

Insiders, Outsiders, and Market Breakdowns

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A simple classical Walrasian framework is proposed for the study of manipulation among asymmetrically informed risk-averse traders in financial markets, and it is used to analyze the occurrence of a market breakdown in the trading system. Such a phenomenon occurs when the outsiders refuse to trade with the insiders because the informational motive for trade of the insider outweighs her hedging motive. We demonstrate the robustness of our results by proving that the market collapse condition extends not only to the linear strategy action, but to the whole class of feasible nonlinear strategy functions. Implications for insider-trading regulation are sketched.

This article finds in closed form the entire set of noisy rational expectations equilibria for a model of an exchange economy characterized by asymmetric information and strategic behavior. The model has the following features. There is an informed risk-averse monopolist and a continuum of competitive risk-averse uninformed traders. The basic structure of the model includes strategic behavior by the informed trader, a Walrasian price formation mechanism in

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which traders submit demand functions, an endogenous motive for trade due to the random endowment of the insider, and rational expectations by the uninformed traders. Although subsets of these features have been included in various models in the literature, the current article is the first to have all of them. Moreover, it is the first to investigate the entire set of continuous equilibria, including nonlinear ones, within a noisy rational expectations Walrasian environment involving only risk-averse individuals.

The primary focus of the article is to examine the conditions that lead to a collapse of trade in a financial market. Glosten and Milgrom (1985), Glosten (1989), and Leach and Madhavan (1989) found that a risk-neutral individual will not voluntarily act as a market-maker if he is at a severe informational disadvantage relative to some other traders. In this article, we extend their result to the case of the risk-averse individual trading in a Walrasian market. It is shown that when this trader is "too uninformed," he will not trade with the more informed; the linear equilibrium fails, as do all of the continuous nonlinear equilibria.¹

The introduction of risk-averse agents with nonzero expected aggregate endowments has three advantages. First, both conditions are realistic, in that aggregate security holdings are strictly positive and investors do worry about the volatility of their positions. Second, the conditions ensure that there is a hedging as well as an informational motive for trade.² Third, these assumptions enable us to focus on the risk premia commanded by the holders of the risky security, and to study how it varies with the insider's position. Noisy rational expectations models with risk-neutral agents cannot be used to examine this issue, but models such as Grossman and Stiglitz (1980), Hellwig (1980), Bray (1981), Admati (1985), and Kyle (1989), which employ only risk-averse agents in a Walrasian environment, are capable of such an analysis. The primary difference between the present article and this literature is that none of the above models examine risk premia outside the linear equilibria, while the present analysis is extended to the full set of equilibria.

The model presented in this article combines the assumptions of the perfect and imperfect competition frameworks employed in the

¹The article of Genotte and Leland (1990) is related to our article, in that they examine nonlinear equilibria. However, they only solve for and analyze a single nonlinear equilibrium. The reason for this specificity is that there exist some traders in their model who use exogenously defined hedging strategies (like portfolio insurance), which are nonlinear functions of the equilibrium price. The nonlinear equilibrium in their article is therefore analogous to the linear equilibrium in most noisy rational expectations models, in that it is the only pricing rule considered.

²The inclusion of risk-neutral agents destroys the risk-sharing concept employed in the classical portfolio models. Further, evidence from Loderer and Sheehan's (1989) study indicates that insiders do a great deal of trading that is not informationally motivated.

noisy rational expectations literature. In the models of perfect competition, individuals believe they can trade any amount without altering the security's price. These studies include Grossman (1977), Grossman and Stiglitz (1980), Verrechia (1982), Glosten and Milgrom (1985), Allen (1987), and Ausubel (1990). In contrast, models of imperfect competition employ traders who believe their transactions influence prices, as illustrated by Kyle (1985, 1989) and Caballe (1989). The present article, like the models of Grinblatt and Ross (1985) and Laffont and Maskin (1990), falls in between, with a large monopolistic Insider and competitive outsiders. What differentiates our article from the last two is that all of our traders are risk-averse. In this respect, the model is a partially revealing version of Kihlstrom and Postlewaite's (1983) fully revealing model of a monopolist and a competitor.

With some exceptions, the above cited articles prevent prices from becoming fully revealing through the use of noise traders or, equivalently, an auctioneer who sets the aggregate demand to a random number.³ In contrast, in the models of Bray (1981), Ausubel (1990), Gale and Hellwig (1988), Glosten (1989), and Laffont and Maskin (1990), prices are not fully revealing because the insider has an informational advantage in two dimensions. In the latter specification, it is possible to form equilibria by having each individual 'remain at his initial endowment. The use of noise traders, or an auctioneer who sets demand to a random number, however, prevents this. Under either assumption, trade must always occur; either trade is necessary to balance out the demands of the noise traders, or it is required to meet the auctioneer's quantity constraint. As the primary focus of this article is to obtain and analyze the no-trade equilibrium, we have opted for the latter modeling technique.

The organization of the article is as follows. Our model is presented in Section 1. In Section 2, we solve for the equilibrium and present its general characteristics; it is in this section that the market breakdown condition is presented. In Section 3, the market risk premium and the transmission of information are considered. In Section 4, we investigate the special case of the linear equilibrium and detail its properties. We conclude in Section 5.

1. Model

The model analyzes a two-date exchange economy. At date 0, endowments are distributed, players submit demand functions, and the Walrasian auctioneer then finds a price-quantity pair to equate supply

³Gale and Hellwig (1988) discuss the different roles of "noise traders," and find that their diverse roles make the results very difficult to interpret.

with demand.⁴ There are no restrictions on the demand schedule an individual must submit, except that it must be a continuous function. If the auctioneer cannot find a price that equates supply and demand, the market collapses and no trade occurs. Whenever two or more market-clearing price-quantity pairs exist, the auctioneer selects one at random and then allocates quantities to satisfy everyone's demand at the designated price. At date 1, all uncertainty is resolved, and the players receive their final payoffs and engage in consumption.

The economy contains two sets of agents (M and C), whose individual members have exponential utility with risk-aversion parameter θ . The set M consists of a single monopolistic informed investor who we take to be a woman, while C contains a continuum of small traders who we refer to as men.

The investors trade in two securities, both of which pay off in the economy's single consumption good. The first security is a riskless bond, the price of which at dates 0 and 1 is set equal to 1. This restriction is simply a normalization, since all consumption takes place on date 1. The second security is a risky stock, whose date-0 price is P (determined by market-clearing), and whose date-1 value (P_1) is generated by three additive factors: $P_1 = \mu_p + \varepsilon + \eta$. Throughout time, both securities have their supplies normalized to unity.

At date 0, prior to trade, the informed investor knows the values of μ_p and ε and has a prior belief that η is normally distributed with mean zero and variance σ_η^2 . In addition, she is also endowed with $1 - S_0$ shares of the stock, $1 - B_0$ shares of the bond, and w shares of a nontraded asset. The nontraded endowment can be interpreted as human capital or any other illiquid asset. While our model takes the expected endowment of this asset to be zero, none of the basic results are altered if a positive mean is assumed. The payoff of this nontraded endowment is correlated with the return of the risky stock; it, therefore, needs to be "hedged." For simplicity, perfect correlation is assumed. Thus, at date 1, she earns an amount wP_1 from the nontraded endowment.⁵

At date 0, the uninformed traders know μ_p and correctly believe that ε , η , and w are independently normally distributed with zero means and variances σ_ε^2 , σ_η^2 , and σ_w^2 , respectively. The uninformed traders are modeled as a continuum of individuals whose set has a finite measure. More precisely, if we let $f(i)$ represent the density of traders of "type" i , then it is assumed that

⁴The market mechanism employed here is similar to Wilson's (1979) auction of shares.

⁵The assumption of perfect correlation between the payoffs of the nontraded asset and the risky asset makes our model equivalent to models where noise is introduced by making the insider's endowment of the risky asset at date 1 unknown to everyone but her.

$$\int_{-\infty}^{\infty} f(i) \, di = n.$$

Let $S_0(i)$ and $B_0(i)$ represent piecewise continuous functions giving the initial stock and bond endowments of uninformed traders. Then their total endowments are

$$\int_{-\infty}^{\infty} S_0(i) f(i) \, di = S_0$$

and

$$\int_{-\infty}^{\infty} B_0(i) f(i) \, di = B_0.$$

The functions $S_0(i)$ and $B_0(i)$ are common knowledge in the economy.

The functions $S(i)$ and $B(i)$ are the respective demand densities for the stock and bond by type i uninformed traders. The aggregate demand is represented by the variables S and B . Thus,

$$\int_{-\infty}^{\infty} S(i) f(i) \, di = S$$

and

$$\int_{-\infty}^{\infty} B(i) f(i) \, di = B.$$

Notice that the informed has an informational advantage because she knows her endowment (w) and two of the factors generating the return of the risky asset (ε and μ_p); whereas the uninformed only know μ_p . Hence, σ_i^2 is a measure of the small investor's "informational disadvantage." A similar metric for "informational disadvantage" appears throughout most of the noisy rational expectations literature.

2. Characterization of Equilibrium

The Bayesian-Nash equilibrium must satisfy two conditions. First, each of the uninformed investors must submit a demand schedule that maximizes his expected utility, subject to his budget constraint and the available information, including P . Second, knowing this demand schedule, the informed investor has to submit a demand function so as to maximize her utility subject to her wealth and informational endowments, which are exogenous. The problem is considerably simplified by noting that there is just one non-price-taker; and that supplies add to unity. Therefore, one can reduce the insider's

problem to that of picking a price P on the aggregate demand schedule of the uninformed traders, $S(P)$.

The next fact is used extensively throughout the article.

Fact 1. If X_1 and X_2 have a bivariate normal distribution, where μ_1 , μ_2 , σ_1 , σ_2 , and ρ are the unconditional means, standard deviations, and correlation of the two random variables, then the conditional distribution of X_1 given that $X_2 = x_2$ is a normal distribution whose mean is $E(X_1 | x_2) = \mu_1 + \rho\sigma_1(x_2 - \mu_2)/\sigma_2$, and variance is $Var(X_1 | x_2) = (1 - \rho^2)\sigma_1^2$.

It is now possible to analyze the problem of the informed monopolist.

2.1 Problem of the informed monopolist

Using the well-known properties of exponential utility functions and normal distributions, the monopolist wishes to maximize over P and B the following value function:

$$\max_{P,B} V = (1 - S + w)(\mu_p + \epsilon) + (1 - B) - 0.5\theta(1 - S + w)^2\sigma_\epsilon^2 \quad (1)$$

subject to her budget constraint

$$[(1 - S) - (1 - S_0)]P + [(1 - B) - (1 - B_0)] = 0. \quad (2)$$

Since the insider is effectively selecting a price on the aggregate demand function generated by the uninformed players, $S(P)$, her optimization problem becomes very simple. Using (2), one eliminates B from (1), and then derives the first-order condition for a maximum of (1) with respect to P as

$$S'(P - \mu_p - \epsilon) + (S - S_0) + \theta\sigma_\epsilon^2 S'(1 - S + w) = 0. \quad (3)$$

A more convenient and informative representation of the above equation is

$$P = \mu_p + \alpha(P) + \tau, \quad (4)$$

where

$$\alpha(P) = (S_0 - S)/S' - \theta\sigma_\epsilon^2(1 - S)$$

and

$$\tau = \epsilon - \theta\sigma_\epsilon^2 w. \quad (5)$$

Equation (4) brings into sharp focus a number of insights devel-

oped in the noisy rational expectations literature. First, given P , the variable τ is fully revealed. This tells us that although there may be a nonlinear term in the price, the price is still a linear function of the normally distributed Information variable of the insider, τ , and it reveals this. Importantly, this revelation would not occur if the aggregate demand function submitted by the outsiders was a correspondence. In the Appendix, we prove that it is not optimal for the outsiders to submit such a correspondence. Second, even though the outsiders know τ , they cannot fully identify ε , since w also enters the equation. The general public is therefore left uncertain as to whether the primary motivation for trading by the informed is "hedging" or "informational." Third, if ε is the only variable 'that the informed has an informational advantage in' ($\sigma_\varepsilon^2 = 0$), the market clearing price P fully reveals ε . Fourth, although the uninformed cannot disentangle ε , they can learn something about P_i from the offer price P . We now proceed to analyze how this learning takes place.

2.2 Problem of the uninformed competitors

Equation (5) tells us that τ is a linear function of two normally distributed random variables, implying that it is also a normally distributed random variable. Simple calculations can then be used to show that $E(\tau) = 0$ and $\text{Var}(\tau) = \sigma_\tau^2 = \sigma_\varepsilon^2 + \theta^2 \sigma_\omega^2 \sigma_\varepsilon^2$. As $P_i = \mu_P + \varepsilon + \eta$ by construction, one also knows that P_i is a normally distributed random variable with $E(P_i) = \mu_P$ and $\text{Var}(P_i) = \sigma_\varepsilon^2 + \sigma_\eta^2$.

Further, the correlation between τ and P_i is

$$\rho(P_i, \tau) = \sigma_\varepsilon^2 / [\sigma_\varepsilon^2 (\sigma_\varepsilon^2 + \sigma_\eta^2)]^{1/2}.$$

Hence, by observing the equilibrium price P , and thereby τ , the uninformed update their priors on P_i . The posterior distribution of P_i given P , using fact 1, is now normally distributed with

$$\begin{aligned} E(P_i | P) &= E(P_i | \tau) \\ &= \mu_P + \sigma_\tau^2 [\tau - 0] / \sigma_\tau^2 = \mu_P + \sigma_\tau^2 [P - (\mu_P + a(P))] / \sigma_\tau^2 \end{aligned} \quad (6)$$

and

$$\text{Var}(P_i | P) = \text{Var}(P_i | \tau) = (\sigma_\varepsilon^2 + \sigma_\eta^2) - \sigma_\tau^4 / \sigma_\tau^2. \quad (7)$$

Equations (6) and (7) make precise the "learning procedure" of the uninformed. Equation (6) gives them the posterior mean of P_i as a function of P , while Equation (7) is used to update the variance. Notice that the posterior precision on P_i is higher than the prior precision, and this improvement does not depend on P (assuming P exists).

The above arguments show that the final period 1 wealth of the i th

uninformed trader, $S(i)P_i + B(i)$, is normally distributed. Therefore, one can use the assumption that their utility functions are negative exponential and rewrite their objective in a mean-variance framework as

$$\max_{B(i), S(i)} V = S(i)E(P_i | P) + B(i) - 0.5\theta[S(i)]^2 \text{Var}(P_i | P), \quad (8)$$

subject to the budget constraint'

$$[S(i) - S_0(i)]P + [B(i) - B_0(i)] = 0. \quad (9)$$

The first-order condition of (8), subject to (9), is then used to determine the individual demand schedule of each uninformed investor. The demand, it should be noted, is the same for all the uninformed.

2.3 Feasible aggregate demand curves

Aggregating all the individual demands we 'get

$$S = \int_{-\infty}^{\infty} S(i) f(i) di = \frac{n[E(P_i | P) - P]}{\theta \text{Var}(P_i | P)}. \quad (10)$$

Substituting for $E(P_i | P)$ from (6) and for $\text{Var}(P_i | P)$ from (7) in (10), we obtain a differential equation in $S(P)$:

$$K_1 + K_2 S + K_3 S' P + K_4 S' S + K_5 S' = 0, \quad (11)$$

where

$$K_1 = S_0, \quad K_2 = -1,$$

$$K_3 = \frac{\sigma_r^2 - \sigma_i^2}{\sigma_i^2} = \frac{\theta^2 \sigma_i^4 \sigma_w^2}{\sigma_i^2},$$

$$K_4 = \theta \sigma_i^2 \left\{ 1 + \frac{1}{n} \left[1 + K_3 + K_3 \frac{\sigma_i^2}{\sigma_i^2} \right] \right\} \geq \theta \sigma_i^2,$$

$$K_5 = -(\mu_p K_3 + \theta \sigma_i^2).$$

The solution of (11) gives us all the possible aggregate demand curves that satisfy the first-order condition of the informed investor. Her second-order condition [obtained by differentiating (3) with respect to P] further restricts this set, and provides the complete set of feasible aggregate demand curves. In order to find the set of equilibria, we analyze the problem in three steps. First, Proposition 1 establishes the set of solutions satisfying Equation (11). Second, Proposition 2 uses the second-order condition to find restrictions that any

solution to the problem must satisfy. Third, Theorem 1 uses Propositions 1 and 2 to determine the set of feasible equilibrium aggregate demand functions of the uninformed.

Proposition 1. *There exists a complex number C such that the set of feasible aggregate demand curves that satisfy Equation (11) can be represented by*

$$(K_3 + K_2)(K_3P + K_5) - K_4K_1 + K_3K_4S[(K_1 + K_2S)^{K_3/K_2}] = C,$$

for any initial condition (S^*, P^*) . In the present application, one can simplify the above equation by substituting out K_1 and K_2 to obtain

$$[(K_3 - 1)(K_3P + K_5) - K_4S_0 + K_3K_4S](S_0 - S)^{-K_3} = C. \quad (12)$$

Proof. Differentiate (12) with respect to P to get back (11). For a detailed exposition of the “method of integrating factors” that was utilized to solve (11), refer to the Appendix. Q.E.D.

While there always exists a curve passing through any (S, P) pair, not all of them satisfy every equilibrium condition imposed by the economics of the problem. As the next proposition shows, the insider’s second-order conditions impose the intuitive restriction that an equilibrium aggregate demand curve cannot have any upward sloping region.

Proposition 2. *In equilibrium, it is necessary and sufficient for an aggregate demand curve to have a sufficiently negative slope for all S , and satisfy (12). The exact requirement is $dP/dS \leq -\theta\pi^{-1} \times \text{Var}(P_1 | P) < 0$ for $S \in (-\infty, \infty)$ in equilibrium.*

Proof: See the Appendix.

Using Propositions 1 and 2, It is now possible to find a general characterization of the set of equilibrium demand curves.

Theorem 1. *Any curve satisfying (12) is an equilibrium demand curve if and only if it passes through a point (S^*, P^*) , where the following conditions are met. If $S^* < S_0$, then*

$$K_3P^* + K_4S^* + K_5 > 0 \quad (13)$$

and

$$(K_3 + K_2)(K_3P^* + K_5) - K_4K_1 + K_3K_4S^* \geq 0. \quad (14)$$

Conversely, if $S^* > S_0$, then

$$K_3 P^* + K_4 S^* + K_5 < 0 \quad (15)$$

and

$$(K_3 + K_2)(K_3 P^* + K_5) - K_4 K_1 + K_3 K_4 S^* \leq 0. \quad (16)$$

Proof. See the Appendix.

Graphically, the conditions can be related to Figure 1. The complete set of feasible aggregate demand curves lies in the shaded region of Figure 1. Equality (13) is line 2, whereas equality (14) is line 1. Note that the shaded region is bounded by the familiar linear demand curve (line 1) and the no-trade demand curve (quantity demanded = S_0 = aggregate endowment of outsiders), and all the feasible nonlinear demand curves lie in between.

2.4 Equilibria

Because an aggregate demand curve must lie entirely within the shaded region of Figure 1, the set of possible equilibria, for a given τ , turns out to be closed, compact, and located along a line segment. The characterization is contained in the next theorem.

Theorem 2. For a given value of τ , the set of all equilibrium pairs (S, P) lies in a bounded segment of the line

$$P = \mu_p + \frac{\tau}{1 + K_3} - S \frac{\theta}{n} \left[\sigma_i^2 + \frac{K_3}{1 + K_3} \sigma_i^2 \right] \quad (17)$$

The bounds are $S \in (A_1, A_2]$, where

$$A_1 = S_0 \quad (18)$$

and

$$\begin{aligned} A_2 = & \{ \theta S_0 \sigma_i^2 (1 + K_3) [(1 + K_3) + n + (n(K_3 - 1)/S_0)] \\ & + \theta S_0 \sigma_i^2 K_3 (1 + K_3) - \tau n (K_3 - 1) K_3 \} \\ & \times \{ \theta K_3 [(2 + n)(1 + K_3) + 2K_3 \sigma_i^2] \}^{-1}. \end{aligned} \quad (19)$$

Graphically, the set of all equilibrium pairs is depicted by line 3 in Figure 2. If the insider buys the risky stock, the top line 3 represents this set; if the insider sells the risky stock, the bottom line 3 represents this set. Note that the equilibrium price-quantity pair chosen (C), if a nonlinear demand curve is played, lies between the no-trade price-quantity pair (A_1) and the price-quantity pair that occurs if the linear demand curve is played (A_2).

Proof. Given the aggregate demand curve $S(P)$, the informed investor chooses a price P . Theorem 1 derives the set of feasible aggregate

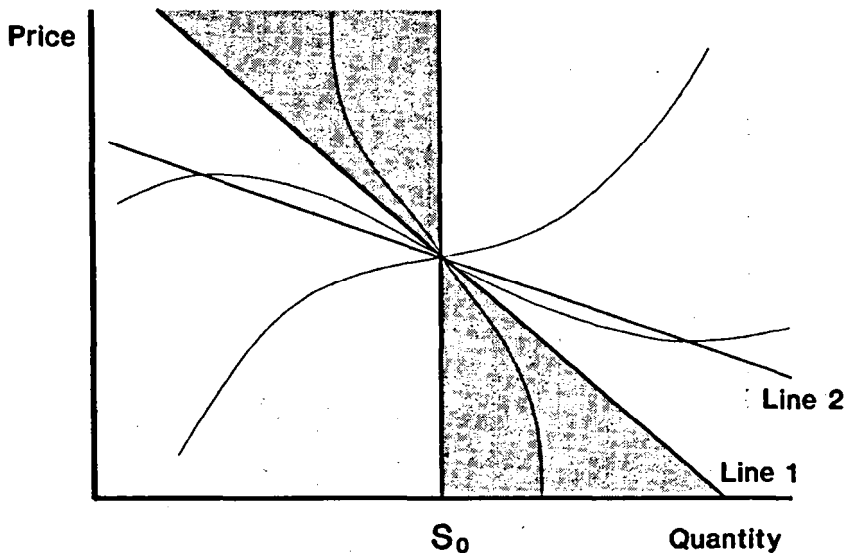


Figure 1
Multiple equilibrium demand curves

The aggregate quantities of shares of the risky stock demanded by the outsiders as a function of the market-clearing price. The multiple equilibrium demand curves lie in the shaded region. Note that the shaded region is bounded by the familiar linear demand curve (line 1) and the no-trade demand curve (quantity demanded = S_0 = aggregate endowment of outsiders), and all the feasible nonlinear demand curves lie in between. The curved lines represent nonlinear solution to (12).

demand curves, which can then be substituted into (4) to give us (17), the offer price P of the informed. Now, equilibrium price-quantity pairs will lie on the points of intersection of the price rule of the informed (17)—given by line 3 in Figure 2—and the feasible aggregate demand curves (12). This means that they will lie between A_1 and A_2 . At A_1 , $S = S_0$. The point A, is the intersection of (17), line 3, and the linear demand curve (20), line 1 (given in Figures 1 and 2). Solving (20) and (17) simultaneously, we obtain (19). Q.E.D.

Theorem 2 determines whether the insider will purchase or sell securities. When $A_2 > S_0$, the insider is a 'net seller, while a value of $A_2 < S_0$ implies that the insider is a net buyer of stock. Since the aggregate supply of the stock is fixed, the uninformed are necessarily forced to take the opposite position.

Many researchers tacitly assume that nonlinear demand functions are implausible under the mean-variance framework with Gaussian assumptions, but Theorem 1 shows that this is not necessarily so. This may appear to be discouraging news since Gale and Hellwig (1988) show that the study of markets as a communication system

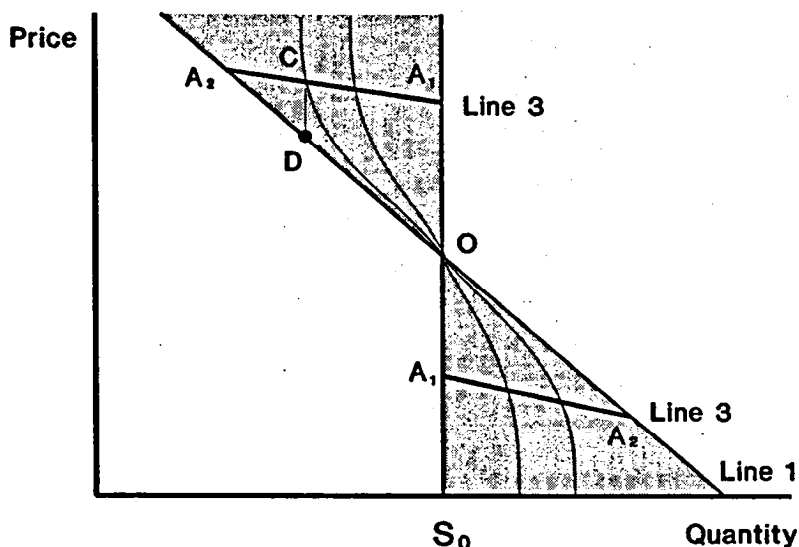


Figure 2

Set of all possible equilibria for a given value of τ

The set of all possible equilibrium price-quantity pairs of the risky stock that can result for a given value of τ . If the insider buys the risky stock, the top line 3 represents this set; if the insider sells the risky stock, the bottom line 3 represents this set. Note that the equilibrium price-quantity pair chosen (C), if a nonlinear demand curve is played, lies between the no-trade price-quantity pair chosen (A_1) and the price-quantity pair that occurs if the linear demand curve is played (A_2).

can depend upon which equilibrium is being considered.⁶ However, the equilibria in this model have some aspects in common. Further, in Section 4, our model shows that if the informed player can choose the equilibrium, she will select the linear one. This helps to refine the analysis further.

2.5 Common properties across equilibria

One feature that is constant across equilibria is the critical value of the “informational disadvantage” at which trade between the informed and uninformed ceases. An examination of Figure 1 shows why this happens. All the equilibria lie in the shaded region, and this region disappears if line 1 rotates far enough so that it has a positive slope. The next proposition provides the exact conditions under which these events occur.

Proposition 3. *If $\theta^2 \sigma_1^2 \sigma_w^2 < \sigma_2^2$ (i.e., $K_3 + K_2 < 0$), then the only equilibrium demand curve is the line $S = S_0$.*

⁶ Kyle’s (1989) paper only examines the equilibria where traders submit symmetric linear demand functions. It is unclear what occurs in the other equilibria.

Proof. Consider the set of points to the left of S_0 . If $K_3 + K_2 < 0$, then there does not exist a point (S^*, P^*) with $S^* < S_0$ such that

$$K_3P^* + K_4S^* + K_5 > 0$$

and

$$(K_3 + K_2)(K_3P^* + K_5) - K_4K_1 + K_3K_4S^* \geq 0.$$

From the proof of Theorem 1, this implies that there does not exist a curve that satisfies (12) and satisfies the second-order equilibrium conditions. To prove the proposition for points to the right of S_0 , simply reverse the sign on the above arguments. Q.E.D.

Proposition 3 puts an upper bound on the “informational disadvantage” the uninformed will tolerate when trading with an insider. If it is crossed, trade between the informed and the uninformed does not ensue.

Throughout this article, we define a market breakdown as occurring whenever $S = S_0$ is the only equilibrium aggregate demand curve. There are two reasons for employing this definition. First, this demand curve implies that the informed cannot trade with the uninformed. Second, the equilibrium strategies that players must use under the aggregate demand curve $S = S_0$ are not very stable. When the aggregate demand curve is vertical, the insider’s holdings become independent of I ? As a result, her utility is independent of her actions and she has no incentive to submit any particular demand function. Maintenance of the equilibrium, however, requires the insider to set P using a particular equilibrium rule, despite the fact that she has no particular incentive to do so. Admittedly, in equilibrium, she does not have an incentive to do otherwise either. However, given the precarious nature of the equilibrium, using Proposition 3 to conclude that there is only a suspension of trade between the informed and the uninformed seems rather conservative. Rather, the instability leads to the more reasonable conclusion that whenever $\theta^2\sigma_1^2\sigma_2^2 < \sigma_1^2$, a complete market collapse results, in the sense that there will not exist a market-clearing price.

A possible interpretation of Proposition 3 is that it provides a condition designating the minimum amount of information the uninformed have to gather before trading with the informed. If they do not have this minimum amount of information, they do not have the “confidence to trade” in the market.

So far, the discussion has concentrated upon regularities relating to the market’s collapse. However, there also exist equilibrium invariant properties when trading takes place. One such attribute is that

the outsider's conditional distribution of P_1 , given P , does not vary across equilibria.

Proposition 4. *The conditional variance and expectation of P_1 , given P , is independent of the equilibrium demand schedule.*

Proof: From (7), $\text{Var}(P_1 | P) = (\sigma_1^2 + \sigma_2^2) - (\sigma_1^4/\sigma_2^2)$. Equations (4) and (6) show that $E(P_1 | P) = \mu_P + \tau(\sigma_2^2/\sigma_2^2)$. Q.E.D.

This proposition implies that neither the "informational content" nor the "bias" of prices depends on the aggregate demand function under consideration. Intuitively, this independence property arises from the fact that the signal to the outsiders (τ) is independent of the equilibrium under consideration. Thus, irrespective of the equilibrium, the outsiders always face the same residual risk and, therefore, demand the same risk premium.

Theorem 2 has already shown that trade between the informed and the uninformed is greatest for the linear demand function (line 1 in Figure 2). Therefore, it is somewhat surprising that trade among the uninformed is independent of the equilibrium being considered.

Proposition 5. *Trade among the uninformed is independent of the equilibrium being considered.*

Proof: Total volume of trade between the uninformed is

$$\int_{-\infty}^{\infty} |[S(t) - S_0(t)]| f(t) dt,$$

which equals

$$\left| \frac{n[E(P_1 | P) - P]}{\theta \text{Var}(P_1 | P)} \right| - S_0.$$

As the conditional distributions are independent of the equilibrium being considered (Proposition 4), so is the total volume of trade among the uninformed. Q.E.D.

One way to understand the proposition is to think of trade among the uninformed separately from trade between the informed and the uninformed. Given the amount held by the insider, the outsiders must now allocate the remaining shares among themselves at the market-clearing price. Since the conditional variance of the stock is independent of the equilibrium under consideration, so are the optimal holdings of each outside agent. As a result, the total volume of trade among the outsiders must also be independent of the equilibrium.

A final result that is common across equilibria is that the informed does not behave competitively, even if she is very small relative to the rest of the market. The next proposition addresses the issue.

Proposition 6. *Holding all else constant, consider the case where the measure of uninformed traders continues to grow without bound ($n \rightarrow \infty$). In the limit, the equilibrium price does not approach the competitive rational expectations equilibrium price.*

Proof. In a competitive rational expectations equilibrium, where everyone is a price-taker, the demand of the informed is

$$(1 - S) = (\mu_P + \epsilon - P)/\theta\sigma_i^2 - w.$$

So $P = a(P) + \tau$, where $a(P) = -\theta\sigma_i^2(1 - S)$ and $\tau = \epsilon - \theta\sigma_i^2w$. Since the problem of the uninformed is basically unchanged, one can again use Equation (10) to calculate the aggregate demand in which

$$E(P_1 | P) = \mu_P + \sigma_i^2[P - (\mu_P + a(P))]/\sigma_i^2$$

and

$$\text{Var}(P_1 | P) = (\sigma_i^2 + \sigma_i^2) - \sigma_i^4/\sigma_i^2.$$

After substituting in the above relationships, one finds that the feasible aggregate demand curve of the uninformed in the competitive rational expectations equilibrium is

$$P = \mu_P - S(\theta/n) (\sigma_i^2 + \sigma_i^2)\sigma_i^2 - \sigma_i^2(n\sigma_i^2 + \sigma_i^2).$$

Notice that, unlike the imperfect competition case, the only feasible demand function of the uninformed is the linear one. As the demands of both add to unity, we can solve for the equilibrium price. Then, as $n \rightarrow \infty$, one obtains this *equilibrium price given* τ to be

$$\lim_{n \rightarrow \infty} P = \mu_P + \frac{(\tau - \theta\sigma_i^2)\sigma_i^2}{\sigma_i^2 - 1}.$$

Now consider the imperfect competition model of this article. From (17), the equilibrium price given τ , as $n \rightarrow \infty$, is

$$P = \mu_P + \frac{\tau}{1 + K_3} = \mu_P + \frac{\tau \sigma_i^2}{\theta^2 \sigma_i^2 \sigma_w^2 + \sigma_i^2}.$$

This is not the same as above.

Q.E.D.

This proposition reinforces the conclusion Gale and Hellwig (1988) draw from their model: the competitive rational expectations model is not invariably the right model even when the informed trader is

small relative to the market.⁷ Intuitively, the insider is always an information monopolist. As a result, no matter how many uninformed participants there are in the market, they must sensitize their demand to the equilibrium price in order to prevent extreme losses to the Insider.

3. Risk Premia and Information Transmission

One of the primary achievements of the noisy rational expectations literature has been to formalize the concept of imperfect information transmission within markets. However, the related issue of risk premia has received very little attention. [Two exceptions are Admati (1985) and Kyle (1989).] A feature of the present model is that it can be used to tie together the concepts of information transmission and risk. Proposition 4 helps in this regard, because it becomes possible to discuss risk premia and information transmission without referring to a particular equilibrium profile. The next proposition shows that whenever the market does not collapse, there exists a risk premium in the economy. In addition, the premium only reflects the uncertainty faced by the *uninformed* investors, and the 'position they hold in equilibrium.

Proposition 7. Let R represent the risk premium $E(P_1 | P) - P$. If the market has not collapsed, then $R = (\theta/n)S \text{Var}(P_1 | P)$.

Proof. Rearrange Equation (10).

Q.E.D.

Intuitively, the risk premium in this article is similar to that produced in the traditional CAPM model. The difference is that here market risk has two components. The first part is the uncertainty about P_1 , faced by the uninformed investors, conditional on the equilibrium price they observe— $\text{Var}(P_1 | P)$. The second element is the amount of risk the small traders must absorb (S). The reason R depends on S is that the risk premium only reflects the level of risk held by the uninformed section of the market. As a result, negative values of S result in negative risk premia.

The next issue we address is the amount of information transmitted by prices. To measure the informativeness of the price and the equilibrium dispersion of information between the insider and the outsiders, consider the metric $m = 1 - [\text{Var}(P_1 | P) - \sigma_1^2]/\sigma_1^2$, which measures the fraction of the insider's information that is transmitted via the price. If prices reveal all of the insider's information, then

⁷Milgrom and Stokey (1982) make the same point: a trader with new information is never small.

$\text{Var}(P_1 | P) = \sigma_\eta^2$; and $m = 1$. When none of the insider's information is revealed, then $\text{Var}(P_1 | P) = \sigma_\eta^2 + \sigma_\epsilon^2$ and $m = 0$.⁸ Now, using Equation (7), it is possible to simplify m , and obtain $m = \sigma_\eta^2 / (\sigma_\eta^2 + \theta \sigma_\epsilon^2 \sigma_\omega^2)$, noting that the θ here refers to the risk-aversion parameter of the informed. The next proposition lists several comparative statics regarding m .

Proposition 8. *If the market has not collapsed, $dm/d(-\sigma_\eta^2) < 0$, $dm/d\theta < 0$, $dm/d\sigma_\epsilon^2 < 0$, $dm/d\sigma_\omega^2 < 0$, and $dm/dn = 0$.*

As Proposition 8 shows, the Insider reveals a smaller percentage of her information if she knows less, is more risk-averse, or faces more risk either from her wages or her ignorance about η . (These conclusions are, of course, conditional on the assumption that the informational differences have not caused the market to collapse.) Counterintuitively, the size of the market (η) has no impact on m . The intuition is that the insider has a "hedging" motive as well as an "informational" motive for trade, and the changes above are causing the "informational" motive to be dominated, thereby decreasing the fraction of information transmitted. To obtain some perspective on our results, we created a similar metric for Subrahmanyam's (1991) model. While it does not have a closed-form representation, one can still show that $\partial m / \partial \theta$ has the same sign in both models. In the case of Kyle's (1985) article, with risk-neutral agents, $m = 0.5$ regardless of the economy's parameters.

4. The Linear Equilibrium

While Theorem 2 provides the complete set of equilibria, it is useful to examine certain special cases. Of particular interest is the case of the linear demand curve since it has figured so prominently in the noisy rational expectations literature.

Proposition 9. *The linear demand curve is a special case of the general solution to (11) that occurs when $C = 0$, as displayed below:*

$$(K_3 + K_2)(K_3P + K_5) - K_4K_1 + K_3K_5S = 0. \quad (20)$$

Proof. Set C equal to zero in Equation (12) to obtain (20). Q.E.D.

⁸ See Admati and Pfleiderer (1987) for a detailed analysis of how information is endogenously allocated in a multisecurity competitive rational expectations equilibrium.

Using Proposition 9, it is easy to establish the relationship between market depth and the asymmetry of information.

Corollary 9.1. *Market depth increases as the informational disadvantage of the outsiders decreases. Formally, let P' represent dP/dS ; then $dP'/d\sigma_i^2 > 0$.*

Proof: Differentiate (20) with respect to S to establish $P' = K_4/(K_3 + K_2)$. Substitute out K_2 , K_3 , and K_4 , and then differentiate P' with respect to σ_i^2 . Q.E.D.

Economists often define a market's liquidity by the slope of the aggregate demand function. Not surprisingly, we predict that liquidity (or depth, as it is referred to in some studies) increases as the "informational disadvantage" decreases.

In symmetric information models, no trader knows more than any other individual in the market. As a result, these models predict that stock prices are perfectly elastic, and portfolio decisions will result in an optimal level of risk-sharing within the economy (e.g., the CAPM). The question then becomes whether such models are good approximations for an economy in which there is very little asymmetric information. The next corollary tells us that they are not.

Corollary 9.2. *When prices become fully revealing, the linear demand curve is represented by*

$$P = \mu_P - (\theta\sigma_i^2/n)S.$$

Proof. Substitute the values of $E(P_1 | P)$ and $\text{Var}(P_1 | P)$ from Proposition 4 in (10) and take the limit as $\sigma_i^2 \rightarrow 0$ to obtain

$$S = (\mu_P - P)/\theta\sigma_i^2/n.$$

Rearranging proves the proposition.

Q.E.D.

Corollary 9.2 makes two points. First, as the informational advantage of the insider disappears, one does not produce a perfectly elastic price function. Second, unlike many previous rational expectations models, markets do not break down here when prices become fully revealing [e.g., Grossman and Stiglitz (1980), Kyle (1985)]. This occurs because the informed trader has a hedging motive to trade, which remains preserved even when prices become almost fully revealing.

The discussion in Section 2 led us to stress the importance of the linear strategy function; liquidity and volume of trade are greatest here. We now give another reason why it may be important to focus exclusively on the linear case.

Proposition 10. *If the insider can choose an aggregate demand function, she will select the linear demand function.*

Proof. From (1) and (2), we find that the informed is effectively maximizing:

$$\begin{aligned} & \{ (1 - S + w)(\mu_p + \epsilon) + (1 - B_0) \} - 0.5 \theta \sigma^2 (1 - S + w)^2 \\ & + (S - S_0)P. \end{aligned} \quad (21)$$

Now, if the informed trader were to be offered the nonlinear demand function of Figure 2, she would choose C on it. However, for the same quantity she is buying at C , she could get a lower price and, hence, be better off at D . This is because Equation (21) tells us that, given $S < S_0$, the informed would like to minimize P . However, as A_2 is the optimal point on the linear demand curve, she is even better off at A_2 . Hence, she would prefer A_2 to C . We reverse the argument when $S > S_0$. Q.E.D.

Proposition 10 establishes that the informed trader's "favorite" equilibrium is the linear one. This might provide some justification for focusing on the linear case, but we leave open the question of what coordinating mechanism the informed trader can use to obtain her preferred aggregate demand schedule. However, as a practical matter, the insider's market power should make it much easier for her to employ a coordinating device when compared with the unorganized uninformed traders.⁹ As Proposition 10 shows, if coordination is accomplished, the result will be the employment of the linear demand schedule by the economy.

5. Conclusion

It has often been argued that equilibrium prices aggregate information effectively, and thus reduce or even eliminate information asymmetries in the economy. The weight of the academic law-and-economics commentary has used this 'argument to disapprove the regulation of insider trading [see Carlton and Fischel (1983) for a succinct treatment]. The results presented here question this conclusion. Based on a simple model constructed from the classical portfolio problem, we conclude that if insider-trading laws do not exist, the market may fail completely as a communication system.

The model presented here thus provides a potential explanation for the widespread existence of insider-trading laws. An examination

⁹ A possible coordinating mechanism is a simple public announcement by the Informed trader. See Laffont and Maskin (1990) for a similar discussion.

of the data in de Caires (1987) shows that in 26 of the 38 biggest stock markets of the world, insiders are required to either abstain from trading or disclose their information before doing so. Of the remaining 12 countries, though no statutory provisions exist, shareholder protection codes are evolving.

Academic interest in the desirability of such extensive insider-trading regulations is only just beginning. Ausubel (1990) finds that a commitment to such laws is beneficial even to the insider; Fishman and Hagerty (1988) argue that the presence of insiders may discourage other information gatherers and thus decrease the informational content of prices; we find that if these laws do not exist, the uninformed might not have the confidence to trade with the insider at all.

Finally, the model also delivered several other results regarding a security's price. It showed that there may exist a continuum of equilibria in the familiar exponential utility-normal distribution framework, and the linear is only a special case. Another result is that the risk premium on a security is completely characterized by the information and stock held by the market's outsiders. A further conclusion is that of all the possible equilibria, the insider's utility is maximized when the linear demand schedule is employed. The market-maker, therefore, has another positive role to play: to choose and coordinate the equilibrium demand schedule.

Appendix

Proof that demand correspondences will not be submitted

The problem of the i th outsider is to

$$\max_{B(i), S(i)} E(U(S(i)P_1 + B(i)) | P),$$

given his budget constraint

$$[S(i) - S_0(i)]P + [B(i) - B_0(i)] = 0.$$

Taking the first derivative with respect to $S(i)$ yields

$$\int_{-\infty}^{\infty} (P_1 - P) U'(\cdot) f(P_1 | P) dP_1,$$

while the second derivative with respect to $S(i)$ is

$$\int_{-\infty}^{\infty} (P_1 - P)^2 U''(\cdot) f(P_1 | P) dP_1.$$

For strictly concave utility functions (e.g., the exponential utility function), the second derivative is always negative. Therefore, there

is a unique $S(i)$ for every P , implying that the aggregate $S(P)$ is a function and not a correspondence. Q.E.D.

Proof of Proposition 1

The goal is to find the solutions to the differential equation

$$K_1 + K_2S + K_3S'P + K_4S'S + K_5S' = 0. \quad (A1)$$

To solve the above problem, we use the method of integrating factors, and this we outline below.

Let $M = K_1 + K_2S$ and $N = K_3P + K_4S + K_5$. Now rewrite (A1) as

$$\mu M + \mu NS' = 0. \quad (A2)$$

The next goal is to find an integrating factor μ that makes (A2) exact, in other words, one wishes to find a function μ such that

$$\frac{d}{dS}(\mu M) = \frac{d}{dP}(\mu N). \quad (A3)$$

If such a μ is found, then the following equation holds:

$$M\mu_S - N\mu_P + \mu(M_S - N_P) = 0. \quad (A4)$$

Assuming that the integrating factor μ is dependent only on S (later we will show that this turns out to be true), one can use (A4) to produce

$$\mu_S = \frac{(N_P - M_S)\mu}{M} = \frac{K_3 - K_2}{K_1 + K_2S}\mu. \quad (A5)$$

Solving the partial differential equation (A5) gives

$$\mu = (K_1 + K_2S)^{(K_3-K_2)/K_2}$$

as the result. Thus, the assumption turns out to be true; μ is not dependent on P . Now, represent the solution of (A1) by

$$\psi(S, P) = C. \quad (A6)$$

Next differentiate (A6) with respect to P to give

$$\psi_P + \psi_S S' = 0. \quad (A7)$$

Comparing (A7) with (A2) shows that

$$\psi_P = \mu M = (K_1 + K_2S) (K_1 + K_2S)^{(K_3-K_2)/K_2} \quad (A8)$$

and

$$\psi_S = \mu N = (K_3P + K_4S + K_5) (K_1 + K_2S)^{(K_3-K_2)/K_2}, \quad (A9)$$

From (A9), one has that

$$\dot{\psi}_s = (K_3 P + K_5) (K_1 + K_2 S)^{(K_3 - K_2)/K_2} + K_4 S (K_1 + K_2 S)^{(K_3 - K_2)/K_2}. \quad (\text{A10})$$

Integrate both sides of (A10) with respect to S . The first part of the right-hand side of the equation gives

$$(K_3 P + K_5) \frac{1}{K_3} (K_1 + K_2 S)^{K_3/K_2}. \quad (\text{A11})$$

The second part of the right-hand side has to be integrated by parts. Doing so produces the following result:

$$S \frac{K_4}{K_3} (K_1 + K_2 S)^{K_3/K_2} - \frac{K_4}{K_3} (K_1 + K_2 S)^{(K_3 + K_2)/K_2} \frac{1}{K_3 + K_2}. \quad (\text{A12})$$

Thus, one obtains

$$\psi(S, P) = (\text{A11}) + (\text{A12}) + b(P) \quad (\text{A13})$$

and

$$\begin{aligned} \psi_P &= (K_1 + K_2 S)^{K_3/K_2} + b'(P) \\ &= (K_1 + K_2 S)^{K_3/K_2}, \end{aligned}$$

where the second equality follows from (AS). Hence, $h'(P) = 0$, which implies that h is an arbitrary constant, C . Thus, simplifying (A13), one obtains the solution to the differential equation (A1):

$$(K_1 + K_2 S)^{K_3/K_2} [(K_3 + K_2)(K_3 P + K_5) - K_4 K_1 + K_3 K_4 S] = C.$$

Proof of Proposition 2

The proof proceeds in four steps. We first prove that the conditions are *necessary* for every P that is selected by the insider for a given τ (step 1). We then prove that every P on the real line is selected for some τ . We do this by showing that P is *continuous* in τ (step 2) and P is *unbounded* in τ (step 3). This leads us finally to prove that the conditions are also *sufficient* (step 4).

Step 1. The conditions are necessary.

Proving that the proposition's conditions are necessary requires an examination of the system's second-order conditions. While the first-order conditions locate extrema, the insider will only trade to a stationary point if it is a maximum. Thus, if a point along the feasible aggregate demand curve is selected for some τ , then at that point the second-order condition of the Insider, obtained by differentiating the

left-hand side of (3), must be weakly negative; that is,

$$S''[P - \mu_P - \epsilon + \theta\sigma_v^2(1 - S + w)] + 2S' - \theta\sigma_v^2(S')^2 \leq 0. \quad (\text{A14})$$

Next examine the first-order conditions. At each extremum one has

$$P - \mu_P - \epsilon + \theta\sigma_v^2(1 - S + w) = (S_0 - S)/S'. \quad (\text{A15})$$

Plugging (A15) into (A14), one finds

$$S''[(S_0 - S)/S'] + 2S' - \theta\sigma_v^2(S')^2 \leq 0. \quad (\text{A16})$$

Differentiating (12) twice with respect to P produces

$$S'' = -\frac{(K_2 + K_3 + K_4S')S'}{K_3P + K_4S + K_5}. \quad (\text{A17})$$

Next rearrange Equation (11) into the following form:

$$(S - S_0)/S' = K_3P + K_4S + K_5. \quad (\text{A18})$$

Substituting (A17) and (A18) into (A16), the second-order condition simplifies to

$$S'[(1 + K_3) + S'(K_4 - \theta\sigma_v^2)] \leq 0.$$

This leads to the following bounds on S' :

$$-\frac{1 + K_3}{K_4 - \theta\sigma_v^2} \leq S' \leq 0.$$

Now substitute out K_3 and K_4 , and then use the fact that $\sigma_v^2 = \sigma_\epsilon^2 + \theta^2\sigma_v^4\sigma_w^2$ to simplify the above expression and obtain $-n\theta^{-1} \times \text{Var}^{-1}(P_1 | P) \leq S' \leq 0$. Rearrangement of this expression implies

$$\frac{dP}{dS} \in \left(-\infty, -\frac{\theta}{n} \text{Var}(P_1 | P) \right]. \quad (\text{A19})$$

Since $[\theta \text{Var}(P_1 | P)]/n$ is a positive number, (A19) shows that the slope of the demand curve must be strictly negative.

This proves that, in equilibrium, it is necessary for the aggregate demand curve to satisfy (12) and strictly decline for all P that are selected by the insider for some τ . ■

Step 2. P is continuous in τ .

The proof that P is continuous in τ is by contradiction.

Suppose the choice of P is discontinuous in τ at τ^* . Then at τ^* the insider must be indifferent between selecting among at least two prices, say P^1 and P^2 . Further, since P^1 and P^2 are maxima for insiders

with signal τ^* , the outsiders know that if either P^1 or P^2 is selected, then $\tau = \tau^*$. Now define a demand function for the outsiders, $S^*(P)$, that satisfies the following equation:

$$S^* = \frac{n[E(P_1 | \tau^*) - P]}{\theta \text{Var}(P_1 | \tau^*)}. \quad (\text{A20})$$

This demand function can be interpreted as the outsiders' demand given they assume that $\tau = \tau^*$ for all P . Since the outsiders know $\tau = \tau^*$ at P^1 and P^2 , one has that $S^* = S$ at both these points.

Next, define the insider's value function, given that the outsiders use $S^*(P)$ as V^* , and consider its derivative. Noting that $dS^*/dP = -n\theta^{-1} \text{Var}^{-1}(P_1 | \tau^*)$ from (A20), the left-hand side of (3) can be rearranged as

$$\begin{aligned} \frac{dV^*}{dP} = & -n\theta^{-1} \text{Var}^{-1}(P_1 | \tau^*)[P - \mu_p - \epsilon + \theta\sigma_v^2(1 - S^* + w)] \\ & + (S^* - S_0). \end{aligned} \quad (\text{A21})$$

Observe from the above equation that the insider's utility, given that the outsiders are using the demand function $S^*(P)$, is strictly concave in P . Thus, if she is indifferent between P^1 and P^2 , the directional derivative of (A21) from P^1 to P^2 at P^1 must be strictly positive. Similarly, the directional derivative from P^2 to P^1 at P^2 must also be strictly positive.

We will now derive a contradiction by proving that at least one of the directional derivatives is negative.

Without loss of generality, assume $P^1 < P^2$. Noting that $S^*(P^1) = S(P^1) > S^*(P^2) = S(P^2)$ from (A20), subtract (3) from (A21) to get

$$\begin{aligned} \frac{dV^*}{dP} = & \left[-\frac{\partial S}{\partial P} - \frac{n}{\theta} \text{Var}^{-1}(P_1 | \tau^*) \right] \\ & \times [P - \mu_p - \epsilon + \theta\sigma_v^2(1 - S^* + w)]. \end{aligned} \quad (\text{A22})$$

Consider the case where $S^*(P^2) = S(P^2) < S^0$. From (3), this implies that the second term in square brackets is negative. Since we have already shown that $-n\theta^{-1} \text{Var}^{-1}(P_1 | P) \leq S' \leq 0$, the first term in square brackets is negative. Hence, the derivative given in (A22) is positive at $S = S^*$. As $P^1 < P^2$ by assumption, the directional derivative from P^2 to P^1 at P^2 is negative. This is a contradiction. For the case

where $S(P^2) > S^0$, take the directional derivative from P^1 to P^2 at P^1 to find that it is again negative. A contradiction results again.

This proves that the selection of P is continuous in τ . ■

Step 3. P is unbounded in τ .

The proof that P is unbounded in τ is by contradiction.

Suppose P is bounded above by P_u and below by P_o . From steps 1 and 2 of the proof, the demand curve is weakly downward sloping around any P selected in equilibrium. This, plus the assumption that the demand functions are continuous and differentiable, implies that $S_i(P_i) \leq S_o(P_o)$ for some $P_i > P_o$. From (1), if the inequality on S is strict, there must exist a τ large enough so that P_i is preferred to P_o . Thus, if the insider does not prefer P_i to P_o , it must mean that as $P \rightarrow P_o$ one has that dS/dP goes to zero.

A similar analysis for the case where P is bounded below leads to the conclusion that P is bounded in τ if and only if as $P \rightarrow P_o$ one has that dS/dP goes to zero. The proof is now completed by showing that dS/dP does not go to zero for any finite P .

For any finite P^* , (11) implies dS/dP does go to zero unless $S(P^*) = S_o$. Assume that at P_o one has $S(P_o) = S_o$. Then from (11), $S(P^*) \neq S_o$ for all $P > P_o$ if there exists any $P > P_o$ such that $S \neq S_o$. A similar conclusion holds for $P < P_o$. Hence, we get a contradiction. Therefore, the set of prices selected in equilibrium is not bounded below if it is possible for the insider to sell stock, and is not bounded above if it is possible for the insider to purchase stock. ■

Step 4. The conditions are sufficient.

Sufficiency requires that if the insider's first-order conditions are met on a demand curve satisfying the proposition, then she is also at a global maximum. The proof proceeds by contradiction.

Suppose there exists a S^1 on a downward sloping demand curve, and this S^1 satisfies the insider's first-order conditions. From the arguments above, S^1 must be a local maximum. Let us assume S^1 is not a global maximum. If S^1 is not a global maximum, then there must exist another quantity S^3 such that $V(S^1) = V(S^3)$ (recall V is the insider's value function). Since V is continuous in S and $V(S^1)$ is a local maximum, there must exist a $S^2 \in (S^1, S^3)$, such that $V(S^2)$ is a local minimum. By the definition of a local minimum, the insider's first-order conditions must be satisfied at S^2 . However, earlier arguments show that if the demand function is everywhere declining, then whenever the insider's first-order conditions are met, she is at a local maximum. This is a contradiction. Hence, the insider must be maximizing globally. ■

Proof of Theorem 1

First consider the set of points such that $S^* < S_0$. We have to show that the feasible aggregate demand curves cannot cross below line 1 or line 2 in Figure 1.

Suppose a candidate demand curve passes through a pair of points (S^*, P^*) , such that $K_3P^* + K_4S^* + K_5 < 0$. From (11), one finds that $dP/dS = (K_3P + K_4S + K_5)/(S - S_0)$. Thus, if $S^* < S_0$ and $K_3P^* + K_4S^* + K_5 < 0$, one has $dP/dS > 0$ at (S^*, P^*) . From Proposition 2, this solution to (11) is therefore not an equilibrium aggregate demand curve.

From the above arguments, an equilibrium aggregate demand curve must never cross below line 2 to the left of S_0 . The next step of the proof shows that a candidate demand curve should also never cross below line 1 to the left of S_0 . Consider a candidate demand curve that passes through a point (S^*, P^*) , such that

$$(K_3 + K_2)(K_3P^* + K_5) - K_4K_1 + K_3K_4S^* < 0.$$

The next step shows that there exists a point (P, S) along this demand curve where the slope is zero. From Equation (2),

$$P = \frac{C(K_1 + K_2S)^{-\kappa_3/\kappa_2} + K_1K_4 - K_3K_4S}{(K_3 + K_2)K_3} - \frac{K_5}{K_3}. \quad (A23)$$

Differentiation of (A23) with respect to S produces

$$\frac{dP}{dS} = -\frac{1}{(K_3 + K_2)} \left[\frac{C(K_1 + K_2S)^{-\kappa_3/\kappa_2}}{S_0 - S} + K_4 \right].$$

Substituting for K_1 and K_2 , we obtain

$$\frac{dP}{dS} = - \left[\frac{C(S_0 - S)^{\kappa_3}}{(S_0 + S)(K_3 - 1)} + \frac{K_4}{(K_3 - 1)} \right]. \quad (A24)$$

From (A24), dP/dS can equal zero only when $[C(S_0 - S)^{\kappa_3}]/(S_0 - S) = -K_4$. Now (12) shows that $C(S_0 - S)^{\kappa_3} = (K_3 + K_2)(K_3P + K_5) - K_4K_1 + K_3K_4S = R$, where R is a real number. From the above calculations, a curve defined by (12) and the initial value (S^*, P^*) will have a region with a zero slope if $R/(S_0 - S) = -K_4$ exists for some value of $S < S_0$. Since $K_4 > 0$, it is necessary to determine only if R is negative. Now, one has $R < 0$ if $(K_3 + K_2)(K_3P^* + K_5) - K_4K_1 + K_3K_4S^* < 0$. Thus, any curve defined by (12) and the initial value (S^*, P^*) will have a region with a zero slope if $(K_3 + K_2)(K_3P^* + K_5) - K_4K_1 + K_3K_4S^* < 0$. Therefore, the candidate demand curve cannot cross below line 1 either.

To complete the proof, we need to show that any curve satisfying the first half of the theorem's conditions never has a slope that violates

(A19). To prove this, note that from the arguments in the previous paragraph, $R = C(S_0 - S)^{\alpha_3} > 0$ if $(K_3 + K_2)(K_3 P^* + K_3) - K_4 K_1 + K_3 K_4 S^* > 0$. Hence, for $S < S_0$, from (A24), the slope of the aggregate demand curve satisfies the condition (A19). This completes the theorem for curves passing through points $S < S_0$.

To verify the theorem for points $S > S_0$, simply repeat the arguments with the signs reversed. Q.E.D.

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