Trading Costs, Liquidity, and Asset Holdings

Ravi Bhushan
Massachusetts Institute of Technology

In this article I develop a model that accounts for interdependence between trading costs in various asset markets arising from the optimizing behavior of liquidity traders. The model suggests that noise trading is an important determinant of the liquidity of asset markets andprovides a positive theory for diversified asset holdings by risk-neu tral liquidity traders.

In this article, I am concerned with the determinants of the liquidity of asset markets and the costs of trading in them. In general, liquidity traders who have discretion to trade in any risky asset will wish to trade larger amounts in assets with lower trading costs. However, when informed traders are also present in these markets, more liquidity trading in an asset may attract more informed trading, which adversely affects the cost of trading in that asset.[1] Furthermore, the. amount of noise trading in the various asset markets is also likely to affect the optimizing behavior of both informed and liquidity traders. Thus, when there are trading costs arising from the presence. of informed traders and when (at least some) liquidity traders have discretion to allocate their liquidity needs across assets, trading costs in the various asset markets

The financial support of the International Financial Services Research Center at M.I.T. is gratefully acknowledged. I would like to thank Michael Brennan, Bhagwan Chowdhry, Paul Healy, Antonio Mello, Stew Myers, John Parsons, David Scharfstein, Chester Spatt, Ro Verrecchia, and the seminar participants at M.I.T. and Wharton for many helpful comments. I am especially grateful to Mark Wolfson and the reviewer, Paul Pfleiderer, for providing me with some key insights on this subject. Address reprint requests to Ravi Bhushan. E52-325F, Sloan School of Management, 50 Memorial Drive, M.I.T.. Cambridge, MA 02139.

The Review of Financial Studies 1991 Volume 4, number 2, pp. 343-360

become interdependent. I develop a multi-asset model to analyze the trading cost and liquidity patterns across assets that emerge in equilibrium.

Following Bagehot (1971), trading costs are regarded as arising from the adverse selection problem that market-makers face in trading with informed traders.[2] Trading cost is thus the price the liquidity traders have to pay to compensate the market-maker for his losses in' trading with informed traders. Glosten and Milgrom (1985), among others, have shown that trading costs resulting from informed trading are likely to be positive even with risk-neutral market-makers, who act competitively and face zero transactions costs.

I consider a single-period economy with many risky assets and three types of traders: noise traders, liquidity traders, and a monopolistic informed trader in each asset. A noise trader who has no discretion in selecting an asset for trading corresponds to an investor who is forced to trade in some particular risky asset because his short or long position in that asset exposes. him to too much risk. I model such trading in an asset as exogenous. A liquidity trader corresponds to an investor with exogenous cash requirements who seeks to minimize his trading costs by optimally allocating his demands across assets. Finally, a monopolistic informed trader corresponds to an investor who invests in information collection and chooses the quantity traded to maximize his trading profits. Liquidity and noise traders provide camouflage to the informed traders. The market-makers in the various asset markets are assumed to make zero expected profits.

My model is based on Kyle (1985), and is similar in spirit to both Admati and Pfleiderer (1988) and Foster and Viswanathan (1990). In the latter two studies, the focus is on variations in trading costs in a *single asset over time.* The focus here is on the variation in trading costs *across assets* at a single point in time.[3]

The model implies that noise trading is an important determinant of the liquidity of asset markets. The profit of the informed trader in a given market, which is equal to the losses sustained by the liquidity and noise traders in that market, is positively related to the information advantage of the informed trader.[4] If there were no noise

[2] Examples of other papers that follow this theme are Copeland and Galai (1983), Glosten and Milgrom (1985), Kyle (1985), and Admati and Pfleiderer (1988). Another approach, following Dansetz's (1968) seminal work, is to model the cast of trading in an asset market as arising due to the inventory costs of dealers or specialists. Garman (1976), Stoll (1978), Amihud and Mendelson (1980). and Ho and Stoll (1981) are some examples of this line of research.

[3] Another interesting case arises when, at a given point in time, the same security trades at multiple locations. See Chowdhry and Nanda (1990) for an analysis of this problem.

[4] Intuitively, the informed trader's Information advantage will be large if the expectation of the absolute deviation between the return conditional on information and the unconditional return is large. In the model, the Informed traders ate assumed to get perfect Information about the firm-specific components of returns, which makes this expectation proportional to the variability of the firm-specific component of return.

traders in any asset, then liquidity traders would minimize their trading losses to the informed by concentrating all their trading in the asset with the lowest information advantage, and all other asset markets would shut down.[5]

The existence of noise trading in some assets provides incentives for liquidity traders to diversify; they shift some of their trading from the "lowest information advantage asset" to these assets to minimize their trading costs. Diversification benefits these traders because the additional losses due to the larger information advantage of the informed traders in these assets are offset by the sharing of the losses with the noise traders.[6] The model thus provides a motivation, In addition to that of risk reduction, for diversification by investors.

The plan of the rest of the article is as follows. In Section 1, I discuss the model with no noise trading. In Section 2, I consider the case of noise trading. In Section 3, I provide a brief discussion of the implications of the model; and, finally, in Section 4, I present some concluding remarks.

1. The Basic Model

I consider a one-period economy in which M risky assets are traded. The end-of-period payoff on a share of the risky asset i, S_i, is given by[7]

$$\tilde{S}_i = 1 + \tilde{\gamma} + \tilde{v}_i, \tag{1}$$

where $\tilde{\gamma}$ and $\tilde{v}_i$ are mutually independent, normally distributed random variables, each with a mean of zero and with variances of σ_γ^2 and σ_i^2, respectively. Equation (1) can be interpreted as a "single factor" representation of security returns, where the variables $\tilde{\gamma}$ and $\tilde{v}_i$, respectively, represent the market and the firm-specific components. The firm-specific components of the returns on the different risky assets are assumed to be mutually independent. The monopolistic insider in asset i observes the true value of $\tilde{v}_i$ before trading starts.[8] No one observes $\tilde{\gamma}$. Thus, although the returns on the different risky assets are correlated through the market factor, the correlation between the information of various insiders is zero since their individual observations are only of the mutually independent firm-specific compo-

[5] This result is similar to the concentrated trading patterns results that arise in Admati and Pfleiderer (1988).

[6] Chowdhry and Nanda (1990) also have similar findings. In analyzing the trading of a single security across multiple locations, they find that liquidity traders minimize their costs by splitting their trading across locations.

[7] See Subrahmanyam (1991) for a similar representation.

[8] Similar results obtain if the assumption that the insider perfectly observes $\tilde{v}_i$ is relaxed.

nents. The assumption of the mutual independence of the $\tilde{v}_i$'s considerably simplifies the analysis.

All traders are risk neutral and the market-makers make zero expected profits. Thus, the market-maker in asset i sets a price that equals the conditional expected value of the asset i given his information, which is the order flow in asset i. I make the simplifying assumption that the market-maker in any asset only observes the order flow in that asset. In Section 4, I discuss the effect of relaxing this assumption and the assumption of the independence of the $\tilde{v}_i$'s, and show by numerical example that the basic results and the intuition derived from this model continue to hold.

Since the insider only observes $\tilde{v}_i$, the quantity traded by him, $\tilde{X}_i$, is a function only of $\tilde{v}_i$. Liquidity trader k $(k = 1,\dots,T)$ purchases a total of $\tilde{y}^k$ shares, which he allocates across the various assets. The random variables $\tilde{y}^k$ are mutually independent and normally distributed, each with mean zero and variance Δ so that $\tilde{Y}\ (=\Sigma_{k=1}^{T}\tilde{y}^k)$, the total liquidity demand in all assets, is distributed with mean zero and variance $T\Delta$. A negative value of $\tilde{y}^k$ corresponds to a sale by the liquidity trader k. I assume that there are no restrictions on short sales. Each liquidity trader minimizes his expected cost of trading by choice of the amounts of the various assets to be traded.

Following Kyle (1985) and Admati and Pfleiderer (1988), I examine the equilibrium in which the pricing strategies of the market-makers and the order submission strategies of the informed traders are linear functions. I also assume that the random variables $\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_M$, $\tilde{y}^1, \tilde{y}^2, \dots, \tilde{y}^T$ are multivariate normal.

I analyze a Nash equilibrium. It is defined as a set of pricing rules for the market-makers and trading strategies for the informed traders and the liquidity traders, such that (1) each trader takes the trading strategies of all other traders and the pricing rules of the market-makers as given; (2) each market-maker earns zero expected profits; (3) each informed trader maximizes his expected trading profits given his information; and (4) liquidity traders minimize their trading costs. In general, multiple Nash equilibria, each with a different level of trading cost; can arise in such a setting. However, equilibria, other than the one with the minimum trading cost, are unstable because in these equilibria some market-makers can earn nonzero profits by offering a new set of pricing rules, which, if offered, break these equilibria down. To eliminate such equilibria, an additional requirement is, therefore, imposed on the equilibrium: (5) there is no (linear) pricing rule that a market-maker can post and that earns positive profits after the liquidity traders and the informed traders adjust their trading strategies to be optimal given this new pricing rule. The addition of (5) above provides a unique equilibrium.

Assume that $\tilde{Z}_i$, the total quantity traded by noise traders in asset i, is normally distributed with mean zero and variance Γ_i. Let Y_i denote the total quantity of the asset i traded by the liquidity traders, and let $\tilde{\omega}_i \equiv \tilde{X}_i + \tilde{Y}_i + \tilde{Z}_i$ be the total order flow to the market-maker in asset i. Let D_i be the depth of the market in asset i in equilibrium (i.e., the order flow necessary to move the price by one dollar). Lemma 1, which is essentially Theorem 1 of Kyle (1985), provides the restriction that is Imposed on each D_i, through the equilibrium behavior of the market-makers and the informed traders.

Lemma 1 (Kyle). *Let $\tilde{P}_i$ be the price of the asset i set by the market-maker and let X_i be the order of the informed traderfor asset i. Then*

$$\tilde{X}_i = D_i \tilde{v}_i / 2 \tag{2}$$

and

$$\tilde{P}_i = 1 + \tilde{\omega}_i / D_i, \tag{3}$$

where D_i satisfies

$$D_i = (2/\sigma_i)\,[\Gamma_i + \mathrm{var}(\tilde{Y}_i)]^{1/2}. \tag{4}$$

The pricing rule of a market-maker, which is completely characterized by the depth D_i set by him, is to set the asset price to equal the conditional expected value of the asset's payoff given the order flow. This conditional expected value, in turn, is equal to the sum of unconditional expected payoff (=1) and the information in the order flow on the firm-specific component.

The expression in square brackets in (4) represents the variance of the total uninformed demands (i.e.; the sum of the variances of the noise. traders' demands and the liquidity traders' demands). The quantity $\mathrm{var}(\tilde{Y}_i)$ is endogenous since it depends on the allocation of the liquidity traders' trades across the various assets. These allocations, in turn, depend on the depths of the various market& since the depth of a market affects the expected cost of trading in it. This cost is equal to the number of shares traded times the expected difference between the price and the value of one share of the asset. Let α_i^k be the fraction of his demand allocated by liquidity trader k to asset i. Trader k faces the problem of optimally choosing the α_i^k's to minimize his expected cost of trading. The next lemma characterizes how the liquidity traders choose these fractions α_i^k.

Lemma 2. *The optimal choice of α_i^k, the fraction of his liquidity demand that the liquidity trader k trades in asset i, is given by*

$$\alpha_i^k = D_i\left[\sum_{j=1}^{M} D_j\right]^{-1}. \tag{5}$$

Proof. Consider liquidity trader k. His expected cost of trading in asset i is given by

$$E\tilde{C}_i = E[\{\tilde{P}_i(\tilde{\omega}_i) - \tilde{S}_i\}\alpha_i^k\tilde{y}^k|\tilde{y}^k]. \tag{6}$$

Substituting for $\tilde{P}_i(\tilde{\omega}_i)$ from (3) in (6), and computing the expected value,

$$E\tilde{C}_i = (\alpha_i^k)^2(\tilde{y}^k)^2/D_i. \tag{7}$$

His total expected cost of trading $E\tilde{C}$ is given by,

$$E\tilde{C} = \sum_{i=1}^{M} E\tilde{C}_i = \sum_{i=1}^{M} \frac{(\alpha_i^k)^2(\tilde{y}^k)^2}{D_i}, \tag{8}$$

so that minimizing $E\tilde{C}$ subject to the constraint $\Sigma\alpha_i^k = 1$ yields expression (5).

The fraction α_i^k is Independent of the realization y^k so that the fraction traded in an asset is the same for all liquidity traders. Henceforth, I will omit the superscript k. Each liquidity trader allocates his liquidity needs across the various assets in proportion to the depths of the markets in those assets.

Consider first the case in which there is no noise trading in any asset ($\Gamma_i = 0$, for all i). Theorem 1 then establishes that the only equilibrium is the one in which all trading takes place in the asset with the least return variability.

Theorem 1. *When there is no noise trading liquidity traders trade only in the asset with the lowest return variability. There is a breakdown of the market in all other assets.*

Proof: Consider 'the asset i. Since all liquidity traders allocate a_i fraction of their demands to asset i,

$$\mathrm{var}(\tilde{Y}_i) = \alpha_i^2 T\Delta. \tag{9}$$

Substituting from (9) in (4), the market depth in asset i is

$$D_i = (2/\sigma_i)\alpha_i(T\Delta)^{1/2}. \tag{10}$$

Substituting for a_i from (5) in (10) implies that for $i = 1,\ldots,M$, D_i must satisfy

$$D_i = 0 \tag{11a}$$

or

$$D_i = \frac{2}{\sigma_i}(T\Delta)^{1/2} - \sum_{j=1, j\neq i}^{M} D_j. \tag{11b}$$

The only way that (11) can hold for each i, is if $D_i \neq 0$ for only one i. A depth of zero in a market is equivalent to that market shutting down. Thus, all markets except one must shut down, and for the open market D_i must satisfy

$$D_i = (2/\sigma_i)(T\Delta)^{1/2}. \tag{12}$$

This implies that there are M different possible equilibria, one for each asset with the liquidity trading concentrated in that asset.

Equation (12) shows that D_i would be largest for the market with the smallest σ_i. Thus, of all the possible equilibria, the equilibrium with the minimum trading costs for the liquidity traders is the one in which liquidity trading is concentrated in the asset with the smallest σ_i. Denote this asset by 1. To see how market forces would lead to only this equilibrium surviving, assume that the existing equilibrium in the market is the one with the market in some other asset j $(j \neq 1)$ open and all other markets shut down. The market-maker in that asset would then set a depth of $(2/\sigma_j)(T\Delta)^{1/2}$ in order to break even. However, this is not a stable equilibrium since it can be broken by any. market-maker in an asset with σ_i lower than σ_j by offering a pricing rule with depth higher than $(2/\sigma_j)(T\Delta)^{1/2}$ but less than $(2/\sigma_i)(T\Delta)^{1/2}$. With this pricing rule, he can expect to earn positive profits by attracting all the liquidity traders' business when the liquidity traders and the informed traders adjust their trading strategies to be optimal given this new rule. Therefore, the only stable equilibrium is the one in which only the market in asset 1 is open with a depth of $(2/\sigma_1)(T\Delta)^{1/2}$, which is the largest possible depth that can be offered.

The unconditional profit of the informed trader in asset i is equal to $\frac{1}{2}\mathrm{var}(Y_i)^{1/2}\sigma_i$, which is increasing in σ_i, the variability of the firm-specific component of the return (hereafter, return variability). Intuitively, an informed trader's information advantage will be large if the return conditional information can deviate significantly from the unconditional return. Thus, information advantage can be measured as the expectation of the absolute deviation between the return conditional on information and the unconditional return. Given that the informed trader observes $\tilde{v}_i$ perfectly, and that $\tilde{v}_i$ is normally distributed, this expectation turns out to be proportional to σ_i, so that σ_i can be considered a measure of information advantage. If σ_i were zero for one asset, then all liquidity trading would take place only in that asset since liquidity traders' losses to the informed from trading.

in that asset would be zero. If there is no asset with σ_i equal to zero, then the losses of the liquidity traders to the informed are minimized if they concentrate all their trading in the asset with the lowest σ_i.

So far I have assumed that there is no noise trading in any asset. In the next section, I show that the existence of noise trading in some assets has a significant impact on the liquidity of markets; and that this result of concentration of trading in one market no longer holds.

2. The Effect of Noise Traders

It is assumed that a noise trader must trade a given number of shares in a given asset during the period, but I do not address the motives of such traders in any great detail here. As an example, such trading behavior can result if a trader's long or short position in some particular asset exposes him to too much risk and he has to trade a given amount in a short span of time to reduce his exposure. Such a trader may very well recognize that his order affects the market price, but if he expects his demand to affect the price by an amount that is acceptable to him for his urgency or risk considerations. then his demands will appear to be insensitive to price and his behavior can be modeled as if his demands are inelastic.

Rewriting $\operatorname{var}(\tilde{Y}_i)$ as $\operatorname{var}(\tilde{Y}_i) = \alpha_i^2 T\Delta$, (4) becomes

$$D_i = (2/\sigma_i)[\Gamma_i + \alpha_i^2 T\Delta]^{1/2} \tag{13}$$

for all i, $i = 1, \ldots, M$. Substituting for a_i from (5) into (13), D_i must satisfy, for each i,

$$\frac{\sigma_i^2 D_i^2}{4} = \Gamma_i + D_i^2 \left[D_i + \sum_{j=1, j\neq i}^{M} D_j \right]^{-2} T\Delta. \tag{14}$$

Equation (14) suggests that, unlike Section 2, there are no trivial solutions to this problem. Theorem 2 below characterizes the equilibrium.

Theorem 2. The equilibrium is characterized by the following.

(a) The market breaks down for each asset with no noise trading (i.e., with $\Gamma_i = 0$), with the possible exception of the asset with the least return variability.

(b) For all assets with nonzero levels of noise trading in equilibrium there would be nonzero levels of liquidity trading also in such assets.

Proof. The proof follows a similar line of reasoning to that used in proving Theorem 1 and is provided in the Appendix.

The existence of noise trading has a significant impact on the behavior of liquidity traders who now diversify across assets. The presence of noise trading in an asset provides a nonzero level of depth to that market. Equation (5) then shows that the liquidity traders' optimal trading strategy is to allocate at least some of their trading in this asset, too. The liquidity traders thus have an incentive to shift some of their trading from the least σ_t asset into assets with noise trading in them.

The presence of noise traders in an asset makes the adverse selection problem less severe in that asset. This reduces a liquidity trader's cost of trading in that asset compared to the case in which there were no noise traders in that asset. The additional losses due to the informed trader's larger information advantage (higher return variability) in such an asset are traded off against the reduction in trading cost that is achieved by the presence of noise traders.

Except for the asset with the least return variability, the market must break down hi all assets with no noise trading because the absence of noise traders implies no possibility of a reduction in trading cost to offset the additional losses due to the larger information advantage of informed traders in such assets. For the least σ_t asset in this category, the market would remain open only if the return variability for this asset is the lowest among all assets, and if this σ_t is so low that the disadvantage to the liquidity traders for tiding in it due to lack of noise trading is offset by the lower trading costs provided by the informed trader's lower information advantage in it.[9]

For most parameter values (i.e., for most economies), the markets for all assets with no noise trading, including the one with the lowest σ_t in this category, break down. In the analysis that follows, I assume that markets are active only in assets which have some noise trading in them.[10] Renumber these assets from $t = 1, \ldots, Q$. In equilibrium, (14) must hold for $t = 1, \ldots, Q$ and can be rewritten as

$$\Gamma_t/D_t^2 = \sigma_t^2/4 - K, \tag{15}$$

where

$$K = T\Delta\left(\sum_{j=1}^{Q} D_j\right)^{-2}. \tag{16}$$

[9] Whether or not the market in this asset remains open depends on several factors, including the number of assets with noise trading, the levels of noise trading in them, and the relative magnitude of this asset's return variability compared to the other σ_t's. The Appendix provides the exact conditions that need to be satisfied for the market in this asset to remain open. The existence of an asset with noise trading and with a lower σ_t than this asset is sufficient to ensure a market shutdown for this asset. Also, if there is a large number of assets with noise trading, or if the level of total noise trading is large, then the market in this asset again shuts down.

[10] This assumption is made only for ease of expression and to reduce notational burden. None of the results relics on this assumption.

Equations (15) and (16) together imply that K must satisfy

$$K = T\Delta\left[\sum_{i=1}^{Q}\left\{\frac{\Gamma_i}{\sigma_i^2/4 - K}\right\}^{1/2}\right]^{-2}. \tag{17}$$

K is thus an economy-wide constant and is a solution to the nonlinear equation (17). Given K, Equation (15) implies that, for $i = 1, \ldots, Q$,

$$D_i = \left[\frac{\Gamma_i}{\sigma_i^2/4 - K}\right]^{1/2}. \tag{18}$$

Equation (18) shows that the depth of a market is proportional to the $(\Gamma_i)^{1/2}$ and varies inversely with σ_i. I summarize these results in Theorem 3.

Theorem 3. *For all assets with noise trading, the depth of the market in an asset is proportional to the standard deviation of the total noise demands in that asset and varies inversely with the asset's return variability.*

Equation (6) implies that the proportional cost of trading varies inversely with the depth of the market in that asset, so that from Theorem 3, the percentage cost of trading in an asset is inversely proportional to $[\Gamma_i]^{1/2}$ and varies directly with σ_i. As in Kyle (1985), there is an inverse relation between trading costs and noise trading because noise trading alleviates the adverse selection problem. Trading costs are positively related to return variability because the informed trader's information advantage, increases with return variability.

From Equation (5), α_i/D_i is the same for all assets $i = 1, \ldots, Q$, so that the fraction of their liquidity demands allocated by liquidity traders to an asset is proportional to the depth of the market in that asset. Theorem 3 then implies that liquidity traders allocate more of their trading to assets with higher levels of noise trading and to assets with lower return variability.

The behavior of liquidity traders leads to complex interactions between the different markets. In general, it is not possible to obtain an explicit solution for the depths in the various asset markets However, an explicit solution exists when each asset has the same return variability σ. Then (17) simplifies to

$$K = T\Delta\frac{\sigma^2}{4}\left[\left(\sum_{i=1}^{Q}\Gamma_i^{1/2}\right)^2 + T\Delta\right]^{-1}. \tag{19}$$

Substitution of this expression for K into (18) and simplification implies, for $i = 1, \ldots, Q$,

$$D_i = \frac{\Gamma_i^{1/2}}{\sigma/2}\left[\left(1 + T\Delta\left\{\sum_{j=1}^{Q}\Gamma_j^{1/2}\right\}^{-2}\right)^{1/2}\right]. \tag{20}$$

Equation (20) implies that the depth in a market can be written as the depth with no liquidity trading (i.e., with $T\Delta = 0$) times a factor F, which is the same for all markets and which accounts for the increase in depth resulting from the presence of liquidity traders. This factor F, which is represented by the terms in square brackets in (20), is an increasing function of the ratio of the standard deviation of the total liquidity demands to the sum of the standard deviations of the total noise demands in all assets. Ceteris paribus, an increase in the level of total liquidity trading implies an increase in F and a proportional increase in the depths of all markets.

It can also. be easily shown that for this Identical return variability case the depths would be zero for assets with no noise trading. Thus, no liquidity trading would ever occur in an asset with no noise trading. Equations (5) and (20) also imply that a_i, the fraction of their demands traded by the liquidity traders in the market i, can be written as

$$\alpha_i = \Gamma_i^{1/2}\left\{\sum_{j=1}^{Q}(\Gamma_j)^{1/2}\right\} \tag{21}$$

so that thefraction traded in an asset is equal to the standard deviation of noise demands in this asset divided by the sum of the standard deviations of noise demands in all assets. Equation (21) also clearly demonstrates that liquidity traders diversify their trading across all assets with nonzero levels of noise trading.

3. Implications and Extensions

In this section, I consider the implications of the model for a theory of portfolio diversification and show that the general implications of the model continue to hold when one takes the correlations in asset returns and order flows into account.

3.1 A positive theory of diversification

In the previous section, the existence of noise traders in the various markets was shown to imply that liquidity traders diversify their trading across assets. Thus, if a liquidity trader starts with positive liquidity demands (i.e., with net buying needs), he will end up with a portfolio of all risky assets with nonzero levels of noise trading. Similarly, if the liquidity trader has net selling needs, he would like to short-sell all assets in which there are markets. In the model, short sales are treated symmetrically with purchases. If, however, short sales involve additional costs due to margin requirements, etc., then a liquidity trader would prefer to hold a diversified portfolio so that liquidity needs can be met at minimum costs by diversified sales across assets without incurring the additional costs of short-selling. Thus, even though all liquidity traders are risk neutral, they will hold diversified portfolios. The model thus leads to a positive theory of diversification even in the face of risk neutrality.

3.2 Correlated returns and correlated order flows

In the model, I assumed that each market-maker could observe the order flow only in his asset. Yet, since all liquidity traders trade the same fraction in a given asset, the order flows in any two assets i and j are $\alpha_i \tilde{Y} + \tilde{X}_i + \tilde{Z}_i$ and $\alpha_j \tilde{Y} + \tilde{X}_j + \tilde{Z}_j$, respectively, and are correlated through the $\alpha_i \tilde{Y}$ and $\alpha_j \tilde{Y}$ terms.[11] This correlation of liquidity trading provides an incentive for the market-makers to try to observe the order flows in other assets because conditioning on order flows in other assets yields a more precise estimate of the informed component of the order flow.

This inference problem becomes complicated if the firm-specific components of returns are also correlated across assets. For then each informed trader would like to trade in assets whose firm-specific components of returns are correlated with the firm-specific component of the return for his asset, so that the informed trading components of the order flows for different assets also become correlated.

Correlation in the order flow thus results from correlation in both the liquidity component as well as the informed component. Market-makers would like to set prices conditional on all observable order flows, and each informed trader would like to trade in all assets. A more general model would take account of these interactions. To gain some intuition as to how they affect the trading strategies and trading cost of liquidity traders, I now consider a simple model that accounts for such interactions.[12] Consider an economy with only two assets, in which the payoff on each asset can still be represented by Equation (1). The variance-covariance matrix of the firm-specific components $\tilde{v}_i$, about which there is private information, is now denoted by Σ_v, which is assumed to be nonsingular and the correlation between the $\tilde{v}_i$'s is represented by ρ_v. Assume that there is only one

[11] A similar situation arises in section 4 in Admati and Pfleiderer (1988). When liquidity traders an abate their trading across different time periods, the order flows across time become correlated.

[12] The analysis and much of the notation used here rely heavily on Caballe and Krishnan (1990). They consider a more general model, but there is no discretionary liquidity trading in their model.

monopolistic informed trader who observes both $\tilde{v}_1$ and $\tilde{v}_2$, and that there is a single competitive market-maker who observes the order flows in both assets and sets prices accordingly.

As before, the noise trading is uncorrelated across the two assets and the variance of noise trading in assets 1 and 2 is Γ_1 and Γ_2, respectively. Let the variance of the total liquidity demands be given by $T\Delta$ (T is the number of liquidity traders), and let a represent the fraction of the demands of liquidity traders that is allocated to asset 1. Let Q represent the variance-covariance matrix of the total uninformed demands (the sum of the liquidity and noise demands, $Y + Z$).

Assuming linear pricing rules and trading strategies, the pricing rule of the market-maker is represented as

$$P(\tilde{\omega}) = A_0 + A_1\tilde{\omega}, \tag{22}$$

where P is the vector of prices set by him and $\tilde{\omega}$ is the vector representing the order flow to him. The trading strategy of the informed trader is represented as

$$X(\tilde{v}) = B_0 + B_1\tilde{v}, \tag{23}$$

where X represents the vector of the amounts traded by the informed trader in the two assets and $\tilde{v}$ represents the vector of the $\tilde{v}_i$'s.

For this setting, the results in Caballe and Krishnan (1990) can easily be adapted to show that the equilibrium solution is characterized by the following:

$$\tfrac{1}{4}M^T\Sigma_v M = I, \tag{24}$$

$$M^TQ^{-1}M = F, \tag{25}$$

$$A_1 = (M^T)^{-1}F^{1/2}M^{-1}, \tag{26}$$

$$B_1 = \tfrac{1}{2}A_1^{-1}, \tag{27}$$

$$A_0 = (1, 1)^T, \tag{28}$$

$$B_0 = (0, 0)^T. \tag{29}$$

Here M is an eigenmatrix that simultaneously diagonalizes the matrices Σ_v and Q^{-1}, and F is a diagonal matrix whose diagonal elements are the roots of $\det[Q^{-1} - \lambda(\tfrac{1}{4})\Sigma_v] = 0$. The informed trader's (unconditional) expected profits are given by trace $(B_1\Sigma_v)$ and the liquidity traders' (unconditional) expected trading costs are given by $T\Delta[(\alpha, 1 - \alpha)A_1(\alpha, 1 - \alpha)^T]$.

Even in this simple setting, the market-maker's pricing rule and the informed trader's trading strategy cannot be represented in closed form, which prevents analytical. treatment of the liquidity traders' optimal trading allocations. However, numerical examples illustrate how the correlations in asset returns and order flows affect the optimizing behavior of the liquidity traders. I focus attention solely on the liquidity traders' optimizing behavior here since the strategies of the market-maker and the informed trader are adequately addressed by Caballe and Krishnan (1990). However, it should be noted that, in general, the informed trader trades in both assets and the market-maker conditions his prices on the order flows in both assets.

In the numerical examples, I assume that each asset has the same return variance $(=\sigma_v^2)$. It can then be shown that the-liquidity traders' trading costs as well as the informed trader's total expected profits are proportional to σ_v. In the examples below I normalize σ_v to unity.

Table 1 shows the effect of correlated asset returns on liquidity traders' optimal trading allocations when the variances of noise trading in each asset and the variance of liquidity demands are all set at unity. In the table, the liquidity traders' unconditional trading costs for various values of correlations in asset returns and for various values of α, the allocation of their demands across the two assets, are listed. Given that the two assets are identical in theirnoise trading and return variance characteristics, and that in the absence of return correlation diversification was found to be optimal, it is not surprising to find that independent of the correlation of asset returns, an $\alpha = 1/2$ is the cost-minimizing strategy.[13]

Table 1 also shows that the benefits from diversification decline as the correlation ρ_v increases. When $\rho_v = 0$, trading costs for the "diversification" strategy with a = ½ are only 57 percent of the trading costs for "concentrations strategy (α = 0 or l), while for $\rho_v = 0.9$, the corresponding number is 88 percent. This happens because as ρ_v increases, the market-maker puts increasing reliance on both order flows in setting the individual asset's prices. This works to the detriment of the diversification strategy, which works best when one asset's price does not respond much to the order flow in the other asset. Nevertheless, the presence of noise traders In both assets ensures that there are some benefits from diversification for the liquidity traders even when the private information is highly correlated across assets.

One final point to be noted from Table 1 is that for $\rho_v = 0$, the minimum trading cost for the liquidity traders is 0.204, which is less than the cost that arises if the market were characterized by the model

[13] Several other cases with different levels of noise trading and liquidity trading were also considered. The finding were consistent with diversification leading to lower trading costs.

Table 1
The effect of the correlation in asset returns on the trading costs of liquidity traders

Correlation in asset returns (ρ_v)	Expected trading costs of liquidity traders Fraction of liquidity traders' demands allocated to asset 1 (α)										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.354	0.304	0.262	0.231	0.211	0.204	0.211	0.231	0.262	0.304	0.354
0.1	0.353	0.307	0.268	0.239	0.220	0.214	0.220	0.239	0.268	0.307	0.353
0.5	0.345	0.314	0.287	0.267	0.254	0.250	0.254	0.267	0.287	0.314	0.345
0.9	0.318	0.306	0.296	0.288	0.283	0.281	0.283	0.288	0.296	0.306	0.318

Parameter values: variance of return for each asset (σ_v^2) = 1.0; variance of noise traders' demands in asset 1 (Γ_1) = 1.0; variance of noise traders' demands in asset 2 (Γ_2) = 1.0; variance of liquidity traders' demands ($T\Delta$) = 1.0.

presented in Section 2. In that case, (21) implies that the optimal choice of α_i's is that $\alpha_1 = \alpha_2 = \frac{1}{2}$; and (20) can then be used to calculate the two D_i's. The (unconditional) trading costs $[= T\Delta(\alpha_1^2/D_1 + \alpha_2^2/D_2)]$ of the liquidity traders equal 0.224. When private information is uncorrelated across assets, the informed trader does not gain much by being allowed to trade in both assets, while the liquidity traders benefit because the market-maker can more easily distinguish between the informed and uninformed components of trading when he bases his pricing rules on both the order flows, which leads to lower trading costs.

The effect of the level of noise trading on the liquidity traders' optimal trading strategies is analyzed in Table 2. The liquidity traders' trading costs for various values of a and Γ_1, keeping Γ_2 constant; are listed. As the relative level of noise trading in asset 1 increases, the optimal policy of the liquidity traders is to allocate more of their trading to asset 1. When Γ_1/Γ_2 is equal to 1, optimal a is ½, but when

Table 2
The effect of noise trading on the liquidity traders' optimal trading allocations

Variance of noise traders' demands in asset 1 (Γ_1)	Expected trading costs of liquidity traders Fraction of liquidity traders' demands allocated to asset 1 (α)										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	0.345	0.314	0.287	0.267	0.254	0.250	0.254	0.267	0.287	0.314	0.345
2.0	0.342	0.307	0.277	0.252	0.234	0.222	0.218	0.223	0.235	0.256	0.284
5.0	0.335	0.298	0.264	0.234	0.209	0.189	0.175	0.169	0.172	0.183	0.202
10.0	0.330	0.292	0.255	0.222	0.192	0.167	0.148	0.137	0.132	0.137	0.150

Parameter values: variance of return for each asset (σ_v^2) = 1.0; correlation between asset returns (ρ_v) = 0.5; variance of noise traders' demands in asset 2 (Γ_2) = 1.0; variance of liquidity traders' demands ($T\Delta$) = 1.0.

Γ_1/Γ_2 is 10, the optimal a is about 0.8.[14] This is consistent with the results in Section 2, that a higher level of noise trading in an asset alleviates more of the adverse selection problem in that asset and consequently attracts more trading by liquidity traders as part of their. cost-minimizing strategy. The analysis in this section thus suggests that the major results of this paper are robust.

4. Conclusion

In this paper, I have examined cross-sectional variation in trading costs and liquidity across assets in a model with many assets and both uninformed and informed traders. In contrast, Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) are concerned with inter-temporal variation in trading costs in a single asset; The results in this paper are analogous to those found in these other papers. Admati and Pfleiderer (1988) find that when each liquidity trader can trade in only one period, liquidity traders minimize their trading costs by concentrating all their trading in one period. Analogously; I find that absent noise trading in all assets, all liquidity trading is concentrated in the asset with the least Information advantage.

Admati and Pfleiderer (1988) do not find a closed-form solution when liquidity traders can allocate their trading across different ‘time periods, but their arguments and numerical ‘examples show that liquidity traders generally allocate their trading across time periods with more concentration in earlier periods. This concentration in earlier periods occurs because the market-maker can infer liquidity demands as time goes on so that the adverse selection problem is less severe in earlier periods. I find that liquidity traders diversify their trading across all assets with noise trading. In my model, more liquidity trading takes- place in assets with more noise trading or less return variability since these are the assets with a less severe adverse selection problem. This result is also similar to that of Foster and Viswanathan (1990), who argue that public disclosure reduces the information advantage of the informed over time so that the uninformed can reduce their trading costs by shifting their trading to later time periods. Roughly, their later time periods correspond to the lower return variability assets in my model.

This paper has demonstrated that there is a motive for diversification across assets even when traders are risk neutral, The amount of noise trading and the information advantage of the informed traders are shown to be important determinants of the liquidity of financial mar-

[14] Although the table only shows results for the case of $\rho_v = 0.5$, the patterns for other values of ρ_v are very similar.

kets. The optimizing behavior of liquidity traders leads to interdependence between trading costs in various asset markets, and ties these markets together. Future research in this area is likely to enhance our understanding of how asset markets function and the interrelationships that exist among them.

Appendix: Proof of Theorem 2

Equation (14) gives the restriction that is imposed on each market-maker's choice of D_i for a Nash equilibrium to hold, in which each market-maker can make zero expected profits. First, consider the assets with no noise trading. It is easily seen that (14) can hold for all assets for which $\Gamma_i = 0$ only if the depth is equal to zero for all such assets except possibly one.

In general, there could be several possible equilibria, one for each such asset with a nonzero depth in it. I consider only the one with the least σ_i in this category, relying on the same logic as used in the proof of Theorem 1. An equilibrium, in which a market other than the one with the least σ_i in this category is open, can be broken by a market-maker in an asset with a lower σ_i, offering a slightly higher depth than currently offered in the open market; this would attract all liquidity traders' business and earn positive expected profits. Therefore, the only possibility is for the market with the least σ_i in this category to remain open. Let the asset with the least σ_i in this category be denoted asset 1.

Now-consider the assets with positive levels of noise trading. Let the total number of these assets by Q. Number these assets as 2, ... Q + 1. Assume first that the market does not break down in asset 1. Then, for asset 1, (14) becomes

$$T\Delta\left(D_1 + \sum_{j=2}^{Q+1} D_j\right)^{-2} = \frac{\sigma_1^2}{4}. \tag{A1}$$

Combining (Al) with -Equation (14) for assets $2, \ldots, Q + 1$ implies that, for $i = 2, \ldots, Q + 1$,

$$D_i = [4\Gamma_i/(\sigma_i^2 - \sigma_1^2)]^{1/2}. \tag{A2}$$

For (A2) to have a solution, $\sigma_1 < \sigma_i$ for all $i \in [2, \ldots, Q + 1]$. If $\sigma_1 < \sigma_i$ for all $i \in [2, \ldots, Q + 1]$, then (A2) can be solved for D_i, for $i = 2, \ldots, Q + 1$. Substituting for these depths in (A1), one can solve for D_l:

$$D_1 = \frac{2(T\Delta)^{1/2}}{\sigma_1} - \sum_{j=2}^{Q+1}\left[\frac{4\Gamma_j}{\sigma_j^2 - \sigma_1^2}\right]^{1/2}. \tag{A3}$$

Thus, if $\sigma_1 < \sigma_i$, for all $i \in [2, \ldots, Q+1]$, and if

$$\frac{2(T\Delta)^{1/2}}{\sigma_1} > \sum_{j=2}^{Q+1} \left[\frac{4\Gamma_j}{\sigma_j^2 - \sigma_1^2}\right]^{1/2}, \tag{A4}$$

then from (A2) and (A3), $D_i > 0$, for $i = 1, \ldots, Q+1$. $D_1 > 0$ implies that the market does not break down in asset 1. 1. $D_i > 0$, for $i = 2, \ldots,$ Q + 1, implies, using (5), that there is nonzero level of liquidity trading in all the assets with nonzero levels of noise trading.

If $\sigma_1 > \sigma_i$, for any $i \in [2, \ldots, Q+1]$, or (A4) does not hold, then (A1) cannot also hold or the market in asset 1 must break down. In that case, (14) still must be satisfied for $i = 2, \ldots, Q+1$. Equation (14) has a unique solution with nonzero D_i's for all these markets. Equation (5) then implies that there is nonzero level of liquidity trading in all these assets.

References

Admati, A. R., and P. Pfleiderer, 1988, "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies,* 1, 3-40.

Amihud, Y., and H. Mendelson, 1980, "Dealership Market: Market-making with Inventory," *Journal of Financial Economics,* 8, 31-53.

Bagehot, W. (pseud.), 1971, "The Only Game in Town," *Financial Analysts Journal,* 22, 12-14.

Caballe, J., and M. Krishnan, 1990, "Insider Trading and Asset Pricing in an Imperfectly Competitive Multi-Security Market," working paper, Purdue University, July.

Chowdhry, B., and V. Nanda, 1990, "Multi-Market Trading and Market Liquidity," working paper, University of California, Los Angeles, June.

Copeland, T. E., and D. Galai, 1983, "Information Effects on the Bid-Ask Spread," *Journal of Fiance,* 38, 1457-1469.

Demsetz, H., 1968, "The Cost of Transacting," *Quarterly Journal of Economics,* 82, 33-53.

Poster, P. D., and S. Viswanathan, 1990, "A Theory of the Interday Variations in Volume, Variance, and Trading Costs In Securities Markets," *Review of Financial Studies,* 3, 593-624.

Garman, M., 1976, "Market Microstructure," *Journal of Financial Economics,* 3, 257-275.

Glosten, L R., and P. R. Milgrom, 1985, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics,* 14, 71-100.

Ho, T.. and H. Stoll, 1981, "Optimal Dealer Pricing under Transactions and Return Uncertainty," *Journal of Financial Economics,* 9, 47-73.

Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica,* 53, 1315-1335.

Stoll, H., 1978, "The Supply of Dealer Services in Securities Markets," *Journal of Finance,* 33, 1133-1151.

Subrahmanyam, A., 1991, " A Theory of Trading in Stock Index Futures," Review of Financial Studies 4, 17-51.