A Theory of Trading in Stock Index Futures

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It is demonstrated that markets in stock index futures or, more generally, in baskets of securities, provide a preferred trading medium for uninformed liquidity traders who wish to trade portfolios, because adverse selection costs are typically lower in these markets than in markets for individual securities. Thus, an explanation isprovided for the immense liquidity and popularity of markets in stock index futures. Implications are also developed for the effect of the introduction of a basket on market liquidity and the informativeness and variability of component security prices, and for the price relationship between the basket and its underlying portfolio.

Recent years have witnessed a remarkable increase in the popularity of markets in broad market portfolios or "baskets" of securities. In a frictionless world, these markets would be completely redundant. However, since the advent of the most popular basket security, the S&P 500 index futures contract in 1982, there has been a spectacular growth in its trading volume [see, e.g., Schwert (1990)]. Other actively traded index futures contracts include the NYSE Composite Index contract and the Major Market Index contract. Instru-

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ments called Index Participations (IPs), in which long holders had a right to cash-out their positions at an amount equal to the value of the underlying index portfolio, were recently introduced simultaneously on the American and Philadelphia Stock Exchanges, and the response to the instruments was claimed to be the best for any derivative instrument introduced during the past 20 years on these exchanges.[1] Basket securities which permit all the stocks in the S&P 500 to be traded in a single transaction recently began trading on the NYSE and the Chicago Board Options Exchange.[2] A proposal for a basket security by Leland, O'Brien, and Rubinstein Associates, Inc. (innovators of portfolio insurance) was recently approved by the SEC.[3] We present a model that accounts for the popularity of markets in baskets and shows that the introduction of a basket security has implications for the informativeness and variability of prices and market liquidity.

Two broadly defined motives for trade in financial markets are information and liquidity. Informed traders trade on the basis of their private information. Liquidity traders, however, trade for reasons not directly related to future payoffs of financial assets, so that their reasons for trade are determined outside the financial market.[4] We demonstrate that the introduction of a market in a basket of securities will typically cause many liquidity traders to concentrate their trading in this market, because their losses due to adverse trades with informed traders will usually be lower in this market than in individual security markets.

Liquidity trades are typically executed either by individuals or by large financial intermediaries whose motives reflect the liquidity needs of their customers. These liquidity needs include the desire for immediate consumption, idiosyncratic wealth shocks, tax planning, risk-exposure adjustment, or the transactions required to synthesize derivative securities (e.g., portfolio insurance). In much of the recent literature on financial markets, liquidity traders are assumed to trade because of their need for immediacy, despite incurring transaction

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[1] "Basket Trading Starts Out Strongly," *New York Times*, May 15, 1989, p. C1. Trading in these baskets was suspended in August 1989 because of Federal court rulings stating that their approval by the SEC was illegal and that they fell under the jurisdiction of the Commodity Futures Trading Commission. See "Court Denies SEC Authority Over 'Baskets'," *Wall Street Journal*, June 19, 1990, p. C1.

[2] "Trading Begins in Stock 'Baskets'," *New York Times*, October 27, 1989, p. C6.

[3] "SEC Approves Rule Curtailing Program Trades," *Wall Street Journal*, July 25, 1990, p. C1. Details of the proposal can be found in "That Old Black Magic? The Portfolio Insurance Crowd's New Idea," *Barron's Business and Financial Weekly*, May 15, 1989, p. 62–63.

[4] Grossman and Miller (1987) and Holden (1990) are examples of works in which liquidity traders are assumed to receive exogenous endowment shocks and endogenously decide on their optimal trades by maximizing expected utility

costs as a result of the presence of informed traders. [For notable examples, see Kyle (1984, 1985), Glosten and Milgrom (1985), Diamond and Verrecchia (1987), and Admati and Pfleiderer (1988) .] This article adopts a similar approach and characterizes the strategic trading decision of liquidity traders who wish to trade portfolios of securities and may trade either in the individual securities or in a basket of these securities.

Our modeling strategy is similar to that of Kyle (1984) and Admati and Pfleiderer (1988). These authors analyze a speculative market in a single security. We extend their models to markets for multiple securities and a basket. In each market, risk-neutral informed traders and liquidity traders are assumed to submit their orders to risk-neutral market-makers who set prices expecting to earn zero profits. Adapting terminology from Admati and Pfleiderer, we postulate the existence of two types of liquidity traders: *discretionary* and *nondiscretionary.* Discretionary liquidity traders wish to trade in several securities simultaneously, and strategically choose to realize their trades either in the individual securities or in a basket of these securities, depending on where their losses to informed traders are minimized.[5] Nondiscretionary liquidity traders are exogenously constrained to trade a given order size in either a particular security or the basket.[6] Informed traders choose their trades strategically while rationally taking into account the effect their trades will have on prices.

Under our initial assumption that informed traders possess only security-specific information, our main result is that the independent orders submitted by informed traders in the basket tend to offset each other when the number of securities comprising the basket is large. This "diversification" benefit reduces the transaction costs of the discretionary liquidity traders when they trade in the basket. We then demonstrate that the tendency for the basket to serve as the lowest transaction cost market for discretionary liquidity traders is strong even when there exist traders who possess private information about systematic factors. Next, we develop some implications of our results for the informativeness and variability of prices and for market liquidity. We also characterize the behavior of the "basis" (the price difference between the basket and its underlying portfolio) and examine "lead-lag" relationships (the tendency for movements in one price to provide predictive information about subsequent movements in another price) between the basket and portfolio.

[5] In contrast, in Admati and Pfleiderer, discretionary liquidity traders optimally choose the *timing* of their trades. In Kyle (1984, 1985), all liquidity trading is nondiscretionary.

[6] Examples of nondiscretionary traders include individuals with stockholdings that do not correspond to the basket and financial institutions which face short-selling restrictions in the individual securities but not so in the basket.

Gorton and Pennacchi (1989) provide a similar rationale for basket securities.[7] Their model endogenizes the optimal portfolio choice of liquidity traders, which is taken to be exogenous in our article. In their model, however, information acquisition is assumed to be exogenous and there is no dichotomization of private information into security-specific and systematic components. In contrast, we endogenize information acquisition, examine how the basket changes incentives to collect different kinds of information, and also analyze the effect of the basket on the informativeness and variability of component security prices and on market liquidity. In other related work, Gammill and Perold (1989), Rubinstein (1989), and Harris (1990) discuss informally many of the issues we address in this article, and Chan (1989b) derives several interesting implications for the time-series price behavior of stock index futures markets and their underlying portfolios in the context of a model similar to ours. Chan's (1989b) analysis, however, does not incorporate strategic liquidity trading, and he focuses on an extant index futures market and its underlying portfolio. For most of our work, we are concerned with the *effect* introducing a basket market on the informativeness and variability of markets for the underlying securities. Our analysis of lead-lag relationships, however, is similar to that in Chan (1989b).

The remainder of this article is organized as follows. Under the assumption that informed traders possess only security-specific information, we outline the strategic trading decision of discretionary liquidity traders in Section 1. In Section 2, we examine the effect of introducing systematic factor information trading into the model. In Section 3, we discuss some implications of our results for price behavior and market liquidity. We conclude in Section 4.

The Analysis with Only Security-Specific Informed Trading

We use a model based on Kyle (1984, 1985) and Admati and Pfleiderer (1988) to characterize the trading decision of strategic liquidity traders who wish to trade in several securities simultaneously and who may realize their trades either in the securities or in a market in a portfolio of securities (e.g., an index futures contract).

Consider N securities simultaneously traded over a single period. Trading takes place at time 0 and the securities are liquidated at time 1. The value of security i ($i = 1, \ldots, N$) at time 0 is denoted by $\bar{S}_i$ (which is public knowledge), and its liquidation value at time 1 (denoted by S_i) is given by

[7] This work was brought to the author's attention after a substantial portion of work on the present article was complete.

$$S_i = \bar{S}_i + \beta_i \gamma + \epsilon_i, \qquad i = 1, \ldots, N, \tag{1}$$

where $\epsilon_1, \ldots, \epsilon_N$, and γ are all mutually independent, normally distributed random variables, each with a mean of zero. We interpret $\beta_i \gamma$ and ϵ_i as the systematic and security-specific (or idiosyncratic) components of the security value innovation, respectively, and β_i as the sensitivity of the value of security f to the "common factor" γ. The specification (1) is thus consistent with a "factor model" representation of security returns.

We also postulate the existence of a securitywith liquidation payoffs identical to a weighted average of the payoffs of the individual securities. We refer to this security as the "basket." The value of the basket at time 1 is given by[8]

$$S_m = \sum_{i=1}^{N} w_i \bar{S}_i + \sum_{i=1}^{N} w_i \beta_i \gamma + \sum_{i=1}^{N} w_i \epsilon_i, \tag{2}$$

where w_i is the weight of security i in the basket such that $0 < w_i < 1$ and $\Sigma_{i=1}^{N} w_i = 1$.

Informed traders observe signals correlated with ϵ_i, $i = 1, \ldots, N$, or γ at time 0 and take positions in individual securities and the basket based on their private information. In this section all informed traders are assumed to possess only security-specific information. In subsection 1.1 below, we discuss the structure of the model and the resulting equilibrium in the context of the market for a particular security i, taking the level of liquidity trading in this market to be exogenous. We extend the analysis to the case of many securities and a basket in subsection 1.2.

1.1 Equilibrium in the market for security *i*

All traders in the model are risk neutral. Each of the informed traders with information about the security observes the realization of ϵ_i with perfect precision at time 0. (The assumption of perfect precision is relaxed in Section 2.) The number of such traders is denoted by k_i. Liquidity traders in the security have no discretion over their order size: they must trade a given number of shares in the security. The net amount of the liquidity demand, z_i, is random. Liquidity trades contain no information about the payoffs of the security, that is, z_i is independent of ϵ_i and γ, and, in addition, is assumed to be normally distributed with a mean of zero.

The k_i informed traders and the liquidity traders submit their orders

[8] If the basket is Interpreted as an index futures contract, the value of the basket would instead be given by the standard no-arbitrage relationship, which relates the cash value to the value of the futures contract.

to a market-maker, not knowing the market clearing price when they do so. Competition is assumed to force the expected profits of the market-maker to zero. The market-maker thus absorbs the net trade in the security, denoted by ω_t, and then sets a price expecting to earn zero profits. The price set by the market-maker at time 0 is, therefore,

$$P_t = E(S_t|\omega_t). \tag{3}$$

As in Kyle (1984, 1985), the market-maker sets prices by a linear pricing rule:

$$P_t = \bar{S}_t + \lambda_t\omega_t. \tag{4}$$

We refer to the quantity λ_t as the "pricing parameter" for the security *i*. Here $1/\lambda_t$ is a measure of the "liquidity" or the "depth" of the market for this security. A low λ_t means a more liquid (or deeper) market in the sense that the cost of a given trade is low.

The (Nash) equilibrium that obtains in this market is described by the following lemma[9] (Unless otherwise stated, the proofs of all lemmas and propositions are in Appendix A.)

Lemma 1. *Each informed trader j submits an order x_{ji}, where*

$$x_{jt} = \epsilon_t/(k_t + 1)\lambda_t. \tag{5}$$

The equilibrium value of λ_t is given by

$$\lambda_t = \frac{1}{k_t + 1}\sqrt{\frac{k_t\,\mathrm{var}(\epsilon_t)}{\mathrm{var}(z_t)}}. \tag{6}$$

Note that λ_t is decreasing in the number of informed traders k_i, because an increase in the number of informed traders intensifies competition between them, leading to a decrease in the adverse selection faced by the market-maker. The other comparative statics associated with the lemma are intuitive. The expected collective losses of the liquidity traders are given by the expected value of the product of their demands, z_i, and the difference between the price of the security and the security's value. These losses are

$$E((P_t - S_t)z_t) = \lambda_t\,\mathrm{var}(z_t). \tag{7}$$

Thus, for a given variance of liquidity trades, the collective losses of the liquidity traders are lower, the lower the pricing parameter.

1.2 The case of many securities and a basket

We now introduce multiple classes of liquidity traders. These classes are denoted by L, $R_1, \ldots, R_N$, and M. Each trader belonging to class

[9] This lemma restates Lemma 1 of Admati and Pfleiderer (1988) in our notation.

L wishes to trade *all* securities and may realize his trades in either the individual securities or the basket. Class L corresponds to mutual fund or index fund portfolio managers satisfying the liquidity needs of their clients. We refer to the traders denoted by L *as discretionary* liquidity traders, since they are assumed to choose the *market(s)* in which to trade. Traders belonging to the classes denoted by $R_1, \ldots,$ R_N, and *Mare* constrained to trade a given order size in securities 1, . . . , N and the basket, respectively. These traders are referred to as *nondiscretionary* traders, since they are constrained to trade a fixed order size in a particular market. Classes $R_1, \ldots, R_N$ represent traders with holdings that do not correspond to the basket, so that they do not wish to trade in it, or noise traders (traders with incorrect or nonrational expectations) in the sense of Black (1986). The class M (which may be empty) is intended to model noise traders or uninformed portfolio traders (e.g., portfolio insurers) who, because of institutional rigidities, such as short-selling restrictions and high margin requirements, cannot realize their trades in the individual securities. In our model, this class prevents the market for the basket from breaking down in the event that the discretionary liquidity traders trade in the individual securities, because market-makers do not wish to trade in the absence of liquidity trading. In the analysis that follows, however, the tendency for the basket to serve as the lowest transaction cost market for the discretionary liquidity traders is strong even when class M is empty.

There are p discretionary liquidity traders, each of whom wishes to trade all securities in proportion to their weights in the basket. We let w_i be the weight of security fin each discretionary liquidity trader's portfolio trade. The demands of these traders for security fare denoted by $w_i l_1, \ldots, w_i l_p$, $i = 1, \ldots, N$. We define $w_i l$ to be the *total* demand of the discretionary liquidity traders for security i, thus $w_i l = w_i \cdot \sum_{q=1}^{p} l_q$. Since each discretionary liquidity trader replicates the portfolio represented by the basket, the total order submitted by discretionary liquidity traders in the basket will equal l. The *total* demands of the nondiscretionary traders in security *f* are denoted by $y_i = w_i r_i$, where r_i, $i = 1, \ldots, N$, are mutually independent and identically distributed with mean zero and $\text{var}(r_i) = \text{var}(r)$.[10] This assumption allows for cross-sectional heterogeneity in the nondiscretionary liquidity trades in the individual securities by postulating that securities that are heavily weighted in the basket have large amounts of nondiscretionary liquidity trading, which is plausible for a value-weighted index, such as the S&P 500. Thus, our assumptions imply

[10] The analysis of this section and the next remains unchanged even if $r_1, \ldots, r_N$ are assumed to be correlated.

that the standard deviations of both the nondiscretionary and discretionary liquidity trade in security i are proportional to w_i, $i = 1, \ldots, N$.[11] The total nondiscretionary liquidity trade in the basket is denoted by m. The random variables l_q, r_i, m, γ, and ε_i, $q = 1, \ldots, p$, $i = 1, \ldots, N$, are assumed to be mutually independent and multivariate normally distributed, each with a mean of zero.

For simplicity, we assume that discretionary liquidity traders must realize their entire trades either in the individual securities or in the basket. Although such an assumption can be justified on the basis of per-trade transaction costs, we show in Appendix B that, provided the number of securities is large, our basic results continue to obtain even if the discretionary liquidity traders are allowed to allocate their trades between the individual securities and the basket.

We now consider the set of Nash equilibria that arise in our model. Note that despite the fact that they do not possess information about systematic factors, risk-neutral informed traders will find it profitable to submit orders in the basket as well as in the individual securities about which they are informed, because possessing information about a security is tantamount to possessing noisy information about the basket. Assuming that the market-maker in a particular market does not observe the order flow in the other markets, we first analyze the depth of the market for the basket for a *given* level of liquidity trades in each market. Let z_m denote the net liquidity trade in the basket. The following lemma characterizes the equilibrium in the market for the basket.

Lemma 2. *Each trader possessing information about security i submits an order x_{im} in the basket, where*

$$x_{im} = w_i \epsilon_i / \lambda_m (k_i + 1). \tag{8}$$

The equilibrium pricing parameter λ_m in the market for the basket is given by

$$\lambda_m = \sqrt{\left(\sum_{i=1}^{N} w_i^2 \operatorname{var}(\epsilon_i) \frac{k_i}{(k_i + 1)^2}\right)[\operatorname{var}(z_m)]^{-1}}. \tag{9}$$

To minimizenotational complexity in the expressions that follow, we now define

$$K_i \equiv (k_i + 1)^2/k_i, \qquad i = 1, \ldots, N.$$

[11] The assumption of proportionality of the standard deviation of the nondiscretionary liquidity trades In the securities to the security weight w_i Is not crucial to our analysis and intuition. The assumption, however, captures in a stylized manner the notion that heavily weighted stocks (which, in general, can be expected to be stocks of large firms) would tend to have large amounts of nondiscretionary liquidity trading. In addition, the assumption slightly simplifies the exposition.

Note that K_i is increasing in the number of informed traders possessing information about security *i*. From Equations (6) and (9), it can be seen that the reciprocal of K_i represents the effect of the number of informed traders on the depths of the various markets.

Consider now a discretionary liquidity trader *q*. Since this trader knows that his demand for security *i* is $w_i l_q$, his losses to informed traders are given by $\sum_{i=1}^{N} w_i^2 \lambda_i (l_q)^2$ or $\lambda_m (l_q)^2$ [see (7)], depending on whether he realizes his trades in the individual securities or the basket. As all pricing parameters decrease monotonically in the total variances of liquidity trading in the respective markets,[12] the losses of each of the discretionary liquidity traders are lowest when they *all* trade in either the individual securities or the basket. The losses of each of the discretionary liquidity traders are minimized when they all trade in the basket if λ_m with all the discretionary liquidity trades concentrated in the basket is less than $\sum_{i=1}^{N} w_i^2 \lambda_i$ with all such trades concentrated in the securities. The following proposition states the condition under which this is true.

Proposition 1. *A necessary and sufficient condition for the losses of each discretionary liquidity trader to be minimized by concentration of trade in the basket is that*

$$\sum_{i=1}^{N} w_i \sqrt{\frac{\operatorname{var}(\epsilon_i)/K_i}{\operatorname{var}(l) + \operatorname{var}(r)}} > \sqrt{\frac{\sum_{i=1}^{N} w_i^2 [\operatorname{var}(\epsilon_i)/K_i]}{\operatorname{var}(l) + \operatorname{var}(m)}}. \quad (10)$$

Under this condition, there always exists a Nash equilibrium in which discretionary liquidity traders all trade in the basket.

Note that the variability (Le., the standard deviation) of private information about the basket $\sqrt{\sum_{i=1}^{N} w_i^2 \operatorname{var}(\epsilon_i)}$ is less than the weighted sum of the variabilities of private information about the individual securities $\sum_{i=1}^{N} w_i \sqrt{\operatorname{var}(\epsilon_i)}$, because of the "diversification" or "information offset" effect of the independent trades of the security-specific informed traders in the basket. Thus, the total effect of informed trading is less damaging to the discretionary liquidity traders in the basket than in the individual securities. However, if the nondiscretionary liquidity trade in the securities is heavy, the tendency for (10) to hold is weakened and the transaction costs of the discretionary liquidity traders may be lower in the securities. A high nondiscre-

[12] This comparative statics result also forms the basis for Admati and Pfleiderer's (1988) "temporal concentration" result; that liquidity traders who can time their trades have a tendency to concentrate their trading in a single period.

tionary trade in the basket makes the market for the basket deeper and strengthens the tendency of (10) to hold.

It is evident that if condition (10) holds, then one Nash equilibrium of our trading game involves all discretionary traders trading in the basket. However, depending on the parameter values, there can exist up to three classes of Nash equilibria for this game. Under the assumption that (10) holds, all these equilibria are described below.

(i) If

$$\sum_{i=1}^{N} w_i \sqrt{\frac{\operatorname{var}(\epsilon_i)/K_i}{\operatorname{var}(l) + \operatorname{var}(r)}} > \sqrt{\frac{\sum_{i=1}^{N} w_i^2[\operatorname{var}(\epsilon_i)/K_i]}{\operatorname{var}(m)}}, \quad (11)$$

then the only Nash equilibrium of our trading game is that all discretionary liquidity traders trade in the basket.

(ii) If the inequality (11) does not hold, there can exist two other classes of Nash equilibria.

(a) n discretionary liquidity traders trade in the basket, and $p - n$ trade in the individual securities, where n is the number such that the variances of the demands of the n discretionary liquidity traders who trade in the basket and the variances of the demands of the $p - n$ discretionary liquidity traders who trade in the individual securities satisfy the relation

$$\lambda_m = \sum_{i=1}^{N} w_i^2 \lambda_i.$$

Because of discreteness, several values of n may satisfy the above equality, and such a value of n may or may not exist.[13]

(b) All discretionary liquidity traders trade in the securities.

Equilibria of the class (ii) (a) are not robust to small perturbations in parameter values in the sense that if an equilibrium of this class exists for a particular set of parameter values, for an arbitrarily close set of parameter values [obtained, e.g., by perturbing var(l)] the only possible Nash equilibria are those in which the discretionary liquidity traders all concentrate their trading either in the individual securities

[13] For example, suppose that $N = 2$, $\operatorname{var}(r) = \operatorname{var}(m) = 0$, $[\operatorname{var}(\epsilon_1)/K_1] = [\operatorname{var}(\epsilon_2)/K_2]$, $w_1 = w_2$, and that there are six discretionary liquidity traders with var(l_q) ($q = 1, \ldots, 6$) = 2, 2, 4, 6, 2, and 2. so that var(l) = 18. It can easily be verified that $\lambda_m = \frac{1}{4}(\lambda_1 + \lambda_2)$ when the allocation of var(l) to the basket and the securities is 6 and 12, respectively. This allocation can be achieved with $n = 1$, n = 2, and $n = 3$ in the above example. In the case in which there are three discretionary liquidity traders with var(l_q) ($q = 1, 2, 3$) = 3, 5, and 10 (again, summing to 18), no value of n satisfies the equality $\lambda_m = \frac{1}{4}(\lambda_1 + \lambda_2)$.

or in the basket.[14] Thus, as in Admati and Pfleiderer (1988), all robust Nash equilibria in our model involve concentration of trading. A general implication of the multiplicity of equilibria is that sufficient liquidity needs to be generated in any market for a new derivative security before it can become attractive for traders who use the underlying securities to realize their trades.[15,16]

For a large N, condition (10) can be expected to hold across a wide range of parameter values; in this case, the Nash equilibrium in which the discretionary liquidity traders all trade in the basket minimizes their trading costs. In the absence of coordination between these traders, however, it is difficult to predict the actual outcome of the game. Note that because of the diversification of security-specific private information in the basket, the tendency for condition (10) to hold is strong for a large N even if there is no nondiscretionary liquidity trading in the basket [i.e., if $\text{var}(m) = 0$]. For the remainder of this article, we will be concerned with the implications of the Nash equilibrium which is characterized by discretionary liquidity traders trading in the basket. In this equilibrium, informed traders trade both in individual security markets and in the basket, the informed trading in individual securities being supported by the nondiscretionary liquidity traders $R_1, \ldots, R_N$.[17]

2. The Effect of Systematic Factor-Informed Trading

In this section, we introduce traders who are informed about the systematic component of security values into the model. The number

[14] This concept of robustness is identical to that used by Admati and Pfleiderer (1988).

[15] Dybvig and Spatt (1983) discuss analogous multiple Nash equilibria that arise in the context of the adoption of innovations or standards. If the benefit to a particular agent from the adoption of an innovation depends on the number of other agents adopting the innovation, this gives rise to two kinds of equilibria: one in which no agent adopts, and another in which most agents do adopt. Milgrom and Roberts (1989) study a general class of "supermodular" games in which the marginal returns to increasing a player's strategy rise with increases in the competitors' strategies,

[16] An institutional example that illustrates the phenomenon of exchanges trying to induce artificial liquidity in markets for their new products is that of the Chicago Board of Trade (CBOT) providing traders with permits that enabled them to trade newly introduced GNMA futures contracts more cheaply than other CBOT products. See "CBOT to Launch Mortgage Contract," *Financial Times,* June 15, 1989, p. 28.

[17] The reader may well wonder whether the model is consistent with arbitrage by traders between the individual securities and the basket to remove price differences. It turns out, however, that there are no arbitrage opportunities in our model. To see this, first note that the price of the basket incorporates all the individual security initial values $\bar{S}_1, \ldots, \bar{S}_N$. Thus, the *unconditional* expected difference between the sum of the prices of the securities and the price of the basket is zero. Traders do not know the market clearing price when they submit their orders and no one group of traders can predict the total order flow in any market with precision. It follows that profitable arbitrage between the markets is not possible. Note also that the link between the markets is maintained by the informed traders, who trade in both the individual securities and the basket in equilibrium. The actual sizes of their trades are determined by the zero-profit condition imposed on the market-makers. For a model characterized by intermarket lags in revelation of information in which index arbitrage issues are analyzed, see Kumar and Seppi (1990).

of these "systematic factor," or, for brevity, "factor"-informed traders is denoted by g. We now relax the assumption that information traders are perfectly informed; however, for simplicity, we assume that all informed traders possessing security-specific information about a particular security and all factor-informed traders observe the same signal. Specifically, the security-specific informed traders in security i ($i = 1, \ldots, N$) are assumed to observe $\varepsilon_i + u_i$, where $\mathrm{var}(u_i) = \theta_i$, $i = 1, \ldots, N$, and the factor-informed traders are assumed to observe $\gamma + v$, where $\mathrm{var}(v) = \kappa$. Further, $u_i, \ldots, u_N, v$ are assumed to be mutually independent and independent of $\varepsilon_1, \ldots, \varepsilon_N, \gamma$, and all the liquidity trades, and are multivariate normally distributed with a mean of zero. Analogous to K_i, we define $G \equiv (g + 1)^2/g$. The following lemma describes the resulting equilibrium for a given level of liquidity trading in each market.

Lemma 3. *With factor-information trading the equilibrium pricing parameters in the basket and in security i for given liquidity trade variances $\mathrm{var}(z_m)$ and $\mathrm{var}(z_i)$ in these markets are, respectively, given by*

$$\lambda_m = \sqrt{\left[\left(\sum_{i=1}^{N} \frac{w_i^2 \,\mathrm{var}^2(\epsilon_i)}{K_i[\mathrm{var}(\epsilon_i) + \theta_i]}\right) + \frac{(\sum_{i=1}^{N} w_i \beta_i)^2 \,\mathrm{var}^2(\gamma)}{G[\mathrm{var}(\gamma) + \kappa]}\right][\mathrm{var}(z_m)]^{-1}}, \tag{12}$$

$$\lambda_i = \sqrt{\left(\frac{\mathrm{var}^2(\epsilon_i)}{K_i[\mathrm{var}(\epsilon_i) + \theta_i]} + \frac{\beta_i^2 \,\mathrm{var}^2(\gamma)}{G[\mathrm{var}(\gamma) + \kappa]}\right)[\mathrm{var}(z_i)]^{-1}}. \tag{13}$$

This lemma reflects the additional component of adverse selection faced by the market-makers due to their trades with the factor-informed traders.

To avoid notational clutter, we now define

$$T_{ei} \equiv \frac{\mathrm{var}^2(\epsilon_i)}{\mathrm{var}(\epsilon_i) + \theta_i}$$

and

$$T_g \equiv \frac{\mathrm{var}^2(\gamma)}{\mathrm{var}(\gamma) + \kappa}.$$

For given variances of the components of security value innovations, changes in the precision of information possessed by the security-specific and factor-informed traders are reflected in changes in T_{ei} and T_g, respectively. We now demonstrate that the tendency for the basket to serve as the lowest transaction cost market for the discretionary liquidity traders is strong even in the presence of traders possessing private information about systematic factors. The following proposition is the analog of Proposition 1.

Proposition 2. *With factor-information trading, a necessary and sufficient condition for the transaction costs of the discretionary liquidity traders to be minimized by concentration of trade in the basket is that*

$$\sum_{i=1}^{N} w_i \sqrt{\left(\frac{T_{ei}}{K_i} + \frac{\beta_i^2 T_g}{G}\right)[\mathrm{var}(l) + \mathrm{var}(r)]^{-1}}$$
$$> \sqrt{\left[\sum_{i=1}^{N} \frac{w_i^2 T_{ei}}{K_i} + \left(\sum_{i=1}^{N} w_i\beta_i\right)^2 \frac{T_g}{G}\right][\mathrm{var}(l) + \mathrm{var}(m)]^{-1}}. \quad (14)$$

Under this condition, there always exists a Nash equilibrium in which discretionary liquidity traders all trade in the basket.

Squaring the left-hand side and right-hand side in condition (14), it is straightforward to show that the condition will always hold if var(r) = var(m).[18] This result can be explained as follows. When the factor sensitivities (β_i's) are all of the same sign, the factor-informed traders trade on the basis of information that is perfectly correlated across securities, so that the effect of systematic factor-informed trading is symmetric in the basket and in the individual securities. When factor sensitivities differ in sign across securities, there is a diversification effect of *systematic* information in the basket which strengthens the tendency of (14) to hold. Since the security-specific informed trades continue to provide a diversification benefit in the basket, the transaction costs of the discretionary liquidity traders are still minimized when they trade in the basket. If var(m) > var(r), condition (14) holds more strongly. This follows because a high var(m) relative to var(r) tends to increase the liquidity of the market for the basket relative to that of individual security markets.

In general, if the diversification effect of security-specific information is sufficiently strong (as can be expected for the case of a basket comprised of a large number of securities), one may expect (14) to hold across a wide range of parameter values. If (14) does

[18] For the Identical weights case, var(r) = var(m) implies that the variances of the nondiscretionary trades in all markets are equal. In the case of heterogeneous weights, this equality implies that the variance of the basket equivalent of a nondiscretionary portfolio trade In the securities equals the variance of the nondicretionary trade in the basket.

hold, and if the discretionary liquidity traders trade in the basket, then $\lambda_m < \sum_{i=1}^{N} w_i^2\lambda_i$, implying that the adverse price impact of uninformed portfolio trades in markets for baskets will be lower than the aggregate adverse price impacts of corresponding trades in the underlying securities. Note that this result obtains because of two reasons: first, the security-specific informed traders provide a diversification benefit in the basket; and, second, liquidity trading is heavier in the basket than in the individual securities.

3. Implications for Price Behavior and Market Liquidity

This section uses the model of the previous section to develop some implications for the informativeness and variability of prices, market liquidity, and the relationship between the prices of the basket and its underlying portfolio. Throughout the analysis, we assume that if a basket is present in the economy, then condition (14) holds and the Nash equilibrium characterized by discretionary liquidity traders trading in the basket obtains. In the absence of the basket, liquidity traders L and $R_1, \ldots, R_N$ trade in the individual securities.

3.1 Informativeness of prices and market liquidity

Informativeness of prices. We begin by making the information acquisition endogenous to the model by assuming that each trader pays a fixed cost to acquire his information.[19] Let g' be the number of factor-informed traders and k'_i be the number of informed traders possessing security-specific information about security *i* in the *absence* of a basket. Define $G' \equiv (g' + 1)^2/g'$ and $K'_i \equiv (k'_i + 1)^2/k'_i$. The following lemma describes the gross expected profits to security-specific and factor-informed traders in the presence and in the absence of a basket, *given* the number of informed traders in each market.

Lemma 4. *(i) In the absence of a basket, the gross expected profits of each factor-informed trader and each informed trader possessing security-specific information about security i [denoted by $\pi'_g(g', k'_1, \ldots, k'_N)$ and $\pi'_i(g', k'_i)$, respectively] are given by*

$$\pi'_g(g', k'_1, \ldots, k'_N) = \sum_{i=1}^{N} \frac{w_i\beta_i^2 T_g}{g'G'} \sqrt{\frac{\text{var}(l) + \text{var}(r)}{T_{ei}/K'_i + [\beta_i^2 T_g]/G'}}, \quad (15)$$

[19] Note that if the class M is nonempty [i.e., if $\text{var}(m) > 0$], then there Is more *aggregate* liquidity trading in the presence of the basket than in its absence. One might wonder whether the results we derive In this subsection are sensitive to this feature of our model, since Increased liquidity trading tends to promote Information collection. However, our results apply even if $\text{var}(m) = 0$, so that the class M is not crucial to the analysis of this section.

$$\pi_i'(g', k_i') = \frac{w_i T_{ei}}{k_i' K_i'} \sqrt{\frac{\mathrm{var}(l) + \mathrm{var}(r)}{T_{ei}/K_i' + [\beta_i^2 T_g]/G'}}. \tag{16}$$

(ii) In the presence of the basket, these profit [denoted by $\pi_g(g, k_1, \ldots, k_N)$ *and* $\pi_i(g, k_1, \ldots, k_N)$, *respectively] are given by*

$$\pi_g(g, k_1, \ldots, k_N) = \sum_{i=1}^{N} \frac{w_i \beta_i^2 T_g}{gG} \sqrt{\frac{\mathrm{var}(r)}{T_{ei}/K_i + [\beta_i^2 T_g]/G}}$$
$$+ \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \frac{T_g}{gG} \sqrt{\frac{\mathrm{var}(l) + \mathrm{var}(m)}{\Sigma_{i=1}^N [w_i^2 T_{ei}/K_i] + (\Sigma_{i=1}^N w_i \beta_i)^2 T_g/G}}, \tag{17}$$

$$\pi_i(g, k_1, \ldots, k_N) = \frac{w_i T_{ei}}{k_i K_i} \sqrt{\frac{\mathrm{var}(r)}{T_{ei}/K_i + [\beta_i^2 T_g]/G}}$$
$$+ \frac{w_i^2 T_{ei}}{k_i K_i} \sqrt{\frac{\mathrm{var}(l) + \mathrm{var}(m)}{\Sigma_{i=1}^N [w_i^2 T_{ei}/K_i] + (\Sigma_{i=1}^N w_i \beta_i)^2 T_g/G}}. \tag{18}$$

The first and second terms in the right-hand side of Equations (17) and (18) denote the profits of the informed traders from trading in the individual securities and the basket, respectively. It can be seen that the profits of a particular type of informed trader decrease in the number of other informed traders of the same type (which is intuitive). Also, in the presence of a basket, the profits of traders possessing security-specific information about security *i increase* in the number, of security-specific informed traders possessing information about the *other* $N - 1$ securities. In addition, the profits of all security-specific informed traders increase in the number of factor-informed traders and vice versa. The latter results are due to the fact that an increase in the number of a particular type of informed traders intensifies competition between traders of that type, and this benefits the *other* types of informed traders.

Let c_i and c_g denote the costs of acquiring security-specific information about security i and systematic factor information, respectively. The equilibrium number of informed traders possessing security-specific information about security i ($i = 1, \ldots, N$) and factor-informed traders are approximately given by either the k_i or g satisfying the equations $\pi_i(g, k_1, \ldots, k_N) = c_i$, $i = 1, \ldots, N$, and $\pi_g(g, k_1, \ldots, k_N) = c_g$ or the k_i' and g' satisfying $\pi_i'(g', k_i') = c_i$, $i = 1, \ldots, N$, and $\pi_g'(g', k_1', \ldots, k_N') = c_g$, respectively, depending on the presence or the absence of a basket. Closed-form solutions to the equilibrium number of security-specific and factor-informed traders are not possible in general. Our goal is to identify cross-sectional implications of our analysis and examine special cases to illustrate the manner in which the basket may be expected to change incentives to collect different types of information, without attempting to solve for the equilibrium number of informed traders of each type.

We define g_c [corresponding to $G_c \equiv (g_c + 1)^2/g_c$] to be the maximum number of factor-informed traders in the presence of the basket that is supported by the model parameters. Thus, g_c is that number that approximately satisfies the equation $\pi_g(g_c, 0, \ldots, 0) = c_g$, and is therefore the number of factor-informed traders that equates the per capita profits of these traders to the cost of acquiring the information under the restriction that there is nosecurity-specific informed trading in any security or the basket. Analogously, k_{ic} [corresponding to $K_{ic} \equiv (k_{ic} + 1)^2/k_{ic}$] is defined to be the maximum number of informed traders possessing security-specific information about security i supported by the model parameters in the presence of the basket. Thus, k_{ic} approximately satisfies the equation $\pi_i(0, 0, \ldots, 0, k_{ic}, 0, \ldots, 0) = c_i$.

First, in an attempt to derive cross-sectional implications from our analysis, we provide sufficient conditions under which the equilibrium number of security-specific informed traders increases or decreases after the introduction of the basket in the following proposition, which is a direct consequence of the comparative statics associated with Lemma 4.

Proposition 3. *Upon introduction of a basket, the number of traders possessing security-specific information about security i increases if*

$$\pi_i(1, \ldots, 1, k_i', 1, \ldots, 1) > \pi_i'(g', k_i') \tag{19}$$

and decreases if

$$\pi_i(g_c, k_{1c}, \ldots, k_{(i-1)c}, k_i', k_{(i+1)c}, \ldots, k_{Nc}) < \pi_i'(g', k_i'). \tag{20}$$

While the tendency for the above conditions to hold clearly depends upon the exogenous parameter values, there is an interesting cross-sectional variation in this tendency. Specifically, upon substituting from (16) and (18) of Lemma 4, it can be seen that the tendency for (19) to hold is stronger for heavily weighted securities than for securities that have small weights in the basket. Conversely, for low-weight securities, condition (20) is likely to hold. The introduction of a basket therefore has a tendency to *increase* the number of security analysts for the *most* heavily weighted securities in the basket (e.g., IBM in the case of the S&P 500 index) and to *decrease* the same for securities that have smaller weights in the index. The intuition for this result is that the profits of security-specific informed traders in heavily weighted securities are substantial in the market for the basket and for a sufficiently large w_i, the number of such traders may increase after a basket is introduced.

We now show that if the heterogeneity in the model parameters T_{ei}, K_i, K_{ic}, w_i, and β_i is sufficiently small, for a large N, the introduction of a basket will typically result in an increase in the number of factor-informed traders and a decrease in the number of security-specific informed traders. To this end, we now consider the stylized special case of homogeneous securities and an equally weighted basket, that is, we assume that $w_i = 1/N$, $\beta_i = 1$, $\text{var}(\varepsilon_i) = \text{var}(\varepsilon)$, $\theta_i = \theta$, $c_i = c$, and $k_i = k$, $i = 1, \ldots, N$, and suppress the i subscript from $T_{ei} K_i$, and K_{ic}. The following proposition, which follows directly from Lemma 4, provides sufficient conditions under which $k < k'$ and $g > g'$.

Proposition 4. *Under the above special assumptions, (i) the number of factor-informed traders increases upon introduction of a basket if*

$$\sqrt{\frac{\text{var}(r)}{T_e/4 + T_g/G'}} + \sqrt{\frac{\text{var}(l+m)}{T_e/4N + T_g/G'}} > \sqrt{\frac{\text{var}(l+r)}{T_e/K' + T_g/G'}}; \tag{21}$$

(ii) the number of security-specific information traders decreases upon introduction of a basket if

$$\sqrt{\frac{\text{var}(r)}{T_e/K' + T_g/G_c}} + \sqrt{\frac{\text{var}(l+m)}{T_e/K' + (N-1)T_e/K_c + N^2 T_g/G_c}} < \sqrt{\frac{\text{var}(l+r)}{T_e/K' + T_g/G'}}. \tag{22}$$

If N is large, the tendency for conditions (21) and (22) to hold is strong. This can be explained as follows. The profits of security-specific informed traders in the basket decrease in N, as a result of the adverse selection induced by other security-specific informed traders and the factor-informed traders in the basket. Thus, these traders cannot profit to a great extent by trading in a basket comprised of a large number of securities. In addition, the profits of these traders in the individual securities decrease because some liquidity traders switch to the basket. Because of these reasons, the introduction of the basket tends to decrease the number of security-specific informed traders. The profits of the factor-informed traders in the basket, however, *increase* in N.[20] This is because the diversification effect of security-specific information in the basket becomes stronger as N increases, which benefits the factor-informed traders. Thus, the introduction of the basket tends to increase the number of factor-informed traders.

We now define the *noisiness* of prices as the variance of the deviations of prices from their fundamental values. Informativeness is thus inversely related to noisiness. Let Q_i, Q_m, and Q_s denote the noisiness of the prices of security i, the basket, and the portfolio of individual securities, respectively. These measures are thus defined as follows:

$$Q_i \equiv \operatorname{var}(S_i - P_i), \qquad 1, \ldots, N,$$

$$Q_m \equiv \operatorname{var}(S_m - P_m),$$

$$Q_s \equiv \operatorname{var}\left(S_m - \sum_{i=1}^{N} w_i P_i\right).$$

It is straightforward to show that, in equilibrium, the values of Q_i, Q_m, and Q_s are given by the following:

$$Q_i = \left(\operatorname{var}(\epsilon_i) - \frac{T_{\epsilon i} k_i}{k_i + 1}\right) + \left(\operatorname{var}(\gamma) - \frac{T_g g}{g + 1}\right), \tag{23}$$

$$\begin{aligned} Q_m = {} & \sum_{i=1}^{N} w_i^2 \left(\operatorname{var}(\epsilon_i) - \frac{T_{\epsilon i} k_i}{k_i + 1}\right) \\ & + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \left(\operatorname{var}(\gamma) - \frac{T_g g}{g + 1}\right), \end{aligned} \tag{24}$$

$$\begin{aligned} Q_s = {} & \sum_{i=1}^{N} w_i^2 \left(\operatorname{var}(\epsilon_i) - \frac{T_{\epsilon i} k_i}{k_i + 1}\right) + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \\ & \cdot \left(\operatorname{var}(\gamma) - \frac{T_g g}{g + 1} - \frac{T_g}{G}\left[\left(\sum_{i=1}^{N} w_i \beta_i\right)^2 - \sum_{i=1}^{N} w_i^2 \beta_i^2\right]\right). \end{aligned} \tag{25}$$

The above expressions imply that the noisiness of prices is decreasing in the number of informed traders and the precision of the informed

[20] This an be easily verified by examining the second term In the right-hand side of Equation (17) under the special assumptions.

traders' signals, in keeping with intuition. *Given* the number of informed traders, the noisiness is independent of the liquidity trades because an increase in the variance of liquidity trades causes informed traders to scale up their activity proportionately and thus leaves the information content of prices unchanged. However, if the number of informed traders is *endogenous,* an increase in the variance of liquidity trading increases the returns to being informed, leading to an entry of more informed traders and, consequently, to an increase in the informativeness of prices.

Each of the expressions in (23)-(25) additively decomposes into two parts: the noisiness in the security-specific component and the noisiness in the systematic component. We first compare the noisiness of the prices of the basket and its underlying portfolio in the following proposition, which follows directly from Equations (24) and (25).

Proposition 5. *If the factor sensitivities β_i are all of the same sign, the price of the basket is more noisy in the systematic component than the price of the portfolio of individual securities. The price of the basket and the price of the portfolio are equally informative in the security-specific component.*

The somewhat surprising result that the price of the portfolio is less noisy in the systematic component than the price of the basket when factor sensitivities are ail of the same sign is due to the fact that there is a diversification effect of the uncorrelated nondiscretionary trades in the securities, which reduces the noise induced by the perfectly correlated trades of the factor-informed traders in the portfolio relative to the basket. If factor sensitivities differ in sign across securities, the diver&cation effect of systematic information counteracts the diversification effect of the nondiscretionary traders; consequently, the difference between the noisiness of the basket price and that of the portfolio price is of ambiguous sign.

In the next proposition, which is a direct consequence of Equations (23)-(25), we examine how the informativeness of prices in the two components of security values changes when a basket is introduced.

Proposition 6. *(a) Upon introduction of a basket, the price of security i becomes less noisy in the systematic component if and only if $g > g'$, and less noisy in the security-specific component if and only if $k_i > k'_t$.*

(b) Upon introduction of a basket, the price of the portfolio of securities becomes less noisy in the systematic component if and only if $g > g'$, and less noisy in the security-specific component if and only if

$$\sum_{i=1}^{N} w_i^2 \frac{T_{ei}k_i'}{k_i'+1} < \sum_{i=1}^{N} w_i^2 \frac{T_{ei}k_i}{k_i+1}. \tag{26}$$

For heavily weighted securities, Proposition 3 implies that the number of security-specific informed traders may increase on introduction of the basket. From (a) of the above proposition, this implies that prices of such securities will become more informative in the security-specific component. Conversely, for securities whose basket weights are smaller, the number of security-specific informed traders tends to decrease and prices of these securities may be expected to become less Informative in the security-specific component. Turning to (b) of the above proposition, since heavily weighted securities dominate the 'weighted sums in (26), if indeed the informativeness of such securities increases, the informativeness of the portfolio in the security-specific component may also increase.

Gammill and Perold (1989) voice the concern that the introduction of markets in baskets can decrease the informativeness of stock prices by decreasing liquidity trading in individual stock markets. Our analysis implies that price informativeness in the security-specific component of stocks that are heavily weighted in the basket may well *increase* because of the increased profits from security-specific information trading for such stocks after the introduction of the basket. For less heavily weighted stocks, our analysis formalizes the intuition in Gammill and Perold (1989) that baskets can reduce the returns to trading on stock-specific information by causing migration of liquidity traders to the basket and, consequently, decrease the price informativeness In the security-specific component.

Finally, if the cross-sectional variation in basket weights is small and N is large, so that the diversification effect of security-specific information in the basket is sufficiently strong, from Proposition 4, the introduction of the basket may be expected to increase the informativeness of individual security prices in the systematic component and decrease their informativeness in the security-specific component.

It is also of interest to examine how the *overall* informativeness of the prices of the portfolio and the individual securities changes when a basket is introduced. The following proposition also follows directly from Equations (23)-(25).

Proposition 7. *(a) Upon introduction of a basket, the overall noisiness of the price of security i, Q_i, decreases if and only if*

$$\frac{T_g g}{g+1} > \frac{T_g g'}{g'+1} + \frac{T_e k_i'}{k_i'+1} - \frac{T_e k_i}{k_i+1}. \tag{27}$$

(b) Upon introduction of a basket, the overall noisiness of the price of the portfolio of individual securities, Q_s, decreases if and only if

$$\frac{T_g g}{g+1} + \frac{T_g}{G}\left[\left(\sum_{i=1}^{N} w_i\beta_i\right)^2 - \sum_{i=1}^{N} w_i^2\beta_i^2\right]$$
$$> \frac{T_g g'}{g'+1} + \frac{T_g}{G'}\left[\left(\sum_{i=1}^{N} w_i\beta_i\right)^2 - \sum_{i=1}^{N} w_i^2\beta_i^2\right]$$
$$- \left(\sum_{i=1}^{N} w_i^2\left[\frac{T_e k_i'}{k_i'+1} - \frac{T_e k_i}{k_i+1}\right]\right)\left[\left(\sum_{i=1}^{N} w_i\beta_i\right)^2\right]^{-1}. \quad (28)$$

Under the homogeneity and identical weight assumptions made in deriving Proposition 4, condition (28) in the above proposition becomes

$$\frac{T_g(g^2 + 2g - g/N)}{(g+1)^2} > \frac{T_g(g'^2 + 2g' - g'/N)}{(g'+1)^2}$$
$$+ \left[\frac{T_e k'}{k'+1} - \frac{T_e k}{k+1}\right]N^{-1}. \quad (29)$$

If $g > g'$, the left-hand side of (29) is greater than the first term of the right-hand side. Therefore, from Proposition 4, if N is large, condition (29) holds across a wide range of parameter values; in this case, the noisiness of the portfolio Q_s may be expected to decrease. The reason for this is that the information possessed by security-specific informed traders is unimportant relative to factor information for a large N and, therefore, the increase in informativeness in the systematic component dominates the decrease in informativeness in the security-specific component. From (27), however, the effect of the introduction of the basket on the overall informativeness of the prices of individual securities is ambiguous and depends on the parameter values.

Market liquidity. The effect of the basket on market liquidity may be analyzed by computing the ratio of the pricing parameters for security i in the presence and absence of the basket. Denoting λ_{ib} and λ_{in} to be the pricing parameters for security i in the presence and the absence of the basket, respectively, this ratio, from (6), is

$$\frac{\lambda_{ib}}{\lambda_{in}} = \sqrt{\frac{T_{ei}/K_i + T_g/G}{T_{ei}/K_i' + T_g/G'}}\sqrt{\frac{\mathrm{var}(I) + \mathrm{var}(r)}{\mathrm{var}(r)}}. \quad (30)$$

If information acquisition is assumed to be exogenous, this ratio is greater than unity (i.e., security markets are less liquid in the presence of a basket than in its absence), and is identical across securities. The empirical implication is that although the basket tends to reduce the liquidity of the component securities, to the extent that large weight securities tend to have large amounts of both discretionary and non-discretionary liquidity trading, the percentage decrease in liquidity across securities will not show any significant variation in cross section.

Under endogenous information acquisition, the effect of a basket on the liquidity of component security markets is ambiguous. We now make the empirically plausible assumption that the adverse selection due to factor information trading (represented by T_g/G' and T_g/G) is small,[21] so that the right-hand side of (30) may be approximated as

$$\sqrt{\frac{K_i'}{K_i}}\ \sqrt{\frac{\operatorname{var}(l) + \operatorname{var}(r)}{\operatorname{var}(r)}}\,.$$

Now, from Proposition 3, K_i'/K_i tends to be less than unity for heavily weighted securities. Thus, if the weights of these securities are sufficiently large, the markets in these securities may actually become more liquid after the basket is introduced. For securities with smaller weights in the basket, the ratio $\lambda_{ib}/\lambda_{in}$ may be expected to increase. The following proposition follows directly from Equation (30) and Proposition 3.

Proposition 8. *If (20) holds for security i, and if the adverse selection induced by factor-information trading is sufficiently small, the liquidity of the market for security i decreases after the basket is introduced.*

Our discussion implies that the basket may have a heterogeneous effect on market liquidities if there is large cross-sectional variation in basket weights–specifically, the ratio of market liquidity in the presence of a basket to that in its absence may be expected to be lower for heavily weighted securities than for securities whose weights in the basket are small. If the cross-sectional variation in weights is small and the number of securities is large, the introduction of a basket may be expected to decrease the liquidity of all the component security markets.

[21] Casual empiricism suggests that the precision of systematic information is considerably lower than that of security-specific information, as systematic private information typically consists of macroeconomic forecasts based on *public* information, whereas security-specific information is usually possessed by insiders in the relevant firm whose information is often very precise relative to that of the general public.

We now informally discuss the effect of the introduction of a basket on the price informativeness and liquidity of securities that are *not* part of the basket in the above framework.[22] Assuming that there is no discretionary liquidity trading in such securities but that their values are correlated with the common factor γ, the discussion following Proposition 4 implies that if the diversification effect of security-specific information is sufficiently strong, the introduction of a basket tends to increase the number of factor information traders in the market for such securities. This increase leads to increased competition between the factor informed traders, which increases incentives to collect security-specific information and results in an increase in the number of security-specific informed traders in nonbasket securities. In this case, prices of nonbasket securities may be expected to become *more* informative and more liquid upon introduction of a basket. This result must be interpreted cautiously, however. First, to the extent that well-diversified portfolios are good substitutes for each other, one would expect some discretionary liquidity trading also to be present in widely held nonbasket stocks. Second, we have assumed that factor information traders trade in all securities and the basket. While it may be plausible to assume that traders exploit private factor information in the widely held stocks, such as those on the Major Market Index or the S&P 500 list, this may not be the case for stocks of smaller of less widely held firms.

3.2 Price variability

To analyze the effect of the introduction of a basket on price variability, we extend our model to many periods. We consider N securities (indexed by $i = 1, \ldots, N$) simultaneously traded over T periods. The initial value of security i is $\bar{S}_i$ and its value at time T is given by

$$S_i = \bar{S}_i + \sum_{t=1}^{T} (\beta_i \gamma_t + \epsilon_{it}). \tag{31}$$

Informed traders observe either $\epsilon_{it+1} + u_{it}$ or $\gamma_{t+1} + v_t$ at time t and take positions based on the same. All liquidity trades, security value innovations, and signal noises are serially uncorrelated, mutually independent, and multivariate normally distributed with zero mean. Private information is thus useful for only one period. The variances of all random variables are assumed to be constant over time. This is the natural extension to, the one-period model of Section 1. Subscripting security value innovations with $t + 1$ and all other random variables and numbers of informed traders of each type with time t,

[22] I am grateful to the referee for suggesting the discussion in this paragraph.

our results generalize in a straightforward manner to many periods. Let $\mathrm{var}(\gamma_t) = \mathrm{var}(\gamma)$ and $\mathrm{var}(\varepsilon_{i\,t}) = \mathrm{var}(\varepsilon_i)$ for all $t = 1, \ldots, T$. It can easily be shown that the variance of the price change V_{it} in a particular security i at time t is given by

$$V_{it} = \frac{\mathrm{var}(\epsilon_i)}{\mathrm{var}(\epsilon_i) + \theta_i}\left(\frac{k_{it}}{1 + k_{it}}\mathrm{var}(\epsilon_i) + \frac{1}{1 + k_{it-1}}\mathrm{var}(\epsilon_i) + \theta_i\right)$$
$$+ \frac{\beta_i^2\mathrm{var}(\gamma)}{\mathrm{var}(\gamma) + \kappa}\left(\frac{g_t}{1 + g_t}\mathrm{var}(\gamma) + \frac{1}{1 + g_{t-1}}\mathrm{var}(\gamma) + \kappa\right). \quad (32)$$

The above expression demonstrates that if the number of informed traders is constant over time, the variance of price changes is also constant and *independent* of the number of informed traders. This result implies that prices reflect information earlier with some informed traders than without any informed traders, but the *rate* of information flow to the market is the same in the presence and in the absence of informed trading.[23] It is easy to construct numerical examples demonstrating that if the number of informed traders is not constant overtime, then the effect of the introduction of a basket on the variance of price changes of individual securities is ambiguous, The following proposition follows directly from (32).

Proposition 9. *If the number of informed traders is constant over time, the introduction of a basket has no effect on the variance of the price changes of individual securities. If the number of informed traders is not constant over time, the effect of the introduction of a basket on individual security price variability is ambiguous.*

The above result implies that the introduction of a basket has no effect on the variance of price changes in component securities if the number of informed traders is constant over time, even if the number of informed traders is different in the presence of a basket than in the absence of one. Note that our results on price variability (summarized by the above proposition) also apply for nonbasket securities.

Extant studies on the effects of the introduction of index futures contracts on cash price volatility have produced ambiguous results. For example, Edwards (1988) finds that the introduction of the S&P 500 futures contract has reduced cash market volatility; Davis and White (1987) conclude that the volatility of the cash market has not been affected by the introduction of the index futures contracts; and Harris (1989) claims that the volatility of S&P 500 stocks have shown

[23] Admati and Pfleiderer (1988) derive a similar result in their article.

a statistically significant increase relative to that of a control group of similar stocks not on the index after the introduction of the S&P 500 futures contract. (Harris states, however, that the increase is negligible in economic terms.) Our model fits in well with this ambiguous evidence.

3.3 The price relationship between the basket and the portfolio
In this subsection, we describe the behavior of the "basis," or the difference between the price of the basket and that of its underlying portfolio, and examine whether price changes in the basket tend to predict subsequent price changes in the portfolio and vice versa. We continue to use the multiperiod model of the previous subsection.

The "basis." Let $B_t \equiv P_{mt} - \Sigma_{i=1}^{N} w_i P_{it}$ denote the basis at time t. Then we have

$$B_t = \lambda_{mt}\omega_{mt} - \sum_{i=1}^{N} w_i \lambda_{it}\omega_{it}. \tag{33}$$

Substituting for the informed trades from appropriately subscripted forms of Equations (A2)-(A5) of Appendix A, we have

$$B_t = \lambda_{mt}(l_t + m_t) - \sum_{i=1}^{N} w_i \lambda_{it} y_{it} = \lambda_{mt}(l_t + m_t) - \sum_{i=1}^{N} w_i^2 \lambda_{it} r_{it}. \tag{34}$$

Under our assumptions, the basis is a random variable that is independent of all security value innovations and consists of only liquidity trade components. Since informed traders have symmetric access to all markets, the order flows in the securities and in the basket are equally informative. Thus, the difference in the prices does not contain any term that involves a fundamental value innovation. The basis is a depth-weighted difference of the liquidity trades in the basket and the weighted sum of the liquidity trades in the securities. Under our assumptions, $\lambda_{mt} < \Sigma_{i=1}^{N} w_i^2 \lambda_{it}$. Thus, we would expect basket prices to move less in response to large liquidity orders than the price of the portfolio of individual securities. The following proposition follows directly from Equation (34) and the above discussion.

Proposition 10. *Under the above assumptions, a portfolio trade $w_i y$ ($i = 1, \ldots, N$) in the securities will induce a larger absolute basis than a corresponding trade y in the basket.*

"Lead-lag" relationships. The tendency of movements in one price to provide predictive information about subsequent movements in another price is commonly termed a "lead-lag" relationship. Some recent articles [e.g., Kawaller, Koch, and Koch (1987) and Chan (1989a)] have empirically examined lead-lag relationships between index futures markets and their underlying portfolios. In the context of our model, we now analyze the regression coefficient of the forecast of one-period ahead price changes on the current period's price changes. Thus, our measures of lead and lag are defined as follows:

$$\xi_{mt} \equiv \operatorname{cov}\left(\Delta \sum_{i=1}^{N} w_i P_{it}, \Delta P_{mt-1}\right)[\operatorname{var}(\Delta P_{mt-1})]^{-1}, \tag{35}$$

$$\xi_{st} \equiv \operatorname{cov}\left(\Delta P_{mt}, \Delta \sum_{i=1}^{N} w_i P_{it-1}, \right)\left[\operatorname{var}\left(\Delta \sum_{i=1}^{N} P_{it-1}\right)\right]^{-1}, \tag{36}$$

where A denotes a change from the previous period. A large ξ_{mt} relative to ξ_{st} would imply a large lead of the basket over the portfolio of individual securities and conversely. We now assume that the number of informed traders is constant over time-specifically, that $k_t = k$ and $g_t = g$ for all $t = 1, \ldots, T$. The following lemma, which follows directly from Equations (A2)-(A5) of Appendix A and Equations (12) and (13), gives the equilibrium values of ξ_{mt} and ξ_{st}.

Lemma 5. *In equilibrum,*

$$\xi_{mt} = \left[\sum_{i=1}^{N} \frac{w_i^2 T_{ei}}{K_i} + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \frac{T_g}{G}\right] \cdot \left[\sum_{i=1}^{N} w_i^2 \operatorname{var}(\epsilon_i) + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \operatorname{var}(\gamma)\right]^{-1} \tag{37}$$

and

$$\xi_{st} = \left[\sum_{i=1}^{N} \frac{w_i^2 T_{ei}}{K_i} + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \frac{T_g}{G}\right] \cdot \left\{\left[\sum_{i=1}^{N} w_i^2 \operatorname{var}(\epsilon_i) + \left(\sum_{i=1}^{N} w_i \beta_i\right)^2 \operatorname{var}(\gamma) - \left[\sum_{i=1}^{N} w_i \beta_i)^2 - \sum_{i=1}^{N} w_i^2 \beta_i^2\right]^2 \frac{T_g}{G}\right\}^{-1}. \tag{38}$$

Note that if the β_i's are all of the same sign, then $\xi_{st} > \xi_{mt}$. The intuition for this result is similar to that given for Proposition 5. Specifically, the diversification effect of the uncorrelated nondiscretionary trades in the securities reduces the variance of price changes in the portfolio of individual securities relative to that in the basket. The following proposition follows directly from (37) and (38).

Proposition 11. *If the factor sensitivities β_i are all of the same sign, the lead from the portfolio to the basket is larger than that from the basket to the portfolio. Movements in both the price of the basket and the price of the portfolio provide information about subsequent movements in the price of the other.*

The above result is broadly consistent with the empirical study of Chan (1989a), who studies the lead--lag relationship between the Major Market Index futures contract and its underlying cash index and finds that both futures prices and cash prices tend to lead each other significantly. The result, however, is inconsistent with the study of Kawaller, Koch, and Koch (1987), who find that the S&P 500 futures market tends to lead the S&P 500 index by 20 to 45 minutes, and that the lead from the index to the futures is less significant and does not last beyond one minute. We note here that institutional frictions, such as nonsynchronous trading in the underlying stocks, may also contribute to the lead of the index futures price over the index cash price. Also, unlike Kawaller, Koch, and Koch, who use the index cash price reported by the Chicago Mercantile Exchange, Chan (1989a) uses actual individual stock transaction data to compute the index cash price and thus avoids the potential delay bias associated with using the reported index.

4. Summary and Concluding Remarks

We have presented a theory of trading in markets for baskets of securities and have demonstrated that these markets allow liquidity traders to realize their trades more efficiently because their losses to informed traders are usually lower in baskets than in the individual securities. Our model explicitly incorporates trading by strategic liquidity traders who have discretion over the markets in which they may execute portfolio trades. Markets in baskets are shown to be advantageous for such traders because the security-specific component of adverse selection tends to get diversified away in such markets. When the factor sensitivities of all securities comprising the basket are of the same sign, the adverse selection induced by factor-information trading is found to be symmetric in the basket and the individual securities. If the factor sensitivities differ in sign across securities, the diversification of systematic information in the basket further reduces the adverse selection in the basket relative to that in the securities. Thus, the basket tends to serve as the lowest transaction cost market for such discretionary liquidity traders under a wide range of parameter values even when both security-specific and systematic components of adverse selection exist. Our analysis thus provides an explanation for the tremendous popularity of markets in stock index futures.

Some implications of our analysis are as follows.

- The adverse selection problem faced by market-makers (reflected in percent bid-ask spreads) can be expected to be less severe in markets for baskets of securities than in markets for individual securities. This implication is consistent with the empirical fact that proportional bid-ask spreads in index futures markets are roughly one tenth those in individual stocks. [See Gammill and Perold (1989).]
- The adverse price impact of trades in component securities may increase upon the introduction of a basket as a result of reduced liquidity trading in the securities. This increase is predicted to be more strongly evident for securities with smaller weights in the basket than for heavily weighted securities. This implication can be tested using individual stock transaction data before and after the introduction of the S&P 500 futures contract [e.g., by using the methodology of Glosten and Harris (1988)].
- If the weights of heavily weighted securities in the basket are large, the number of security analysts (and the informativeness of prices in the security-specific component) tends to increase for securities that are most heavily weighted In the basket and to decrease for securities with smaller weights.
- Provided the cross-sectional variation in basket weights across securities is sufficiently small and the number of securities is large, the introduction of a basket may be expected to (i) increase the overall informativeness of the price of the underlying portfolio and make this price more responsive to new systematic information and (ii) make individual security prices (and the price of the portfolio) less informative in the security-specific component and more informative in the systematic component.
- The introduction of a basket will have no effect on the variability of price changes of individual securities.
- There will be a strong tendency for both movements in the price of the basket and movements in the price of the portfolio to provide predictive information about subsequent price movements in the other.

In practice, however, not all securities are represented in a basket. For example, the Major Market Index futures contract is based on just 20 stocks. There is, however, a large degree of substitutability between various securities for diversification purposes. Therefore, we would expect many liquidity traders desirous of trading portfolios that do not correspond to the basket to realize their trades in the basket in order to minimize their adverse selection costs, The phenomenon of liquidity traders switching to the basket may therefore be expected to also cover many securities not in the basket. We also note that this article does not model the optimal choice of basket contracts by exchanges. Further research would attempt to explain reasons for the existence for a variety of basket contracts on different broad market indexes (e.g., the NYSE Composite, the Value Line, the Major Market, an the S&P 500 index future contracts, and the S&P 100 index option).

It has been alleged that the introduction of index futures contracts may destabilize prices by encouraging irrational speculation (noise trading). (See, e.g., "Playing With Fire," *Business Week,* September 16, 1985.) Our model implies that this need not necessarily be a cause for concern. In this model [as in that of Kyle (1984)], an increase in noise trading actually makes prices more informative by increasing the returns to being informed and thereby facilitating the entry of more informed traders.

Appendix A

Proof of Lemma 1. Note that normality and the zero profit condition imposed on the market-maker imply that λ_i is the regression coefficient in the forecast of $\varepsilon_i + \gamma$ on the order flow ω_i. Since informed traders observe only ε_i, we have

$$\lambda_i = \mathrm{cov}(\epsilon_i, \omega_i)/\mathrm{var}(\omega_i). \tag{A1}$$

The informed trader's objective function is

$$E(x_{ji}(S_i - P_i) \mid \epsilon_i).$$

Let the informed trader *j* conjecture that the order of all other informed traders is given by $\beta\varepsilon_i$. Thus, he maximizes

$$E((x_{ji}\epsilon_i - x_{ji}\lambda_i(x_{ji} + (k_i - 1)\beta\epsilon_i + z_i)) \mid \epsilon_i).$$

Maximizing this expression with respect to x_{ji} yields

$$x_{ji} = \left(\frac{1}{2\lambda_i} - \frac{(k_i - 1)\beta}{2}\right)\epsilon_i.$$

Setting this equal to $\beta\varepsilon_i$ yields $\beta = 1/(k_i + 1)\lambda_i$. Thus, we get (5). Substituting for x_{ji} in (A1), we get a quadratic equation in λ_i, the positive root of which yields (6). ■

Proof of Lemma 2. Let the security-specific informed trader (informed about security *i*) make the consistent conjecture that the demand of each of the other security-specific traders informed about security i in the basket is given by $\beta_i w_i \epsilon_i$ and that the demands of all other informed traders (given by x_{jm}, $j \neq i$) are independent of ε_i. Then this informed trader's objective function is

$$E\left(\left(x'_{im} \sum_{i=1}^{N} w_i \epsilon_i - x'_{im}\lambda_m\left(x'_{im} + (k_i - 1)\beta_i w_i(\epsilon_i) + \sum_{j \neq i} x_{jm}\right)\right)\Bigg|\epsilon_i\right).$$

The optimal trade of this trader is then

$$x_{im} = \left[\frac{1}{2\lambda_m} - \frac{(k_i - 1)\beta_i}{2}\right](w_i \epsilon_i).$$

Setting this equal to $\beta_i w_i \epsilon_i$ yields

$$\beta_i = 1/(k_i + 1)\lambda_m$$

and so $x_{im} = w_i \epsilon_i/(k_i + 1)\lambda_m$, $i = 1, \ldots, N$, yielding (8).

Noting that $\lambda_m = \mathrm{cov}(\Sigma_{i=1}^{N} w_i \epsilon_i, \omega_m)/\mathrm{var}(\omega_m)$ where ω_m is the total order flow in the market for the basket, substituting for x_{im} from above, and taking the positive root of the resulting quadratic equation yields (9). ∎

Proof of Proposition 1. The pricing parameter in security i if discretionary liquidity traders all trade in the securities is given by

$$\lambda_i = \sqrt{\frac{\mathrm{var}(\epsilon_i)}{w_i^2[\mathrm{var}(l) + \mathrm{var}(r)]K_i}}.$$

The pricing parameter in the basket when they trade in the basket is given by

$$\lambda_m = \sqrt{\left(\sum_{i=1}^{N} \frac{\mathrm{var}(w_i \epsilon_i)}{K_i}\right)[\mathrm{var}(l) + \mathrm{var}(m)]^{-1}}.$$

The condition $\lambda_m < \Sigma_{i=1}^{N} w_i^2 \lambda_i$ holds if and only if Equation (10) holds. The second part of the proposition follows from the fact that if discretionary liquidity traders all trade in the basket, then $\lambda_m < \Sigma_{i=1}^{N} w_i^2 \lambda_i$ [from (10)] and therefore do not wish to switch to the individual securities. ∎

Proof of Lemma 3. Let the security-specific informed trader (informed about security i) make the consistent conjecture that the demand of each of the other security-specific traders informed about security i in the basket is given by $\beta_i w_i(\epsilon_i + u_i)$ and that the demands of all other informed traders (again, given by x_{jm}, $j \neq i$) are independent of $\varepsilon_i + u_i$. Denoting this trader's order by x'_{im} and that of each of the factor-informed traders as x_{sm}, this informed trader's objective function is

$$E\left(\left(x'_{im}\sum_{i=1}^{N} w_i\epsilon_i - x'_{im}\lambda_m\left(x'_{im} + (k-1)\beta_i w_i(\epsilon_i + u_i) + \sum_{j\neq i} x_{jm} + gx_{sm}\right)\right)\middle|\,\epsilon_i + u_i\right).$$

The optimal trade of this trader is then

$$x_{im} = \left[\frac{\mathrm{var}(\epsilon_i)}{2\lambda_m(\mathrm{var}(\epsilon_i + \theta_i))} - \frac{(k_i - 1)\beta_i}{2}\right] w_i(\epsilon_i + u_i).$$

Setting this equal to $\beta_i w_i(\epsilon_i + u_i)$ yields

$$\beta_i = \mathrm{var}(\epsilon_i)/(k_i + 1)\lambda_m[\mathrm{var}(\epsilon_i) + \theta_i],$$

and so

$$x_{im} = (\epsilon_i + u_i)w_i\mathrm{var}(\epsilon_i)/(k_i + 1)\lambda_m[\mathrm{var}(\epsilon_i) + \theta_i]. \tag{A2}$$

Similarly, it can be shown that

$$x_{sm} = \frac{(\gamma + v)\sum_{i=1}^{N} w_i\beta_i\mathrm{var}(\gamma)}{(g + 1)\lambda_m[\mathrm{var}(\gamma) + \kappa]}. \tag{A3}$$

Noting that $\lambda_m = \mathrm{cov}(\sum_{i=1}^{N} w_i\epsilon_i + \sum_{i=1}^{N} w_i\beta_i\gamma, \omega_m)/\mathrm{var}(\omega_m)$, where ω_m is the total order flow in the market for the basket, substituting for x_{im} and x_{sm} from above, and taking the positive root of the resulting quadratic equation yield (12). Using a method similar to the above for the ith security market and denoting x_i and x_{si} to be the orders of each of the security-specific and factor-informed traders, we get

$$x_i = (\epsilon_i + u_i)\mathrm{var}(\epsilon_i)/(k_i + 1)\lambda_i[\mathrm{var}(\epsilon_i) + \theta_i], \tag{A4}$$

and

$$x_{si} = (\gamma + v)\beta_i\,\mathrm{var}(\gamma)/(g + 1)\lambda_i[\mathrm{var}(\gamma) + \kappa]. \tag{A5}$$

Now $\lambda_i = \mathrm{cov}(\epsilon_i + \beta_i\gamma, \omega_i)/\mathrm{var}(\omega_i)$, where ω_i is the total order flow in security i. Substituting for x_i and x_{si}, and taking the positive root of the resulting quadratic equation yields (13). ■

Proof of Proposition 2. This proof is identical to that of Proposition 1 in that it involves comparing $\sum_{i=1}^{N} w_i^2\lambda_i$ when discretionary liquidity traders all trade in the securities to λ_m when they all trade in the basket. ■

Proof of Lemma 4. Equations (15)-(18) are readily obtained by taking unconditional expectations of the expressions for the profits of each type of informed traders. ■

Appendix B

This appendix demonstrates that our basic results continue to obtain in the limit for a large number of securities even if discretionary liquidity traders are allowed to allocate their demands between the securities and the basket. Throughout this appendix, we assume that the number of informed traders is exogenously specified. Let us assume that a discretionary liquidity trader q trades a fraction f of his total demand in the securities and $1 - f$ in the basket. The trader minimizes his losses, taking the pricing parameters in each of the markets as given. His objective function is

$$\left(f^2 \sum_{i=1}^{N} w_i^2 \lambda_i + (1-f)^2 \lambda_m\right)(l_q^2).$$

This is minimized when

$$f = \frac{\lambda_m}{\sum_{i=1}^{N} w_i^2 \lambda_i + \lambda_m}. \tag{B1}$$

Note that f depends only on the pricing parameters and the weights; thus, all discretionary liquidity traders choose the same f. Since the variance of the liquidity trades in security i is proportional to w_i^2, we can write $\mathrm{var}(z_i) \equiv w_i^2\psi$. Denote $\mathrm{var}(z_m) \equiv \psi_m$. Consider first the case of Section 1, where only security-specific informed traders trade. Substituting for λ_i and λ_m from (6) and (9) in (B1), we have

$$f = \sqrt{\frac{\sum_{i=1}^{N} w_i^2[\mathrm{var}(\epsilon_i/K_i)]}{\psi_m}} \cdot\left(\sum_{i=1}^{N} \sqrt{\frac{w_i^2[\mathrm{var}(\epsilon_i/K_i)]}{\psi}} + \sqrt{\frac{\sum_{i=1}^{N} w_i^2[\mathrm{var}(\epsilon_i/K_i)]}{\psi_m}}\right)^{-1}. \tag{B2}$$

Note that since

$$\sum_{i=1}^{N} \sqrt{\frac{w_i^2 \mathrm{var}(\epsilon_i)}{K_i}} > \sqrt{\sum_{i=1}^{N} \frac{w_i^2 \mathrm{var}(\epsilon_i)}{K_i}},$$

the adverse selection in the basket is lower than that in the securities. We do not attempt to solve for ψ_m and ψ as this is a complicated algebraic exercise and it is not, in general, possible to find them in closed form. Instead, we examine limiting results for the case of homogeneous securities and an equally weighted basket as $N \to \infty$. Imposing the symmetry assumptions $w_i = 1/N$, $\mathrm{var}(\varepsilon_i) = \mathrm{var}(\varepsilon)$, and $K_i = K$, (B2) reduces to

$$f = \frac{\sqrt{\mathrm{var}(\epsilon)/K\psi_m}}{\sqrt{N\,\mathrm{var}(\epsilon)/K\psi} + \sqrt{\mathrm{var}(\epsilon)/K\psi_m}}. \tag{B3}$$

It is apparent from (B3) that as $N \to \infty$, $f \to 0$, which means that the discretionary liquidity trading becomes more and more concentrated in the basket as the number of securities becomes large. Thus, we have the following proposition.

Proposition B1. *As the number of securities become large, the fraction of discretionary liquidity trades realized in the individual securities approaches zero.*

It may be worthwhile to quantify the fraction of trades realized in the basket for different values of N. Among actively traded baskets, the one with the smallest number of stocks is the Major Market Index futures contract, which consists of 20 stocks. For the parameterization $\mathrm{var}(\varepsilon) = \psi_m = \psi$, if $N = 20$, the fraction of trades by discretionary 'liquidity traders in the basket is approximately 82 percent. Thus, even if the discretionary liquidity traders allocate their trades, the fraction of their trades in the basket may be large even for small values of N.

Repeating the same exercise as above for the case of Section 2, in which both factor and security-specific informed traders trade, we have (under the additional assumptions that $\beta_i = 1$ and $T_{ei} = T_e$, $i = 1, \ldots, N$)

$$f = \sqrt{\frac{T_e/(NK) + T_g/G}{\psi_m}} \cdot \left(\sqrt{\frac{T_e/K + T_g/G}{\psi}} + \sqrt{\frac{T_e/(NK) + T_g/G}{\psi_m}} \right)^{-1}. \tag{B4}$$

As $N \to \infty$,

$$f \to \frac{\sqrt{T_g/\psi_m G}}{\sqrt{(T_e/K + T_g/G)/\psi} + \sqrt{T_g/\psi_m G}}.$$

Thus, with $N = \infty$, $f \to 0$ as $T_g/T_e \to 0$ and/or $K/G \to 0$. This leads us to Proposition B2.

Proposition B2. *If the number of securities large, the discretionary liquidity trades tend to be more and more concentrated in the basket as the precision ofsystematic information becomes very small relative to the precision ofsecurity-specific information and/or as the number of security-specific informed traders becomes very small relative to the number of factor informed traders.*

In this case, for $N = 20$, if $K = G$, $\psi_m = \psi$, and $\mathrm{var}(\varepsilon) = \mathrm{var}(\gamma) = 1$, then, assuming that security-specific informed traders possess perfect information $(T_e = 1)$ and that $T_g = \frac{1}{4}$, we obtain $1 - f = 67$ percent, whereas for $T_g = \frac{1}{20}$, $1 - f$ increases to 76 percent. This suggests that when discretionary liquidity traders can allocate their trades, then if the precision of systematic information is low (an empirically plausible assumption), the fraction of discretionary liquidity trades realized in the basket can be considerably large even if the number of securities is small.

Thus, we have shown that our basic results continue to hold in the limit as the number of securities becomes large, even if discretionary liquidity traders are allowed to allocate their trades between the securities and the basket.

References

Admati, A. R., and P. Pfleiderer, 1988, “A Theory of Intraday Patterns: Volume and Price Variability,” *Review of Financial Studies,* 1, 3-40.

Black, F., 1986, “Noise,” *Journal of Finance,* 41, 529-543.

Chan, K., 1989a, “A Further Analysis of the Lead Lag Relationship Between the Cash Market and the Stock Index Futures Market,” working paper, Ohio State University, Columbus.

Chan, K., 1989b, “Information in the Stock Market and Stock Index Futures Market,” working paper, Ohio State University, Columbus.

Davis, C. D., and A. P. White, 1987, “Stock Market Volatility,” staff study no. 153 by the staff of the Board of Governors of the Federal Reserve System.

Diamond, D. W., and R. E. Verrecchia, 1987, “Constraints on Short-Selling and Asset Price Adjustment to Private Information,” *Journal of Financial Economics* 18, 277-311.

Dybvig, P. H., and C. S. Span, 1983. “Adoption Externalities as Public Goods,” *Journal of Public Economics,* 20, 231-247.

Edwards, F. R., 1988, “Does Futures Trading Increase Stock Market Volatility?,” *Financial Analysts Journal,* Jan.-Feb., 63-69.

Gammill, J. F., and A. F. Perold, 1989, ‘The Changing Character of Stock Market Liquidity,” *The Journal of Portfolio Management*, Spring, 13-18.

Glosten L. R., and Harris 1988, "Estimating the components of the Bid/Ask Spread," *Journal of Financial Economics*, 19, 123-142.

Glosten, L. R., and P. R. Milgron, 1985, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," Journal of Financial Economics 14, 71-100.

Gorton, G. and G. Pennacchi, 1989, "Security Baskets and Index Linked Securities," working paper, Wharton School, University of Pennslvania, Philadelphia.

Grossman, S. J., and M. H. Miller, 1987. "Liquidity and Market Structure." *Journal of Finance,* 43, 617-633.

Harris, L., 1989, "S&P 500 Cash Stock Price Volatilities," *Journal of Finance,* 44, 1155-1175.

Harris, L., 1990, "The Economics of Cash Index Alternatives," *The Journal of Futures Markets,* 10, 179-194.

Holden, C. W., 1990, "A Theory of Arbitrage in Financial Market Equilibrium," working paper, School of Business, Indiana University, Bloomington.

Kawaller, I. G., P. D. Koch, and T. W. Koch, 1987, "The Temporal Relationship Between the S&P 500 Futures and the S&P 500 Index," *Journal of Finance,* 42, 1309-1330.

Kumar, P., and D. J. Seppi, 1990, "Information and Index Arbitrage," working paper, Carnegie Mellon University, Pittsburgh, Pa.

Kyle, A. S. 1984, "Market Structure, Information," in G. G. Storey, A. Schmitz, and A. H. Sarris (eds.), *International Agricultural Trade: Advanced Readings in Price Formation, Market Structure and Price Instability,* Westview Press, Boulder, Col., pp. 45-64.

Kyle, A. S., 1985, "Continuous Auctions and Insider Trading," *Econometrica,* 53, 1315-1335.

Milgrom, P. R. and J. Roberts, 1989, "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," working paper, Stanford University, Stanford, Cal.

Rubinstein, M. 1989, "Market Basket Alternatives," *Financial Analysts Journal,* Sept.-Oct., 20, 29.

Schwert, G. W. 1990, "Stock Market Volatility," in *Market Volatility and Investor Confidence,* New York Stock Exchange, New York.