Volume, Volatility, and the Dispersion of Beliefs

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I examine a two-period noisy rational expectations model of futures market and show that the dispersion of expectations about a weighted average of future prices measures both the additional volatility and the additional expected volume of trade associated with noisy information. The role played by dispersion helps clarify several stylized facts concerning volume and price behavior. Specifically, dispersion can be a factor contributing to the positive correlation between volume and absolute price changes, and the positive correlation between consecutive absolute price changes.

The sources of price volatility and trading volume and their interrelation are topics of enduring interest in financial economics. Numerous empirical studies have examined the contemporaneous behavior of volume and absolute price changes and have documented a positive correlation between the two; for a survey up to 1987, see Karpoff (1987). More recent empirical investigations, of which Gerety and Mulherin (1989) is the most comprehensive, have focused on the intraday patterns in volume and price volatility and have found similar correlations. The reaction of prices and traders to information flows has also been analyzed in nonstrategic models [Clark (1973), Pfleiderer (1984), Holthausen and Verrecchia (1988), Grundy and McNichols (1989), Kim and Verrecchia (1991)] and in strategic models [Admati and Pfleiderer (1988)

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and Foster and Viswanathan (1990)]. These models provide theoretical underpinnings for the empirical results. Reflecting on the body of work on volume and volatility, Gallant, Rossi, and Tauchen (1992) nevertheless conclude that "there seems to be no model with dynamically optimizing heterogeneous agents that can jointly account for major stylized facts-serially correlated volatility, contemporaneous volume-volatility correlation, and excess kurtosis of price changes" (p. 202).

I examine a two-period noisy rational expectations model of a futures market and show that the dispersion of past and current beliefs help to explain a number of stylized facts concerning price volatility and the volume of trade. The key observation underlying this conclusion is that the dispersion of beliefs measures both the excess volatility and the excess volume of trade induced by the "noisy" liquidity demand of futures hedgers. As in all models of this class, a random liquidity demand masks the private information embedded in prices and therefore preserves the dispersion of expectations. I show that dispersion contributes to the variance of price changes (my measure of price volatility), the expected volume of trade, the correlations between absolute price changes and both contemporaneous volume and lagged volume, and the correlation between consecutive absolute price changes.

The intuition behind the positive link between price volatility and the dispersion of price expectations involves a familiar signal extraction problem. Speculators filter from current prices private information concerning the sequence of future prices. When liquidity demand is uncertain, however, speculators confuse price variation caused by changes in liquidity demand and price variation caused by private information. Because of this confusion, individual estimates of future prices are overresponsive to shocks in liquidity demand. It follows that there is an excess covariance between average estimates of future prices and liquidity demand. This creates excess price variability because equilibrium prices are linear combinations of average estimates of future prices and of liquidity demand.[1] The measure of this excess price variability turns out to be the dispersion of expectations. To see why, recall that rational estimate errors are orthogonal to the equilibrium futures price. This means that the covariance between estimate errors and the risk-weighted liquidity demand is exactly compensated for by the covariance between estimate errors and the average price estimate. The latter is equal to the dispersion of expec-

[1] More specifically, the equilibrium price at time 0 is based on a weighted average of the average estimates of time 1 and time 2 futures prices. The equilibrium price at time 1 is just based on an average estimate of the futures price at time 2.

tations. In effect, dispersion measures the tendency for price volatility to increase because of noisy information. Of course, there is a contrary tendency as well, since less uncertainty is resolved when information is noisy.[2] Because of this contrary tendency, the net measurable relationship between volatility and dispersion is ambiguous a priori. Some empirical evidence on this question will be reviewed.

It is well understood that speculative trade in financial markets depends on divergent beliefs. Consistent with this, I show that the expected volume of trade by a speculator depends on a linear combination of the dispersions of expectations in the current and preceding round of trade, where the weight of each dispersion depends on the precision in that round of the speculator's estimate of the next futures price and on the risk aversion coefficient. If speculators have different estimate precisions, then the expected volume of trade by a speculator also depends on the difference between his individual estimate precision and the average precision. Two limiting cases are instructive in interpreting this result. In the first case, analogous to a one-period example analyzed by Pfleiderer (1984), traders have homogeneous expectations in the first round of trade but have divergent expectations in the second round. The expected volume of speculative trade in the second round depends on the final precision-weighted dispersion. In the second limiting case, analogous to a model of Varian (1986), speculators start from dispersed expectations and end with identical expectations. Trade occurs to close out the speculative bets of the first round, so the expected volume of trade is related to the initial precision-weighted dispersion of expectations. In each of these limiting cases, the dispersion measures the average expected absolute change in the differential beliefs of speculators from one round of trade to the next. The weight of the dispersion depends on the precision of their estimates of the futures price in the next round of trade and on the risk aversion coefficient. This precision determines the size of speculative bets in each round. If speculators have private information of different precision, the absolute change in a speculator's differential belief also depends on the difference between his individual estimate precision and the average precision. In general, dispersion persists over time, and, just as in these two limiting cases, new information generates speculative trade by changing the extent of disagreement between speculators. The average expected absolute change in the precision-weighted differential beliefs of speculators is proportional to a linear combination

[2] Holthausen and Verrecchia (1988) consider a model where the problem of adverse selection between noise and information plays no role because speculators make no inferences from prices. They find that price volatility is inversely related to dispersion.

of the dispersions of expectations in the current and preceding rounds of trade.

Clearly, this characterization of expected trading volume does not mean that positive dispersions per se are sufficient to have trade. There is no retrading if precision-weighted differential beliefs are not changed by new information. The expected volume of trade depends on the changes in precision-weighted differential beliefs of individual speculators through the dispersions of expectations in the current and preceding rounds of trade. Milgrom and Stokey (1982) showed that, in complete markets, there is no retrading when expectations about forthcoming private (public) information are concordant (essentially concordant). Specifically, they showed that Pareto-optimal allocations remain Pareto optimal if expectations about new information are concordant. Concordance roughly means that speculators agree on how to interpret future information. Grundy and McNichols (1989) examine a setting, similar to that in this article, of a single risky asset, negative exponential utility, and multivariate normally distributed hedging demand shocks and nonprice signals. They characterize the information structure under which there exists an equilibrium in which markets are effectively complete despite the heterogeneity of information, speculators have concordant (essentially concordant) beliefs about forthcoming private (public) signals, and there is no retrading among speculators at the second round. Consistent with Milgrom and Stokey, trade by speculators is limited to accommodating any change in liquidity demand. One property of the information structure is that speculators have information of identical precision. Grundy and McNichols also show that for this information structure the equilibrium is not unique. There exists a second equilibrium in which speculators do not reach a Pareto-optimal allocation at the first round (the market is not effectively complete) and do not have even essentially concordant beliefs about the equilibrium price in the second round. Thus, Grundy and McNichols show that the conditions of the Milgrom-Stokey no-trade theorem are themselves partially endogenous. In the Grundy-McNichols setting, when speculators conjecture that there will (will not) be trade when a public signal about which speculators have essentially concordant beliefs is released, the allocations from the first round are not (are) a Pareto optimum and traders have nonconcordant (concordant) beliefs concerning the second-round price. Kim and Verrecchia (1991) show that when speculators have information of differing precision, Pareto-optimal allocations are not achieved in the first round (the market is not even essentially complete) and, irrespective of whether speculators have concordant beliefs about future information releases, the conditions of the Milgrom-Stokey no-trade theorem are not satisfied.

In the Kim-Verrecchia model the expected volume of trade is proportional to an average measure of the differences in precision. With equal precision, Kim and Verrecchia's model simplifies to the example in section 4 of Grundy and McNichols.

An important observation made by Grundy and McNichols is that future market prices are part of the information set relevant to speculators. Thus, in their model, expectations about future market prices must also be concordant to prevent retrading. In keeping with this, the dispersions of expectations in our model pertain to a weighted average of all subsequent prices, and trade occurs because speculators update their precision-weighted differential expectation of this average. This implies that even if they agree about the distribution of ultimate asset values, traders can speculate on the basis of different beliefs about the future liquidity demands that affect prices in the following periods.

These results on the relationship between dispersion, trade, and price volatility complement the results of Pfleiderer (1984), Grundy and McNichols (1989), and Kim and Verrecchia (1991). Recent empirical work by Frankel and Froot (1990) supports our findings concerning the role of dispersion. In one of the few studies to measure the dispersion of expectations, Frankel and Froot (1990) examine foreign exchange survey data previously examined by Dominguez (1986). They find a positive relation between dispersion and price volatility and a positive relation between dispersion and volume.

The other components of expected volume and price volatility that emerge from our analysis are familiar. Expected volume and volatility both include components derived from adjustments in liquidity demands. Expected volume is a function of the variances of changes in the shares of total hedging demand held by speculators and the variances of the demands of individual hedgers. Similarly, volatility is a function of the variance of innovations in weighted per-capita hedging demand. This particular link between volume and volatility has been fully analyzed in previous models, such as Pfleiderer (1984).

A final source of volatility is the price uncertainty, which is resolved between trading rounds. There is no corresponding component of expected volume. Information affects the expected volume of trade only indirectly, through its effects on the dispersion of information and on estimate precisions.[3] This captures the conventional perception that the volume of trade is a noisier indicator of the flow of information than is price volatility.

The next two sections outline the model and equilibrium condi-

[3] With a different parametrization than ours, information also would generate trade through Income effects.

tions. These conditions are applied in Section 4 to two special examples set out in Section 3. In Example 1, optimal speculative demands are myopic in the sense that they do not hedge future speculative opportunities. In Example 2, demands are nonmyopic, but different speculators have equally precise information. The basic difference between these examples is that in Example 1 volume and volatility are driven by the dispersion of expectations about the liquidating futures price, whereas in Example 2 they are driven by the dispersion of expectations about a weighted average of all subsequent futures prices. The dispersion of expectations is defined and characterized in Section 4. Our main results are presented in Sections 5 and 6. Some concluding remarks are contained in Section 7.

1. Model

My model is similar to the two-period noisy rational expectations models of Pfleiderer (1984), Grundy and McNichols (1989), and Kim and Verrecchia (1991), but its information structure is less restrictive. There is a single futures contract that matures at the end of two periods, at time 2. The futures contract is traded in two successive rounds at times 0 and 1, and final settlement occurs at time 2. The futures price at time t is $\tilde{p}_t$, $t = 0,1,2$. To prevent arbitrage, the final futures price must be equal to the spot price of the futures commodity at time 2. I assume that the latter is generated exogeneously by a sequence of independent normal shocks $\tilde{s}_t$ with zero mean so that

$$\tilde{p}_2 = E(\tilde{p}_2) + \sum_{t=0}^{2} \tilde{s}_t.$$

Here $E(\tilde{p}_2)$ denotes the unconditional expectation of $\tilde{p}_2$. Similar notation is used throughout the article. Thus, $E(\tilde{x})$ is the unconditional expected value of $\tilde{x}$, $\mathrm{var}(\tilde{x})$ is the unconditional variance of $\tilde{x}$, $\mathrm{cov}(\tilde{x}, \tilde{y})$ is the unconditional covariance between $\tilde{x}$ and $\tilde{y}$, and $r(\tilde{x}, \tilde{y})$ is the unconditional correlation between $\tilde{x}$ and $\tilde{y}$. When the operators E, var, and cov appear with time subscripts and agent superscripts, they represent conditional expectations. For example, $E_t^i(\tilde{x})$ is the ith speculator's expectation of $\tilde{x}$ conditional on his information at time t. Operators with a time subscript but no agent superscript represent weighted averages of the individual operators at time t. I will define these averages more precisely in the next section. Also, I use tildes to denote random variables that have not yet been realized. I should point out that the specification of the spot price above is not a critical feature of the general model, and that it is only imposed as a restriction in the first of two special examples that I consider later.

Traders are classified into *J* hedgers and *I* speculators. Hedgers are liquidity traders endowed with spot market risks. They stand for the commercial producers and processors of the futures commodity. The random futures demand of the *jth* hedger at time *t* is specified exogenously as $\tilde{h}^j_t$, a normally distributed random variable with mean zero. The exogenous specification of liquidity demand is typical of noisy rational expectations models. This simplification seems somewhat artificial when the liquidity traders are hedgers. However, endogenizing hedging demands would provide little additional insight into the relationship between information and expected volume and volatility. For instance, the size and direction of the effect of an increase in the dispersion of information on hedging demands are unpredictable. Per-capita hedging demand is $\tilde{h}_t = (I)^{-1} \Sigma_j \tilde{h}^j_t$. In contrast to hedgers, speculators have no prior spot market risks. Their utility for final wealth is a negative exponential function with an identical[4] risk aversion parameter α.

The *ith* speculator cannot observe the realization of the shock $\tilde{s}_t$ at time *t,* but he can observe information about this shock prior to trading.[5] This information consists of a vector of random variables, or "signals," that are correlated with the shocks $\tilde{s}_t$. Some signals are private, others public. For example, past and current futures prices are public information. In general, speculators may also have some direct information on hedging demand, but this information is imperfect. It follows that hedging demand is a source of noise that prevents equilibrium prices from revealing private information. This is the reason that expectations remain dispersed. The joint distribution of information variables with the shocks $\tilde{s}_t$ and with hedging demands is also normal; in particular, only linear equilibrium price functions are examined. The set of restrictions on the distributions of random variables, on the preferences of speculators, and on the form of equilibrium functions yields a tractable linear model. As pointed out by Grundy and McNichols (1989), these restrictions rule out certain variance-covariance matrices for the signals and the hedging demands.

Futures positions require no investment and are marked to market.

[4] This assumption is made for convenience of notation. The effect of different risk aversions is analogous to the effect of different precisions of information.

[5] Since final settlement occurs at time 2, speculators do not trade on the information about shock $\tilde{s}_2$ observed at this time. Also note that, prior to time 2, speculators have no information about $\tilde{s}_2$ because the shocks 3, ate independent. Hence, at times 0 and 1 every speculator's expectation of $\tilde{s}_2$ is equal to zero. This is why $\tilde{s}_2$ does not appear in equations such as Equation (3). The shock $\tilde{s}_2$ only has an effect on the variances of prices.

For simplicity, we assume that rates of interest are equal to zero.[6] Thus, with a single period left, the *ith* speculator selects the futures position f_1^i, where $f_1^i > 0$ represents a long position, that maximizes

$$\max_{f_1^i} E_1^i(-\exp(-\alpha(f_0^i(p_1 - p_0) + f_1^i(\tilde{p}_2 - p_1)))) \tag{1}$$

Similarly, at time 0, the speculator chooses f_0^i to maximize

$$\max_{f_0^i} E_0^i(-\exp(-\alpha(f_0^i(\tilde{p}_1 - p_0) + \tilde{f}_1^i(\tilde{p}_2 - \tilde{p}_1)))). \tag{2}$$

2. Equilibrium Conditions

In this section, I present the sequence of equilibrium demands and prices generating the expected volume of trade and price volatility. Equilibrium prices are found by substituting the solutions to expressions (1) and (2) into market-clearing conditions. Equilibrium demands are equal to the optimal demands valued at the equilibrium prices. I begin with a few preliminary definitions. Define

$$\beta^i = -\frac{\operatorname{cov}_0^i(\tilde{p}_1, \tilde{p}_2 - \tilde{p}_1)}{\operatorname{var}_0^i(\tilde{p}_2 - \tilde{p}_1)}.$$

Thus β^i reflects the sensitivity of the price change in the first period to that in the second period. Since the price change in the second period embodies future speculative opportunities, this sensitivity changes the perceived riskiness of the first price change. It follows that time 0 optimal demands and clearing prices will depend on β^i. Also let

$$\rho_1^i = (\alpha \operatorname{var}_1^i(\tilde{p}_2))^{-1}$$

denote the precision of the *ith* speculator's estimate of $\tilde{p}_2$ at time **1** weighted by his risk tolerance, and let

$$\rho_0^i = (\alpha(\operatorname{var}_0^i(\tilde{p}_1) - (\beta^i)^2 \operatorname{var}_0^i(\tilde{p}_2 - \tilde{p}_1)))^{-1}$$

be the analogous weighted precision at time 0, where the precision of the estimate of $\tilde{p}_1$ is adjusted to take into account the correlation of the first-period return with the second-period return. The "market" weight of the *ith* speculator is

$$w_t^i = \frac{\rho_t^i}{I\rho_t},$$

where $\rho_t = I^{-1}(\Sigma_i \rho_t^i)$ is the arithmetic average of the ρ_t^i's. The weighted

[6] When rates of interest differ from zero but are nonrandom, optimal futures demands at time zero are simply adjusted by an interest rate discount factor.

average of the conditional expectations of the final futures price at time 1 is

$$E_1(\tilde{p}_2) = \sum_i w_1^i E_1^i(\tilde{p}_2).$$

The weights w_t^i reflect the fact that speculators with less precise information trade less intensely on their differential beliefs. The same weights are used to define the averages of individual conditional variances, such as $\mathrm{var}_t(\tilde{p}_2) = \Sigma_i w_t^i \, \mathrm{var}_t^i(\tilde{p}_2)$.

We are now ready to summarize the equilibrium conditions for our general model, in Proposition 1. These conditions will be applied to two special examples in the next section. The proofs of Proposition 1 and all later propositions and lemmas are presented in the Appendix.

Proposition 1. Equilibrium Conditions. *Equilibrium futures prices are*

$$p_0 = E_0(\tilde{p}_1) + \sum_i \beta_i w_0^i E_0^i(\tilde{p}_2 - \tilde{p}_1) + \rho_0^{-1} h_0$$

and

$$p_1 = E_1(\tilde{p}_2) + \rho_1^{-1} h_1.$$

The corresponding equilibrium demands are

$$\begin{aligned} f_0^i = \rho_0^i \Bigg(& (1 - \beta^i)(E_0^i(\tilde{p}_1) - E_0(\tilde{p}_1)) + \beta^i(E_0^i(\tilde{p}_2) - E_0(\tilde{p}_2)) \\ & + \sum_k \beta^k w_0^k E_0^k(\tilde{p}_1) - \beta^i E_0(\tilde{p}_1) + \beta^i E_0(\tilde{p}_2) \\ & - \sum_k \beta^k w_0^k E_0^k(\tilde{p}_2) \Bigg) - w_0^i I h_0 \end{aligned}$$

and

$$f_1^i = \rho_1^i (E_1^i(\tilde{p}_2) - E_1(\tilde{p}_2)) - w_1^i I h_1.$$

Equilibrium conditions at time 1 are standard in one-period models of futures markets with risk-averse speculators. The equilibrium price yields an average conditional expected return equal to per-capita hedging demand weighted by the risk tolerance and average precision of speculators. The equilibrium demand is the sum of two components. The first component is the bet waged by the speculator against the average of all conditional estimates of the liquidating futures price. The second component is the speculator's share of hedging demand at time 1. Since I assumed identical risk tolerances, this share is uniquely determined by the relative precision of the speculator's information.

With two periods, the equilibrium conditions are slightly more complicated because the correlation between price changes in the first and second periods must be taken into account. Optimal demands include a component proportional to β^i [this is the second term on the right-hand side of Equation (A7)]. The equilibrium demands can be decomposed as the sum of three components. The first component of equilibrium demand is a weighted average of two distinct bets against the average market estimates of futures prices at times 1 and 2. The second component of demand depends on the difference between the precision of the speculator's information and the average precision of information. This is the second line of f_0^i, which reduces to zero when speculators have equal precisions. The third component of demand is the speculator's share of hedging demand at time 0.

It is important to notice that since speculators do not know hedging demand at time 0, they cannot infer the average conditional expected values of $\tilde{p}_1$ and $\tilde{p}_2$ from the equilibrium price p_o. Similarly, at time 1, they cannot infer the average conditional expectation of $\tilde{p}_2$ from p_1. This has two implications. First conditional expectations remain dispersed because equilibrium prices are "noisy"; that is, prices are not sufficient statistics for the private signals observed by speculators. Second, from the viewpoint of an econometrician armed with knowledge of the hedging demand, speculators' conditional expectations are biased. This is because, in making inferences from equilibrium prices, speculators are unable to isolate the influence of the unknown hedging demands from that of private information. For instance, speculators cannot tell if a high futures price is caused by favorable private signals or by a high hedging demand. Hence, they tend to overestimate (underestimate) payoffs when hedging demand is high (low). This bias induces an excess covariance between hedging demands and conditional price expectations. We will see in Section 4 that this excess covariance is proportional to the dispersion of expectations. This is the mechanism that induces a positive relation between price volatility and dispersion.

3. Two Special Examples

I now apply the equilibrium conditions of Section 2 to two examples with specialized information structures that lead to illustrative characterizations of the expected volume of trade and price volatility.

Example 1: "Myopic" futures demands. In this first example, I impose restrictions that essentially reduce the model to a sequence of one-period models. The trick is to eliminate the elements of demands and prices that depend on the correlation of first-period returns with second-period returns. While these restrictions may appear artificial, they greatly clarify the factors of expected volume and volatility. Furthermore, qualitative results will remain the same when we lift these restrictions in Example 2.

Assume that the payoff shocks $\tilde{s}_t$ are disclosed at time $t + 1$ and that speculators have no information on future payoff shocks or on future hedging demands. It is implicit in this last assumption that hedging demands are independent over time and independent of the payoff shocks $\tilde{s}_t$. Since speculators have no information on future shocks or on future hedging demands, and since the payoff shocks are revealed after one period, the only piece of information about which speculators disagree at time t is the realization of the current shock $\tilde{s}_t$. Hence,

$$E_t^i(\tilde{p}_2) - E_t(\tilde{p}_2) = E_t^i(\tilde{s}_t) - E_t(\tilde{s}_t).$$

Futhermore, conditional expectations of future payoff shocks and hedging demand are equal to zero because speculators have no information on these variables and their unconditional means are equal to zero by assumption. One can therefore write

$$E_0^i(\tilde{p}_1) = E_0^i(\tilde{E}_1(\tilde{p}_2) + \rho_1^{-1}\tilde{h}_1) = E_0^i(\tilde{E}(\tilde{p}_2) + \tilde{s}_0 + \tilde{E}_1(\tilde{s}_1))$$
$$= E(\tilde{p}_2) + E_0^i(\tilde{s}_0) = E_0^i(\tilde{p}_2). \tag{3}$$

Equality (3) tells us that the conditional expectations of $\tilde{p}_1$ and $\tilde{p}_2$ are equal. It follows that the speculator's two bets on these prices are equal and that their weighted sum reduces to a single bet on $\tilde{p}_2$ (or, equivalently, a single bet on $\tilde{p}_1$). Furthermore, the component of f_0^i that is correlated with time 1 speculative opportunities reduces to zero [see (A7)]. This is the sense in which we are calling the speculative demand f_0^i myopic. However, the coefficient ρ_0^i still depends on β^i. Thus, demands differ slightly from the demands obtained in a model where speculators just look one period ahead. With this simplification, equilibrium conditions at time 0 reduce to

$$p_0 = E_0(\tilde{p}_2) + \rho_0^{-1}h_0 \tag{4}$$

and

$$f_0^i = \rho_0^i(E_0^i(\tilde{p}_2) - E_0(\tilde{p}_2)) - w_0^i I h_0. \tag{5}$$

A comparison of Equation (4) with the expression for p_1 in Proposition 1 shows that the equilibrium futures price evolves very simply as the sum of the risk-adjusted per-capita hedging demand and the average conditional estimate of the liquidating futures price $\tilde{p}_2$.

A similar comparison of Equation (5) with f_1^i shows that the change in speculative demands from time 0 to time 1 is determined first by changes in the shares of hedging demand held by speculators, and second by the change in speculative bets against the average estimate of the final price. Hence, barring changes in hedging demand, trade in the second round is generated because speculators' adjustments of their risk-weighted estimates of $\tilde{p}_2$ diverge.

Example 2: **General model with equal precisions.** In contrast to the previous example, the payoff shocks $\tilde{s}_t$ in this example are not disclosed at time $t + 1$. This assumption implies that the futures demands at time 0 include a component correlated with speculative opportunities at time 1. I also allow the possibility that speculators have information about future hedging demands and about future payoff shocks $\tilde{s}_t$. However, I now assume that speculators have price estimates of equal precision. Since price estimates are of equal precision, we have $\mathrm{var}_t^i(\tilde{p}_\tau) = \mathrm{var}_t(\tilde{p}_\tau)$, $\beta^i = \beta$, and $\rho_t^i = \rho_t$. It follows that a change of variable to the weighted average of future prices

$$\tilde{p} = (1 - \beta)\tilde{p}_1 + \beta\tilde{p}_2$$

reduces the equilibrium conditions in Proposition 1 to

$$p_0 = E_0(\tilde{p}) + \rho_0^{-1}h_0, \tag{6}$$

$$p_1 = E_1(\tilde{p}) + \beta\rho_1^{-1}h_1, \tag{7}$$

$$f_0^i = \rho_0(E_0^i(\tilde{p}) - E_0(\tilde{p})) - h_0, \tag{8}$$

$$f_1^i = \rho_1(\beta)^{-1}(E_1^i(\tilde{p}) - E_1(\tilde{p})) - h_1. \tag{9}$$

The format of conditions (6) to (9) is exactly the same as the format of the equilibrium conditions in Example 1. The futures demand at t is equal to the sum of the speculator's bet on $\tilde{p}$ and of his share of hedging demand. The price at time t evolves as the sum of the average conditional expectation of $\tilde{p}$ and of the risk-weighted per-capita hedging demand at time t. It is immediate from this analogy that results on the volume of trade and price volatility obtained in Example 2 must be closely patterned after the corresponding results in Example 1. However, $\tilde{p}$, a weighted average of $\tilde{p}_1$ and $\tilde{p}_2$, is substituted for the liquidating price, $\tilde{p}_2$, and the relevant dispersion is the dispersion of expectations about $\tilde{p}$. This illustrates Grundy and McNichol's (1989) point that future market prices—in this case, $\tilde{p}_1$—are part of the future information on which traders speculate. In particular, they may agree about the distribution of the liquidating price but still speculate about future liquidity demands affecting nonliquidating prices.

4. The Dispersion of Expectations

In this section, I define a measure of the dispersion of expectations and show that the excess covariance between the random risk-weighted hedging demand and average price expectations is equal to this measure. This is a first indication that price volatility may be positively related to the dispersion of expectations because equilibrium prices are linear combinations of hedging demands and average price estimates. I conclude the section with a brief review of known results on the exogenous determinants of the dispersion of expectations.

Let

$$D_t(\tilde{x}) = E\left(\sum_i w_t^i(\tilde{E}_t^i(\tilde{x}) - \tilde{E}_t(\tilde{x}))^2\right)$$

denote the dispersion of expectations about a random variable 3 at t, where the weights w_t^i are defined in the same way as in Section 2. The dispersion $D_t(\tilde{x})$ is the average distance between individual expectations of $\tilde{x}$ and the average expectation of $\tilde{x}$. (When speculators have information of the same precision, individual distances from the average expectations are all equal to the dispersion.) The following lemma describes an interesting property of the dispersion of expectations.

Lemma 1.

$$D_t(\tilde{x}) = \mathrm{cov}(\tilde{x} - \tilde{E}_t(\tilde{x}), \tilde{E}_t(\tilde{x})).$$

To clarify Lemma 1, consider that when rational speculators have the same estimate of $\tilde{x}$, this estimate must be orthogonal to the estimate error $\tilde{x} - \tilde{E}_t(\tilde{x})$. Lemma 1 says that when speculators have different estimates of $\tilde{x}$ and their average estimate is thus not known, the covariance between this average estimate and the average estimate error is equal to the dispersion of expectations about $\tilde{x}$. In the examples we consider, Lemma 1 implies that the dispersion of expectations has the following additional property.

Lemma 2. *The excess covariance of the average price expectation of speculators with the risk-weighted hedging demand is equal to the dispersion of expectations. That is, in Example 1,*

$$D_0(\tilde{p}_2) = \mathrm{cov}(\tilde{E}_0(\tilde{p}_2) - \tilde{p}_2, \tilde{h}_0/\rho_0) = \mathrm{cov}(\tilde{E}_0(\tilde{p}_2), \tilde{h}_0/\rho_0$$

and

$$D_1(\tilde{p}_2) = \mathrm{cov}(\tilde{E}_1(\tilde{p}_2) - \tilde{p}_2, \tilde{h}_1/\rho_1) = \mathrm{cov}(\tilde{E}_1(\tilde{p}_2), \tilde{h}_1/\rho_1),$$

and in Example 2,

$$D_0(\tilde{p}) = \text{cov}(\tilde{E}_0(\tilde{p}) - \tilde{p}, \tilde{h}_0/\rho_0)$$

and

$$D_1(\tilde{p}) = \text{cov}(\tilde{E}_1(\tilde{p}) - \tilde{p}, (\beta/\rho_1)\tilde{h}_1).$$

Lemma 2 is a consequence of Lemma 1. In every period the average error of the price estimate is orthogonal to the known futures price $\tilde{p}_t$. [In Example 1 this average error is $\tilde{p}_2 - \tilde{E}_t(\tilde{p}_2)$, and in Example 2 it is $\tilde{p} - \tilde{E}_t(\tilde{p})$.] However, the average error of the price estimate is correlated with the average price estimate and with hedging demand because speculators cannot separate out these two components of $\tilde{p}_t$. Therefore, the excess covariance of the average estimate error with risk-weighted hedging demand must be exactly offset by the covariance of the average estimate error with the average price estimate. By Lemma 1, the latter is equal to the dispersion of expectations about this price.

Pfleiderer (1984) has analyzed the exogenous determinants of the dispersion of expectations in a one-period example where traders observe private signals of equal precision. In his model, the precision of private information and the dispersion of expectations are determined by a single parameter. This parameter is the standard deviation of the error terms in private signals. Pfleiderer shows that the dispersion of expectations increases with the variance of asset supply. Pfleiderer also shows that the dispersion of expectations as a function of the dispersion of private information is initially increasing and subsequently decreasing. The rough intuition behind this is as follows. As the standard deviation of the errors in private signals begins to increase from 0, expectations begin to diverge. However, when this standard deviation becomes too large, traders find their private signals too noisy and begin to put more weight on the market price. The dispersion of expectations therefore decreases. In more general models, traders observe successive waves of private and public signals of different precisions, so the relationship between the dispersion of expectations and precisions is more complex. The exact functional form of this relationship depends on the details of the informational structure, and it appears difficult to draw broad conclusions. Even the positive relationship between asset supply variability and dispersion may not hold up in all examples.

5. Volume and Volatility: “Myopic” Equilibrium

In the next two sections, I present my main results characterizing the determinants of trading volume and price volatility. These results indicate that there are two distinct possible sources of positive association between trading volume and price volatility. The first source is readily understood. Both volume and volatility are directly affected by shocks to liquidity demand: volume because speculators absorb the demand of hedgers each period, and volatility because the return that speculators expect for taking on this risk is proportional to the size of hedging demand.

The second possible source of positive association between volume and volatility is the dispersion of expectations. The association between volume and dispersion is intuitive. Volume is generated by intertemporal changes in differential beliefs, and the average size of these changes is related to the dispersions of expectations in successive periods. The possibility of a positive link between price volatility and dispersion is less obvious. Indeed, such a link may seem surprising because average price estimates are less variable when individual estimates are dispersed. What explains the additional price variability associated with dispersion is the fact that equilibrium prices are linear combinations of average future price estimates and of the risk-weighted hedging demand. The noise-related excess covariance between these two terms is precisely measured by the dispersion of expectations.

Proposition 2 describes the factors of expected trading volume and price volatility in Example 1. The virtue of this example is that equilibrium prices reduce to average estimates of a single future price adjusted by a hedging premium.

Let $\tilde{\Delta}_t f^i = \tilde{f}^i_t - \tilde{f}^i_{t-1}$ and $\tilde{\Delta}_t b^j = \tilde{b}^j_t - \tilde{b}^j_{t-1}$ denote the changes in the demands of speculators and hedgers at time t, and let $\tilde{\Delta}_t p = \tilde{p}_t - \tilde{p}_{t-1}$ denote the price change at time t. Our measure of price volatility is $\text{var}(\tilde{\Delta}_t p)$.

Proposition 2. Expected Volume of Trade and Volatility in Example 1. *(a) The expected volume of trade at time* 1 *is equal to*

$$E(\text{vol}) = \frac{1}{2}E\left(\sum_i |\tilde{\Delta}_1 f^i| + \sum_j |\tilde{\Delta}_1 b^j|\right)$$

$$= \frac{1}{\sqrt{2\pi}}\left(\sum_i (\text{var}(\tilde{\Delta}_1 f^i))^{1/2} + \sum_j (\text{var}(\tilde{\Delta}_1 b^j))^{1/2}\right),$$

where

$$\text{var}(\tilde{\Delta}_1 f^i) = (\rho_0^i)^2 D_0(\tilde{p}_2) + (\rho_1^i)^2 D_1(\tilde{p}_2) + (\rho_0^i)^2(\text{var}_0(\tilde{p}_2) - \text{var}_0^i(\tilde{p}_2))$$
$$+ (\rho_1^i)^2(\text{var}_1(\tilde{p}_2) - \text{var}_1^i(\tilde{p}_2)) + \text{var}\left(\frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right).$$

(b) The variance of price changes at time 1 *is equal to*

$$\operatorname{var}(\tilde{\Delta}_1 p) = D_0(\tilde{p}_2) + D_1(\tilde{p}_2) + \operatorname{var}_0(\tilde{p}_2) - \operatorname{var}_1(\tilde{p}_2) \;\; + \operatorname{var}\left(\frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right).$$

Proposition 2 first confirms that the expected volume of trade and price volatility both depend on changes in liquidity demands. This dependence is reflected in several terms proportional to the variances of weighted hedging shocks.

The second and more novel implication of Proposition 2 is that the expected trading volume and price volatility at time 1 are both related to the dispersions of expectations about $\tilde{p}_2$ at times 0 and 1. As shown by Lemma 2, the dispersion $D_0(\tilde{p}_2)$ in the expression for price volatility measures the excess covariance between time 0 risk-weighted hedging demand and the average estimate of $\tilde{p}_2$. This excess covariance is caused by the confusion between information shocks and hedging shocks. The dispersions in volatility therefore represent a component of volatility directly attributable to informational noise. The dispersion terms in the expression for the expected volume of trade are part of the change in the speculative bets made by traders at times 0 and 1. The sizes of these bets depend on the average discrepancy between individual and average estimates of $\tilde{p}_2$; that is, they depend on dispersion. The sizes of speculative bets also depend on the precision (adjusted precision) of individual estimates of $\tilde{p}_2$ $(\tilde{p}_1)$. These precisions appear as the coefficients ρ_t of the dispersions in the expression for expected volume. Note that the effect of precision on demands can confound empirical verification of the positive link between dispersion and expected volume. A decrease in the precision of private information can increase the dispersion of expectations while decreasing the precision of price estimates. If the latter effect is dominant, expected volume decreases while dispersion increases. An example of this is given in Pfleiderer (1984).

The effect of differences in estimate precisions on individual speculative demands may be seen in the second line of the expression for $\operatorname{var}(\tilde{\Delta}_1 f^i)$. This line shows that differences in the precision of private information affect the aggregate expected volume of trade. However, the direction of this effect is not clear. Speculators who are better informed than average naturally take on larger positions. This is offset by the fact that speculators who are less well informed than average trade with less intensity. The net effect on aggregate expected volume is therefore ambiguous, particularly since variations in the spread of precisions are not independent of changes in dispersion.

The third implication of Proposition 2 is that the quantity of price uncertainty resolved between periods, given by the expression $\operatorname{var}_0(\tilde{p}_2) - \operatorname{var}_1(\tilde{p}_2)$, is an explicit component of volatility, but that it is not a component of the expected volume of trade. New information generates trade only indirectly, through its effect on dispersion and on estimate precisions. Volume is therefore a fuzzier indicator of information than volatility. On the other hand, the influence of the quantity of information on price volatility clouds the empirical relation between volatility and dispersion. Volatility can decrease in spite of an increase in dispersion, just because less uncertainty is resolved.

The effect of the dispersion of expectations can also be detected in the contemporaneous and lagged relations between volume and price volatility. Proposition 3 describes these lagged relations, which are explained by the lagged dependence of time t volume and volatility on the dispersion of expectations at time $t - 1$. Proposition 3 relies on the following lemma.

Lemma 3. *The covariance between the absolute values of two normally distributed variables $\tilde{x}$ and $\tilde{y}$ with correlation $r(\tilde{x}, \tilde{y})$ and means equal to zero is equal to*

$$\operatorname{cov}(|\tilde{x}|, |\tilde{y}|) = 2(\operatorname{var}(\tilde{x})\operatorname{var}(\tilde{y}))^{1/2}\pi^{-1}((1 - r^2(\tilde{x}, \tilde{y}))^{1/2} - 1 + r(\tilde{x}, \tilde{y})\arcsin r(\tilde{x}, \tilde{y})).$$

Proposition 3. *(a) The covariance between contemporaneous absolute price changes and the volume of trade is positive and equal to*

$$\frac{1}{2}\left(\operatorname{cov}(|\tilde{\Delta}_1 p|, \sum_i |\tilde{\Delta}_1 f^i| + \sum_j |\tilde{\Delta}_1 b^j|)\right)$$
$$= \frac{1}{2}\left(\sum_i \operatorname{cov}(|\tilde{\Delta}_1 p|, |\tilde{\Delta}_1 f^i|) + \sum_j \operatorname{cov}(|\tilde{\Delta}_1 p|, |\tilde{\Delta}_1 b^j|)\right),$$

where $\operatorname{cov}(|\tilde{\Delta}_1 p|, |\tilde{\Delta}_1 f^i|)$ and $\operatorname{cov}(|\tilde{\Delta}_1 p|, |\tilde{\Delta}_1 b^j|)$ are obtained from Lemma 3 and the correlation $r(\tilde{\Delta}_1 p, \tilde{\Delta}_1 f^i)$ is derived from the covariance

$$\operatorname{cov}(\tilde{\Delta}_1 f^i, \tilde{\Delta}_1 p) = -(\rho_0^i D_0(\tilde{p}_2) + \rho_1^i D_1(\tilde{p}_2) + \rho_0^i(\operatorname{var}_0(\tilde{p}_2) - \operatorname{var}_0^i(\tilde{p}_2)) + (\rho_0^i/\rho_0^2)\operatorname{var}(\tilde{b}_0) + (\rho_1^i/\rho_1^2)\operatorname{var}(\tilde{b}_1)).$$

(b) The covariance between absolute-price changes and lagged volume of trade is found by substituting $\tilde{\Delta}_2 p$ for $\tilde{\Delta}_1 p$ in the expression for the covariance between contemporaneous volume and absolute price changes given in (a). The correlation $r(\tilde{\Delta}_1 f^i, \tilde{\Delta}_2 p)$ is derived from the covariance

$$\operatorname{cov}(\tilde{\Delta}_1 f^i, \tilde{\Delta}_2 p) = \rho_1^i D_1(\tilde{p}_2) + (\rho_1^i/\rho_1^2)\operatorname{var}(\tilde{b}_1) + \rho_1^i(\operatorname{var}_1(\tilde{p}_2) - \operatorname{var}_1^i(\tilde{p}_2)).$$

The covariances between absolute price changes and volumes of trade in (a) and (b) are all positive because each of them is equal to a sum of positive covariances between an absolute price change and the trading volumes of speculators and hedgers. As shown by Lemma 3, this positivity is simply a feature of the covariance between the absolute changes of any two normally distributed variables with means equal to zero and a nonzero correlation. Together, the expressions for the covariances between absolute price changes and volumes of trade and the expressions for the covariances between raw price and demand changes show that part of the correlation between absolute price changes and speculative volumes is explained by the effect of liquidity demands and that the other part is explained by the dispersion of expectations. The association between dispersion and these correlations is another manifestation of the impact of informational noise on changes in futures demands, on the one hand, and on changes in futures prices, on the other hand. The latter are related to the confusion between noise and information. The correlation between the volume of trade and the absolute price change at time t depends on the dispersions at times t and $t - 1$, while the correlation between the absolute price change at time t and the volume of trade at time $t - 1$ depends on the dispersion of expectations at time $t - 1$.

Proposition 4 is in the same vein as Proposition 3. It shows that the correlation between consecutive absolute price changes is also a positive function of the time-l dispersion of expectations about $\tilde{p}_2$. The reason is that the absolute price changes at times 1 and 2 both depend on this dispersion. Intuitively, informational noise at time t induces a local serial correlation in volatility by perturbing the price changes in the periods preceding and following time t.

Proposition 4 (Serial Correlation between Absolute Price Changes). *The correlation between the absoluteprice changes* $|\tilde{\Delta}_1 p| = |\tilde{p}_1 - \tilde{p}_0|$ *and* $|\tilde{\Delta}_2 p| = |\tilde{p}_2 - \tilde{p}_1|$ *is positive and equal to*

$$\frac{2}{\pi - 2}\left(\left(1 - r^2(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p)\right)^{1/2} - 1 + r(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p)\arcsin r(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p)\right).$$

In Example 1, the correlation $r(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p)$ *is drawn from the covariance*

$$\operatorname{cov}(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p) = -(D_1(\tilde{p}_2) + \operatorname{var}(\tilde{b}_1/\rho_1)).$$

6. Volume and Volatility in Example 2

In this section, I adapt[7] Proposition 2 to the more general Example 2. Recall that the essential difference between the two examples is

[7] Propositions 3 and 4 can be adapted in a similar fashion.

that, in Example 2, equilibrium demands and prices are based on a weighted average of the distinct conditional expectations of $\tilde{p}_1$ and $\tilde{p}_2$. In Example 1 these conditional expectations are equal. Proposition 5 is therefore guided by the same intuition regarding the role of dispersion as the results of the preceding section. The difference is that dispersion now concerns expectations about a weighted average of future prices. This leads to an important distinction between Examples 1 and 2: the dispersion of expectations about future hedging demand has a tangible effect on expected volume and volatility. In Example 1 this dispersion is equal to zero. This adds a realistic touch: speculators can agree about "fundamentals," such as the distribution of the liquidating futures price, yet trade on the basis of divergent beliefs about future liquidity demands. This type of trading is generated by what Kraus and Smith (1989) call "market created risk."

Proposition 5. *Assume that speculators have information of identical precision. Then the following statements bold.*

(a) The expected volume of trade at time 1 is given by the same expression as in Proposition 2(a), but

$$\operatorname{var}(\tilde{\Delta}_1 f^i) = \operatorname{var}(\tilde{b}_1 - \tilde{b}_0) + (\rho_0)^2 D_0(\tilde{p}) + (\rho_1/\beta)^2 D_1(\tilde{p}) - 2\rho_0(\rho_1/\beta)\operatorname{cov}(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1(\tilde{p}), \tilde{E}_0^i(\tilde{p})).$$

(b) The variance of price changes is equal to

$$\operatorname{var}(\tilde{\Delta}_1 p) = D_0(\tilde{p}) + D_1(\tilde{p}) + \operatorname{var}_0(\tilde{p}) - \operatorname{var}_1(\tilde{p}) + \operatorname{var}\left(\beta\frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right) + 2\operatorname{cov}\left(\tilde{p} - \tilde{E}_0(\tilde{p}), \beta\frac{\tilde{b}_1}{\rho_1}\right),$$

where we recall that $\operatorname{var}_t(\tilde{p})$ *is the average conditional variance of* $\tilde{p}$ *at t.*

7. Conclusion

I have shown that the dispersion of expectations gives a measure of both the excess volume and the excess price variability associated with market "noisiness." This result suggests that a number of salient facts about volume and price volatility are related to dispersion. Specifically, the model predicts that the dispersion of expectations contributes to (1) positive correlations between trading volume and contemporaneous and future absolute price changes, and (2) a positive correlation between consecutive absolute price changes. These are novel predictions that can be tested with survey data similar to the data in Frankel and Froot (1990).

These results also provide some insight on certain seasonal patterns of volume and volatility. For example, the peak volume and volatility observed at the opening of trade may be a consequence of greater dispersion. Since information is more diffuse and markets tend to be noisier after an interruption of trade, dispersion is likely to be greater at this time.[8] Another piece of empirical evidence consistent with our results is described in Neal (1988). He finds that the ratio of volatility to expected volume is greatest at the open and tends to decrease throughout the day. Since volatility directly depends on the price uncertainty resolved over time, while volume does not, the ratio of volatility to volume is a rough proxy for the information accumulated between trading rounds. The latter naturally tends to be greatest at the opening of trade.

The relation between the dispersion of expectations and price volatility is not as fully recognized as the relation between dispersion and the volume of trade. Typically, price volatility is more closely connected with information or liquidity flows; this may be why dramatic price changes unaccompanied by equally dramatic news or liquidity shocks are sometimes found puzzling, and also why such changes tend to be ascribed to the nonrational behavior of some traders. An alternative explanation is that noisy markets tend to be more volatile. The role of dispersion illustrated in this model pins down this particular influence of noise on price volatility.

Appendix: Proofs

The following proofs will use the equalities

$$\operatorname{cov}(\tilde{x}, E_t^i(\tilde{x})) = \operatorname{var}(\tilde{E}_t^i(\tilde{x})) = \operatorname{var}(\tilde{x}) - \operatorname{var}_t^i(\tilde{x}). \tag{A1}$$

Proof of Proposition 1. This proposition is obtained by recursive optimization and two successive applications of the properties of moment generating functions of normal variables to the expectation in (2).

We first expand expression (1) as

$$-\exp(-\alpha(f_0^i(p_1 - p_0) - f_1^i p_1))E_1^i(\exp(-\alpha f_1^i \tilde{p}_2)), \tag{A2}$$

where the solution f_1^i to the maximization problem in (1) is a standard one-period result:

[8] A burst of volume and volatility at the open is also be expected from adjustments in liquidity demands. Similar liquidity adjustments also cause an increase in volume and volatility at the close. This idea has been formalized by Brock and Kleidon (1992) and tested by Gerety and Mulherin (1992).

$$f_1^i = \rho_1^i(E_1^i(\tilde{p}_2) - p_1), \tag{A3}$$

Note that the term $E_1^i(\exp(-\alpha f_1^i \tilde{p}_2))$ is the moment generating function of $\tilde{p}_2$ at $-\alpha f_1^i$.

This term is therefore equal to

$$\exp(-\alpha f_1^i E_1^i(\tilde{p}_2) + \tfrac{1}{2}(\alpha f_1^i)^2 \operatorname{var}_1^i(\tilde{p}_2)).$$

Now substitute the optimal value off: given by (A3) into this term and substitute the resulting expression back in (A2) to obtain

$$-\exp\left(-\alpha f_0^i(p_1 - p_0) - \frac{(E_1^i(\tilde{p}_2) - p_1)^2}{2\operatorname{var}_1^i(\tilde{p}_2)}\right). \tag{A4}$$

Define the vector $x = (x_1, x_2) = (p_1, E_1^i(\tilde{p}_2))$, the function

$$Q(x) = -\alpha f_0^i x_1 - \frac{(x_2 - x_1)^2}{2\operatorname{var}_1^i(\tilde{p}_2)},$$

and expression (2), the expected utility at time 0, can be reexpressed as

$$-\exp(\alpha f_0^i p_0) E_0^i(\exp(Q(\tilde{x}))). \tag{A5}$$

The first and second derivatives of *Q(x)* are respectively equal to

$$Q'(x) = \left(-\alpha f_0^i - \frac{x_1 - x_2}{\operatorname{var}_1^i(\tilde{p}_2)}, \frac{x_1 - x_2}{\operatorname{var}_1^i(\tilde{p}_2)}\right)$$

and

$$Q''(x) = \frac{1}{\operatorname{var}_1^i(\tilde{p}_2)}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Now substitute the Taylor expansion of *Q(x)* into the expected utility at time 0 to obtain

$$\begin{aligned} -\exp(\alpha f_0^i p_0 + Q(E(\tilde{x})))E_0^i(\exp(Q'(E(\tilde{x}))(\tilde{x} - E(\tilde{x})) + \tfrac{1}{2}(\tilde{x} - E(\tilde{x})) \\ \times Q''(E(\tilde{x}))(\tilde{x} - E(\tilde{x}))^T)), \end{aligned} \tag{A6}$$

where the superscript *T* indicates the transpose of a vector or matrix. Let Λ represent the covariance matrix of $\tilde{x}$, and note that

$$\Lambda = \begin{pmatrix} \operatorname{var}_0^i(\tilde{p}_1) & \operatorname{cov}_0^i(\tilde{p}_1, \tilde{p}_2) \\ \operatorname{cov}_0^i(\tilde{p}_1, \tilde{p}_2) & \operatorname{var}_0^i(\tilde{p}_2) - \operatorname{var}_1^i(\tilde{p}_2) \end{pmatrix}.$$

Next, define $\Sigma^{-1} = \Lambda^{-1} - Q''(E(\tilde{x}))$. It can be verified that

Using equality (A1) and the fact that the estimate errors $\tilde{x} - E_t(\tilde{x})$ and $\tilde{x} - E_t^i(\tilde{x})$ are orthogonal to their respective conditioning information concludes the proof of Lemma 1.

Proof of Lemma 2. This result is an immediate consequence of Lemma 1 and of the orthogonality of pricing errors at t with the equilibrium price at t. In Example 1,

$$\mathrm{cov}(\tilde{p}_2 - E_t(\tilde{p}_2), \tilde{p}_t) = 0.$$

Substitute the equilibrium value of $\tilde{p}_t$, apply Lemma 1, and rearrange terms to obtain

$$\mathrm{cov}\left(\tilde{E}_t(\tilde{p}_2), \frac{\tilde{b}_t}{\rho_t}\right) = D_t(\tilde{p}_2) + \mathrm{cov}\left(\tilde{p}_2, \frac{\tilde{b}_t}{\rho_t}\right).$$

The orthogonality between $\tilde{b}_t$ and $\tilde{p}_2$ concludes the proof. In Example 2, similarly, use the equality

$$\mathrm{cov}(\tilde{p} - E_t(\tilde{p}), \tilde{p}_t) = 0,$$

substitute the equilibrium value of $\tilde{p}_t$, and rearrange terms to obtain

$$\mathrm{cov}\left(\tilde{E}_0(\tilde{p}), \frac{\tilde{b}_0}{\rho_0}\right) = D_0(\tilde{p}) + \mathrm{cov}\left(\tilde{p}, \frac{\tilde{b}_0}{\rho_0}\right)$$

and

$$\mathrm{cov}\left(\tilde{E}_1(\tilde{p}), \frac{\beta}{\rho_1}\tilde{b}_1\right) = D_1(\tilde{p}) + \mathrm{cov}\left(\tilde{p}, \frac{\beta}{\rho_1}\tilde{b}_1\right).$$

Proof of Proposition 2

(a) Expected volume of trade. The volume of trade at t is half the sum of the absolute values of the change in traders' futures positions since t – 1. For a speculator, this change is $\tilde{\Delta}_1 f^i$. For the *jth* hedger, it is $\tilde{\Delta}_1 b^j$. By assumption, these changes are normally distributed variables. Also by assumption $E(\tilde{\Delta}_1 b^j) = 0$, from which it follows that $E(\tilde{\Delta}_1 f^i) = 0$. The expectation of the absolute value of a normal variable with mean equal to zero[9] is

$$E(|\tilde{x}|) = ((2/\pi)\mathrm{var}(\tilde{x}))^{1/2}. \tag{A9}$$

[9] We have

$$E(|\tilde{x}|) = \left(\frac{2}{\pi}\mathrm{var}(\tilde{x})\right)^{1/2} \exp\left(-\frac{(E(\tilde{x}))^2}{2\,\mathrm{var}(\tilde{x})}\right) + E(\tilde{x})(1 - 2\mathrm{CF}_x),$$

where CF_x is the cumulative distribution of $\tilde{x}$ at 0. If $E(\tilde{b}^j) \neq 0$, all qualitative results about the expected volume of trade continue to hold because $E(|\tilde{x}|)$ *is* an increasing function of $\mathrm{var}(\tilde{x})$.

Substituting $\tilde{\Delta}_1 f^i$ and $\tilde{\Delta}_1 b^j$ i into (A9) reduces the expected volume to the sum of standard deviations of demand changes given at the top of Proposition 2.

Next, we must find var$(\tilde{\Delta}_1 f^i)$. First substitute the equilibrium demands from Proposition 1 into this variance. This leads to

$$\begin{aligned}\operatorname{var}(\tilde{\Delta}_1 f_i) = {} & (\rho_0^i)^2 \operatorname{var}(\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2)) \\ & + (\rho_1^i)^2 \operatorname{var}(\tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2)) \\ & - 2\rho_0^i \rho_1^i \operatorname{cov}(\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2)) \\ & - 2\rho_1^i \operatorname{cov}\left(\tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2), \frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right) \\ & + 2\rho_0^i \operatorname{cov}\left(\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right) \\ & + \operatorname{var}\left(\frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right).\end{aligned}$$

Note that the second line of this equation reduces to zero. This follows from the independence between payoff shocks and the fact that in Example 1, $\tilde{E}_t^i(\tilde{p}_2) - \tilde{E}_t(\tilde{p}_2) = \tilde{E}_t^i(\tilde{s}_t) - \tilde{E}_t(\tilde{s}_t)$. Furthermore, the covariance between $\tilde{b}_0$ and $\tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2)$ and the covariance between $\tilde{b}_1$ and $\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2)$ are also equal to zero by assumption. Using these facts and regrouping terms, we obtain

$$\begin{aligned}\operatorname{var}(\tilde{\Delta}_1 f_i) = {} & (\rho_0^i)^2 \operatorname{cov}(\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2) - 2(\tilde{b}_0/\rho_0)) \\ & + (\rho_1^i)^2 \operatorname{cov}(\tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2), \tilde{E}_1^i(\tilde{p}_2) - \tilde{E}_1(\tilde{p}_2) - 2(\tilde{b}_1/\rho_1)) \\ & + \operatorname{var}\left(\frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right).\end{aligned}$$

Proposition 1 and Equation (4) imply that

$$\tilde{b}_1/\rho_1 = \tilde{p}_1 - \tilde{E}_1(\tilde{p}_2)$$

and

$$\tilde{b}_0/\rho_0 = \tilde{p}_0 - \tilde{E}_0(\tilde{p}_2). \tag{A10}$$

Using (A10), the orthogonality of time 0 estimate errors with $\tilde{p}_0$, and the orthogonality of time 1 estimate errors with $\tilde{p}_1$ leads to

$$\begin{aligned}\operatorname{var}(\tilde{\Delta}_1 f_i) = {} & (\rho_0^i)^2(\operatorname{var}(\tilde{E}_0^i(\tilde{p}_2)) - \operatorname{var}(\tilde{E}_0(\tilde{p}_2))) \\ & + (\rho_1^i)^2(\operatorname{var}(\tilde{E}_1^i(\tilde{p}_2)) - \operatorname{var}(\tilde{E}_1(\tilde{p}_2)))\end{aligned}$$

$$+ \operatorname{var}\left(\frac{\rho_1^i}{\rho_1}\tilde{b}_1 - \frac{\rho_0^i}{\rho_0}\tilde{b}_0\right).$$

To conclude the proof of Proposition 2(a), use Lemma 1 and the equalities var $(\tilde{E}_t^i(\tilde{p}_2)) = \operatorname{var}(\tilde{p}_2) - \operatorname{var}_t^i(\tilde{p}_2)$ and $\operatorname{cov}(\tilde{p}_2, \tilde{E}_t(\tilde{p}_2)) = \operatorname{var}(\tilde{p}_2) - \operatorname{var}_t(\tilde{p}_2)$ for $t = 0,1$. These equalities follow from (Al).

We now turn to proving (b).

(b) Variance of price changes. Substitute the prices from Proposition 1 and Equation (4) to obtain

$$\begin{aligned}\operatorname{var}(\tilde{p}_1 - \tilde{p}_0) &= \operatorname{var}(\tilde{E}_1(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2)) + \operatorname{var}\left(\frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right)\\ &\quad + 2\operatorname{cov}\left(\tilde{E}_1(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right).\end{aligned}$$

Use Proposition 1 and Equation (4) again to substitute for the hedging demands in the second line of this expression. This yields

$$\begin{aligned}\operatorname{var}(\tilde{p}_1 - \tilde{p}_0) &= \operatorname{var}(\tilde{E}_1(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2)) + \operatorname{var}\left(\frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right)\\ &\quad + 2\operatorname{cov}(\tilde{E}_1(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \tilde{p}_1 - \tilde{p}_0).\end{aligned}$$

Since forecasting errors at times 0 and 1 are orthogonal to p_0, the last line of this expression reduces to

$$2\operatorname{cov}(\tilde{E}_1(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2), \tilde{p}_1). \tag{A11}$$

Further notice that

$$\operatorname{cov}(\tilde{E}_1(\tilde{p}_2), \tilde{p}_1) = \operatorname{cov}(\tilde{p}_2, \tilde{p}_1). \tag{A12}$$

Now replace $\tilde{p}_1$ in (A12) by its equilibrium value, and substitute (A12) into (A11) and (A11) into the variance of price changes to obtain

$$\begin{aligned}\operatorname{var}(\tilde{p}_1 - \tilde{p}_0) = &-\operatorname{var}(\tilde{E}_1(\tilde{p}_2)) - \operatorname{var}(\tilde{E}_0(\tilde{p}_2))\\ &+ 2\operatorname{cov}(\tilde{E}_1(\tilde{p}_2), \tilde{E}_0(\tilde{p}_2)) + \operatorname{var}\left(\frac{\tilde{b}_1}{\rho_1} - \frac{\tilde{b}_0}{\rho_0}\right)\\ &+ 2\operatorname{cov}(\tilde{p}_2, \tilde{E}_1(\tilde{p}_2)) - 2\operatorname{cov}(\tilde{E}_1(\tilde{p}_2), \tilde{E}_0(\tilde{p}_2))\\ &+ 2\operatorname{cov}\left(\tilde{p}_2 - \tilde{E}_0(\tilde{p}_2), \frac{\tilde{b}_1}{\rho_1}\right).\end{aligned}$$

In Example 1, hedging shocks are assumed independent of each other and of payoff shocks. This implies that the last line of (A13) reduces to 0. Using Lemma 1 and the last equality used in the proof of part (a) of this proposition concludes the proof of part (b).

Proof of Lemma 3. Since $\tilde{x}$ and $\tilde{y}$ have a bivariate normal distribution and their means are zero, the covariance of their absolute values can be written as

$$\frac{1}{2\pi|M|^{1/2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|xy|\exp(\alpha x^2+\beta y^2+\gamma xy)\,dx\,dy$$

$$-\frac{2}{\pi}(\mathrm{var}(\tilde{x})\mathrm{var}(\tilde{y}))^{1/2}, \qquad \text{(A14)}$$

where

$$\alpha=-\frac{\mathrm{var}(\tilde{y})}{2|M|},\qquad \beta=-\frac{\mathrm{var}(\tilde{x})}{2|M|},\qquad \gamma=\frac{\mathrm{cov}(\tilde{x},\tilde{y})}{|M|},$$

and $|M|$ is the determinant of the covariance matrix of $\tilde{x}$ and $\tilde{y}$:

$$\begin{aligned}|M|&=\mathrm{var}(\tilde{x})\mathrm{var}(\tilde{y})-\mathrm{cov}^2(\tilde{x},\tilde{y})\\&=\mathrm{var}(\tilde{x})\mathrm{var}(\tilde{y})(1-r^2(\tilde{x},\tilde{y})).\end{aligned}$$

To do this integration, first transform to polar coordinates R and θ. The integration with respect to R yields readily to the substitution $u = \exp(kR^2)$, where k is a polynomial in sin θ and cos θ. The double integral then reduces to a rational function of $\sin(2\theta)$ and $\cos(2\theta)$ which is handled by the substitution $v = \tan\theta$. Dividing the resulting covariance by the standard deviations of $|\tilde{x}|$ and $|\tilde{y}|$, respectively equal to $\sqrt{((\pi-2)/\pi)\mathrm{var}(\tilde{x})}$ and $\sqrt{((\pi-2)/\pi)\mathrm{var}(\tilde{y})}$, leads to the expression for the correlation between $|\tilde{x}|$ and $|\tilde{y}|$. This correlation is positive for all nonzero values of $r(\tilde{x},\tilde{y})$.

[The reader may also find it interesting to compare Lemma 3 with the results of Krishnaiah, Hagis, and Steinberg (1963).]

Proof of Proposition 3. The covariance between volume and the absolute price change at time 1 is the sum of the covariances between the absolute price change and individual trading volumes. We only need to find one of these covariances. It follows immediately from Lemma 3 that

$$\begin{aligned}\tfrac{1}{2}\mathrm{cov}(|\tilde{\Delta}_1 p|,|\tilde{\Delta}_1 f^i|)&=\pi^{-1}(\mathrm{var}(\tilde{\Delta}_1 p)\mathrm{var}(\tilde{\Delta}_1 f^i))^{1/2}\\&\quad\times((1-r^2(\tilde{\Delta}_1 f^i,\tilde{\Delta}_1 p))^{1/2}\\&\quad-1+r(\tilde{\Delta}_1 f^i,\tilde{\Delta}_1 p)\arcsin r(\tilde{\Delta}_1 f^i,\tilde{\Delta}_1 p)).\end{aligned}$$

The covariance from which the correlation $r(\tilde{\Delta}_1 f^i,\tilde{\Delta}_1 p)$ is derived is found as follows. $\tilde{\Delta}_1 f^i$ is equal to the difference between the speculative bets at times 1 and 0 minus the change in the speculator's shares of hedging demand. Since $\tilde{\Delta}_1 p$ is part of the information set of every trader at time 1, and $\tilde{p}_0$ is part of their information set at time 0, the covariance $\mathrm{cov}(\tilde{\Delta}_1 p, \tilde{\Delta}_1 f^i)$ reduces to

$$-\mathrm{cov}(\rho_0^i(\tilde{E}_0^i(\tilde{p}_2) - \tilde{E}_0(\tilde{p}_2)), \tilde{p}_1) - \mathrm{cov}\left(\tilde{\Delta}_1 p, \frac{\rho_1^i}{\rho_1}\tilde{h}_1 - \frac{\rho_0^i}{\rho_0}\tilde{h}_0\right). \quad \text{(A15)}$$

The final expression for the covariance is obtained by substituting the equilibrium prices from Proposition 1 and Equality (4) into (A15), and using the independence between hedging shocks and payoff shocks, Lemma 2 and orthogonality conditions between forecasting errors and observable variables. Recall that at time 1, $\tilde{s}_0$ is observable.

Substituting the solution to each of the individual covariances and summing them leads to the expression shown for the aggregate covariance between the contemporaneous volume and absolute price change. The covariance between lagged volume and the absolute price change is obtained in a similar manner by substitution of equilibrium demands and prices from Proposition 1 and Equations (4) and (5), and by applying Lemma 2 and the orthogonality conditions between forecasting errors and observable variables.

Proof of Proposition 4. Lemma 3 is applied to find the correlation between consecutive absolute price changes. The correlation $r(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p)$ between consecutive price changes is derived from their covariance. This is given by the formula

$$\mathrm{cov}(\tilde{\Delta}_1 p, \tilde{\Delta}_2 p) = \mathrm{cov}(\tilde{p}_2 - \tilde{E}_1(\tilde{p}_2) - \tilde{h}_1/\rho_1, \tilde{\Delta}_1 p). \quad \text{(A16)}$$

Since $\tilde{\Delta}_1 p$ is in the information set of every trader at time 1, (A16) reduces to

$$-\mathrm{cov}(\tilde{\Delta}_1 p, \tilde{h}_1/\rho_1).$$

If we substitute the equilibrium price functions into this expression and use the same orthogonality conditions as in previous proofs, the independence of hedging shocks and payoff shocks and Lemma 2 conclude the proof.

Proof of Proposition 5.

(a) Expected volume of trade. The term $\mathrm{var}(\tilde{\Delta}_1 f^i)$ in the expression for the expected volume of trade is the only one that differs from Example 1. Recall that with the change of variable $\tilde{p} = (1 - \beta)\tilde{p}_1 + \beta\tilde{p}_2$, we have

$$\tilde{\Delta}_1 f^i = (\rho_1/\beta)(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1^i(\tilde{p})) - \tilde{h}_1 - (\rho_0(\tilde{E}_0^i(\tilde{p}) - \tilde{E}_0(\tilde{p})) - \tilde{h}_0).$$

Except for the different coefficients, this expression is identical to $\tilde{\Delta}_1 f^i$ in Example 1. The proof therefore proceeds much like the proof of Proposition 2 (a). That is, it uses the substitution of the equilibrium demands from (8) and (9), as well as equilibrium prices (6) and (7), Lemma 1, and the orthogonality conditions between estimate errors and any observable variables (e.g., past and current prices) at time *t*. In this example, simplifications obtained from the assumptions specific to Example 1 (i.e., the independence of hedging from payoff shocks and the disclosure of s_0 at time 1) no longer hold. On the other hand, a number of terms drop out because all speculators have information of the same precision. First expand $\mathrm{var}(\tilde{\Delta}_1 f^i)$ and regroup terms to obtain

$$\begin{aligned}\mathrm{var}(\tilde{f}_1^i - \tilde{f}_0^i) = {} & \mathrm{var}(\tilde{h}_1 - \tilde{h}_0) + \rho_0^2 \mathrm{var}(\tilde{E}_0^i(\tilde{p}) - \tilde{E}_0(\tilde{p})) \\ & + \left(\frac{\rho_1}{\beta}\right)^2 \mathrm{var}(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1(\tilde{p})) \\ & - 2\frac{\rho_1}{\beta}\rho_0 \mathrm{cov}(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1(\tilde{p}), \tilde{E}_0^i(\tilde{p}) - \tilde{E}_0(\tilde{p})) \\ & - 2\,\mathrm{cov}\left(\frac{\rho_1}{\beta}(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1(\tilde{p})) \right. \\ & \qquad \left. - \rho_0(\tilde{E}_0^i(\tilde{p}) - \tilde{E}_0(\tilde{p})), \tilde{h}_1 - \tilde{h}_0\right).\end{aligned}$$

Because all speculators have information of the same precision, the last term drops out and the second to last term simplifies to

$$-2(\rho_1/\beta)\rho_0 \mathrm{cov}(\tilde{E}_1^i(\tilde{p}) - \tilde{E}_1(\tilde{p}), \tilde{E}_0^i(\tilde{p})).$$

Also, as seen in Section 4, individual distances between individual and average expectations of a random variable $\tilde{x}$ are all equal to the dispersion of expectations about $\tilde{x}$. Hence the second term is equal to $(\rho_0)^2 D_0(\tilde{p})$, and the third term is equal to $\rho_1/\beta)^2 D_1(\tilde{p})$.

(b) Variance of price changes. Substitute the prices from Equations (6) and (7):

$$\begin{aligned}\mathrm{var}(\tilde{\Delta}_1 p) = {} & \mathrm{var}(\tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p})) + \mathrm{var}\left(\frac{\beta}{\rho_1}\tilde{h}_1 - \frac{\tilde{h}_0}{\rho_0}\right) \\ & + 2\,\mathrm{cov}\left(\tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p}), \frac{\beta}{\rho_1}\tilde{h}_1 - \frac{\tilde{h}_0}{\rho_0}\right).\end{aligned}$$

Expand the last line after reexpressing hedging demands in terms of prices differences by using (6) and (7). Note that $\mathrm{cov}(\tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p}), \tilde{p}_0) = 0$, and cancel offsetting terms to arrive at

$$\operatorname{var}(\tilde{\Delta}_1 p) = -\operatorname{var}(\tilde{E}_1(\tilde{p})) - \operatorname{var}(\tilde{E}_0(\tilde{p})) + \operatorname{var}\left(\frac{\beta}{\rho_1}\tilde{b}_1 - \frac{\tilde{b}_0}{\rho_0}\right)$$
$$+ 2\operatorname{cov}(\tilde{p}_1, \tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p})) + 2\operatorname{cov}(\tilde{E}_0(\tilde{p}), \tilde{E}_1(\tilde{p})).$$

Apply Lemma 1 to the first line and use the orthogonality between $\tilde{p} - \tilde{E}_1(\tilde{p})$ and $\tilde{p}_1$ to replace $\operatorname{cov}(\tilde{p}_1, \tilde{E}_1(\tilde{p}))$ by $\operatorname{cov}(\tilde{p}_1, \tilde{p})$. Then expand $\tilde{p}_1$ by using Equation (7) to reduce the variance to

$$\operatorname{var}(\tilde{\Delta}_1 p) = D_0(\tilde{p}) + D_1(\tilde{p}) + \operatorname{var}\left(\frac{\beta}{\rho_1}\tilde{b}_1 - \frac{\tilde{b}_0}{\rho_0}\right)$$
$$+ \operatorname{cov}(\tilde{p}, \tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p})) + 2\operatorname{cov}\left(\tilde{p} - \tilde{E}_0(\tilde{p}), \frac{\beta}{\rho_1}\tilde{b}_1\right).$$

To conclude the proof note that

$$\operatorname{cov}(\tilde{p}, \tilde{E}_1(\tilde{p}) - \tilde{E}_0(\tilde{p})) = \operatorname{var}(\tilde{p}) - \operatorname{var}_1(\tilde{p}) - \operatorname{var}(\tilde{p}) + \operatorname{var}_0(\tilde{p}).$$

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