

Asymmetric Predictability of Conditional Variances

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We show that there is an asymmetry in the predictability of the volatilities of large versus small firms. Using both univariate and multivariate ARMA-GARCH-M parameterizations, we find that volatility surprises to large market value firms are important to the future dynamics of their own returns as well as the returns of smaller firms. Conversely, however, shocks to smaller firms have no impact on the behavior of either the mean or the variance of the returns of larger capitalization companies.

There is substantial evidence that short-horizon security returns are predictable [see, e.g., Keim and Stambaugh (1986) and Lo and MacKinlay (1988,1990a)]. In particular, Conrad and Kaul (1988, 1989) show that time variation in expected returns accounts for substantial proportions (in excess of 25 percent for small firms) of the variance of both weekly and monthly portfolio returns. However, Lo and MacKinlay (1990a) and Mech (1990) uncover a rather striking aspect of

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the predictability of stock returns: there is an asymmetry in the ability to predict returns of stocks of different market value. Specifically, returns of large capitalization stocks can be used to predict reliably the returns of smaller stocks, but not vice versa. Lo and MacKinlay conclude that this asymmetry requires “. . . further investigation of mechanisms by which aggregate shocks to the economy are transmitted from large capitalization companies to small ones” (1990a, p, 198).

The asymmetry in the predictability of mean returns does not necessarily imply that information is transmitted (with a lag) from large to small capitalization companies. However, such an asymmetry does suggest that there are important differences in the dynamics of the stock prices of firms categorized on ‘the basis of market value. To investigate further the predictable price behavior of different securities, in this paper we examine empirically the differential predictability of the *conditional variances* of the returns of large versus small firms. The motivation for analyzing conditional volatilities follows directly from Ross (1989), who shows that the variance of price changes is related directly to the rate of flow of information. Hence, studying the differential predictability of volatilities will shed light on the process by which information is assimilated across firms of different market value.

We use both univariate and multivariate specifications of the GARCH family of statistical processes to model conditional variances and to estimate the interaction between the conditional volatilities of different securities. An important contribution of this article is the use of a new multivariate parameterization. This multivariate methodology is a significant improvement over earlier univariate methods used to assess the interaction between the conditional volatilities of different asset markets [see, e.g., Hamao, Masulis, and Ng (1990)]. As opposed to the univariate approach, which utilizes only the information in an asset’s own history of returns, the multivariate approach uses the information in the entire variance-covariance matrix of the returns of all assets. This, in turn, leads to more precise estimates of the parameters of the model. Also, we use a new multivariate (positive-definite) parameterization of the GARCH model [see Baba et al. (1989)] which leads to a substantial reduction in the number of parameters compared to the vector representation typically used in the literature [see, e.g., Bollerslev, Engle, and Wooldridge (1988)]. The reduction in the number of parameters is obtained by imposing suitable cross-equation restrictions. For comparison purposes, we use both univariate and multivariate parameterizations to estimate the linkages between the conditional variances of different market value securities.

The data used in this paper are weekly returns on three size-based portfolios of NYSE and AMEX stocks over the 1962-1988 period. The evidence shows a distinct asymmetry in the predictability of the volatilities of large versus small firms: a volatility “surprise” to larger firms can be used to predict reliably the volatility of smaller market value firms, but not vice versa. For ease of exposition, we refer to this asymmetric predictability as volatility “spillover effects” from larger to smaller firms. The unidirectional spillover effects in conditional volatilities remain unaltered even when we take into account the asymmetry in the predictability of mean returns uncovered by Lo and MacKinlay (1990a) and Mech (1990).

To gauge whether our results are specific to a particular time period, we conduct all tests for two equal-length subperiods, 1962-1975 and 1976-1988. Though- the asymmetry in volatility spillover effects is stronger in the first subperiod, the same (statistically significant) patterns are also observed during the second subperiod. We also show that infrequent/nonsynchronous trading cannot be the source of the asymmetric predictability of conditional variances. Thus, the evidence in this paper suggests an intriguing aspect of the price dynamics of large versus small firms. Shocks to larger firms are important not only to the future dynamics of their own returns, but also to the behavior of the returns of smaller firms. Conversely, however, shocks to smaller firms have little relevance for the behavior of both the conditional mean and variance of the returns of larger firms. Any model of the time-varying moments of stock returns needs to take into account these asymmetric effects.

In Section 1, we describe the methodology used in the paper and present some descriptive statistics of the data in Section 2. A detailed analysis of the ARMA-GARCH-M models used to evaluate volatility spillover effects across securities is contained in Section 3. We also evaluate the potential role of nonsynchronous trading in generating the asymmetric predictability of conditional variances. A brief summary and some concluding remarks are presented in Section 4.

1. The Methodology

We use the GARCH family of statistical processes to model the conditional mean and variance of security returns. Engle (1982) introduces the ARCH model, which is generalized to a GARCH specification by Bollerslev (1986) [see also Engle and Bollerslev (1986)]. We use both a univariate approach and a new, more efficient, multivariate parameterization to estimate the parameters of the model.

1.1 Univariate model

We first discuss the univariate approach and then introduce the multivariate methodology. The specification used in this paper is an **ARMA(1,1)-GARCH(1,1)-M process**¹

$$R_{it} = \delta_0 + \beta_i R_{it-1} + \phi_i R_{it-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad (1)$$

where $\epsilon_{it} | \Psi_{t-1} \sim N(0, b_{it})$,

$$b_{it} = m_i + b_i b_{it-1} + c_i \epsilon_{it-1}^2, \quad (2)$$

where R_{it} is the return of security i in period t ,

$$m_i > 0, \quad b_i c_i \geq 0, \quad b_i + c_i < 1,$$

and Ψ_{t-1} is the set of all information available at time $t-1$.²

Equation (1) models security returns as an ARMA(1,1) process with a GARCH-M term. An ARMA(1,1) process for returns is parsimonious and appears to be well-specified [see Conrad and Kaul (1988)]. Note that we introduce a GARCH-M term in our model [see Equation (1)], although we use size-based portfolios, and not the market portfolio, in our tests. The inclusion of the GARCH-M term can be viewed as an attempt to use all past information, including the information contained in the conditional variance, to obtain expected return estimates. Clearly, we are not imposing any equilibrium model (e.g., the CAPM) on the mean equation. In any event, our conclusions about the asymmetric assimilation of information across securities are insensitive to the inclusion of the GARCH-M term in Equation (1).

The 'conditional variance h_{it} in Equation (2) is a function of last period's squared errors and conditional variance. The conditions $m_i > 0$ and $b_i c_i \geq 0$ are required to ensure that h_{it} is positive for all t , and $b_i + c_i < 1$ ensures that h_{it} is a stationary process. A number of studies have shown that stock return variances change through time and that a GARCH(1,1) process appears to model conditional variances adequately [see Bollerslev (1987); French, Schwert, and Stambaugh (1987); Akgiray (1989); and Schwert and Seguin (1990)]. The GARCH specification has the advantage of incorporating the findings of Mandelbrot (1963) and Fama (1965) that large price changes are

¹ Engle, Lillen, and Robins (1987) extend the GARCH model to allow the conditional mean to be a function of the conditional variance at time t .

² One potential problem in modeling conditional variances using the GARCH process is that it does not take into account the asymmetric response of conditional variances to past returns. The evidence in Black (1976), French, Schwert, and Stambaugh (1987), and Schwert (1990) indicates that negative stock returns lead to higher volatility than equivalent positive returns. Such an asymmetry in conditional variances may be better captured by alternative parametric [Nelson (1991)] or non-parametric [Pagan and Schwert (1990)] models. However, in this paper, we are primarily interested in the asymmetric predictability of conditional volatilities and the GARCH specification may be adequate for this purpose.

followed by large changes, and small by small, of either sign. It allows conditional variances to depend on past realized variances and hence is consistent with the actual volatility pattern of the stock market.

To address formally the issue of volatility spillovers across securities, we estimate univariate ARMA(1,1)-GARCH(1,1)-M models for each security separately and then introduce the lagged squared errors for security j as an exogenous variable in the conditional variance equation of security i . This two-stage procedure is similar to the one used by Hamao, Masulis, and Ng (1990). In particular,

$$R_{it} = \delta_{i0} + \beta_i b_{it} + \phi_i R_{it-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad (3)$$

where $\epsilon_{it} | \Psi_{t-1} \sim N(0, b_{it})$,

$$b_{it} = m_i + b_i b_{it-1} + c_i \epsilon_{it-1}^2 + k_{ji} \epsilon_{jt-1}^2, \quad j \neq i, \quad (4)$$

where k_{ji} measures the impact of past volatility "surprise" to security j , ϵ_{jt-1}^2 , $j \neq i$, on the conditional variance of security i , b_{it} .

A similar specification, with coefficient k_{ji} , can be used to gauge the effect of past volatility surprise to security i , ϵ_{it-1}^2 , on the conditional variance of security j , b_{jt} . The relative magnitudes of k_{ji} and k_{ji} in turn, can help determine whether aggregate shocks have differential effects across securities. Also, the impact of shocks at longer lags can be evaluated using the same approach.

1.2 Multivariate model

The second approach uses a new multivariate ARMA(1,1)-GARCH(1,1)-M parameterization to assess the differential predictability of the volatilities of different securities. The extension from a univariate ARMA-GARCH-M model to an n -variate model requires allowing the conditional variance-covariance matrix $\mathbf{H}_t$ of the n -dimensional zero mean random variables ϵ_t to depend on elements of the information set Ψ_{t-1} . A positive definite parameterization of $\mathbf{H}_t$ can be written as

$$R_{it} = \delta_{i0} + \beta_i b_{it} + \phi_i R_{it-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad (5)$$

where $\epsilon_t | \Psi_{t-1} \sim MVN(0, \mathbf{H}_t)$,

$$\mathbf{H}_t = \mathbf{M}'\mathbf{M} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B} + \mathbf{C}'\epsilon_{t-1}\epsilon_{t-1}'\mathbf{C}, \quad (6)$$

where $\mathbf{h}_t \sim \text{vec}[\mathbf{H}_t]$ is the $L \times 1$ vector parameterization of $\mathbf{H}_t$, where $L = \frac{1}{2}n(n+1)$, $\mathbf{M}$ is the $n \times n$ constant symmetric matrix in the variance-covariance matrix $\mathbf{H}_0$, $\mathbf{B}$ is the $n \times n$ GARCH coefficient matrix on the lagged variance and covariance terms, and $\mathbf{C}$ is, the $n \times n$ ARCH coefficient matrix on the lagged squared error terms.³

³The conditions for stationarity of the multivariate GARCH process are similar to the ones for a vector autoregressive process. In particular, $\mathbf{H}_t$ can be written as a vector autoregressive process of

Previous papers that adapt the ARCH or GARCH methodology to multiple assets work with $\mathbf{h}_t$, the vector parameterization of $\mathbf{H}_t$, in the estimation of the model [see, e.g., Bollerslev, Engle, and Wooldridge (1988)]. However, for any parameterization to be sensible, we must require that $\mathbf{H}_t$ be positive definite for all realizations of ϵ_t . These restrictions are difficult to impose if the vector parameterization is used. The parameterization specified in Equation (6), however, is an alternative representation of $\mathbf{H}_t$ that imposes such restrictions easily without eliminating interesting models entertained by the vector representation. Clearly, $\mathbf{H}_t$ as characterized in Equation (6) will be positive definite under very weak conditions. Moreover, in a recent paper, Baba et al. (1989) show that the vector parameterization implied by the positive definite representation in Equation (6) is *unique*, while the converse is not true. Hence, we can estimate Equations (5) and (6) and obtain the implied vector representation of the variance-covariance matrix $\mathbf{H}_t$.

The parameterization of $\mathbf{H}_t$ in Equation (6) is also appealing from a practical standpoint. This representation imposes cross-equation restrictions and, hence, drastically reduces the number of parameters to be estimated. For example, for three assets the GARCH model in Equation (6) has 18 unknown parameters (excluding the parameters in the symmetric constant matrix $\mathbf{M}$). In contrast, the number of parameters to be estimated in the corresponding vector representation increases to 72.⁴

The advantage of the multivariate approach is that, unlike the univariate approach, it estimates the ARMA(1,1)-GARCH(1,1)-M models for all securities simultaneously, thus utilizing the information in the entire variance-covariance matrix of the errors. This, in turn, leads to more precise estimates of the parameters of the model. Moreover, since all parameters are estimated jointly, this approach avoids the "generated regressor" problem associated with the univariate approach [see, e.g., Pagan (1984)].

finite dimension p of the form

$$\sum_{j=0}^p \mathbf{A}_j \mathbf{x}_{t-j} = \epsilon_t$$

where the ϵ_t are uncorrelated k -dimensional random variables and the $\mathbf{A}_j$ are $k \times k$ matrices with $\mathbf{A}_0 = \mathbf{I}$ and $\mathbf{A}_p \neq \mathbf{0}$. The vector autoregressive process will be stationary if roots of the determinantal equation

$$|\mathbf{A}_0 \mathbf{m}^p + \mathbf{A}_1 \mathbf{m}^{p-1} + \dots + \mathbf{A}_p| = 0$$

are less than unity in absolute value.

⁴In general, with n assets, the number of parameters to be estimated in the GARCH representation in Equation (6) is $2n^2$ (excluding the constant part of $\mathbf{H}_t$). On the other hand the number of free parameters in the vector representation is given by $2[n(n+1)/2]^2$. See Baba et al. (1989) for a detailed discussion of alternative representations of $\mathbf{H}_t$ in multivariate GARCH models.

2. Some Descriptive Characteristics of the Data

2.1 Data

We use the Center for Research in Security Prices (CRSP) daily return file which contains data for both the American and the New York Stock Exchanges to calculate weekly portfolio returns for the 1962-1988 period. The choice of a weekly sampling interval is largely a compromise between the relatively few monthly observations and the potential biases associated with nontrading, the bid-ask effect, etc., in daily data. We use portfolio returns, rather than individual security returns, because (a) it is much more difficult to extract the (expected return) signal from noisy returns of an individual security [see Conrad, Kaul, and Nimalendran (1990)] and (b) it is feasible to estimate the multivariate ARMA(1,1)-GARCH(1,1)-M model only for a few assets.

At the end of each year, we sort all the NYSE/AMEX firms on the basis of market value (price times number of shares outstanding). We then combine the 100 smallest, the 100 intermediate, and the 100 largest market value stocks into three portfolios. For each week (Wednesday close to Wednesday close) of the following year, one-week simple returns of each security within each portfolio are value-weighted to form three series of portfolio returns. Compounding individual-security daily returns to obtain weekly returns before calculating weekly portfolio returns, and value-weighting the security returns, minimizes the biases present in the daily rebalancing approach to calculating portfolio returns [see Blume and Stambaugh (1983) and Roll (1983)]. For example, bid-ask errors in closing stock prices lead to an upward bias in simple returns. Value-weighting should reduce this bias because the (proportional) spreads of larger firms are less than the spreads of smaller firms.

For tractability reasons, we use only three value-weighted portfolios in our analysis and label them portfolios 1, 2, and 3. Note that even the parsimonious n -asset multivariate ARMA(1,1)-GARCH(1,1)-M model in Equations (5) and (6) involves estimation of $4n + \frac{1}{2}n(n+1) + 2n^2$ parameters of *both* the mean and conditional variance equations. Hence, it is critical to restrict the number of assets in order to estimate the parameters of the multivariate model feasibly.

2.2 Summary statistics

Table 1 shows the summary statistics for the weekly portfolio returns, squared returns, and absolute returns. The Kolmogorov-Smirnov D -statistic (reported in the last column) leads to the rejection of normality for the returns of all three portfolios. This is not surprising given the presence of significant kurtosis in all portfolio returns. Portfolio 1 also exhibits significant skewness.

Table 1

Summary statistics of weekly realized returns, squared returns, and absolute returns of three value-weighted portfolios of the 100 smallest, 100 intermediate, and the 100 largest stocks on the New York and American Stock Exchanges formed by rankings of market value of equity outstanding at the end of the previous year, July 1962–December 1988 ($n = 1382$)

Variable (x)	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	Q	$\bar{x} \times 10^2$	$s(x) \times 10^2$	Skewness	Kurtosis	D
R_t	0.369	0.203	0.168	0.083	0.014	0.042	296.921 [0.000]	0.5895	3.0468	1.681	16.676	0.094 [<0.01]
R_t^2	0.255	0.104	0.061	0.036	0.023	0.031	114.484 [0.000]	0.2912	2.4544	-0.228	7.683	0.051 [<0.01]
R_t	0.022	-0.017	0.002	-0.020	-0.011	-0.005	1.832 [0.935]	0.2016	2.0156	-0.133	5.678	0.034 [<0.01]
$(R_t)^2$	0.174	0.025	0.077	0.057	0.035	0.022	57.780 [0.000]	0.0962	0.3834			
$(R_t)^2$	0.337	0.147	0.092	0.055	0.068	0.066	215.303 [0.000]	0.0610	0.1548			
$(R_t)^2$	0.239	0.144	0.097	0.070	0.077	0.130	159.632 [0.000]	0.0410	0.0876			
$ R_t $	0.330	0.131	0.129	0.128	0.110	0.080	245.684 [0.000]	2.0633	2.3174			
$ R_t $	0.293	0.202	0.151	0.129	0.149	0.148	291.831 [0.000]	1.8287	1.6621			
$ R_t $	0.201	0.187	0.140	0.135	0.138	0.193	235.111 [0.000]	1.5341	1.3219			

R_t , R_t^2 , and $|R_t|$ are weekly portfolio returns of the 100 smallest, 100 intermediate, and the 100 largest NYSE and AMEX stocks, respectively, $\bar{x}$ and $s(x)$ are the sample mean and standard deviation of the variable, and $\hat{\rho}_t$ is the estimated autocorrelation at lag t . Under the hypothesis that the true autocorrelations are zero, the standard error of the estimated autocorrelations is about 0.027. The Q -statistics (with p values in brackets) are to test the hypothesis that all autocorrelations up to lag 6 are jointly zero. The D -statistic is the Kolmogorov-Smirnov test statistic for normality, with p values in brackets.

The first-order autocorrelations of weekly returns of portfolios 1 and 2 are large and several standard errors' from zero and higher-order autocorrelations remain statistically different from zero, but decay rapidly across longer lags. The autocorrelation structure of returns also displays a consistent pattern as we go from the smallest portfolio (portfolio 1) to the largest (portfolio 3): the magnitude and persistence of the autocorrelations decline monotonically. However, higher-order autocorrelations remain significant for all but the largest portfolio (which exhibits no autocorrelation). Since we use value weights in forming portfolios, the autocorrelation structure of the returns of portfolio 3 is dominated by the return characteristics of the largest market value firms, which exhibit virtually no autocorrelation.

The -autocorrelation structure of returns indicates that expected returns vary through time. Moreover, estimates of ARMA(1,1) models for returns show that time variation in expected returns explains substantial proportions (in excess of 25% for small firms) of return variance [see Conrad and Kaul (1988)].

Table 1 shows that the autocorrelations of the series $\{R_t^2\}$ and $\{|R_t|\}$ are also statistically significant. These autocorrelations, however, decay fairly slowly at longer lags. The autocorrelation in absolute returns is generally larger than that in squared returns. This evidence confirms the findings of Mandelbrot (1963) and Fama (1965) that large price changes are followed by large changes, and small by small, of either sign.

Table 2 presents autocorrelations of the residuals, squared residuals, and absolute residuals of an ARMA(1,1) model for each portfolio, that is,

$$R_{it} = \delta_{i0} + \delta_{i1}D_t + \phi_i R_{it-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad i = 1, 2, 3, \quad (7)$$

where $D_t = 1$, for the first week in January (ending on a Wednesday), and $D_t = 0$, 1 for all other weeks.

A stationary AR(1) process for expected returns appears to be well-specified: the residuals of the model exhibit no significant autocorrelation as indicated by the estimated autocorrelations and the Q-statistics. However, the squared and absolute residuals again are strongly autocorrelated, which suggests the need to model the implied persistence in conditional variances.

Table 3, panel A presents the first-order lagged cross correlations between the three value-weighted portfolio returns. The (i, j) th element of the correlation matrix is the correlation between $R_{i,t-1}$ and $R_{j,t}$. The numbers reported in Table 3 reflect the same asymmetric pattern in cross-correlations found by Lo and MacKinlay (1990a) and Mech (1990). Specifically, the elements below the diagonal of the matrix are invariably larger than those above the diagonal. This implies

Table 2
Summary statistics of the residuals, squared residuals, and absolute residuals from the model in which returns follow a stationary ARMA(1,1) process, July 1962–December 1988 ($n = 1382$)

Variable (x)	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	Q	$\bar{x} \times 10^3$	$s(x) \times 10^3$	Skewness	Kurtosis	D
$\hat{\epsilon}_1$	0.006	-0.035	0.056	0.008	-0.058	0.033	12.211 [0.057]	-0.0001	2.6557	1.288	14.407	0.071 [<0.01]
$\hat{\epsilon}_2$	0.002	-0.014	0.014	0.008	0.003	0.023	1.392 [0.966]	-0.0001	2.3594	0.146	8.141	0.057 [<0.01]
$\hat{\epsilon}_3$	-0.002	0.004	-0.016	-0.005	-0.024	0.006	1.262 [0.974]	-0.0000	2.0126	-0.120	5.573	0.033 [<0.01]
$(\hat{\epsilon}_1)^2$	0.144	0.069	0.057	0.022	0.035	0.011	42.373 [0.000]	0.0705	0.2577			
$(\hat{\epsilon}_2)^2$	0.314	0.201	0.076	0.033	0.039	0.071	211.402 [0.000]	0.0556	0.1485			
$(\hat{\epsilon}_3)^2$	0.251	0.158	0.103	0.077	0.082	0.133	178.704 [0.000]	0.0405	0.0865			
$ \hat{\epsilon}_1 $	0.240	0.134	0.106	0.082	0.093	0.067	147.633 [0.000]	1.8416	1.9128			
$ \hat{\epsilon}_2 $	0.255	0.206	0.140	0.110	0.117	0.150	243.260 [0.000]	1.7103	1.6246			
$ \hat{\epsilon}_3 $	0.217	0.198	0.154	0.143	0.146	0.198	264.841 [0.000]	1.5140	1.3253			

The model is

$$R_{it} = \delta_0 + \delta_1 D_t + \phi_i R_{it-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad i = 1, 2, 3.$$

R_{1t} , R_{2t} , and R_{3t} are portfolio returns for week t of the 100 smallest, 100 intermediate, and the 100 largest NYSE and AMEX stocks, respectively, formed by rankings of market value of equity outstanding at the end of the previous year. $D_t = 1$, for the first week in January, and $D_t = 0$, for all other weeks. ϕ and θ are the autoregressive and moving average parameters. $\hat{\epsilon}_1$, $\hat{\epsilon}_2$, and $\hat{\epsilon}_3$ are the residuals from the estimated ARMA(1,1) model for realized returns of the smallest, intermediate, and largest portfolios, respectively. $\bar{x}$ and $s(x)$ are the sample mean and standard deviation of the variable and $\hat{\rho}_i$ is the estimated autocorrelation at lag i . Under the hypothesis that the true autocorrelations are zero, the standard error of the estimated autocorrelations is about 0.027. The Q -statistics (with p values in brackets) are to test the hypothesis that all autocorrelations up to lag 6 are jointly zero. The D -statistic is the Kolmogorov-Smirnov test statistic for normality, with p values in brackets.

Table 3

Weekly estimates of first-order lagged cross-correlations between (I) the returns of three value-weighted portfolios of NYSE/AMEX stocks (panel A) and (II) the residuals from models in which returns follow a stationary ARMA(1,1) process

Variable (x)	A: Return cross-correlations		
	R_{1t}	R_{2t}	R_{3t}
$R_{1,t-1}$	0.369	0.188	0.014
$R_{2,t-1}$	0.367	0.255	0.033
$R_{3,t-1}$	0.227	0.195	0.022
	B: Residual cross-correlations		
	$\hat{\epsilon}_{1t}$	$\hat{\epsilon}_{2t}$	$\hat{\epsilon}_{3t}$
$\hat{\epsilon}_{1,t-1}$	0.006	-0.031	0.017
$\hat{\epsilon}_{2,t-1}$	0.127	0.002	0.011
$\hat{\epsilon}_{3,t-1}$	0.096	0.015	-0.002

The three value-weighted portfolios are of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges, formed by rankings of market value of equity outstanding at the end of the previous year, July 1962–December 1988 ($n = 1381$). R_{1t} , R_{2t} , and R_{3t} are weekly portfolio returns of the three portfolios, and $\hat{\epsilon}_{1t}$, $\hat{\epsilon}_{2t}$, and $\hat{\epsilon}_{3t}$ are residuals from weekly estimates of ARMA(1,1) models for the returns. Under the hypothesis that the true cross-correlations are zero, the standard error of the estimated cross-correlation is about 0.027.

a distinct asymmetry in the predictability of the returns of different securities: the return of larger stocks can be used to predict reliably the returns of small stocks, but not vice versa. The relations between the returns of the largest firms (portfolio 3) and the smallest firms (portfolio 1) are particularly noteworthy. The first-order lagged cross-correlation between last week's return on large stocks [$R_{3,t-1}$] and this week's return on small stocks [R_{1t}] is 22.7 percent, whereas the cross-correlation between $R_{1,t-1}$ and R_{3t} is only 1.4 percent. This asymmetry is intriguing and does suggest further investigation into whether information, as measured by changes in conditional variances, has similar asymmetric effects on the stock prices of large versus small capitalization companies.

Before addressing the issue of asymmetric responses of conditional variances to aggregate shocks, it is critical to ensure that all asymmetry in the predictability of mean returns is purged from the raw data. This is necessary because the predictability of mean returns in Table 3, panel A could show up as asymmetric responses of conditional variances to shocks. Panel B presents the first-order lagged cross-correlations between the residuals of estimated ARMA(1,1) models [see Equation (7)] for each portfolio. There is a significant reduction in the asymmetric predictability of the residuals, with the asymmetry between portfolios 3 and 2 disappearing completely. This is not surprising because conditional on a small portfolio's own past history of returns, the lagged return of the large portfolio contains little additional information about the smaller portfolio's future returns.

Recall that over 25 percent of variation in the small portfolio's returns can be explained by its own lagged returns [see Conrad, Kaul, and Nimalendran (1990) for details]. To account for any remaining asymmetry in return cross-correlations, in our subsequent analysis, the mean equation of a particular portfolio includes the lagged returns of the remaining two portfolios. This procedure renders all lagged cross-correlations between the residuals of the mean equations to be symmetric and indistinguishable from zero?

3. Model Estimates

In this section, we present both univariate and multivariate estimates of the ARMA(1,1)-GARCH(1,1)-M specification and we analyze the differential impact of aggregate shocks on the three market value portfolios.

3.1 Univariate ARMA(1,1)-GARCH(1,1)-M models

We first estimate the univariate ARMA(1,1)-GARCH(1,1)-M model for each portfolio to assess the appropriateness of this specification for weekly stock returns. We use a modified form of the model [see Equations (1) and (2)] to take account of the turn-of-the-year effect in both the mean and variance equations, and we include the lagged returns of all the portfolios to account for any asymmetric lagged cross-correlations that remain in ϵ_u 's. The estimated model is

$$R_{it} = \delta_0 + \delta_n D_t + \beta_i b_{it} + \phi_i R_{i,t-1} + \sum_{j=1}^3 \gamma_{ij} R_{j,t-1} + \epsilon_{it} - \theta_i \epsilon_{i,t-1}, \quad i, j = 1, 2, 3, \quad (8)$$

$$b_{it} = m_i + b_i b_{i,t-1} + c_i \epsilon_{i,t-1}^2 + d_n D_t + d_n D_{t-1}, \quad (9)$$

where $D_t = 1$, for the first week in January (ending on a Wednesday), and $D_t = 0$, for all other weeks,⁶ and the γ_{ij} 's measure the joint impact of the lagged returns of the two remaining portfolios on the conditional mean of portfolio i .

⁶ Use of this procedure depends on the assumption that there are no nonlinear dependencies in the conditional mean of portfolio returns. However, this assumption appears to be reasonable [see Pagan and Schwert (1990)].

⁷ Note that since Equation (9) is a GARCH specification for the conditional variance, we include the lagged January dummy D_{t-1} in addition to the contemporaneous dummy variable for the first week of January. This specification allows the effects of the January dummy variable to be interpreted as the response to a simple pulse. The immediate effect of D_t on b_{it} is measured by d_n . Because of the GARCH(1,1) parameterization, this first-week-of-January effect will lead to an expected increase/decrease in $b_{i,t+1}$ (January's second week) of $d_n(b_i + c_i)$, which is offset by the d_n term [see Baillie and Bollerslev (1989)].

Nonlinear optimization techniques are used to calculate the maximum likelihood estimates based on the BHHH algorithm [see Berndt, Hall, Hall, and Hausman (1974)]. The estimated models are shown in Table 4. No indications of serious model misspecification are observed since the Ljung-Box statistics for all three portfolios show a lack of serial correlation in both the normalized residuals and the squared residuals. The normalized residuals exhibit no skewness, while there is some indication of kurtosis for portfolios 1 and 2. However, the levels of kurtosis are significantly lower than those observed in raw returns (see Table 1). We also estimated more complicated GARCH specifications, but these estimates indicate that the parsimonious GARCH (1,1) model is well-specified and is an adequate representation of weekly portfolio returns.

Estimates of the GARCH(1,1) models provide strong evidence of changing conditional variances for all three portfolios. The estimates of b_i and c_i are always statistically significant, with the former markedly greater than the latter. The sum $\hat{b}_i + \hat{c}_i$ is always less than unity, which suggests that the conditional variances follow a stationary process. However, there is some evidence that 'the conditional variance of portfolio 3 may follow an integrated GARCH process. We reestimate the GARCH (1,1) model for each portfolio imposing the restriction for integration in variances (i.e., $b_i + c_i = 1$). Likelihood ratio tests reject this restriction for portfolios 1 and 2, but not for portfolio 3. These tests should, however, be interpreted with caution because the appropriate procedure for testing for integration in variance is not yet clear. It is possible that all the well-known problems associated with tests for unit roots in the mean apply to such tests for the variance (see Engle and Bollerslev (1986)).

Some additional aspects of the results in Table 4 are noteworthy. First, there is a January effect in both the conditional mean and the conditional variance of portfolios 1 and 2. Hence, not only is the mean return higher in January, but there is also a marked increase in variance around the turn of the year. Second, the GARCH-M effect is always positive, but never significant at conventional levels. The positive effect agrees with the findings of French, Schwert, and Stambaugh (1987) and further discussion of the significance of these results is contained in Section 3.3. Finally, the only lagged cross-effect that is still significant in the mean equation is that of $R_{2,t-1}$ on $R_{1,t}$; all remaining lagged cross-effects are indistinguishable from zero. For brevity, we do not report the lagged coefficients (the $\hat{\gamma}_i$'s) in Table 4.

3.2 Spillover effects across securities

To analyze the effects of aggregate shocks on securities of different market value, we estimate two modified versions of Equations (3)

Table 4

Weekly estimates of univariate ARMA(1,1)-GARCH(1,1)-M models for three value-weighted portfolios of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges, formed by rankings of market value of equity outstanding at the end of the previous year, July 1962–December 1988 ($n = 1381$)

Portfolio	Log likelihood	$\delta_0 \times 10^3$	δ_1	β	ϕ	θ	$m \times 10^3$	b	c	$d_1 \times 10^3$	$d_2 \times 10^3$	Skew- ness	Kurtosis	LB1(6)	LB2(6)
1	3267.252	0.574 (0.710)	0.069 (8.588)	0.672 (0.433)	0.459 (8.632)	0.345 (5.504)	0.054 (7.325)	0.713 (27.079)	0.190 (7.756)	1.609 (7.610)	-0.920 (-4.534)	0.118	7.957	9.781 [0.134]	4.493 [0.611]
2	3349.993	0.677 (0.897)	0.019 (4.477)	2.255 (1.611)	0.449 (4.034)	0.216 (1.834)	0.021 (4.452)	0.810 (40.610)	0.159 (8.840)	0.238 (2.262)	-0.107 (-0.978)	-0.581	6.983	5.551 [0.476]	5.092 [0.532]
3	3568.835	2.774 (2.274)	0.001 (0.147)	3.259 (0.978)	-0.515 (-1.626)	0.539 (1.762)	0.004 (1.743)	0.880 (52.480)	0.114 (7.208)	0.044 (0.462)	-0.023 (-0.265)	-0.130	3.689	2.844 [0.829]	4.233 [0.645]

The model is

$$R_{it} = \delta_{i0} + \delta_{i1}D_t + \beta_i R_{it-1} + \sum_{j=1}^2 \gamma_{ij} R_{jt,t-1} + \epsilon_{it} - \theta_i \epsilon_{i,t-1}, \quad i, j = 1, 2, 3, \quad \text{where } \epsilon_{it} | \Psi_{t-1} \sim N(0, b_{it}),$$

$$b_{it} = m_i + b_i b_{i,t-1} + c_i \epsilon_{i,t-1}^2 + d_{i1} D_t + d_{i2} D_{t-1},$$

where the γ_{ij} 's measure the joint impact of the lagged returns of the two remaining portfolios on the conditional mean of portfolio i . R_{1t} , R_{2t} , and R_{3t} are portfolio returns for week t of the 100 smallest, 100 intermediate, and 100 largest NYSE and AMEX stocks, respectively. $D_t = 1$, for the first week in January, and $D_t = 0$, for all other weeks. All estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t -statistics. The skewness and kurtosis coefficients are for the normalized residuals of the model. LB1(6) and LB2(6) are the Ljung-Box statistics, with six degrees of freedom, for the normalized residuals and normalized squared residuals, respectively. The p values of the $\chi^2(6)$ statistics are reported in brackets below each statistic.

and (4). First, we estimate the relation between the conditional variance of portfolio i and *each* of the two remaining portfolios' lagged squared shocks:

$$R_{it} = \delta_{i0} + \delta_{in}D_t + \beta_i b_{it} + \phi_i R_{it-1} + \sum_{\substack{j=1 \\ j \neq i}}^3 \gamma_{ij} R_{jt-1} + \epsilon_{it} - \theta_{it}, \quad i, j = 1, 2, 3, \quad (10)$$

$$b_{it} = m_i + b_i b_{it-1} + c_i \epsilon_{it-1}^2 + d_{in}D_t + d_{i2}D_{t-1} + k_{ij}\epsilon_{jt-1}^2, \quad j \neq i, \quad (11)$$

where k_{ij} measures the impact of past volatility surprises to *each* of the remaining two portfolios ϵ_{jt-1}^2 , $j \neq i$, on the conditional variance of portfolio i , b_{it} , and the γ_{ij} 's measure the joint impact of the lagged returns of the two remaining portfolios on the conditional mean of portfolio i .

We also estimate the *simultaneous* effects of volatility surprises to the two remaining portfolios on the conditional volatility of portfolio i by estimating the following equations:

$$R_{it} = \delta_{i0} + \delta_{in}D_t + \beta_i b_{it} + \phi_i R_{it-1} + \sum_{\substack{j=1 \\ j \neq i}}^3 \gamma_{ij} R_{jt-1} + \epsilon_{it} - \theta_{it} \epsilon_{it-1}, \quad i, j = 1, 2, 3, \quad (12)$$

$$b_{it} = m_i + b_i b_{it-1} + c_i \epsilon_{it-1}^2 + d_{in}D_t + d_{i2}D_{t-1} + \sum_{\substack{j=1 \\ j \neq i}}^3 k_{ij} \epsilon_{jt-1}^2, \quad (13)$$

where the k_{ij} 's measure the joint impact of past volatility surprises to *both* the remaining portfolios, the ϵ_{jt-1}^2 's, $j \neq i$, on the conditional variance of portfolio i , b_{it} .

Note that in this univariate approach, the lagged surprises of the other portfolios are treated as exogenous variables and the volatility spillover effects for each portfolio are estimated separately. Hence, the information in the variance-covariance matrix of the errors of the three portfolios is not taken into account. The statistical power of this approach, therefore, is likely to be weaker than the multivariate approach. Also, since estimates of ϵ_{jt-1}^2 are generated from an auxiliary model, the standard errors of the k_{ij} 's may be incorrect [see, e.g., Pagan (1984)]. Hence, caution must be exercised in drawing inferences based on estimates of Equations (10) and (11), and (12) and (13). Nevertheless, we first present the univariate estimates because the parameters, particularly the k_{ij} 's, have an easy and intuitive inter-

Table 5

Weekly estimates of pairwise volatility spillover effects at lag 1 using univariate ARMA(1,1)-GARCH(1,1)-M models for three value-weighted portfolios of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges, formed by rankings of market value of equity outstanding at the end of the previous year, July 1962–December 1988 ($n = 1380$)

Variable (x)	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\epsilon_{1,t-1}^2$	0.19046 (7.756)	0.00053 (0.095)	0.00217 (0.622)
$\epsilon_{2,t-1}^2$	0.19675 (8.627)	0.15917 (8.840)	0.00390 (0.631)
$\epsilon_{3,t-1}^2$	0.09947 (6.606)	0.10256 (7.372)	0.11413 (7.208)

The diagonal elements are estimates of the ARCH(1) coefficient obtained by estimating a GARCH(1,1) model for each portfolio and the off-diagonal elements are estimates of b_{ij} in the model

$$R_{it} = \delta_{i0} + \delta_{i1}D_t + \beta_i b_{ii} + \phi_i R_{i,t-1} + \sum_{j=1}^3 \gamma_{ij} R_{j,t-1} + \epsilon_{it} - \theta_i \epsilon_{i,t-1}, \quad i, j = 1, 2, 3,$$

where $\epsilon_{it} | \Psi_{t-1} \sim N(0, b_{ii})$,

$$b_{ii} = m_i + b_i b_{i,t-1} + c_i \epsilon_{i,t-1}^2 + d_{i1} D_t + d_{i2} D_{t-1} + k_{ij} \epsilon_{j,t-1}^2, \quad j \neq i,$$

where k_{ij} measures the impact of past volatility surprises to each of the two remaining portfolios, $\epsilon_{j,t-1}^2$, $j \neq i$, on the conditional variance of portfolio i , b_{ii} , R_{1t} , R_{2t} , and R_{3t} are portfolio returns for week t of the 100 smallest, 100 intermediate, and 100 largest NYSE and AMEX stocks, respectively. $D_t = 1$, for the first week in January, and $D_t = 0$, for all other weeks. $\hat{b}_{ij}$ = estimated conditional variance of portfolio i in week t . $\epsilon_{j,t-1}^2$ = squared residuals of portfolio j at lag 1. The numbers in parentheses below the estimated coefficients are t -statistics.

pretation. We then present the multivariate estimates that do not suffer from the potential problems associated with the univariate results.

Univariate estimates of pairwise volatility spillover effects at lag 1 are presented in Table 5. For brevity, we do not report estimates of all parameters in Equations (10) and (11). The format of Table 5 is identical to that of Table 3. The striking aspect of the evidence is the distinct asymmetry in the spillover effects across securities. Both in absolute and statistical terms, there is a significant asymmetric spillover effect from larger to smaller firms. For example, 'the estimated effect of $\epsilon_{3,t-1}^2$ on b_{1t} is almost 50 times larger than the effect of $\epsilon_{1,t-1}^2$ on b_{3t} , (0.09947 vs 0.00217). Moreover, this asymmetric pattern is displayed consistently across all three size-based portfolios, since elements below the diagonal of the matrix are always much larger than those above the diagonal. In fact, the asymmetry is most pronounced between medium (portfolio 2) and small firms.

Table 5 shows pairwise effects. However, it may be more appropriate to gauge the simultaneous effects (on the conditional volatility of a particular portfolio) of shocks to the other portfolios. Table 6 shows the estimates of the simultaneous effects. The distinct asymmetry in the spillover effects remains: the volatility of smaller firms can be predicted by shocks to larger firms, but not vice versa. Although the

Table 6

Weekly estimates of simultaneous volatility spillover effects at lag 1 using univariate ARMA(1,1)-GARCH(1,1)-M models for three value-weighted portfolios of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges, formed by rankings of market value of equity outstanding at the end of the previous year, July 1962-December 1988 ($n = 1379$)

Variable (x)	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$
$\hat{\epsilon}_{1,t-1}^2$	0.12338 (6.381)	-0.00135 (-0.226)	0.00248 (0.599)
$\hat{\epsilon}_{2,t-1}^2$	0.19102 (4.814)	0.10695 (5.315)	0.00290 (0.395)
$\hat{\epsilon}_{3,t-1}^2$	0.00146 (0.054)	0.10371 (7.305)	0.11133 (6.932)

The elements of the matrix are obtained by estimating a GARCH(1,1) model for each portfolio with squared shocks, $\epsilon_{j,t-1}^2$, of both the remaining portfolios

$$R_{it} = \delta_{i0} + \delta_{i1}D_t + \beta_i R_{it} + \phi_i R_{i,t-1} + \sum_{j=1}^3 \gamma_{ij} R_{j,t-1} + \epsilon_{it} - \theta_i \epsilon_{i,t-1}, \quad i, j = 1, 2, 3,$$

where $\epsilon_{it} | \Psi_{t-1} \sim N(0, b_{it})$,

$$b_{it} = m_i + b_i b_{i,t-1} + c_i \epsilon_{i,t-1}^2 + d_{i1} D_t + d_{i2} D_{t-1} + \sum_{j=1}^3 k_{ij} \epsilon_{j,t-1}^2,$$

where the k_{ij} 's measure the joint impact of past volatility surprises to both the remaining portfolios, the $\epsilon_{j,t-1}^2$'s, $j \neq i$, on the conditional variance of portfolio i , b_{it} , R_{1t} , R_{2t} , and R_{3t} are portfolio returns for week t of the 100 smallest, 100 intermediate, and 100 largest NYSE and AMEX stocks, respectively. $D_t = 1$, for the first week in January, and $D_t = 0$, for all other weeks. b_{it} = estimated conditional variance of portfolio i in week t . $\epsilon_{j,t-1}^2$ = squared residuals of portfolio j at lag 1. The numbers in parentheses below the estimated coefficients are t -statistics.

spillover effects of medium-sized firms on the small firms dominate those of the large firms, there is a statistically significant spillover effect from large to medium firms. In contrast, the data never reveal any significant spillover effects in the opposite direction, that is, from smaller to larger firms.

Finally, we also estimate the cross-effects of shocks at lag 2 on the conditional volatilities of the three portfolios. The asymmetry remains as pronounced as in Tables 5 and 6. In fact, the effect on small firms of shocks to large firms at lag 2 dominates the lag 1 effect: the (joint) effects of $\epsilon_{3,t-1}^2$ and $\epsilon_{3,t-2}^2$ on b_1 , are 0.004 and 0.058, with t -statistics of 0.301 and 2.505, respectively. Also, all the results in Tables 5 and 6 (i.e., the volatility spillover effects) are not altered if lagged returns of the two other portfolios are excluded from the mean equation of portfolio i [i.e., the γ_{ij} 's are restricted to be equal to zero in Equations (10) and (12)].

3.3 The multivariate ARMA(1,1)-GARCH(1,1)-M model

We next present estimates of the multivariate ARMA(1,1)-GARCH(1,1)-M model for the three size-based portfolios. The advan-

Table 7

Weekly estimates of a multivariate ARMA(1,1)-GARCH(1,1)-M model for three value-weighted portfolios of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges, formed by rankings of market value of equity outstanding at the end of the previous year, July 1962-December 1988 ($n = 1381$)

Portfolio	$\hat{\delta}_0 \times 10^3$	$\hat{\delta}_1$	$\hat{\beta}$	$\hat{\phi}$	$\hat{\theta}$	Skewness	Kurtosis	LB1(6)	LB2(6)
1	-0.859 (-1.385)	0.061 (17.278)	3.294 (3.524)	0.512 (19.454)	0.412 (14.690)	0.360	8.224	10.671 [0.099]	5.754 [0.452]
2	0.099 (0.182)	0.019 (11.078)	2.615 (2.997)	0.576 (11.567)	0.403 (8.158)	-0.126	3.620	11.772 [0.067]	4.843 [0.565]
3	2.200 (2.336)	0.004 (2.447)	5.019 (2.254)	-0.349 (-1.723)	0.283 (1.331)	0.133	4.505	2.590 [0.858]	7.362 [0.289]

The model is

$$R_{it} = \delta_{i0} + \delta_{i1}D_t + \beta_i R_{it-1} + \sum_{j=1}^3 \gamma_{ij} R_{jt-1} + \epsilon_{it} - \theta_i \epsilon_{it-1}, \quad i, j = 1, 2, 3, \quad \text{where } \epsilon_{it} | \Psi_{t-1} \sim MVN(0, H_t),$$

$$H_t = MM' + B'H_{t-1}B + C'\epsilon_{t-1}\epsilon'_{t-1}C + D'D_tD_t.$$

R_{1t} , R_{2t} , and R_{3t} are portfolio returns for week t of the 100 smallest, 100 intermediate, and 100 largest NYSE and AMEX stocks, respectively. $D_t = 1$, for the first week in January, and $D_t = 0$, for all other weeks. All estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t -statistics. The skewness and kurtosis coefficients are for the normalized residuals of the model. LB1(6) and LB2(6) are the Ljung-Box statistics, with six degrees of freedom, for the normalized residuals and the normalized squared residuals, respectively. The p values of the $\chi^2(6)$ statistics are reported in brackets below each statistic.

tage of the multivariate approach is that it estimates all parameters simultaneously, thus utilizing the information in the entire variance-covariance matrix of the errors of the three portfolios. This approach is particularly appealing when testing for volatility spillover effects. Unlike the univariate approach, volatility surprises to a specific portfolio are not regarded as exogenous determinants of the conditional variances of the other portfolios. Instead, all cross-effects between the errors of the three portfolios are determined simultaneously. This, in turn, leads to more powerful tests of the effects of aggregate shocks on different securities. Apart from leading to an increase in power, 'the multivariate parameterization is also a more appropriate specification (as compared to the univariate parameterization) to test for spillover effects *across* securities.

We estimate a modified version of the multivariate model in Equations (5) and (6) to take into account the turn-of-the-year effect in both the means and the variances⁷ and any remaining cross-correlations in the returns of the three portfolios. Estimates of the parameters of the mean equations for all three portfolios are presented in Table 7. For brevity, we do not report estimates of the lagged cross-effects—that is, the $\hat{\gamma}_H$'s. The multivariate model is well-specified: the Ljung-Box statistics for all three portfolios show a lack of serial correlation in both the normalized residuals and squared residuals of all three portfolios.⁸

The most interesting difference between the univariate and multivariate parameter estimates of the mean equations is that the GARCH-M effect (i.e., the effect of the conditional variance of a portfolio on its conditional mean) is larger and statistically significant in the multivariate results. These results are consistent with the evidence in French, Schwert, and Stambaugh (1987) who find a significantly positive relation between the monthly expected returns and conditional variances of S&P 500 and CRSP value-weighted indices. There are two reasons for the differences in the univariate and multivariate GARCH-M effects. First, recall that although expected returns of a particular portfolio are only a function of its own conditional variance even in the multivariate system, the conditional variance itself is a function of not only its own past realized variances, but also past realized variances of the other portfolios. Second, the mean equations are estimated jointly in the multivariate approach. Hence, utilization of the information in the variance-covariance matrix of the errors of

⁷ To account properly for the first-week-of-January effect, as in the univariate estimation, we include lagged dummy variables in the conditional variance equations.

⁸ The parameters of the multivariate model are estimated using the MGARCH program provided by Kenneth Kroner of the University of Arizona. This program uses the BHHH algorithm to maximize the likelihood function.

the three portfolios leads to different multivariate estimates of the GARCH-M effects.

We also use forecasts generated from estimates of the mean equations to determine whether they are conditionally unbiased and account for significant proportions of the variance of weekly realized returns: The forecasts from the multivariate model are similar in spirit to those estimated by Conrad and Kaul (1988), except that they are generated simultaneously and take account of the GARCH-M effect. The regressions of realized returns on the forecasts yield intercepts close to zero and slope coefficients close to unity for each portfolio. This suggests that the forecasts are unbiased. Moreover, the system- R^2 is 14.6 percent compared to a system- R^2 of 8.1 percent when the forecasts are generated, independently for each portfolio.⁹ Hence, simultaneous estimation of the portfolio return forecasts (with GARCH-M effects) appears to yield better expected return measures.

In Table 8, we present estimates of the partial cross-effects of lagged squared errors on the current conditional variances of the three portfolios. For brevity, we do not report estimates of the entire $\mathbf{M}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ matrices. However, to calculate the estimates (and the corresponding t -statistics) of the effects of lagged squared residuals of other portfolios on a particular portfolio's current conditional variance, we use the *unique* vector parameterization of the estimated representation of $\mathbf{H}_t$ in Equation (6). In estimating the vector representation of $\mathbf{H}_t$, we impose the cross-equation restrictions implied by the representation in Equation (6).

The format used in Table 8 is identical to the format of Tables 5 and 6. In particular, the (i, j) th element of each of the matrices measures the cross-effect of the squared shock of portfolio i at time $t-1$ on the conditional volatility of portfolio j at time t . The estimates in Table 8 confirm the univariate results of a distinct asymmetry in the 'effects of aggregate shocks at lag 1 across securities of different market value. The volatility of smaller firms can be predicted by shocks to larger firms, but not vice versa. As in the univariate estimates, the elements below the diagonal in Table 8 are consistently larger than elements above the diagonal. In fact, the effect of a (lag 1) volatility surprise to medium-sized firms (portfolio 2) on the conditional volatility of small firms (portfolio 1) is almost twice as large as the effect of the small firms' own lagged squared shocks, $\epsilon_{1,t-1}^2$, on $b_{1,t}$.

Although the spillover effects of medium firms on small firms dominate the spillover effects of the large firms on small firms, there is,

⁹ The system- R^2 's are measures of fit when all portfolio returns are combined and run as a single regression. However, the 8.1 percent estimate is obtained based on least-squares regression residuals, while 14.6 percent is based on seemingly unrelated regression residuals. The SAS SYSLIN procedure was used to obtain the system- R^2 . [For details, see Judge et al. (1980).]

Table 8

Weekly estimates of partial effects, using a multivariate ARMA(1,1)-GARCH(1,1)-M model, of lagged squared residuals on conditional variances of three value-weighted portfolios of the 100 smallest, 100 intermediate, and 100 largest stocks on the New York and American Stock Exchanges formed by rankings of market value of equity outstanding at the end of the previous year, July 1962-December 1988 ($n = 1381$)

Variable (x)	$\hat{b}_{1i}$	$\hat{b}_{2i}$	$\hat{b}_{3i}$
$\hat{\epsilon}_{i,t-1}^2$	0.08899 (9.723)	0.00088 (0.281)	0.00008 (0.029)
$\hat{\epsilon}_{j,t-1}^2$	0.15896 (12.801)	0.20052 (24.767)	0.00755 (1.788)
$\hat{\epsilon}_{k,t-1}^2$	0.02821 (1.846)	0.02587 (2.031)	0.08099 (11.954)

$\hat{b}_{ji}$ = estimated conditional variance of portfolio i in week t $\hat{\epsilon}_{j,t-1}^2$ = squared residuals of portfolio j at lag 1. The reported numbers (with t -statistics in parentheses) are calculated using the unique vector representation of the multivariate GARCH(1,1) model in Equation (6).

in turn, a statistically significant spillover effect from large to medium firms. The magnitude of this spillover effect is small because it reflects the cross-effect at lag 1 only; Estimates of the cross-effects beyond lag 1 (not reported) show that the squared unexpected returns of large firms at-lag 2 and beyond continue to have a statistically significant impact on the conditional volatility of medium firms, but not vice versa. The importance of the asymmetric cross-effects of aggregate shocks¹⁰ on large versus small firms is best revealed by the fact that shocks, at any lag, to smaller firms never have a statistically significant impact on larger firms. However, the effects of lagged shocks to larger firms on the current conditional variance of smaller firms are always statistically significant.¹¹

Finally, we reestimate the multivariate model without the lagged returns of portfolio j , $i \neq j$, in the mean equation of portfolio i . Estimates of the partial cross-effects are virtually identical to those reported in Table 8. Hence, the asymmetric response of conditional variances to information remains unaltered irrespective of whether we take into account the asymmetry in the predictability of mean returns.

To determine whether the evidence presented in the previous section is specific to a particular subperiod, we conduct all tests for two equal-length subperiods July 1962-September 1975 and October 1975-

¹⁰ These shocks are "aggregate" because we are dealing with portfolios of a large number of securities. Hence, the effects of idiosyncratic shocks to individual firms are likely to be diversified.

¹¹ Although the unconditional distribution corresponding to the GARCH model with conditionally normal errors is leptokurtic, Bollerslev (1987) argues that this model may not account sufficiently for the observed leptokurtosis in asset returns. Following Bollerslev (1987), we also estimate our multivariate model allowing ϵ_{it} to follow a fat-tailed t distribution. The asymmetric spillover effects remain unaltered even under this alternative specification.

December 1988. Subperiod analysis is also indicated because stock return data may not exhibit covariance stationarity over long periods [see Pagan and Schwert (1990)]. Although the absolute magnitudes of the asymmetric spillover effects are larger during the first subperiod, the asymmetry remains statistically significant even in the second subperiod. Detailed results are provided in Conrad, Gultekin, and Kaul (1990).¹²

3.4 Nonsynchronous trading and asymmetric crosseffects¹³

One possible source of the asymmetric volatility spillover-effects is nonsynchronous trading of securities on the stock market. Asynchronous prices of different securities can lead to spurious autocorrelation and lagged cross-correlation in portfolio returns [see, e.g., Fisher (1966), Scholes and Williams (1977), and Cohen et al. (1986)]. Although nonsynchronous trading has not been shown to generate similar spurious effects in squared returns, it remains a potential cause for the asymmetric predictability of conditional variances. To gauge the importance of nonsynchronous trading, we derive the magnitudes of spurious lagged cross-correlations between squared (unexpected) returns using the nontrading model of Lo and MacKinlay (1990b).

Suppose the unobservable "true" continuously compounded returns R_{it} of N securities are generated by the following model:

$$R_{it} = \alpha_i + \beta_i M_t + \epsilon_{it}, \quad i = 1, \dots, N, \quad (14)$$

where M_t is a zero-mean common factor and ϵ_{it} is zero-mean idiosyncratic noise that is temporally and cross-sectionally independent.

Given that all securities do not trade at each point in time, observed returns R_{it}^o will be a weighted average of past true returns,

$$R_{it}^o = \sum_{k=0}^{\infty} X_{it}(k) R_{it-k}, \quad i = 1, \dots, N, \quad (15)$$

where the random weights $X_{it}(k)$ are defined as products of no-trade indicators:

$$X_{it}(k) = (1 - \delta_{it})\delta_{it-1}\delta_{it-2} \cdots \delta_{it-k}$$

12 We also conduct all tests on the smallest, the intermediate, and the largest quintiles of NYSE/AMEX stocks. The results are similar to the evidence for the 100-firm portfolios reported in this paper, though the magnitudes of the asymmetric cross-effects are not as large. We also exclude the 1986-1988 period to assess the impact of the 1987 crash on our results. The with- and without-crash samples provide similar estimates of the cross-effects in conditional variances. Detailed results may be obtained from the authors.

13 The material in this section is based on the specific simulation model suggested by Craig MacKinlay. We are thankful to him for also providing us the simulation program used to obtain the results.

$$= \begin{cases} 1, & \text{with probability } (1 - p)p^t, \\ 0, & \text{with probability } 1 - (1 - p)p^t, \end{cases}$$

where the δ_{it} 's are independently and identically distributed Bernoulli random variables that take on the value 1 when security i does not trade at time t , and 0 otherwise. Details of this model are in Lo and MacKinlay (1990b).

We simulate the cross-effects in squared (unexpected) returns using the model in Equations (14) and (15). We consider 30 assets and three portfolios. The β_i 's of all assets are set equal to unity and $\text{Var}(M_t) = \text{Var}(\epsilon_{it})$ for all assets. We choose the nontrading probabilities of securities in the three portfolios as 0.60, 0.30, and 0.00, respectively. Hence, the three portfolios being simulated are comparable to the three size-based portfolios in our sample.

We simulate 5000 daily returns for each asset using Equation (14) and also a nontrading history for each asset using a uniform distribution and the nontrading probabilities mentioned above. Based on the simulated returns and the nontrading histories, we obtain a series of observed returns R_{it} . We form geometrically weighted portfolio returns and aggregate the 5000 daily returns into 1000 five-period (weekly) returns.

For each portfolio $j = 1, 2, 3$, the residuals are obtained from the following regression:

$$R_{jt}^o = \gamma_{0j} + \gamma_{1j}R_{1,t-1}^o + \gamma_{2j}R_{2,t-1}^o + \gamma_{3j}R_{3,t-1}^o + u_{jt}^o, \quad (16)$$

where γ_{0j} is an intercept and γ_{1j} , γ_{2j} , and γ_{3j} are the coefficients of the lagged returns of portfolios 1, 2, and 3, respectively. Hence, Equation (14) is similar to the model for the mean equation used in this paper: the lagged returns of all three portfolios are included as regressors.

Table 9 shows the average lagged cross-correlations of the squared residuals of Equation (16), that is, $\rho_{jk} = \text{Corr}(\hat{u}_{j,t-1}^2, \hat{u}_{k,t-1}^2)$, $j = 1, 2, 3$, $k = 1, 2, 3$. The cross-correlations are calculated under the assumptions of a homoskedastic (panel A) and a heteroskedastic (panel B) distribution for M_t . Estimates in each of the panels are based on 1000 replications and the format for reporting the numbers is identical to the one used in Table 5.

Table 9 shows that nontrading is an unlikely source of the asymmetric volatility spillover effects documented in this paper. For the homoskedastic specification of M_t , all cross-correlations are close to zero. However, for our purposes, the more realistic numbers may be in panel B. Here again, however, there is virtually no asymmetry in the cross-correlations. In fact, unlike the evidence in Tables 5 and 8, none of the estimates of cross-effects are statistically distinguishable from zero.

Table 9

Simulated weekly lagged cross-correlations with nontrading between the squared residuals of the observed returns of three portfolios R_{jt}^2 , $j = 1, 2, 3$, regressed on the lagged returns of all three portfolios

Variable (x)	A: $M_t \sim \text{iid } N(0, \sigma^2)$		
	$\hat{u}_{1t}^2$	$\hat{u}_{2t}^2$	$\hat{u}_{3t}^2$
$\hat{u}_{1,t-1}^2$	-0.002 (0.031)	0.000 (0.032)	-0.002 (0.032)
$\hat{u}_{2,t-1}^2$	-0.001 (0.032)	-0.001 (0.032)	0.002 (0.032)
$\hat{u}_{3,t-1}^2$	-0.001 (0.032)	0.000 (0.033)	-0.003 (0.032)
	B: $M_t \sim N(0, \sigma_t^2)$, $\sigma_t^2 = 0.10 + 0.90 M_{t-1}^2$		
	$\hat{u}_{1t}^2$	$\hat{u}_{2t}^2$	$\hat{u}_{3t}^2$
$\hat{u}_{1,t-1}^2$	0.108 (0.104)	0.111 (0.113)	0.095 (0.098)
$\hat{u}_{2,t-1}^2$	0.109 (0.109)	0.108 (0.109)	0.095 (0.101)
$\hat{u}_{3,t-1}^2$	0.131 (0.120)	0.133 (0.125)	0.115 (0.109)

The model is

$$R_{jt}^2 = \gamma_{0j} + \gamma_{1j} R_{1,t-1}^2 + \gamma_{2j} R_{2,t-1}^2 + \gamma_{3j} R_{3,t-1}^2 + u_{jt}^2 \quad j = 1, 2, 3,$$

where γ_{0j} is an intercept and γ_{1j} , γ_{2j} , and γ_{3j} are the coefficients of the lagged returns of portfolios 1, 2, and 3, respectively. The individual security returns in each portfolio are $R_{jt}^2 = \sum_{k=1}^K X_{jk}(k) R_{k,t-1}$, and the "true" returns R_{jt} are generated from the following stochastic model:

$$R_{jt} = \alpha_j + \beta_j M_t + \epsilon_{jt}$$

where M_t is the zero-mean common factor and ϵ_{jt} is zero-mean idiosyncratic noise that is both temporally and cross-sectionally independent. In panel A, the variance of M_t is assumed to be homoskedastic, while in panel B it follows a time-varying process. The numbers in the table are averages of the cross-correlations obtained from 1000 replications. The simulated standard errors are in parentheses.

4. Summary and Conclusions

In this article, we show that there is a distinct asymmetry in 'the predictability of the volatilities of large versus small firms. Shocks to larger firms are important to the future dynamics of their own returns as well as the returns of smaller firms. Conversely, however, shocks to smaller firms have no impact on the behavior of both the conditional mean and variance of the returns of larger firms. Since the volatility of returns is directly related to the rate of flow of information [Ross (1989)], one possible explanation for this evidence is that aggregate information first affects large firms and is then impounded with a lag in the prices of small capitalization companies [see, e.g., Lo and MacKinlay (1990a) and Mech (1990)]. However, though consistent with the evidence, the asymmetric predictability of the mean and

volatility of stock returns does not necessarily imply an asymmetry in the timing of the effects of shocks on large versus small firm.¹⁴ The asymmetric volatility spillover effects are also consistent, for example, with an economy in which the conditional volatilities of both large and small stocks are driven by the same factor(s), without any differences in the timing of the effects of these factors. It may simply be the case that the factors are more closely associated with the squared return shocks to larger firms.

Clearly, further research needs to investigate the source(s) of the rich autocorrelation and cross-correlation patterns observed in stock price data. However, the findings of our paper do suggest that 'any model of time-varying expected returns and volatilities needs to take into account the asymmetric predictability of the mean and variance of different capitalization companies.

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