

Stock-Price Manipulation

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It is generally agreed that speculators can make profits from insider trading or from the release of false information. Both forms of stock-price manipulation have now been made illegal. In this article, we ask whether it is possible to make profits from a different kind of manipulation, in which an uninformed speculator simply buys and sells shares. We show that in a rational expectations framework, where all agents maximize expected utility, it is possible for an uninformed manipulator to make a profit, provided investors attach a positive probability to the manipulator being an informed trader.

Historically, the possibility of artificially influencing stock prices has been an important issue.¹ Soon after the Amsterdam Stock Exchange was founded at the beginning of the seventeenth century, brokers discovered that they could profitably manipulate stock prices. They would engage in a concentrated bout of selling. Frightened investors would then also sell, prices would fall, and the brokers could buy back stock to restore their original positions at a lower price. The brokers also found that the profitability of these “bear raids” could be increased by spreading

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¹ For accounts of the history of stock-price manipulation, see Sobel (1965) and Twentieth Century Fund (1935).

false rumors about the poor prospects of the firm.

This type of manipulation occurred in all stock markets that were established during the following years. In some countries such activities were judged to be illegal.² However, manipulation continued and there are many colorful accounts of how fortunes were made on Wall Street during the nineteenth century through stock price manipulation. Jacob Little, who was nicknamed the "Great Bear of Wall Street," was said "to gorge and digest more stock in one day than the weight of the bulk of his whole body in certificates."³

Bear raids were not always successful, since they created the possibility that another speculator would corner the market. One widely recounted example is the Harlem Railway corner, in which the short sellers were unable to cover their positions and were forced to buy back shares at an inflated price.⁴ In this case, the short sellers (the New York City Council) had taken an action that they thought would reduce the value of the stock. On occasion, managers of firms also would manipulate the value of their own stock. For example, in 1901 the managers of American Steel and Wire Company (the forerunner of USX) shorted the firm's stock and then closed its steel mills. When the closure was announced, the stock price fell from around \$60 to around \$40 per share. The managers then covered their short positions and reopened the mills, at which point the stock price rose to its previous level.⁵

The many instances of stock-price manipulation that were revealed over the years led to considerable discussion of the issue. In fact, Huebner (1934, p. 397) argued that stock-price manipulation was the most widely discussed aspect of stock markets. After the Great Crash of 1929, there was widespread public concern that the fall in prices had been caused by bear raids. As a result of this concern, the Senate Committee on Banking and Currency conducted extensive investi-

² During the Napoleonic Wars, the prices of bonds and stocks on the London Stock Exchange were sensitive to the progress of the fighting. Manipulators would operate in conjunction with newspapers to spread false information about the war and profit from the resulting changes in prices. A number of people were tried for a conspiracy to manipulate prices [see Twentieth Century Fund (1935), p. 449, and Flynn (1934), p. 213].

³ Stedman (1905, p. 101), quoted by Sobel (1965, p. 60). Little would sell short shares that he did not own and then spread rumors about the insolvency of the company. After he had forced the price down, he would cover his short position.

⁴ At the beginning of 1863, Commodore Cornelius Vanderbilt bought stock in the Harlem Railway at around \$8 to \$9 a share. He took an interest in running the company and its stock price advanced to \$50 per share. In April 1863, the New York City Council passed an ordinance allowing the Harlem Railway to build a streetcar system the length of Broadway and, as a result, the stock price went to \$75. Members of the council then conspired to sell the stock short, repeal the ordinance, and thus force the price down. However, Vanderbilt discovered the plot and managed to buy the entire stock of the company in secret. When the members of the council tried to cover their short positions after the repeal of the ordinance, they discovered that none of the stock could be purchased. Vanderbilt forced them to settle at \$179 per share. See Eiteman, Dice, and Eiteman (1966, p. 562) for an account of this episode.

⁵ See Wycoff (1968, p. 77-78) for an account of this incident.

gations into the operations of the security markets. Although they uncovered little evidence of bear raids during the Great Crash, they did uncover extensive evidence of other types of manipulation. In particular, many witnesses outlined the operation of “trading pools” during the 1920s. A group of investors would combine: first to buy a stock, then to spread favorable rumors about the firm, and finally to sell out at a profit.⁶

The evidence uncovered by the Senate Committee led to extensive provisions in the Securities Exchange Act of 1934 to eliminate manipulation. The kinds of manipulation that the Act effectively outlawed fall naturally into two categories. The first can be described as *action-based manipulation*, that is, manipulation based on actions that change the actual or perceived value of the assets. Examples of action-based manipulation are the Harlem Railway and American Steel and Wire Company cases described above. The second category can be described as *information-based manipulation*, that is, manipulation based on releasing false information or spreading false rumors. The trading pools run by John J. Levinson (see note 6) are examples of information-based manipulation.

The Securities Exchange Act attempted to eradicate action-based manipulation by, among other things, making it illegal for directors and officers to sell short the securities of their own firm. There were also a number of provisions of the Act designed to eliminate information-based manipulation. Firms were required to issue information to the public on a regular basis so that the spreading of rumors would be more difficult. The Act also made it illegal for anybody to attempt to raise or depress stock prices by making statements which they knew to be false.

Although there have been a number of well-publicized exceptions, the Act has by and large been fairly successful in eradicating these two categories of manipulation.⁷ However, there is a third category of manipulation that is much more difficult to eradicate. We refer to this third category as *trade-based manipulation*. It occurs when a trader attempts to manipulate a stock simply by buying and then selling, without taking any publicly observable actions to alter the value of the firm or releasing false information to change the price.

⁶ Among other things, “bulling” stocks in this way had the advantage of avoiding a corner, since with margin trading (which is the counterpart of short selling in a bear raid) settlement is made in cash rather than shares. On a number of occasions, trading pools would act in concert with journalists who would write favorable stories about the stock being manipulated in return for a share in the profits. For example, Sobel (1965, pp. 248-249) recounts how John J. Levinson, a pool manager, and Raleigh T. Curtis, who wrote a column entitled “The Trader” in the *New York Daily News*, made profits of over \$1 million per year in this way.

⁷ See, for example, Securities and Exchange Commission (1959)

On the face of it, it would seem that trade-based manipulation cannot be profitable. The argument is simple. When a trader tries to buy a stock, he drives up the price. When he tries to sell it, he drives down the price. Thus, any attempt to manipulate the price of a stock simply by buying and selling requires the trader to “buy high” and “sell low.” This is the reverse of what is required to make a profit. Jarrow (1992) has formalized this argument, showing that, under certain conditions, profitable manipulation is impossible in an efficient market. Our purpose in the current article is to investigate whether profitable, trade-based manipulation is theoretically possible in a model where all agents are rational.

Hart (1977) and Jarrow (1992) have analyzed manipulation formally in the context of dynamic models of asset markets. Hart considers conditions under which profitable speculation is possible in a deterministic setting. He finds that if the stationary equilibrium is unstable or demand functions are nonlinear and satisfy some technical conditions, speculators can trade profitably. Jarrow (1992) extends Hart’s analysis to a stochastic setting and derives similar results. He shows that profitable speculation is possible if there is “price momentum,” so that an increase in price caused by the speculator’s trade at one date tends to increase prices at future dates. In addition, he shows that profitable manipulation is possible if the speculator can corner the market. In both articles, the form of the investors’ demand functions is taken as exogenous, rather than being derived from expected-utility-maximizing behavior. So, it is not clear whether and under what conditions manipulation is consistent with rationality.

In contrast, we develop a model with asymmetric information where all agents have rational expectations and maximize expected utility. Also, we work in a finite horizon framework, where bubbles are ruled out by construction. It is shown that profitable price manipulation is possible, even though there is no price momentum and no possibility of a corner. Incomplete information is crucial to our argument. Investors are uncertain whether a large trader who buys the share does so because he knows it is undervalued or because he intends to manipulate the price. It is this pooling that allows manipulation to be profitable.

A number of other authors have considered manipulation. Vila (1989) and Bagnoli and Lipman (1990) consider the possibility of profitable action-based manipulation. Vila presents an example where a manipulator pools with somebody who is purchasing stock prior to a takeover bid in which the value of the firm will be increased. Bagnoli and Lipman also develop a model where the manipulator pools with someone who can take actions that alter the true value of the firm. In their model, the manipulator takes a position, announces a takeover

bid, and then unwinds the initial position. They are able to show that the relationship between incentives to search and the possibility of manipulation are such that the welfare effects of banning manipulation are ambiguous.

In addition, Vila (1989) presents an example of information-based manipulation where the manipulator shorts the stock, releases false information, and then buys back the stock at a lower price. Benabou and Laroque (1992) also consider information-based manipulation. They show that if a person has privileged information about a stock and his statements are viewed as credible by investors, he can profitably manipulate the stock price by making misleading announcements and trading.

Kumar and Seppi (1992) develop a model of trade-based manipulation. The manipulator initially takes a position in the futures market at a time when it is known that everybody is uninformed so prices are not affected. She then pools with an informed trader on her trades in the stock market and alters the stock price. This change improves her position in the futures market and, although she makes losses in the spot market, she more than makes up for this with her profits in the futures market.

In this article, we only consider trade-based manipulation in which the manipulator simply buys and sells the stock without taking a position in any other market. We do not allow the possibility of a purchaser of the firm taking actions that alter the value of the firm or releasing false rumors.

In another article, Vila (1987) considers a model of corners and squeezes in a futures market. The price can be manipulated by anyone who obtains a sufficiently large fraction of the supply. Corners and squeezes are rather special phenomena, which depend on different factors from trade-based manipulation in our sense. In any case, corners do not play a role in our analysis.

We proceed as follows. In Section 1, we describe a three-period asset market. Characterization of the third-period equilibrium is trivial, since the value of the assets is revealed by then. The second-period equilibrium is characterized in Section 2. In Section 3, we characterize those first-period equilibria in which profitable trade-based manipulation occurs. A discussion of the model and results is contained in Section 4, and concluding remarks can be found in Section 5. A more thorough discussion of the model and the proofs is contained in the Appendix.

1. The Basic Model

As we pointed out in the introduction, our objective in this article is to demonstrate the *theoretical* possibility of profitable stock-price

manipulation. To do this, we use the simplest possible model. Even in this highly simplified context, the analysis becomes somewhat involved, and extreme assumptions are used to make the argument more transparent. Many of these assumptions could be relaxed, however, with a consequent increase in difficulty. The robustness of the model is discussed in Section 4.

Although the model is not intended to be a realistic description of an actual stock market, we believe it provides a plausible “scenario” for stock-price manipulation. The kind of thing we have in mind is the following. We start with a company whose shares are not very actively traded. Most of the stockholders are small, “passive” investors who regard their shares as a long-term investment. The fraction of their wealth held in this stock is large enough to make them risk averse.⁸

In addition to passive investors, there are also “large traders” who are continually searching for stocks to invest in. Their cost of gathering information is sufficiently small and their wealth is sufficiently large that they hold many (but not all) stocks and are well diversified. As a result, they can be treated as approximately risk neutral.

The large traders continually search for stocks in which they can profitably invest. Suppose a firm has attracted attention. The large traders fall into two categories. The first type we refer to as an “informed trader,” and the second type as a “manipulator.” The informed trader anticipates, perhaps because of research that he has undertaken or inside information, that favorable information about the stock will be released in the near future. The manipulator does not have any special knowledge about the particular stock; he is uninformed. It is not possible for other investors to determine whether a large trader is informed or uninformed in any particular case.

Suppose the large trader is informed. Relative to his information, the stock is undervalued. In order to profit from his informational advantage, he must buy the stock before the information becomes public. As the informed trader buys the stock, the price will begin to rise, incorporating his information. If the price rises to the point where it equals the expected value of the stock, conditional on the informed trader’s information, it will be impossible for him to make a profit. However, the investors may be willing to sell the stock for less than

⁸ There is considerable evidence that most stockholders fit this pattern. For example, Blume, Crockett, and Friend (1974) find that the average degree of portfolio diversification is roughly equivalent to having an equally weighted portfolio with two stocks. Blume and Friend (1978) find that a large proportion of stockholders have only one or two stocks in their portfolios, and very few have more than 10. This limited amount of diversification can be explained by the costs of gathering information about stocks, which are so large for most people that it is optimal for them to invest in a small number of stocks. Of course, diversification is available at low cost through mutual funds and pension plans. We assume the investors in our model believe they can earn a higher return by managing their own (undiversified) portfolios.

the informed trader's expected value. In the first place, they are uncertain about his motives in buying the stock. He may have good information about the stock's true value, but he may also be mistaken. He may simply be manipulating the stock price for his own advantage. For all these reasons, the risk-averse investors may be willing to take their profits and sell after the price has risen only a modest amount.

As time passes, more information becomes available. If the informed trader is right and the stock was initially undervalued, the stock price will continue to rise. At some point the informed trader, who is impatient to move on to another market opportunity, will take his profits and sell the stock at a higher price than he paid for it. Eventually, the true value becomes known to everyone.

We claim that, in this setting, a manipulator who is uninformed can make a profit simply from buying and selling the stock. He will initially buy the stock, imitating the informed trader. He can buy at a low price because the risk aversion of the investors limits the initial increase. Later, imitating the informed trader who knows a good announcement will be forthcoming, he sells the stock at a higher price than he paid for it. In this way he makes an overall profit even though he has the same information about the stock as investors.

This, in brief, is the setting for the analysis. We describe it more formally as follows.

Time: Trading takes place at three dates, indexed by $t = 1, 2, 3$.

Assets: There are two assets: cash, which serves as the numeraire, and a *stock*.

Asset returns: The stock pays a single dividend at the end of the last period. We assume it can take two values, a high value V_H or a low value V_L . Cash is assumed to be riskless and has zero yield (i.e., \$1 held at the end of date t is worth \$1 at the beginning of date $t + 1$).

Traders: There are three types of traders: a continuum of identical *investors* and two large traders, an *informed trader* and a *manipulator*. Each of the investors has a negligible holding of the stock, so they all behave like price-takers. For practical purposes, there is no loss of generality in replacing the continuum of investors with a single, representative investor. The informed trader has private information about the stock; the manipulator does not.

Information: With probability a , an announcement concerning the value of the stock is made. The informed trader knows when an announcement is forthcoming; the other traders do not. If an announcement is made, with probability π , it is good news (i.e., the value is announced to be V_H), and with probability $(1 - \pi)$ it

is bad news (i.e., the value is announced to be V_L). If the announcement is good news, it is made at date 3; if it is bad news, it is made at date 2.

Entry and exit: Prior to the first date, only investors are in the stock market. At date 1, a large trader may enter. The informed trader enters only if he anticipates an announcement (i.e., he enters with probability a). If there is to be no announcement, so that the informed trader does not enter, the manipulator may enter. The (unconditional) probability that he enters is β . With probability $1 - a - \beta$, no large trader enters. Once a large trader enters the market, he stays until the second date. At the second date, he liquidates his holding of stock and leaves the market. The investors remain in the market until the last date.

Endowments: Initially, the investors hold all of the stock. The representative investor's initial endowment is $E > 0$. The informed trader and the manipulator have none. Without loss of generality, we can normalize all traders' endowments of cash to zero.

Preferences: All traders maximize the expected utility of their final wealth. In the case of large traders, this means wealth at date 2; for the investors, wealth at date 3. The investor is *risk averse* and his risk preferences are represented by a von Neumann-Morgenstern utility function U . We assume that U is continuously differentiable, strictly increasing, and strictly concave. The informed trader and the manipulator are *risk neutral*.

The stochastic structure and the information structure of the model are summarized in Figure 1.

Some of the assumptions above are made for simplicity; others are crucial. The role of particular assumptions and the possibility of relaxing them are discussed in Section 4. The reader should, however, note a couple of assumptions that play an important role in what follows. The first is the assumption that good news is revealed after bad news. We can do without this assumption in other contexts (see the discussion in Section 4) but it is crucial here. The second is the assumption that the manipulator enters only if no information is expected. In other words, the manipulator knows that there will be no informed trader in the market, something the investors do not know. In fact, this asymmetry is inessential and is made for convenience. In Section 4, we indicate how it can be removed.

The general pattern of trade in equilibrium is as follows. We assume that if a large trader enters the market, he purchases a positive amount of the stock. In the event that a large trader does not enter the market at date 1, there is no trade; this fact reveals to the investors that no large trader is present and the stock's true value is V_L . In that case,

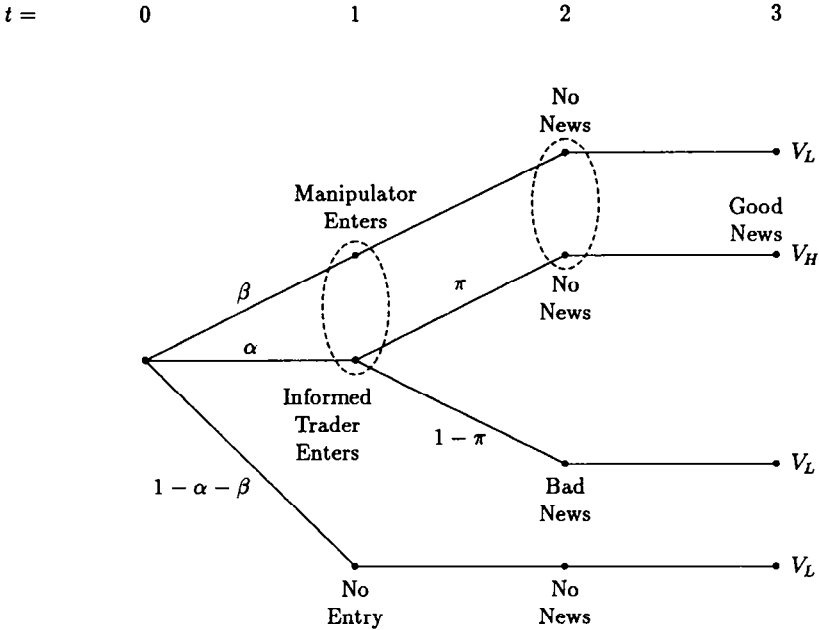


Figure 1
The stochastic structure and information structure

A large trader may enter the market at date 1. If there is no entry, the stock price falls to V_L . If the large trader purchases $B > 0$ units, the stock price rises. Investors do not know whether the entrant is an informed trader or a manipulator (as indicated by the dotted lines). At date 2, there will either be bad news, in which case the price falls to V_L , or no news, in which case the price rises again. The large trader then sells his entire holding of B and leaves the market. If there is no news at date 2, there is either good news at date 3, in which case the price rises to V_H , or there is no news, in which case investors realize the large trader is a manipulator and the price falls to V_L .

there is no trade at any date and the equilibrium price is always V_L . If a large trader does enter the market, he purchases $B > 0$ units of stock, regardless of his type (the informed trader and the manipulator pool at date 1). At date 2, he sells his entire holding of B units and leaves the market. If there is an announcement at date 2, it reveals the true value of the stock to be V_L and this becomes the equilibrium price at which the large trader sells (in this case, the large trader must be informed). If there is no announcement, there is a positive probability that the large trader is informed and the stock's value must be high. In this case, no news is good news. However, there is also a positive probability that the large trader is the manipulator, in which case the stock's true value is low. (The informed trader and manipulator are pooling.) The equilibrium price will reflect the investors' uncertainty about the stock's value.

At the last date, the investors are alone in the market, holding the initial endowment of the stock. Either there is an announcement that

the stock's value is high or there is no announcement, in which case the value must be low. Since the investors are alone in the market, there can be no trade. The equilibrium price will equal the true value of the stock, which has been revealed either through an announcement or by the absence of an announcement.

In analyzing the equilibriums of the model, we assume that the investors can observe the quantities bought and sold by the large traders. There is no loss of generality in this. If the investors could not observe these quantities directly, they would be able to infer them from prices. It is simply more convenient to express all the equilibrium variables as functions of these quantities.

2. Equilibrium at Date 2

Equilibrium is characterized by backward induction. Since further analysis of equilibrium at date 3 is not required, we begin with the second date. To characterize the equilibrium of the trading game at date 2, we distinguish several cases.

2.1 A large trader did not enter at date 1

In this case, no trade occurs at date 1: We have assumed that the absence of trade reveals the absence of a large trader and this in turn reveals that the stock's value is V_L . Then at date 2, the equilibrium price equals V_L and there is no trade.

2.2 A large trader entered the market at date 1

If a large trader enters the market at date 1, he is assumed to buy a positive amount of stock $B > 0$ at a price of $P_1(B)$. The investor's beliefs are represented by the probability $Q_1(B)$ that the large trader is informed. Since the informed trader and manipulator pool at date 1, $Q_1(B) = \gamma = a/(a + \beta)$.

Two subcases have to be distinguished, according to whether there is an announcement at date 2 or not.

2.2.1 An announcement is made at date 2. In this case, the true value of the stock is announced to be V_L . Since there is no uncertainty, the investors are willing to purchase any amount of the stock at the equilibrium price $P_2 = V_L$. The large trader, who must be informed, will sell the entire B units at this price.

2.2.2 No announcement is made at date 2. In this case, the investor is uncertain about the identity of the large trader. He might be informed, in which case there will be an announcement at date 3 and the stock's value is V_H , or he might be the manipulator, in which case

the value of the stock is V_L . With probability γ , the large trader is informed and, given that he is informed, the probability that no announcement is made is π . So the probability that the large trader is informed and no announcement is made at date 2 is $\gamma\pi$. The probability that no announcement is made is $\gamma\pi + 1 - \gamma$, so the posterior probability that the large trader is informed, given that no announcement is made, is $\delta \equiv \gamma\pi/(\gamma\pi + 1 - \gamma)$.

The large trader is assumed to offer B units of stock for sale. (If he offers less, investors infer that he is the manipulator and will only pay V_L for the stock.) In order to clear the market, the equilibrium price must satisfy the investor's first-order condition:

$$P_2(B) = \frac{\delta U'(EV_H + M(B))V_H + (1 - \delta)U'(EV_L + M(B))V_L}{\delta U'(EV_H + M(B)) + (1 - \delta)U'(EV_L + M(B))}, \quad (1)$$

where $M(B) \equiv (P_1(B) - P_2(B))I$ is the investors' holding of cash at the end of date 2.

3. Equilibrium at Date 1

As we have seen, if a large trader does not enter the market at date 1, there is no trade and the equilibrium price is V_L . No further analysis is required. In the case where a large trader does enter the market at date 1, we assume the informed trader and manipulator pool and purchase the same quantity of stock. Let the quantity of stock purchased be $B > 0$ and the price paid be P_1 . In the second period, one of three things will happen to the representative investor.

(a) With probability $(1 - \pi)\gamma$ an announcement is made that the value of the stock is low. In that case, his final wealth will be

$$W_L(B) \equiv EV_L + (P_1 - V_L)B = (E - B)V_L + P_1B.$$

(b) With probability $\pi\gamma$, there is no announcement and the value of the stock is high (although he does not know it yet). In that case, his final wealth will be

$$W_H(B) \equiv EV_H + (P_1 - P_2(B))B.$$

(c) With probability $(1 - \gamma)$, there is no announcement and the value of the stock is low. In that case, his final wealth will be

$$W_M(B) \equiv EV_L + (P_1 - P_2(B))B.$$

Using these formulas, we can assign a unique expected utility for the representative investor to each quantity of stock purchased by the large trader. Let $U^*(B)$ denote the equilibrium payoff to the repre-

sentative investor if B units of stock are purchased at date 1. Then $U^*(B)$ is given by the formula

$$U^*(B) = \gamma\pi U(W_H(B)) + \gamma(1 - \pi)U(W_L(B)) + (1 - \gamma)U(W_M(B)).$$

In order to clear the market, the equilibrium price P_1 will have to satisfy the investors' first-order condition:

$$P_1 = \frac{\gamma\pi U'(W_H(B))V_H + \gamma(1 - \pi)U'(W_L(B))V_L + (1 - \gamma)U'(W_M(B))V_L}{\gamma\pi U'(W_H(B)) + \gamma(1 - \pi)U'(W_L(B)) + (1 - \gamma)U'(W_M(B))}. \quad (2)$$

In the preceding construction, we have implicitly assumed that B is the equilibrium choice, so the investors' beliefs about the identity of the large trader are correct. For any other value of B , let us assume the investors believe that the large trader is the manipulator. In that case, the equilibrium price is V_L at date 1 and date 2. It is impossible for the large trader to make an arbitrage profit so his payoff is zero. These beliefs will support any choice of P that gives both the manipulator and the informed trader nonnegative profit. In other words, B is optimal for both types if the informed trader's payoff is nonnegative:

$$\pi P_2(B)B + (1 - \pi)V_L B - P_1(B)B \geq 0. \quad (3)$$

This obviously implies that the manipulator's payoff $P_2(B)B - P_1(B)B$ is positive.

A *pooling equilibrium* is described by a quantity of stock B and a price P_1 satisfying (2) and (3).

Proposition. *As long as the investors are sufficiently risk averse and the probability of manipulation β is sufficiently small, there exists a pooling equilibrium at date 1 in which the manipulator achieves strictly positive profits.*

Proof See the Appendix.

The possibility of profitable manipulation

It is important to notice in what sense the pooling equilibrium represents an example of successful stock-price manipulation. The manipulator is a speculator in the traditional sense that he begins with a zero holding of the stock and eventually unwinds the total position. Furthermore, by pooling, the manipulator has an effect on the stock price that he could not have had if his type had been revealed at date 0. Had his type been revealed, the price profile would be been $\{V_L, V_L, V_L\}$ rather than

$$\left\{ P_1(B), P_2(B), \begin{Bmatrix} V_H \\ V_L \end{Bmatrix} \right\}.$$

The manipulator makes a profit in the sense that his final wealth is greater in the pooling equilibrium than in any equilibrium in which he reveals his type.

4. Discussion

The simplicity of the model we have been analyzing makes the factors supporting profitable manipulation fairly transparent. Many of these assumptions could be relaxed, however, at the cost of greater complexity. In this section, we discuss the ways in which our strong assumptions could be relaxed and what effect, if any, this would have on the analytical results.

4.1 Information and trade-based manipulation

In this article, we have attempted to show that profitable, *trade-based* manipulation is possible in a plausible stock market equilibrium with rational agents. In this equilibrium, the manipulator does not have an informational advantage over investors about the returns to the stock. Moreover, no false information is released, which distinguishes the situation from *information-based* manipulation. However, the incompleteness of information does play a role. There must be an informed trader for the manipulator to imitate in order for manipulation to be profitable. If it is common knowledge that the large trader in the market is an uninformed manipulator, there is no way the manipulator can make profits simply from trading with a given set of investors. Hence, it is unavoidable with the type of trade-based manipulation considered that manipulators have information (about themselves) that investors do not.

Still, it could be argued that the manipulator does possess information about returns that investors do not. Specifically, as noted earlier, he knows that when he is in the market there is no informed trader around and, therefore, no announcement will be forthcoming. In fact, this assumption is inessential as we can easily show. Suppose we had assumed instead that the entrances of the informed trader and the manipulator were *independent* events, which occur with probabilities a and β , respectively. Therefore, with probability $a(1 - \beta)$, the informed trader is the only large trader in the market; with probability $\beta(1 - a)$, the manipulator is the only large trader; with probability $a\beta$, there are two large traders; and with probability $(1 - a)(1 - \beta)$, there are none. All that has changed is that we have added a new event at date 1—namely, the event that two large traders enter the market at the same time.

Because equilibrium is analyzed in each of these events separately, the analysis of the other three events is the same as before. In the

fourth event, when the two large traders enter they are ignorant of each other's presence. They choose the same quantity B as in the other events. Then the demand for the stock will be $2B$, and this reveals the presence of two large traders. In that case, both the investors and the manipulator know that information will be revealed at the second date. The manipulator can make money in this event for the same reason that the informed traders makes money (viz., because of the variance of the future stock price, which reduces the current stock price below its future expected value).

To sum up, we have sketched an equilibrium story in which at each point in time the manipulator has exactly the same information as the investors-except, of course, that the manipulator always knows that he is the manipulator and not the informed trader; however, there is nothing we can do about that.

4.2 The role of risk aversion

When the large trader enters the market, the purchase of shares will signal that good news is anticipated (with positive probability) and this drives up the price. The increase in the price is limited by several factors. In the first place, even if there is an informed trader in the market, it is possible that the anticipated good news will not arrive. Second, the large trader may be a manipulator. Conditional on a large trader entering the market, the expected value of the stock is

$$\gamma\pi V_H + \gamma(1 - \pi)V_L + (1 - \gamma)V_L = \gamma\pi V_H + (1 - \gamma\pi)V_L,$$

which is higher than the initial expected value

$$\alpha\pi V_H + \alpha(1 - \pi)V_L + \beta V_L + (1 - \alpha - \beta)V_L = \alpha\pi V_H + (1 - \alpha\pi)V_L$$

but less than V_H . A third factor limiting the increase in price is the investors' risk aversion. Because the investors are uncertain what the true value of the stock is, they will be willing to sell for less than the expected value as long as they hold a positive amount of the stock. The more of the stock the large trader purchases, the less the investors hold and the closer the market-clearing price will be to the expected value; however, generally, it will be less than the expected value.

The manipulator can make money even if the investors are risk neutral, since in that case, $P_1 = \gamma\pi V_H + (1 - \gamma\pi)V_L$, $P_2 = \delta V_H + (1 - \delta)V_L$, and $\gamma\pi < \delta$. For the informed trader to make money, however, risk aversion is essential. The reason is that the informed trader sometimes sells the stock after bad news has been announced. With probability π , he sells at a price

$$P_2 = \delta V_H + (1 - \delta)V_L;$$

with probability $(1 - \pi)$, he sells at a price $P_2 = V_L$. Thus, his average selling price is

$$\begin{aligned}\bar{P}_2 &= \pi(\delta V_H + (1 - \delta)V_L) + (1 - \pi)V_L \\ &= V_L + \delta\pi(V_H - V_L) < V_L + \gamma\pi(V_H - V_L) = P_1,\end{aligned}$$

since $\delta < \gamma$. The problem is that the informed trader suffers because of the presence of the manipulator. The manipulator helps the informed trader at date 1, by depressing the price of stock, but he hurts him even more at date 2 by pooling with the informed trader at the high price. This is not surprising: since the investors are risk neutral, the sum of the manipulator's and the informed trader's profits must be zero.

If the investors are risk averse, on the other hand, they will be willing to sell at a lower price at date 1 in order to insure themselves against the riskiness of the stock. By adding in this risk premium, it is possible for the informed trader to make money as well.

Our main result states that profitable manipulation is possible if investors are sufficiently risk averse. How risk averse the investors need to be depends on other parameters of the model, including the probability of manipulation. It turns out that profitable manipulation does not require extreme values of risk aversion or other parameters, as Figure 2 illustrates. The figure shows the trade-off between the probability of manipulation and the degree of risk aversion that is necessary to support profitable manipulation. There is a large range set of parameter values for which manipulation is possible. Figure 3 illustrates the path followed by prices for the same parameter values.

4.3 Information structure

One of our strongest simplifying assumptions is embodied in the model's information structure. We assume the informed trader enters only if he knows that an announcement is forthcoming. This is clearly unnecessary: as long as he knows that an announcement is forthcoming with positive probability, profitable manipulation will be possible for some parameter values. Similarly, the assumption that bad news is announced before good news can be relaxed. To carry through the analysis, it is only necessary that with positive probability the bad news arrives before the good news. The conditions for profitable manipulation become more complicated, but the qualitative features of the analysis remain unchanged.

A more important question is, "How could this information structure arise?" The kind of setting we have in mind is typified by the following story. Suppose a firm is undertaking pilot studies to see whether a new manufacturing process is feasible at commercial levels of production. The development process consists of trying successively larger volumes until commercial production levels are achieved and proved to be viable. The firm may discover relatively early that

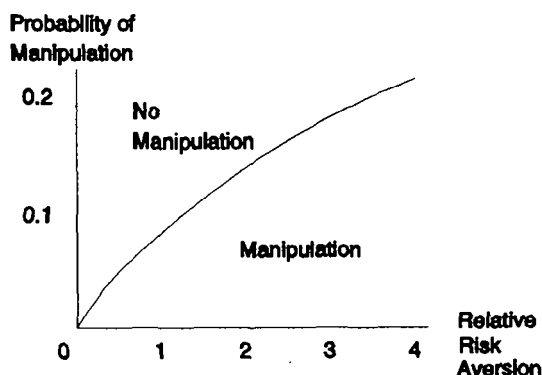


Figure 2

An illustration of the parameter values where there is manipulation

In the example, investors have a constant relative risk-aversion utility function and the parameters are $V_H = 12$, $V_L = 10$, $\alpha = 0.5$, $\pi = 0.5$, $E = 1$, and $B = 0.45$. The figure shows the probabilities of manipulation (i.e., $1 - \gamma$) and the values of relative risk aversion for which manipulation is profitable. If investors are risk averse, the risk premium is large and manipulation can be profitable even if there is a significant probability of manipulation. Conversely, if investors are not very risk averse, the risk premium is small and the probability of manipulation needs to be low for there to be manipulation.

the process will not work. But it will not be clear until several iterations have been successfully completed whether the process is viable at commercial levels. If the informed trader has undertaken research into the company's activities, he will know that bad news is more likely in the early stages and that good news cannot be expected until the firm has carried out development for some considerable time. The more time that passes without an announcement, the more likely the announcement will eventually be good news.

Timing is important in this story. If good news were released before bad news, then the absence of an announcement at date 2 would signal either no news or bad news at date 3. In that case, the manipulator could not profit by selling his stock at date 2. It might be thought that a bear raid could succeed in these circumstances, where the manipulator would sell his stock at date 1 and buy it back at a low price at date 2. But there is an interesting asymmetry between the two scenarios. A bear raid will not make money simply because the sale of stock at date 1 will lower the price below the expected value at date 2. Thus, the informed trader will be losing money selling the stock and buying it back.

The advantage of the timing asymmetry here is that it simplifies and sharpens the analysis. Although it plays a crucial role in the present story, the assumption is clearly not essential. In Allen and Gale (1990), we investigated a different scenario, in which the timing of information revelation plays no role, and we obtained results similar to those reported here.

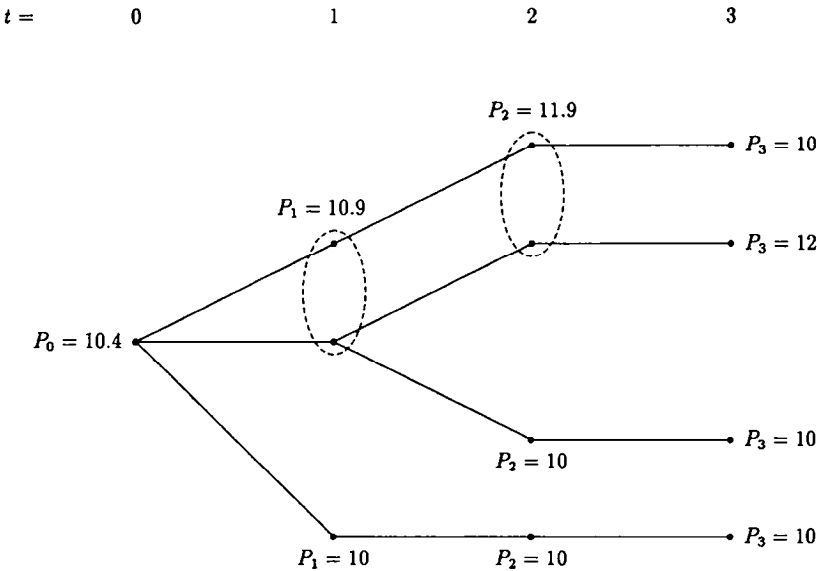


Figure 3
An illustration of the price path
The example is the same as for Figure 2, with a relative risk aversion of 2 and a probability of manipulation $(1 - \gamma)$ of 0.02. The initial price of the stock at date 0 before there is any trading is 10.4. This price is calculated from the investors' first-order condition on the assumption that they hold all of the stock. If there is no entry at date 1, the price falls to 10, however, if there is entry, it rises to 10.9. At date 2, there is either an announcement of bad news and the price falls to 10, or there is no news and the price rises to 11.9. Finally, at date 3, the price rises to 12 if there is good news, or falls to 10 if there is no announcement.

4.4 Impatience

The manipulator and the informed trader are both assumed to have a rather extreme form of impatience that requires them to sell their entire holdings at date 2. This assumption is really only required for the informed trader, since the manipulator will find it optimal to imitate the informed trader and sell his entire holding at date 2, even if it is not strictly necessary for him to do so. The assumption of extreme impatience for the informed trader is motivated by the idea that he has more profitable uses for his funds because of his comparative advantage in gathering information. What we have in mind is that the informed trader continually evaluates opportunities to profit by buying and selling stock before information becomes public. At the first date, his best opportunity is to enter the market we analyze in the article. By the second date, other opportunities have come along and the informed trader is eager to exploit them. If he does not move promptly, his opportunity may be lost. For this reason, he wants to liquidate his holding of the first stock as quickly as possible. To be more precise, only if a large capital gain is anticipated between

date 2 and date 3 would it be profitable for him to remain in his present market. As the numerical example illustrates, by date 2 the stock price may have risen so high that it is not worth retaining the stock until date 3 simply to capture the relatively small price appreciation that occurs between dates 2 and 3. He is better off selling at date 2 to take advantage of his other profitable opportunities.

While this story seems plausible (to us), it does not justify the extreme form of impatience assumed in the article. What it does justify is the assumption that the informed trader has a smaller discount factor than the investors, which is consistent with him getting some value from stock retained until date 3. One way to model the informed trader's impatience to liquidate his stockholding is to assume he maximizes the expected value of his terminal wealth, but that he earns a higher rate of return on cash than the investors do. With these less restrictive preferences, the analysis can be carried out with essentially the same results as before. What will change is the set of parameter values for which manipulation can be profitable. For example, if the increase in price between dates 2 and 3 is too great, the informed trader may wish to retain his stock until the third period. In that case, the manipulator cannot profit by selling his stock at date 2. So, in order to support profitable manipulation, we must have a second-period price that is not too far below the third-period price. This will be the case in the equilibriums we have analyzed if certain restrictions on the parameters are satisfied, for example, if γ is not too large.

4.5 Risk neutrality

In the model, we have assumed the large traders are both risk neutral. This can obviously be relaxed without changing the results. What matters is that the large traders be less risk averse than the small investors, a not implausible assumption. Risk neutrality is assumed in the article because it greatly simplifies the characterization of equilibrium.

4.6 Robustness of equilibrium

Because of the incompleteness of information, the model has a large number of equilibriums, all of them supported by appropriate off-the-equilibrium-path beliefs. A natural question to ask is whether the equilibriums we have characterized are plausible. In particular, do they satisfy some natural refinement of Nash equilibrium. We have not addressed this question for the present model; however, in another version [Allen and Gale (1990)], we showed that profitable manipulation occurred in equilibriums that did satisfy a refinement related to the Cho-Kreps Intuitive Criterion [Cho and Kreps (1987)].

5. Concluding Remarks

A number of authors have argued that stock-price manipulation was an important phenomenon in U.S. stock markets up until the 1930s. Concern about the harmful effects of manipulation led to the passage of the Securities Exchange Act of 1934. This made a number of practices that facilitated manipulation, such as short selling by managers and the announcement of false information, illegal. There is some evidence that these restrictions have largely eliminated action-based and information-based manipulation. Stock-price manipulation has therefore been considered to be a phenomenon that is mainly of historical interest.

The results presented above show that trade-based stock-price manipulation, which is very difficult to prohibit, is consistent with rational utility-maximizing behavior. In the particular model considered, manipulation is possible without actions to alter the true value of the firm or the release of false information. To the extent these are possible, they will increase investors' beliefs that the trader is informed and will make manipulation more profitable.

The importance of trade-based manipulation schemes is, of course, an empirical question. However, casual observation suggests they might be important. Large traders frequently buy and then sell substantial blocks of stock, even though they are apparently not interested in taking over the firms. Some part of the profits from this trade may be the result of manipulation of the type discussed in this article.

In addition to the empirical importance of trade-based manipulation, another important issue that remains to be analyzed is whether such manipulation is undesirable or not. Although a number of authors regard all types of manipulation as undesirable,⁹ this was not the position that lay behind the framing of the Securities Exchange Act of 1934. This did not rule out all types of manipulation per se, since this would have prevented the type of "price pegging" sometimes done by underwriting syndicates when new securities are issued [see Twentieth Century Fund (1935, pp. 499-502) for a discussion of the desirability of this type of manipulation]. In fact, as mentioned in the introduction, Bagnoli and Lipman (1990) have shown that action-based manipulation can be socially desirable. Thus, it remains an open question whether or not the type of trade-based manipulation considered is undesirable.

⁹ For example, in his discussion of the Securities Exchange Act, Flynn (1934, p. 284) states, "It is, of course, absolutely essential that all forms of manipulation shall be eliminated from the securities markets."

Appendix

In this appendix, we present precise definitions of the trading games discussed in the main body of the article and prove the proposition.

A.1 Equilibrium at date 2

The analysis of equilibrium at date 2 is trivial in two cases: first, where no large trader enters the market at date 1; second, where the stock's true value is announced at date 2. Accordingly, we focus on the case where a large trader has entered the market and traded at date 1 and there is no announcement at date 2.

The initial conditions at date 2 are the quantity of stock purchased at date 1, the price at which stock is traded at date 1, and the investors' subjective probability that the large trader is informed. Let (B, P_1, g) be a fixed but arbitrary set of initial conditions; taking these as given, we define a pooling equilibrium for the market at date 2.

In the case of interest, there are two types of large trader, the informed trader (I) and the manipulator (M). The large trader chooses a quantity of stock $0 \leq S \leq B$ to put on the market. The representative investor conditions his beliefs about the large trader's type on the quantity of stock sold. Let $Q_2(S)$ denote the investor's belief (subjective probability) that the large trader is in fact informed. Let $P_2(S)$ denote the market-clearing price when the large trader sells S units of stock and the investor's beliefs are $Q_2(S)$.

The large trader's objective, whatever his type, is to maximize his revenue $P_2(S)S$ from the sale of stock. The representative investor chooses the optimal quantity of stock to buy, taking as given the prevailing price. In equilibrium, the representative investor must purchase S units of stock, giving him a net stock position of $E - B + S$ and a net cash position of $P_1B - P_2(S)S$. Of course, his final wealth at date 3 will depend on the true value of the stock. If the value of the stock is high, his wealth is

$$W_H(S) \equiv (E - B + S)V_H + P_1B - P_2(S)S;$$

if the true value is low, his final wealth is

$$W_L(S) \equiv (E - B + S)V_L + P_1B - P_2(S)S.$$

Since the investor knows that the true value of the stock is high if and only if the large trader is in fact informed, he expects the high value with probability $Q_2(S)$.

A *pooling equilibrium* is defined by the quantity S^* , the price function $P_2(S)$, and the beliefs $Q_2(S)$. To be an equilibrium, the triple $(S^*, P_2(S), Q_2(S))$ must satisfy the following conditions. First, the quantity S^* must maximize the large trader's payoff, given the price function $P_2(S)$:

$$S^* \in \arg \max_{0 \leq S \leq B} P_2(S) S. \quad (A1)$$

Second, the investor's beliefs must be consistent with the equilibrium strategy of the large trader and with his prior beliefs. Since both types of the large trader pool at S^* , the investor gets no new information from observing the quantity traded. However, "no news" is good news in this case, so he updates his prior if no announcement is made:

$$Q_2(S^*) = Q_1 \pi / (Q_1 \pi + 1 - Q_1). \quad (A2)$$

For values of S other than S^* , beliefs are arbitrary. Third, the investors must be willing to purchase any amount of stock S at the price $P_2(S)$, given that their beliefs are given by $Q_2(S)$. The price $P_2(S)$ will clear the market if and only if it satisfies the representative investor's first-order condition when we assume the investor has purchased S units of stock:

$$P_2(S) = \frac{Q_2(S) U'(\mathbf{W}_H(S)) V_H + (1 - Q_2(S)) U'(\mathbf{W}_L(S)) V_L}{Q_2(S) U'(\mathbf{W}_H(S)) + (1 - Q_2(S)) U'(\mathbf{W}_L(S))}, \quad \forall S. \quad (A3)$$

In the body of the article we have focused on one particular equilibrium outcome, the one in which the large trader sells all of his stock at date 2. This outcome is supported by a number of pooling equilibria, which differ only in the prices and beliefs observed off the equilibrium path. We can show that such an equilibrium exists by constructing an example. First, put $S^* = B$ and define beliefs by putting

$$Q_2(S) = \begin{cases} 0, & \text{if } S \neq S^*; \\ Q_1 \pi / (Q_1 \pi + 1 - Q_1), & \text{if } S = S^*. \end{cases}$$

That is, the investor assumes that any deviation is made by the manipulator. By construction, (A2) is satisfied. The price function is defined by the equilibrium condition (A3). For $S \neq S^*$, the market-clearing condition (A3) collapses to

$$P_2(S) = V_L.$$

For $S = S^*$, we need to show that $P_2(S^*)$ is well defined. Putting $S = S^*$ and $P_2(S^*) = p$ in the expression on the left-hand side, we get a function $f(p)$ that is continuous on the interval $[V_L, V_H]$ and has a range contained in $[V_L, V_H]$. There exists at least one value p^* such that $p^* = f(p^*)$. Put $P_2(S^*) = p^*$. By construction, (A3) is satisfied.

To show that (A1) is satisfied, simply note that for all $S < B$, $V_L S < P_2(B)B$.

Note that, although we have given a fairly complete characterization of a particular equilibrium outcome at date 2, we have not charac-

terized a unique outcome. The indeterminacy arises from the possibility that $P_2(B)$ is not uniquely determined by (A3). Although uniqueness is not crucial for our approach, it will make things easier if we ignore the indeterminacy for the time being. Therefore, we adopt the following *auxiliary assumption*.

Assumption. For every initial condition (B, P_1, Q_1) , $P_2(B)$ is uniquely determined by (A3).

Also, uniqueness together with the assumptions about the investor's preferences implies that the price $P_2(B)$ varies continuously as the initial conditions (B, P_1, Q_1) are varied.

We have defined a unique equilibrium outcome $(B, P_2(B), Q_2(B))$ for every initial condition (B, P_1, Q_1) . There are, of course, many other equilibrium outcomes, all supported by appropriate beliefs and prices off the equilibrium path. Rather than undertake a formal analysis of equilibrium selection, we make the following observations, which point in the direction of the equilibrium we have selected.

(1) Since unsold stock is not valued by the large trader, it is not sensible for him to purchase more at date 1 than he can sell at date 2. Of course, one could imagine an equilibrium in which it would be optimal for the large trader to purchase more than he could profitably sell, simply because the investor's beliefs would be adversely affected if he attempted to buy less. But this story is rather contrived and does not appear to have any realistic counterpart. It seems more reasonable to suppose that the large trader would be expected by the investor to sell what he had purchased (alternatively, to purchase only what he knew he could sell profitably), in which case, no adverse effect on beliefs should be attached to the attempt to sell all B units.

(2) A more difficult point is the possibility that the large trader might actually be better off selling less than B units in equilibrium. For standard monopolistic-pricing reasons, the large trader may be able to raise his revenue by restricting supply, assuming no adverse effects on the investor's beliefs. In that case, an equilibrium with $S^* < B$ might be preferred by the large trader, and this might be used as an argument why this equilibrium should be observed. Still, the large trader would have been even better off if he had bought less at date 1, and this argues for an equilibrium in which $S^* = B$.

(3) In the game form studied here, the large trader submits a market order S and the market clears at the price $P_2(S)$. Trade occurs only once. One could also assume that after trade has occurred once, the large trader can submit another order and so on, repeatedly. Introducing dynamic elements often greatly complicates the analysis of games; however, in the present circumstances, the analysis seems to be quite straightforward: there is only one intuitive outcome. The

large trader will always want to sell some of his remaining units if the price is positive. Every equilibrium outcome will involve the sale of all B units. Since there is nothing to prevent pooling, and the manipulator will always want to imitate the informed trader, beliefs must be constant and equal to Q_1 along the equilibrium path. Since investors will only buy at the lowest price, trade can only occur at a single price, and that price must be determined by the equilibrium condition (A3), with $S = B$. Thus, we can argue that the possibility of repeated trade also points to the outcome we have selected as the unique plausible outcome.

A.2 The reduced-form game at date 1

For each set of initial conditions (B, P_1, Q_1) , we have selected a unique, plausible equilibrium outcome at date 2. This means that the equilibrium payoffs to the representative investor, the informed trader, and the manipulator are well defined as a function of (B, P_1, Q_1) . By defining payoff functions in this way, we can characterize an equilibrium at date 1 without explicit reference to the nature of equilibrium at later dates.

Suppose that the manipulator enters the market at date 1. Let $V_M^*(B, P_1, Q_1)$ denote the manipulator's payoff if the initial conditions at date 2 are given by (B, P_1, Q_1) . Since the manipulator pools with the informed trader at date 2, he buys at P_1 and sells at $P_2(B, P_1, Q_1)$, where $P_2(B, P_1, Q_1)$ is defined by (A3) with $S = B$ and $Q_2(B) = Q_1\pi / (Q_1\pi + 1 - Q_1)$. Then his equilibrium payoff is

$$V_M^*(B, P_1, Q_1) = P_2(B, P_1, Q_1)B - P_1B.$$

Note that in the event that $Q_1 = 0$, so that the investor "knows" that the large trader is the manipulator, $P_2(B, P_1, Q_1) = V_L$ and the formula gives the correct payoff for this case, too.

We can define the informed trader's payoff similarly, except that here we have to take account of the possibility of an announcement at date 2. The informed trader's payoff is denoted by $V_R^*(B, P_1, Q_1)$ and defined by

$$V_R^*(B, P_1, Q_1) = \pi\{P_2(B, P_1, Q_1)B + (1 - \pi)V_LB\} - P_1B.$$

If $Q_1 = 0$, the announcement will come as a surprise, but the formula clearly gives the right payoff, $(V_L - P_1)B$ in this case, too.

When there is no large trader in the market, the representative investor's payoff is $U(EV_I)$. Recall that we are assuming that the presence or absence of the large trader is always signaled by the presence or absence of trade. However, in the absence of a large trader, equilibrium is trivial anyway, so no more needs to be said.

Suppose then that a large trader has entered the market. Given the

initial conditions (B, P_1, Q_1) , the representative investor faces three possible outcomes: (i) with probability $Q_1 \pi$, the large trader is informed and the stock's value will be high; (ii) with probability $Q_1 (1 - \pi)$, the large trader is informed and the stock's value will be low, or (iii) with probability $(1 - Q_1)$, the large trader is the manipulator and the stock's value will be low. Whatever happens at date 2, the investor will repurchase B units of stock and end up holding E units. His final wealth depends on both the value of the stock and the prices at which he has traded. With probability $Q_1 \pi$, his final wealth is

$$W_H(B, P_1, Q_1) \equiv EV_H + (P_1 - P_2(B, P_1, Q_1))B;$$

with probability $(1 - \pi) Q_1$, his final wealth is

$$W_L(B, P_1, Q_1) \equiv EV_L + (P_1 - V_L)B;$$

with probability $(1 - Q_1)$, his final wealth is

$$W_M(B, P_1, Q_1) \equiv EV_L + (P_1 - P_2(B, P_1, Q_1))B.$$

Then the investor's payoff can be denoted by $U^*(B, P_1, Q_1)$ and defined by

$$\begin{aligned} U^*(B, P_1, Q_1) = & Q_1 \pi U(W_H(B, P_1, Q_1)) \\ & + (1 - \pi) Q_1 U(W_L(B, P_1, Q_1)) \\ & + (1 - Q_1) U(W_M(B, P_1, Q_1)). \end{aligned}$$

Note that since the representative investor is a price-taker, an increase in his holding of stock is assumed to leave the price $P_2(B, P_1, Q_1)$ unchanged, even though in equilibrium, $P_2(B, P_1, Q_1)$ does depend on B . The investor perceives that an increase in his sale of stock at date 1 by one unit will change his expected utility by

$$\begin{aligned} & \pi Q_1 U'(W_H(B, P_1, Q_1))(P_2(B, P_1, Q_1) - P_1) \\ & + (1 - \pi) Q_1 U'(W_L(B, P_1, Q_1))(V_L - P_1) \\ & + (1 - Q_1) U'(W_M(B, P_1, Q_1))(P_2(B, P_1, Q_1) - P_1). \end{aligned}$$

A.3 Equilibrium at date 1

A *pooling equilibrium at date 1* is defined by the quantity of stock B^* , a price function $P_1(B)$, and the beliefs $Q_1(B)$, satisfying the following conditions. First, B^* simultaneously maximizes the payoffs of the informed trader and the manipulator:

$$B^* \in \arg \max_{B \geq 0} V_R^*(B, P_1(B), Q_1(B));$$

$$B^* \in \arg \max_{B \geq 0} V_M^*(B, P_1(B), Q_1(B)). \quad (A4)$$

Second, the investor's beliefs must be consistent with his prior beliefs and the equilibrium strategies of the large trader. Since both types pool, there is no new information and his beliefs must equal his prior at the equilibrium trade:

$$Q_1(B^*) = \gamma. \quad (A5)$$

Finally, for any value of B , the price function $P_1(B)$ must clear the market. It will do this if and only if it satisfies the representative investor's first-order condition, given his beliefs and the sale of B units of stock:

$$\begin{aligned} P_1(B) \{ & \pi Q_1 U'(\mathcal{W}_H(B, P_1, Q_1)) + (1 - \pi) Q_1 U'(\mathcal{W}_L(B, P_1, Q_1)) \\ & + (1 - Q_1) U'(\mathcal{W}_M(B, P_1, Q_1)) \} \\ = \{ & \pi Q_1 U'(\mathcal{W}_H(B, P_1, Q_1)) P_2(B, P_1, Q_1) \\ & + (1 - \pi) Q_1 U'(\mathcal{W}_L(B, P_1, Q_1)) V_L \\ & + (1 - Q_1) U'(\mathcal{W}_M(B, P_1, Q_1)) P_2(B, P_1, Q_1) \}. \end{aligned} \quad (A6)$$

A.4 Proof of Proposition 1

In order to prove Proposition 1, we show the existence of a particular form of pooling equilibrium at date 1. Let B^* denote some positive value of stock purchased at date 1, and specify the investor's beliefs as follows:

$$Q_1(B) = \begin{cases} 0, & \text{if } B \neq B^*; \\ \gamma, & \text{if } B = B^*. \end{cases}$$

For any $B \neq B^*$, the market-clearing price will be

$$P_1(B) = V_L.$$

For $B = B^*$, the equilibrium price is defined by the equilibrium condition (A6). To see that there exists such a function, replace $P_1(B)$ by p in (A6). Note that the right-hand side of (A6), divided by the expression in braces on the left-hand side, is a continuous function of p that maps $[V_L, V_H]$ into itself. Thus, it possesses a fixed point p^* , which is the required value of $P_1(B^*)$. Since (A5) and (A6) are satisfied by construction, it only remains to show that (A4) is satisfied.

For any $B \neq B^*$, the investors are certain the large trader is a manipulator, and these beliefs will not be revised at date 2. Thus, at date 2 we shall have $P_2(B, P_1(B), Q_1(B)) = V_L$. Consequently,

$$V_R^*(B, P_1(B), Q_1(B)) = V_M^*(B, P_1(B), Q_1(B)) = 0,$$

for any $B \neq B^*$. To satisfy (A4) it suffices to show that the large traders of each type get a positive payoff at $B = B^*$. For certain parameter values, this will indeed be the case. Choose a fixed but arbitrary value of $0 < B^* < E$. For this value of B^* , (A3) implies that

$$P_2(B^*, P_1(B^*), \gamma) \rightarrow V_H \text{ as } \gamma \rightarrow 1 \text{ (i.e., as } \beta \rightarrow 0).$$

Thus, by choosing β sufficiently small, we can make sure the stock price at date 2 is arbitrarily close to V_H in the event that no announcement is made. Next, note that from (A6) for a fixed value of B^* and β sufficiently small, $P_1(B) \rightarrow V_L$ as the investors' degree of relative risk aversion increases without bound. Thus, for β sufficiently small and for U sufficiently risk averse,

$$\begin{aligned} V_R^*(B^*, P_1(B^*), Q_1(B^*)) &\approx \pi V_H B^* + (1 - \pi) V_L B^* - V_L B^* \\ &= \pi(V_H - V_L) B^* > 0 \end{aligned}$$

and

$$\begin{aligned} V_M^*(B^*, P_1(B^*), Q_1(B^*)) &= V_H B^* - V_L B^* \\ &= (V_H - V_L) B^* > 0. \end{aligned}$$

This completes the proof of the proposition.

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