factor pricing functions, is indifferent between the two if the relation between their prices is (i) exactly linear and (ii) constant over time. These conditions are necessary and sufficient for producers with production inflexibility who face normally distributed prices.

[12] Hedging substitution also depends on price elasticities of agents' demand functions [Hirshleifer (1988b, 1989)]. Duffie and Jackson (1989) develop a model of optimal innovations of futures contracts.

deviations from linearity in their prices versus differences in the costs of trading in them.[13] Futures contracts with different maturities would thus actively trade simultaneously when the deviations from linearity in their prices are large or when these price relations are unstable over time. But when the price relations are close to linear and are stable over time, distant maturity futures contracts would only be thinly traded. These results apply to the existence of hedging markets on different assets as well. Therefore, the absence of an explicit hedging market for a certain asset or of long-term futures contracts should not be taken prima facie as evidence of inefficiencies or market failures.

A comparison of the concentration of hedging in the lumber and Standard and Poor's 500 (S&P) futures markets provides empirical support for the foregoing implications for temporal hedging patterns. The lumber futures market is chosen because it is characterized by relatively volatile futures price spreads [Fama and French (1987)].[14] In contrast, the relations between consecutive S&P futures prices are approximately linear. Moreover, being a financial futures market, the S&P futures market has a high fraction of hedgers with a short period of commitment relative to commodity futures markets. Consider the months of July during 1984-1987. In July, the nearest-maturity contracts in both markets are the September contracts, and futures contracts maturing six months later, in March, are traded in both markets. (Subsequent lumber futures contracts are for November, January, and March, whereas subsequent S&P futures contracts are for December and March.) The squared coefficient of correlation between daily changes in September and March S&P futures prices in the sample period was between .985 to .998, indicating that the relation between September and March S&P prices was close to linear. In contrast, the squared coefficient of correlation between daily changes in September and March lumber futures prices was between .842 and .882, suggesting that the relation between September and March lumber prices deviated considerably from linearity. Open interest is the number of (long or short) futures contracts that are open (i.e., have not been liquidated by an offsetting trade or fulfilled by delivery). The open interest in a futures contract is largely determined by hedging needs [Gray and Peck (1981)]. Consistent with our analysis, open interest in the September contract constituted 94 to 97 percent of total open interest in the S&P market, but only 40 to 58 percent of total open

[13]Section 4 suggests that all producers with production flexibility revise their hedging positions on date 2. The cost of hedging in distant contracts rather than a near contract can thus be substantial.

[14]While lumber is a storable commodity, the delivery specifications in the futures market are such that lumber delivered in one month cannot be redelivered in a subsequent month. Consequently, the cost of a cash-and-carry arbitrage is very high.

interest in the lumber market. (The data are from the *Yearbook* of the Chicago Mercantile Exchange.)

When can one expect the price relation between ${}_2P_2$ and ${}_2P_3$ to be approximately linear? One case is when the spot asset is storable and the marginal cost of storage is a linear function of the prevailing spot price.[15] Two factors can cause the relation between ${}_2P_2$ and ${}_2P_3$ to deviate from linearity: (i) stochastic interest rates and (ii) a dependence of the marginal cost of storage on inventory levels. Working's (1949) supply-of-storage theory suggests that the marginal cost of storage includes a convenience yield that is positive for low inventory levels and zero for large inventory levels. Because interest rate risk can be hedged in financial (futures) markets, the convenience yield emerges as the critical factor in the relations between futures and spot prices and between prices of consecutive commodity futures contracts.[16] Thus, the hedging and informational substitution of near and distant commodity futures contracts, and intertemporal hedging patterns, depend on the levels of inventories. One would expect the relation between consecutive futures prices to deviate considerably from linearity when inventory levels are very low.

The open interest in a futures contract typically reaches a peak as the contract becomes the nearest to maturity and starts falling when the delivery date approaches. Because the nearest to delivery futures contract is usually the least costly contract, some hedgers with longer-term spot commitments hedge in the nearest-maturity futures contract and roll over their hedges to a subsequent contract as the delivery **date** approaches. This analysis suggests that, ceteris paribus, one would expect hedgers with long-term spot commitments to constitute a smaller fraction of open interest in nearest-maturity contracts when inventory levels are very low than when inventory levels are high.

The behavior of the wheat market from 1976 through 1982 supports this analysis.[17] The wheat market experienced a period of very low inventories in the first half of 1979. As a result, March futures prices on the first trading day in March 1979 were higher than May futures prices by about 7 percent, reflecting a high convenience yield. In contrast, March futures prices were lower than May prices on that day in 1976-1978 and 1980-1982, the three-year periods before and after 1979. (The data are from the *Statistical Annual* of the Chicago Board of Trade.) Consider the ratios of the amount of open interest in the March wheat futures contract on the first delivery day in March to the

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[15] Let γ be 1 plus the riskless interest rate, and let a be the (constant) marginal cost of physical storage. Then, with costless arbitrage, ${}_2P_3 = \alpha + \gamma\, {}_2P_2$.

[16] Shalen (1989) studies the effects of sensitivity to convenience yield by different types of production activities on optimal maturity of hedges,

[17] Gray and Peck (1981) is a detailed study of the wheat market during this period.

amount of open interest in this contract on the first trading days in January and February. (The contract becomes the nearest to maturity in December.) In 1979 the ratios of March over January and March over February were 0.36 and 0.43, respectively. These values are much higher than in 1976-1978 and 1980-1982. The March over January ratio in 1976-1978 and 1980-1982 was between 0.12 and 0.22, and the March over February ratio in these years was between 0.16 and 0.29. That is, in 1979, when convenience yield was an important determinant of futures prices, the fraction of hedging in the March contract by hedgers with longer-term commitments was considerably smaller than in the other years. Therefore, as the delivery month approached, the extent of liquidation of March futures contracts was much smaller than in other years. Consistent with this explanation, the ratios of open interest in the March wheat futures contract over open interest in all other wheat futures contracts on the first trading days in January and February of 1979 (0.61 and 0.43) were lower than the corresponding values in 1976-1978 and 1980-1982 (between 0.69 and 1.07 in January, and between 0.48 and 0.72 in February).

Such analysis suggests that the degree to which short-term futures contracts span long-term contracts, and the ability of actual futures markets to achieve stochastic separation of production, vary from “production year” to “production year” and within a “production year,” depending on the level of inventories and, more generally, on deviations of futures prices from cash and carry values. Furthermore, parameters that affect the length of the period of spot commitments and the degree of nonlinearity in the relation between prices of consecutive futures contracts can be used to predict the degree to which long-term contracts will be traded.

3. The Effects of Futures Trading on Optimal Output

This section examines the effects of futures trading on the optimal output level. To make the comparison with earlier studies more apparent, we assume that only futures contracts with delivery on date 3 are traded on date 1 and that $C(X_1, X_2) = C_1X_1 + C_2X_2$, where X_2 is unknown on date 1 (though the analysis is easily generalized to the case of intertemporally-complementary factor pricing function).

Proposition 2. *Consider a risk-averse producer with production flexibility and a deterministic production function whose optimalfutures positions on dates 1 and 2 are short positions. A futures market on her output will induce her to produce a higher level of output than she would in the absence of a futures market, if her production decisions are dynamically stochastically separable. Futures trading* *need not, however, increase her output level when her production decisions are not stochastically separable.*

Holthausen (1979) shows, in a two-date model, that producers who hold short futures position would produce more in a presence of a futures market than in its absence.[18] Proposition 2 extends the result to a dynamic world. It shows that if the short futures position of a producer with dynamic stochastic separation were originally set at the optimal level and then restricted below that level, the producer would respond by reducing her output. The introduction of unrestricted futures trading allows the producer to produce as if she lived in a world of certainty with an output price equal to the prevailing futures price. Preventing her from optimally hedging prevents her from producing in this manner, and she produces less. When the production decision is not stochastically separable, the incremental revenues from the output produced by X_1 are uncertain even with unrestricted futures trading, and restricting the optimal futures position need not decrease output.[19]

4. Optimal Hedging Positions in the Futures and Stock Markets

Sections 4 and 5 examine the implications of production flexibility for optimal hedging in the futures and stock markets and equilibrium futures prices. A three-date mean-variance model is studied, extending the two-date mean-variance models of Stoll(1979) and Hirshleifer (1988a), where agents can also trade in the stock market. Following Stoll (1979) and Hirshleifer (1988a), we assume that shares in the production process are not traded in the stock market, but producers can hold M_t units of a "stock market portfolio" on date t. In this way, we study the implications for hedging and equilibrium prices of both systematic and residual (nonmarketable) production risks, as well as examine the interaction between the two risks. Although dynamic stochastic separation was the important factor in the preceding analysis, it is irrelevant in the subsequent analysis in the sense that the results hold for both producers with intertemporally-complementary production functions and for producers with dynamic stochastic sep-

[18] The optimal futures position is a sum of a production-hedging term and a speculative term. As Section 4 shows, the hedging term in this model is always short.

[19] First, Proposition 2 need not hold for producers facing stochastic production functions and uncertain demand [Anderson and Danthine (1983a)]. Second, Proposition 2, like the two-date result, is based on a partial analysis of a single producer. A similar statement regarding aggregate output requires an equilibrium model. Two-date equilibrium models, one with only supply uncertainty [Newbery and Stiglitz (1981)] and one with only demand uncertainty [Kawai (1983)]. show that futures trading increases aggregate output.

aration. The flexibility or inflexibility of production is the critical factor.

The net return on date $t + 1$ on a unit of the stock market portfolio held from t to $t + 1$ is denoted by R_{t+1} (which, like other cash flows, is valued on date 3) and is uncertain on date t. It is assumed that on every date the producer maximizes $E_t(\pi) - \frac{1}{2}\lambda \mathrm{Var}_t(\pi)$ where λ is a positive constant reflecting risk aversion, and $E_t(\pi)$ and $\mathrm{Var}_t(\pi)$ are the expectation and variance, on date t, of profits. The producer's profit function on date 1 is now

$$\begin{aligned}\pi = {}_3P_3\, G(X_1, X_2) - C_1X_1 - C_2X_2 + {}_2F_3({}_3P_3 - {}_2P_3) \\ + {}_1F_3({}_2P_3 - {}_1P_3) + M_1R_2 + M_2R_3. \end{aligned} \tag{7}$$

The optimal futures and stock positions on each date are each the sum of a production-hedging term and a speculative term (representing speculation on futures price changes and stock returns). Because this section focuses on production hedging, it ignores the speculative terms. (The entire futures and stock positions are derived in Appendix B.) The optimal production-hedging term in the futures market on date 2 is ${}_2F_3^b = -G(X_1, X_2)$. The stock market has no production-hedging term on date 2 because the futures market eliminates all production uncertainty on date 2.

Let $\pi_{p,2} \equiv {}_2P_3G(X_1, X_2) - C_1X_1 - C_2X_2$ denote the revenues from production valued at ${}_2P_3$. The optimal production-hedging futures position on date 1 of a producer with production flexibility is

$${}_1F_3^b = -\frac{\mathrm{Cov}_1({}_2P_3, \pi_{p,2})\mathrm{Var}_1(R_2) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1(R_2, \pi_{p,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2}. \tag{5}$$

The optimal position is equal to minus the coefficient of ${}_2P_3$ in the regression of $\pi_{p,2}$ on ${}_2P_3$ and R_2. The optimal hedge is not equal to expected output even for producers with dynamically stochastically separable production decisions, but it is adjusted to account for the relation between the uncertainty of incremental revenues from the output produced by X_2 with the interim futures price and stock market return.[20] Because ${}_2F_3^b = -G(X_1, X_2)$, the optimal production-hedging positions on dates 1 and 2 are different, even when realized output equals expected output. In contrast, with production inflexibility, ${}_1F_3^b = {}_2F_3^b = -G(X_1, X_2)$. Hence, production flexibility creates a pro-

[20] It is often argued (e.g., Rolfo (1980)] that the hedging effectiveness of futures contracts for producers with deterministic production functions is high because they suffer only from price risk, but the hedging effectiveness of futures contracts for producers with stochastic production functions is low because they suffer from both price and output risks. With production flexibility, the hedging effectiveness for all producers with deterministic production functions (even with dynamic stochastic separation) also depends on their effectiveness in hedging both price and output risks.

duction-hedging incentive for producers to retrade futures contracts on date 2.

The optimal production-hedging position in the stock market on date 1 of a producer with production flexibility is

$$M_1^b = -\frac{\mathrm{Cov}_1(R_2, \pi_{p,2})\mathrm{Var}_1({}_2P_3) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1({}_2P_3, \pi_{p,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2}. \tag{9}$$

The optimal position is equal to minus the coefficient of R_2 in the regression of $\pi_{p,2}$ on ${}_2P_3$ and R_2. With production flexibility, $\pi_{p,2}$ is stochastic on date 1 even for producers with dynamically stochastically separable production decisions. Moreover, because $\pi_{p,2}$ is not a linear function of ${}_2P_3$, the stock market can have a production-hedging role even when ${}_2P_3$ and R_2 are uncorrelated but not independent. In contrast, with production inflexibility, $\pi_{p,2}$ is certain on date 1 and the stock market has no production-hedging role. Production flexibility thus creates a production-hedging role for the stock market even for producers with dynamically stochastically separable production decisions.

5. Futures Market Equilibrium

Equilibrium in the futures market requires that the net supply of futures contracts be zero. Suppose that the futures market consists of m identical speculators (agents who do not participate in the production process) and n identical hedgers (producers), where speculators and hedgers can differ in their risk aversion.[21] Let superscripts s and h denote speculators and hedgers, respectively. Assume that all agents have identical beliefs and behave rationally in the sense of Muth (1961). For simplicity, assume that the commodity being produced is nonstorable so that the effects of inventories can be ignored.

Letting $G^s(X_1, X_2) = 0$, summing Equation (B5) over all agents, and equating the sum to zero, we find that the equilibrium futures price on date 2 is

$${}_2P_3 = E_2({}_3P_3) - n\theta G^b(X_1, X_2)\mathrm{Var}_2({}_3P_3)(1 - \rho_2^2) - E_2(R_3)\beta_2, \tag{10}$$

where

$$\theta \equiv \frac{\lambda^s \cdot \lambda^b}{n\lambda^s + m\lambda^b}, \quad \beta_2 \equiv \frac{\mathrm{Cov}_2({}_3P_3, R_3)}{\mathrm{Var}_2(R_3)}, \quad \rho_2^2 \equiv \frac{[\mathrm{Cov}_2({}_3P_3, R_3)]^2}{\mathrm{Var}_2({}_3P_3)\cdot\mathrm{Var}_2(R_3)}.$$

The futures risk premium on date 2 is the sum of two terms. The

[21] See Hirshleifer (1988a) for a model with an endogenous number of speculators

first term, $n\theta G^b(X_1, X_2)\mathrm{Var}_2({}_3P_3)(1 - \rho_2^2)$, is a premium for the residual risk, where $nG^b(X_1, X_2)$ is the net aggregate production-hedging position. The second term, $E_2(R_3)\beta_2$, is a premium for the systematic risk of *speculating* in the futures market (by both hedgers and speculators). Because the stock market has no production-hedging role on date 2, the systematic risk premium is independent of net hedging.

Proposition 3. *The equilibrium futures price on date 1 in the dynamic economy is*

$$\begin{aligned} {}_1P_3 = {} & E_1({}_2P_3) - n\theta \,\mathrm{Cov}_1({}_2P_3, \pi_{p,2}^b) \\ & + n\theta \,\mathrm{Cov}_1(R_2, \pi_{p,2}^b)\beta_1 - E_1(R_2)\beta_1 + \mathrm{SRP}. \end{aligned} \tag{11}$$

The equilibrium futures price on date 1 in the static economy is

$$\begin{aligned} {}_1P_3 = {} & E_1({}_2P_3) - n\theta G^b(X_1, X_2)\mathrm{Var}_1({}_2P_3)(1 - \rho_1)^2 \\ & - E_1(R_2)\beta_1 + \mathrm{SRP}, \end{aligned} \tag{12}$$

where

$$\beta_1 \equiv \frac{\mathrm{Cov}_1({}_2P_3, R_2)}{\mathrm{Var}_1(R_2)}; \qquad \rho_1^2 \equiv \frac{[\mathrm{Cov}_1({}_2P_3, R_2)]^2}{\mathrm{Var}_1({}_2P_3)\cdot\mathrm{Var}_1(R_2)};$$

and SRP, a risk premium reflecting covariances of ${}_2P_3$ and R_2 with revenues from speculative positions taken on date 2, is

$$\begin{aligned} \mathrm{SPR} \equiv {} & -\{n\theta \,\mathrm{Cov}_1({}_2P_3, \pi_{s,2}^b) + m\theta \,\mathrm{Cov}_1({}_2P_3, \pi_{s,2}^s)\} \\ & + \{n\theta \,\mathrm{Cov}_1(R_2, \pi_{s,2}^b) + m\theta \,\mathrm{Cov}_1(R_2, \pi_{s,2}^s)\}\beta_1, \end{aligned}$$

where

$$\pi_{s,2} \equiv [{}_2F_3 + G(X_1, X_2)]({}_3P_3 - {}_2P_3) + M_2R_3.$$

Production flexibility has important implications for tests of futures risk premia. First, there is a debate on whether futures risk premia depend on net aggregate hedging positions. Keynes's (1930) theory of normal backwardation suggests that futures risk premia depend on net hedging. Using the Sharpe (1964)-Vintner (1965) CAPM (i.e., assuming that systematic risk premia are independent of net hedging), Dusak (1973) finds no evidence of systematic risk premia in grain futures prices. Carter, Rausser, and Schmitz (1983), on the other hand, assuming that the systematic risk premia depend on net hedging, find that grain futures prices contain systematic risk premia. Hirshleifer (1988a), however, shows that the systematic risk premium is independent of net hedging but that the futures price also includes a residual risk premium that depends on net hedging. Bessembinder (1992) tests for futures risk premia in two ways: the first is based on

Hirshleifer (1988a), and the second on Carter, Rausser, and Schmitz (1983). He finds that foreign currency and commodity futures prices contain residual risk premia that depend on net hedging and systematic risk premia that sometimes depend on net hedging.[22]

Proposition 3 provides new insights into the relation between risk premia and net hedging positions. The systematic risk premium in the dynamic economy includes $n\theta\,\mathrm{Cov}_1(R_2, \pi^b_{p,2})\beta_1$, which *depends on the net aggregate production-hedging position.* In contrast, the systematic risk premium in the static economy is independent of net hedging because the stock market has no production-hedging role. The residual risk premia, on the other hand, depend on the net hedging position in both economies.

Second, production flexibility also has important implications for the functional relation of futures risk premia with the systematic and residual risks. All the CAPM-based empirical studies mentioned assume, like the static model, that the futures systematic risk premium is a linear function of beta and of beta alone. Yet, $n\theta\,\mathrm{Cov}_1(R_2, \pi^b_{p,2})\beta_1$ is *nonlinear* with β_1 because $\pi^b_{p,2}$ is also a function of ${}_2P_3$. The nonlinearity reflects two factors that affect the systematic risk of the cash flow on the futures hedging position in a multiplicative way. First, for a given position, β_1 measures the systematic risk of futures price changes. Second, ${}_1F^b_3$ also depends on $\mathrm{Cov}_1({}_2P_3, R_2)$ [Equation (8)].[23] Because the cash flow on the futures hedging position is ${}_1F^b_3 \cdot ({}_2P_3 - {}_1P_3)$, the two factors enter the risk premium in a multiplicative way. More importantly, *beta alone is not the appropriate measure of systematic risk* in the dynamic model; rather, it is $\mathrm{Cov}_1(R_2, \pi^b_{p,2}) \cdot \beta_1$. Lastly, the residual risk premium also differs in the two economies. In the static economy, as in the two-date case, this premium is $n\theta G^b(X_1, X_2)\,\mathrm{Var}_1({}_2P_3)(1 - \rho_1)^2$. In contrast, in the dynamic economy the premium is $n\theta\,\mathrm{Cov}_1({}_2P_3, \pi^b_{p,2})$, reflecting both price and output risks.

Futures contracts, unlike equity, have an expiration date. The foregoing analysis suggests that tests of futures risk premia may be sensitive to the time to delivery and to the length of hedging and production periods. Both the characteristics of the equilibrium risk premia and the functional relation between the premia and the systematic and residual risks change with time to delivery. Tests for risk premia should hence control for both the length of the period and the time to delivery, with different regressions required on dates 1 and 2. The length of the period over which firms make binding commitments in

[22] Using nonparametric tests, Chang (1985) finds that grain futures prices contain risk premia that depend on net hedging.

[23] First, production revenues are an increasing function of ${}_2P_3[\partial\pi^b_{p,2}/\partial\,{}_2P_3] = G(X_1, X_2)$, using Equation (4). Second, for producers with intertemporally-complementary production functions, the optimal level of X_1 depends on $\mathrm{Cov}_1[R_2, {}_2P_3 \cdot G_1(X_1, X_2)]$, where $G_1(X_1, X_2)$ is a function of ${}_2P_3$.

their production process (i.e., over which no input level can be adjusted) emerges as a critical factor. This length varies among firms, depending on the costs and benefits of changes in output levels. Empirical studies of futures prices typically use equal lengths of periods ("observation horizons") for commodity and financial futures markets. Yet, the length of the period of commitments is likely to be shorter for financial futures than for commodity futures. Also, in some futures markets the length of the period of commitments can vary across contracts. In agricultural futures markets, for example, the length of "a period" for within-crop contracts is likely to differ from the length of "a period" for across-crop (old-crop, new-crop) contracts. Consequently, the optimal way to test for risk premia and the appropriate length of the "observation horizon" can vary from one futures market to another, depending on the underlying industry, and sometimes across contracts within a market.[24]

6. Conclusions

This article extends the analysis of the behavior of producers under price uncertainty in the presence of futures markets into a production world, where producers can adjust their inputs at interim periods. It studies the ability of futures contracts to enable producers to produce as if they lived in a world of certainty where the output price they face is the prevailing futures price. The ability depends on intertemporal separabilities and inflexibilities in the production processes but not on linearities in risk preferences. When a production process is spanned only by consecutive single-period futures contracts, this ability also requires exact linearity in the relation between consecutive futures prices. When these conditions fail, the length of the period of commitment in the output market and the maturity dates of the futures contracts emerge as the crucial determinants. Hence, theoretical and empirical studies of futures markets should be careful to choose the period of "real" commitment and contract duration that are most appropriate for the case being studied.

The study presents empirical evidence illustrating the importance of these conditions for intertemporal hedging patterns. First, it compares the lumber futures market, which is characterized by relatively volatile futures price spreads, with the S&P futures market, which is characterized by approximately linear price relations. Also, the S&P market has a high fraction of hedgers with a short period of com-

[24]Consistent with this analysis, Chang (1985) finds that "the validity of [Keynes'] theory seems to be in different degrees in different markets and in different periods" (page 208), and Caner, Rausser, and Schmitz (1983) find that "the degree of systematic risk is not constant across contracts for any of the commodities" (page 328).

mitment relative to commodity markets. The concentration of hedgers in nearest-maturity contracts was substantially larger in the S&P market than in the lumber market, Second, a study of the wheat futures market shows that when the deviation from linear price relations was relatively large, the fraction of hedging in the nearest-maturity contract by hedgers with longer-term commitments was considerably smaller than average.

The dynamic model yields results that differ considerably from current studies of equilibrium futures prices. Both the characteristics of the systematic risk premium and the functional relation between the premium and systematic risk depend on the flexibility to adjust output levels at interim dates and the length of the period of commitment in the output market, and they change with time to delivery. Unlike the static model, the systematic risk premium is not a function of beta alone, but of beta times the covariance of market returns with production revenues. Depending on the underlying industry, the optimal way to test for risk premia and the appropriate length of the "observation horizon" can vary from one futures market to another and across contracts within a market.

Appendix A

Proof of Proposition 1. The optimal input level on date 2, X_2^*, is a function of X_1 and ${}_2P_3$, and the optimal futures position on date 2, ${}_2F_3^*$, is a function of X_1, ${}_1F_3$, and ${}_2P_3$. Denote these functions by $X_2^* = A(X_1, {}_2P_3)$ and ${}_2F_3^* = B(X_1, {}_1F_3, {}_2P_3)$. On date 1 the producer chooses the levels of X_1 and ${}_1F_3$ that maximize $E_1[U(\pi)]$ given ${}_2F_3^*$ and X_2^*. The first-order conditions with respect to X_1 and ${}_1F_3$ yield

$$\begin{aligned} &E_1\{U'(\pi)[{}_3P_3G_1(X_1, X_2) - C_1(X_1, X_2)]\} \\ &\quad = -E_1[U'(\pi)B_1(X_1, {}_1F_3, {}_2P_3)({}_3P_3 - {}_2P_3)] \\ &\qquad - E_1\{U'(\pi)A_1(X_1, {}_2P_3)[{}_3P_3G_2(X_1, X_2) - C_2(X_1, X_2)]\}, \qquad \text{(A1)} \\ &E_1[U'(\pi)({}_2P_3 - {}_1P_3)] \\ &\quad = -E_1[U'(\pi)B_2(X_1, {}_1F_3, {}_2P_3)({}_3P_3 - {}_2P_3)]. \qquad \text{(A2)} \end{aligned}$$

But

$$\begin{aligned} &E_1[U'(\pi)B_i(X_1, {}_1F_3, {}_2P_3)({}_3P_3 - {}_2P_3)] \\ &\quad = E_1\{B_i(X_1, {}_1F_3, {}_2P_3)E_2[U'(\pi)({}_3P_3 - {}_2P_3)]\} = 0 \end{aligned}$$

for $i = 1, 2$. The first equality follows from the fact that $B_i(X_1, {}_1F_3, {}_2P_3)$ is nonstochastic at date 2 and from the properties of marginal and conditional distributions. The second equality follows from Equation (3). Likewise, because $A_1(X_1, {}_2P_3)$ is nonstochastic on date 2,

$$E_1\{U'(\pi)A_1(X_1, {}_2P_3)[{}_3P_3G_2(X_1, X_2) - C_2(X_1, X_2)]\} = 0$$

[using Equation (2)]. Hence, (A1) and (A2) become

$$E_1[U'(\pi)C_1(X_1, X_2)] = E_1[U'(\pi)G_1(X_1, X_2){}_3P_3], \tag{A3}$$

$$ {}_1P_3E_1[U'(\pi)] = E_1[U'(\pi){}_2P_3]. \tag{A4}$$

When the production function and factor pricing functions are intertemporally separable, $G_1(X_1, X_2)$ and $C_1(X_1, X_2)$ are nonrandom on date 1. Equation (A3) then becomes

$$\begin{aligned} C_1(X_1, X_2)E_1[U'(\pi)] &= G_1(X_1, X_2)E_1[U'(\pi){}_3P_3] \\ &= G_1(X_1, X_2)E_1[U'(\pi){}_2P_3], \end{aligned}$$

where the second equality uses $E_1[U'(\pi){}_3P_3] = E_1\{E_2[U'(\pi){}_3P_3]\}$ and Equation (3). From (A4) it follows that $C_1(X_1, X_2) = {}_1P_3G_1(X_1, X_2)$. ■

Proof of Proposition 2. Totally differentiating (A3) and using $dX_2/dX_1 = -G_{21}(X_1, X_2)/G_{22}(X_1, X_2)$ from Equation (4) yields

$$\begin{aligned} \frac{dX_1}{d_1F_3} = -[&E_1\{U''(\pi)G_1(X_1, X_2)({}_2P_3 - {}_1P_3)^2\} \\ &+ E_1\{U''(\pi)[{}_1P_3G_1(X_1, X_2) - C_1]({}_2P_3 - {}_1P_3)\}]/D, \end{aligned}$$

where

$$\begin{aligned} D \equiv\ &E_1\{U''(\pi)[{}_2P_3G_1(X_1, X_2) - C_1]^2\} + E_1[U'(\pi){}_2P_3G_{11}(X_1, X_2)] \\ &- E_1\left[U''(\pi){}_2P_3\frac{G_{12}(X_1, X_2)G_{21}(X_1, X_2)}{G_{22}(X_1, X_2)}\right]. \end{aligned}$$

When the production function is intertemporally complementary, the denominator is negative (the first and second terms are negative and the last term is positive), but the numerator can be positive or negative. (The first term is negative, but the sign of the second term is ambiguous: $E_1[{}_1P_3G_1(X_1, X_2) - C_1]$ can be positive or negative, and although $E_1[{}_1P_3G_1(X_1, X_2) - C_1]$ depends on ${}_2P_3$ the sign of this relation is unclear.) Consequently, the sign of dX_1/d_1F_3 cannot be determined.

For an intertemporally separable production function,

$$\frac{dX_1}{d_1F_3} = -\frac{G_1(X_1, X_2)E_1\{U''(\pi)({}_2P_3 - {}_1P_3)^2\}}{E_1\{U''(\pi)[{}_2P_3G_1(X_1,X_2) - C_1]^2\} + G_{11}(X_1,X_2)E_1[U'(\pi){}_2P_3]}.$$

Both denominator and numerator are negative, yielding dX_1/d_1F_3

< 0. When ${}_1F_3 < 0$, restricting ${}_1F_3$ below the optimal level reduces optimal output. When ${}_1F_3 > 0$, it increases optimal output. ∎

Proof of Proposition 3. To get (11); sum Equation (B11) over n hedgers and m speculators, with $\pi^s_{p,2} = 0$. Equate the sum to zero.

To get (12); production inflexibility implies that the first term in (B11) is equal to $-G(X_1, X_2)$. Sum (B11) with this first term and with $G(X_1, X_2) = 0$ over n hedgers and m speculators. Equate the sum to zero. ∎

Appendix B

Derivation of optimal positions in futures and stock market on dates *2* and 1

Derivation of optimal positions on date 2. The expectation and variance, on date 2, of the producer's profit function are

$$E_2[\pi] = G(X_1, X_2)E_2({}_3P_3) - C_1X_1 - C_2X_2 + {}_2F_3[E_2({}_3P_3) - {}_2P_3] + {}_1F_3({}_2P_3 - {}_1P_3) + M_1R_2 + M_2E_2(R_3), \tag{B1}$$

$$\mathrm{Var}_2[\pi] = [G(X_1, X_2) + {}_2F_3]^2\mathrm{Var}_2({}_3P_3) + M_2^2\mathrm{Var}_2(R_3) + 2M_2[G(X_1, X_2) + {}_2F_3]\mathrm{Cov}_2({}_3P_3, R_3). \tag{B2}$$

The first-order conditions with respect to ${}_2F_3$ and M_2 yield

$${}_2F_3 = -G(X_1, X_2) - M_2\frac{\mathrm{Cov}_2({}_3P_3, R_3)}{\mathrm{Var}_2({}_3P_3)} + \frac{E_2({}_3P_3) - {}_2P_3}{\lambda\,\mathrm{Var}_2({}_3P_3)}, \tag{B3}$$

$$M_2 = -[G(X_1, X_2) + {}_2F_3]\frac{\mathrm{Cov}_2({}_3P_3, R_3)}{\mathrm{Var}_2(R_3)} + \frac{E_2(R_3)}{\lambda\,\mathrm{Var}_2(R_3)}. \tag{B4}$$

Combining (B4) and (B3) yields

$${}_2F_3 = -G(X_1, X_2) + \frac{[E_2({}_3P_3) - {}_2P_3]\mathrm{Var}_2(R_3) - \mathrm{Cov}_2({}_3P_3, R_3)E_2(R_3)}{\lambda\{\mathrm{Var}_2({}_3P_3)\mathrm{Var}_2(R_3) - [\mathrm{Cov}_2({}_3P_3, R_3)]^2\}}, \tag{B5}$$

$$M_2 = \frac{E_2(R_3)\mathrm{Var}_2({}_3P_3) - \mathrm{Cov}_2({}_3P_3, R_3)[E_2({}_3P_3) - {}_2P_3]}{\lambda\{\mathrm{Var}_2({}_3P_3)\mathrm{Var}_2(R_3) - [\mathrm{Cov}_2({}_3P_3, R_3)]^2\}}. \tag{B6}$$

Derivation of optimal positions on date 1. Let $\pi_{p,2} \equiv {}_2P_3G(X_1, X_2) - C_1X_1 - C_2X_2$ denote the revenues from production valued at ${}_2P_3$, and let $\pi_{s,2} \equiv [{}_2F_3 + G(X_1, X_2)][{}_3P_3 - {}_2P_3] + M_2R_3$ denote the revenues from optimal speculation on date 2. The expectation and variance, on date 1, of the producer's profit function are

$$E_1(\pi) = E_1(\pi_{p,2}) + E_1(\pi_{s,2}) + {}_1F_3[E_1({}_2P_3) - {}_1P_3] + M_1E_1(R_2), \tag{B7}$$

$$\begin{aligned}\mathrm{Var}_1(\pi) = {} & \mathrm{Var}_1(\pi_{p,2}) + \mathrm{Var}_1(\pi_{s,2}) + {}_1F_3^2\mathrm{Var}_1({}_2P_3) \\ & + M_1^2\mathrm{Var}_1(R_2) + 2{}_1F_3\mathrm{Cov}_1({}_2P_3, \pi_{p,2}) \\ & + 2{}_1F_3\mathrm{Cov}_1({}_2P_3, \pi_{s,2}) + 2{}_1F_3M_1\mathrm{Cov}_1({}_2P_3, R_2) \\ & + 2M_1\mathrm{Cov}_1(R_2, \pi_{p,2}) + 2M_1\mathrm{Cov}_1(R_2, \pi_{s,2}) \\ & + 2\mathrm{Cov}_1(\pi_{p,2}, \pi_{s,2}).\end{aligned} \tag{B8}$$

Because X_2, M_2, and ${}_2F_3$ are not functions of ${}_1F_3$ or M_1, the first-order condition with respect to ${}_1F_3$ and M_1 *are*

$$\begin{aligned}E_1({}_2P_3) - {}_1P_3 - \lambda\{ & {}_1F_3\mathrm{Var}_1({}_2P_3) + \mathrm{Cov}_1({}_2P_3, \pi_{p,2}) \\ & + \mathrm{Cov}_1({}_2P_3, \pi_{s,2}) + M_1\mathrm{Cov}_1({}_2P_3, R_2)\} = 0,\end{aligned} \tag{B9}$$

$$\begin{aligned}E_1(R_2) - \lambda\{ & M_1 \cdot \mathrm{Var}_1(R_2) + \mathrm{Cov}_1(R_2, \pi_{p,2}) \\ & + {}_1F_3\mathrm{Cov}_1({}_2P_3, R_2) + \mathrm{Cov}_1(R_2, \pi_{s,2})\} = 0.\end{aligned} \tag{B10}$$

Combining (B9) and (B10) yields

$$\begin{aligned}{}_1F_3 = {} & -\frac{\mathrm{Cov}_1({}_2P_3, \pi_{p,2})\mathrm{Var}_1(R_2) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1(R_2, \pi_{p,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2} \\ & + \frac{[E_1({}_2P_3) - {}_1P_3]\mathrm{Var}_1(R_2) - \mathrm{Cov}_1({}_2P_3, R_2)E_1(R_2)}{\lambda\{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2\}} \\ & - \frac{\mathrm{Cov}_1({}_2P_3, \pi_{s,2})\mathrm{Var}_1(R_2) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1(R_2, \pi_{s,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2},\end{aligned} \tag{B11}$$

$$\begin{aligned}M_1 = {} & -\frac{\mathrm{Cov}_1(R_2, \pi_{p,2})\mathrm{Var}_1({}_2P_3) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1({}_2P_3, \pi_{p,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2} \\ & + \frac{E_1(R_2)\mathrm{Var}_1({}_2P_3) - \mathrm{Cov}_1({}_2P_3, R_2)[E_1({}_2P_3) - {}_1P_3]}{\lambda\{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2\}} \\ & - \frac{\mathrm{Cov}_1(R_2, \pi_{s,2})\mathrm{Var}_1({}_2P_3) - \mathrm{Cov}_1({}_2P_3, R_2)\mathrm{Cov}_1({}_2P_3, \pi_{s,2})}{\mathrm{Var}_1({}_2P_3)\mathrm{Var}_1(R_2) - [\mathrm{Cov}_1({}_2P_3, R_2)]^2}.\end{aligned} \tag{B12}$$

Equations (B11) and (B12) represent the optimal futures and stock position as the sum of a pure production-hedging position (the first term in either equation) and a speculative position (the last two terms in either equation). The speculative position reflects two factors. The first term, which is similar to the optimal speculative futures position on date 2, represents speculation on ${}_2P_3$ and R_2. The speculative position, however, need not be zero even when $E_1({}_2P_3) = {}_1P_3$ and $E_1(R_2) = 0$. Nonzero speculative positions in the futures and stock markets can still be taken on date 1 to account for the correlation of ${}_2P_3$ and R_2 with $\pi_{s,2}$.

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